MCB166 — Fall 2017— Problem Set 5

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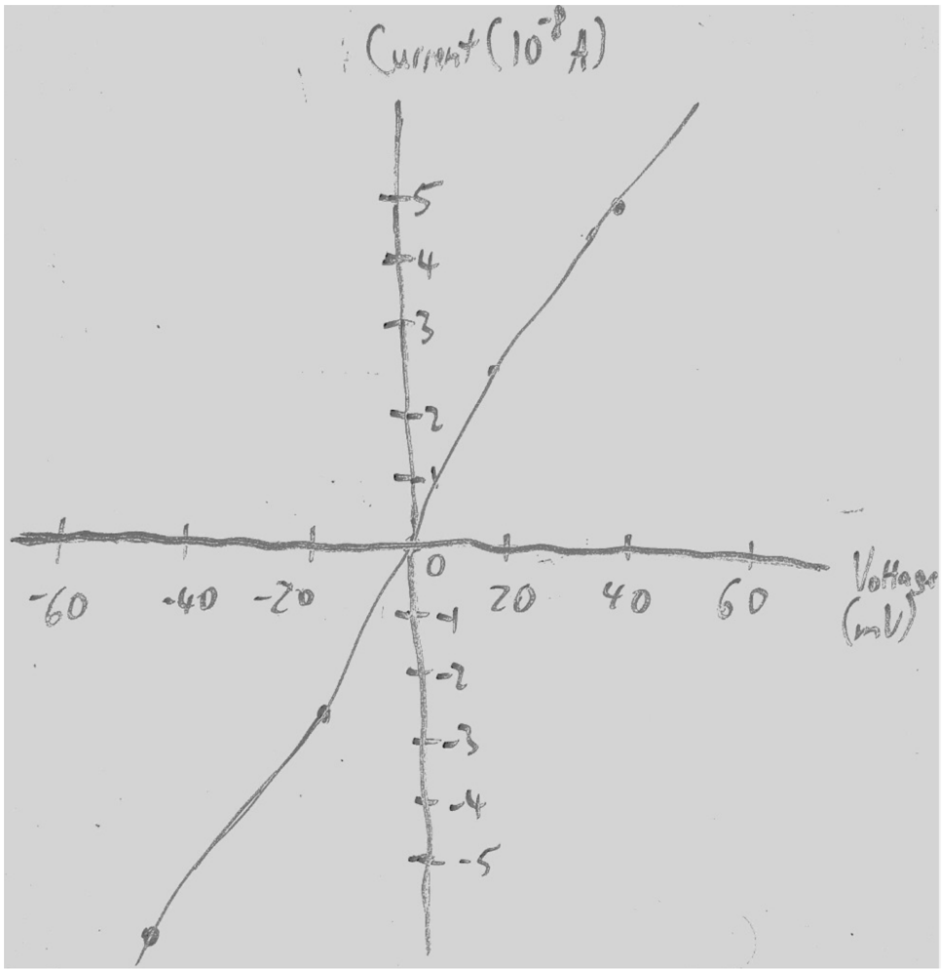
1. The coxal muscle receptor of the crab is innervated by two large nerve fibers (Roberts & Bush, 1971). One of them, the T-fiber, is about 60μm in diameter and 3mm long. Two microelectrodes were inserted close together into the middle region of the fiber, one to pass stimulating currents and one for recording. Fig. A shows some of the results of one experiment. You may assume that the space constant is long relative to the fiber.

   (a) Plot the results as a voltage-current graph.



   (b) What is the input resistance of the fiber?

   Since the plot basically gives a straight line, I will use the points (0mV, 0A) and $\left(20\text{mV}, 2.5 \times 10^{-8}\text{A}\right)$ to calculate the resistance:

$$
\begin{aligned}
R_{in} &= \frac{\Delta V}{\Delta I} \\
&= \frac{20\text{mV} - 0\text{mV}}{2.5 \times 10^{-8}\text{A} - 0\text{A}} \\
&= 800000\Omega \\
&= 800\text{k}\Omega
\end{aligned}
$$

   Therefore, the input resistance of the fiber is about $800\text{k}\Omega$.

(c) Give an estimate of the specific membrane resistance.

Since the input resistance is about 800000Ω and we were given that the T-fiber's diameter is about $60\mu\text{m} = 6 \times 10^{-5}\text{m}$ and its length is $3\text{mm} = 0.003\text{m}$:

$$\begin{aligned} r_m &= R_{in} \times A \\ &= 800000\Omega \times (\pi d l) \\ &= 800000\Omega \times \left(\pi \times 6 \times 10^{-5}\text{m} \times 0.003\text{m}\right) \\ &\approx 800000\Omega \times 5.655 \times 10^{-7}\text{m}^2 \\ &\approx 0.452\Omega\text{m}^2 \end{aligned}$$

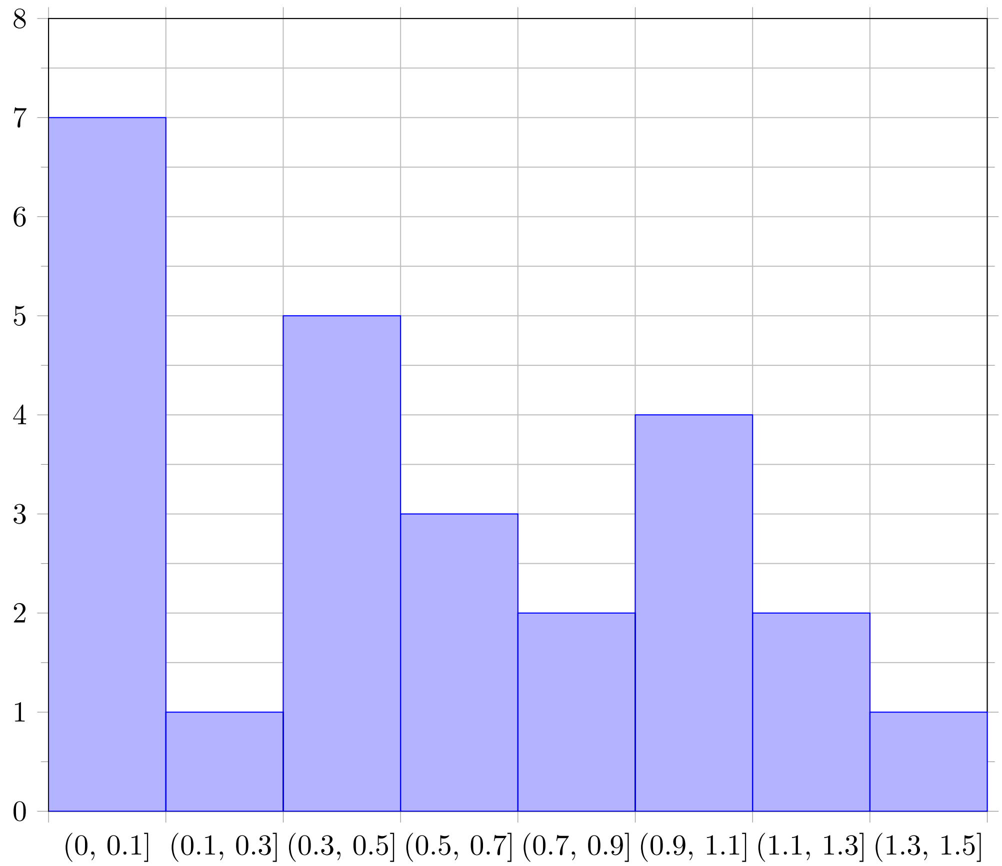
Therefore, the specific membrane resistance is about $0.452\Omega\text{m}^2$.

(d) If the assumption is untrue and the space constant is comparable to or shorter than the fiber, would the specific resistance of the membrane be higher or lower? Give brief reasons.

If the assumption was false and the space constant is comparable to or shorter than the fiber, then the area itself would have been smaller. Since the specific membrane resistance is proportional to the area given the input resistance, then a smaller area would result in a smaller specific membrane resistance.

2. A neuromuscular junction is stimulated 25 times. The amplitudes of the EPPs for each of the 25 trials are (in mV): 0.3, 0, 0.5, 0.7, 0, 1.1, 1.5, 0, 1.1, 0.9, 0, 0, 1.3, 1.1, 0.5, 0.5, 0.7, 0, 0.7, 1.1, 0.5, 0.5, 0, 0.9, 1.3.

   (a) Plot an amplitude histogram of the responses using a bin width of 0.2mV.



   (b) In other experiments you determined that the mean amplitude of the mEPP was 0.5mV. Calculate m by at least two methods, assuming Poisson statistics for the release process.

   Using the Direct Method:

$$\begin{aligned} m &= \frac{\bar{V}}{\bar{q}} \\ &= \frac{\frac{0.3+0+0.5+0.7+0+1.1+1.5+0+1.1+0.9+0+0+1.3+1.1+0.5+0.5+0.7+0+0.7+1.1+0.5+0.5+0+0.9+1.3}{25}}{0.5} \\ &= 1.216 \end{aligned}$$

   Using the Failure Method:

$$\begin{aligned} m &= \ln \frac{N}{N_0} \\ &= \ln \frac{25}{7} \\ &\approx 1.273 \end{aligned}$$

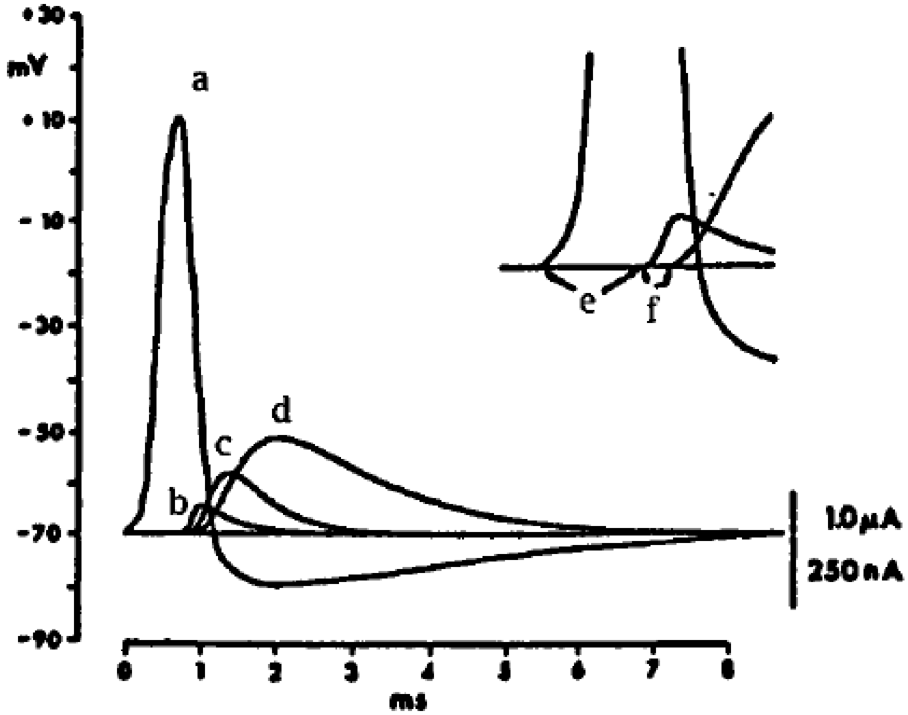
(c) Given your value for m, how many times would you predict that 3 quanta would be released in an experiment in which the nerve was stimulated 200 times?

From the Direct Method and Failure Method above, I got $m = 1.216$ and $m \approx 1.273$. So, I will use the average between the two to get $m = 1.2445$ and use this in my calculation below:

$$\begin{aligned} N_3 &= \frac{m^3 e^{-m}}{6} \times N \\ &= \frac{(1.2445)^3 e^{-1.2445}}{6} \times 200 \\ &\approx 18.509 \end{aligned}$$

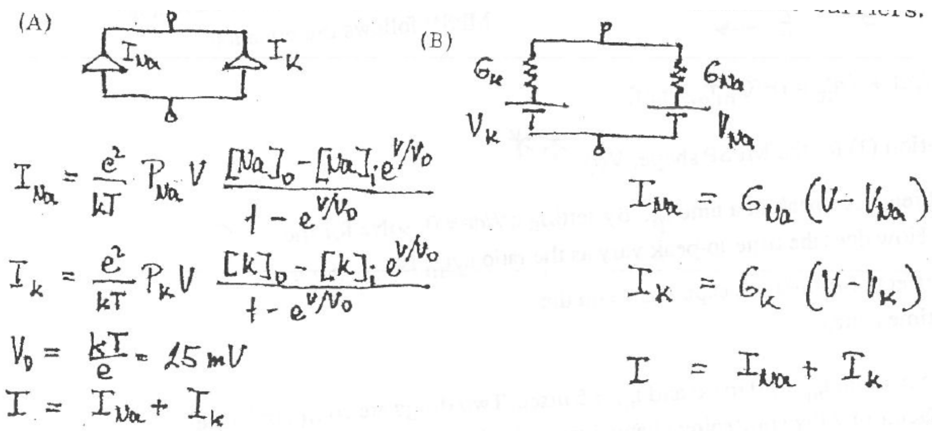
Since $N_3 \approx 18.509$, I would predict 18 times.

3. Reproduced below is part of figure 12.16. It depicts different aspects of synaptic transmission as studied at the squid giant synapse. Label and describe all parts of the figure.



Part (a) of the graph shows the action potential, part (b) shows the presynaptic Ca^{2+} current, part (c) shows the postsynaptic current (EPSC), and part (d) shows the postsynaptic potential (EPSP). Part (e) of the blowup figure shows the delay between the action potential and the onset of the Ca^{2+} current. Part (f) of the blowup figure shows the delay between the Ca^{2+} current and the onset of the postsynaptic current of part (c).

4. We are going to compare two models for Na and K currents in parallel. In the first model (A) the conductors are <u>constant-field rectifiers</u>. In the second model (B), they are linear conductors with Nernst-potential barriers.



(a) For each model, write the expression for the resting potential, i.e. the potential at which $I_{\text{Na}} + I_{\text{K}} = 0$.

At the resting membrane potential, the total current is 0 so for Model A:

$$I_{\text{Na}} + I_{\text{K}} = 0$$

$$\frac{e^2}{kT} P_{\text{Na}} V \frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + \frac{e^2}{kT} P_{\text{K}} V \frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} = 0$$

$$P_{\text{Na}} \left([\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}} \right) + P_{\text{K}} \left([\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}} \right) = 0$$

$$(P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0) - (P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i)\, e^{\frac{V}{\frac{kT}{e}}} = 0$$

$$e^{\frac{V}{\frac{kT}{e}}} = \frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

$$\frac{V}{\frac{kT}{e}} = \ln \frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

$$V = \frac{kT}{e} \ln \frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

For Model B:

$$I_{\text{Na}} + I_{\text{K}} = 0$$

$$G_{\text{Na}}\left(V - V_{\text{Na}}\right) + G_{\text{K}}\left(V - V_{\text{K}}\right) = 0$$

$$G_{\text{Na}}V - G_{\text{Na}}V_{\text{Na}} + G_{\text{K}}V - G_{\text{K}}V_{\text{K}} = 0$$

$$G_{\text{Na}}V + G_{\text{K}}V = G_{\text{Na}}V_{\text{Na}} + G_{\text{K}}V_{\text{K}}$$

$$V = \frac{G_{\text{Na}}V_{\text{Na}} + G_{\text{K}}V_{\text{K}}}{G_{\text{Na}} + G_{\text{K}}}$$

(b) For model (A), show that as $V \to \infty$ or $V \to -\infty$ (i.e. become very large), $I(V)$ becomes linear but with different slopes. The ratio these slopes is called the rectification ratio. Write the resting potential for rectification ratio = 1.

The current for Model A is represented as:

$$\begin{aligned} I(V) &= \frac{e^2}{kT}P_{\text{Na}}V\frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + \frac{e^2}{kT}P_{\text{K}}V\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} \\ &= \frac{e^2}{kT}V\left(P_{\text{Na}}\frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + P_{\text{K}}\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}}\right) \end{aligned}$$

Basically, we need to show that $\left(P_{\text{Na}}\frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + \frac{e^2}{kT}P_{\text{K}}\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}}\right)$ is constant as $V \to \pm\infty$.

For $\lim_{V\to-\infty} I(V)$:

$$\begin{aligned} \lim_{V\to-\infty} I(V) &= \frac{e^2}{kT}V\left(P_{\text{Na}}\frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + P_{\text{K}}\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}}\right) \\ &= \frac{e^2}{kT}V\left(P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0\right) \end{aligned}$$

For $\lim_{V\to+\infty} I(V)$:

$$\begin{aligned} \lim_{V\to-\infty} I(V) &= \frac{e^2}{kT}V\left(P_{\text{Na}}\frac{[\text{Na}]_0 - [\text{Na}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} + P_{\text{K}}\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}}\right) \\ &= \frac{e^2}{kT}V\left(P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i\right) \end{aligned}$$

Since I can represent $I(V)$ when $V \to -\infty$ and $V \to +\infty$ as m_1V and m_2V respectively, I can define $\frac{m_1}{m_2} = e^{\frac{V}{V_0}}$ as the rectification ratio:

$$m_1 = \frac{e^2}{kT}\left(P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0\right)$$

$$m_2 = \frac{e^2}{kT}\left(P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i\right)$$

$$e^{\frac{V}{V_0}} = \frac{m_1}{m_2} = \frac{\frac{e^2}{kT}\left(P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0\right)}{\frac{e^2}{kT}\left(P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i\right)} = \frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

Therefore, I get:

$$\frac{V}{V_0} = \ln\frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

$$V = V_0 \ln\frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

$$V = \frac{kT}{e}\ln\frac{P_{\text{Na}}[\text{Na}]_0 + P_{\text{K}}[\text{K}]_0}{P_{\text{Na}}[\text{Na}]_i + P_{\text{K}}[\text{K}]_i}$$

(c) Show that model (B) can be written as a single element $I = \bar{G}(V - \bar{V})$. Write expressions for $\bar{G}$ and $\bar{V}$.

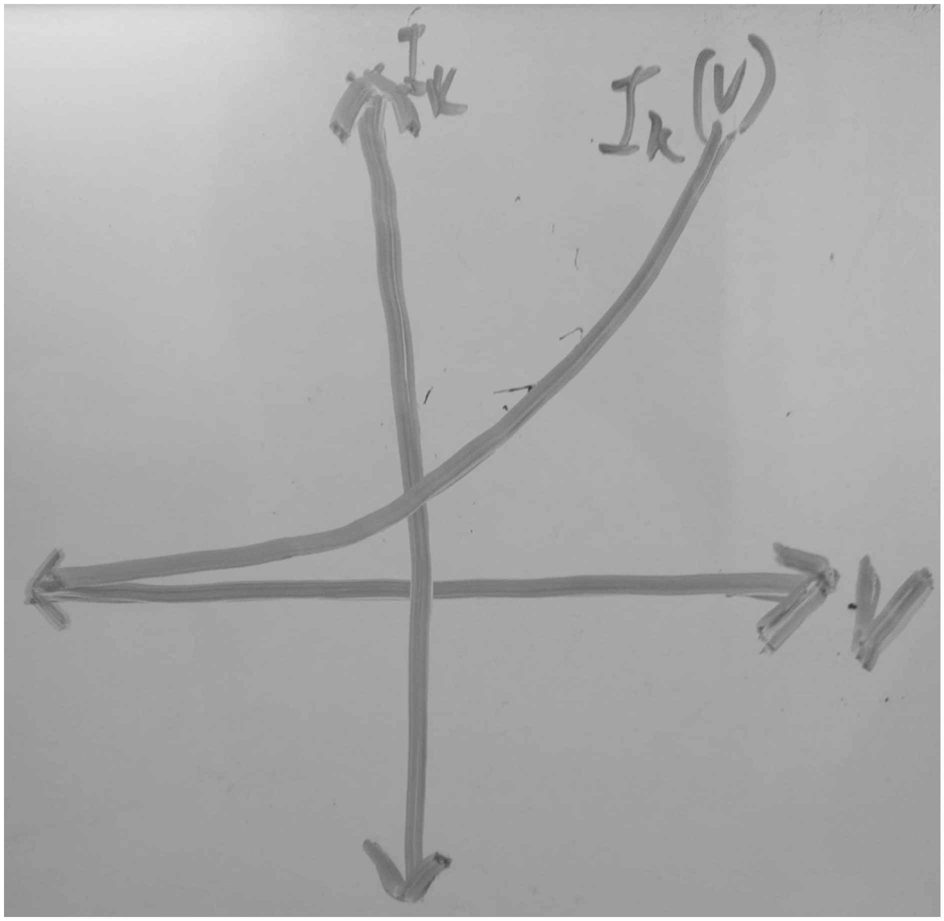
$$\begin{aligned} I &= G_{\text{Na}}\left(V - E_{\text{Na}}\right) + G_{\text{K}}\left(V - E_{\text{K}}\right) \\ &= \left(G_{\text{Na}} + G_{\text{K}}\right)\left(V - \frac{G_{\text{Na}}E_{\text{Na}} + G_{\text{K}}E_{\text{K}}}{G_{\text{Na}} + G_{\text{K}}}\right) \end{aligned}$$

(d) Model (B) makes no sense when any relevant ionic concentration goes to zero. This is because one of the Nernst potentials will become infinite. Explain what is wrong with the model for such a condition. Does model (A) have this trouble? If not, why not? Sketch a curve of $I_{\text{K}}(V)$ when $[K]_0 = 0$, and describe the current.

In Model A, we get that V is the log of sums while in Model B, we get that it is the sum of logs. Since the log of sums is not the same as the sum of logs, this is why the Model B is incorrect. Model A does not have this issue since even if there was no potassium, you can still get a valid answer with sodium only.

When $[K]_0 = 0$:

$$\begin{aligned} I_{\text{K}}(V) &= \frac{e^2}{kT}P_{\text{K}}V\frac{[\text{K}]_0 - [\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} \\ &= \frac{e^2}{kT}P_{\text{K}}V\frac{-[\text{K}]_i e^{\frac{V}{\frac{kT}{e}}}}{1 - e^{\frac{V}{\frac{kT}{e}}}} \\ &= \frac{e^2}{kT}[\text{K}]_i P_{\text{K}}\frac{V}{1 - e^{-\frac{V}{\frac{kT}{e}}}} \end{aligned}$$



I_k
I_k(V)
V