# CS170 — Fall 2017— Homework 7 Solutions

Jonathan Sun, SID 25020651

## 0. Who Did You Work With?

Collaborators: Kevin Vo, Aleem Zaki, Jeremy Ou

# 1. A HeLPful Introduction

(a) The necessary and sufficient conditions on real numbers $a$ and $b$ that make the linear program infeasible do not exist. This is because no matter what $a$ and $b$ are, $x$ and $y$ can both equal 0 and this will always satisfy the constraints.

(b) The necessary and sufficient conditions on real numbers $a$ and $b$ that make the linear program unbounded are if $a < 0$ or $b < 0$. This is because the conditions to maximize $x + y$ and $ax + by \leq 1$ with either $a < 0$ or $b < 0$ would make the region's area infinite. In other words, if the slope of the line for $ax + by \leq 1$ is positive, then there is no maximum for either $x$ or $y$ making the region's area infinite. If the slope of the line is negative, then both $x$ and $y$ will have individual maximum values and so $x + y$ will be bounded. Therefore, the to make the linear program unbounded, either $a$ or $b$ will need to be less than zero to make the slope positive.

(c) The necessary and sufficient conditions on real numbers $a$ and $b$ that allow the linear program to have a unique optimal solution are that $a > 0$, $b > 0$, and $a \neq b$. This allows for the slope of the line for $ax + by \leq 1$ to be negative, which will give individual maximum bounds to both $x$ and $y$. Furthermore, $a \leq b$ makes the slope not equal to $-1$, which would cause the maximum value of $x$ to equal the maximum value of $y$. This would give multiple solutions since $maxx + y$ would all be the same value if the slope is $-1$. Therefore, we need the slope to be negative and also not equal to $-1$.

## 2. TeaOne

(a) The cost of creating a ZestyJuice is $5 \times 0.1 + 1 \times 0.2 + 8 \times 0.01 = 0.78$. Since a ZestyJuice sells for 4.5, the profit of selling one is $4.5 - 0.78 = 3.72$. The cost of creating a MilkTea is $12 \times 0.1 + 16 \times 0.2 = 4.4$. Since a MilkTea sells for 5, the profit of selling one is $5 - 4.4 = 0.6$. With this, I will represent ZestyJuice as $z$ and MilkTea as $m$. The linear program will be:

$$\max 3.72z + 0.6m$$

$$z \leq 60$$

$$m \leq 40$$

$$0.78z + 4.4m \leq b$$

$$z \geq 0,\ m \geq 0$$

(b)

$$\min 60y_1 + 40y_2 + 6y_3$$

$$y_1 + 0.78y_3 \geq 3.72$$

$$y_2 + 4.4y_3 \geq 0.6$$

$$y_1 \geq 0,\ y_2 \geq 0,\ y_3 \geq 0$$

(c) There are three types of solutions for this part since we can have a budget that is so large that we can afford to make all 60 ZestyJuice and 40 MilkyTea, a limited budget so we maximize ZestyJuices since they make the most profit, and another limited budget where we maximize MilkyTea because they turn a profit.

$$z = \frac{b}{0.78},\ m = 0 \text{ for } b < 46.8$$

$$z = 60,\ m = \frac{b - 46.8}{4.4} \text{ for } 46.8 \leq b \leq 222.8$$

$$z = 60,\ m = 40 \text{ for } b > 222.8$$

## 3. Mountain pass

(a)

(b)

## 4. The Hungry Caterpillar

## 5. Star-shaped polygons in 2D

The idea for this problem is that the point $x$ will be able to see all the points of the polygon, which means that there can be a direct line between each point of the polygon to the point $x$ and none of these points will intersect or be the same equation as each other between each point of the polygon and $x$. In other words, the linear program needs to show whether there exists a point $x$ that is the intersection of all the lines that go from each point of the polygon to $x$. This can further "simplified" by having the linear program show whether there exists a point $x$ that is the intersection of all the lines that go from each CORNER of the polygon to $x$.

Therefore, the linear program can be represented as:

$$\max None$$

$$a_i x + b_i y > c_i \text{ where } i \in \{1, 2, ..., n-1\}$$

## 6. All Knight-er

**Four-Part Solution:**

**Main Idea:**
ok

**Pseudocode:**

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ok
```

**Proof of Correctness:**
ok □

**Run Time:**
ok

**Justification:**
ok