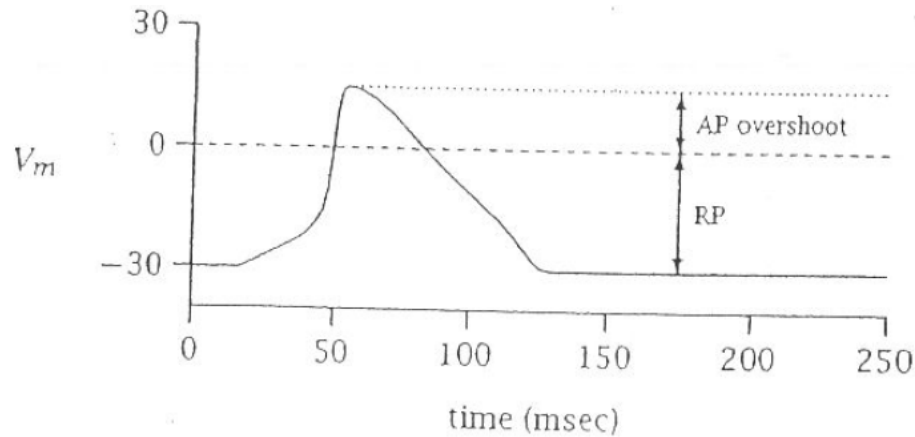


MCB166 — Fall 2017— Problem Set 3

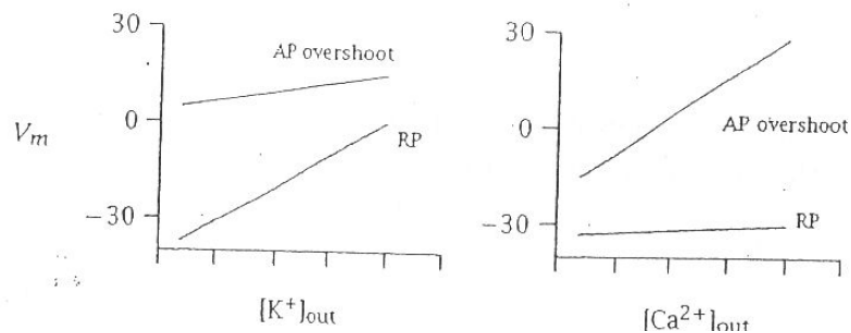
Jonathan Sun, SID 25020651

Tuesday 17th October, 2017

1. The unicellular organism *Paramecium caudatum* shows a resting potential (RP) and an action potential (AP) that are similar in many respects to corresponding neural potentials. With the cell in “typical pond water,” the following measurements were made with an intracellular electrode:



If one varies $[K^+]_{out}$ only, or $[Ca^{2+}]_{out}$ only, one observes the following:



In the following questions, assume that the membrane of *P. caudatum* is normally permeable only to K^+ , Ca^{2+} , and water.

- (a) In the resting state, which of these is true? Explain concisely.

- $P_K > P_{Ca}$
- $P_K = P_{Ca}$
- $P_K < P_{Ca}$

The correct answer is **i. $P_K > P_{Ca}$** . This is because the slope for the resting potential in the V_m vs. $[K^+]_{out}$ graph is much greater than the slope for the resting potential in the V_m vs. $[Ca^{2+}]_{out}$ graph even though they both started at around $V_m = -30$ mV. Therefore, the V_m of the resting potential as $[K^+]_{out}$ increases is much higher than the V_m of the resting potential as $[Ca^{2+}]_{out}$ increases. This means that the resting membrane potential depends more on $[K^+]_{out}$ than $[Ca^{2+}]_{out}$.

- (b) Which is true during the peak of the AP? Explain concisely.

During the peak of the action potential, $P_K < P_{Ca}$ because the V_m for $[Ca^{2+}]_{out}$ increases a lot more than the V_m for $[K^+]_{out}$, which means that P_{Ca} is influenced a lot more than P_K . This means that the action potential depends more on $[Ca^{2+}]_{out}$ than $[K^+]_{out}$.

- (c) Compared to the ionic concentrations of “typical pond water,” is $[K^+]_{in}$ greater than, equal to, or less than $[K^+]_{out}$? Explain.

Compared to the ionic concentrations of “typical pond water,” $[K^+]_{in} > [K^+]_{out}$. This is because in part (a) we have determined that at the resting potential, $P_K > P_{Ca}$, which means that the resting potential depends more on $[K^+]_{out}$ than $[Ca^{2+}]_{out}$. Therefore, we will disregard $[Ca^{2+}]_{out}$ from our calculations. Since $V_{rest} \approx -30mV$ and this is close to E_K , I know that I should get a negative number for E_K . Calculating for E_K we get:

$$V_{rest} \approx E_K = \frac{RT}{zF} \ln \frac{P_K[K^+]_{out}}{P_K[K^+]_{in}} \approx -30mV$$

Since the only way for $\frac{RT}{zF} \ln \frac{P_K[K^+]_{out}}{P_K[K^+]_{in}}$ to be negative is for the numerator of the natural log to be smaller than the denominator of the natural log, this means that $[K^+]_{in} > [K^+]_{out}$.

- (d) Compare also $[Ca^{2+}]_{in}$ with $[Ca^{2+}]_{out}$.

Compared to the ionic concentrations of “typical pond water,” $[Ca^{2+}]_{in} < [Ca^{2+}]_{out}$. This is because in part (b) we have determined that at the action potential, $P_K < P_{Ca}$, which means that the action potential depends more on $[Ca^{2+}]_{out}$ than $[K^+]_{out}$. Therefore, we will disregard $[K^+]_{out}$ from our calculations. Since $V_{AP} \approx +15mV$ and this is close to E_{Ca} , I know that I should get a positive number for E_{Ca} . Calculating for E_{Ca} we get:

$$V_{AP} \approx E_{Ca} = \frac{RT}{zF} \ln \frac{P_{Ca}[Ca^{2+}]_{out}}{P_{Ca}[Ca^{2+}]_{in}} \approx +15mV$$

Since the only way for $\frac{RT}{zF} \ln \frac{P_{Ca}[Ca^{2+}]_{out}}{P_{Ca}[Ca^{2+}]_{in}}$ to be positive is for the numerator of the natural log to be greater than the denominator of the natural log, this means that $[Ca^{2+}]_{in} < [Ca^{2+}]_{out}$.

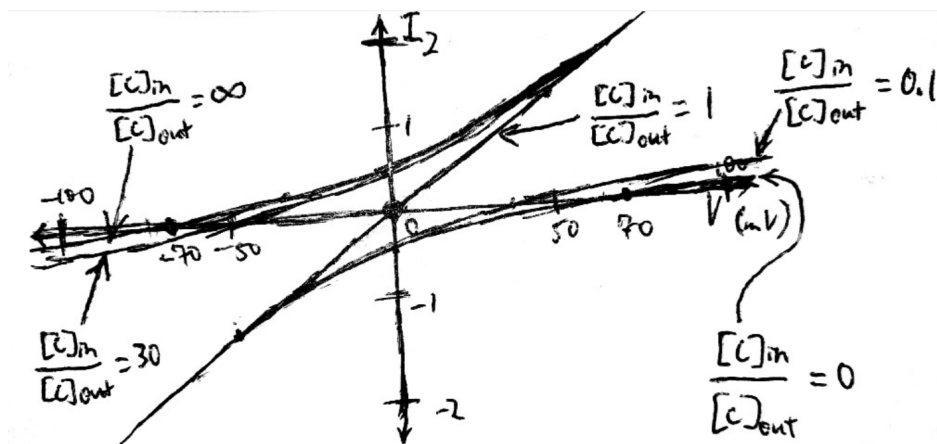
- (e) When the posterior end of the organism is mechanically tapped, the membrane transiently hyperpolarizes. What permeability change(s) might be responsible? Explain.

When the membrane transiently hyperpolarizes, this may be caused by the increase in P_K . This is because at resting potential, we found that $P_K > P_{Ca}$ and so the increase in P_K would have more effect than the decrease in P_{Ca} for hyperpolarization.

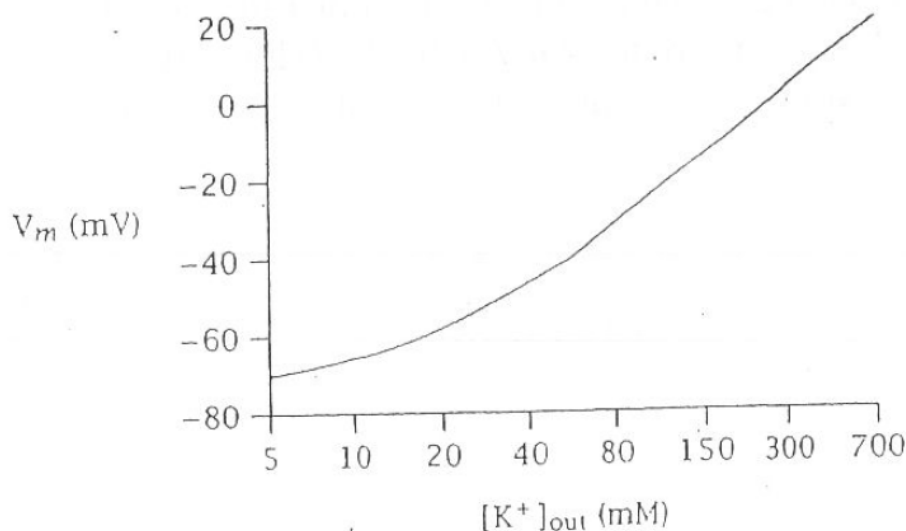
2. (a) Briefly state the assumptions for the constant field model.

The constant field model assumes that ion movement within the membrane obeys the Nernst-Planck equation, ions move across the membrane independently without interacting with each other, and the electric field in the membrane is constant (the electric potential drops linearly across the membrane).

- (b) Sketch approximately the I - V relations predicted by the constant field model for various ratios of intracellular and extracellular ion concentrations, i.e., when $\frac{[C]_{in}}{[C]_{out}} = 0, 0.1, 1, 30, \text{ or } \infty$.



- (c) Using the data provided in the figure below, calculate the ratio of P_{Na}/P_K that predicts the resting potential as a function of $[K^+]_{out}$ for the *Myxicola* neuron. Note: $[Na^+]_{out} = 430\text{mM}$, $[Na^+]_{in} = 12\text{mM}$, $[K^+]_{in} = 270\text{mM}$, and $P_{Cl} = 0$.



Using the data, I will guesstimate that $V_m = -68\text{mV}$ when $[K^+]_{out} = 5\text{mM}$. From this, I can solve for $\frac{P_{Na}}{P_K}$.

$$V_m = \frac{RT}{F} \ln \frac{P_K[K^+]_{out} + P_{Na}[Na^+]_{out} + P_{Cl}[Cl^-]_{in}}{P_K[K^+]_{in} + P_{Na}[Na^+]_{in} + P_{Cl}[Cl^-]_{out}}$$

Since $P_{\text{Cl}} = 0$:

$$V_m = \frac{RT}{F} \ln \frac{P_{\text{K}}[\text{K}^+]_{\text{out}} + P_{\text{Na}}[\text{Na}^+]_{\text{out}}}{P_{\text{K}}[\text{K}^+]_{\text{in}} + P_{\text{Na}}[\text{Na}^+]_{\text{in}}}$$

$$V_m = \frac{RT}{F} \ln \frac{[\text{K}^+]_{\text{out}} + \frac{P_{\text{Na}}}{P_{\text{K}}}[\text{Na}^+]_{\text{out}}}{[\text{K}^+]_{\text{in}} + \frac{P_{\text{Na}}}{P_{\text{K}}}[\text{Na}^+]_{\text{in}}}$$

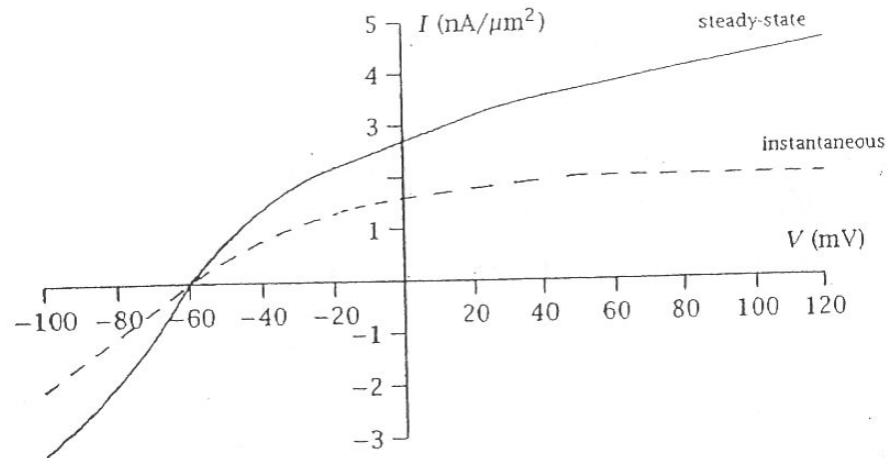
Plugging in the values, we get:

$$-68 = 58 \log \frac{5 + \frac{P_{\text{Na}}}{P_{\text{K}}} 430}{270 + \frac{P_{\text{Na}}}{P_{\text{K}}} 12}$$

Solving for $\frac{P_{\text{Na}}}{P_{\text{K}}}$ we get:

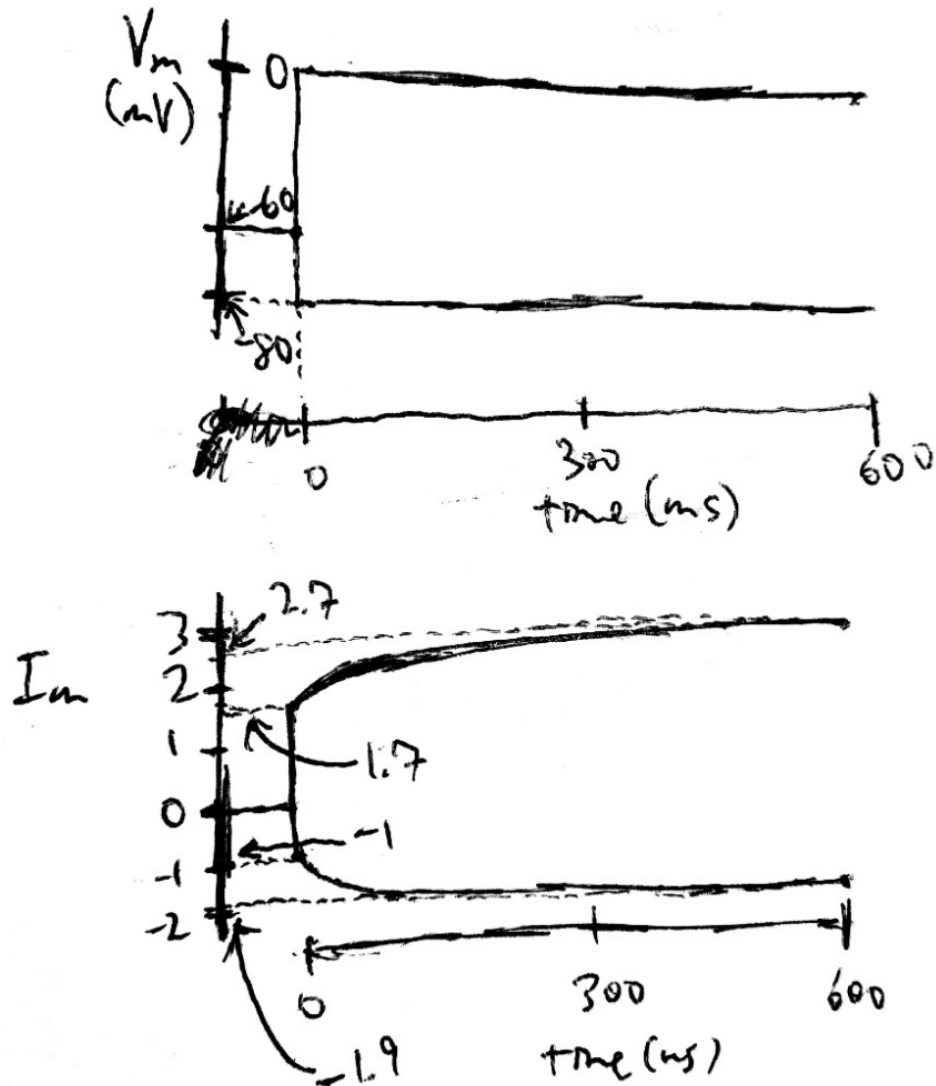
$$\frac{P_{\text{Na}}}{P_{\text{K}}} \approx 0.03$$

3. The instantaneous and steady-state $I - V$ relations of a neuron obtained from voltage-clamp experiments are shown below:



The time-dependent current follows first-order kinetics with the time constant $\tau = 0.1$ sec.

- (a) Draw the membrane current with respect to time after the membrane voltage is stepped from $V_H = -60$ mV to $V_c = 0$ mV and to $V_c = -80$ mV. Label the current and time axes with the appropriate units.



(b) Repeat (a) after the membrane voltage is stepped from $V_H = +50\text{mV}$ to $V_c = +100\text{mV}$.

To solve this, I will use the formula:

$$I(t) = I_o + I_{max} \left(1 - e^{-\frac{t}{\tau}}\right)$$

Since $I_o = 0$ and $\tau = 0.1\text{seconds}$:

$$I(t) = I_{max} \left(1 - e^{-\frac{t}{0.1}}\right)$$

Using this formula, I can solve for what percentage of I_{max} the current is at various time steps to get:

