OCES 2003 sort of finals, Spring 2023

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Set on: Tues 16th May; due: Tues 16th May

Blurb

- The final has a maximum mark out of 30, although there are a maximum of 34 marks available
  - 24 is roughly around the A- boundary
  - anything below 14 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

  - zero on the question concerned
  - a fixed penalty starting from around 1/3 of the total marks
  - zero for the whole assignment
  - zero for the whole course
  - academic suspension, expulsion etc.

  The following counts as plagiarism (and is a non-exhaustive list):

  - copying from others and/or websites like Chegg; when found, both the copier and (where relevant) the person copied from will at a minimum get zero for the assessment (in line with university policy), with possibility for failing the whole course, and possibly with academic suspension (repeated cases will lead to expulsion)
  - copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
    * use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
    * any more than around three usages in text is probably excessive
  - copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
  - changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and will most likely lead to an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html\#b`)
- You do not have to cite lecture materials from this course, unless you want to

Problems

1. **(8 marks)** The question generally relates to Rossby waves.

   (a) In which direction do planetary Rossby waves propagate in (N, E, W, S, some combination thereof)?

   [1 mark]

   (b) What is the restoring force associated with Rossby waves?

   [1 mark]

   (c) Counter-propagating Rossby waves have been invoked as a mechanism for shear instabilities. Consider the configuration of two counter-propagating Rossby waves in an unstable configuration as sketched below:

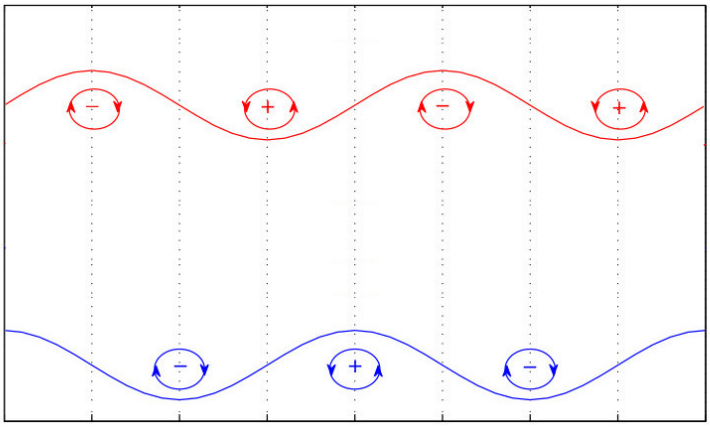

Figure 1: CRW configuration.

   Explain the Rossby wave propagation mechanism in terms of vorticity anomalies and induced velocity anomalies. With that, which direction is the red and the blue wave propagating towards by its self-induced acceleration? Justify your answer (by drawing, describing and/or otherwise).

   [4 marks]

   (d) Show how the configuration above really leads to a constructive interference, and serves as a mechanistic explanation for instability. On the other hand, what would happen if one of the waves are shifted by half a wavelength? (Hint: for the second part, you could just state the answer for full credit without necessarily needing to draw out the configuration, but it might help.)

   [2 marks]

2. **(8 marks)** Fig. 2 shows some sort of eddy kinetic energy profile associated with the observationally derived surface geostrophic flow from satellite observations, which is related to the variance (square of the deviation from the mean; *time mean* is used here, but that's not hugely relevant for this question).

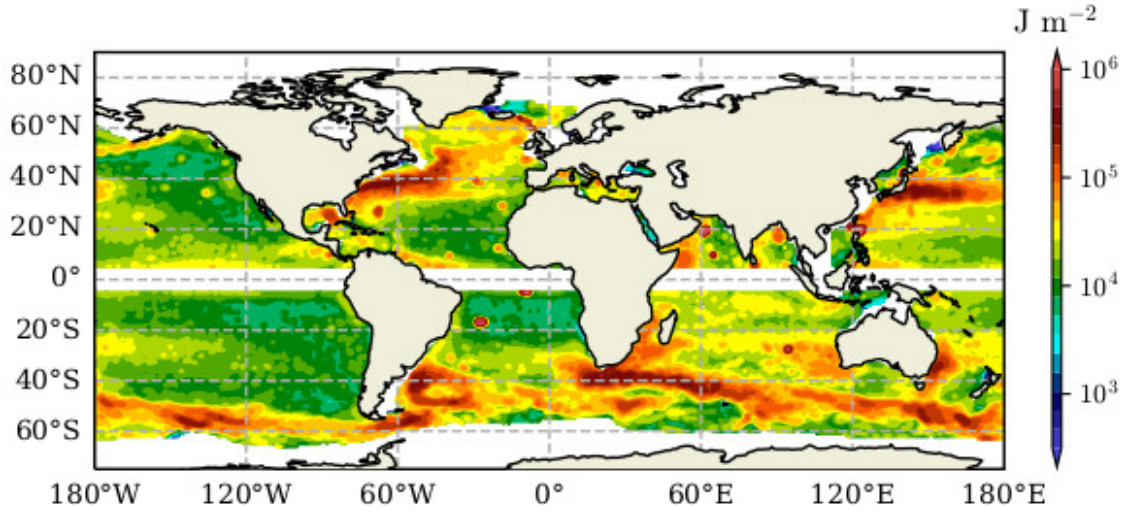

Figure 2: (Depth-integrated) EKE from observationally derived surface geostrophic flow. From Xiaoming Zhai (UEA).

(a) State in no more than 30 words what geostrophic balance means.

[1 mark]

(b) State in no more than 30 words what hydrostatic balance means.

[1 mark]

(c) Explain briefly in around 100 words how satellite altimeters function, i.e. how does it measure the sea surface height (in this case relative to the ellipsoid).

[1 mark]

(d) With the above three parts, explain in no more than 200 words or so how one derives the geostrophic flow from satellite altimeter observations.

[3 marks]

(e) Given your previous answers so far, provide explanations on why the high latitudes and equatorial region are blanked out in Fig. 2.

[2 marks]

3. **(7 marks)** Recall that gravitational attraction is given by

$$F = G\frac{M_1 M_2}{r^2},$$

where F is the force, G is some constant, $M_{1,2}$ are the relevant masses, and r is the separation between the two masses.

Tidal forcing on Earth is mostly by the moon circulating around the Earth, and could be regarded as an essentially depth-independent forcing on the Earth's ocean.

(a) Give an explanation why the tidal forcing on the Earth's ocean can be regarded as depth-independent to a good approximation.

[2 marks]

(b) Without doing any computation, it should be relatively simple to see that the gravitational attraction force of the moon on the Earth's ocean is absolutely tiny compared to the Earth's attraction of the ocean. Why is that? (Hint: The equation above should help. There are two reasons, and both possible answers are acceptable, but one is significantly more important than the other; maybe a bonus mark for you if you can say which one is more important *and why*.)

[1 marks]

(c) (Hard-ish) Since the gravitational attraction force of the moon on the Earth's ocean is tiny, the associated *vertical* accelerations at the point directly below the moon are also going to be tiny. Yet, when we humans struggle to jump a meter or so from rest, the ocean regularly experiences tidal excursions of several meters. Why is that? (Hint: see italic text in this question, and noting that water is squishy.)

[2 marks]

(d) (Hard-conceptually, but actually quite easy) Suppose for no particularly good reason a freak accident occurs and humans end up blowing up the moon, such that the moon of mass M_{moon} splits into two smaller moons each of mass $M_{\text{moon}}/2$, and they end up orbiting the Earth exactly at the opposite end of each other, such as that given in Fig. 3. Explain why the equilibrium tides on Earth remain exactly the same. (Hint: redo what I did in the lectures but with two moons of equal but half the original mass, assuming we have linearity).

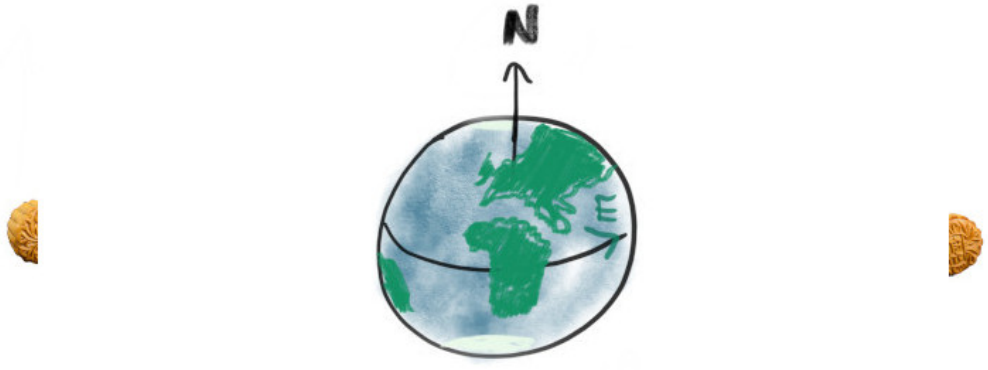

Figure 3: Two moon(cakes) with equal mass exactly at antipodes (or half a wavelength apart).

[2 marks]

4. **(7 marks)** As above, tidal forcing generates roughly depth-independent sloshing of water, that ends up transferring into and/or exciting internal waves, where the dispersion relation is approximately given by

$$\omega = \pm\sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}}.$$

   (a) Give a one or two (short-ish) sentence definition for f_0, N, k_x and k_z, and say how k_x and k_z are related to the horizontal and vertical wavelength.

   [2 marks]

   (b) Explain how phase and group velocities $\boldsymbol{c}_p$ and $\boldsymbol{c}_g$ describe different propagation of wave quantities.

   [1 marks]

   (c) Given internal waves have the property that $\boldsymbol{c}_p \cdot \boldsymbol{c}_g = 0$, explain why this tells us that internal waves are *dispersive* waves.

   [2 marks]

   (d) In contrast to surface waves where the vertical displacement is on the order of tens of centimeters to meters (ignoring cases of freak waves and tsunamis), internal waves generally can have a larger vertical excursion on the order of tens of meters. What is the physical reason that allows this? Explain your answer. (Hint: what is the relevant restoring force for these waves?)

   [2 marks]

5. **(4 marks)** As you may know there are people in the world who believe the world is flat, but they also believe in the Gulf Stream. Suppose the world really was flat, and the rotation axis is pointing up from the disc illustrated in Fig. 4. Go through the argument relating to gyre circulation for Stommel's model, and show that there is no Western intensification and therefore no Western Boundary Currents (such as the Gulf Stream) on a flat Earth, and their beliefs are contradictory in this case (among various others...)

Figure 4: Illustration of flat Earth, with rotation axis pointing up from center of the disc.

[4 marks]

(Note: While it is entirely possible to get full marks in this question with no words and just three symbols, that's probably a bit risky to try, and a bit of explanation would be better.)