Path Analysis

Lecture #2

PSQF 6249: Factor Analysis and Structural Equation Modeling

September 3, 2025

Key Questions for Today's Lecture

- What distinguishes path models from multivariate regression models?
- What are the identification conditions for path models?
- What is an indirect effect? What is a total effect?
- What are standardized coefficients?

Today's Lecture

- Path analysis
  - Starting with multivariate regression…
  - …then arriving at our final destination

- Path analysis details:
  - Standardized coefficients
  - Model modification
  - Direct and indirect effects

- Additional issues in path analysis
  - Estimation types
  - Variable considerations

Today's Data Example

- Data are simulated based on the results reported in:

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: a path analysis. *Journal of Educational Psychology, 86*, 193-203.

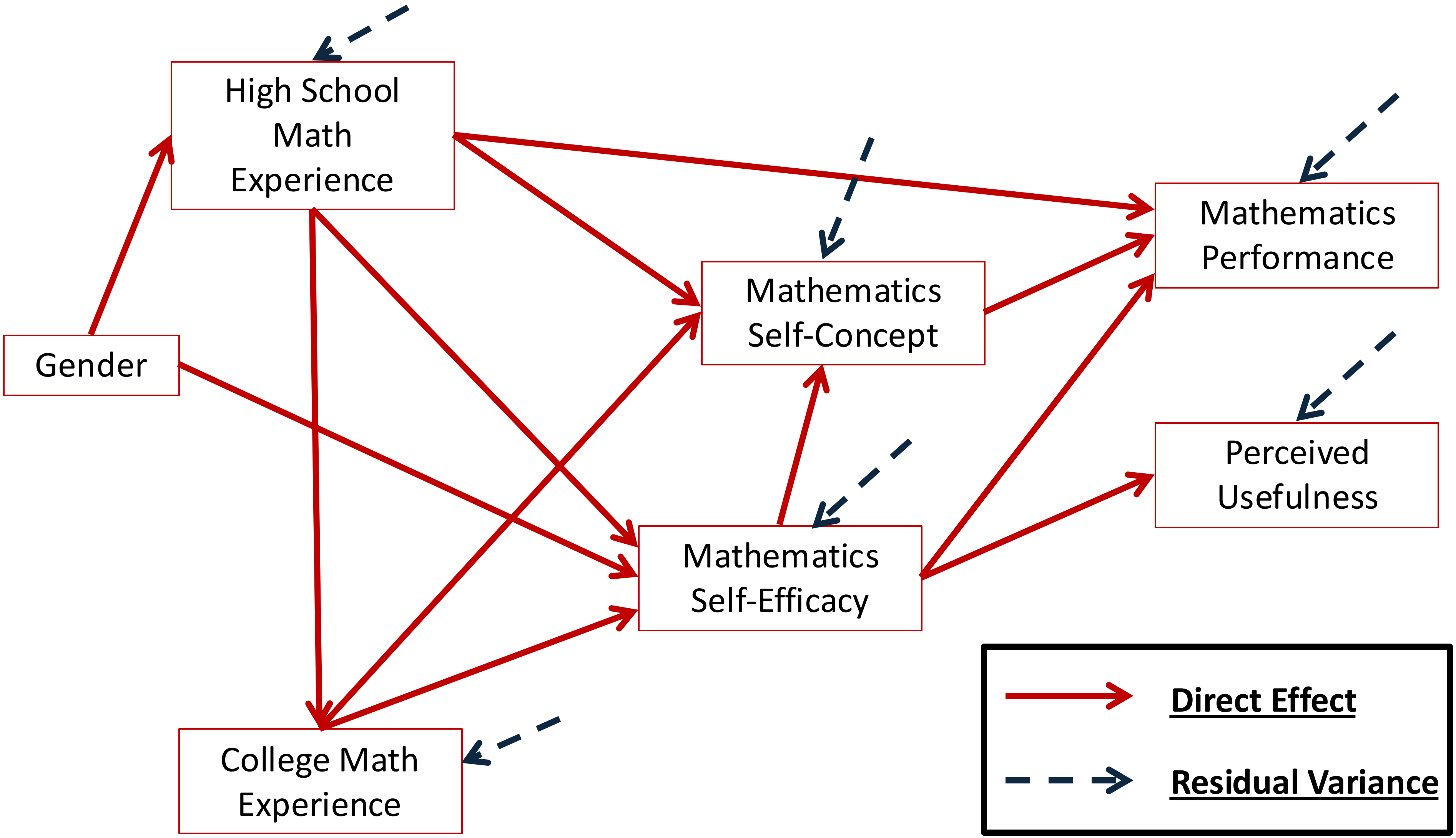
- Sample of 350 undergraduates (229 women, 121 men)
  - In simulation, 10% of variables were missing (using missing completely at random mechanism)

- Note: simulated data characteristics differ from actual data (some variables extend beyond their official range)
  - Simulated using Multivariate Normal Distribution
    - Some variables had boundaries that simulated data exceeded
  - Results will not match exactly due to missing data and boundaries

Variables of Data Example

- Gender (1 = male; 0 = female)
- Math Self-Efficacy (MSE)
  - Reported reliability of .91
  - Assesses math confidence of college students
- Perceived Usefulness of Mathematics (USE)
  - Reported reliability of .93
- Math Anxiety (MAS)
  - Reported reliability ranging from .86 to .90
- Math Self-Concept (MSC)
  - Reported reliability of .93 to .95
- Prior Experience at High School Level (HSL)
  - Self report of number of years of high school during which students took mathematics courses
- Prior Experience at College Level (CC)
  - Self report of courses taken at college level
- Math Performance (PERF)
  - Reported reliability of .788
  - 18-item multiple choice instrument (total of correct responses)

Our Destination: Overall Path Model



The Big Picture

- For our class today, our path analyses assumes the variables in our analysis are multivariate normally distributed
  - Mean vectors
  - Covariance matrices

- By specifying simultaneous regression equations (the core of path models), a very specific covariance matrix is implied

- Much like MANOVA and multilevel models, the key to path analysis is finding an effective approximation to the unstructured (saturated) covariance matrix
  - With fewer parameters, if possible

- The art to path analysis is in specifying models that blend theory and statistical evidence to produce generalizable results

MULTIVARIATE REGRESSION

Multivariate Regression

- We will now simultaneously model two variables from our example data that we wish to describe:
  - Mathematics performance (PERF)
  - Perceived usefulness (PERF)

- We will assume these to be continuous variables

- Initially, we will only look at an empty model with these two variables
  - Empty models are baseline models
  - We will use these to show how such models look based on the characteristics of the multivariate normal distribution
  - We will also show the bigger picture when modeling multivariate data: how we must be sure to model the covariance matrix correctly

Multivariate Empty Model: The Notation

- The multivariate model for PERF and USE is given by two regression models, which are estimated simultaneously:

$$PERF_i = \beta_0^{PERF} + e_i^{PERF}$$
$$USE_i = \beta_0^{USE} + e_i^{USE}$$

- As there are two variables, the error terms have a joint distribution that will be a multivariate normal:

$$\begin{bmatrix} e_i^{PERF} \\ e_i^{USE} \end{bmatrix} \sim N_2\left(\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \sigma_{e:PERF}^2 & \sigma_{e:PERF,USE} \\ \sigma_{e:PERF,USE} & \sigma_{e:USE}^2 \end{bmatrix}\right)$$

- Each error term has its own variance but now there is a covariance between error terms
  - We will soon see that the overall **R** matrix structure can be modified

Data Model

- Before showing the syntax and the results, we must first describe how the multivariate empty model implies how our data should look
  - This will be true for this week…next week we will have Y show up on either side of the equals sign which changes the math a little

- Multivariate model with matrices:

$$\begin{bmatrix} PERF_i \\ USE_i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_0^{PERF} \\ \beta_0^{USE} \end{bmatrix} + \begin{bmatrix} e_i^{PERF} \\ e_i^{USE} \end{bmatrix}$$

$$\mathbf{Y}_i = \mathbf{X}_i \mathbf{B} + \mathbf{e}_i$$

- Using expected values and linear combination rules, we can show that:

$$\mathbf{Y}_i \sim N_2(\mathbf{X}_i \mathbf{B}, \mathbf{V}_i)$$

$$\begin{bmatrix} PERF_i \\ USE_i \end{bmatrix} \sim N_2 \left(\boldsymbol{\mu}_i = \begin{bmatrix} \beta_0^{PERF} \\ \beta_0^{USE} \end{bmatrix}, \mathbf{V}_i = \mathbf{R} = \begin{bmatrix} \sigma_{e:PERF}^2 & \sigma_{e:PERF,USE} \\ \sigma_{e:PERF,USE} & \sigma_{e:USE}^2 \end{bmatrix} \right)$$

Lavaan Multivariate Regression Model Syntax

$$\begin{bmatrix} PERF_i \\ USE_i \end{bmatrix} \sim N_2\left(\begin{bmatrix} \beta_0^{PERF} \\ \beta_0^{USE} \end{bmatrix}, \begin{bmatrix} \sigma^2_{e:PERF} & \sigma_{e:PERF,USE} \\ \sigma_{e:PERF,USE} & \sigma^2_{e:USE} \end{bmatrix}\right)$$

```
model02.syntax = "
#Variances:
  perf ~~ perf
  use  ~~ use

#Covariance:
  perf ~~ use

#Means:
  perf ~ 1
  use  ~ 1
"
```

$\sigma^2_{e:PERF}$

$\sigma^2_{e:USE}$

$\sigma_{e:PERF,USE}$

β_0^{PERF}

β_0^{USE}

This covariance matrix is said to be **saturated**: All parameters are estimated

It is also called an **unstructured** covariance matrix

No other structure for the covariance matrix can fit better (only as well as)

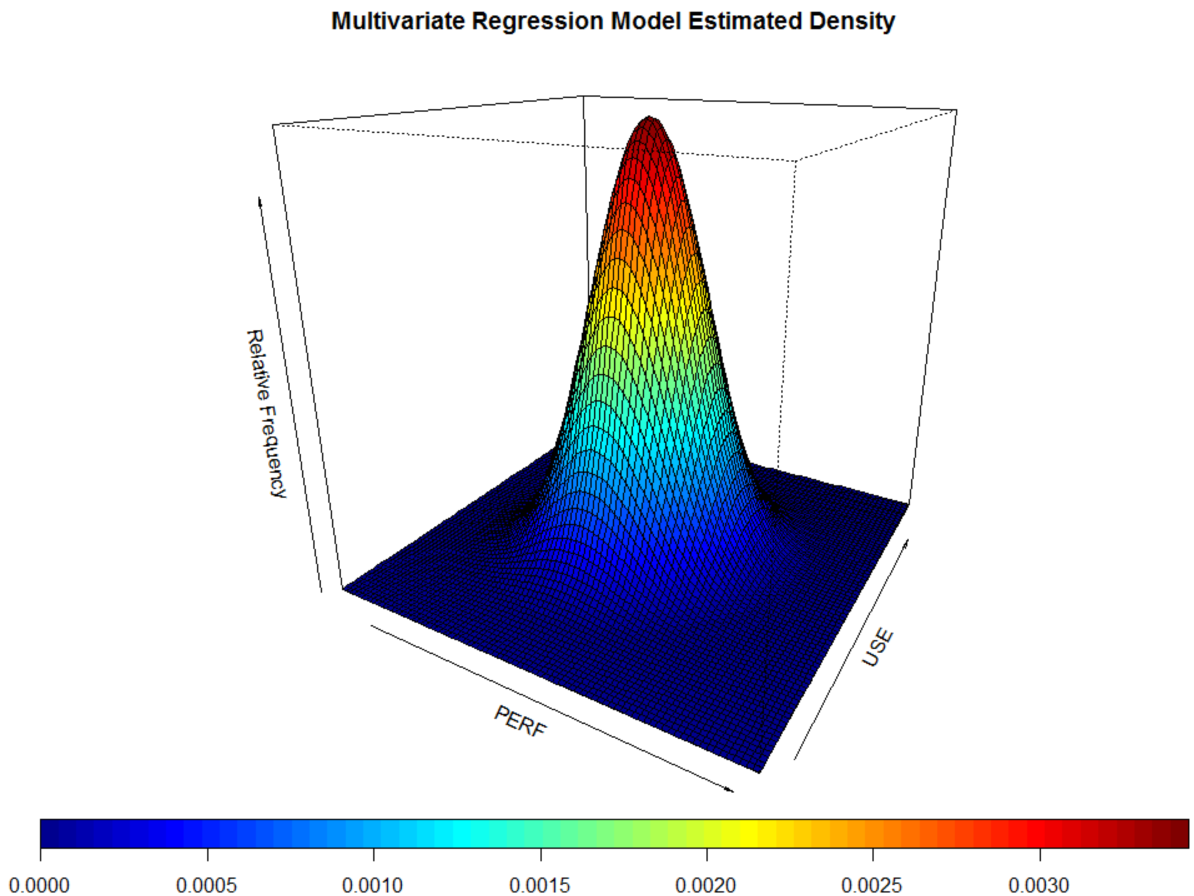
Multivariate Regression Model Results

- The estimated values:
  - What is the estimated correlation between PERF and USE?

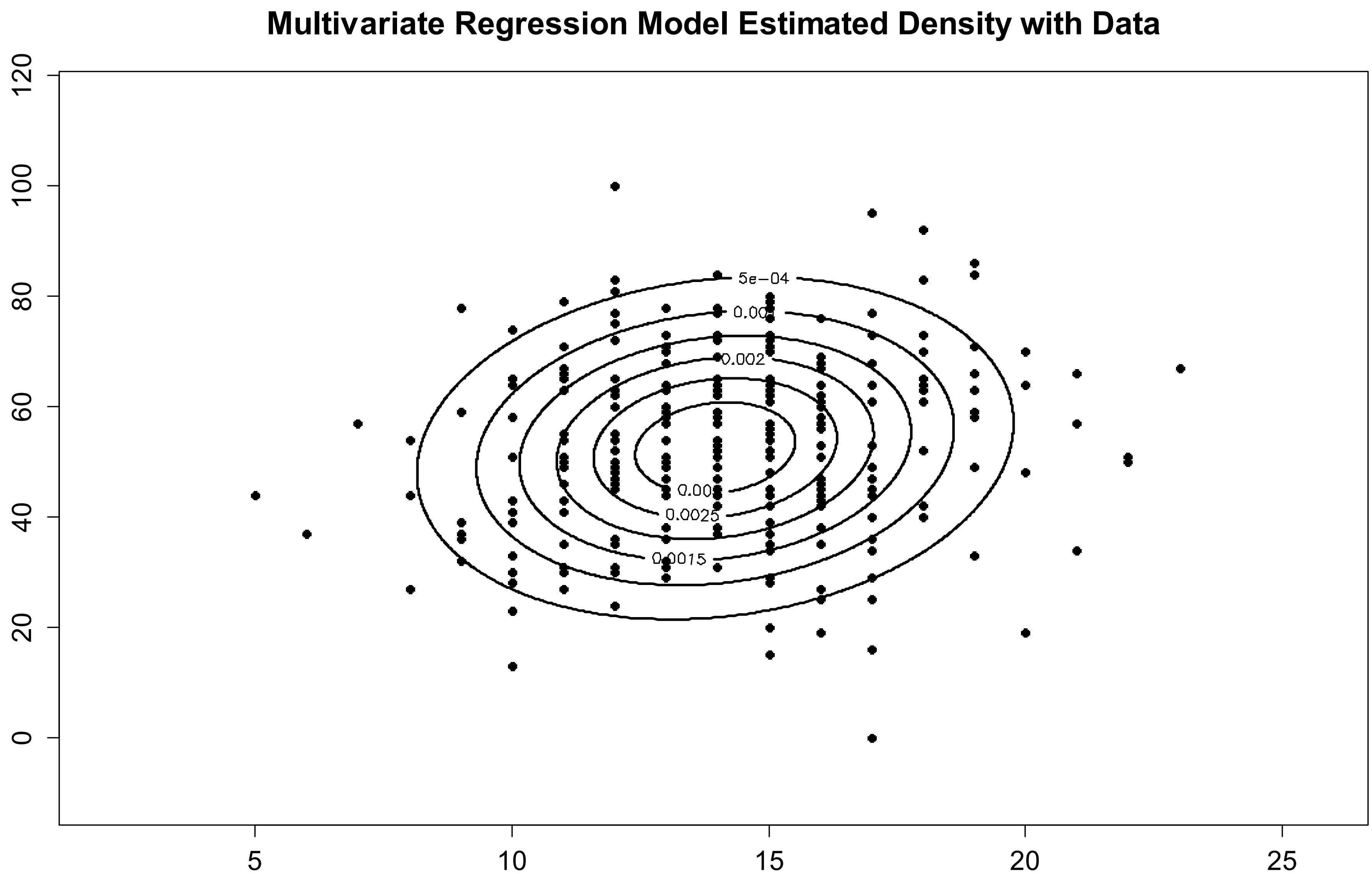
```
                  Estimate  Std.err  Z-value  P(>|z|)
Covariances:
  perf ~~
    use              6.847    2.850    2.403    0.016

Intercepts:
    perf            13.959    0.174   80.442    0.000
    use             52.440    0.872   60.140    0.000

Variances:
    perf             8.742    0.754   11.596    0.000
    use            249.245   19.212   12.973    0.000
```

Plotting the Model Estimated Results

$$\begin{bmatrix} PERF_i \\ USE_i \end{bmatrix} \sim N_2\left(\begin{bmatrix} 13.959 \\ 52.440 \end{bmatrix}, \begin{bmatrix} 8.742 & 6.847 \\ 6.847 & 249.245 \end{bmatrix}\right)$$



Comparing Model with Data



METHODS OF EXAMINING MODEL FIT

Methods of Model Fit

- Model-data fit is of utmost concern when building models with multivariate outcomes

- If a model does not fit the data:
  - Parameter estimates may be biased
  - Standard errors of estimates may be biased
  - Inferences made from the model may be wrong

- Examining model fit is the first step in multivariate models

- That said, not all "good-fitting" models are useful…
  - …model fit just allows you to talk about your model…there may be nothing of significance (statistically or practically) in your results, though

Types of Model Fit Information

- Model fit information for models where outcomes are conditionally MVN* come in several types, but all are based on the premise that any model mean and covariance structure must fit as well as the saturated mean vector and covariance matrix model

  ***If model outcomes are not conditionally MVN, model fit is very different**

- All possible models/structures **are nested within** the saturated mean vector and covariance matrix model
  - Most model fit statistics come from comparing any model/structure with the saturated model

- Indices shown first are called "global" model fit indices
  - Report fit of model globally (as opposed to locally for specific parameters)

Model Fit Using Our Example Empty MV Model

- We will evaluate the model fit of four models that change some assumptions with our empty multivariate model:

Model #	Mean Vector Structure	Mean Vector Estimates	Covariance Matrix Structure	Covariance Matrix Estimates
00	**Saturated**	$\begin{bmatrix} \mathbf{13.959} \\ \mathbf{52.440} \end{bmatrix}$	**Saturated**	$\begin{bmatrix} \mathbf{8.742} & \mathbf{6.847} \\ \mathbf{6.847} & \mathbf{249.245} \end{bmatrix}$
01	Saturated	$\begin{bmatrix} 13.959 \\ 52.440 \end{bmatrix}$	Variance Components	$\begin{bmatrix} 136.002 & 0 \\ 0 & 136.002 \end{bmatrix}$
02	Saturated	$\begin{bmatrix} 13.959 \\ 52.440 \end{bmatrix}$	Independent Variables	$\begin{bmatrix} 8.751 & 0 \\ 0 & 249.201 \end{bmatrix}$
03	Saturated	$\begin{bmatrix} 13.959 \\ 52.440 \end{bmatrix}$	Compound Symmetry	$\begin{bmatrix} 136.004 & 7.207 \\ 7.207 & 136.004 \end{bmatrix}$

  - Note: most structural equation models have saturated mean vectors (often not thought of for model fit)

- All model fit information is contained in lavaan output when using the summary() function with the fit.measures = TRUE option

```
#display empty model output
summary(model02.fit, fit.measures=TRUE)
```

Example lavaan Model Fit Output

```
> summary(model03.fit, fit.measures=TRUE)
lavaan 0.6-19 ended normally after 22 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         5
  Number of equality constraints                     1

                                                  Used       Total
  Number of observations                           348         350
  Number of missing patterns                         3

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               603.527     444.560
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.358
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                                 6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
  Scaling correction factor                                  1.088

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                    -117.981     -95.998

  Robust Comparative Fit Index (CFI)                         0.000
  Robust Tucker-Lewis Index (TLI)                         -112.245

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2386.796   -2386.796
  Scaling correction factor                                  0.756
      for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
      for the MLR correction

  Akaike (AIC)                                4781.591    4781.591
  Bayesian (BIC)                              4797.000    4797.000
  Sample-size adjusted Bayesian (SABIC)       4784.311    4784.311

Root Mean Square Error of Approximation:

  RMSEA                                          1.316       1.129
  90 Percent confidence interval - lower         1.229       1.054
  90 Percent confidence interval - upper         1.405       1.206
  P-value H_0: RMSEA <= 0.050                    0.000       0.000
  P-value H_0: RMSEA >= 0.080                    1.000       1.000

  Robust RMSEA                                               1.432
  90 Percent confidence interval - lower                     1.344
  90 Percent confidence interval - upper                     1.523
  P-value H_0: Robust RMSEA <= 0.050                         0.000
  P-value H_0: Robust RMSEA >= 0.080                         1.000

Standardized Root Mean Square Residual:

  SRMR                                           6.723       6.723
```

The fit.measures=TRUE Model Fit Statistics

- **Unlabeled section**
  - **Likelihood ratio test versus the saturated model**
  - **Testing if your model fits as well as the saturated model**

- Model test baseline model
  - Likelihood ratio test pitting the saturated model against the independent variables model
  - Testing whether any variables have non-zero covariances (significant correlations)

- User model versus baseline model
  - CFI
  - TLI

- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)

- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Indices of Global Model Fit

- Primary: obtained model $\boldsymbol{\chi}^2$ (from Model test baseline model) – here we use the MLR rescaled $\boldsymbol{\chi}^2$ from the "Robust" Column
  - ➢ χ^2 is evaluated based on model df (difference in parameters between your CFA model and the saturated model)
  - ➢ Tests null hypothesis that ***this*** model (H_0) fits equally to ***saturated model*** (H_1) so significance is undesirable (smaller χ^2, bigger p-value is better)
    - Means saturated model is estimated **automatically** for each model analyzed
  - ➢ Just using χ^2 is insufficient, however:
    - Distribution doesn't behave like a true χ^2 if sample sizes are small (or, if not using MLR, if items are non-normally distributed)
    - Obtained χ^2 depends largely on sample size
    - Some mention this is an unreasonable null hypothesis (perfect fit??)

- Because of these issues, alternative measures of fit are usually used in conjunction with the χ^2 test of model fit
  - ➢ Absolute Fit Indices (besides χ^2)
  - ➢ Parsimony-Corrected; Comparative (Incremental) Fit Indices

Chi-Square Test of Model Fit

- The Chi-Square Test of Model Fit provides a likelihood ratio test comparing the current model to the **saturated (unstructured) model**:
  - The value is -2 times the difference in log-likelihoods (rescaled if MLR)
  - The degrees of freedom is the difference in the number of estimated model parameters
  - The p-value is from the Chi-square distribution
- **If this test has a significant p-value:**
  - The current model (H_0) is rejected – the model fit is significantly worse than the full model
  - In latent variable models, this test is usually ignored
    - Said to be overly sensitive
- **If this test does not have a significant p-value:**
  - The current model (H_0) is not rejected – **fits equivalently to full model**

Model Results: Comparing Global Fit vs. Saturated Via LRT

- Model 00 (saturated covariance matrix):

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                0.000        0.000
Degrees of freedom                                 0            0
Minimum Function Value               0.0000000000000
Scaling correction factor                                      NA
  for the Yuan-Bentler correction (Mplus variant)
```

- Model 01 (variance components covariance matrix):

```
Estimator                                         ML       Robust
Minimum Function Test Statistic              604.272      804.743
Degrees of freedom                                 2            2
P-value (Chi-square)                           0.000        0.000
Scaling correction factor                                   0.751
  for the Yuan-Bentler correction (Mplus variant)
```

- Model 02 (independent variables covariance matrix):

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

- Model 03 (compound symmetry covariance matrix):

```
Estimator                                         ML       Robust
Minimum Function Test Statistic              603.527      444.560
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.000        0.000
Scaling correction factor                                   1.358
  for the Yuan-Bentler correction (Mplus variant)
```

Where the Saturated Model Test Comes From

- The saturated model LRT comes from a likelihood ratio test of the current model with the saturated model

- If using MLR (Robust method), then this LRT is rescaled based on the estimated scaling factors of both models

- This same information can be obtained from:
  - Loglikelihood model output section
  - anova() function comparing fit for current and saturated models

```
> # likelihood ratio tests:
> anova(model00.fit, model01.fit)

Scaled Chi-Squared Difference Test (method = “satorra.bentler.2001”)

lavaan->lavTestLRT():
   lavaan NOTE: The “Chisq” column contains standard test statistics, not the robust
   test that should be reported per model. A robust difference test is a function of two
   standard (not robust) statistics.
            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
model00.fit  0 4180.1 4199.3   0.00
model01.fit  2 4780.3 4791.9 604.27     804.74       2  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(model00.fit, model02.fit)

Scaled Chi-Squared Difference Test (method = “satorra.bentler.2001”)

lavaan->lavTestLRT():
   lavaan NOTE: The “Chisq” column contains standard test statistics, not the robust
   test that should be reported per model. A robust difference test is a function of two
   standard (not robust) statistics.
            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
model00.fit  0 4180.1 4199.3 0.0000
model02.fit  1 4184.1 4199.5 6.0641     5.5729       1    0.01824 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(model00.fit, model03.fit)

Scaled Chi-Squared Difference Test (method = “satorra.bentler.2001”)

lavaan->lavTestLRT():
   lavaan NOTE: The “Chisq” column contains standard test statistics, not the robust
   test that should be reported per model. A robust difference test is a function of two
   standard (not robust) statistics.
            Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
model00.fit  0 4180.1 4199.3   0.00
model03.fit  1 4781.6 4797.0 603.53     444.56       1  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Calculating the LRT for Global Fit Test for Model 04

- From the lavaan output:

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)            -2088.064   -2088.064
  Scaling correction factor                               1.012
    for the MLR correction
  Loglikelihood unrestricted model (H1)    -2085.032   -2085.032
  Scaling correction factor                               1.028
    for the MLR correction

  Number of free parameters                        4           4
```

- Calculation:
  - 4 parameters in our model; 5 in saturated model
  - Scaling correction factor:

$$c_{LR} = \left|\frac{(q_{restricted})(c_{restricted}) - (q_{full})(c_{full})}{(q_{restricted} - q_{full})}\right| = \left|\frac{(4 \times 1.012) - (5 \times 1.028)}{4 - 5}\right| \approx 1.088$$

  - $\chi^2 = -\frac{2^*(-2{,}088.064 - -2{,}085.032)}{1.088} = \frac{6.064}{1.088} = 5.573$
  - DF = 1

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

Saturated Model LRT and Loglikelihood Output

- Look at the following output from the saturated model (model 00):

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                0.000        0.000
Degrees of freedom                                 0            0
Minimum Function Value               0.0000000000000
Scaling correction factor                                      NA
  for the Yuan-Bentler correction (Mplus variant)

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2085.032    -2085.032
  Loglikelihood unrestricted model (H1)      -2085.032    -2085.032

  Number of free parameters                          5            5
```

- If the loglikelihoods of the current model ("User model" or H_0) are equal to the loglikelihoods of the saturated model ("Unrestriced model" or H_1), then you are running a model that is equivalent to the saturated model
  - **<u>No other model fit will be available or useful</u>**

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~

- **Model test baseline model**
  - **Likelihood ratio test pitting the saturated model against the independent variables model**
  - **Testing whether any variables have non-zero covariances (significant correlations)**

- User model versus baseline model
  - CFI
  - TLI

- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)

- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Model Test Baseline Model

- The "model test baseline model" section provides a LRT:
  - Comparing the saturated (unstructured) model (MODEL 00) with an independent variables model (called the baseline model) (MODEL 03)
  - Note: this is equal to previous LRT in Model 03 output:

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

- Here, the "null" model is the baseline (the independent variables model)
  - If the test is significant, this means that at least one (and likely more than one) variable has a significant covariance (and correlation)
  - If the test is not significant, this means that the independence model is appropriate
    - This is not likely to happen
    - But if it does, there are virtually no other models that will be significant
- Not often reported as it is likely variables are correlated

Model Results: Comparing Fit of Baseline and Saturated

- Model 00 (saturated covariance matrix):

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

- Model 01 (variance components covariance matrix):

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

- Model 02 (independent variables covariance matrix):

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

- Model 04 (compound symmetry covariance matrix):

```
Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018
```

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~

- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~

- **User model versus baseline model**
  - **CFI**
  - **TLI**

- Loglikelihood and Information Criteria
  - Likelihood ratio tests (nested models)
  - Information criteria comparisons (non-nested models)

- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

User Model Versus Baseline Model Section

- The “User model versus baseline model” section provides two additional measures of model fit comparing the current (user) model to the baseline (independent variables) model

```
User model versus baseline model:

  Comparative Fit Index (CFI)                     1.000       1.000
  Tucker-Lewis Index (TLI)                        1.000       1.000
```

- CFI stands for Comparative Fit Index
  - Higher is better (above .95 indicates good fit)

- TLI stands for Tucker Lewis Index
  - Higher is better (above .95 indicates good fit)

Comparative (Incremental) Fit Indices

- Fit evaluated relative to a 'null' model (of 0 covariances)
  - Relative to that, your model should be great!

T = target (current/estimated) model
N = null (baseline/independent variables) model

- **CFI: Comparative Fit Index**
  - Based on idea of the chi-square non-centrality parameter: (χ^2 – df)
  - $CFI = 1 - \frac{\max(\chi^2_T - df_T, 0)}{\max(\chi^2_T - df_T, \chi^2_N - df_N, 0)}$
  - From 0 to 1: bigger is better, > .90 = "acceptable", > .95 = "good"

- **TLI: Tucker-Lewis Index** (= **Non-Normed Fit Index**)
  - $TLI = \frac{\frac{\chi^2_N}{df_N} - \frac{\chi^2_T}{df_T}}{\frac{\chi^2_N}{df_N} - 1}$
  - From <0 to >1, bigger is better, >.95 = "good"

Comparative Fit Index Calculation: Model 00

The estimated model
(Model 02; the target)

```
Estimator                                          ML      Robust
Minimum Function Test Statistic                 0.000       0.000
Degrees of freedom                                  0           0
Minimum Function Value                 0.0000000000000
Scaling correction factor                                      NA
  for the Yuan-Bentler correction (Mplus variant)
```

The independent variables model
(Model 04; the null model)

```
Estimator                                          ML      Robust
Minimum Function Test Statistic                 6.064       5.573
Degrees of freedom                                  1           1
P-value (Chi-square)                            0.014       0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

Compute numerator:

$$\max(\chi_T^2 - df_T, 0) = \max(0 - 0{,}0) = 0$$

Compute denominator:

$$\max(\chi_T^2 - df_T, \chi_N^2 - df_N, 0)$$
$$= \max(0{,}5.573 - 1{,}0) = 4.573$$

Compute CFI:

$$CFI = 1 - \frac{\max(\chi_T^2 - df_T, 0)}{\max(\chi_T^2 - df_T, \chi_N^2 - df_N, 0)} = 1 - \frac{0}{4.573}$$
$$= 1.000$$

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000
```

Tucker-Lewis Index Calculation

The estimated model
(Model 02; the target)

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                0.000        0.000
Degrees of freedom                                 0            0
Minimum Function Value               0.0000000000000
Scaling correction factor                                      NA
  for the Yuan-Bentler correction (Mplus variant)
```

Compute Target Model ratio:

$$\frac{\chi^2_T}{df_T} = \frac{0}{0} = ?$$

The independent variables model
(Model 04; the null model)

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

Compute Null Model Ratio:

$$\frac{\chi^2_N}{df_N} = \frac{5.573}{1} = 5.573$$

Compute TLI:

$$TLI = \frac{\frac{\chi^2_N}{df_N} - \frac{\chi^2_T}{df_T}}{\frac{\chi^2_N}{df_N} - 1} = \frac{5.573 - ?}{5.573 - 1} = ?$$

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000       1.000
  Tucker-Lewis Index (TLI)                       1.000       1.000
```

Information Criteria Output

- The information criteria output provides relative fit statistics:

```
Information Criteria

          Akaike (AIC)                    7172.878
          Bayesian (BIC)                  7223.031
          Sample-Size Adjusted BIC        7181.790
            (n* = (n + 2) / 24)
```

  - AIC: Akaike Information Criterion
  - BIC: Bayesian Information Criterion (also called Schwarz's criterion)
  - Sample-size Adjusted BIC

- These statistics weight the information given by the parameter values by the parsimony of the model (the number of model parameters)
  - For all statistics, the smaller number is better
- The core of these statistics is -2*log-likelihood

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~
- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~
- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~
- **Loglikelihood and Information Criteria**
  - **Likelihood ratio tests (nested models)**
  - **Information criteria comparisons (non-nested models)**
- Root Mean Square Error of Approximation
  - How far off a model is from the saturated model, per degree of freedom
- Standardized Root Mean Square Residual
  - How far off a model’s correlations are from the saturated model correlations

Comparing Information Criteria

- Information criteria are relative tests of fit

```
Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2085.032   -2085.032
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032

  Number of free parameters                          5           5
  Akaike (AIC)                                4180.064    4180.064
  Bayesian (BIC)                              4199.325    4199.325
  Sample-size adjusted Bayesian (BIC)         4183.464    4183.464
```

- The are calculated based on the log-likelihood of the model, factoring in a penalty for number of parameters (plus other things)

- They should never be used to compare nested models
  - The likelihood ratio test is the most powerful test statistic to use for nested models

- When comparing non-nested models, first choose a statistic
  - AIC, BIC, or Sample-size Adjusted BIC are what are given by default

- The preferred model is the one with the lowest value of that statistic

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~

- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~

- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~

- ~~Loglikelihood and Information Criteria~~
  - ~~Likelihood ratio tests (nested models)~~
  - ~~Information criteria comparisons (non-nested models)~~

- **Root Mean Square Error of Approximation**
  - **How far off a model is from the saturated model, per degree of freedom**

- Standardized Root Mean Square Residual
  - How far off a model's correlations are from the saturated model correlations

Indices of Global Model Fit

Parsimony-Corrected: **RMSEA**

- **Root Mean Square Error of Approximation**
- Uses comparison with CFA model and saturated model
  - χ^2 listed here from first part of lavaan output
- Relies on a non-centrality parameter (NCP)
  - Indexes how far off your model is → χ^2 distribution shoved over
  - NCP → $d = (\chi^2 - df) / (N-1)$ Then, $RMSEA = SQRT(d/df)$
    - df is difference between # parameters in CFA model and saturated model
  - RMSEA ranges from 0 to 1; smaller is better
    - < .05 or .06 = "good", .05 to .08 = "acceptable", .08 to .10 = "mediocre", and >.10 = "unacceptable"
  - In addition to point estimate, get 90% confidence interval
  - RMSEA penalizes for model complexity – it's discrepancy in fit per df left in model (but not sensitive to N, although CI can be)
  - Test of "close fit": null hypothesis that RMSEA ≤ .05

RMSEA (Root Mean Square Error of Approximation)

- The RMSEA is an index of model fit where 0 indicates perfect fit (smaller is better; model 04 shown):

```
Root Mean Square Error of Approximation:

  RMSEA                                         0.121        0.115
  90 Percent Confidence Interval         0.044  0.220        0.040  0.210
  P-value RMSEA <= 0.05                         0.063        0.073
```

- RMSEA is based on the approximated covariance matrix
  - More on this in two weeks
- The goal is a model with an RMSEA less than .05
  - Although there is some flexibility
- The result above indicates our model fits well (RMSEA of .026)
  - Expected for 13 parameters (out of 14 possible)

RMSEA from Our Example Model 04

- From lavaan:

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)

                                                       Used
        Number of observations                          348
```

Create Non-Centrality Parameter $d = \frac{\chi^2 - df}{N-1} = \frac{5.573-1}{348-1} = 0.013$

Calculate RMSEA:

$$\text{RMSEA} = \sqrt{\frac{d}{df}} = \sqrt{\frac{0.013}{1}} = 0.115$$

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.121        0.115
  90 Percent Confidence Interval          0.044  0.220        0.040  0.210
  P-value RMSEA <= 0.05                          0.063        0.073
```

The fit.measures=TRUE Model Fit Statistics

- ~~Unlabeled section~~
  - ~~Likelihood ratio test versus the saturated model~~
  - ~~Testing if your model fits as well as the saturated model~~
- ~~Model test baseline model~~
  - ~~Likelihood ratio test pitting the saturated model against the independent variables model~~
  - ~~Testing whether any variables have non-zero covariances (significant correlations)~~
- ~~User model versus baseline model~~
  - ~~CFI~~
  - ~~TLI~~
- ~~Loglikelihood and Information Criteria~~
  - ~~Likelihood ratio tests (nested models)~~
  - ~~Information criteria comparisons (non-nested models)~~
- ~~Root Mean Square Error of Approximation~~
  - ~~How far off a model is from the saturated model, per degree of freedom~~
- **Standardized Root Mean Square Residual**
  - **How far off a model's correlations are from the saturated model correlations**

Standardized Root Mean Squared Residual

- The SRMR (standardized root mean square residual) provides the average standardized difference between:
  - The estimated covariance matrix of the saturated model
  - The estimated covariance matrix of the current model

```
Standardized Root Mean Square Residual:

  SRMR                                           0.066       0.066
```

- The model-estimated covariance matrix for model 04:

```
> model04.estimates$cov
     perf    use
perf   8.751
use    0.000 249.201
```

- The model estimated covariance matrix for model 02:

```
> model02.estimates$cov
     perf    use
perf   8.742
use    6.847 249.245
>
```

- Lower is better (some suggest less than 0.08)

Model Fit Comparisons

- For our example, we have four models
  - One is the saturated model which is guaranteed to fit

- Therefore, we have to ask the following set of questions:
  - Does model 01 fit?
  - Does model 02 fit?
  - Does model 03 fit?

- Of those that fit, which is preferred?

Model 01 Fit Statistics

```
> summary(model03.fit, fit.measures=TRUE)
lavaan (0.5-17) converged normally after   6 iterations

                                                  Used       Total
  Number of observations                           348         350

  Number of missing patterns                         3

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              604.272     804.743
  Degrees of freedom                                 2           2
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.751
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                     -58.465     -86.772

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2387.168   -2387.168
  Scaling correction factor                                  1.212
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
    for the MLR correction

  Number of free parameters                          3           3
  Akaike (AIC)                                4780.336    4780.336
  Bayesian (BIC)                              4791.893    4791.893
  Sample-size adjusted Bayesian (BIC)         4782.376    4782.376

Root Mean Square Error of Approximation:

  RMSEA                                          0.930       1.074
  90 Percent Confidence Interval          0.869  0.993       1.003  1.147
  P-value RMSEA <= 0.05                          0.000       0.000

Standardized Root Mean Square Residual:

  SRMR                                           6.723       6.723
```

Model 02 Fit Statistics

```
> summary(model04.fit, fit.measures=TRUE)
lavaan (0.5-17) converged normally after  18 iterations

                                                  Used       Total
  Number of observations                           348         350

  Number of missing patterns                         3

  Estimator                                         ML      Robust
  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.014       0.018
  Scaling correction factor                                  1.088
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                      -0.000      -0.000

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2088.064   -2088.064
  Scaling correction factor                                  1.012
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
    for the MLR correction

  Number of free parameters                          4           4
  Akaike (AIC)                                4184.128    4184.128
  Bayesian (BIC)                              4199.537    4199.537
  Sample-size adjusted Bayesian (BIC)         4186.848    4186.848

Root Mean Square Error of Approximation:

  RMSEA                                          0.121       0.115
  90 Percent Confidence Interval          0.044  0.220       0.040  0.210
  P-value RMSEA <= 0.05                          0.063       0.073

Standardized Root Mean Square Residual:

  SRMR                                           0.066       0.066
```

Model 03 Fit Statistics

```
> summary(model05.fit, fit.measures=TRUE)
lavaan (0.5-17) converged normally after  21 iterations

                                                  Used       Total
  Number of observations                           348         350

  Number of missing patterns                         3

  Estimator                                         ML      Robust
  Minimum Function Test Statistic              603.527     444.560
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.358
    for the Yuan-Bentler correction (Mplus variant)

Model test baseline model:

  Minimum Function Test Statistic                6.064       5.573
  Degrees of freedom                                 1           1
  P-value                                        0.014       0.018

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.000       0.000
  Tucker-Lewis Index (TLI)                    -117.981     -95.998

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2386.796   -2386.796
  Scaling correction factor                                  0.945
    for the MLR correction
  Loglikelihood unrestricted model (H1)      -2085.032   -2085.032
  Scaling correction factor                                  1.028
    for the MLR correction

  Number of free parameters                          4           4
  Akaike (AIC)                                4781.591    4781.591
  Bayesian (BIC)                              4797.000    4797.000
  Sample-size adjusted Bayesian (BIC)         4784.311    4784.311

Root Mean Square Error of Approximation:

  RMSEA                                          1.316       1.129
  90 Percent Confidence Interval          1.229  1.405       1.054  1.206
  P-value RMSEA <= 0.05                          0.000       0.000

Standardized Root Mean Square Residual:

  SRMR                                           6.723       6.723
```

LOCAL MODEL FIT MEASURES

"Local" Measures of Model (Mis)Fit

- Local measures of model (mis)fit are statistics that point to the location (typically of a covariance matrix) where a model may not fit well
  - As opposed to "global" measures that indicate a model fit overall

- Local measures of model (mis)fit are typically of two types:
  - Residual covariance matrices (unstandardized, standardized, or normalized)
    - The difference between the model's estimated covariance matrix and the saturated model's estimated covariance matrix
    - These were used for the SRMR
  - Model "modification indices"
    - 1-degree of freedom hypothesis tests for the improvement of the model LRT if one more parameter was allowed to be estimated

Residual Covariance Matrices

- The model estimated covariance matrices for model 02:

```
> model02.estimates$cov
        perf     use
perf   8.751
use    0.000 249.201
```

- The "raw" or "unstandardized" residual covariance matrix for model 04 (literally taking model04 – model02):
  - Shows that the biggest difference is the covariance between PERF and USE

```
> model02.residuals = residuals(model02.fit, type="raw")
> model02.residuals$cov
       perf    use
perf -0.009
use   6.847  0.044
```

- I often prefer "normalized" versions of these matrices
  - We can inspect the normalized residual covariance matrix (like z-scores) to see where our biggest misfit occurs
  - This indicates we should estimate the covariance (> 1.96)

```
> residuals(model02.fit, type="normalized")$cov
       perf    use
perf -0.012
use   2.403  0.002
```

Modification Indices: More Help for Fit

- As we used Maximum Likelihood to estimate our model, another useful feature is that of the modification indices
  - Modification indices, also called Score or LaGrangian Multiplier tests, attempt to suggest the change in the log-likelihood for adding a given model parameter (larger values indicate a better fit for adding the parameter)

```
> modindices(model02.fit)
   lhs op rhs    mi   epc sepc.lv sepc.all sepc.nox
3 perf ~~ use 6.239 7.125   7.125    0.153    0.153
```

- <u>mi column: </u>the expected value of the LRT of the current model (model 04) and a model where this parameter was added
- <u>mi.scaled column</u>: the scaled (robust) LRT
  - Should be bigger than 3.84 for 1 df
  - Practice is to find values that are much higher (say 10 or more)
- <u>epc column: </u>expected value of the parameter in the model where this parameter was added

MI Follow Up

- As the covariance parameter was suggested to be added by the MI, here is what happens when we add the parameter and re-estimate the model

```
> modindices(model02.fit)
   lhs op rhs    mi   epc sepc.lv sepc.all sepc.nox
3 perf ~~ use 6.239 7.125   7.125    0.153    0.153
```

- By adding that parameter, we get model 00:

```
Estimator                                         ML       Robust
Minimum Function Test Statistic                6.064        5.573
Degrees of freedom                                 1            1
P-value (Chi-square)                           0.014        0.018
Scaling correction factor                                   1.088
  for the Yuan-Bentler correction (Mplus variant)
```

LRT between
Model 00 and Model 02

```
                   Estimate  Std.err  Z-value  P(>|z|)
  Covariances:
    perf ~~
      use              6.847    2.850    2.403    0.016
```

Estimated Covariance Parameter

Multivariate Regression

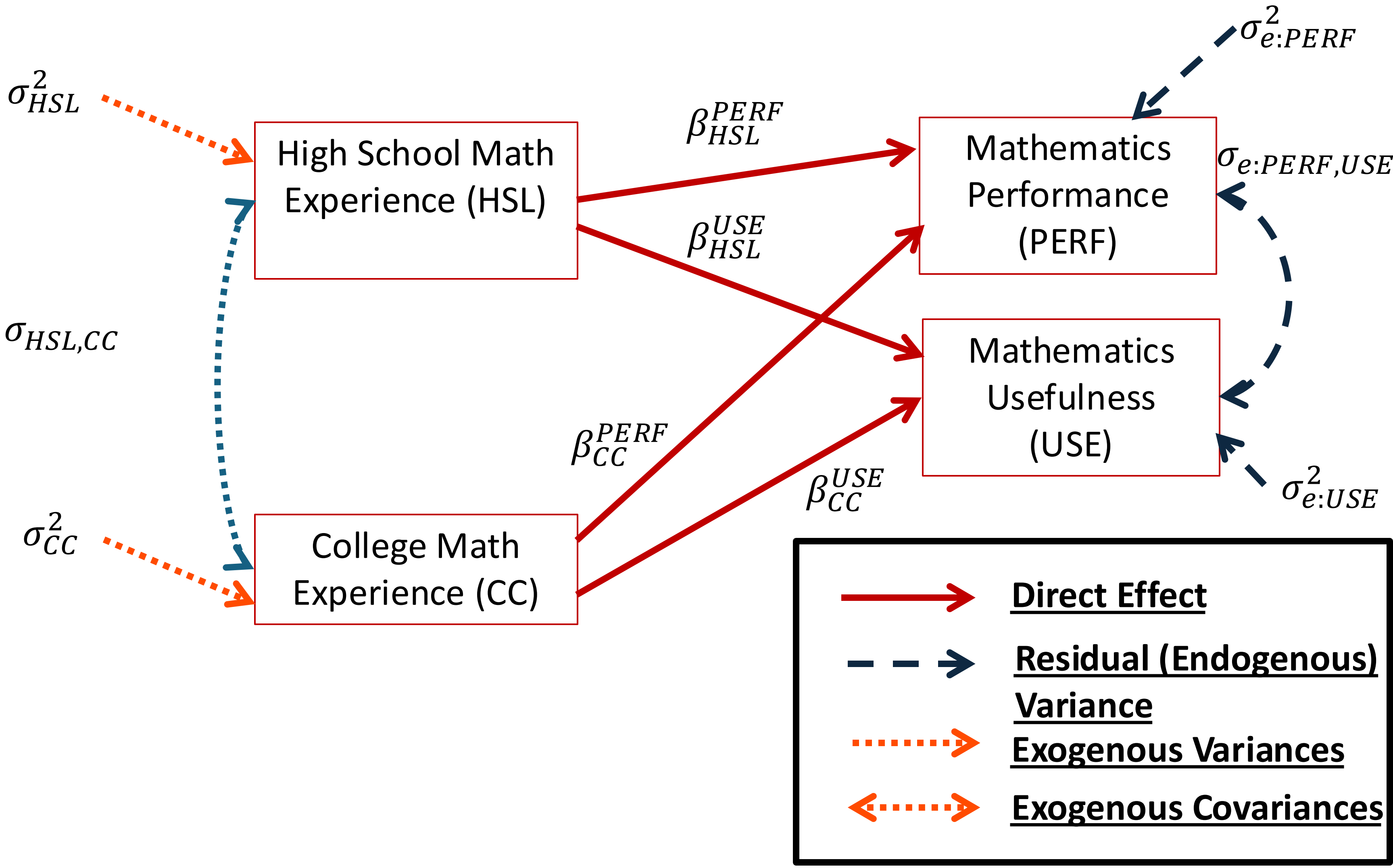
- Next will add predictors to both variables at once:
  - Predicting mathematics performance (PERF) with high school (HSL) and college (CC) experience
  - Predicting perceived usefulness (USE) with high school (HSL) and College (CC) experience

$$PERF_i = \beta_0^{PERF} + \beta_{HSL}^{PERF} HSL_i + \beta_{CC}^{PERF} CC_i + e_i^{PERF}$$
$$USE_i = \beta_0^{USE} + \beta_{HSL}^{USE} HSL_i + \beta_{CC}^{USE} CC_i + e_i^{USE}$$

- We denote the residual for PERF as e_i^{PERF} and the residual for USE as e_i^{PERF}
  - Here, we assume the residuals are Multivariate Normal:

$$\begin{bmatrix} e_i^{PERF} \\ e_i^{USE} \end{bmatrix} \sim N_2\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{e:PERF}^2 & \sigma_{e:PERF,USE} \\ \sigma_{e:PERF,USE} & \sigma_{e:USE}^2 \end{bmatrix} \right)$$

Multivariate Linear Regression Path Diagram



Types of Variables in the Analysis

- An important distinction in path analysis is between endogenous and exogenous variables

- Endogenous variable(s): variables whose variability ***is explained*** by one or more variables in a model
  - In linear regression, the **dependent variable** is the only endogenous variable in an analysis
    - Mathematics Performance (PERF) in our example

- Exogenous variable(s): variables whose variability ***is not explained*** by any variables in a model
  - In linear regression, the **independent variable**(s) are the exogenous variables in the analysis
    - High school (HSL) and college (CC) experience

Labeling Variables

- The endogenous (dependent) variables are:
    - Performance (PERF) and Usefulness (USE)

- The exogenous (independent) variables are:
    - High school (HSL) and college (CC) experience

Behind the Scenes with Exogenous Variables

- Lavaan also puts the exogenous variables into the likelihood function
  - Could be problematic if these are not MVN (like gender)
  - We will ignore this for now

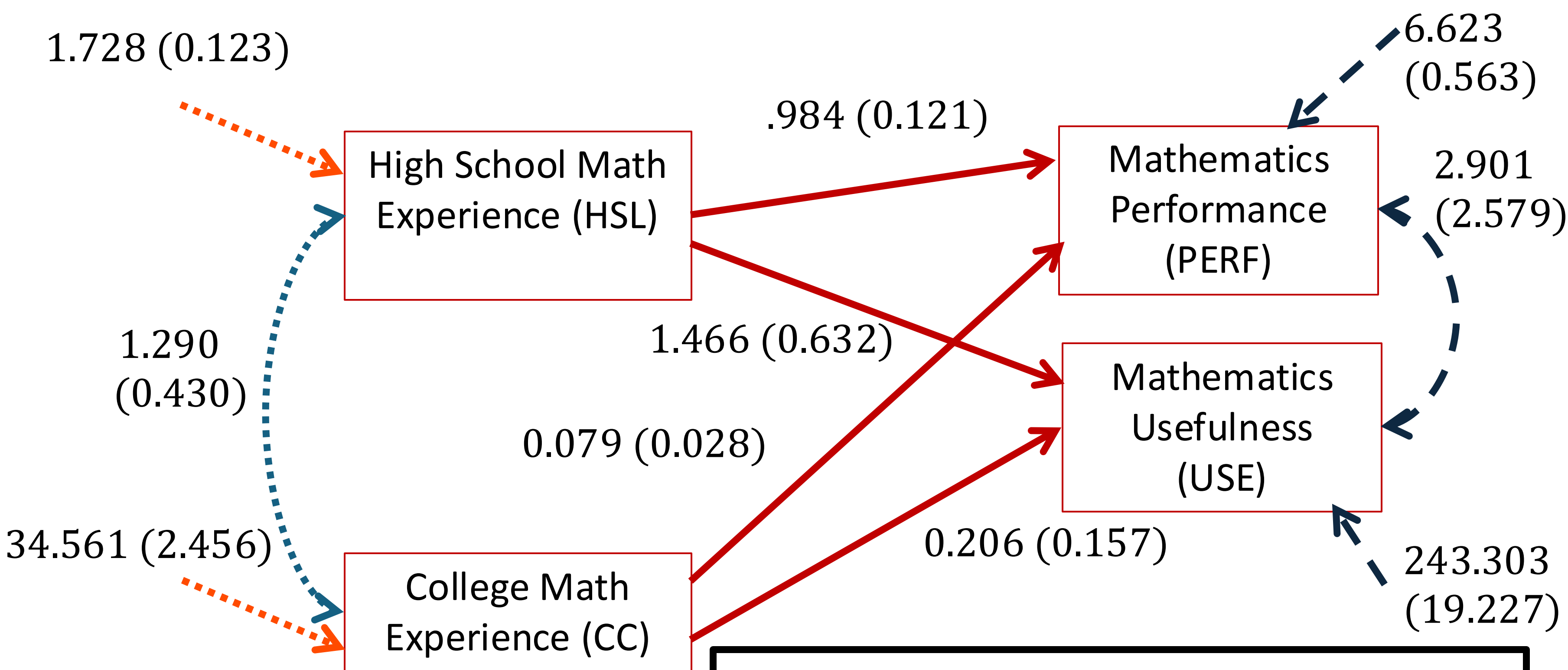
- The fixed.x = TRUE option takes the sample means/variances of these variables and plugs them into the likelihood
  - We'll do this explicitly through syntax and turn this option off

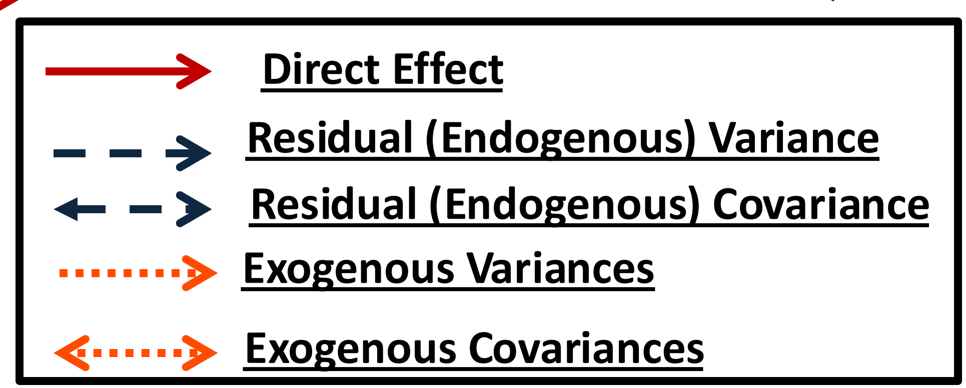
```
model05.syntax = "
#endogenous variable equations
  perf ~ hsl + cc
  use ~ hsl + cc

#endogenous variable intercepts
  perf ~ 1
  use ~ 1

#endogenous variable residual variances
  perf ~~ perf
  use ~~ use

#endogenous variable residual covariances
  perf~~ use

#exogeneous variables put into likelihood function:

#exogeneous means(intercepts)
hsl ~ 1
cc ~ 1

#exogeneous variances
hsl ~~ hsl
cc ~~ cc

#exogeneous covariances
hsl ~~ cc
"
```

Multivariate Regression Model Parameters

- If we considered all four variables to be part of a multivariate normal distribution, our unstructured (saturated) model would have 14 parameters:
  - 4 means
  - 4 variances
  - 6 covariances (4-choose-2 or 4*(4-1)/2))
- The model itself has 14 parameters:
  - 4 intercepts
  - 4 slopes
  - 2 residual variances
  - 1 residual covariance
  - 2 exogenous variances
  - 1 exogenous covariance
- Therefore, this model will fit perfectly – no model fit statistics will be available
  - Even without model fit, interpretation of parameters can proceed

Multivariate Linear Regression Path Diagram





Not Shown On Path Diagram:

- $\beta_0^{PERF} = 8.264\ (0.594)$
- $\beta_0^{USE} = 43.129\ (3.338)$
- $\mu_{HSL} = 4.922\ (0.073)$
- $\mu_{CC} = 10.330\ (0.331)$

Standardized Coefficients

- The scale of the (unstandardized) slope coefficients is given in terms of UNITS of Y (SD Y) per UNITS of X (SD X)
  - Y goes up β_X^Y UNITS of Y for every UNIT of X
    - HSL has SD of 1.31; CC has SD of 5.88
  - If the UNITS of X differ for the various IVs in a model, it can be hard to compare relative strengths of coefficients
    - $\beta_{HSL}^{PERF} = .984$ (but HSL has SD of 1.31)
    - $\beta_{CC}^{PERF} = .079$ (but CC has SD of 5.88)
- Standardized coefficients are the coefficients that would be obtained if Y and X were standardized:
  - Standardized = variance of 1 (i.e. z-scores used for analysis)
- Standardized coefficients are useful for comparing the relative effects of each IV in the model

Standardization in lavaan

- To get standardized estimates, add the "standardized=TRUE" option to the summary statement

```
#standardized parameter estimates:
summary(model01.fit, fit.measures=TRUE, standardized=TRUE)
```

- Under the output section, the last two columns are the standardized estimates (std.lv and std.all)
  - Also available from the function standardizedSolution (adds no.x type)

```
standardizedSolution(model01.fit, type = "std.all")
```

- Three types of standardizations are given:
  - **std.all**: These are the standardized regression coefficients; use these for continuous IVs (used for our current analysis)
  - **std.nox**: These only standardize based on variance of Y (the DV). Use when binary variables are IVs (like gender dummy coding) as unit of X has no meaning
  - **std.lv:** Discussed when we get to models with latent variables

Interpreting Multivariate Regression Results for PERF (nearly identical results)

- $\beta_0^{PERF} = 8.264$: the intercept for PERF – the value of PERF when all predictors are zero (HSL = 0 and CC = 0)

- $\beta_{HSL}^{PERF} = 0.986$: the slope for HSL predicting PERF. Indicates that for every one-unit increase in HSL (holding CC constant), PERF increases by .986
  - The standardized coefficient was .438

- $\beta_{CC}^{PERF} = 0.079$: the slope for CC predicting PERF. Indicates that for every one-unit increase in CC (holding HSL constant), PERF increases by .079
  - The standardized coefficient was .157

Interpreting Multivariate Regression Results for USE

- $\beta_0^{USE} = 43.129$: the intercept for USE – the value of USE when all predictors are zero (HSL = 0 and CC = 0)

- $\beta_{HSL}^{USE} = 1.466$: the slope for HSL predicting USE. Indicates that for every one-unit increase in HSL (holding CC constant), USE increases by 1.466
  - The standardized coefficient was .122

- $\beta_{CC}^{USE} = 0.206$: the slope for CC predicting USE. Indicates that for every one-unit increase in CC (holding HSL constant), USE increases by .206. This was found to be not significant, meaning college experience did not predict perceived usefulness
  - The standardized coefficient was .077

Interpretation of Residual Variances and Covariances

- $\sigma^2_{e:PERF} = 6.623$: the residual variance for PERF
  - The R^2 for PERF was .240 (the same as before)

- $\sigma^2_{e:USE} = 243.303$: the residual variance for USE
  - The R^2 for USE was .024 (a very small effect)

- $\sigma_{e:PERF,USE} = 2.901$: the residual covariance between USE and PERF
  - This value was not significant, meaning we can potentially set its value to zero and re-estimate the model

- Each describes the amount of variance not accounted for in each dependent (endogenous) variable

Regression Model Explained Variance

- After adding both independent variables HSL and CC, the residual variance of performance was $\sigma^2_{e:PERF} = 6.631$
  - Value of PERF's variance estimate from Model 01

- Therefore, the inclusion of these variables reduced the variance of PERF from 8.722 to 6.631, for an

$$R^2 = \frac{8.722 - 6.631}{8.722} = .24$$

- lavaan reports this value by using the inpsect() function:

```
> inspect(model01.fit, what="r2")
 perf   use
0.240 0.024
```

IDENTIFICATION OF PATH MODELS

Identification of Path Models

- Model identification is necessary for statistical models to have meaningful results
  - From the error on the previous slide, we essentially had too many unknown values (parameters) and not enough places to put the parameters in the model

- For path models, identification can be a very difficult thing to understand (we will stick to the basics here)

- Because of their unique structure, path models must have identification in two ways:
  - "Globally" – so that the total number of parameters does not exceed the total number of means, variances, and covariances of the endogenous and exogenous variables
  - "Locally" – so that each individual equation is identified

- Identification is guaranteed if a model is both "globally" and "locally" identified

Global Identification: "T-rule"

- A necessary but not sufficient condition for path models is that of having equal to or fewer model parameters than there are distributional parameters

- As the path models we discuss assume the multivariate normal distribution, we have two matrices of parameters with which to work
  - Distributional parameters: the elements of the mean vector and (or more precisely) the covariance matrix

- For the MVN, the so-called T-rule states that a model must have equal to or fewer parameters than the unique elements of the covariance matrix of all endogenous and exogenous variables (the sum of all variables in the analysis)
  - Let $s = p + q$, the total of all endogenous (p) and exogenous (q) variables
  - Then the total unique elements are $\frac{s(s+1)}{2}$

More on the "T-rule"

- The classical definition of the "T-rule" counts the following entities as model parameters:
  - Direct effects (regression slopes)
  - Residual variances
  - Residual covariances
  - Exogenous variances
  - Exogenous covariances

- Missing from this list are:
  - The set of exogenous variable means
  - The set of intercepts for endogenous variables

- Each of the missing entities are part of the lavaan likelihood function, but are considered "saturated" so no additional parameters can be added
  - These do not enter into the equation for the covariance matrix of the endogenous and exogenous variables

Global Identification of Our Example

Model	Endogenous Variables (p)	Exogenous Variables (q)	Unique Covariance Matrix Elements	Model Parameters (excluding Exo. Var. means)	Identification Status
Multivariate Regression Full Model (slide 39)	2	2	$\frac{4^{*}(4+1)}{2} = 10$	10: $\sigma^2_{HSL}, \sigma_{HSL,CC}, \sigma^2_{CC}, \beta^{PERF}_{HSL}, \beta^{PERF}_{CC}, \beta^{USE}_{HSL}, \beta^{USE}_{CC}, \sigma^2_{e:PERF}, \sigma^2_{e:USE}, \sigma_{e:PERF,USE}$	Just Identified

T-rule Identification Status

- **Just-Identified:** number of covariances = number of model parameters
  - Necessary for identification, but no model fit indices available
- **Over-Identified:** number of covariances > number of model parameters
  - Necessary for identification; model fit indices available
- **Under-Identified:** number of covariances < number of model parameters
  - **Model is <u>NOT IDENTIFIED</u>:** No results available
  - Do not pass go…do not collect $200

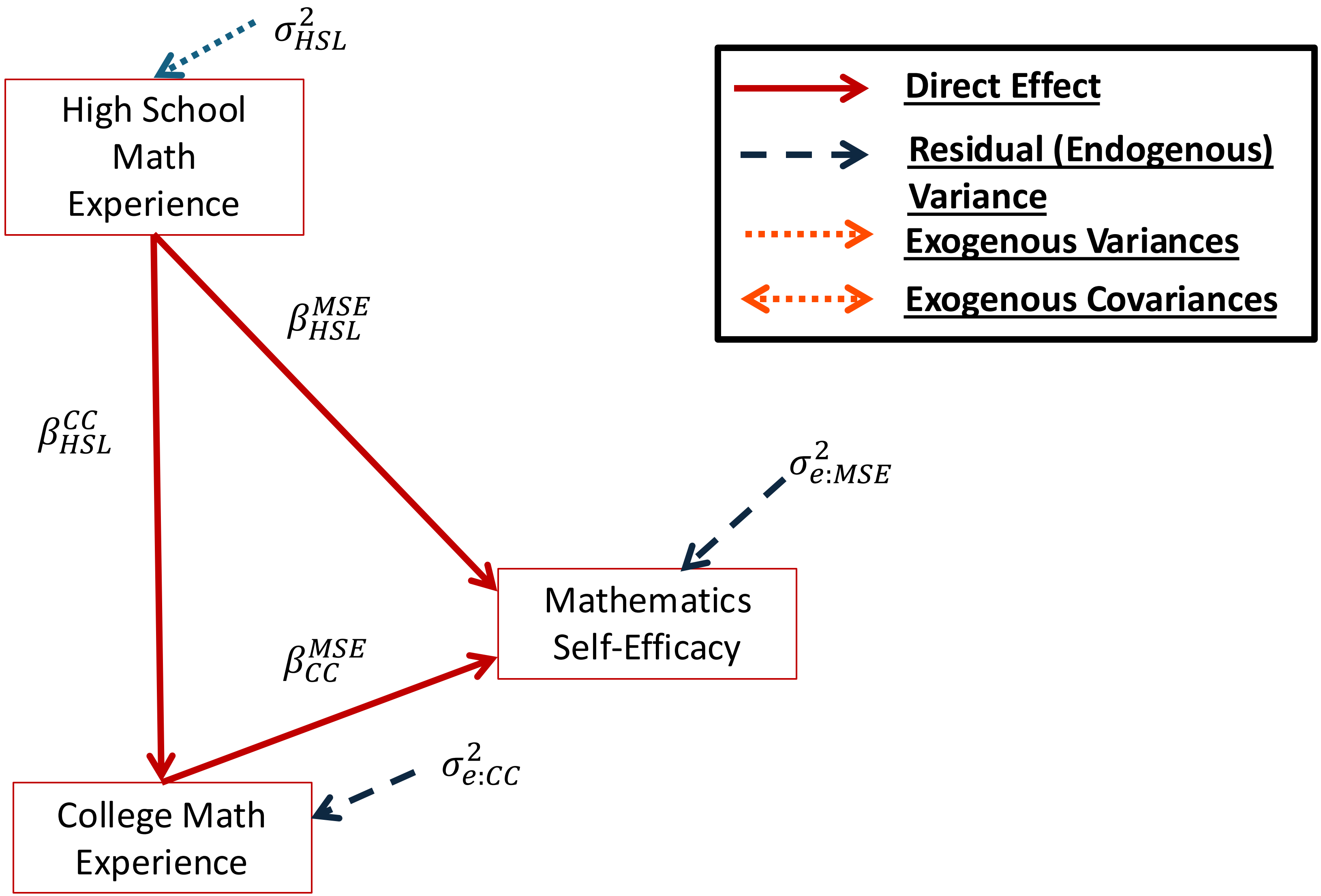
Moving from Global to Local Identification: Types of Path Models

- For most research designs, global identification will suffice
  - For the most part, **recursive path models** will be identified if the “t-rule” is met

- A **recursive path model** is one where the direct effects are unidirectional – no feedback loops
  - Our path model is an example of a recursive path model

- A **non-recursive path model** is one where the direct effects are bidirectional for some variables – feedback loops are present
  - Difficult to envision using cross-sectional data
  - More frequent in econometrics
  - Different estimation algorithms used (see the next few slides)

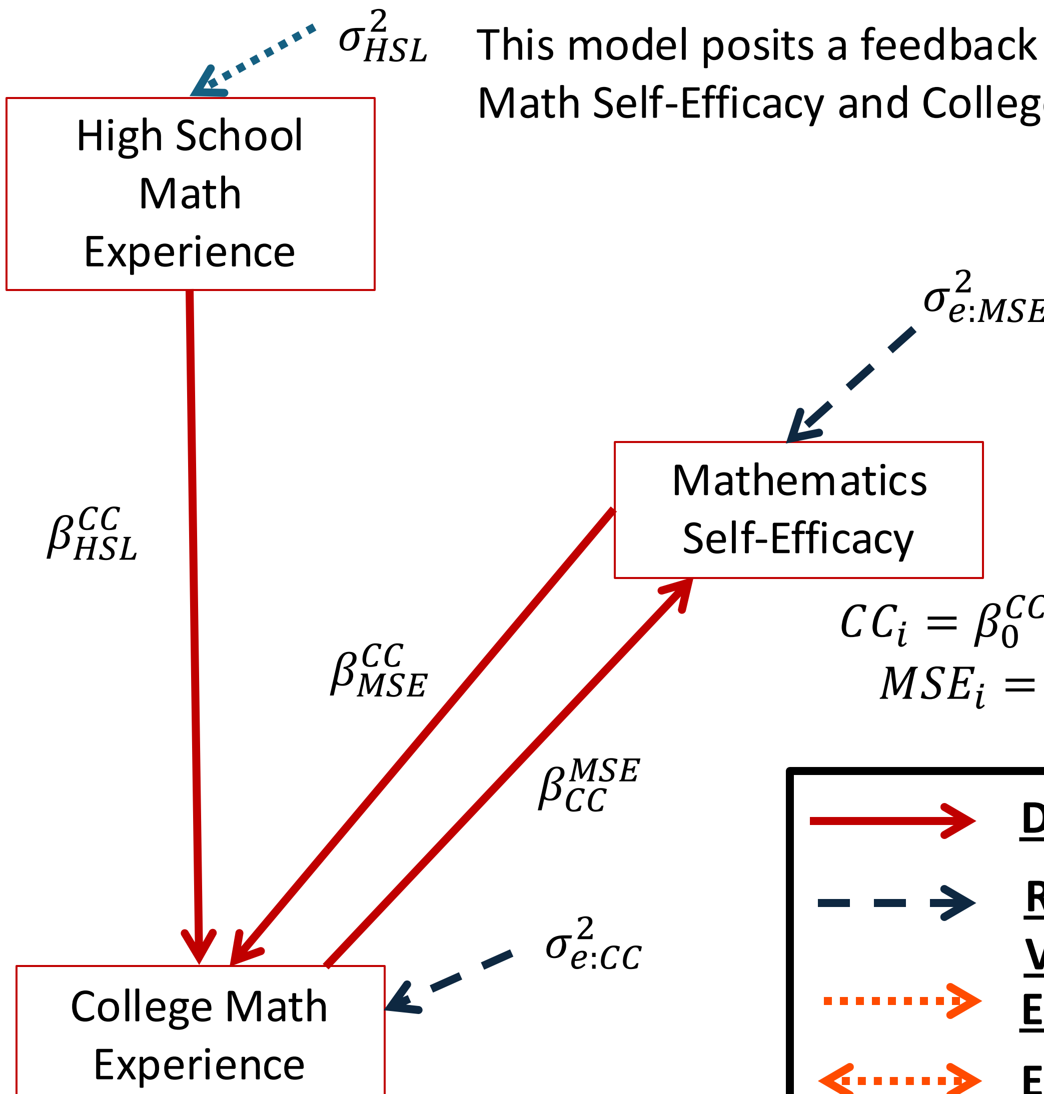
Basic Path Model: Recursive



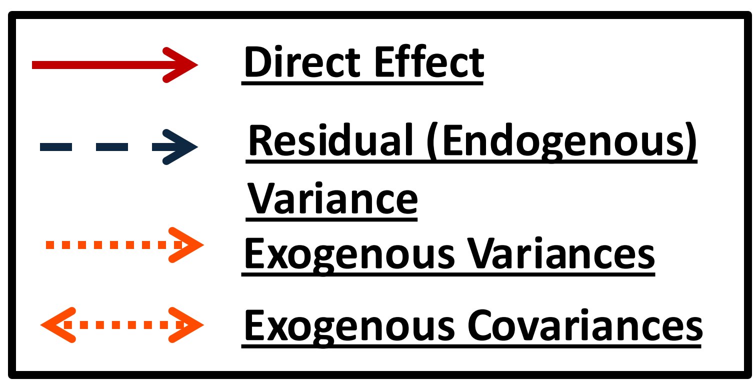
A Non-Recursive Path Model

This model posits a feedback relationship between Math Self-Efficacy and College Math Experience

Global Identification is met: 6 unique covariance elements; 6 model parameters



$$CC_i = \beta_0^{CC} + \beta_{HSL}^{CC} HSL_i + \beta_{MSE}^{CC} + e_i^{CC}$$
$$MSE_i = \beta_0^{MSE} + \beta_{CC}^{MSE} CC_i + e_i^{MSE}$$



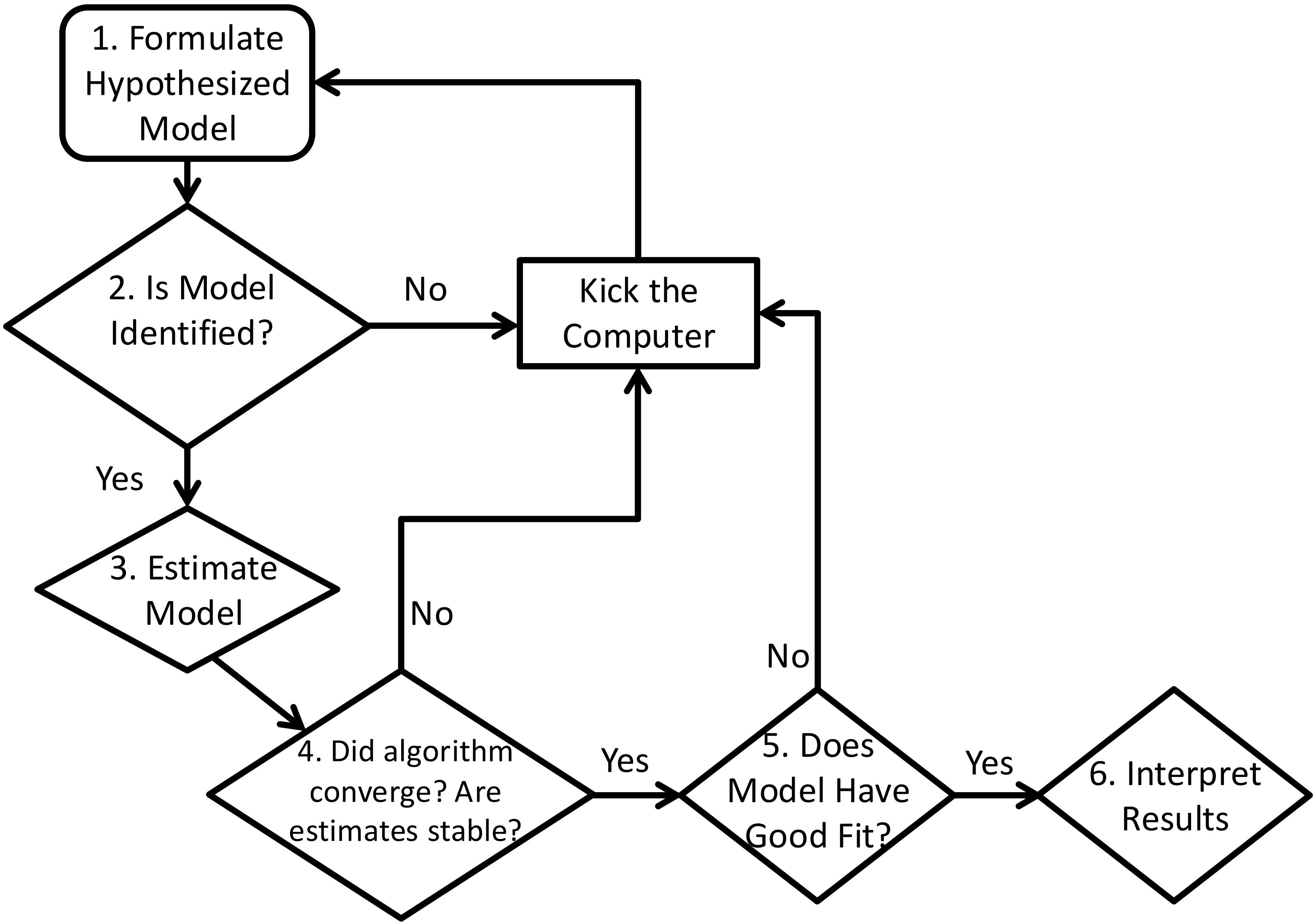
Guiding Identification Principals

- If you have a recursive model (no feedback loops) make sure:
  - # of model parameters $\leq$ # of unique covariance elements
  - No undirected paths (residual covariance) connecting variables with direct effects
    - Does not make sense to say one variable causes another yet their correlation is unexplained

- If you have a non-recursive model (feedback loops):
  - Think critically about whether such a model can be investigated by your data (cross-sectional versus longitudinal)
  - Attempt to determine if the model meets the rank condition
  - Investigate model output for irregularities (very large effects relative to the scale of the variables

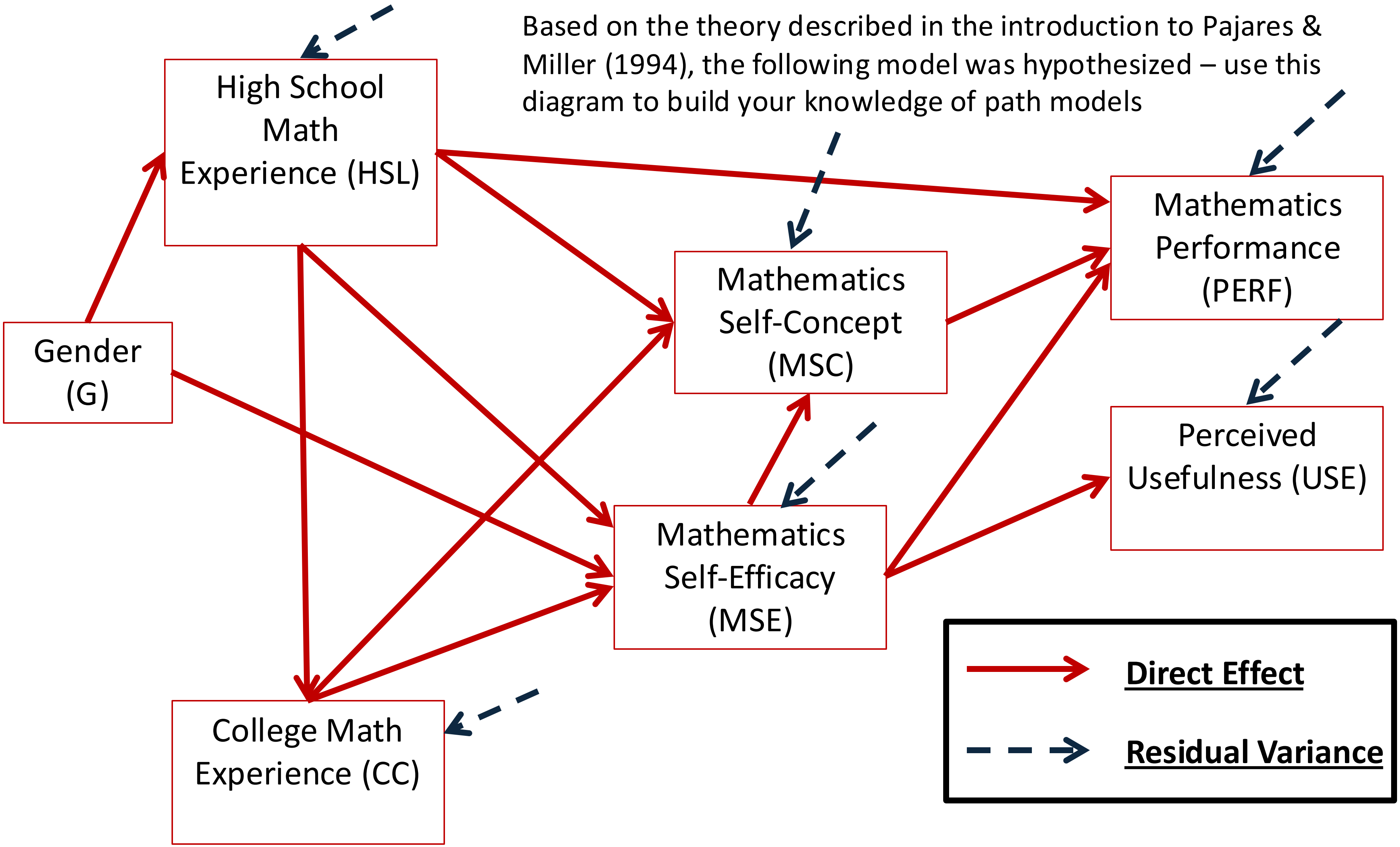
THE FINAL PATH MODEL: PUTTING IT ALL TOGETHER

A Path Model of Path Analysis Steps



Our Destination: Overall Path Model

Based on the theory described in the introduction to Pajares & Miller (1994), the following model was hypothesized – use this diagram to build your knowledge of path models



Path Model Setup – Questions for the Analysis

- How many variables are in our model? 7
  - Gender, HSL, CC, MSC, MSE, PERF, and USE
- How many variables are endogenous? 6
  - HSL, CC, MSC, MSE, PERF, and USE
- How many variables are exogenous? 1
  - Gender
- Is the model recursive or non-recursive?
  - Recursive – no feedback loops present

Path Model Setup – Questions for the Analysis

- Is the model identified?
  - Check the t-rule first (and only as it is recursive)
  - How many covariance terms are there in the all-variable matrix?

$$\frac{7^{*}(7+1)}{2} = 28$$

  - How many model parameters are to be estimated?
    - 12 direct paths
    - 6 residual variances
    - 1 variance of the exogenous variable
    - **(19 model parameters for the covariance matrix)**
    - 6 endogenous variable intercepts
      - Not relevant for t-rule identification, but counted in Mplus
- **The model is over-identified**
  - 28 total variance/covariances but 19 model parameters
  - We can use lavaan to run our analysis

Overall Hypothesized Path Model: Equation Form

- The path model from can be re-expressed in the following 6 endogenous variable regression equations:

1. $HSL_i = \beta_0^{HSL} + \beta_G^{HSL} G_i + e_i^{HSL}$
2. $CC_i \quad = \beta_0^{CC} + \beta_{HSL}^{CC} HSL_i + e_i^{CC}$
3. $MSE_i = \beta_0^{MSE} + \beta_G^{MSE} G_\mathrm{i} + \beta_{HSL}^{MSE} HSL_i + \beta_{CC}^{MSE} CC_i + e_i^{MSE}$
4. $MSC_i = \beta_0^{MSC} + \beta_{HSL}^{MSC} HSL_i + \beta_{CC}^{MSC} CC_i + \beta_{MSE}^{MSC} MSE_i + e_i^{MSC}$
5. $USE_i = \beta_0^{USE} + \beta_{MSE}^{UES} MSE_i + e_i^{USE}$
6. $PERF_i = \beta_0^{PERF} + \beta_{HSL}^{PERF} HSL_i + \beta_{MSE}^{PERF} MSE_i + \beta_{MSC}^{PERF} MSC_i + e_i^{PERF}$

Path Model Estimation in lavaan

- Having (1) constructed our model and (2) verified it was identified using the t-rule and that it is a recursive model, the next step is to (3) estimate the model with lavaan

```
model06.syntax = "
#endogenous variable equations
  perf ~ hsl + msc + mse
  use  ~ mse
  mse  ~ hsl + cc + gender
  msc  ~ mse + cc + hsl
  cc   ~ hsl
  hsl  ~ gender

#endogenous variable intercepts
  perf ~ 1
  use  ~ 1
  mse  ~ 1
  msc  ~ 1
  cc   ~ 1
  hsl  ~ 1

#endogenous variable residual variances
  perf ~~ perf
  use  ~~ use
  mse  ~~ mse
  msc  ~~ msc
  cc   ~~ cc
  hsl  ~~ hsl

#endogenous variable residual covariances
  #none specfied in the original model so these have zeros:
  perf ~~ 0*use + 0*mse + 0*msc + 0*cc + 0*hsl
  use  ~~ 0*mse + 0*msc + 0*cc + 0*hsl
  mse  ~~ 0*msc + 0*cc + 0*hsl
  msc  ~~ 0*cc + 0*hsl
  cc   ~~ 0*hsl

#exogeneous variables put into likelihood function:

  #means(intercepts)
  gender ~ 1

  #variances
  gender ~~ gender

"
```

Model Fit Evaluation

- First, we check convergence:

```
> summary(model06.fit, fit.measures=TRUE, standardized=TRUE)
lavaan 0.6-19 ended normally after 68 iterations
```

  - lavaans's algorithm converged

- Second, we check for abnormally large standard errors
  - None too big, relative to the size of the parameter
  - Indicates identified model

- Third, we look at the model fit statistics:

Model Fit Statistics

```
Model Test User Model:
                                              Standard      Scaled
  Test Statistic                                58.896      58.913
  Degrees of freedom                                 9           9
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.000
    Yuan-Bentler correction (Mplus variant)

Model Test Baseline Model:

  Test statistic                               619.926     629.882
  Degrees of freedom                                21          21
  P-value                                        0.000       0.000
  Scaling correction factor                                  0.984

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.917       0.918
  Tucker-Lewis Index (TLI)                       0.806       0.809

  Robust Comparative Fit Index (CFI)                         0.918
  Robust Tucker-Lewis Index (TLI)                            0.809

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -6126.013   -6126.013
  Scaling correction factor                                  0.937
      for the MLR correction
  Loglikelihood unrestricted model (H1)      -6096.565   -6096.565
  Scaling correction factor                                  0.953
      for the MLR correction

  Akaike (AIC)                               12304.025   12304.025
  Bayesian (BIC)                             12404.332   12404.332
  Sample-size adjusted Bayesian (SABIC)      12321.850   12321.850

Root Mean Square Error of Approximation:

  RMSEA                                          0.126       0.126
  90 Percent confidence interval - lower         0.096       0.096
  90 Percent confidence interval - upper         0.157       0.157
  P-value H_0: RMSEA <= 0.050                    0.000       0.000
  P-value H_0: RMSEA >= 0.080                    0.994       0.994

  Robust RMSEA                                               0.140
  90 Percent confidence interval - lower                     0.108
  90 Percent confidence interval - upper                     0.175
  P-value H_0: Robust RMSEA <= 0.050                         0.000
  P-value H_0: Robust RMSEA >= 0.080                         0.999

Standardized Root Mean Square Residual:

  SRMR                                           0.056       0.056
```

This is a likelihood ratio (deviance) test comparing our model (H_0) with the saturated model – The saturated model fits much better (but that is typical).

This compares the independence model (H_0) to the saturated model (H_1) – it indicates that there is significant covariance between variables

The CFI estimate is .918 and the TLI is .809. Good fit is considered 0.95 or higher.

The RMSEA estimate is 0.126. Good fit is considered 0.05 or less.

The average standardized residual covariance is 0.056. Good fit is less than 0.05.

Based on the model fit statistics, we can conclude that our model does not do a good job of approximating the covariance matrix – so we cannot make inferences with these results (biased standard errors and effects may occur)

Model Modification

- Now that we have concluded that our model fit is poor we must modify the model to make the fit better
  - Our modifications are purely statistical – which draws into question their generalizability beyond this sample
- ***<u>Generally, model modification should be guided by theory</u>***
  - However, we can inspect the normalized residual covariance matrix (like z-scores) to see where our biggest misfit occurs

```
> residuals(model06.fit, type="normalized")
$type
[1] "normalized"

$cov
         perf    use    mse    msc     cc    hsl gender
perf   -0.076
use    -0.159  0.041
mse    -0.071 -0.110 -0.086
msc     0.059  5.051 -0.039  0.043
cc     -0.028  0.720 -0.377 -0.161  0.046
hsl     0.006  0.559  0.085  0.105 -0.034  0.039
gender -1.522 -0.027 -0.422 -1.452 -2.567  0.091  0.000

$mean
  perf    use    mse    msc     cc    hsl gender
-0.014  0.126  0.012  0.211  0.009  0.004  0.000
```

Two normalized residual covariances are bigger than +/-1.96:
MSC with USE and
CC with Gender

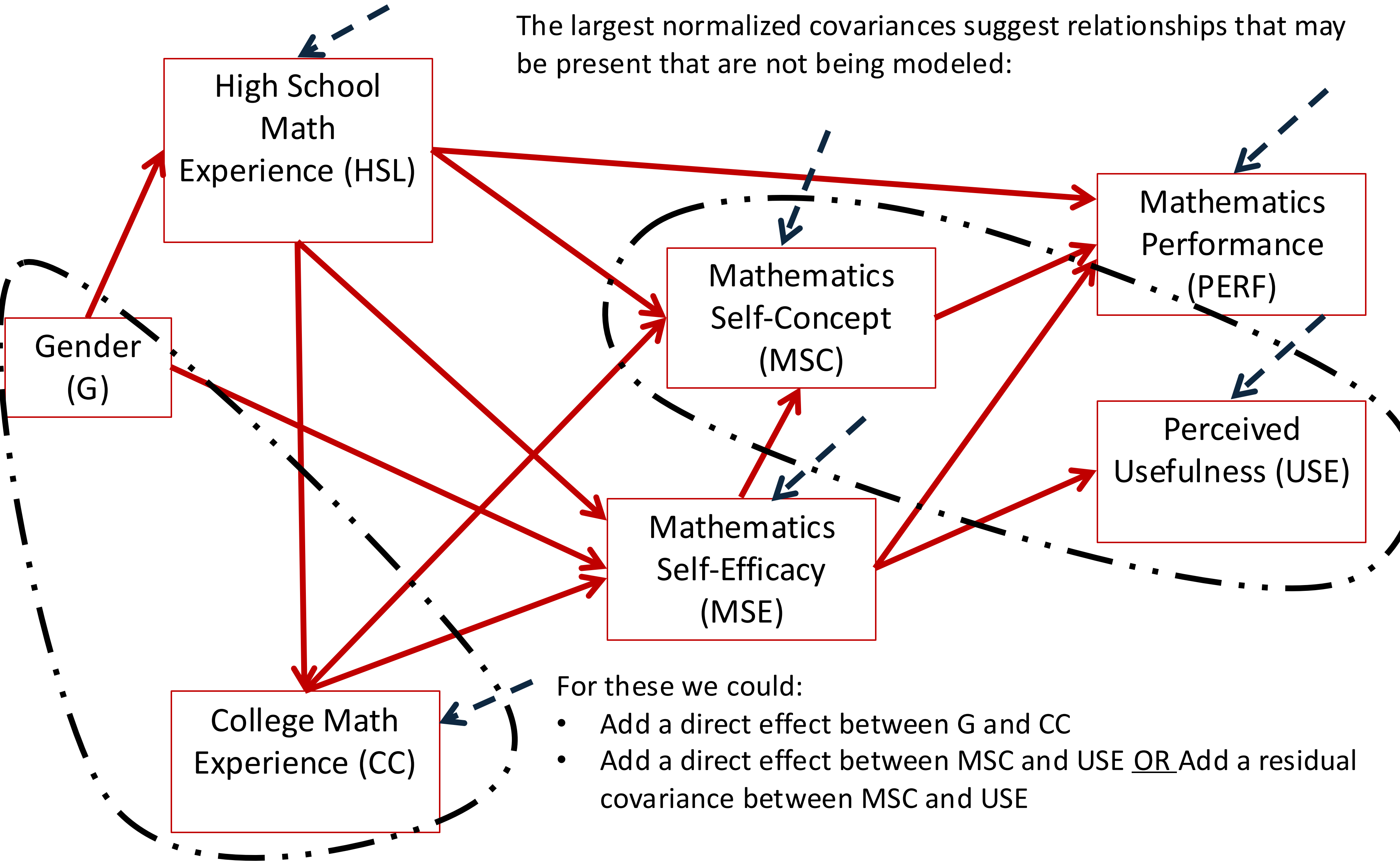
Modification Indices: More Help for Fit

- As we used Maximum Likelihood to estimate our model, another useful feature is that of the modification indices
  - Modification indices are actually Score (LaGrangian Multiplier) tests that attempt to suggest the change in the log-likelihood for adding a given model parameter (larger values indicate a better fit for adding the parameter)

```
> modindices(model06.fit)
     lhs op   rhs     mi    epc sepc.lv sepc.all sepc.nox
31   use ~~   msc 41.517 70.912  70.912    0.386    0.386
52   use  ~   msc 40.032  0.451   0.451    0.490    0.490
60   msc  ~   use 41.517  0.299   0.299    0.275    0.275
```

Our Destination: Overall Path Model

The largest normalized covariances suggest relationships that may be present that are not being modeled:



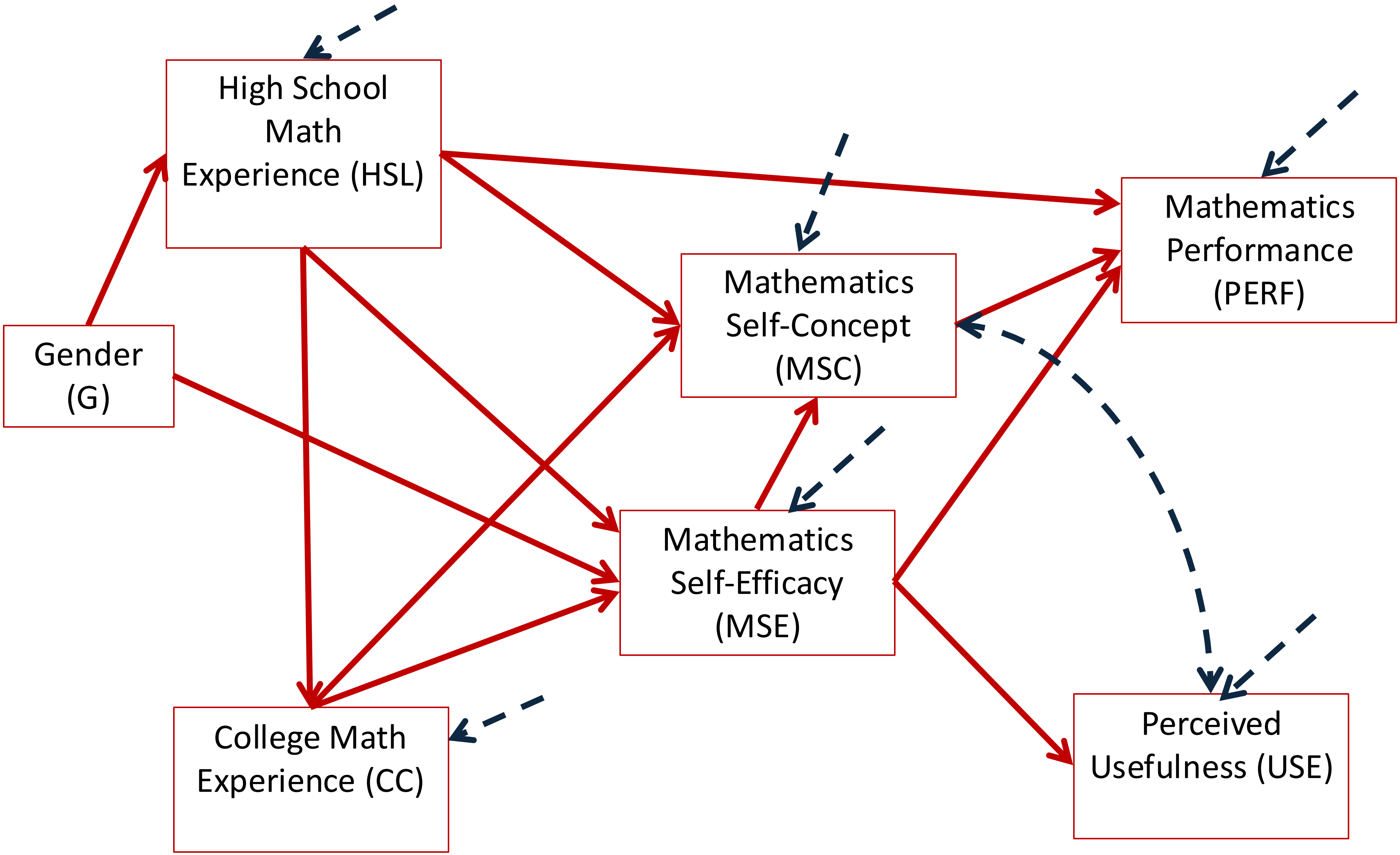
For these we could:

- Add a direct effect between G and CC
- Add a direct effect between MSC and USE <u>OR</u> Add a residual covariance between MSC and USE

Modification Indices Results

- The modification indices have three large values:
  - A direct effect predicting MSC from USE
  - A direct effect predicting USE from MSC
  - A residual covariance between USE and MSC
- Note: the MI value is -2 times the change in the log-likelihood and the EPC is the expected parameter value
  - The MI is like a 1 DF Chi-Square Deviance test
    - Values greater than 3.84 are likely to be significant changes in the log-likelihood
- Because all three happen for the same variable, we can only choose one
  - This is where theory would help us decide
- As we do not know theory, we will choose to add a residual covariance between USE and MSC
  - Their covariance is **unexplained** by the model – not a great theoretical statement (but will allow us to make inferences if the model fits)
  - MI = 41.529
  - EPC = 70.912

Modified Model



Assessing Model fit of the Modified Model

- Now we must start over with our path model decision tree
  - The model is identified (now 20 parameters < 28 covariances)
  - lavaan estimation converged; Standard errors look acceptable
- Model fit statistics:

```
Estimator                                         ML       Robust
Minimum Function Test Statistic               14.827       14.393
Degrees of freedom                                 8            8
P-value (Chi-square)                           0.063        0.072
Scaling correction factor                                   1.030
  for the Yuan-Bentler correction (Mplus variant)
```

The comparison with the saturated model suggests our model fits statistically

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.049       0.048
  90 Percent Confidence Interval          0.000  0.088       0.000  0.086
  P-value RMSEA <= 0.05                          0.457       0.484
```

The RMSEA is 0.048, which indicates good fit

```
User model versus baseline model:

  Comparative Fit Index (CFI)                    0.989       0.990
  Tucker-Lewis Index (TLI)                       0.970       0.972
```

The CFI and TLI both indicate good fit

```
Standardized Root Mean Square Residual:

  SRMR                                           0.035       0.035
```

The SRMR also indicates good fit

Therefore, we can conclude the model adequately approximates the covariance matrix – meaning we can now inspect our model parameters…

Model Parameter Investigation

	Estimate	Std.err	Z-value	P(>|z|)	Std.lv	Std.all
Regressions:						
perf ~						
hsl	0.153	0.107	1.432	0.152	0.153	0.068
msc	0.037	0.009	4.147	0.000	0.037	0.215
mse	0.139	0.013	10.700	0.000	0.139	0.557
use ~						
mse	0.277	0.073	3.803	0.000	0.277	0.209
mse ~						
hsl	4.138	0.406	10.203	0.000	4.138	0.459
cc	0.393	0.105	3.723	0.000	0.393	0.194
gender	4.168	1.160	3.593	0.000	4.168	0.166
msc ~						
mse	0.736	0.066	11.119	0.000	0.736	0.512
cc	0.519	0.117	4.434	0.000	0.519	0.179
hsl	2.824	0.593	4.764	0.000	2.824	0.218
cc ~						
hsl	0.662	0.247	2.686	0.007	0.662	0.149
hsl ~						
gender	0.208	0.154	1.348	0.178	0.208	0.075
Covariances:						
use ~~						
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Intercepts:						
perf	1.023	0.782	1.309	0.191	1.023	0.344
use	32.162	5.393	5.964	0.000	32.162	2.035
mse	47.738	2.237	21.339	0.000	47.738	4.006
msc	-23.369	4.484	-5.211	0.000	-23.369	-1.364
cc	7.072	1.268	5.576	0.000	7.072	1.201
hsl	4.843	0.091	53.390	0.000	4.843	3.663
gender	0.346	0.025	13.599	0.000	0.346	0.727
Variances:						
perf	3.763	0.309	12.173	0.000	3.763	0.427
use	238.854	19.097	12.507	0.000	238.854	0.956
mse	97.294	7.758	12.541	0.000	97.294	0.685
msc	142.912	10.793	13.241	0.000	142.912	0.487
cc	33.923	2.456	13.813	0.000	33.923	0.978
hsl	1.738	0.126	13.813	0.000	1.738	0.994
gender	0.226	0.008	28.835	0.000	0.226	1.000

Some parameters are not significant…we could omit those, but we'll leave them for now

Model #2 Standardized Parameter Estimates

- We can interpret the std.all standardized parameter estimates for all variables except gender
  - It is not continuous so SD of gender does not make sense

- A 1-SD increase in HSL means CC increases by 0.149 SD

```
                   Estimate  Std.err  Z-value  P(>|z|)   Std.lv  Std.all
Regressions:
  perf ~
    hsl               0.153    0.107    1.432    0.152    0.153    0.068
    msc               0.037    0.009    4.147    0.000    0.037    0.215
    mse               0.139    0.013   10.700    0.000    0.139    0.557
  use ~
    mse               0.277    0.073    3.803    0.000    0.277    0.209
  mse ~
    hsl               4.138    0.406   10.203    0.000    4.138    0.459
    cc                0.393    0.105    3.723    0.000    0.393    0.194
    gender            4.168    1.160    3.593    0.000    4.168    0.166
  msc ~
    mse               0.736    0.066   11.119    0.000    0.736    0.512
    cc                0.519    0.117    4.434    0.000    0.519    0.179
    hsl               2.824    0.593    4.764    0.000    2.824    0.218
  cc ~
    hsl               0.662    0.247    2.686    0.007    0.662    0.149
  hsl ~
    gender            0.208    0.154    1.348    0.178    0.208    0.075

Covariances:
  use ~~
    msc              70.249   10.358    6.782    0.000   70.249    0.380
```

Model #2 std.nox Interpretation

- The STDY standardization does not standardize by the SD of the X variable
  - So its interpretation makes sense for Gender (1 = male)

Here, males have an average MSE (intercept) that is .166 SD higher than females

```
> standardizedSolution(model03.fit, type="std.nox")
     lhs op    rhs est.std    se      z pvalue
1   perf  ~    hsl   0.068 0.048  1.432  0.152
2   perf  ~    msc   0.215 0.052  4.147  0.000
3   perf  ~    mse   0.557 0.052 10.700  0.000
4    use  ~    mse   0.209 0.055  3.803  0.000
5    mse  ~    hsl   0.459 0.045 10.203  0.000
6    mse  ~     cc   0.194 0.052  3.723  0.000
7    mse  ~ gender   0.166 0.046  3.593  0.000
8    msc  ~    mse   0.512 0.046 11.119  0.000
9    msc  ~     cc   0.179 0.040  4.434  0.000
10   msc  ~    hsl   0.218 0.046  4.764  0.000
11    cc  ~    hsl   0.149 0.055  2.686  0.007
12   hsl  ~ gender   0.075 0.055  1.348  0.178
```

Overall Model Interpretation

- High School Experience is a significant predictor of College Experience
  - More High School Experience means more College Experience
- High School Experience, College Experience, and Gender are significant predictors of Math Self-Efficacy
  - More High School and College Experience means higher Math Self-Efficacy
  - Men have higher Math Self-Efficacy than Women
- High School Experience, College Experience, and Math Self-Efficacy are significant predictors of Math Self-Concept
  - More High School and College Experience and higher Math Self-Efficacy mean higher Math Self-Concept

Overall Model Interpretation, Continued

- Higher Math Self-Efficacy means significantly higher Perceived Usefulness

- Higher Math Self-Efficacy and Math Self-Concept result in higher Math Performance scores
  - High school experience was not significantly related to performance

- Math Self-Concept and Perceived Usefulness have a significant residual covariance

Model Interpretation: Explained Variability

- The R^2 for each endogenous variable:

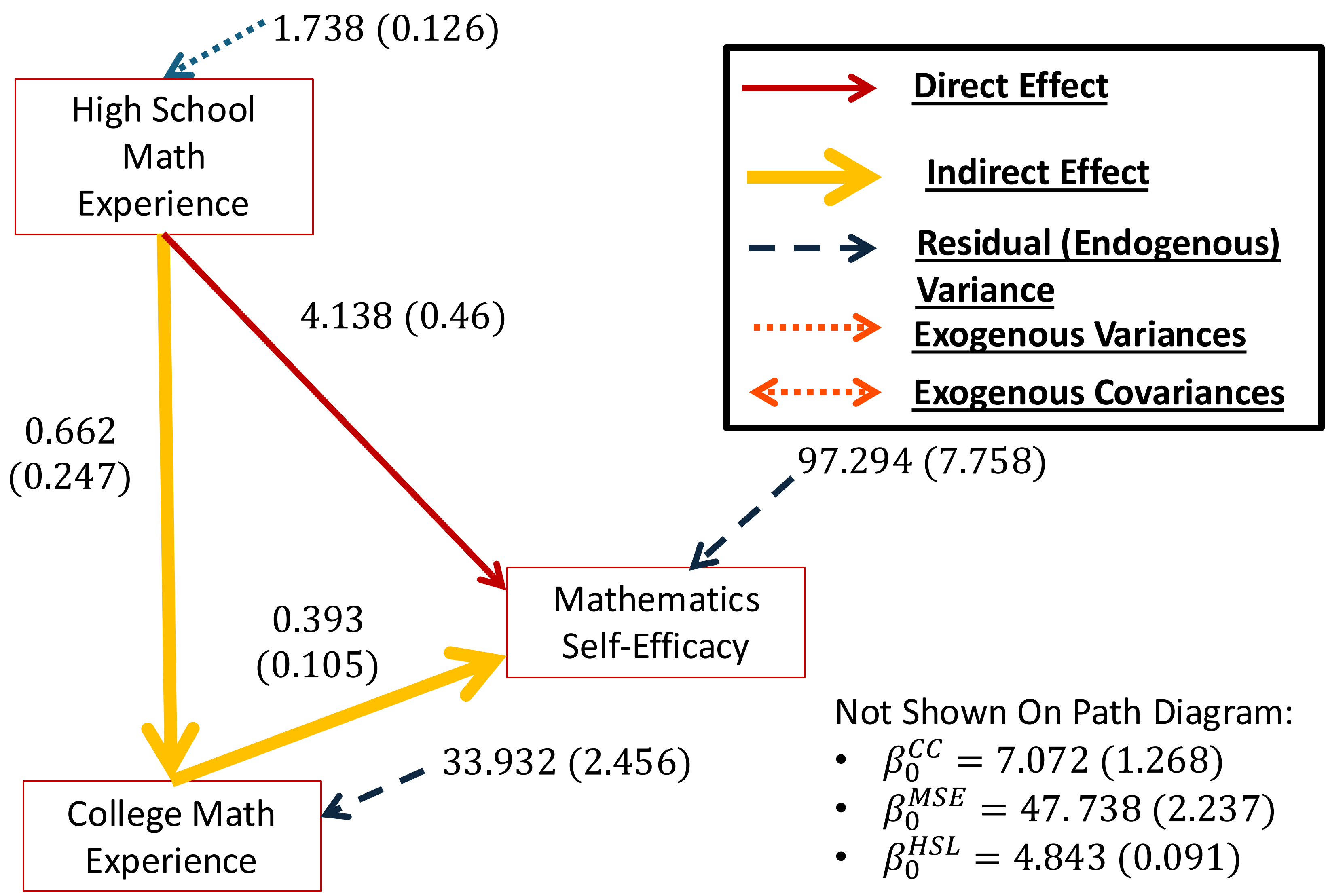
```
> inspect(model03.fit, what="r2")
 perf   use   mse   msc    cc   hsl
0.573 0.044 0.315 0.513 0.022 0.006
```

- Note how high school experience, college experience, and perceived usefulness both have low percentages of variance accounted for by the model
    - We could have increased the R^2 for USE by adding the direct path between MSC and USE instead of the residual covariance

Indirect Paths

- Because High School Experience (HSL) predicted College Experience (CC) and College Experience (CC) predicted Math Self-Efficacy (MSE), an indirect path between HSL and MSE exists
  - An indirect path represents the effect of one variable on another, as mediated by one or more variables
- The indirect path suggests that the relationship between High School Experience (HSL) and Math Self-Efficacy is mediated by College Experience (CC)
  - More formally, the mediational relationship is hypothesized by the path model, a formal test of hypothesis is needed to establish College Experience as a mediator of High School Experience and Math Self-Efficacy
- A number of other indirect paths exist in the model

Direct and Indirect Effects of HSL on MSE (Part of Model 3)



Not Shown On Path Diagram:

- $\beta_0^{CC} = 7.072\ (1.268)$
- $\beta_0^{MSE} = 47.738\ (2.237)$
- $\beta_0^{HSL} = 4.843\ (0.091)$

Calculation of Indirect Effects

- The indirect effect of High School Experience on Math Self-Efficacy is the combination of two path coefficients:
  - The path between High School (HSL) and College (CC) Experience: $\beta_{HSL}^{CC} = 0.662$
  - The path between College Experience (CC) and Math Self-Efficacy (MSE): $\beta_{CC}^{MSE} = 0.393$

- The **indirect effect** of HSL on MSE is the product of these two terms: $\beta_{HSL}^{CC}\beta_{CC}^{MSE} = 0.662^{*}0.393 = 0.260$

- The indirect effect is the amount of increase in the outcome variable (MSE in this case) that comes indirectly by a one-unit increase in the initiating variable (HSL in this case)
  - As HSL increases by one unit, CC increases by 0.662 (the direct effect of HSL on CC)
  - Then, as CC increases by 0.662, HSL increases by 0.393 (the direct effect of CC on MSE)

- Indirectly, MSE increases by 0.260 (the multiplication of the two direct effects) for every one unit increase of HSL

Total Effects

- Finally, of concern in mediational models and general path models is the total effect of one variable on another
- The **total effect** is the sum of all direct and indirect effects
  - It represents the **total** increase in the outcome variable for a one-unit increase in the initiating variable
- In our example, the total effect of High School Experience (HSL) on Math Self-Efficacy (MSE) is the sum of the direct and indirect effects:

$$\beta_{HSL}^{MSE} + \beta_{HSL}^{CC}\beta_{CC}^{MSE} = 4.138 + 0.662^{*}0.393 = 4.398$$

- This means that for every one-unit increase in HSL, the total increase in MSE is 4.398
  - The direct effect represents the increase holding CC constant, which is implausible in this model

Hypothesis Tests for Indirect and Total Effects in lavaan

- Of importance in the understanding of mediating variables is the test of hypothesis for the indirect effect
  - If the indirect effect (the product of the two direct effects) is significant, then the third variable is said to be a mediator

- Hypothesis tests for the indirect effect have become a hot topic in recent years
  - This test uses a bootstrap (resampling) technique to get the p-value

```
#Model 08: Adding indirect effects to full path model -

model08.syntax = "
#endogenous variable equations
  perf ~ hsl + msc + mse
  use  ~ mse
  mse  ~ b_hsl_mse*hsl + b_cc_mse*cc + gender
  msc  ~ mse + cc + hsl
  cc   ~ b_hsl_cc*hsl
  hsl  ~ gender
```

- In lavaan, first label parameters:

```
#indirect effect of interest:
  ind_hsl_mse := b_hsl_cc*b_cc_mse

#total effect of interest:
  tot_hsl_mse := b_hsl_mse + (b_hsl_cc*b_cc_mse)
```

- Then add effects:

lavaan Output

- Lavaan provides the total and indirect effects between terminating and originating variables
  - If the standardized=TRUE command is included in the summary() function call, the standardized versions of these effects are also given (the increase in standard deviations)

```
                     Estimate  Std.err  Z-value  P(>|z|)   Std.lv  Std.all
Defined parameters:
    ind_hsl_mse         0.260    0.111    2.339    0.019    0.260    0.029
    tot_hsl_mse         4.398    0.414   10.618    0.000    4.398    0.488
```

- Here, our output suggests the indirect effect is significant, so we say that CC **mediates** the relationship between HSL and MSE

ADDITIONAL MODELING CONSIDERATIONS IN PATH ANALYSIS

Additional Modeling Considerations

- The path analysis we just ran was meant to be an introduction to the topic and the field
  - It is much more complex than what was described

- In particular, our path analysis assumed all variables to be
  - Continuous and Multivariate Normal
  - Measured with perfect reliability

- In reality, neither of these are true

- Structural equation models (path models with latent variables) will help with variables with measurement error

- Modifications to model likelihoods or different distributional assumptions will help with the normality assumption

About Causality

- You will read a lot of talk about path models indicating causality, or how path models are causal models

- It is important to note that causality can rarely, if ever, be inferred on the basis of observational data
  - Experimental designs with random assignment and manipulations of factors will help detect causality

- With observational data, about the best you can say is that IF your model fits, then causality is ONE reason
  - But realistically, you are simply describing covariances of variables in more fancy ways/parameters

- If your model does not fit, the causality is LIKELY not occurring
  - But still could be possible if important variables are omitted

WRAPPING UP AND REFOCUSING

Key Questions for Today's Lecture

- What distinguishes path models from multivariate regression models?
- What are the identification conditions for path models?
- What is an indirect effect? What is a total effect?
- What are standardized coefficients?

Path Analysis: An Introduction

- In this lecture, we discussed the basics of path analysis
  - Model specification/identification
  - Model estimation
  - Model fit (necessary, but not sufficient)
  - Model modification and re-estimation
  - Final model parameter interpretation

- There is a lot to the analysis – but what is important to remember is the over-arching principal of multivariate analyses: covariance between variables is important
  - Path models imply very specific covariance structures
  - The validity of the results hinge upon accurately finding an approximation to the covariance matrix

Where We Are Heading…

- Over the next few weeks, we will be doing path models, but with unobserved latent variables
  - These are more commonly called factor models or structural equation models
- As with path models, structural equation models are multivariate analysis techniques
  - Models make specific implications for the covariance matrix
- Factor models shift the focus from prediction of observed variables to measurement of unobserved variables
- In the end, we will combine both – factor models for measuring unobserved variables and path models for predicting observed and unobserved variables