CS170 — Fall 2017— Homework 7 Solutions

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0. Who Did You Work With?

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1. A HeLPful Introduction

(a) The necessary and sufficient conditions on real numbers a and b that make the linear program infeasible do not exist. This is because no matter what a and b are, x and y can both equal 0 and this will always satisfy the constraints.

(b) The necessary and sufficient conditions on real numbers a and b that make the linear program unbounded are if $a < 0$ or $b < 0$. This is because the conditions to maximize $x + y$ and $ax + by \leq 1$ with either $a < 0$ or $b < 0$ would make the region's area infinite. In other words, if the slope of the line for $ax + by \leq 1$ is positive, then there is no maximum for either x or y making the region's area infinite. If the slope of the line is negative, then both x and y will have individual maximum values and so $x + y$ will be bounded. Therefore, the to make the linear program unbounded, either a or b will need to be less than zero to make the slope positive.

(c) The necessary and sufficient conditions on real numbers a and b that allow the linear program to have a unique optimal solution are that $a > 0$, $b > 0$, and $a \neq b$. This allows for the slope of the line for $ax + by \leq 1$ to be negative, which will give individual maximum bounds to both x and y. Furthermore, $a \leq b$ makes the slope not equal to -1, which would cause the maximum value of x to equal the maximum value of y. This would give multiple solutions since $max x + y$ would all be the same value if the slope is -1. Therefore, we need the slope to be negative and also not equal to -1.

2. TeaOne

(a)

(b)

(c)

3. Mountain pass

(a)

(b)

4. The Hungry Caterpillar

5. Star-shaped polygons in 2D

6. All Knight-er