OCES 2003 Assignment 3, Spring 2023

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Set on: Fri 14th Apr; due: Fri 21st Apr

Model solutions and mark scheme

Problems

1. (a) Under conditions where hydrostatic and geostrophic balance hold, the vertical flow shear is related to horizontal gradients in the density.

   (1 mark for hydrostatic and geostrophic balance (no half marks here), and 1 mark for the second part of the sentence. Deduct half marks for every 10 words over the 30 word limit.)

   (b) Since

   $$f\frac{\partial u_g}{\partial z} = \frac{g}{\rho_0}\frac{\partial \rho}{\partial y},$$

   we have $\partial\rho/\partial y < 0$ with the usual convention. Given $f < 0$, $\partial u_g/\partial z > 0$, so geostrophic flow increases as you go from depths to the surface.

   (1 mark for some correct statement relating the zonal velocity to the meridional gradient of ρ or b or some other related quantity, and 1 mark for positive $\partial u_g/\partial z$ because of $f < 0$ in the Southern Hemisphere. Stating equations by themselves get no marks, unless there is some sign arguments involved. Deduct half marks for every 10 words over the 100 word limit.)

   (c) Expect no strong circumpolar current, so by thermal wind shear relation results in a flattening of isopycnals. Given the connectivity of isopycnals throughout the globe, this might in a shoaling of the substantial weakening of the (A)MOC (either from shoaling of isopycnals, or from the fact there is nothing bringing water back up as the Southern Ocean pathway is shut off).

   (0.5 mark for weakening/removal of circumpolar current, 1 mark for flattening of isoypcnals in Southern Ocean, 0.5 mark for weakening of (A)MOC. Give 0.5 marks for relevant citations but total credit for this question is to be capped at 2, i.e. 2.5/2 = 2/2.)

2. (a) Kelvin waves propagate from West to East. *(1 mark. No half marks.)*

   (b) Rossby waves exist because the planetary β is non-zero, or similar explanation. *(1 mark. No half marks.)*

   (c) Kelvin waves are non-dispersive, because ω/k and $\partial\omega/\partial k$ is the same since the dispersion relation is linear.

   (0.5 mark for non-dispersive and 0.5 mark for some sort of sensible reasoning.)

   (d) This is actually exactly Q3b of last year's assignment 3, and we have $c_p = \sqrt{10 \times 4000} = 200$ m s^{-1}, so $T = L/U = 2 \times 10^7/200 = 10^5$ s, which is $10^5/3600 \approx 28$ hours.

   (1 mark for working out the velocity, 1 mark for giving the answers in hours. Give 0.5 marks if answer not at the correct unit or specified degree of accuracy.)

(e) Flipping the above around we have $U = L/T = 2 \times 10^7/(60 \times 3600 \times 24) = 3.85...\ \mathrm{m\ s^{-1}} \approx 4\ \mathrm{m\ s^{-1}}$. (This is higher than the 2.8 $\mathrm{m\ s^{-1}}$ that I get quoted from searching "baroclinic Kelvin wave crossing time" in Google).

(1 mark. Give 0.5 marks as necessary.)

3. (a) $L_d = NH/f_0 = 10^{-2} \times 1000/(2\Omega \sin(10)) = 395943...\ \mathrm{m} = 400\ \mathrm{km}$, for $\Omega = 2\pi/(24 \times 3600)$.

   (1 mark for unit conversion and 1 mark for final answer. Give 0.5 marks as necessary.)

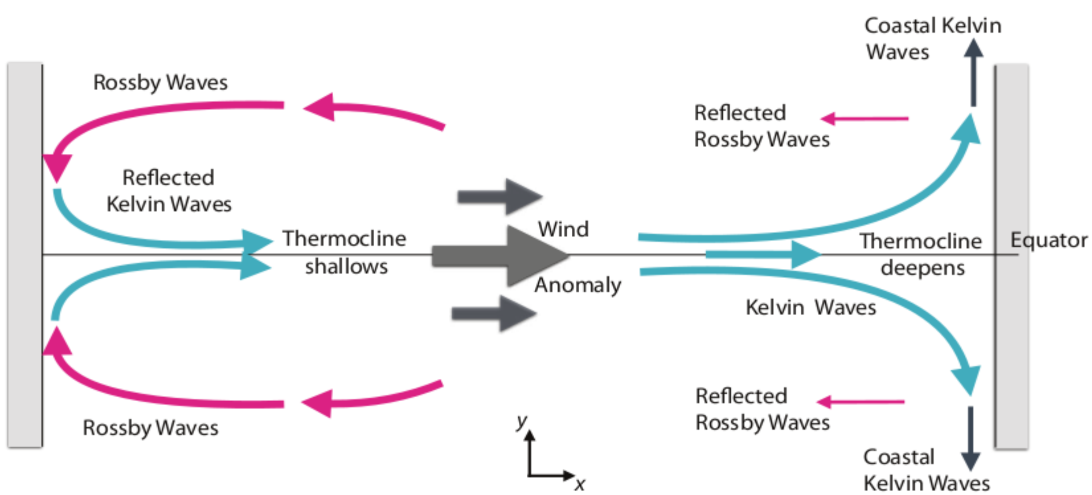
   (b) The zonal wavenumber is $k = 2\pi/\lambda = 2\pi/(4 \times 10^6) = 1.57 \times 10^{-6}$ cycles m^{-1}. The question states that we are dealing with a single Rossby wave, so it is the phase speed that we want, which is given by (getting rid of the sign)

$$c_{p,x} = \frac{10^{-11}}{(1.57 \times 10^{-6})^2 + 0^2 + (4 \times 10^5)^{-2}} = 1.147\ldots\ \mathrm{m\ s^{-1}}.$$

   Then $T = L/U = 2 \times 10^7/1.147\ldots = 17429800\ldots$ s $= 6.72\ldots$ months ≈ 7 months. (This is higher than the 200 days ≈ 6 months that I get quoted from searching "Rossby wave crossing time" in Google).

   (1 mark for computing the wavenumber and giving this in units of per meter; 0.5 marks for wavenumber in units of per kilometer, but not any other choice. 1 mark for about 1.1 $\mathrm{m\ s^{-1}}$*; give full marks for answers between* 1.0 *and* 1.2 $\mathrm{m\ s^{-1}}$*. 1 mark for computing the crossing time and converting this into months. 1 mark for giving the final answer to the nearest month. Given 0.5 marks as appropriate if reasoning is sensible but answer is wrong.)*

4. Basically this diagram from Vallis (Fig 22.19)[1]:



- Thermocline anomalies triggered by whatever reason is propagated eastward by Kelvin waves (where the crossing time is about two months, as calculated above).

[1] Pretty sure there is a typo in that caption, "...Kelvin wave that carries the anomaly *east*ward'...'

- Once the Kelvin waves hit the east coast, some of it is carried off polewards as coastal Kelvin waves.
- Some of the disturbance radiates back as Rossby waves, carrying a signal westward, and takes longer to travel (since the crossing time is about half a year).
- Once these signals hit the west coast, they excite Kelvin waves, and the cycle repeats, giving rise to the oscillatory nature of mixed layer depth, possibly made possible because of the asymmetry in the different wave propagation speeds that can lead to a 'build up' effect (cf. recharge-discharge oscillators).

(1 mark for referencing and citations, 1 mark for eastward propagation of Kelvin waves, 1 mark for westward propagation of Rossby waves, 1 mark for some sort of mention about reflection. Take up to 2 marks off *for extremely irrelevant content, wrong content, and/or going over the specified page limit, capped below by 0 (e.g. -2/4 is still 0/4).)*

!? (Bonus question, no marks + for interest only) See §3.6 of Vallis, 2nd edn. It's just a straightforward application of geostrophic and hydrostatic balance to get from $\partial p'/\partial x$ to v' and h' to b' respectively, where I have been lazy and deliberately dropped the factors of f, some other constants, and various signs (hence the squiggle).