**Multilevel Measurement Model Equations**

**Abbreviations and Subscripts:**

* = free/reduced lunch, = math, = item, = person = student, = cluster = school
* WS = within school, BS = between school
* last subscript (with comma before it) captures what is being predicted by each term
* = latent trait intercept fixed to 0 for identification
* Salient changes between models are marked in red font

**Definitions for Within-Level-1 Model Terms:**

* = overall (fixed + random) latent cluster intercept for free/reduced lunch
* = overall (fixed + random) cluster intercept for math item
* = overall (fixed + potential random) factor loading for WS trait predicting math item
* = overall (fixed + potential random) slope for WS free/reduced lunch 🡪 WS trait
* = person-specific residual for WS trait (with variance = )

**Definitions for Between-Level-2 Model Terms:**

* = (fixed) sample mean of latent cluster mean for free/reduced lunch
* = (random) cluster-specific intercept deviation for free/reduced lunch (with variance = )
* = (fixed) intercept for expected response to math item   
   (conditional at WS trait = 0 and BS trait = 0)
* = (fixed) factor loading for BS trait predicting math item
* = (random) cluster-specific intercept deviation for math item (with variance = )
* = (fixed) slope for BS free/reduced lunch 🡪 BS trait
* = (random) cluster-specific residual for BS trait (with variance = )
* = (fixed) factor loading for WS trait predicting math item
* = (fixed) slope for WS free/reduced lunch 🡪 WS trait

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model01.stan:** Empty (Non-Measurement) Two-Level Model with Correlated Random Item Intercepts

**Within-level-1:**

response to 1…10:

**Between-level-2:**

response to 1…10:

where: 🡪 all possible variances and covariances

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model02.stan:** Within-School Measurement Model with Correlated Random Item Intercepts and Within-School Discriminations Fixed=1

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10:

where: 🡪 all possible variances and covariances

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model03.stan:** Within-School Measurement Model with Correlated Random Item Intercepts and Estimated Within-School Discriminations using Standardized Theta

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10:

within-school loadings:

where: 🡪 all possible variances and covariances

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model04.stan:** Within-School Measurement Model with Correlated Random Item Intercepts and Estimated Within-School Discriminations using Item1=Marker

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10:

within-school loadings:

where: 🡪 all possible variances and covariances,

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model05.stan:** Within-School and Between-School Measurement Model with Uncorrelated Random Item Intercepts and Estimated Level-Specific WS (Item1=Marker) and BS (Item10=Marker) Discriminations

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10:

between-school trait:

within-school loadings:

where: , , , and

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model06.stan:** Within-School and Between-School Measurement Model with Uncorrelated Random Item Intercepts and Estimated Level-Constrained WS (Item1=Marker) and BS (Item1=Marker) Discriminations

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10:

between-school trait:

within-school loadings:

where: , ,   
 , and for

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model07.stan:** Within-School and Between-School Measurement Model without Random Item Intercepts and with Estimated Level-Constrained WS (Item1=Marker) and BS (Item1=Marker) Discriminations

**Within-level-1:**

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

response to 1…10: + \_\_\_\_\_\_

between-school trait:

within-school loadings:

where: , , and for

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**03\_model08.stan:** Within-School and Between-School Measurement Model with Uncorrelated Random Item Intercepts and Free/Reduced Lunch MLM Predictor and Estimated Level-Constrained WS (Item1=Marker) and BS (Item1=Marker) Discriminations

**Within-level-1:**

free/reduced lunch:

response to 1…10:

within-school trait:   
where:

**Between-level-2:**

free/reduced lunch:

response to 1…10:

between-school trait:

within-school loadings:

🡪 within-school slope:

where: , , ,   
 , and for