Market Structure, Investment and Technical Efficiencies in Mobile Telecommunications*

Jonathan Elliott,[†] Georges V. Houngbonon,[‡]
Marc Ivaldi,[§] Paul Scott[¶]

March 2021

Abstract

We develop a model of competition in prices and infrastructural investment among mobile network providers. Market shares and service quality (download speed) are simultaneously determined, for demand affects the network load just as delivered quality affects consumer demand. While consolidation typically has adverse impacts on consumer surplus, economies of scale (which we derive from physical principles) push in the other direction, and we find that consumer surplus is maximized at a moderate number of firms, and that the optimal number of firms is higher for lower income consumers. Our modeling framework allows us to quantify the marginal social value of allocating more spectrum to mobile telecommunications, finding it is roughly five times an individual firm's willingness to pay for a marginal unit of spectrum.

Keywords: Market structure, scale efficiency, antitrust policy, infrastructure, endogenous quality, queuing, mobile telecommunications.

JEL Classification: D21, D22, L13, L40.

*We thank Stéphane Ciriani, Chris Conlon, Mohamed Karray, Paul LaFontaine, Marc Lebourges, Julienne Liang, Maher Said, David Salant, Alan Sorenson, Patrick Sun, and Aleks Yankelevich for many insightful comments, discussions, and suggestions. The usual disclaimer applies. All estimation and simulation code can be found at https://github.com/jonathantelliott/mobile-telecommunications.

†New York University, jonathan.elliott@nyu.edu
‡World Bank, gvivienh@gmail.com
§Toulouse School of Economics, marc.ivaldi@tse-fr.eu
¶New York University, ptscott@stern.nyu.edu

1 Introduction

Numerous contentious policy questions have arisen recently in the mobile telecommunications industry, including mergers and spectrum allocation. Common to these debates is the question of how changes to the market structure impact prices and quality of service.

In this paper, we develop a structural model of the mobile telecommunications industry to capture the impact of changes in industry structure (such as mergers or changes in spectrum allocation), on equilibrium outcomes such as prices, investment, quality of service, and welfare. This allows us to assess the trade-off between market power and economies of scale, both in the traditional sense, where consolidation may result in higher or lower prices (Williamson, 1968), and in understanding how consolidation affects quality of service, which is endogenously determined by firms' investments, spectrum allocation, and the load on the network imposed by data consumption.[1] Our model is also well-suited to addressing questions of spectrum allocation; we use the model to compute the value of allocating more bandwidth to mobile telecommunications.

We model the determination of quality of service using an engineering model (Blaszczyszyn, Jovanovicy and Karray, 2014) with three pieces, each having important economic implications: path loss, information theory, and queuing theory. Our study thus falls within the tradition of engineering production functions of Chenery (1949).

Path loss (i.e., the reduction in power of electromagnetic waves as they travel) results in *economies of density*: a mobile network operator can serve a densely populated area more efficiently (meaning either higher download speed at a given cost or the same download speed at a lower cost) than a sparsely populated area.[2] Noting that mergers can effectively increase the population density served by firms, these economies of density create the potential for cost or quality synergies associated with mergers.

Queuing theory allows us to understand how the allocation of shared resources leads to economies of scale, in the spirit of Mulligan (1983). When two merging mobile network operators combine both their customer bases and owned bandwidth, the merged firm can more efficiently allocate network capacity among customers, leading to higher download speeds.

Information theory, specifically the Shannon-Hartley theorem, tells us how a firm's channel

[1] Quality of service has featured prominently in recent merger cases. For instance, the Sprint/T-Mobile merger was allowed based on the finding "that quality benefits and dynamic competition serve as countervailing forces to the static analysis that substantially address its predicted harmful price effects" (Federal Communications Commission, 2019).

[2] For example, suppose that the number of base stations per person is held constant across different population densities, so that less population-dense areas have lower base station density. Because signals in the sparsely populated areas will have to travel further on average, they will experience greater path loss, and sparsely populated areas will have inferior service despite receiving the same level of investment per capita.

capacity (i.e., the maximum rate of data transmission) depends on the bandwidth (amount of spectrum) operated. By explicitly modeling this dependence, we can understand the impacts of spectrum allocation. For example, we can determine the impact of firms combining their spectrum holdings through a merger or the impact of regulators modifying the industry spectrum allocation.

We embed the engineering model of data transmission within an equilibrium model of competition among firms that operate mobile networks. Firms choose prices of various mobile service plans and the level of investment in infrastructure. Consumers respond in both their mobile contract choices and their data consumption decisions to prices and the download speeds.

A challenge for accurately modeling quality of service is the fact that consumer demand for data and download speeds are simultaneously determined. Consumer demand for a network operator's services depends on its quality of service, and its quality of service depends on consumer demand due to congestion externalities. Most demand models for mobile services do not model the simultaneous determination of demand and quality of service (including Bourreau, Sun and Verboven (2018), Cullen, Schutz and Shcherbakov (2016), Fan and Yang (2016), Sinkinson (2020), Sun (2015), Weiergräber (2018)). Only El Azouzi, Altman and Wynter (2003) and Lhost, Pinto and Sibley (2015) model the simultaneous determination of service quality and choice of service provider using queuing theory like we do. Our study builds on these by incorporating path loss (and therefore economies of density) and by estimating a product-level demand model using detailed consumption and quality data.

We estimate a model of demand for mobile plans and data consumption based on the French market in 2015. Our estimation relies on a unique data set from the French mobile market. We obtain data on choices and consumption by nearly 15 million customers in October 2015.[3] We also secure data on quality of mobile broadband, measured as the actual speed experienced by users. We complement these data with information on network deployment from the spectrum frequency regulator (ANFR), and income distribution from the statistical office (INSEE). While we only observe consumers subscribing to one operator (Orange Mobile), we observe the prices and characteristics for all contracts available in the market, and we prove that the estimation strategy of Berry, Levinsohn and Pakes (1995) can be employed in this setting.[4]

We use the estimated models of demand and infrastructure to compute counterfactual equi-

[3]In accordance with data protection and privacy concerns, we were provided with commune-level statistics rather than accessing the detailed consumer-level data directly.

[4]Our model predicts shares for all products from all providers in the market, but we only require that the model rationalize product-level market shares for Orange. For other firms, we impose firm-level demand shocks and require the model to rationalize firm-level market shares.

libria under different numbers of firms. We find that consumer surplus is maximized at a moderate number of firms, but that aggregate consumer surplus masks considerable heterogeneity across consumers of different income levels. Consumers of different income levels value a marginal increase in download speeds differently, and we find that low income individuals prefer a market with more firms than do high income individuals. We also explore the marginal social value of allocating more spectrum to the mobile telecommunications industry and compare this value with an individual firm's willingness to pay for a marginal unit of spectrum.[5] We find that the marginal social value is about five times greater than an individual firm's willingness to pay.

The remainder of this paper is organized as follows. Section 2 presents the data along with some descriptive statistics on usage and quality of mobile data. Section 3 presents the model of infrastructural investment, and section 4 lays out the demand and cost models. Section 5 presents the estimation strategy and results. Sections 6 and 7 present some counterfactual analyses.

2 Data and Descriptive Statistics

The French telecommunications market hosts four mobile network operators (MNOs): Orange (market share of 37%), SFR (29%), Bouygues (17%), and Free Mobile (17%). 92% of the population above 12 years old are mobile users according to a survey by CREDOC. Mobile services include voice and data communications as well as short message services (SMS) and multimedia services (MMS). Statistics provided by the national regulator show that voice and SMS usage had stabilized by 2015, the period we focus on to estimate our model. In contrast, the volume of data per user was growing rapidly, reaching an average of 800 megabytes (MB) in 2015, up from 100 MB in 2010 (see Figure 21 in Appendix C.1). The provision of high quality data services has been a major concern in recent antitrust cases and regulatory discussions.

2.1 Data description

This study relies on data from several sources. A supplementary data appendix (Appendix C) provides a detailed description of these data sources.

Our main data source is a proprietary data set of 15 million residential mobile customers of one operator, Orange, in October 2015. This data set includes information on the contract subscribed to and the usage of mobile voice and data services. In the remainder of the paper, we focus on data services because network investment since 2013 has typically been made in order to improve the quality of data services, and with the deployment of 4G technology voice

[5] This willingness to pay is the value to which firms' bids in a simple spectrum auction will be related.

and data services can draw on the same network resources.

The customer data set is complemented by data on the quality of mobile data services, defined as download speed. Unlike fixed broadband Internet access, the quality of mobile data is hard to measure due to congestion and users' mobility. Congestion arises because the available bandwidth is shared among users and, as a result, the greater the number of users, the lower the quality (as measured by download speed). In the meantime, the number of users also depends on quality. In our counterfactuals, we will employ a model in which demand and quality of service are simultaneously determined, but for the purpose of estimation, we rely on a direct measure of download speeds as our measure of quality. Speedtest is a service offered by the firm Ookla that allows users to check their download and upload internet speeds. The data include measured download speed, the time of the speed test, the location of the user, and the mobile network operator. We use a proprietary data set provided by Ookla on over one million speed tests in France in the fourth quarter of 2015 to construct a measure of experienced download speeds for each mobile network operator in each municipality. The data appendix explains the construction of this quality measure in detail.

Markets are defined as municipalities (communes), and we limit our analysis to urban markets, defined as those with a population greater than 10 000, for a total of 589 markets.[6] Municipality-level market size is estimated using the population above age 12 (obtained from the French National Institute of Statistics and Economic Studies, INSEE) together with surveys conducted by CREDOC that provide the share of mobile users in the population above 12. We also obtain the income distribution by municipality from INSEE.

We collect tariff data from online quarterly catalogs of offers proposed by the four MNOs and the largest mobile virtual network operator (MVNO). For Orange in particular, we collect all tariff data from offers prior to November 2013 in order to match observations about customers that keep their old offers. Tariff characteristics include monthly prices, data allowances, and voice allowances.

Most of the MNOs offer contracts that vary based on characteristics that are beyond the scope of our model, including bundling with home internet and television services. Because we want to focus on the choice of mobile data services, we aggregate contracts according to monthly data allowance categories: less than 500 megabytes, 500–3 000 MB, 3 000–7 000 MB, and more than 7 000 MB. These data limits are "soft", in the sense that customers can still use data services once the limit is exceeded, but download speeds will be throttled significantly. Our

[6] We limit ourselves to urban markets because antenna coverage is not limited to the boundary of municipalities, particularly in rural areas. In order to obtain a reliable measure of quality at the lowest geographical level, we need to define a market as a large municipality. There are 592 municipalities with a population greater than 10 000, and we drop three of those municipalities due to insufficient download speed tests to construct quality measures. This yields a total of 589 markets in our sample.

demand model will take the softness of data limits into account.

For each data limit category, a representative contract is selected, and for the purposes of our demand estimation, we assume that all consumers selecting a product within a category are selecting the representative product for that category. Table 1 presents the representative contracts, which in most cases have 24-month commitment durations and are not bundled with home internet or television services. Representative products typically have unlimited voice allowances, except for the lowest data limit categories. For MVNO's, our choice set includes one representative contract for each category; that is, we effectively assume there is one representative MVNO firm. Appendix C.5 describes in detail how we select representative contracts.

To be clear, the representative products in our model's choice set have the characteristics of products actually available in the market. The only characteristic that is adjusted from what is actually observed in the market is the monthly price; when a representative contract is associated with a handset subsidy, the monthly price is adjusted to reflect the value of that handset subsidy (see the data appendix for details). Each actual product is then assigned to a representative product, and our estimation takes the market shares of the representative products to be the aggregate market share of all the actual products assigned to that representative product. For instance, our econometric model features one high-data-limit contract for Orange. We treat the price of this product as 38.74 euros. This price corresponds to an observed price of 54.99 euros for this contract and an adjustment of 16.25 euros for the value of the associated handset subsidy. However, we measure the market share of this representative product as the sum of market shares of eleven high-data-limit contracts offered by Orange that are associated with various home internet and television bundles.

Finally, we obtain detailed data on infrastructure from the national frequency regulator (ANFR) and data on telecommuncation antennas from the national telecommunications regulator (ARCEP). These data describe the locations of all base stations with the number of antennas and frequencies operated by firm. We use these infrastructure data along with frequency bandwidth deployment data from Orange to recover a municipality-specific, operator-invariant coefficient of the efficiency of data transmission (i.e., spectral efficiency).

2.2 Descriptive statistics

Table 2 provides summary statistics of the main variables of interest from the data sets described in the previous section.

Measured quality (download speeds) varies substantially both across markets and within markets. Across markets, the average standard deviation for an operator is 9.56 Mbps, and

Table 1: The Choice Set

Operator	**Price**	**Data Limit**	**Unlimited Voice**	**Contracts Represented**	**Min Price**	**Max Price**	**Min Limit**	**Max Limit**
Orange	12.07	50	No	11	4.99	30.99	0	50
Orange	14.99	1000	No	4	14.99	14.99	1000	1000
Orange	22.91	1000	Yes	2	22.91	24.99	1000	1000
Orange	30.91	4000	Yes	5	19.99	48.99	3000	5000
Orange	38.74	8000	Yes	11	38.74	166.0	8000	20000
Bouygues	8.070	0	No	6	3.99	11.32	0	20
Bouygues	14.99	1000	No	3	14.99	14.99	1000	1000
Bouygues	20.91	3000	Yes	4	19.99	29.99	3000	5000
Bouygues	33.74	10000	Yes	4	32.70	72.70	10000	20000
Free Mobile	2	50	No	1	2.00	2.00	50	50
Free Mobile	19.99	3000	Yes	1	19.99	19.99	3000	3000
SFR	12.07	100	No	5	5.990	14.99	100	200
SFR	14.99	1000	No	3	14.99	19.99	1000	1000
SFR	22.91	1000	Yes	3	22.91	29.99	1000	1000
SFR	31.91	5000	Yes	5	19.99	43.99	3000	5000
SFR	37.74	10000	Yes	9	36.70	150.0	10000	20000
MVNO	7.990	No	0	13	7.990	18.99	0	200
MVNO	17.99	1000	No	5	9.990	17.99	500	1000
MVNO	19.99	500	Yes	10	19.99	35.99	500	2000
MVNO	42.99	5000	Yes	13	12.99	61.99	3000	5000
MVNO	64.99	10000	Yes	4	64.99	76.99	10000	10000

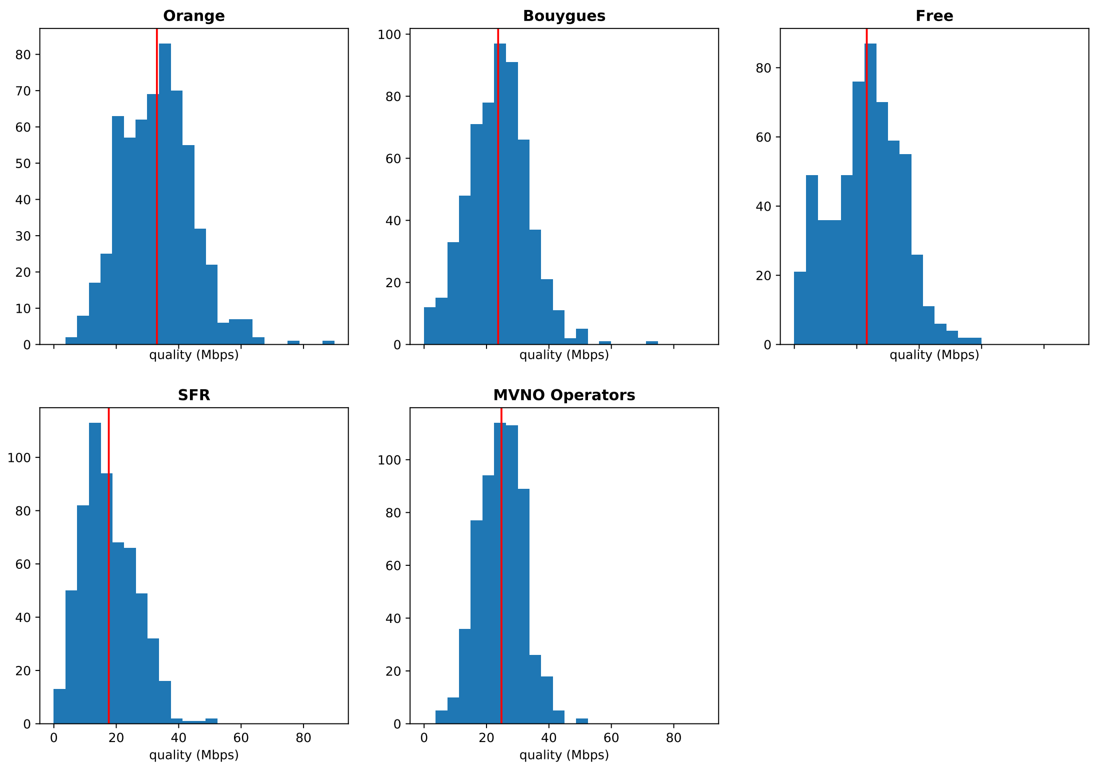
Each row corresponds to an object in the choice set, i.e., a representative product. The minimum and maximum prices and data limits are over the set of contracts represented by each representative product in the choice set.

Table 2: Summary Statistics

	Mean	**Std. Dev.**	**Min.**	**Max.**
Customer data (Orange)				
Market Average Usage (MB)	1 043	194	554	1701
Fraction Users above Data Limit	0.18	0.03	0.10	0.28
Num. customers		4 425 831		
Quality and market data				
Quality Orange (Mbps)	33.02	11.35	3.97	89.87
Quality Bouygues (Mbps)	23.73	9.69	0.60	72.97
Quality Free (Mbps)	23.21	11.08	1.56	57.26
Quality SFR (Mbps)	17.60	8.60	0.39	52.30
Quality MVNO (Mbps)	24.79	7.12	5.13	49.06
Median income (Euros)	13 035	3 179	5 152	31 320
Number of markets		589		
Tariff data				
Price	23.47	14.57	2.00	64.99
Price (Orange)	23.92	11.06	12.07	38.74
Price (Others)	23.33	15.83	2.00	64.99
Data limit	3 081	3 570	0	10 000
Num. products		21		

across operators, the average standard deviation for a market is 7.92 Mbps. Figure 1 displays histograms of measured quality across markets for each mobile network operator. Data usage is positively correlated with measured quality. Figure 2 plots the relationship between Orange market qualities and observed average data usage for three different data limits.[7] The average fraction of the data limit that is consumed is decreasing in the size of the data limit, as demonstrated in figure 3, which plots the histograms of average data consumption for three different data limits.[8]

Figure 1: Histograms of qualities by operator



[7]The correlations for data limits 1 000 MB, 4 000 MB, and 8 000 MB are, respectively, 0.147, 0.271, 0.246.

[8]For the data limits 1 000 MB, 4 000 MB, and 8 000 MB, the fraction of the data limit that is consumed is, respectively, on average, 0.656, 0.578, and 0.533.

Figure 2: Average data usage vs. measured quality across markets

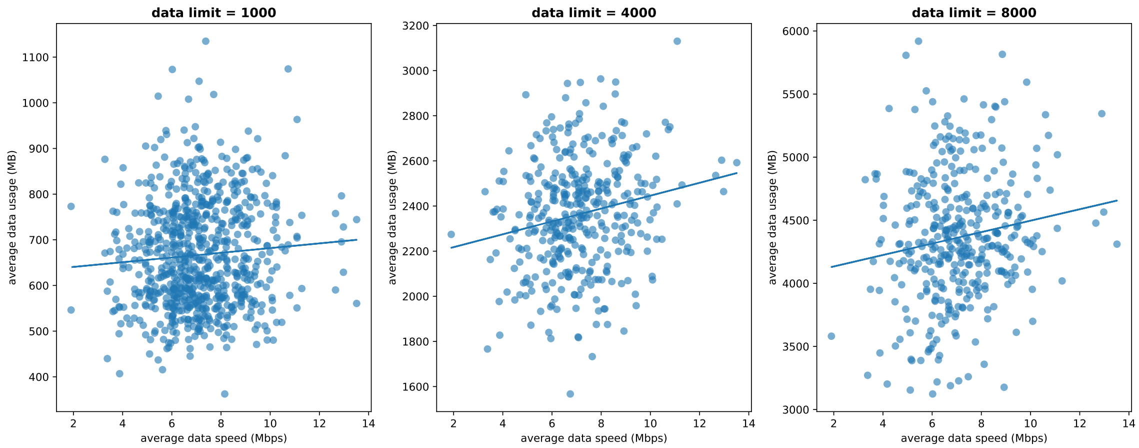
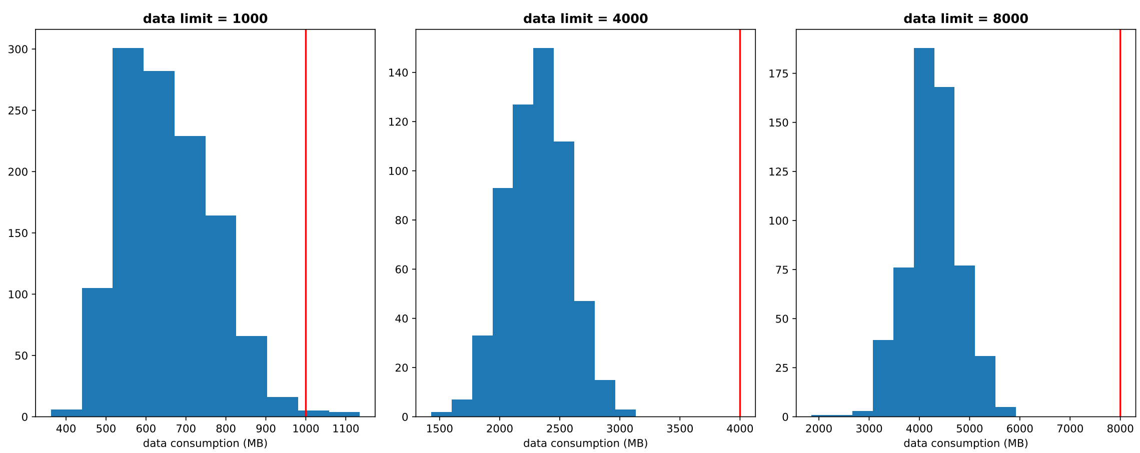


Figure 3: Average data usage across markets



Median incomes are correlated with products market shares. Figure 4 plots the relationship between median income and market shares of the three most expensive contracts offered by Orange, which correspond to the same three contracts depicted in figures 2 and 3. Median incomes are positively correlated with the market shares of the most expensive contracts; however, median incomes are negatively correlated with the average data consumption of those contracts, as depicted in figure 5.[9]

[9] Correlation coefficients for median incomes and market shares are, following the order of the graphs, 0.441, 0.519, 0.278. Correlation coefficients for median incomes and mean data consumption are, in the same order, -0.41, -0.445, -0.568.

Figure 4: Median income vs. expensive contract market shares

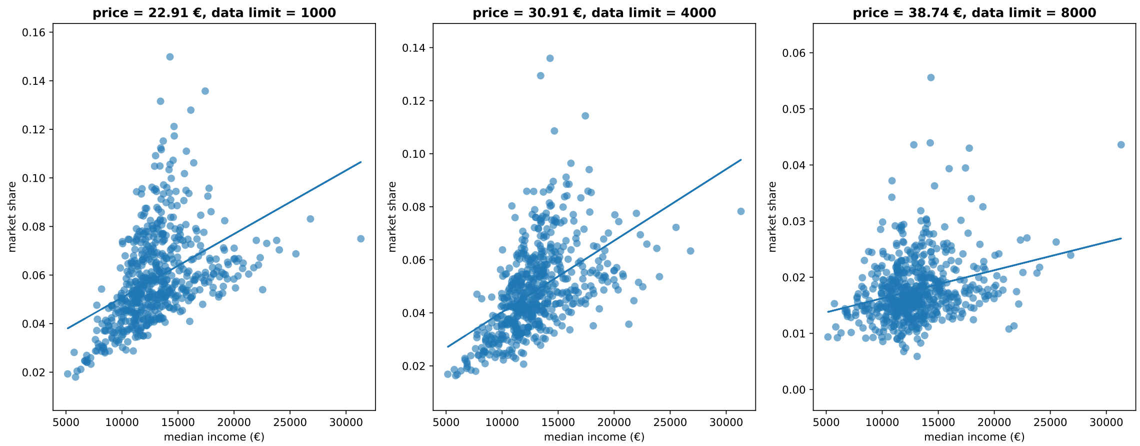
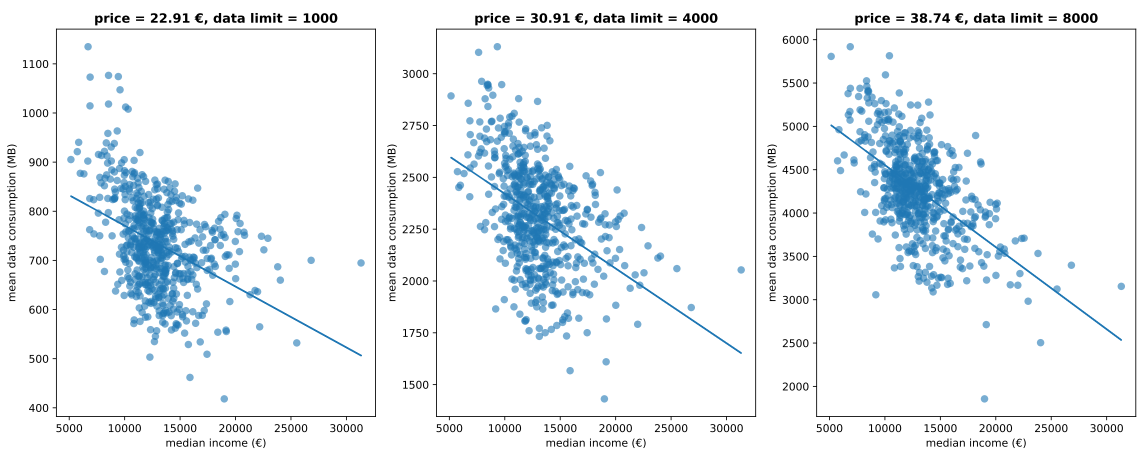


Figure 5: Median income vs. mean data consumption



3 Industry Model

In this section, we describe a formal model of how download speeds are jointly determined by bandwidth allocations, infrastructure investment decisions, and the load imposed on a network by consumers. We rely on standard telecommunications engineering models and are particularly indebted to Blaszczyszyn, Jovanovicy and Karray (2014). Table 14 in the appendix provides a list of all parameters used in the industry and demand models and their definitions.

In this model, firms own and operate their own networks with no sharing of infrastructure. In practice, network sharing occurs when an MNO dedicates a part of its network resources to another MNO. *Passive network sharing* involves the physical structure of base stations and the cost of electric power, but not the resources that transmit and receive signals and affect quality determination. In contrast, *active network sharing* occurs when equipment that transmits data is shared. During 2015, active network sharing occurred primarily in areas with low population density. Because we want to associate each firm's quality of service with the firm's own investment decisions, we ultimately focus on the higher-density areas of France in our demand analysis.

3.1 Base station infrastructure and data transmission

We begin by deriving engineering relationships that apply to each firm's network. For now, we omit firm subscripts and consider the engineering relationships in the abstract. Let B be the bandwidth being operated by a network of identical base stations serving distinct and identical cells. Let R be the cell radius in this network; later, we will think of bandwidth and cell radius as the choice variables in a firm's infrastructural investment decision.[10]

For a given consumer i, the theoretical maximum download speed $q(r_i)$ achieved by a unit of bandwidth depends on the consumer's distance r_i from the base station. Download speeds scale linearly with bandwidth, so if a consumer is allocated b_i units of bandwidth, their theoretical maximum download speed will be $b_i q(r_i)$. We will elaborate on the $q(\cdot)$ function below.

To aggregate download speeds over consumers, it would not be correct to compute the ordinary mean of $q(r)$ because users who receive lower quality signal require more resources for a given download; that is, for a download of a given size, they will either tie up the base station's capacity for longer or they will require a relatively larger fraction of the bandwidth to receive the same download speed as consumers closer to the antenna. Consequently, average download speeds should be derived from harmonic means.

To be more explicit, begin by imagining that all users are guaranteed the same download speed, $\bar{Q}$, and for now we ignore queuing issues and assume constant aggregate demand. Then, a user at distance r will require bandwidth $\bar{Q}/q(r)$. Assuming users are uniformly distributed over the cell, the total bandwidth required to serve the cell is

$$B = G(R)^{-1} \int_0^R \frac{\bar{Q}}{q(r)} g(r)\, dr,$$

where R is the radius of the cell, and $g(r)$ and $G(R)$ reflect its geometry (e.g., $g(r) = 2\pi r$ and $G(R) = \pi R^2$ with circular cells, but we use hexagonal cells, which tessellate).[11] Rearranging

[10]In principle, firms could also choose signal strength, which would also affect data download speeds, but firms typically operate at the maximum signal strength allowed by regulation.

[11]The area of a hexagon is given by $G(R) = \frac{3\sqrt{3}}{2}R^2$, where R is the hexagon's side length. When we actually

the above equation to solve for the average download speed that can be sustained by a given bandwidth, we have

$$\bar{Q}\left(R,B\right)=\frac{B}{G\left(R\right)^{-1}\int_{0}^{R}\frac{g(r)}{q(r)}dr}. \tag{1}$$

This equation expresses *channel capacity*, describing how feasible download speeds are influence by the firm's choice of cell radius R and bandwidth B. Implicitly, we have assumed there is a unit density of users. If the density of users is D, then the channel capacity per consumer would be equal to $\bar{Q}\left(R,B\right)/D$. Intuitively, feasible download speeds depend on the level of demand. Below, we will consider more precisely how demand affects delivered download speed using queuing theory. We will also consider how the demand level depends on delivered download speed, since consumers presumably are more likely to subscribe to a firm and download more data when a firm offers better download speeds. Thus, in equilibrium, demand and download speeds are simultaneously determined.

Next, we consider the individual download speed function $q\left(\cdot\right)$. The Shannon-Hartley theorem tells us that the theoretical upper bound to download speed is given by:

$$q\left(r\right)=b\log_{2}\left(1+SINR\left(r\right)\right) \tag{2}$$

where b is bandwidth and $SINR\left(r\right)$ is the signal-to-noise-and-interference ratio. This ratio is given by the ratio of signal power to the sum of noise and interference power:

$$SINR\left(r\right)=\frac{S\left(r\right)}{N+I}, \tag{3}$$

where $S\left(r\right)$ is signal power, N is noise power, and I is interference power. We now consider each of these three objects in turn.

As the signal travels, its power diminishes (path loss). We take this into account by using the Hata model of path loss. Ultimately, we assume that the signal power is equal to

$$\ln S\left(r\right)=-18.012-3.522\ln\left(r\right). \tag{4}$$

Notice that this entails a path loss exponent of approximately 3.522. In contrast, signal strength in a vacuum would have a path loss exponent of 2 − signals decay more quickly on the Earth's surface. The Hata model that delivers this path loss function takes the signal's frequency as an input. Later, we will use a higher path loss exponent when considering 5G technology; 5G technology is planned to be implemented with higher frequency signals, and

integrate over hexagonal cells, we do not actually use a formula for $g(r)$. Instead, we compute a double integral, integrating over the hexagon's apothem and perpendicular to the apothem.

such signals decay more quickly [add cross-reference].[12]

Noise power N is set equal to Johnson-Nyquist noise, -107.01 dBm per 5Mhz of bandwidth. Interference power is set equal to 2% of the signal power $S(R/2)$ at a distance equal to half the cell radius. During peak hours, average capacity utilization is approximately 2%. Thus, interference power is endogenous to the choice of radius, but not to realizations of demand.

3.2 Queuing

Another concern is that consumers download requests will not arrive uniformly over time. This means that $\overline{Q}$ derived above will not represent the actual delivered download speed in practice but a theoretical upper bound referred to as *channel capacity*.

To derive a relationship between channel capacity and average delivered download speed, we follow Blaszczyszyn, Jovanovicy and Karray (2014) and assume that download requests arrive according to a Poisson process and that download requests are served through a M/M/1 queue. Then, the average download speed will be

$$Q = \overline{Q} - Q^D, \tag{5}$$

where Q^D is the arrival rate of download requests. Each of the terms in equation 5 should be understood as rates, e.g., as quantities measured in Megabits per second.[13]

3.3 Transmission equilibrium

We now consider how the engineering relationships described above come together with demand to determine delivered download speeds in equilibrium. To be clear, at this point we are considering equilibrium in terms of download speeds and consumer demand, taking prices and infrastructure as given. This can be thought of as a final-stage equilibrium. Below, we will consider how prices are set in anticipation of this transmission equilibrium and subsequently how infrastructure is determined in anticipation of price and transmission equilibria.

Formally, the equilibrium we now consider is conditional on a vector of prices $\mathbf{P}$ and infrastructure variables $(\mathbf{R}, \mathbf{B})$. If each firm offers only one contract, and if price is the only contractual choice variable, then $\mathbf{P}$ is a F-dimensional vector, where F is the number of firms. Ultimately, we will consider multi-product firms and other contractual variables besides price

[12]The specific values in our path loss equation can be derived as follows. We begin with the Hata model for urban environments, and we assume a base station height of 30m. We assume the signal frequency of 1900 Mhz, which is approximately the median operated frequency in France in 2015. Finally, we assume a signal power of 61 dBm (or 1259 W) per 5 Mhz of bandwidth at the base station, which corresponds to the regulated limit on effective isotropic radiated power.

[13]For a derivation of this formula, see Taylor, Karlin and Taylor (1998), pp. 548-549 for example.

(i.e., data limits), in which case $\mathbf{P}$ can represent a higher-dimensional vector including prices and non-price characteristics of all products. In either case, $\mathbf{R}$ and $\mathbf{B}$ are both F-dimensional vectors – each firm employs only one network to serve all its products with. The f-subscripts will denote firm-specific variables.

The demand for downloads on firm f's network can be broken down into the product of three terms:

$$Q_f^D\left(Q_f, \mathbf{P}_f, \mathbf{Q}_{-f}, \mathbf{P}_{-f}\right) = D \times S_f\left(Q_f, \mathbf{P}_f, \mathbf{Q}_{-f}, \mathbf{P}_{-f}\right) \times \bar{x}_f\left(Q_f, \mathbf{P}_f, \mathbf{Q}_{-f}, \mathbf{P}_{-f}\right),$$

where D is the density of consumers, $S_f(\cdot)$ represents firm f's total market share as a function of the average download speeds for each firm and prices for each product, and $\bar{x}_f(\cdot)$ represents the average data consumption among firm f's subscribers. The market share and average download speed functions will be derived from a discrete-continuous model of demand, specified below, in which consumers choose which product to subscribe to and how much data to consume.

Combining equations (1) and (5), we have

$$\forall f = 1, \ldots F: \qquad Q_f = B_f\left[G(R_f)^{-1}\int_0^{R_f}\frac{g(r)}{q\left(r\right)}dr\right]^{-1} - Q_f^D\left(Q_f, \mathbf{P}_f, \mathbf{Q}_{-f}, \mathbf{P}_{-f}\right). \tag{6}$$

If prices and the infrastructure variables are given, then we have F equations and F download speeds Q_f to solve for, so under appropriate conditions on the demand system, the above equation uniquely defines a vector of equilibrium download speeds $\mathbf{Q}^*$.

We have now defined the transmission equilibrium as a function of prices and infrastructure, $\mathbf{Q}^*\left(\mathbf{P}, \mathbf{R}, \mathbf{B}\right)$. Subsequently, we will consider the price equilibrium as a function of infrastructure, $\mathbf{P}^*\left(\mathbf{R}, \mathbf{B}\right)$, and then finally the equilibrium in infrastructural investment.

3.4 Price competition

We can understand the network equilibrium model above as holding at the market level m with potentially different infrastructural variables in each market, $(\boldsymbol{R}_m, \boldsymbol{B}_m)$. However, prices are set nationally, so we will not introduce subscripts on the price vectors. From now on, when the infrastructure variables appear without market subscripts, they refer to the stacked vector of infrastructure variables for all markets.

Each firm f sets prices to maximize its variable profits. We define

$$\mathbf{P}_f^*\left(\mathbf{R}, \mathbf{B}\right) = \arg\max_{\boldsymbol{P}_f}\left\{\left(\boldsymbol{P}_f - \mathbf{1}_f c_u\right)\cdot\sum_m N_m \boldsymbol{S}_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right)\right\}, \tag{7}$$

where c_u is the variable cost per customer, $\mathbf{1}_f$ is a vector of ones with length equal to the number of products operated by f, N_m is the size of market m, and $\mathbf{S}_{mf}$ denotes a vector of product-level shares for products owned by firm f. The market share function is derived from the demand system and the transmission equilibrium function as follows:

$$\mathbf{S}^*_{mf}(\mathbf{P}, \mathbf{R}_m, \mathbf{B}_m) = \mathbf{S}_{mf}\left(Q^*_{mf}(\mathbf{P}, \mathbf{R}_m, \mathbf{B}_m), \mathbf{P}_f, \mathbf{Q}^*_{m,-f}(\mathbf{P}, \mathbf{R}_m, \mathbf{B}_m), \mathbf{P}_{-f}\right),$$

where the $\mathbf{S}_{mf}(Q_{mf}, \mathbf{P}_f, \mathbf{Q}_{m,-f}, \mathbf{P}_{-f})$ function stacks firm f's product-level market shares as a function of prices and download speeds. The market share function has an m-subscript to allow for variation in local demographics. The equilibrium download speed function has an m-subscript because the efficiency of data transmission, γ_m, introduced above, may vary by market.

Note that equilibrium download speeds depend on price, so the first-order condition for optimal price-setting must not only take into account the direct effect of lowering price on consumer demand, but also the indirect effect of endogenous download speeds. The indirect effect lowers price elasticities because as demand for firm f falls, f's download speeds increase due to reduced network load, which has a positive effect on demand, thereby dampening the demand reduction.

3.5 Infrastructure competition

The first stage of competition involves firms deciding on their infrastructural investments in each market. Infrastructure costs in market m are given by the following function:

$$C_{mf}(R_{mf}, B_{mf}) = (c_{fc,m} + c_{bw}B_{mf})\frac{A_m}{G(R_{mf})}, \tag{8}$$

where A_m is the land area of market m, $c_{fc,m}$ captures fixed costs per base station (which may vary by market), and c_{bw} captures how the cost of a base-station varies with bandwidth operated. This cost function is motivated by practical considerations. The fixed cost component $c_{fc,m}$ is intended to capture costs of the inactive (non-transmitting) structure of a base station as well as fees paid to property owners, which plausibly vary across markets. The other component, c_{bw}, reflects the fact that to transmit over a larger range of frequencies, more telecommunications equipment must be added to the base station.

Next, we can define market-level profits as follows:

$$\Pi_{mf}(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m) = (\boldsymbol{P}_f - \mathbf{1}_f c_u) \cdot \sum_m N_m \boldsymbol{S}_{mf}(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m) - C_{mf}(R_{mf}, B_{mf}). \tag{9}$$

Finally, we can define the national profit function for each firm f:

$$\Pi_f\left(\boldsymbol{R},\boldsymbol{B}\right)=\sum_m \Pi_{mf}\left(\boldsymbol{P}^*\left(\boldsymbol{R},\boldsymbol{B}\right),\boldsymbol{R}_m,\boldsymbol{B}_m\right), \tag{10}$$

where $\boldsymbol{P}^*\left(\boldsymbol{R},\boldsymbol{B}\right)$ is the solution to the national pricing game defined above.

Equation (10), taken for each firm f, defines the payoffs for the first-stage game in infrastructural investment. Each firm f chooses $(\mathbf{R}_f,\mathbf{B}_f)$ to maximize national profits, taking others firms' $(\mathbf{R}_{-f},\mathbf{B}_{-f})$ as given:

$$\max_{\mathbf{R}_f,\mathbf{B}_f}\left\{\Pi_f\left(\mathbf{R},\mathbf{B}\right)\right\}.$$

In this section, we have considered market shares and data demand as abstract functions of delivered download speeds and prices. In the following section, we will be explicit about the model of consumer behavior. Following that, we will consider the estimation of the demand model and the cost parameters $(c_u,\mathbf{c}_{fc},c_{bw})$.

3.6 Economies of Scale

A merger between two firms in this model will exhibit cost efficiencies if, post merger, the merged firm is able to provide the same quality at a lower cost or higher quality at the same cost. Our model allows for cost efficiencies that result from *economies of density* and *economies of pooling*.

3.6.1 Economies of Pooling

It has long been recognized in the economics literature that "there are economies of scale in servicing a stochastic market" (Carlton, 1978).[14] In operations management, the same phenomenon has been referred to as the "Pooling Principle" (Cattani and Schmidt, 2005). Thus, we use "economies of pooling" to describe economies of scale coming from consolidating bandwidth.

It is easy to see how economies of scale result from our queuing theory model. Equation (5) holds that delivered download speed corresponds to the difference between channel capacity and the download demand rate. Crucially, channel capacity is linear in bandwidth. Thus, if two identical firms combine their bandwidth and their customer bases (holding download demand rate per customer fixed), then both terms on the right-hand side of equation (5) would double. Consequently, download speeds (the left-hand side) would also double.

[14] Robinson (1948) was perhaps the first to describe the phenomenon, under the heading of "the economy of the large machine." De Vany (1976) was an early application using queuing theory to derive economies of scale. Mulligan (1983) shows formally how economies of scale result from queuing theory.

3.6.2 Economies of Density

Due to path loss, captured by the function $q(\cdot)$, the closer consumers are to a base station, the more efficiently that station can serve consumers. Thus, if we increase the density of consumers served by a firm while keeping constant the numbers of consumers per base station, consumers will be closer to base stations serving them on average, improving download speeds. If two network operators were to merge and combine their customer bases, the merged entity would effectively serve a higher population density of consumers. This creates the opportunity for the merged firm to deliver higher download speeds to its customers with the same total investment level of the pre-merger firms.[15]

4 Demand Model

Notation Individuals are indexed by i, the various contracts are denoted by $j \in \mathcal{J} = \{1, \ldots, J\}$, and geographic markets are indexed by $m \in \mathcal{M} = \{1, \ldots, M\}$. Quality of service, measured as average download speed, is captured by $Q_{m,f}$, where f denotes a firm. Quality is constant across $j \in \mathcal{J}_f$ in commune m, where $\mathcal{J}_f$ represents the set of products produced by firm f. We write $f(j)$ to denote the firm associated with product j.

We consider the consumer's static consumption decision. A consumer's indirect utility from a contract j, consuming x megabytes of data, in market m, is denoted by

$$v_m(j, x; \theta_i, \vartheta_i, \varepsilon_i) \equiv u_j\left(x, Q_{m,f(j)}; \vartheta_i, \theta_i\right) + \theta_v v_j - \theta_{pi} p_j + \xi_{jm} + \varepsilon_{ij}, \tag{11}$$

where p_j is the contract price; $u_j(\cdot)$ maps the contract j, data consumption x, and data quality $Q_{m,f(j)}$ into the utility from consumption of mobile services; v_j is a dummy variable equal to one if plan j has an unlimited voice allowance; ξ_{jm} is the product-market-specific demand shock; and θ and ϑ are parameters describing preferences. Idiosyncratic tastes ε_{ij} are realized before the choice of contract j is made. The preference parameter ϑ_i is a random variable capturing how much agent i values data; it is realized *after* the choice of contract is made, but an agent chooses the contract with knowledge of its distribution. The object is to estimate the distribution of preference parameters θ_i and ϑ_i.

Consumer Behavior To maximize utility, the consumer chooses a plan j and data usage x. We first consider what the agent's usage behavior would be, conditional on contract. Usage behavior depends on the consumer's ϑ_i, which is a random variable, reflecting that consumers may be unable to perfectly forecast their utility for data when choosing a phone

[15] Here we ignore the dynamics of merging two firms and integrating their existing infrastructure; we are making statements about what would happen with a given level of investment spread across two firms in comparison to what one integrated firm would achieve with the same level of total investment.

contract. We then consider the optimal choice of contract, which consumers choose after forming expectations over $u_j(\cdot;\vartheta_i,\theta_i)$.

Mobile Data Consumption To rationalize finite data consumption even when additional data consumption entails no monetary cost, our functional form includes a term which corresponds to the disutility of download times. This disutility is proportional to the amount of data downloaded and inversely proportional to the download speed; it can be thought of as the opportunity cost of time spent downloading. Consumers will consume data until the marginal utility of extra data corresponds to the disutility of additional download time.

A consumer's utility of data consumption is given by the following functional form:

$$u_j\left(x,Q;\vartheta_i,\theta_i\right)=\vartheta_i\log\left(1+x\right)-\theta_c c_j\left(x,Q\right), \tag{12}$$

where $c_j(\cdot)$ is the opportunity cost of time spent downloading and is given by the following formula:

$$c_j\left(x,Q\right)=\begin{cases}\frac{x}{Q} & \text{if } x\leq\bar{x}_j\\ \frac{\bar{x}_j}{Q}+\frac{\bar{x}_j-x}{Q^L} & \text{if } x>\bar{x}_j,\end{cases} \tag{13}$$

There is a discontinuity in download speeds when a consumer reaches their monthly data limit, $\bar{x}_j$. Data consumed after reaching the data limit downloads at the throttled speed $Q^L\ll\mathbf{Q}$, where $\mathbf{Q}$ is stacked firm-market-specific download speeds. This creates a discontinuity in the marginal cost of data consumption. Let

$$x_j^*\left(Q;\vartheta_i,\theta_c\right)\equiv\arg\max_{x\in\mathbb{R}_+}\left\{u_j\left(x,Q;\vartheta_i,\theta_i\right)\right\}$$

be the data choice that maximizes data utility for contract j. The first order condition and the structure of the marginal cost of data consumption yield four possible cases that determine the optimal data consumption:[16]

$$x_j^*\left(Q_{f(j)};\vartheta_i,\theta_i\right)=\begin{cases}0 & \text{if } \vartheta_i\leq\frac{\theta_c}{Q_{f(j)}}\\ \frac{\vartheta_i Q_{f(j)}}{\theta_c}-1 & \text{if } \frac{\theta_c}{Q_{f(j)}}\leq\vartheta_i<\frac{\theta_c}{Q_{f(j)}}\left(\bar{x}_j+1\right)\\ \bar{x}_j & \text{if } \frac{\theta_c}{Q_{f(j)}}\left(\bar{x}_j+1\right)\leq\vartheta_i<\frac{\theta_c}{Q^L}\left(\bar{x}_j+1\right)\\ \frac{\vartheta_i Q^L}{\theta_c}-1 & \text{if } \vartheta_i\geq\frac{\theta_c}{Q^L}\left(\bar{x}_j+1\right).\end{cases} \tag{14}$$

The first case captures consumer types ϑ_i that would not consume any data.[17] The second

[16] We are using here the assumption that $Q^L\ll\mathbf{Q}$, which holds in our data.

[17] We interpret such consumers as those that unexpectedly do not need their mobile plan (e.g., they went out of the country for the month). Indeed, in our data, we observe a point mass of consumers that consume zero data—even among those that adopt high data limit plans.

case captures consumer types that consume less than $\bar{x}_j$ even without throttling. The third case captures consumer types that would consume greater than $\bar{x}_j$ if data speeds were not throttled, but under throttling, the marginal cost of an additional unit of data is greater than the marginal benefit, so they choose to consume exactly the data limit. Finally, the fourth case captures consumer types that would consume greater than $\bar{x}_j$ even under throttled data speeds.[18]

Contract Decision The consumer chooses the contract that maximizes her expected utility. The expectation is with respect to the data consumption utility parameter ϑ_i, which we will assume is distributed

$$\vartheta_i \sim \mathit{Exponential}\left(\theta_{di}\right).$$

While the consumer does not know her ϑ_i *ex ante*, she does know her θ_{di}, which we allow to vary by i. Each market has an outside option, $j = 0$, which has indirect utility normalized to ε_{i0}. Following **?**, we introduce a nested structure on the idiosyncratic taste shocks. Specifically,

$$\varepsilon_{ij} = \zeta_{ig} + (1 - \sigma)\,\eta_{ij},$$

where η_{ij} is i.i.d. extreme value and ζ_{ig} has the distribution such that ε_{ij} is extreme value. The value $\sigma \in [0, 1)$ is the nesting parameter, and all contracts (but not the outside option) belong to a single nest.[19] The addition of a nest for all contracts allows for more flexible substitution patterns to the outside option.

The consumer observes θ_i, as well as ε_{ij}, but must choose a contract before observing the ϑ_i. Thus,

$$j^*_{i,m} = \arg\max_{j \in \mathcal{J} \cup \{0\}} \left\{ \mathbb{E}\left[v_m\left(j, x^*_j\left(Q_{m,f(j)}; \vartheta_i, \theta_i\right); \theta_i, \vartheta_i, \varepsilon_{ij}\right)\right]\right\},$$

where the expectation is taken over ϑ_i conditional on θ_{di}.

See Appendix A.2 for an analytic expression of expected utility from data conditional on contract and θ_i.

5 Estimation

We estimate the demand model using a modified version of Berry, Levinsohn and Pakes (1995), described below. After estimating demand, we infer firm's costs based on the assumption that firms set prices and invest in quality optimally.

[18]Small data limit plans have hard data limits (i.e., there is no throttling). We therefore impose that all contracts with data limits less than 500 MB cannot consume greater than the associated data limit.

[19]Note that if $\sigma = 0$, the model is equivalent to one without nesting.

5.1 Demand estimation

We seek to estimate the distribution of consumer parameters θ_i. Specifically, we have the following parameters

$$\theta_i = [\theta_{pi}, \theta_c, \theta_{di}, \theta_v]'.$$

Note that we have two heterogeneous parameters that we will allow to vary by income. Specifically,

$$\begin{pmatrix} \log(\theta_{pi}) \\ \log(\theta_{di}) \end{pmatrix} = \begin{pmatrix} \theta_{p0} \\ \theta_{d0} \end{pmatrix} + \begin{pmatrix} \theta_{pz} \\ \theta_{dz} \end{pmatrix} z_i, \tag{15}$$

where z_i is the consumer's income.[20] The specifications for θ_{pi} and θ_{di} are with respect to the log parameters in order to ensure the correct sign: for all income levels, the price coefficient must be negative and the rate parameter must be positive.

5.1.1 Unobserved demand component

As is standard in the demand estimation literature, we use market shares to back out the unobserved demand components $\boldsymbol{\xi}$. The standard BLP contraction mapping used to solve for $\boldsymbol{\xi}$ does not apply in our setting, however. We observe the set of products offered by all firms, but we only observe detailed market share data for Orange. Specifically, we observe market shares for each of Orange's products for each market m, but we only observe firm-level national market shares for the other firms.

Our modified estimation technique rationalizes product-level market shares for Orange products and only the firm-level aggregate market shares for the other firms. Formally, we assume

$$\forall j \in \mathcal{J}_{-O}, \forall m: \quad \xi_{jm} = \xi_{f(j)},$$

where $\mathcal{J}_{-O}$ is the set of non-Orange products, and $f(j)$ is the firm that corresponds to product j. Appendix A.1 shows that a modified version of the BLP contraction mapping still applies in our context that is capable of solving for the unique vector $\boldsymbol{\xi}$ under the above assumption.

5.1.2 Elasticity imputations

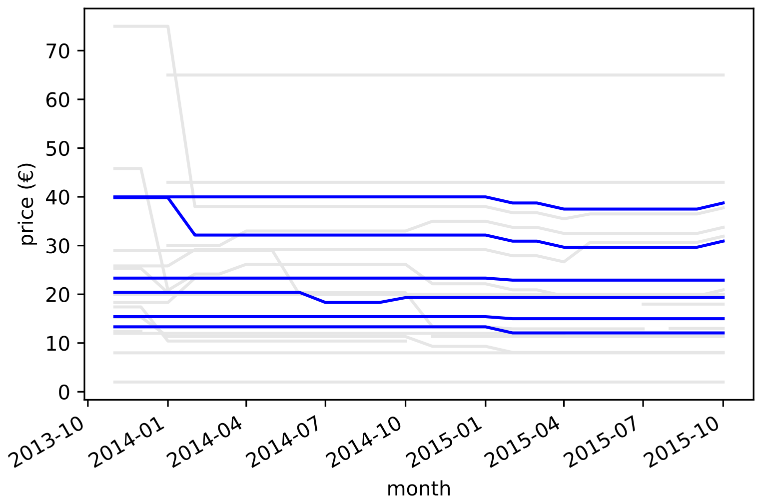
Prices are set nation-wide and do not vary by market. Moreover, prices varied very little over time around our sample period.[21] See Figure 6 for prices over the two years prior to

[20] When we estimate the parameters, z_i will be measured in 10 000 €.

[21] Note that Bourreau, Sun and Verboven (2018) consider a time period that includes the entry of Free Mobile in 2012. Following this entry, there were substantial price changes as the incumbent MNOs reacted to the new low-cost competitor. In contrast, during the two years leading up to our sample period, price variation was quite limited.

our sample period. Prices of Orange contracts are in blue, and the prices of other operator contracts are in light gray. Given the lack of price variation, it is difficult to identify price elasticities from the data.

Figure 6: Prices of Orange contracts over two years



We therefore take an approach where we impute price elasticities over a wide range of possible elasticities. For each elasticity considered, we impose that the price elasticity of Orange products corresponds to the imposed elasticity. Formally, we calculate the implied Orange products price elasticity in market m, defined as follows:

$$e_m^O(\theta) = \frac{\mathbf{s}_{m,O}(1.01\boldsymbol{p}_O, \boldsymbol{p}_{-O}, \mathbf{Q}_m; \theta) - \mathbf{s}_{m,O}(\boldsymbol{p}_O, \boldsymbol{p}_{-O}, \mathbf{Q}_m; \theta)}{0.01\mathbf{s}_{m,O}(\boldsymbol{p}_O, \boldsymbol{p}_{-O}, \mathbf{Q}_m; \theta)},$$

where $s_{m,O}(\cdot)$ is the share of contracts in m produced by Orange,

$$\mathbf{s}_{m,O}(\boldsymbol{p}_O, \boldsymbol{p}_{-O}, \mathbf{Q}_m; \theta) = \sum_{j \in \mathcal{J}_O} \int s_{ijm}(\boldsymbol{p}_O, \boldsymbol{p}_{-O}, \mathbf{Q}_m; \theta_i) dF_m(\theta_i),$$

and $s_{ijm}(\cdot)$ gives the share of consumers of type θ_i in market m who purchase contract j, and $F_m(\theta_i)$ denotes the CDF of consumer types θ_i in market m. The vector $\boldsymbol{p}_O$ represents the vector of Orange product prices, and $\boldsymbol{p}_{-O}$ represents the prices of those produced by non-Orange firms.

For a range of price elasticities $E \in \mathcal{E}$, we impose

$$\mathbb{E}\left[e_m^O(\theta) - E\right] = 0$$

as a moment in our estimation procedure, described below.

5.1.3 Identification

For each Orange contract, we observe monthly data consumption. We identify the data utility parameters θ_{d0}, θ_{dz}, and θ_c, in part, by matching predicted data consumption with observed data consumption. Formally, from the data we construct $\bar{x}_{jm}$, which is the average data consumption across consumers using product j in market m. Given θ, we can construct the mean data consumption across consumers in market m that chose product j:

$$\bar{x}_{jm}(\theta) \equiv (s_{jm}(\boldsymbol{p}, \mathbf{Q}_m; \theta))^{-1} \int\int s_{ijm}(\boldsymbol{p}, \mathbf{Q}_m; \theta_i) x^*_{jm}(\vartheta_i) dF(\vartheta_i|\theta_i)\, dF_m(\theta_i).$$

Appendix A.2 shows how to integrate over ϑ_i analytically.

Matching observed and predicted data consumption effectively identifies the average θ_{di}. To identify both θ_{d0} and θ_{dz}, which controls how θ_{di} varies with income, we use a moment interacting the difference between predicted and observed data consumption and median market income.

Simply matching mean data consumption and shares consuming above limits does not identify the level of the data utility (and therefore θ_c). The level of data utility comes from the tradeoff between the data utility and the contract's other components (price and voice allowance), which is identified by imposing that demand shocks $\boldsymbol{\xi}$ are uncorrelated with data limits (which are correlated with data utility).

The imputed elasticity moment effectively identifies the average θ_{pi}, and we separately identify θ_{p0} and θ_{pz} by imposing that the demand shocks $\boldsymbol{\xi}$ are uncorrelated with median incomes. Voice allowances are assumed to be uncorrelated with the demand shocks.

In summary, we have the following moments that we use to identify the distribution of preference parameters θ. Note that the moments are only imposed for Orange products since we only observe data consumption and product-market shares for Orange.

Moments
$\mathbb{E}\left[e^O_m(\theta) - E\right] = 0$
$\mathbb{E}\left[\xi_{jm}(\theta) inc^{med}_m\right] = 0$
$\mathbb{E}\left[\bar{x}_{jm}(\theta) - \bar{x}_{jm}\right] = 0$
$\mathbb{E}\left[\left(\bar{x}_{jm}(\theta) - \bar{x}_{jm}\right) inc^{med}_m\right] = 0$
$\mathbb{E}\left[\xi_{jm}(\theta) v_j\right] = 0$
$\mathbb{E}\left[\xi_{jm}(\theta) \bar{x}_j\right] = 0$

We use two-stage efficient GMM to estimate θ, searching for θ in an outer loop and solving for $\boldsymbol{\xi}(\theta)$, $\boldsymbol{e}^O(\theta)$, and $\bar{\boldsymbol{x}}(\theta)$ in an inner loop. Further details can be found in Appendix A.3. Results can be found in Appendix B.1.

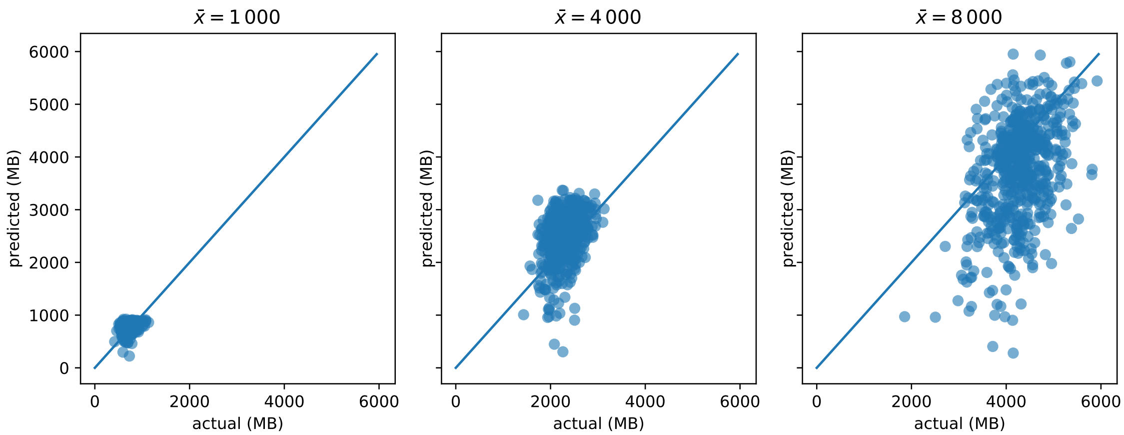
5.2 Results

Demand parameter estimates are listed in table 3 in Appendix B.1 for a range of imputed price elasticities and imputed nesting parameters. The price elasticity implied by Bourreau, Sun and Verboven (2018) is -2.5, the middle imputed price elasticity, which we regard as our preferred specification. For all imputations, price sensitivity is decreasing in income. The data utility parameter is increasing in income, which implies an inverse relationship between income and the value of data consumption, suggesting a higher opportunity cost of time spent downloading for higher income individuals. The variance parameter is increasing in income. While signs are consistent across elasticities, the parameter estimates appear to be sensitive to the price elasticity chosen, especially price, voice allowance, and Orange dummy coefficients.

To interpret the results above, tables 4–6 in Appendix B.1 convert the parameter estimates into willingness to pay for certain contract characteristics across income percentiles. Figure 7 depicts predicted versus actual average data consumption across markets for three Orange contracts with different data limits.[22] The diagonal line is a 45-degree line. Markets in which predicted average consumption equals observed average consumption will lie upon the line. Our estimated model correctly predicts the average level, even though this level is not a constant fraction of the data limit. While it predicts across-market heterogeneity less well, it does weakly predict high data consumption for markets with high observed data consumption and low data consumption for markets with low observed data consumption. The correlation coefficients between actual and predicted consumption for the three contracts across markets are, respectively, 0.305, 0.386, and 0.405.

[22] The predicted average data consumption is based on parameter estimates for the imputed elasticity -2.5 and a nesting parameter of 0.8.

Figure 7: Predicted vs. actual average data consumption



5.3 Cost Estimation

There are three costs parameters to be estimated: c_u, the cost per user; $c_{fc,m}$, the fixed cost per base station in market m; and c_{bw}, which dictates how base station costs vary with bandwidth operated.

5.3.1 Costs per user

From equation (7), the first-order condition from the price setting game is

$$\sum_m N_m \boldsymbol{S}_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right) + \left(\sum_m N_m J_f \boldsymbol{S}_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right)\right)\left(\boldsymbol{P}_f - \mathbf{1}_f c_d\right) = 0, \qquad (16)$$

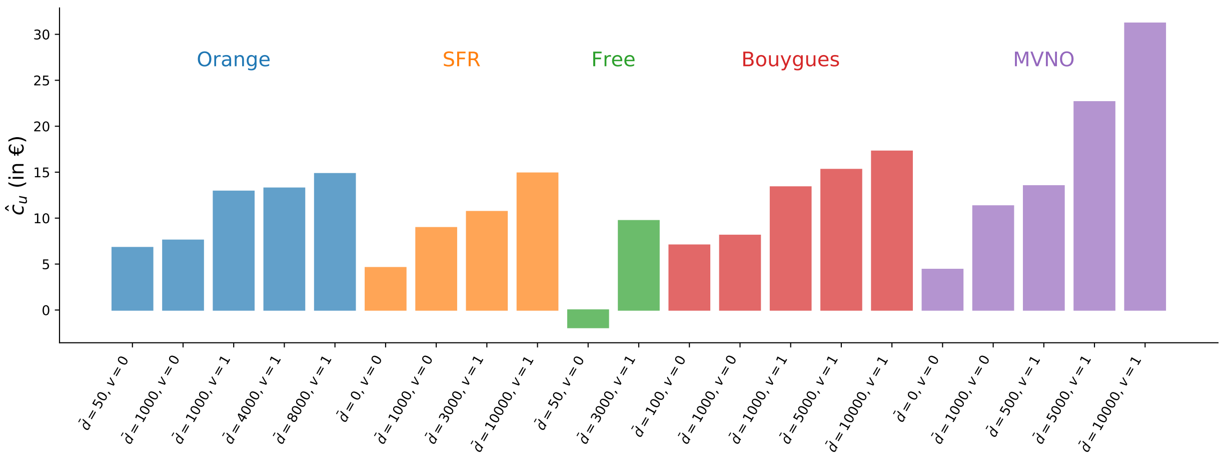
where J_f represents the Jacobian operator with respect to $\boldsymbol{P}_f$.

Therefore, an estimate of marginal cost is given by

$$\hat{\mathbf{c}}_{u,f} = \boldsymbol{P}_f + \left(\sum_m N_m J_f \boldsymbol{S}_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right)\right)^{-1} \sum_m N_m \boldsymbol{S}_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right).$$

Estimated costs for our elasticity and nesting parameter imputations are given in Table 7 in Appendix B.2. Figure 8 displays the estimated per-user costs for each product for an imputed price elasticity of -2.5 and a nesting parameter of 0.6.

Figure 8: Per-user estimated costs



5.3.2 Infrastructure costs

Given the demand estimates, and the model of how the infrastructure variables $(\mathbf{R}, \mathbf{B})$ map into delivered quality, we can simulate how equilibrium revenues change as the infrastructure is changed. Intuitively, we can measure the marginal revenue of infrastructure, and this allows us to infer the marginal cost of infrastructure.

Formally, we compute the marginal operating income from each market based on a 1% change in cell radius and bandwidth:

$$MR_{mf,R}\left(\boldsymbol{R}_m, \boldsymbol{B}_m\right) = \frac{\Pi_{mf}\left(\boldsymbol{P}, \left(.01 + R_{mf}, \boldsymbol{R}_{m,-f}\right), \boldsymbol{B}_m\right) - \Pi_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right)}{.01},$$

$$MR_{mf,B}\left(\boldsymbol{R}_m, \boldsymbol{B}_m\right) = \frac{\Pi_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \left(.01 + B_{mf}, \boldsymbol{B}_{m,-f}\right)\right) - \Pi_{mf}\left(\boldsymbol{P}, \boldsymbol{R}_m, \boldsymbol{B}_m\right)}{.01}.$$

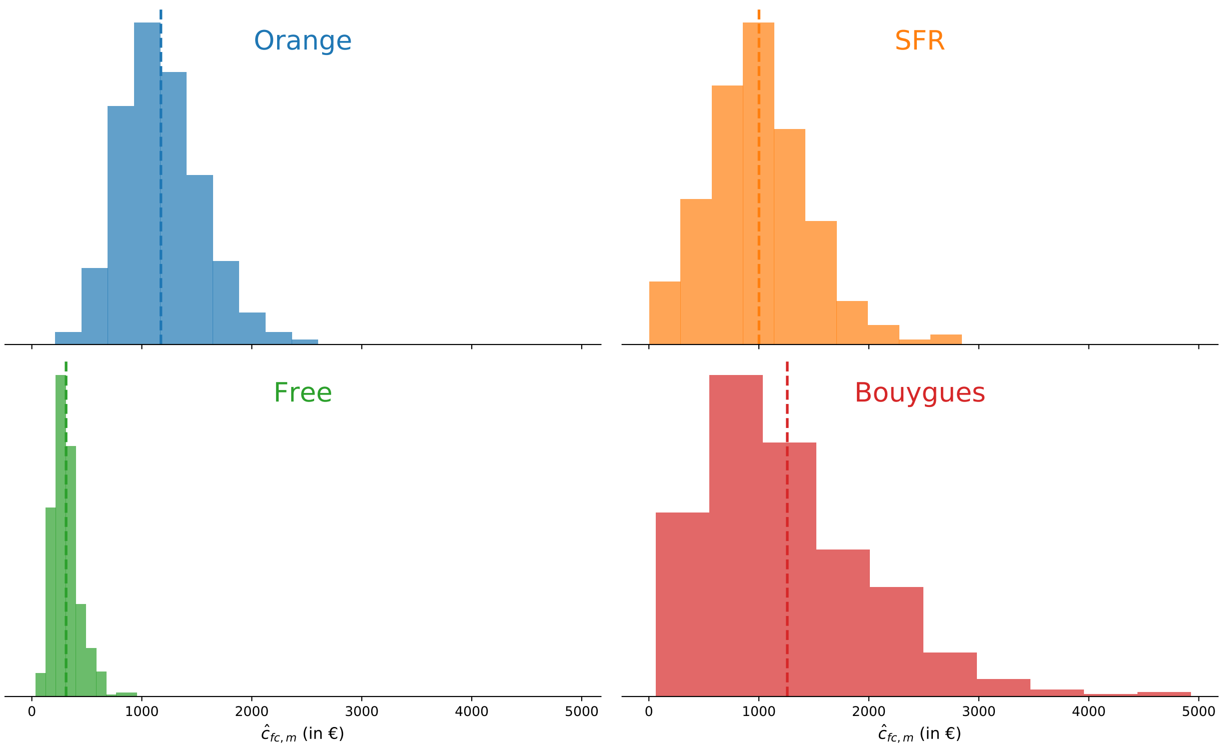
For these calculations, we use the prices observed in equilibrium, implicitly assuming that the equilibrium prices (which are set nationally) will not respond to a change in infrastructure in a single market m. As each commune is quite small, this is a plausible approximation. Note that these profit functions are defined in terms of the equilibrium download speeds that result from the infrastructure and prices. Thus, the above expressions for marginal revenue should be understood as implicitly taking into account how quality implicitly changes as infrastructural investment is changed.

Next, assuming that infrastructure investments are chosen to maximize profits, we can use the marginal revenues above to recover the remaining cost function parameters using equation (8).

The marginal cost of increasing R_{mf} is obtained by differentiating the cost function in equation (8). Therefore, setting marginal cost equal to marginal revenue, we can set the derivative of (8) equal to the marginal revenue with respect to R. This gives us one moment to identify cost function parameters.

Estimated infrastructure costs for our elasticity and nesting parameter imputations are given in Table 8 in Appendix B.2. Figure 9 displays the estimated per-tower costs for an imputed price elasticity of -2.5 and a nesting parameter of 0.6.

Figure 9: Per-tower estimated costs



6 Market Power and Scale Efficiencies

In this section, we explore the trade-off between market power and economies of scale (or density) by considering the optimal number of firms in a static equilibrium. Given the gradual nature of network deployment in the industry, this exercise cannot hope to capture the short-run impacts of potential merger, but it captures the long-run trade-offs associated with consolidation.

We simulate counterfactual equilibria using the estimated cost and demand parameters assuming symmetric firms. Each firm offers two data plans: one with a 2 GB limit and another with a 10 GB limit. The former represents a moderate data allowance; the latter, an extremely generous data allowance (in 2015). All plans are assumed to have unlimited voice.

As we vary the number of firms, total bandwidth available to the industry is divided equally among firms. We compute equilibria based on a representative municipality with median characteristics across municipalities in our sample. We present results below for two possible price elasticities and a nesting parameters of 0.6. Results for other possible nesting parameters are located in Appendix B.3, and it does not appear that the choice of nesting parameter affects our results in a meaningful way.

Figure 10 displays endogenous variables for symmetric equilibria as we vary the number of firms. As a robustness check, we display different elasticity imputations, as discussed in section 5.

Equilibrium prices have a non-standard relationship with the number of firms. At low numbers of firms, prices do decline with the number of firms, but as we get beyond a few firms, prices can increase. The reason for this non-standard relationship has to do with the non-standard nature of price elasticities in this setting. Figure 11 displays *partial price elasticities*, the price elasticity holding quality of service fixed, evaluated at equilibrium prices. These elasticities display the typical relationship with the number of firms. However, this partial price elasticity is not the relevant price elasticity for firms' price setting.

As a firm lowers its price, it attracts more customers, causing the load on its network to increas, lowering download speeds, and dampening the appeal of the lowered price for consumers. In other words, the relevant elasticity for the purposes of setting optimal prices involves a full derivative that takes into account the indirect effect of changing prices on download speeds. Figure 11 also displays these *full price elasticites*, which decline less with the number of firms than the partial elasticities. The reason for the divergence between the full and partial price elasticities is the worsening of the indirect quality effect as the number of firms grows. When there are many firms, a firm's own capacity is small relative to the number of consumers that they can potentially attract from other firms, making quality of service degrade more for a given price increase.

Figure 12 considers welfare compared to the monopoly case as the number of firms is varied. Interestingly, for our preferred elasticity imputation, the optimal number of firms is two in terms of total surplus, and three in terms of consumer surplus.

However, as Figure 13 illustrates, consumers do not agree on the optimal number of firms. The optimal number of firms is decreasing with income in all cases.

Figure 10: Counterfactual prices and qualities

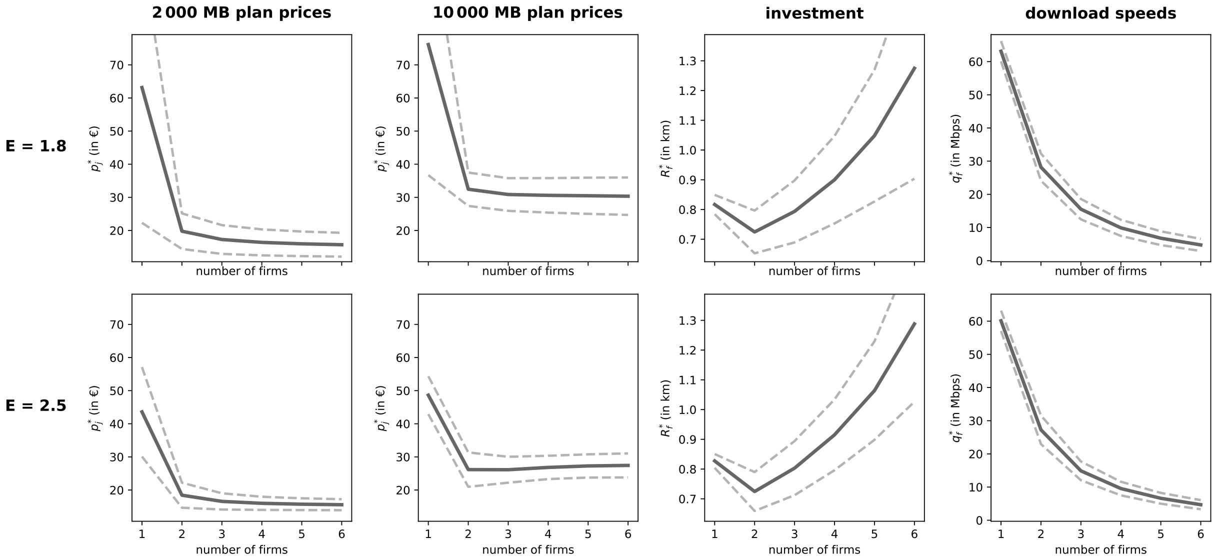


Figure 11: Full and partial price elasticities

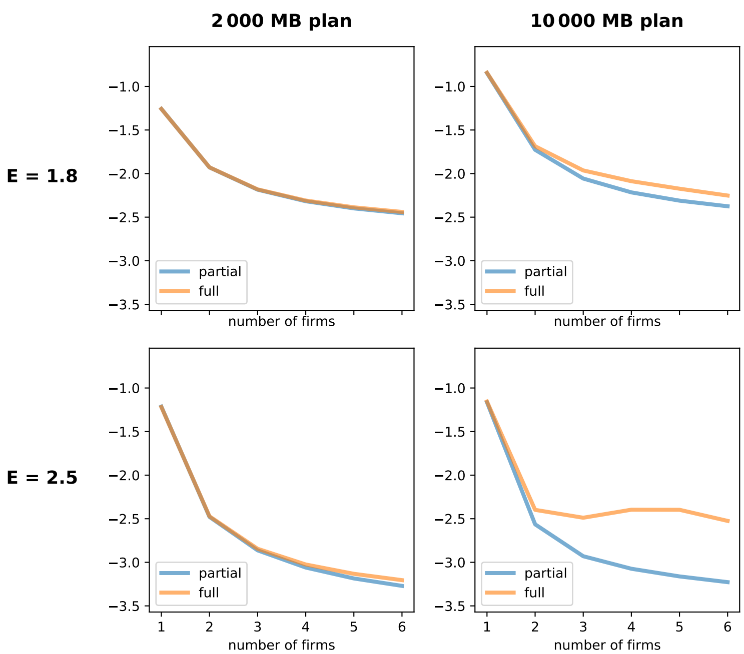


Figure 12: Counterfactual welfare

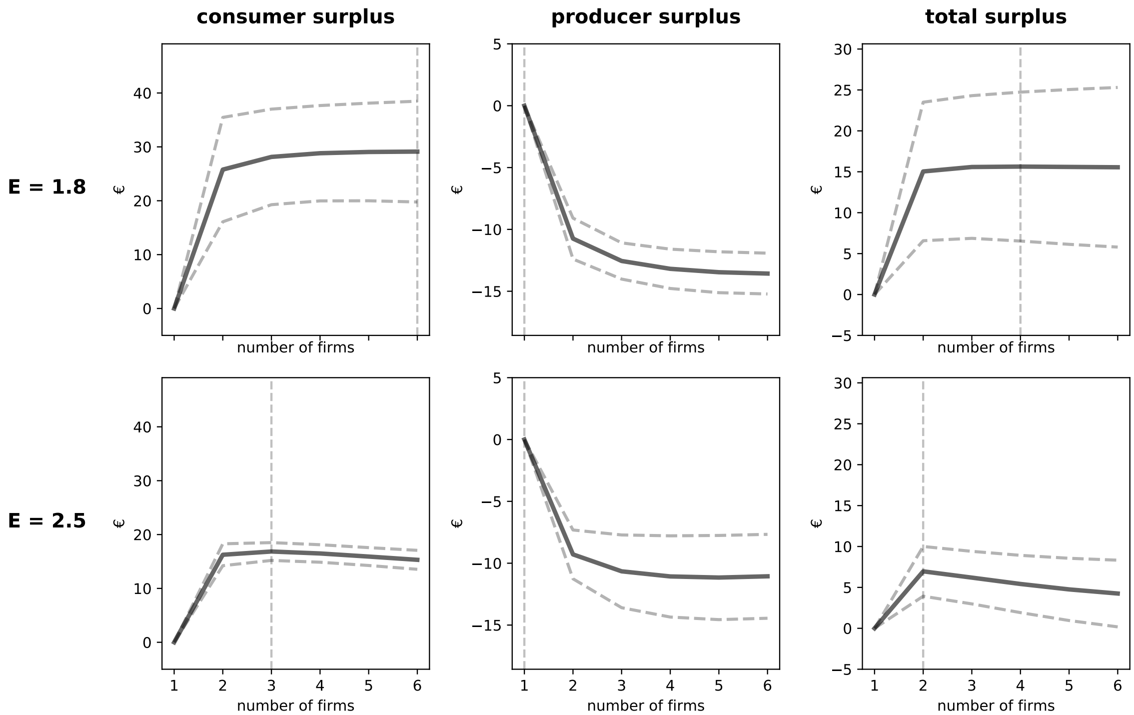
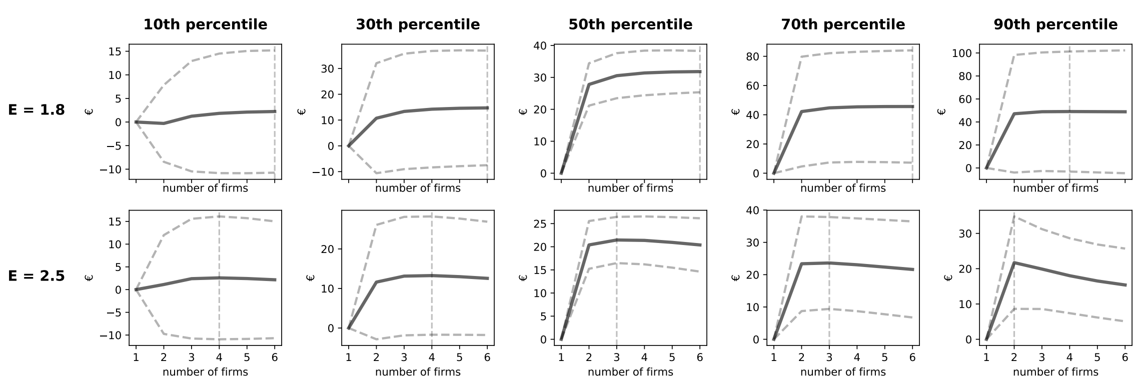


Figure 13: Counterfactual welfare by income level



7 Bandwidth Allocation

Regulators such as the FCC in the US and ANFR in France are tasked with bandwidth allocation, determining which industries are allowed to operate which frequencies of electromagnetic spectrum for which sorts of purposes. It is therefore crucial for such agencies to understand how allocating bandwidth to mobile telecommunications will affect social welfare.[23]

[23] The FCC's mandate is explicitly in "the public interest."

In this section, we quantify how allocating more bandwidth to the telecommunications industry affects firm profits, consumer welfare, and total surplus.

First, let's consider how a firm's profits change when just that firm receives a larger bandwidth allocation. The derivative

$$\frac{\partial \Pi_f \left(\mathbf{R}^* \left(b_f, \mathbf{b}_{-f}\right), \left(b_f, \mathbf{b}_{-f}\right)\right)}{\partial b_f} \tag{17}$$

captures an individual firm's willingness to pay for more bandwidth at the margin.

Next,

$$\frac{\partial \Pi_f \left(\mathbf{R}^* \left(b\mathbf{1}\right), b\mathbf{1}\right)}{\partial b} \tag{18}$$

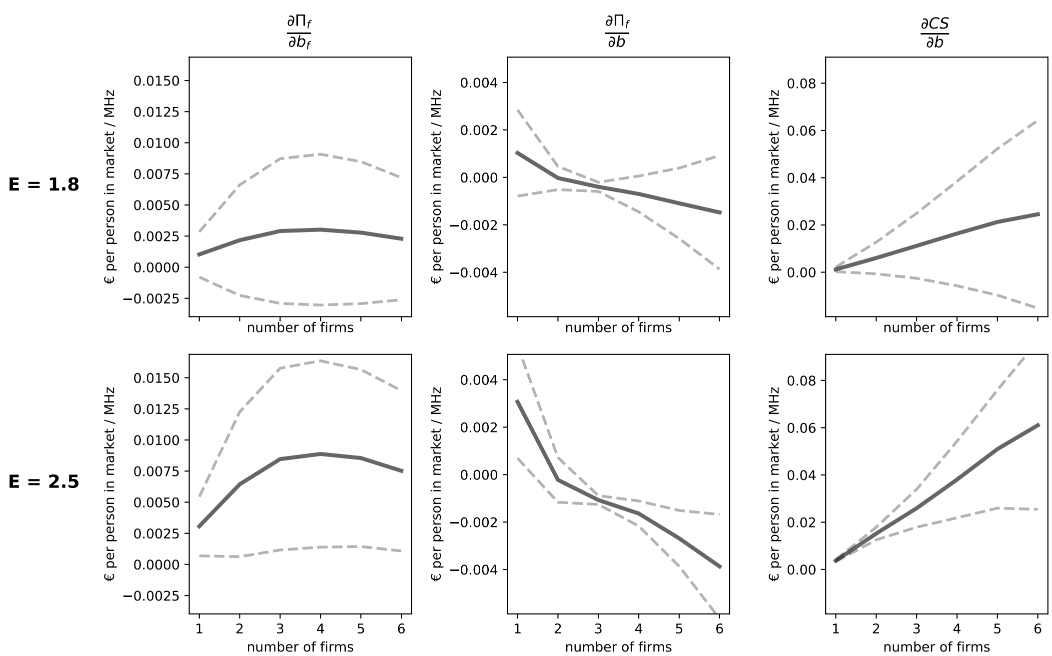
captures how the profits of an individual firm changes when all firms are allocated more bandwidth.

Finally, we can consider how consumer surplus changes as all firms are allocated more bandwidth

$$\frac{\partial CS_f \left(\mathbf{R}^* \left(b\mathbf{1}\right), b\mathbf{1}\right)}{\partial b}. \tag{19}$$

In a simple spectrum auction, the firms' bids will be related to (17). However, the regulator's spectrum decision should be based on comparing (18) and (19) to the marginal social value to allocation spectrum to other industries and purposes.

Figure 14: Bandwidth derivatives



As Figure 14 shows, the firm's willingness to pay for additional bandwidth (the left panel) is about four times less than a unit of bandwidth allocated to the industry would add to consumer surplus (the right panel). This reflects the importance of using a structural model such as ours to quantify the social value of bandwith. While evidence of firms' willingness to pay for spectrum may be relatively easy to observe, such measures should not be taken as measures of the social value of telecommunications spectrum.

References

Berry, Steven, James Levinsohn, and Ariel Pakes. 1995. "Automobile Prices in Market Equilibrium." *Econometrica*, 63(4): 841–90.

Berry, Steven T. 1994. "Estimating Discrete-Choice Models of Product Differentiation." *RAND Journal of Economics*, 25(2): 242–262.

Blaszczyszyn, Bartlomiej, Miodrag Jovanovicy, and Mohamed Kadhem Karray. 2014. "How user throughput depends on the traffic demand in large cellular networks." 611–619, IEEE.

Bourreau, Marc, Yutec Sun, and Frank Verboven. 2018. "Market Entry, Fighting Brands and Tacit Collusion: The Case of the French Mobile Telecommunications Market."

Carlton, Dennis W. 1978. "Market behavior with demand uncertainty and price inflexibility." *The American Economic Review*, 68(4): 571–587.

Cattani, Kyle, and Glen M Schmidt. 2005. "The pooling principle." *INFORMS Transactions on Education*, 5(2): 17–24.

Chenery, Hollis B. 1949. "Engineering Production Functions." *The Quarterly Journal of Economics*, 63(4): 507–531.

Cullen, Joseph, Nicolas Schutz, and Oleksandr Shcherbakov. 2016. "Welfare analysis of equilibria with and without early termination fees in the US wireless industry."

De Vany, Arthur. 1976. "Uncertainty, waiting time, and capacity utilization: A stochastic theory of product quality." *Journal of Political Economy*, 84(3): 523–541.

El Azouzi, Rachid, Eitan Altman, and Laura Wynter. 2003. "Telecommunications network equilibrium with price and quality-of-service characteristics." In *Teletraffic science and engineering.* Vol. 5, 369–378. Elsevier.

Fan, Ying, and Chenyu Yang. 2016. "Competition, product proliferation and welfare: A study of the us smartphone market."

Federal Communications Commission. 2019. "Memorandum opinion and order, declaratory ruling, and order of proposed modification." *WT Docket 18-197.*

Lhost, Jonathan, Brijesh Pinto, and David Sibley. 2015. "Effects of spectrum holdings on equilibrium in the wireless industry." *Review of Network Economics*, 14(2): 111–155.

Mulligan, James G. 1983. "The economies of massed reserves." 73(4): 725–734.

Robinson, Edward Austin Gossage. 1948. "Structure of competitive industry."

Sinkinson, Michael. 2020. "Pricing and entry incentives with exclusive contracts: Evidence from smartphones."

Sun, Patrick. 2015. "Quality competition in mobile telecommunications: Evidence from Connecticut."

Taylor, H.M., S. Karlin, and H.E. Taylor. 1998. *An Introduction to Stochastic Modeling.* Elsevier Science.

Weiergräber, Stefan. 2018. "Network effects and switching costs in the US wireless industry: Disentangling sources of consumer inertia." SFB/TR 15 Discussion Paper.

Williamson, Oliver E. 1968. "Economies as an Antitrust Defense: The Welfare Tradeoffs." *The American Economic Review*, 58(1): 18–36.

A Technical Appendix

A.1 Contraction mapping

Here we consider an alternative version of the Berry, Levinsohn and Pakes (1995) (BLP) contraction mapping. We observe market shares at the product-market level for Orange products but only aggregate firm-level market shares for the other products. We first show that if we observe market shares at the firm-market level, the problem can be rewritten in such a way that the BLP contraction mapping proof holds. We then show that if we observe some firm market shares only at the aggregate level (as is our case), the problem no longer fits into the BLP proof setup, but that the standard function used to recover mean utilities is still a contraction mapping.

A.1.1 Standard BLP contraction mapping setup

We will start with the standard BLP setting in order to introduce notation.

For the standard BLP setting, with products $j \in \mathcal{J} = \{1, \ldots, J\}$, we observe market shares ς_{jm} for each product.

We can express an individual's utility for a product as follows:

$$u_{ijm} = \delta_{jm} + \mu_{ijm} + \varepsilon_{ijm},$$

and the type-specific market shares are as follows:

$$s_{ijm} = \frac{\exp\left(\delta_{jm} + \mu_{ijm}\right)}{\sum_{j'} \exp\left(\delta_{j'm} + \mu_{ij'm}\right)}.$$

The contraction mapping takes for granted the distribution over the heterogeneous component of preferences $F\left(\mu_m\right)$. That is, given a conjectured parameter value θ, we have $F\left(\mu\right)$, and we want to use the contraction mapping to recover mean utilities δ_{jm}.

Specifically, aggregate market shares are

$$s_{jm}\left(\delta\right) = \int \frac{\exp\left(\delta_{jm} + \mu_{ijm}\right)}{\sum_{j'} \exp\left(\delta_{j'm} + \mu_{ij'm}\right)} dF\left(\mu_m\right).$$

The existence of the contraction mapping implies that there is a unique vector δ such that $s_m\left(\delta\right) = \varsigma_m$ for any observed vector of shares s_m.

A.1.2 Grouped products extension

Our setting is one in which market shares are observed only for certain groupings of products. That is, let $\mathcal{J}$ be partitioned into subsets $\mathcal{J}_f$ with $f \in \mathcal{F} = \{1, 2, \ldots F\}$. For each f, we observe only the market share ς_{ft} for all the products within $\mathcal{J}_f$.

The subsets $\mathcal{J}_f$ may include individual products (i.e., in our application each Orange product would have its own $\mathcal{J}_f$ set), or several products (i.e., each non-Orange firm has one $\mathcal{J}_f$ group that includes all that firm's products).

Providing a parametric form:

$$\delta_{jm} = \theta_1 x_{jm} + \xi_{jm},$$

where θ_1 would capture what is often referred to as "linear parameters"; i.e., parameters that can typically be estimated outside of the contraction mapping because they only shift the mean utility component δ_{jm} that the contraction mapping aims to recover. In this extension, the θ_1 parameters must be included in the contraction mapping.

We definitely cannot recover δ_{jm} (or ξ_{jm}) separately for different $j \in \mathcal{J}_f$. So, let's assume $\xi_{jm} = \xi_{fm}$ for all $j \in \mathcal{J}_f$, and for each f.

Let $\bar{x}_{fm}$ be the mean value of x_{fm} for those products within $\mathcal{J}_f$. Then, we have

$$\delta_{jm} = \theta_1 \bar{x}_{fm} + \theta_1 x^d_{jm} + \xi_{fm},$$

where $x^d_{jm} := x_{jm} - \bar{x}_{fm}$.

Now, define

$$\begin{aligned} \widetilde{\delta}_{fm} &= \theta_1 \bar{x}_{fm} + \xi_{fm}, \\ \widetilde{\mu}_{ijm} &= \theta_1 x^d_{jm} + \mu_{ijm}. \end{aligned}$$

This very nearly allows us to re-define the model in terms where we could apply the original BLP proof strategy to establish the contraction mapping. The only problem is that $\widetilde{\mu}_{ijm}$ is defined over j, where we would need it to be defined over f in order to apply the same proof strategy. Let's consider the aggregation over j to f:

$$s_{ifm}\left(\widetilde{\delta}\right) = \sum_{j \in \mathcal{J}_f} \frac{\exp\left(\widetilde{\delta}_{fm} + \widetilde{\mu}_{ijm}\right)}{\sum_{j' \in \mathcal{J}} \exp\left(\widetilde{\delta}_{f(j')m} + \widetilde{\mu}_{ij'm}\right)},$$

where $f(j')$ refers to the f associated with product j'.

Notice that

$$\sum_{j\in\mathcal{J}_f}\exp\left(\widetilde{\delta}_{fm}+\widetilde{\mu}_{ijm}\right)=\exp\left(\widetilde{\delta}_{fm}\right)\sum_{j\in\mathcal{J}_f}\exp\left(\widetilde{\mu}_{ijm}\right).$$

Now, let's define

$$\widetilde{\mu}_{ifm}\equiv\log\left(\sum_{j\in\mathcal{J}_f}\exp\left(\widetilde{\mu}_{ijm}\right)\right).$$

It follows that

$$\sum_{j\in\mathcal{J}_f}\exp\left(\widetilde{\delta}_{fm}+\widetilde{\mu}_{ijm}\right)=\exp\left(\widetilde{\delta}_{fm}+\widetilde{\mu}_{ifm}\right),$$

and therefore

$$s_{ifm}\left(\widetilde{\delta}\right)=\sum_{j\in\mathcal{J}_f}\frac{\exp\left(\widetilde{\delta}_{fm}+\widetilde{\mu}_{ifm}\right)}{\sum_{f'}\exp\left(\widetilde{\delta}_{f'm}+\widetilde{\mu}_{if'm}\right)}.$$

We can then aggregate up to market-level shares s_{fm} by integrating over the $\widetilde{\mu}_{ifm}$, and we have rewritten our extended setting in a way that allows us to apply the BLP proof strategy.

A.1.3 Grouped products extension with nested logit

In the more general random coefficients nested logit (RCNL) model of **?** (GV) used in our demand model, the market share equations as well as the formulas for $\tilde{\delta}$ and $\tilde{\mu}$ no longer hold. We can, however, construct analogous formulas that will allow us to recover firm-specific mean demands $\tilde{\delta}$.

In the RCNL model, an individual's utility for a product is as follows:

$$u_{ijm}=\delta_{jm}+\mu_{ijm}+\bar{\varepsilon}_{ijm},$$

where

$$\bar{\varepsilon}_{ijm}=\zeta_{ig(j)m}+(1-\sigma)\,\varepsilon_{ijm},$$

where $\sigma\in[0,1)$ is the nesting parameter, the function $g(j)$ returns the group identifier to which j belongs, ε_{ijm} is i.i.d. type-1 extreme value, and $\zeta_{ig(j)}$ has the distribution such that $\bar{\varepsilon}_{ijt}$ is extreme value.

Type-specific market shares are as follows:

$$s_{ijm}=\frac{\exp\left(\frac{\delta_{jm}+\mu_{ijm}}{1-\sigma}\right)}{\exp\left(\frac{I_{ig(j)}}{1-\sigma}\right)}\frac{\exp\left(I_{ig(j)}\right)}{\exp\left(I_i\right)},$$

where

$$
\begin{aligned}
I_{ig} &= (1-\sigma)\log\left(\sum_{j\in\mathcal{J}_g}\exp\left(\tfrac{\delta_{jm}+\mu_{ijm}}{1-\sigma}\right)\right),\\
I_i &= \log\left(1+\sum_{g\in\mathcal{G}}\exp\left(I_{ig}\right)\right).
\end{aligned}
$$

In this model, we must redefine $\tilde{\delta}_{fm}$ and $\tilde{\mu}_{ifm}$ to incorporate ρ. Define

$$
\begin{aligned}
\widetilde{\delta}_{fm} &= \tfrac{\theta_1\bar{x}_{fm}+\xi_{fm}}{1-\sigma},\\
\widetilde{\mu}_{ijm} &= \tfrac{\theta_1 x^d_{jm}+\mu_{ijm}}{1-\sigma}.
\end{aligned}
$$

Then note that

$$
s_{ifm}=\sum_{j\in\mathcal{J}_f}s_{ijm}=\exp\left(\tilde{\delta}_{fm}\right)\sum_{j\in\mathcal{J}_f}\frac{\exp\left(\tilde{\mu}_{ijm}\right)}{\exp\left(\frac{I_{ig(j)}}{1-\sigma}\right)}\frac{\exp\left(I_{ig(j)}\right)}{\exp\left(I_i\right)}.
$$

We will assume that products produced by the same firm belong to the same product group. Formally, $g(j)=g(j')$ for all $(j,j')\in\mathcal{J}_f^2$ for all f. This assumption implies that the product-relevant inclusive values $\{I_{ig}\}_{g\in\mathcal{G}}$ are common within firms, and we can write the product group identifier function $g(\cdot)$ as a function of the firm identifier rather than the product identifier. In our context where all contracts belong to the same group, this assumption holds. It follows that

$$
s_{ifm}=\frac{\exp\left(\tilde{\delta}_{fm}\right)}{\exp\left(\frac{I_{ig(f)}}{1-\sigma}\right)}\frac{\exp\left(I_{ig(f)}\right)}{\exp\left(I_i\right)}\sum_{j\in\mathcal{J}_f}\exp\left(\tilde{\mu}_{ijm}\right). \tag{20}
$$

Let's define

$$
\tilde{\mu}_{ifm}=\log\left(\sum_{j\in\mathcal{J}_f}\exp\left(\tilde{\mu}_{ijm}\right)\right).
$$

Then

$$
s_{ifm}=\frac{\exp\left(\tilde{\delta}_{fm}+\tilde{\mu}_{ifm}\right)}{\exp\left(\frac{I_{ig(f)}}{1-\sigma}\right)}\frac{\exp\left(I_{ig(f)}\right)}{\exp\left(I_i\right)}
$$

and

$$
I_{ig}=(1-\sigma)\log\left(\sum_{f\in\mathcal{F}_g}\exp\left(\tilde{\delta}_{fm}+\tilde{\mu}_{ifm}\right)\right),
$$

where $\mathcal{F}_g=\{f\in\mathcal{F}:g(f)=g\}$. This is now in terms of firm-specific variables.

Equation 20 is similar to that of GV, except in the numerator of the first fraction. GV note that, substituting in our notation,

$$f\left(\tilde{\delta}\right) = \tilde{\delta} + \log\left(\varsigma\right) - \log\left(s\left(\tilde{\delta}\right)\right),$$

where ς is observed shares, is a contraction mapping if[24]

$$1 - \frac{1}{s_f}\frac{\partial s_f}{\partial \tilde{\delta}_f} \geq 0.$$

Unlike in GV, this holds in our case. Explicitly,

$$\frac{\partial s_f}{\partial \tilde{\delta}_f} = \left(1 - \frac{\sigma}{1-\sigma} s_{f|g} - s_f\right) s_f,$$

and so

$$\frac{\partial s_f}{\partial \tilde{\delta}_f} = \frac{\sigma}{1-\sigma} s_{f|g} + s_f \geq 0 \quad \Leftrightarrow \quad \sigma s_{f|g} + (1-\sigma)\, s_f \geq 0.$$

This holds for all $\sigma \in [0, 1)$, and so therefore iterating on the standard BLP contraction mapping using Equation 20 will yield the unique vector $\tilde{\delta}$.

A.1.4 Market aggregation extension

The firm-aggregation provided in the previous sections still does not apply to our setting because we observe market shares only at the aggregate level for certain firms. We can still proceed by imposing the more restrictive assumption $\xi_{jm} = \xi_{f(j)}$ for all j, m. This will allow us to recover ξ_f for each f.

Analogous to the previous setup, let $\bar{x}_f$ be the mean value of x_{jm} across products $j \in \mathcal{J}_f$ and markets m. That is

$$\bar{x}_f := \frac{1}{MJ_f} \sum_m \sum_{j \in \mathcal{J}_f} x_{jm},$$

Then

$$\delta_{jm} = \theta_1 \bar{x}_{f(j)} + \theta_1 x^d_{jm} + \xi_{f(j)},$$

where we now define $x^d_{jm} := x_{jm} - \bar{x}_{f(j)}$.

Define

$$\tilde{\delta}_f := \theta_1 \bar{x}_f + \xi_f,$$
$$\tilde{\mu}_{ijm} := \theta_1 x^d_{jm} + \mu_{ijm}.$$

[24]GV also note a few other conditions that must hold, but these conditions do not differ between our modified setup and theirs, and so we therefore do not include them here.

If we sum market shares across products within a firm, we get

$$s_{ifm} = \sum_{j \in \mathcal{J}_f} \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ijm}\right)}{\sum_{j' \in \mathcal{J}} \exp\left(\tilde{\delta}_{f(j')} + \tilde{\mu}_{ij'm}\right)}.$$

This is very similar to what we had in the previous section, except that here we have $\tilde{\delta}_f$ instead of $\tilde{\delta}_{fm}$ because we are additionally averaging over markets.

Now let's average shares across markets, weighting by population,

$$\bar{s}_{if} = \sum_m w(m) \sum_{j \in \mathcal{J}_f} \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ijm}\right)}{\sum_{j' \in J} \exp\left(\tilde{\delta}_{j'm} + \tilde{\mu}_{ij'm}\right)}.$$

Note that

$$\sum_{j \in \mathcal{J}_f} \exp\left(\tilde{\delta}_f + \tilde{\mu}_{ijm}\right) = \exp\left(\tilde{\delta}_f\right) \sum_{j \in \mathcal{J}_f} \exp\left(\tilde{\mu}_{ijm}\right).$$

Now define

$$\tilde{\mu}_{ifm} := \log\left(\sum_{j \in \mathcal{J}_f} \exp\left(\tilde{\mu}_{ijm}\right)\right).$$

Then

$$\sum_{j \in \mathcal{J}_f} \exp\left(\tilde{\delta}_f + \tilde{\mu}_{ijm}\right) = \exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right),$$

so

$$\bar{s}_{if}(\tilde{\delta}) = \sum_m w(m) \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right)}{\sum_{f'} \exp\left(\tilde{\delta}'_f + \tilde{\mu}_{if'm}\right)}.$$

We can aggregate up to aggregate firm shares $\bar{s}_f$ by integrating over $\tilde{\mu}_{ifm}$, i.e.,

$$\bar{s}_f = \int \sum_m w(m) \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right)}{1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)} dF(\tilde{\mu}_{ifm}), \tag{21}$$

where we have normalized the utility of the outside option to 0.

We next need to show that the BLP contraction mapping holds in this case. Consider the following BLP-style fixed point that gives the mean utilities $\tilde{\delta}$ that set theoretical shares to observed market shares $\bar{\varsigma}$:

$$\delta = \underbrace{\delta + \log\left(\bar{\varsigma}\right) - \log\left(\bar{s}\left(\tilde{\delta}; \theta\right)\right)}_{=: f(\tilde{\delta})}. \tag{22}$$

The proof that $f(\cdot)$ is a contraction mapping closely follows that of BLP. In short, if we recognize that averaging across markets is simply integrating over $\tilde{\mu}_{ijm}$ in another dimension, we can rewrite Equation 21 as follows:

$$\bar{s}_f = \int \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right)}{1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)} dG(\tilde{\mu}_{ifm}).$$

The BLP proof depends on the integrand, not the integral itself, so the proof holds for any arbitrary distribution in an arbitrary number of dimensions. The full proof is provided below.

Berry, Levinsohn and Pakes (1995) show that $f : \mathbb{R}^K \to \mathbb{R}^K$ in the metric space $(\mathbb{R}, d)$, where d is the sup-norm, is a contraction mapping if

1. for all $x \in \mathbb{R}^K$, $f(x)$ is continuously differentiable with, for all j and k,

$$\frac{\partial f_j(x)}{\partial x_k} \geq 0$$

and

$$\sum_{k=1}^{K} \frac{\partial f_j(x)}{\partial x_k} < 1;$$

2. $\min_j \inf_x f(x) > -\infty$; and

3. there exists a value $\bar{x}$ with the property that if for any j, $x_j \geq \bar{x}$, then for some k, $f_k(x) < x_k$.

We will now show that $f(\cdot)$ is a contraction mapping.

Proof. Beginning with (1), it is clear from the definition of $\bar{s}_f(\tilde{\delta})$ that $f(\tilde{\delta})$ is continuously differentiable. The derivatives of $f(\tilde{\delta})$ are as follows:

$$\frac{\partial f_j}{\partial \tilde{\delta}_j}(\tilde{\delta}) = 1 - \frac{1}{\bar{s}_j(\tilde{\delta})} \frac{\partial \bar{s}_j}{\partial \tilde{\delta}_j}(\tilde{\delta}),$$
$$\frac{\partial f_j}{\partial \tilde{\delta}_k}(\tilde{\delta}) = -\frac{1}{\bar{s}_j(\tilde{\delta})} \frac{\partial \bar{s}_j}{\partial \tilde{\delta}_k}(\tilde{\delta}) \quad \text{for all } k \neq j,$$

where

$$\frac{\partial \bar{s}_f}{\partial \tilde{\delta}_f}(\tilde{\delta}) = \int \sum_m w(m) \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right)\left[1 + \sum_{f' \neq f} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)\right]}{\left[1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)\right]^2} dF(\tilde{\mu}_{ifm}),$$

$$\frac{\partial \bar{s}_f}{\partial \tilde{\delta}_{f'}}(\tilde{\delta}) = -\int \sum_m w(m) \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}\right)\exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)}{\left[1 + \sum_{f''} \exp\left(\tilde{\delta}_{f''} + \tilde{\mu}_{if''m}\right)\right]^2} dF(\tilde{\mu}_{ifm}) \quad \text{for all } g \neq f.$$

Note that $\frac{\partial \bar{s}_f}{\partial \tilde{\delta}_g}(\tilde{\delta})$ is negative for all $\tilde{\delta} \in \mathbb{R}^K$, so $\frac{\partial f_j}{\partial \tilde{\delta}_k}(\tilde{\delta})$ is positive since $\bar{s}(\tilde{\delta}) \gg 0$. Next note that $\frac{\partial f_j}{\partial \tilde{\delta}_j}(\tilde{\delta})$ is positive if and only if $\frac{1}{\bar{s}_j(\tilde{\delta})}\frac{\partial \bar{s}_j}{\partial \tilde{\delta}_j}(\tilde{\delta}) < 1$. Comparing the integrands of $\frac{\partial \bar{s}_j}{\partial \tilde{\delta}_j}$ and $\bar{s}_j$, since

$$1 + \sum_{f' \neq f} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if't}\right) < 1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if't}\right),$$

we have that $\frac{\partial \bar{s}_j}{\partial \tilde{\delta}_j}(\tilde{\delta}) > \bar{s}_j(\tilde{\delta})$ for all $\tilde{\delta} \in \mathbb{R}^K$, and therefore $\frac{\partial f_j}{\partial \tilde{\delta}_k}(\tilde{\delta})$ is positive. Lastly, we need to show that $\sum_{k=1}^K \frac{\partial f_j(x)}{\partial x_k} < 1$:

$$\begin{aligned}
\sum_{k=1}^K \frac{\partial f_j(x)}{\partial x_k} &= 1 - \frac{1}{\bar{s}_j(\tilde{\delta})} \sum_k \frac{\partial \bar{s}_j}{\partial \tilde{\delta}_k}(\tilde{\delta}) \\
&= 1 - \frac{1}{\bar{s}_j(\tilde{\delta})} \int \sum_m w(m) \frac{\exp\left(\tilde{\delta}_j + \tilde{\mu}_{ijm}\right)}{\left[1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)\right]^2} dF(\tilde{\mu}_{ifm}) \\
&< 1.
\end{aligned}$$

Next we must show that show that $f(\cdot)$ satisfies Assumption (2). Note we can rewrite $f(\tilde{\delta})$ as

$$f(\tilde{\delta}) = \log(\bar{\varsigma}) - \log\left(\int \sum_m w(m) \frac{\exp(\tilde{\mu})}{1 + \sum_{f'} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}\right)} dF(\tilde{\mu})\right).$$

We showed earlier that f is increasing in $\tilde{\delta}_k$ for all k. Therefore, let $\tilde{\delta}_k \to -\infty$ for all k. Then

$$\underline{f} = \log(\tilde{\bar{s}}) - \log\left(\int \sum_t w(t) \exp(\tilde{\mu})\, dF(\tilde{\mu})\right).$$

Note $\underline{f} \gg -\infty$, so we have satisfied (2).

Finally, we turn to (3). Following Berry (1994), consider a firm f. Set $\tilde{\delta}_{f'} = -\infty$ for all $f' \neq f$. Define $\bar{\tilde{\delta}}_f$ as the value that sets the market share of the outside good $\bar{s}_0(\tilde{\delta}) = \bar{\varsigma}_0$, the observed market share. How do we know that such a $\bar{\tilde{\delta}}_f$ exists? Note that $g(x) = \frac{1}{1+\exp(x+\mu)}$ maps from $\mathbb{R}$ into $(0,1)$ for any $\mu \in \mathbb{R}$, and $\bar{\varsigma}_0 \in (0,1)$. Define $\bar{\tilde{\delta}} > \max_f \bar{\tilde{\delta}}_f$. Consider $\tilde{\delta} \in \mathbb{R}^K$

such that for some f, $\tilde{\delta}_f > \bar{\tilde{\delta}}$. From our definition of $\bar{\tilde{\delta}}$, this will yield $\bar{s}_0(\tilde{\delta}) < \bar{\varsigma}_0$, and therefore $\sum_f \bar{s}_f(\tilde{\delta}) > \sum_f \tilde{\bar{s}}_f$. In order for that inequality to hold, there must be some f such that $\bar{s}_f(\tilde{\delta}) > \bar{\varsigma}_f$. Then $f_f(\tilde{\delta}) < \tilde{\delta}_f$, satisfying (3). □

We therefore can iterate on the following

$$\tilde{\delta}_f^{(k)}(\theta_1, \theta_2) = \tilde{\delta}_f^{(k-1)}(\theta_1, \theta_2) + \log\left(\bar{\varsigma}_f\right) - \log\left(\bar{s}_f\left(\tilde{\delta}_f^{(k-1)}; \theta_1, \theta_2\right)\right) \tag{23}$$

to obtain (approximately) the unique fixed point $\tilde{\delta}$ that rationalizes the observed shares $\tilde{\varsigma}$.

A.1.5 Implementation

The setup outlined in the above section is more restrictive than is necessary given our data. We observe product-level market shares for every market for Orange products. We can therefore allow ξ_{jm} to differ by product and market for all $j \in \mathcal{J}_O$, where O denotes Orange. This setup is isomorphic to one in which we treat each $(j, m)_{j \in \mathcal{J}_O, m \in \mathcal{M}}$ as a separate firm, so long as we ensure that the set of "firms" differs across markets (because (j, m) will not be available in $(j, m'))$.

To be more explicit, let $f(j, m)$ give a unique identity for each $(j, m)_{j \in \mathcal{J}_O, m \in \mathcal{M}}$, but for a non-Orange firm $f \in \mathcal{F}_{-O}$, $f(j, m) = f(j', m')$ for all $j, j' \in \mathcal{J}_f$ for all m, m'. Now let's denote the set of "firms" $\mathcal{F} \equiv \{f(j, m) : j \in \mathcal{J}, m \in \mathcal{M}\}$.

In our predicted market share equation, we must be sure to include in the terms corresponding to market m only the firms $f \in \mathcal{F}$ such that there exists $j \in \mathcal{J}$ such that $f(j, m) = f$. An Orange product-market f corresponding to market m will only show up in the denominator of equation (21) in the mth term. This also holds for the numerator so that $\bar{s}_f$ for an Orange f is an average across markets only in a vacuous sense since there will only be one term.

To be explicit, in this case we will have "firm"-specific market weights:

$$w_f(m) = \begin{cases} w_f(m) & \text{if } \exists j \in \mathcal{J}_{-O} : f(j, m) = f \\ 1 & \text{if } \exists j \in \mathcal{J}_O : f(j, m) = f \\ 0 & \text{otherwise.} \end{cases}$$

We can rewrite equation (21) as

$$\bar{s}_f(\tilde{\delta}; \theta) = \int \sum_m w_f(m) \frac{\exp\left(\tilde{\delta}_f + \tilde{\mu}_{ifm}(\theta_1)\right)}{\sum_{f' \in \mathcal{F}(f)} \exp\left(\tilde{\delta}_{f'} + \tilde{\mu}_{if'm}(\theta_1)\right)} dF(\tilde{\mu}(\theta_1); \theta_2), \tag{24}$$

where $\mathcal{F}(f) \equiv \{f' \in \mathcal{F} : M(f) \cap M(f') \neq \emptyset\}$, where $M(f) = \{m \in \mathcal{M} : \exists j \in$

$\mathcal{J}$ such that $f(j,m) = f\}$.[25]

A.2 Expectation Expressions

As demonstrated in Section 4,

$$x_j^*\left(Q_{f(j)};\vartheta_i,\theta_i\right) = \begin{cases} 0 & \text{if } \vartheta_i \leq \frac{\theta_c}{Q_{f(j)}} \\ \frac{\vartheta_i Q_{f(j)}}{\theta_c} - 1 & \text{if } \frac{\theta_c}{Q_{f(j)}} \leq \vartheta_i < \frac{\theta_c}{Q_{f(j)}}\left(\bar{x}_j + 1\right) \\ \bar{x}_j & \text{if } \frac{\theta_c}{Q_{f(j)}}\left(\bar{x}_j+1\right) \leq \vartheta_i < \frac{\theta_c}{Q^L}\left(\bar{x}_j+1\right) \\ \frac{\vartheta_i Q^L}{\theta_c} - 1 & \text{if } \vartheta_i \geq \frac{\theta_c}{Q^L}\left(\bar{x}_j+1\right). \end{cases}$$

We will use this to derive analytic expressions for expected utility from data consumption and predicted average data consumption, integrating over ϑ_i, which is distributed

$$\vartheta_i \sim Exponential\left(\theta_{di}\right),$$

meaning that the pdf of ϑ_i for an agent of type i is

$$f_i\left(\vartheta_i\right) = \begin{cases} \theta_{di} e^{-\theta_{di}\vartheta_i} & \text{if } \vartheta_i \geq 0 \\ 0 & \text{if } \vartheta_i < 0. \end{cases}$$

A.2.1 Expected utility from data consumption

We assume that consumers select plans knowing only their θ_{di} type; their ϑ_i value is realized after choosing a plan.

Their expected value of data consumption $\mathbb{E}\left[u_j\left(x^*\left(Q;\theta_i\right),Q\right)\right]$ is

$$\begin{aligned} \mathbb{E}\left[u_j\left(x^*\left(Q;\vartheta_i,\theta_i\right),Q;\vartheta_i,\theta_i\right)\right] = & \int_{\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\left(\vartheta_i \log\left(\frac{\vartheta_i Q}{\theta_c}\right) - \vartheta_i + \frac{\theta_c}{Q}\right) dF_i\left(\vartheta_i\right) \\ & + \int_{\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\left(\vartheta_i \log\left(1+\bar{x}_j\right) - \theta_c\frac{\bar{x}_j}{Q}\right) dF_i\left(\vartheta_i\right) \\ & + \int_{\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\left(\vartheta_i \log\left(\frac{\vartheta_i Q^L}{\theta_c}\right) - \theta_c\left[\frac{\bar{x}_j}{Q} + \frac{\vartheta_i Q^L/\theta_c - 1 - \bar{x}_j}{Q^L}\right]\right) dF_i\left(\vartheta_i\right), \end{aligned}$$

[25] Note that in this setup, we have to make a few adjustments to the contraction mapping proof. First note that $\frac{\partial f_k(x)}{\partial \tilde{\delta}_j}(\tilde{\delta})$ might be zero not positive, but that is still permissible under the sufficient conditions for the contraction mapping. The partial derivatives will still sum to less than one. Thus, we still satisfy (1). This setup doesn't change the argument for (2). Finally, we must note the following for condition (3). Consider $\bar{\tilde{\delta}}_f$. In this setup, if f corresponds to an Orange (j,m), then $\tilde{\delta}_f$ shows up in only one of the market-specific terms. The function still maps from $\mathbb{R}$ to $(0,1)$, so we can still find a $\bar{\tilde{\delta}}_f$. The rest of the argument holds, so therefore (3) does as well, and we have a contraction mapping even in this setup with product-market-specific demand shocks for Orange and firm-specific demand shocks for all other firms.

where

$$
\begin{aligned}
&\textstyle\int_{\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\left(\vartheta_i\log\left(\frac{\vartheta_i Q}{\theta_c}\right)-\vartheta_i+\frac{\theta_c}{Q}\right)dF_i\left(\vartheta_i\right)\\
&\quad=\tfrac{\exp(-\theta_{di}\vartheta_i)}{\theta_{di}Q}\left(-\left(\theta_{di}Q\vartheta_i+Q\right)\log\left(\tfrac{Q\vartheta_i}{\theta_c}\right)-\theta_c\theta_{di}+Q\exp\left(\theta_{di}\vartheta_i\right)\mathrm{Ei}\left(-\theta_{di}\vartheta_i\right)+\theta_{di}Q\vartheta_i\right)\Big|_{\vartheta_i=\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\\
&\quad=\tfrac{\exp(-\theta_{di}\theta_c(\bar{x}_j+1)/Q)}{\theta_{di}Q}\left(-\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)+Q\right)\log\left(\bar{x}_j+1\right)+Q\exp\left(\tfrac{\theta_{di}\theta_c(\bar{x}_j+1)}{Q}\right)\mathrm{Ei}\left(\tfrac{-\theta_{di}\theta_c(\bar{x}_j+1)}{Q}\right)+\theta_{di}\theta_c\bar{x}_j\right)\\
&\qquad-\tfrac{\exp(-\theta_{di}\theta_c/Q)}{\theta_{di}Q}Q\exp\left(\theta_{di}\theta_c/Q\right)\mathrm{Ei}\left(-\theta_{di}\theta_c/Q\right)\\
&\textstyle\int_{\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\left(\vartheta_i\log\left(1+\bar{x}_j\right)-\theta_c\frac{\bar{x}_j}{Q}\right)dF_i\left(\vartheta_i\right)\\
&\quad=\tfrac{\exp(-\theta_{di}\vartheta_i)}{\theta_{di}Q}\left(\theta_c\theta_{di}\bar{x}_j-\log\left(\bar{x}_j+1\right)\left(\theta_{di}Q\vartheta_i+Q\right)\right)\Big|_{\vartheta_i=\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\\
&\quad=\tfrac{\exp\left(-\theta_{di}\theta_c(\bar{x}_j+1)/Q^L\right)}{\theta_{di}Q}\left(\theta_c\theta_{di}\bar{x}_j-\log\left(\bar{x}_j+1\right)\left(\tfrac{\theta_{di}Q\theta_c(\bar{x}_j+1)}{Q^L}+Q\right)\right)\\
&\qquad-\tfrac{\exp(-\theta_{di}\theta_c(\bar{x}_j+1)/Q)}{\theta_{di}Q}\left(\theta_c\theta_{di}\bar{x}_j-\log\left(\bar{x}_j+1\right)\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)+Q\right)\right)\\
&\textstyle\int_{\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\left(\vartheta_i\log\left(\frac{\vartheta_i Q^L}{\theta_c}\right)-\theta_c\left[\frac{\bar{x}_j}{Q}+\frac{\vartheta_i Q^L/\theta_c-1-\bar{x}_j}{Q^L}\right]\right)dF_i\left(\vartheta_i\right)\\
&\quad=\tfrac{\exp(-\theta_{di}\vartheta_i)}{\theta_{di}QQ^L}\Big(-\theta_c\theta_{di}\left(-Q^L\bar{x}_j+Q^2+Q\bar{x}_j\right)-QQ^L\left(\theta_{di}\vartheta_i+1\right)\log\left(\tfrac{Q^L\vartheta_i}{\theta_c}\right)\\
&\qquad\qquad+QQ^L\exp\left(\theta_{di}\vartheta_i\right)\mathrm{Ei}\left(-\theta_{di}\vartheta_i\right)+\theta_{di}QQ^L\vartheta_i\Big)\Big|_{\vartheta_i=\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\\
&\quad=-\tfrac{\exp\left(-\theta_{di}\theta_c(\bar{x}_j+1)/Q^L\right)}{\theta_{di}QQ^L}\Big(-\theta_c\theta_{di}\left(-Q^L\bar{x}_j+Q^2+Q\bar{x}_j\right)-QQ^L\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q^L+1\right)\log\left(\bar{x}_j+1\right)\\
&\qquad\qquad+QQ^L\exp\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q^L\right)\mathrm{Ei}\left(-\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q^L\right)+\theta_{di}Q\theta_c\left(\bar{x}_j+1\right)\Big)
\end{aligned}
$$

where $\mathrm{Ei}(\cdot)$ is the exponential integral defined as $\mathrm{Ei}(x)=-\int_{-x}^{\infty}\frac{\exp(-t)}{t}dt$.

A.2.2 Mean data consumption

Mean data consumption is

$$
\begin{aligned}
\mathbb{E}\left[x_j^*\left(Q;\vartheta_i,\theta_i\right)\right]=&\int_{\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\left[\frac{\vartheta_i Q}{\theta_c}-1\right]dF_i\left(\vartheta_i\right)\\
&+\int_{\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\bar{x}_j dF_i\left(\vartheta_i\right)\\
&+\int_{\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\left[\frac{\vartheta_i Q^L}{\theta_c}-1\right]dF_i\left(\vartheta_i\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\textstyle\int_{\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\left[\frac{\vartheta_i Q}{\theta_c}-1\right]dF_i\left(\vartheta_i\right)&=\tfrac{\exp(-\theta_{di}\vartheta_i)}{\theta_c\theta_{di}}\left(\theta_c\theta_{di}-Q\left(\theta_{di}\vartheta_i+1\right)\right)\Big|_{\vartheta_i=\theta_c/Q}^{\theta_c(\bar{x}_j+1)/Q}\\
&=\tfrac{\exp(-\theta_{di}\theta_c(\bar{x}_j+1)/Q)}{\theta_c\theta_{di}}\left(\theta_c\theta_{di}-Q\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q+1\right)\right)\\
&\quad-\tfrac{\exp(-\theta_{di}\theta_c/Q)}{\theta_c\theta_{di}}\left(\theta_c\theta_{di}-Q\left(\theta_{di}\theta_c/Q+1\right)\right)\\
\textstyle\int_{\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\bar{x}_j dF_i\left(\vartheta_i\right)&=-\bar{x}_j\exp\left(-\theta_{di}\vartheta_i\right)\Big|_{\vartheta_i=\theta_c(\bar{x}_j+1)/Q}^{\theta_c(\bar{x}_j+1)/Q^L}\\
&=-\bar{x}_j\exp\left(-\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q^L\right)\\
&\quad+\bar{x}_j\exp\left(-\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q\right)\\
\textstyle\int_{\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\left[\frac{\vartheta_i Q^L}{\theta_c}-1\right]dF_i\left(\vartheta_i\right)&=\tfrac{\exp(-\theta_{di}\vartheta_i)}{\theta_c\theta_{di}}\left(\theta_c\theta_{di}-Q^L\left(\theta_{di}\vartheta_i+1\right)\right)\Big|_{\vartheta_i=\theta_c(\bar{x}_j+1)/Q^L}^{\infty}\\
&=-\tfrac{\exp\left(-\theta_{di}\theta_c(\bar{x}_j+1)/Q^L\right)}{\theta_c\theta_{di}}\left(\theta_c\theta_{di}-Q^L\left(\theta_{di}\theta_c\left(\bar{x}_j+1\right)/Q^L+1\right)\right).
\end{aligned}
$$

A.3 Demand Estimation Details

The moments listed in Section 5.1.3 are imposed only for Orange products. The MVNO demand shock ξ_{MVNO} is normalized to 0, so the moments presented in Section 5.1.3 are not correctly specified if Orange demand shocks differ from the MVNO demand shock. Therefore, we add an Orange dummy variable O_j defined as follows

$$O_j = \begin{cases} 1 & \text{if } f(j) = \text{Orange} \\ 0 & \text{otherwise,} \end{cases}$$

and O_j enters utility additively so that Equation 11 becomes

$$v(j, x, m; \theta_i, \vartheta_i, \varepsilon_i) \equiv u_j\left(x, Q_{m,f(j)}; \vartheta_i, \theta_i\right) + \theta_v v_j - \theta_{pi} p_j + \theta_O O_j + \xi_{jm} + \varepsilon_{ij}.$$

The inclusion of the term $\theta_O O_j$ allows Orange products to differ in a systematic way from the products offered by other firms, restoring the validity of moments of the form presented in Section 5.1.3. To identify the parameter θ_O, we impose the following additional moment

$$\mathbb{E}\left[\xi_{jm}(\theta)\, O_j\right] = 0,$$

which is equivalent to imposing

$$\mathbb{E}\left[\xi_{jm}(\theta)\right] = 0,$$

since the moments are only imposed for Orange products.

In addition to the parameterization in Equation 15, we make an additional transformation to ensure the correct sign of θ_c. Let θ^m denote the model θ, presented in Section 4, and let θ^p denote the parameter that we estimate. Equation 25 below provides the mapping between θ^p and θ^m.

$$\theta_c^m = \exp\left(\theta_c^p\right). \tag{25}$$

Incomes are in units of 10,000 €. Data limits are in GB and quality measures are in GBps.[26]

B Supplementary Results

B.1 Demand Estimation Results

Demand parameter estimates are listed in table 3 for a range of imputed price elasticities and nesting parameters.

To interpret the results above, the following tables convert the parameter estimates into willingness to pay for certain contract characteristics across income percentiles. Each percentile corresponds to the

[26] Note that quality measures are in Giga*bytes* per second (GBps), not Giga*bits* per second (Gbps). This conversion is needed so that the second term in Equation 13 has the interpretation of seconds spent downloading data.

Table 3: Demand Parameter Estimates

Elasticity	**Nesting Parameter**	$\hat{\theta}_{p0}$	$\hat{\theta}_{pz}$	$\hat{\theta}_{v}$	$\hat{\theta}_{O}$	$\hat{\theta}_{d0}$	$\hat{\theta}_{dz}$	$\hat{\theta}_{c}$
−4.0	0.0	−0.119	−0.669	2.098	5.161	−1.759	0.315	−6.313
		(0.461)	(0.195)	(0.093)	(0.825)	(0.112)	(0.067)	(0.097)
	0.2	−0.341	−0.67	1.684	4.453	−1.532	0.314	−6.538
		(0.461)	(0.197)	(0.077)	(0.709)	(0.106)	(0.066)	(0.107)
	0.4	−0.627	−0.672	1.269	3.782	−1.239	0.312	−6.828
		(0.461)	(0.2)	(0.069)	(0.583)	(0.098)	(0.064)	(0.118)
	0.6	−1.031	−0.674	0.853	3.172	−0.826	0.31	−7.237
		(0.471)	(0.21)	(0.071)	(0.44)	(0.091)	(0.062)	(0.133)
	0.8	−1.721	−0.679	0.431	2.658	−0.124	0.308	−7.932
		(0.585)	(0.279)	(0.086)	(0.292)	(0.087)	(0.063)	(0.174)
	0.9	−1.22	−1.302	0.256	2.737	−0.064	0.707	−9.124
		(0.729)	(0.284)	(0.034)	(0.334)	(0.131)	(0.078)	(0.044)
−2.5	0.0	−0.474	−0.804	1.562	3.447	−0.787	0.324	−7.283
		(0.532)	(0.248)	(0.046)	(0.587)	(0.096)	(0.067)	(0.197)
	0.2	−0.69	−0.809	1.256	3.117	−0.557	0.322	−7.511
		(0.538)	(0.254)	(0.045)	(0.518)	(0.106)	(0.066)	(0.216)
	0.4	−0.969	−0.815	0.949	2.819	−0.26	0.32	−7.804
		(0.548)	(0.264)	(0.05)	(0.437)	(0.122)	(0.063)	(0.236)
	0.6	−1.364	−0.822	0.639	2.566	0.156	0.318	−8.215
		(0.59)	(0.291)	(0.062)	(0.346)	(0.143)	(0.061)	(0.26)
	0.8	−2.045	−0.833	0.323	2.381	0.856	0.317	−8.912
		(0.822)	(0.426)	(0.083)	(0.255)	(0.169)	(0.069)	(0.332)
	0.9	−2.173	−1.142	0.182	2.437	1.738	0.335	−9.868
		(1.203)	(0.566)	(0.068)	(0.269)	(0.09)	(0.068)	(0.165)
−1.8	0.0	−0.649	−0.949	1.324	2.743	0.514	0.326	−8.578
		(0.641)	(0.323)	(0.031)	(0.506)	(0.497)	(0.062)	(0.633)
	0.2	−0.859	−0.958	1.066	2.57	0.76	0.325	−8.821
		(0.653)	(0.333)	(0.034)	(0.454)	(0.56)	(0.061)	(0.692)
	0.4	−1.131	−0.969	0.806	2.425	1.074	0.323	−9.132
		(0.673)	(0.349)	(0.042)	(0.39)	(0.624)	(0.059)	(0.751)
	0.6	−1.519	−0.983	0.542	2.318	1.505	0.322	−9.56
		(0.739)	(0.391)	(0.056)	(0.318)	(0.689)	(0.059)	(0.821)
	0.8	−2.193	−1.0	0.274	2.267	2.217	0.322	−10.271
		(1.057)	(0.58)	(0.078)	(0.243)	(0.816)	(0.07)	(0.996)
	0.9	−1.694	−0.86	0.247	1.583	1.202	0.352	−9.722
		(0.407)	(0.166)	(0.04)	(0.2)	(0.082)	(0.046)	(0.044)

Table 4: Willingness to pay to go from 1 000 MB data plan to 4 000 MB plan

Elasticity	**Nesting Parameter**	10th %ile	30th %ile	50th %ile	70th %ile	90th %ile
-4.0	0.0	4.68 €	5.26 €	5.74 €	6.27 €	7.20 €
	0.2	4.66 €	5.25 €	5.72 €	6.26 €	7.21 €
	0.4	4.63 €	5.22 €	5.70 €	6.25 €	7.22 €
	0.6	4.59 €	5.19 €	5.68 €	6.24 €	7.25 €
	0.8	4.53 €	5.15 €	5.65 €	6.23 €	7.30 €
	0.9	3.53 €	4.51 €	5.38 €	6.39 €	7.91 €
-2.5	0.0	2.63 €	3.15 €	3.61 €	4.16 €	5.31 €
	0.2	2.60 €	3.12 €	3.58 €	4.15 €	5.33 €
	0.4	2.56 €	3.09 €	3.55 €	4.13 €	5.36 €
	0.6	2.51 €	3.05 €	3.53 €	4.12 €	5.40 €
	0.8	2.47 €	3.02 €	3.51 €	4.12 €	5.47 €
	0.9	1.32 €	1.87 €	2.44 €	3.28 €	5.64 €
-1.8	0.0	0.90 €	1.15 €	1.40 €	1.72 €	2.49 €
	0.2	0.87 €	1.12 €	1.36 €	1.69 €	2.47 €
	0.4	0.83 €	1.09 €	1.33 €	1.66 €	2.46 €
	0.6	0.80 €	1.05 €	1.30 €	1.63 €	2.45 €
	0.8	0.78 €	1.03 €	1.28 €	1.61 €	2.47 €
	0.9	1.40 €	1.74 €	2.05 €	2.44 €	3.36 €

estimated willingness to pay for an individual with an income that is the average across all markets of that percentile.[27] Table 4 presents willingness to pay for an increase from a 1 000 MB plan to a 4 000 MB plan, with quality equal to the median data speed observed in our data (24.3 Mbps). Table 5 presents willingness to pay for a unlimited voice allowance. Finally, Table 6 presents willingness to pay for an increase in data speeds from 10 Mbps to 20 Mbps on a 10 000 MB plan.

B.2 Cost Estimation Results

Tables 7 and 8 present per-user and per-tower cost estimates, respectively, for our imputed parameters. Table 7 presents the estimated costs per-user, averaged across products with similar data limits. Table 8 presents estimated costs per-tower for each MNO, averaged across markets.

[27] Specifically, the 10th percentile is 3 759 €, the 30th percentile is 8 705 €, the 50th percentile is 13 015 €, the 70th percentile is 18 101 €, and the 90th percentile is 28 096 €.

Table 5: Willingness to pay for unlimited voice allowance

Elasticity	**Nesting Parameter**	10th %ile	30th %ile	50th %ile	70th %ile	90th %ile
−4.0	0.0	2.98 €	4.21 €	5.53 €	7.54 €	13.87 €
	0.2	2.99 €	4.21 €	5.54 €	7.56 €	13.92 €
	0.4	3.00 €	4.23 €	5.57 €	7.60 €	14.02 €
	0.6	3.02 €	4.27 €	5.62 €	7.68 €	14.19 €
	0.8	3.05 €	4.32 €	5.70 €	7.81 €	14.49 €
	0.9	1.36 €	2.65 €	4.51 €	8.25 €	27.01 €
−2.5	0.0	3.32 €	5.01 €	6.96 €	10.10 €	21.01 €
	0.2	3.32 €	5.02 €	6.98 €	10.16 €	21.23 €
	0.4	3.32 €	5.04 €	7.03 €	10.25 €	21.54 €
	0.6	3.33 €	5.07 €	7.09 €	10.39 €	21.98 €
	0.8	3.34 €	5.12 €	7.19 €	10.58 €	22.59 €
	0.9	2.38 €	4.27 €	6.80 €	11.54 €	32.68 €
−1.8	0.0	3.52 €	5.73 €	8.44 €	13.10 €	31.10 €
	0.2	3.51 €	5.74 €	8.48 €	13.22 €	31.64 €
	0.4	3.50 €	5.75 €	8.54 €	13.38 €	32.36 €
	0.6	3.49 €	5.77 €	8.62 €	13.60 €	33.33 €
	0.8	3.48 €	5.81 €	8.74 €	13.90 €	34.58 €
	0.9	1.82 €	2.82 €	4.01 €	5.97 €	13.08 €

Table 6: Willingness to pay for increase from 10 Mbps to 20 Mbps

Elasticity	**Nesting Parameter**	10th %ile	30th %ile	50th %ile	70th %ile	90th %ile
−4.0	0.0	3.58 €	4.34 €	5.00 €	5.81 €	7.50 €
	0.2	3.57 €	4.33 €	4.99 €	5.80 €	7.50 €
	0.4	3.55 €	4.31 €	4.98 €	5.79 €	7.51 €
	0.6	3.52 €	4.29 €	4.96 €	5.79 €	7.54 €
	0.8	3.49 €	4.26 €	4.95 €	5.79 €	7.59 €
	0.9	1.49 €	2.44 €	3.47 €	4.89 €	7.98 €
−2.5	0.0	2.02 €	2.61 €	3.16 €	3.89 €	5.59 €
	0.2	2.00 €	2.59 €	3.14 €	3.87 €	5.60 €
	0.4	1.97 €	2.56 €	3.12 €	3.86 €	5.63 €
	0.6	1.94 €	2.53 €	3.10 €	3.85 €	5.67 €
	0.8	1.91 €	2.51 €	3.08 €	3.85 €	5.74 €
	0.9	0.98 €	1.51 €	2.10 €	3.02 €	5.86 €
−1.8	0.0	0.69 €	0.96 €	1.23 €	1.61 €	2.63 €
	0.2	0.67 €	0.93 €	1.20 €	1.58 €	2.61 €
	0.4	0.64 €	0.90 €	1.17 €	1.55 €	2.60 €
	0.6	0.62 €	0.88 €	1.14 €	1.53 €	2.59 €
	0.8	0.60 €	0.86 €	1.13 €	1.52 €	2.61 €
	0.9	0.85 €	1.17 €	1.48 €	1.92 €	3.03 €

Table 7: Per-user cost estimates

Elasticity	Nesting Parameter	$\bar{x} < 1\,000$	$1\,000 \leq \bar{x} < 5\,000$	$\bar{x} \geq 5\,000$
−4.0	0.0	9.37 €	7.20 €	1.75 €
	0.2	9.38 €	7.23 €	1.71 €
	0.4	9.40 €	7.27 €	1.67 €
	0.6	9.43 €	7.33 €	1.67 €
	0.8	9.49 €	7.42 €	1.72 €
	0.9	9.32 €	6.70 €	1.07 €
−2.5	0.0	11.09 €	9.70 €	4.02 €
	0.2	11.08 €	9.72 €	3.96 €
	0.4	11.08 €	9.75 €	3.92 €
	0.6	11.08 €	9.79 €	3.91 €
	0.8	11.08 €	9.86 €	3.93 €
	0.9	10.72 €	10.26 €	4.79 €
−1.8	0.0	9.90 €	9.12 €	3.59 €
	0.2	9.89 €	9.14 €	3.54 €
	0.4	9.88 €	9.18 €	3.50 €
	0.6	9.86 €	9.23 €	3.48 €
	0.8	9.83 €	9.29 €	3.51 €
	0.9	15.12 €	13.26 €	6.67 €

Table 8: Per-base station cost estimates

Elasticity	Nesting Parameter	Orange	SFR	Free	Bouygues
−4.0	0.0	358 204 €	424 580 €	106 442 €	414 102 €
	0.2	357 964 €	421 934 €	102 025 €	413 292 €
	0.4	357 706 €	418 519 €	97 011 €	412 256 €
	0.6	357 503 €	414 118 €	91 164 €	410 952 €
	0.8	357 415 €	409 276 €	84 906 €	409 327 €
	0.9	367 083 €	285 266 €	10 806 €	447 130 €
−2.5	0.0	237 300 €	209 904 €	73 649 €	255 961 €
	0.2	236 503 €	207 199 €	70 274 €	254 722 €
	0.4	235 611 €	203 951 €	66 439 €	253 275 €
	0.6	234 705 €	200 258 €	62 269 €	251 683 €
	0.8	233 919 €	196 560 €	58 148 €	250 097 €
	0.9	198 975 €	125 062 €	12 892 €	211 574 €
−1.8	0.0	93 678 €	71 990 €	29 444 €	98 346 €
	0.2	91 959 €	69 857 €	27 593 €	96 399 €
	0.4	90 058 €	67 462 €	25 572 €	94 236 €
	0.6	88 172 €	64 990 €	23 513 €	92 061 €
	0.8	86 673 €	62 810 €	21 638 €	90 271 €
	0.9	155 446 €	26 413 €	5 757 €	102 698 €

We estimate base station costs using using monthly profits. To recover the cost of long-lived base stations, we assume the static game is infinitely repeated with a monthly discount rate of 0.5%. The above results are therefore $\frac{1}{1-0.995} = 200$ times the per-base station costs we recover.

B.3 Counterfactual Nesting Parameter Imputations

Figure 15: Counterfactual prices and qualities

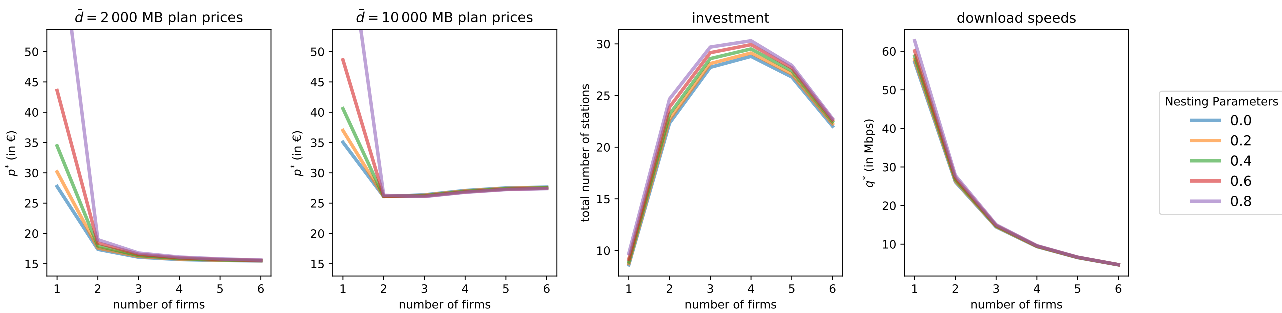


Figure 16: Full and partial price elasticities

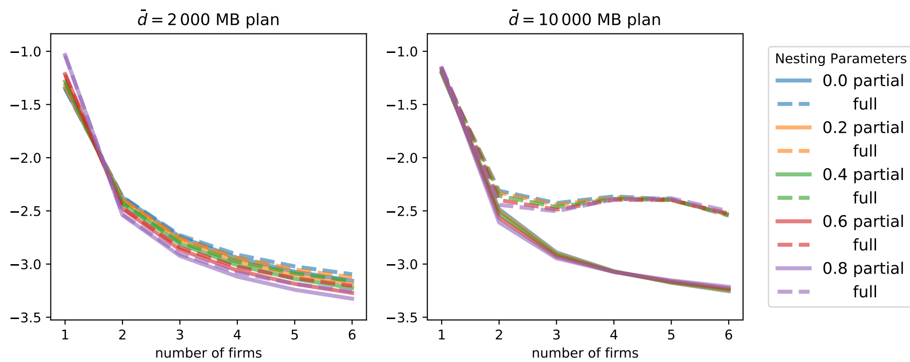


Figure 17: Counterfactual welfare

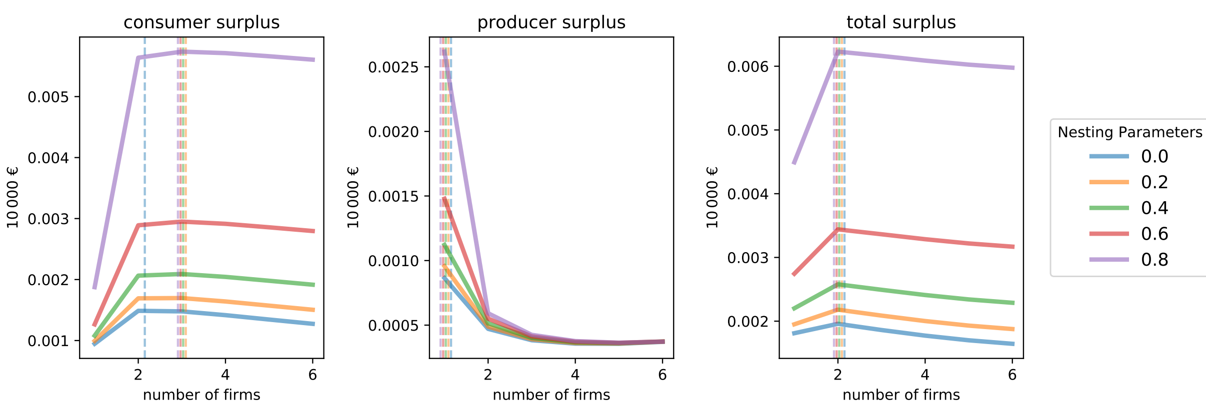


Figure 18: Counterfactual welfare by income level

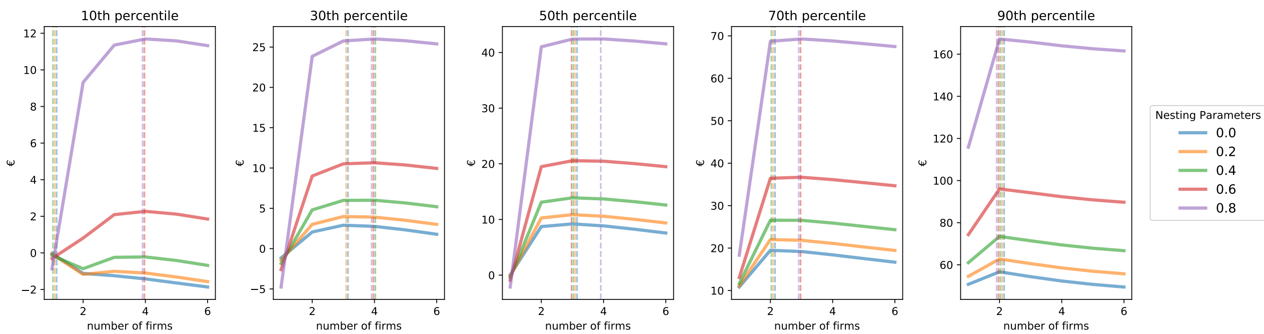
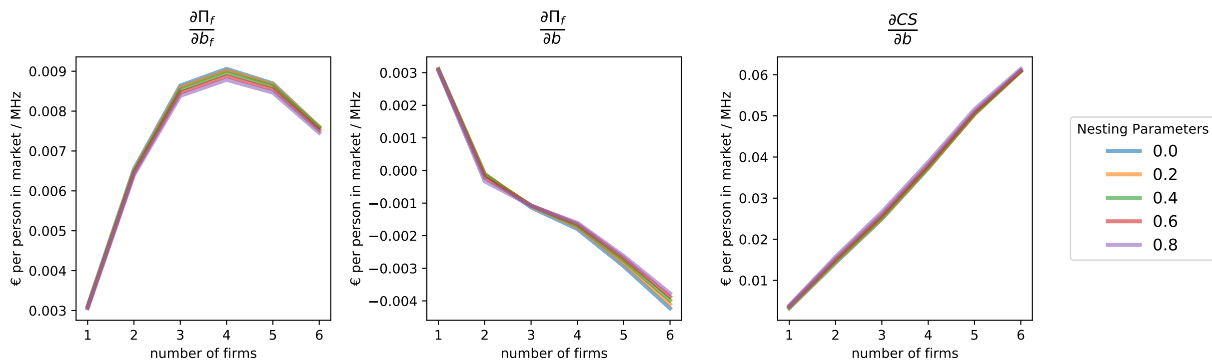


Figure 19: Bandwidth derivatives



C Data Appendix

This appendix describes the French telecom industry, the main datasets and variables, and the construction of the statistical inputs. It is organized into five sections. Section C.1 presents the scope of the study as well as some background information about market structure and network sharing in the French mobile industry. Section C.2 presents the characteristics of mobile tariffs and the tariff dataset. Section C.3 describes the Orange customers dataset and socio-economic characteristics. Section C.4 describes the measurement of the quality of mobile data. Finally, section C.5 presents the main statistical inputs of the estimation model: market characteristics, choice set and empirical moments.

C.1 Industry Background

C.1.1 Market Description

The French telecommunications market includes both fixed and mobile services. Fixed services cover fixed telephony, internet, and television over the internet, and generate slightly more than half of operators' total revenue. Mobile services include voice, data, and short messages services (SMS) and generated 44 percent of total telecommunications revenues in 2015.[28] We focus on mobile services.

Mobile services are supplied using network technology that improves regularly. Each improvement corresponds to a generation of mobile network technology. In our year of study, 2015, French MNOs had largely already deployed 4G technology, especially in the urban areas we consider.

The penetration rate of mobile services reached 110% in 2015 (see figure 20). According to surveys conducted by the French research institute CREDOC, mobile users represent 92% of the population above 12 years old in 2015. We rely on the statistics from these surveys in order to determine the market size. A decomposition according to the technology shows that mobile data is less popular than voice, particularly 4G mobile data whose penetration rate was 34% in 2015.

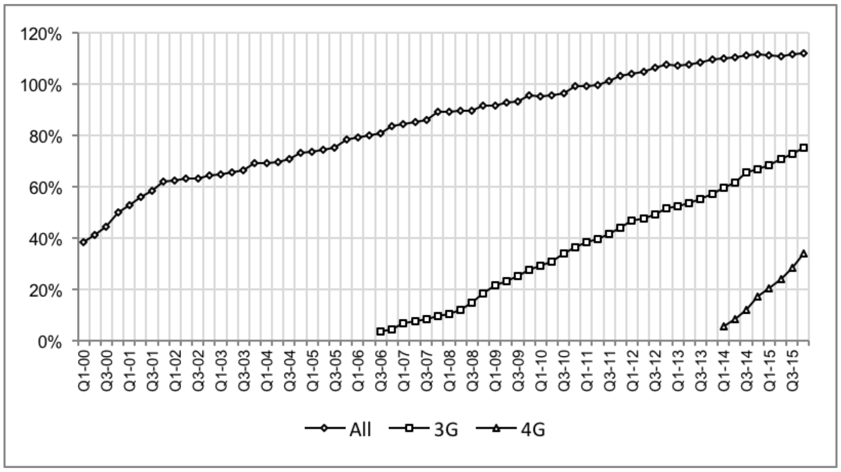
In terms of usage, figure 21 shows that voice and SMS consumption has reached a plateau since 2011. There is roughly no more growth in the number of minutes consumed, which stabilizes around 3 hours per consumer per month. The monthly number of SMS per consumer stays around 250 since 2011. The picture looks quite different for mobile data consumption. As shown in figure 21, the average monthly consumption of data has been growing so far. The volume of monthly data rises from 100 MB in 2010 to 800 MB in 2015.

Mobile services are typically purchased under two types of contracts, postpaid and prepaid, and by two types of customers, residential and business. Postpaid contracts require the subscriber to pay a monthly fee for a certain allowance of mobile services, and they may or may not involve a multi-month commitment in the contract. In contrast, prepaid contracts require consumers to pay as they consume and do not involve long-term commitments. Postpaid contracts represent a large majority of the mobile market, 83% as of December 2015.[29]

Unlike residential customers, business customers can bargain over their contracts and, therefore, exert some buyer power. Residential customers represent 89% of the mobile market in 2015.[30] We focus only on the market for residential contracts.

[28]Source: ARCEP, Series Chronologiques Trimestrielles, April 2016.
[29]Source: ARCEP, Series Chronologiques Trimestrielles, April 2016.
[30]Source: ARCEP, Series Chronologiques Trimestrielles, April 2016.

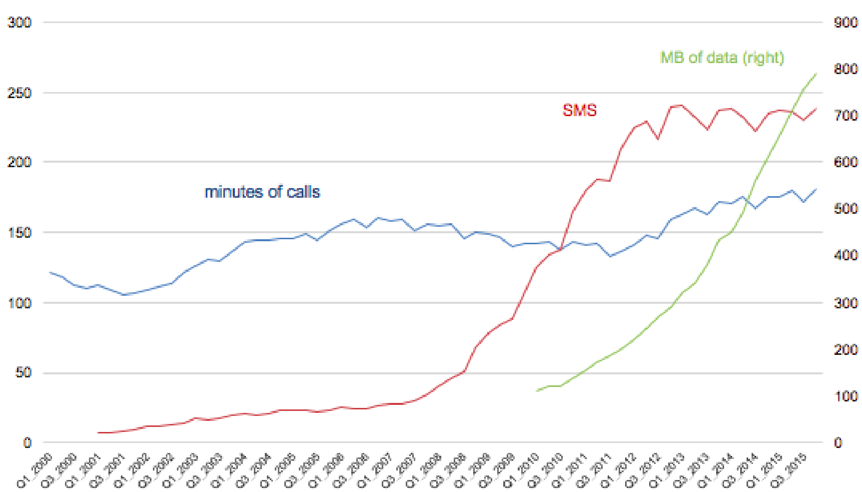
Figure 20: Penetration rate (ARCEP)



Note: Ratio of the number of active SIM cards (postpaid and prepaid excluding MtoM) to the population size.

Source: Own computations using data released by the regulator (ARCEP) - Series Chronologiques Trimestrielles (April 2016). Population data provided by INSEE.

Figure 21: Monthly usage of voice, SMS and data per subscriber



Note: Monthly usage is estimated as quarterly volume divided by 3. This monthly volume is divided by the number of active SIM cards (postpaid and prepaid excluding MtoM) at the end of the quarter.

Source: Own computations using data released by the regulator (ARCEP) - Series Chronologiques Trimestrielles (April 2016).

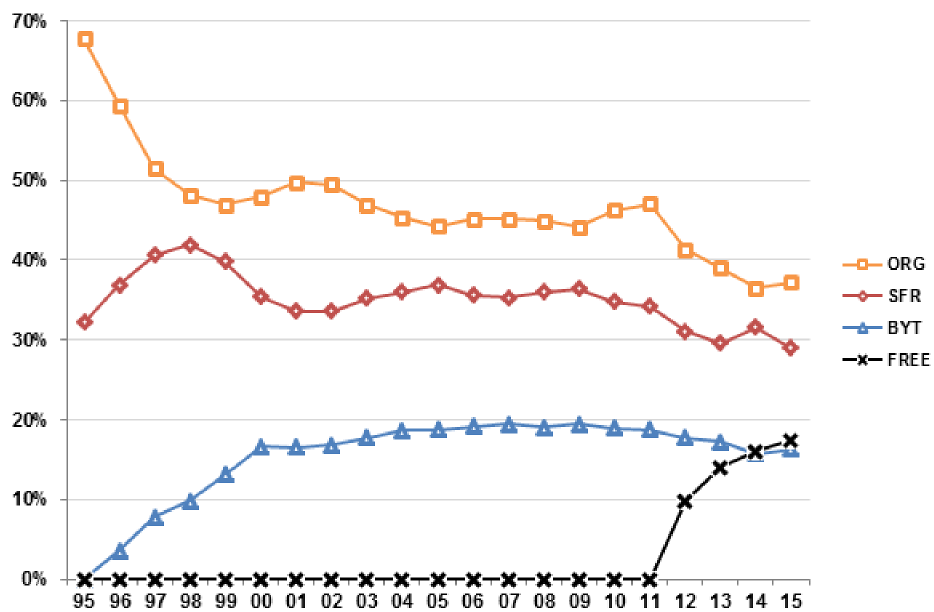
C.1.2 Market structure

During the period we study, the French mobile industry comprised four mobile network operators (MNO): Orange (ORG), SFR-Numericable (SFR), Bouygues Telecom (BYT) and Free Mobile (FREE). FREE entered the market in January 2012 and experienced a sharp increase in market share, up to 16% in four years (see figure 22). Right before FREE's entry, the three incumbents operators introduced their own low-cost brands: SOSH for ORG, RED for SFR and B&YOU for BYT. Contracts sold under these brands are postpaid without commitment.

MNOs own their networks contrary to mobile virtual network operators (MVNO) who typically rent access to MNOs' networks. Providing network access to MVNO is mandatory and enforced by regulation, but the access charge is freely negotiated with the MNO. According to figures from the national regulator ARCEP, there are more than 30 MVNOs in France in 2015, representing 10.6% of the mobile market, hosted by ORG, SFR and BYT.

On top of hosting MVNOs, MNOs also share their network infrastructure, particularly in less dense areas. The next section presents the features of network sharing among MNOs.

Figure 22: Evolution of postpaid residential market shares



Source: World Cellular Information Services (Ovum).

C.1.3 Network sharing

Network sharing occurs when a network operator shares a part or the whole of its network resources with a retail competitor. These resources can be passive network elements, such as antenna supports, masts, or active network elements, such as frequency bandwidths. Passive network sharing affects coverage differentiation but not necessarily quality differentiation. It typically consists of operators sharing the same tower or the cost of electricity. In general, it is any agreement between MNOs that do not involve the sharing of available frequency bandwidths.

In contrast, under active network sharing (Radio Access Network-Sharing), operators cannot differentiate in terms of quality, defined as the frequency bandwidth available per customer. Typically, it consists of the sharing of frequency bands and core network elements. Roaming agreements, whereby an operator's customers rely on the network of a host operator to communicate, is the highest level of active network sharing. It does not offer any possibility for quality or coverage differentiation.

Table 9 below presents the network sharing agreements reached between 2012 and 2015. These agreements apply to two types of areas according to their population density. "White Areas" or "Zones Blanches" correspond to areas where population density is so low that network deployment by several operators is not profitable. They are identified by the regulator on the basis of criteria which can change. These areas are typically rural, and represent roughly 1% of the population and 10% of the national surface. Only ORG, SFR and BYT have invested in these areas.

The most widespread network technologies in the White Areas are 2G, EDGE and GPRS. [31] However, 3G technology has been recently deployed. As of the end of December 2015, half of ORG and BYT's networks in these areas were covered by 3G, compared to 35% for SFR. In general, only one operator invests in a given White Area, and 64% of antennas in these areas are involved in a roaming agreement. Rival operators roam over the network of the only operator which invests in the area. As a result, there is no quality differentiation. For the remaining 36% of antennas, operators share passive network elements.

At the national level, FREE's customers can roam over ORG's 2G and 3G networks as long as there is no FREE antenna nearby. As a result, FREE cannot differentiate from ORG on 2G and 3G technologies, except when a FREE antenna is nearby its customer. In addition, FREE does not have access to networks in ZBs where BYT or SFR is the leader. MVNOs have roaming agreements with their hosts and therefore cannot differentiate in terms of quality or coverage.

Our model focuses on high-density areas to avoid the need to explicitly model network sharing.

[31] EDGE and GPRS are suitable for low speed mobile data services.

Table 9: Network sharing agreements 2012-2015

		FREE		ORG		SFR		BYT
Zone Blanche	Roaming: 64% of 2G & 3G antenna				↔			
	Passive sharing: 36% of antenna				↔			
Low Density	2G and 3G RAN-Sharing	✗		✗			↔	
	4G Roaming	✗		✗			→	
High Density		✗		✗		✗		✗
National	Passive sharing				↔			
	2G and 3G Roaming		→			✗		✗

Source: Summary from discussions with ORG's experts (HOSPITAL Jean-Jacques).

Note: ↔: two-way (reciprocal) sharing, $A \to B$ one-way sharing hosted by operator B.

C.2 Tariff data

C.2.1 Characteristics of Mobile Contracts

Mobile operators offer a variety of contracts that rely on voice and data services that can be bundled with fixed telephony, fixed broadband or television. A typical mobile contract comprises a fixed price (prepaid or postpaid) for given allowances (data, voice and SMS), prices per unit above allowances, commitment duration and handset subsidy.

Mobile contracts typically involve a fixed price for a given allowance, and may involve charges per unit for usage above allowances. Such extra charges are typically associated with plans with small allowances. Some contracts include unlimited SMS and voice allowances, and for contracts with large data allowances, further consumption may be blocked or throttled as consumers hit the limit of their data allowance. In these cases, customers willing to continue data consumption beyond their allowance have to purchase add-ons as one-shot data packages, valid until the end of the contract, or recurrent data packages with a price discount.

For contracts with a data throttling limit, the download speed is reduced for usage above allowance if no add-on is purchased. The maximal download speed under throttling is typically 128 Kbps. With this download speed, it would take over half and hour to download a 30 MB file, compared to 2 minutes under a theoretical unthrottled speed of 2 Mbps in a 3G network, and 24 seconds given a moderate 4G download speed of 10 Mbps. Basically, only emails and light web pages can be opened under throttling. As presented in table 10 below, this download speed is not always specified by operators in their contracts. When it is, it may depend on the location of the usage (local or abroad). The actual download speed experienced by customers is function of the number of simultaneous users, its location and handset. In our demand model, however, we assume that any data consumption over the data limit yields a speed of exactly 128 Kbps.

Mobile tariffs may include a discount for multi-month commitments, a premium for handset subsidy and additional discounts for purchasing fixed or television services on top of mobile services. Customers under commitment can terminate their contracts pursuant to a 2008 act labeled "*Loi Chatel*." According to this act, customers that have been under commitment for more than 12 months can terminate their contract by paying a penalty equal to the quarter of the bill over the remaining commitment period. If the customer has spent less than 12 months, she has to pay the whole remaining bill until the twelfth month, and the quarter of the remaining.

There is no penalty for changing contract with the same brand. However, customers switching between two brands of the same operator may incur a penalty if they were initially under commitment. This is typically the case for ORG's customers switching from standard ORG contracts to the low-cost brand SOSH. Operators introduced contracts without commitment in 2011. FREE does not offer long term commitment contracts, nor handset subsidy.

Table 10: Maximal download speed under throttling (Kbps)

Operator	National	Roaming
ORG	128*	ns
SFR	ns	ns
BYT	128	32
FREE	ns	ns

*:except video streaming.

ns ≡ not specified.

Source: operators' contracts

C.2.2 Tariff dataset

We collected data on contracts released between November 2013 and October 2015 along with their characteristics. It includes postpaid contracts from the four MNOs and the largest MVNO (EI Telecom) as well as their prepaid contracts.[32] Promotional contracts, typically released during summer and Christmas, are not included in the dataset.

Characteristics of contracts have been retrieved from operators' quarterly catalogues. Contract characteristics include tariff, voice and data limits, price per unit of consumption above allowance, international voice or data roaming, handset subsidy, length of commitment, bundling with fixed services.

C.3 Customer data

C.3.1 Choice and usage data

Contract choice and usage data come from ORG's customer database. We therefore only observe choice and usage data for ORG and not customers of the other firms, an issue we address in Section 5. This database contains observations of all postpaid residential customer choices and data/voice usage

[32]ORG's contracts include not only those that are sold through its main brand, but also others sold under alternative brands such as SOSH, BNP Paribas Mobile, FNAC Mobile, Click Mobile, Carrefour Mobile, etc.

in October 2015. Customers can cancel, keep, renew or choose a new contract with ORG. Table 11 presents the number of subscribers according to their status.

Table 11: Number of subscribers

Subscribers	cancel (%)	keep (%)	renew (%)	new (%)	Total (%)
14 992 631	0.9	94.5	3.6	1.1	100

The contract data contain information on the contract and handset characteristics,[33] as well as customer characteristics, including the residence of the customer, which we use to construct market shares. Usage data include SMS, voice, and data consumption.

C.3.2 Socio-economic data

Socio-economic characteristics are generated from the 2011's population census conducted by the French office of statistics (INSEE). These statistics include the deciles of income at municipality level. Income is measured as the fiscal revenue of households living in a given municipality in 2011.

C.4 Quality data

Quality measures are constructed using download speed test results provided by Ookla. Test results come from users who use Ookla's free Internet speed test, called simply "Speedtest," using a web browser or within an app. Using speed tests in France in the fourth quarter of 2015 yields 1 056 285 individual speed tests. Each speed test records the download speed, mobile network operator, and the user's location. We aggregate speed tests by averaging measured download speeds over tests for a given operator and geographic market, yielding an operator-market quality measure. An operator-market quality measure is, on average, an average of 284 test results.

C.5 Statistical inputs

This section presents the main statistical inputs of the estimation procedure: market characteristics and the choice set.

C.5.1 Market characteristics

This section starts with the definition of markets, and then presents the construction of market size and income distribution within markets. A market is defined as the geographical level at which quality measures can be reliable. Specifically, we need a market definition that yields sufficient speed tests to construct accurate measures of quality. As a result, we define market as either a large (urban) municipality, that is with more than 10 000 inhabitants. This definition collapses the initial 36 664 municipalities into 592 markets, and we discard three of these markets due to insufficient speed test results for at least one operator in the market.

[33] In some cases, information on prices, voice and data limits is not consistent with our data from tariffs catalogues. We change these characteristics to be in line with those from tariffs catalogues.

Market size is defined as the population above 12 years old using mobile communications. Table 12 reports the share of mobile users in the population above age 12 according the size of their municipalities of residence. Monthly population size is estimated using the geometric mean of the annual population growth rates obtained from INSEE population data.

Income distribution within markets was presented in section C.3.2. This distribution corresponds to income per capita in 2011.

Table 12: Share of mobile users among population above 12

	2011	2012	2013	2014	2015
< 2 000 inhab.	82	85	85	86	91
2 000 - 20 000 inhab.	83	85	87	84	89
20 - 100 000 inhab.	81	89	87	90	91
> 100 000 inhab.	87	89	90	92	93
Paris	88	91	94	92	96
France	**84**	**89**	**89**	**89**	**92**

Source: CREDOC Surveys

C.5.2 Choice set

We use tariff data presented in section C.2.2 to construct the choice set which includes the postpaid contracts of each MNO, the postpaid contracts of the largest MVNO (El Telecom), or the outside option of not using mobile communications.

Table 13 presents the market shares of the alternatives in the choice set during our sample period. Contracts included in the choice set are available in all markets; however, quality differs across markets. We construct market-specific choice sets by adding the quality data measured in each market. Quality of postpaid MVNO offers is estimated as the simple average of the quality of the hosts (BYT, ORG and SFR).

Table 13: Aggregate market shares of alternatives (%)

	Alternatives							
market size (millions)	ORG	SFR	BYT	FREE	MVNO	Prepaid	Non-users	Total
56.5	26.7	20.9	11.7	12.4	13.0	7.2	8.0	100

As noted previously, the choice set would consist of more than 1 700 alternatives if we included all contracts listed in the tariffs catalog, making the demand estimation cumbersome. We overcome this hurdle by focusing on data limits as the most important attribute. Indeed, recent investment in mobile networks in France are primarily made in order to improve the supply of mobile data services. Therefore, we employ a size reduction strategy that removes the less relevant contract's components and focus on the most significant variation in data limits.

Specifically, we define categories of contracts according to their level of data limits: less than 500 MB, 500–3 000 MB, 3 000–7 000 MB and more than 7 000 MB. These thresholds have been chosen following discussions with the industry experts and the statistical distribution of chosen contracts. The second data limit category—that is, contracts with 500–3 000 MB—have been further split according to their voice allowance: unlimited or not, making a total of five categories of contracts. Low data limit contracts typically do not have unlimited voice, and high data limit contracts typically come with unlimited voice allowance, so we do not split these categories by the voice limit.

Next, we exclude contracts bundled with fixed broadband or television, as they generally come with their mobile standalone version. We then choose the least expensive contract per category as the category's representative contract. Some customers keep old contracts that are no longer available, so we fill these missing data by using the most chosen old contracts within the same category. While some contracts with handset subsidies have corresponding standalone versions, some do not. We adjust the prices of these latter contracts using data on the price of handsets and the upfront cost required by Orange. These data were collected for both iPhone and Samsung, the two most popular handsets. We then distribute the handset cost over 24 months, and update the monthly contract price by subtracting off the monthly cost of the handset. In addition, we assume that Orange's handset subsidies apply to other operators because we do not observed their upfront costs.

C.5.3 Mean data consumption

We use the Orange customer data presented in section C.3.1 to construct market-level measures of mean data consumption for each Orange contract. Note that because we only observe data consumption for consumers of Orange contracts, we cannot construct these measures for contracts of other firms. Contracts are aggregated based on the associated data limit and whether or not the voice allowance is unlimited, as detailed in section C.5.2. Constructing market-contract-level measures of mean data consumption is complicated by the fact that the aggregated contracts in the choice set incorporate contracts with different data limits. For example, the Orange 4 000 MB data limit contract in the choice set incorporates contracts in the customer data with data limits ranging from 3 000 MB to 7 000 MB.

Since we use the mean data consumption in the data to discipline the predicted data consumption in our demand model, which is based on the data limit from the choice set, simply averaging the data consumption observed in the customer data can lead to biased estimates in the data consumption coefficients. For example, using the same 4 000 MB aggregated contract as before, if many customers in this category have contracts with data limits above 4 000 MB, they may consume well above 4 000 MB without hitting their data limit. Simply averaging data consumption for this category might give mean data consumption above 4 000 MB, which our demand estimation would interpret as either being insensitive to download speeds (because they are willing to consume even at the very slow throttled speed) or heavily weight the amount of data consumed (because they are consuming large amounts of data despite the slow throttled speed). In fact, it might be that neither of those conclusions is consistent with consumers' data consumption decisions under their actual data limit.

In order to account for the fact that realized data consumption decisions reflect heterogeneous data limits within a single data limit category, we construct the following measure of mean market shares, $\bar{x}_{jm}$,[34]

$$\bar{x}_{jm} = \frac{1}{|\mathcal{I}_j|} \sum_{i \in \mathcal{I}_j} \min\left\{\frac{x_i}{\bar{x}_i}, 1\right\} \bar{x}_j + \max\left\{0, x_i - \bar{x}_i\right\},$$

where $\mathcal{I}_j$ is the set of consumers with contracts that aggregate to j, x_i is consumer i's data consumption, and $\bar{x}_i$ is her data limit. The value $\bar{x}_j$ is the data limit associated with the aggregate contract j. We separate these two terms rather than simply using the fraction of the data limit consumed times the aggregated contract's data limit because, conditional on bypassing the data limit, the data limit is irrelevant for further data consumption.

[34]For contracts belonging to the group characterized by data limits of less than 500 MB, we impose that consumption cannot be greater than the data limit. For this category of contracts, add-on data packages are a common way of increasing one's data limit. Since we do not observe data package purchases, we simply assume that any consumer that consumed above the data limit did so with a purchased data package and that without one, he would have consumed as much as the data limit allowed. Our demand model reflects this, imposing that contracts in this category cannot consume above the data limit at a reduced speed (as they are able to do for high data limit contracts).

Table 14: Notation

Symbol	Description
B	bandwidth (hertz)
c_u	cost per user
$c_{fc,m}$	cost per base station at zero bandwidth
c_{bw}	cost per base station per unit of bandwidth operated
D	mass of consumers per unit area that are downloading
f	indexes firms
F	used for CDFs
g	density of consumers at given radius
i	indexes consumers
j	indexes products
$\mathcal{J}$	product set
m	indexes markets
p_j	price of contract j
$\overline{Q}$	channel capacity (Mbits/second)
q	data transmission speed as function of distance
Q	download speed (Mbits/second)
Q^L	throttled download speed
Q^D	demand requests (Mbits/second)
r	distance from antenna (km)
R	radius of area served by one base station (km)
s_j	market share
$\boldsymbol{s}$	vector of market shares
u	utility from data consumption over course of month
v	utility of a contract
x	monthly data consumption
γ_m	data transmission efficiency in market m
ε_{ij}	idiosyncratic, consumer-contract-level demand shock
θ	demand parameters
σ	nesting parameter
θ_{pi}	price coefficient
θ_{p0}	parameter controlling the mean of the price coefficient
θ_{pz}	parameter controlling the heterogeneity in the price coefficient
θ_v	coefficient on dummy for unlimited voice
θ_O	coefficient on dummy for Orange products
θ_c	opportunity cost of time spent downloading data coefficient
θ_{di}	parameter of log-normal distribution that defines distribution from which a consumer's utility of data consumption is drawn
θ_{d0}	parameter controlling the mean of θ_{di}
θ_{dz}	parameter controlling the heterogeneity in θ_{di}
ϑ_i	random shock to consumer's utility of data consumption, distributed exponentially with parameter θ_{di}
ξ_{jm}	market-level demand shock