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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 8: Mechanical forcing 2 (rotation/Coriolis)

# Outline

- rotation of Earth, Coriolis “force” (recall from OCES 2001)
  - $\rightarrow$ rotation axis
  - $\rightarrow$ consequences for flow
  - $\rightarrow$ Rossby number + geostrophic balance
- thermal wind balance
  - $\rightarrow$ hydrostatic (vertical) + geostrophic (horizontal) balance
  - $\rightarrow$ SSH anomaly example revisited (see Lec. 6 + 7)

**Key terms**: Coriolis “force”, Rossby number, geostrophic balance/flow, thermal wind (balance)

# Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to <u>numerous</u> approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left( \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{D}_{\boldsymbol{u}} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left( \frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left( \frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

# Recap: hydrostatic pressure

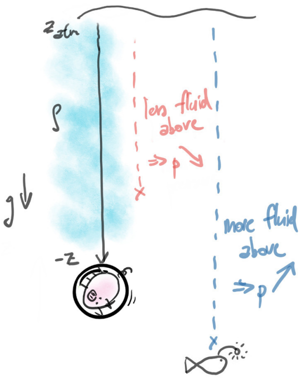



**Figure:** Schematic of hydrostatic pressure

- hydrostatic approximation: pressure **approximately equal** to weight above when static

  → weight is $F = mg$ so for force balance,

$$F = mg = \boxed{g \int_{-z}^{z_{\text{atm}}} \rho \, \mathrm{d}z = p},$$

with $g \approx 9.81 \text{ m s}^{-2}$

→ if $\rho = \text{const}$ then $p = \rho g z + p_{\text{atm}}$

$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

# Recap: pressure gradients and flows

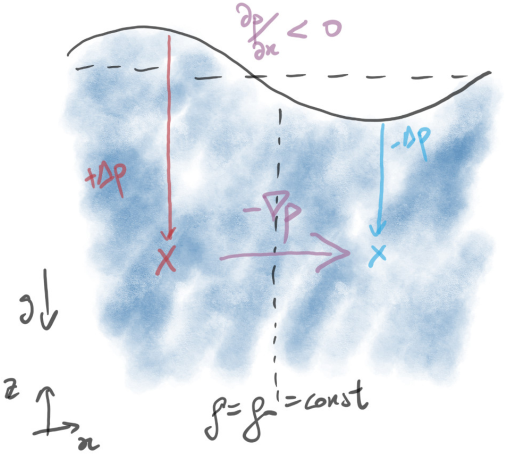
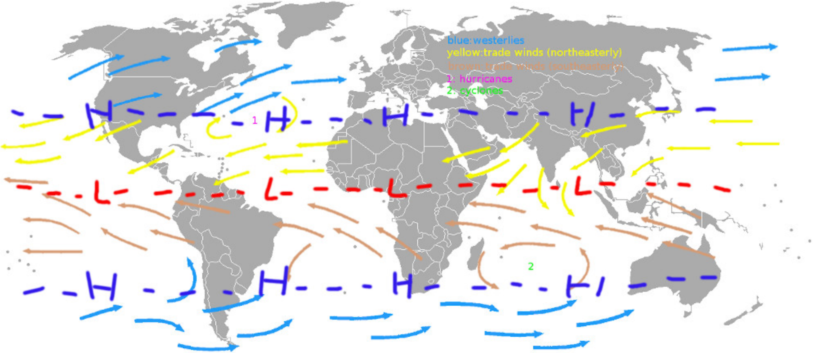



**Figure:** Horizontal effect because of hydrostatic pressure.

- assuming hydrostatic balance, water moves from $+\Delta p$ to $-\Delta p$ because there is a **net force** (negative pressure gradient $-\nabla p$)

  $\rightarrow$ important for geostrophic flows

# Geostrophic flows: atmosphere




**Winds do not go direct from high to low $p$?** (more on wind patterns next Lec.)

# Geostrophic flows: atmosphere

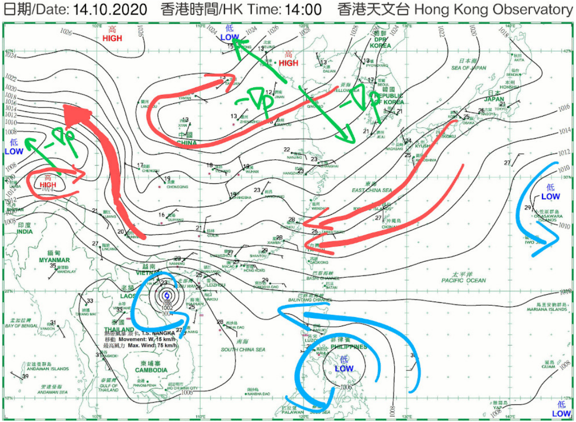



**Figure:** Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

- note that flow doesn't go in the direction of $-\nabla p$!

  $\rightarrow$ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

# Geostrophic flows: ocean

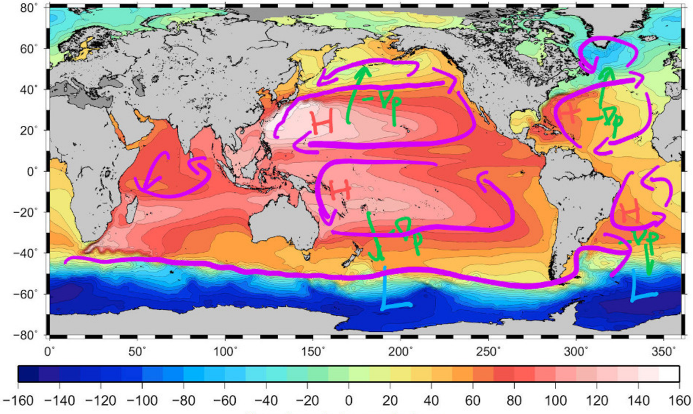



**Figure:** Time-mean global SSH (also called mean dynamic topography, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via hydrostatic balance

  → flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

# Coriolis effect

The Earth rotates around the rotation axis

- the geographical North (as opposed to magnetic north)
- **rate** of rotation is the angular frequency $\Omega$ (units: $\mathrm{s}^{-1}$), with

$$\Omega = \frac{2\pi}{T}$$

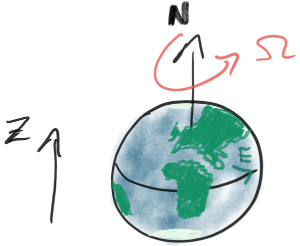



**Figure:** Rotation axis and angular frequency $\Omega$.

$\rightarrow$ $T$ is the period (see again in Lec. 15 - 18), time needed to do one rotation ($2\pi$ radians or 360°)

$\rightarrow$ for Earth (units!)

$$\Omega = \frac{2\pi}{3600 \times 24} \approx 7.29 \times 10^{-5}\ \mathrm{s}^{-1}$$

# Coriolis effect: co-ordinates

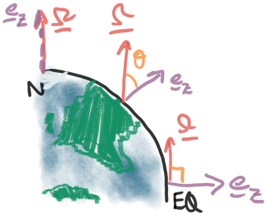



**Figure:** Mis-alignment of $\boldsymbol{\Omega}$ and $\boldsymbol{e}_z$ used locally for depth.

- for a spherical Earth we take rotation axis to be $z$-axis, i.e. $\boldsymbol{\Omega} = \Omega \boldsymbol{e}_z$ (this a vector), but locally, $z$ is depth...
- introduce the latitudinally varying Coriolis parameter

$$f = 2\Omega \sin(\text{latitude})$$

- to take into account of mis-alignments between $\boldsymbol{\Omega}$ and the local $\boldsymbol{e}_z$ for depth

  $\rightarrow$ Coriolis = $-2\boldsymbol{\Omega} \times \boldsymbol{u}$ (global case, $z$ is North)

  $\rightarrow$ Coriolis = $-f\boldsymbol{e}_z \times \boldsymbol{u}$ (local case, $z$ is depth) (mostly going to use this one)

# Frame of reference

(sort of) a demonstration of a frame of reference

- one could work in global picture (with $\Omega$ and $z$ being North) or local picture (with $f$ and $z$ being depth)

  $\rightarrow$ change in point-of-view, co-ordinate system

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- underlying physics should still be the **same**, although description might **look** different (e.g. $\boldsymbol{\Omega}$ vs. $f\boldsymbol{e}_z$)
- **freedom in choice of frame**!

  $\rightarrow$ e.g. frame rotating with the planet, others...

  $\rightarrow$ fine as long as we keep consistency

## Coriolis effect

You can think of it as a “hack”

- Newton’s laws are formulated for **inertial frames**

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**can’t use Newton’s laws then?**

- “Hack”: put in a fictitious forces to compensate the fact we are not in an inertial frame

  $\rightarrow$ can then treat non-inertial frames **as if it were inertial**

  $\rightarrow$ proceed as normal with that caveat

# Coriolis effect

Earth's spin has a major influence on large-scale winds

- extra "force" from the spinning: Coriolis "force" $2\boldsymbol{\Omega} \times \boldsymbol{u}$

  (Gaspard-Gustave de Coriolis, 1792-1843, French mathematician)

  $\rightarrow$ apparent deflection $\Leftrightarrow$ a net force on it

- apparent "force" because not being in inertial frame

  $\rightarrow$ can think of it arising only because of perspective...

  $\rightarrow$ note Coriolis "force" does **no work** (see assignment)

- try it yourself! (seriously this really helped me...)

  $\rightarrow$ on a piece of paper, try drawing a straight line while rotating the paper underneath

  $\rightarrow$ something similar but on e.g. a basketball

# Coriolis effect

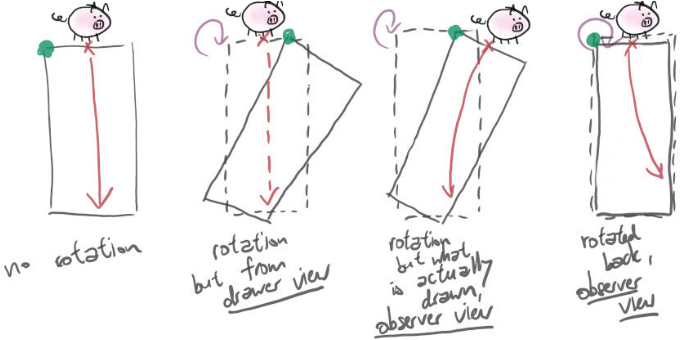


**Figure:** Schematic of apparent deflection from Coriolis “force”. From the drawer perspective, the drawer is doing a straight line and sees a straight line. By from observer’s perspective, there is a deflection. The action is the same, but to describe it from the observer’s point of view, we need to additionally describe this apparently deflection arising from the system’s rotation.

- on a piece of paper, try drawing a straight line while rotating the paper underneath

# Coriolis effect

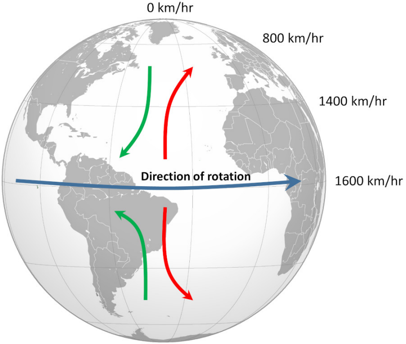


**Figure:** Schematic of apparent deflection from Coriolis "force", from Vallis (2011).

- apparent deflection to the right in NH
  $\rightarrow$ looking down from North Pole, rotating anti-clockwise
- apparent deflection to the left in SH
  $\rightarrow$ looking down from South Pole, now rotating clockwise

# Rossby number

Note there is a competition between fluid velocity (intended path) and rotation

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- matter of time-scales

  $\rightarrow$ if fluid moves quickly relative to system spinning (inertial period), then Coriolis influence small

  $\rightarrow$ if fluid moves relatively slowly, more time for Coriolis effect to act

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- measured by the Rossby number: for $f = 2\Omega\sin(\text{latitude})$,

  (Carl-Gustav Rossby, 1898-1957, Swedish meteorologist)

$$\mathrm{Ro} = \frac{U/L}{f} = \frac{1/T}{f} = \frac{\text{advection}}{\text{rotation}}$$

$\rightarrow$ note $f$ decreases to zero at EQ (mis-alignment of rotation axis, recall $-2\boldsymbol{\Omega} \times \boldsymbol{u}$)

# Rossby number

$$\mathrm{Ro} = \frac{U/L}{f} = \frac{1/T}{f} = \frac{\text{advection}}{\text{rotation}}$$

- For large-scale motion in Earth's atmosphere in mid-lats (say 50°N) and $\Omega = 2\pi/\text{day}$,

$$\mathrm{Ro} = \frac{10 \text{ m s}^{-1}/1000 \text{ km}}{2 \times 2\pi \text{ day}^{-1} \times \sin(50^\circ)} \approx \frac{10^1 \times 10^{-6}}{10^{-4}} = 0.1,$$

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- $\mathrm{Ro} \sim O(0.01)$ for Jupiter `https://www.youtube.com/watch?v=exkeE6aIlpc`
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- flows in the interior of the Sun, $\mathrm{Ro} = O(1)$
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? what about in a toilet bowl? (see assignment)

# Geostrophic flow

After non-dimensionalisation (see OCES 3301 or ask me), momentum equation is (other numbers appear for forcing + dissipation, assume small; see Lec 9 + 10)

$$\mathrm{Ro}\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) + \rho 2\boldsymbol{\Omega}\times\boldsymbol{u} = -\nabla p + \ldots$$

If $\mathrm{Ro} \ll 1$, dominant force balance is:

$$\boxed{2\boldsymbol{\Omega}\times\boldsymbol{u}_g = -\frac{1}{\rho}\nabla p}$$

- geostrophic balance

  $\rightarrow$ given $p$ and $\boldsymbol{\Omega}$, $\boldsymbol{u}_g$ (the geostrophic flow) has to be **something** so resulting forces balance

**what is the implied velocity $\boldsymbol{u}$ then?**

# Geostrophic flow: rationalisation

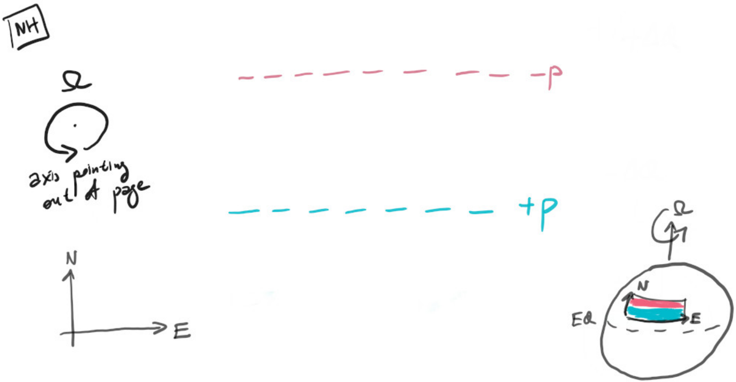


**Figure:** Geostrophic balance and resulting geostrophic flow $\boldsymbol{u}_g$ in Northern Hemisphere. Note $\boldsymbol{u}_g$ is **along** (rather than **across**) isobars.

- in NH top-down view, rotation is **anti**-clockwise

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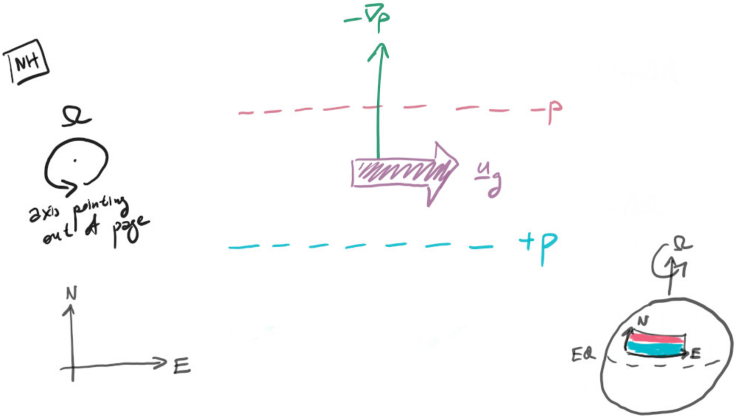


**Figure:** Geostrophic balance and resulting geostrophic flow $\boldsymbol{u}_g$ in Northern Hemisphere. Note $\boldsymbol{u}_g$ is **along** (rather than **across**) isobars.

- in NH, deflection to the **right** of $-\nabla p$

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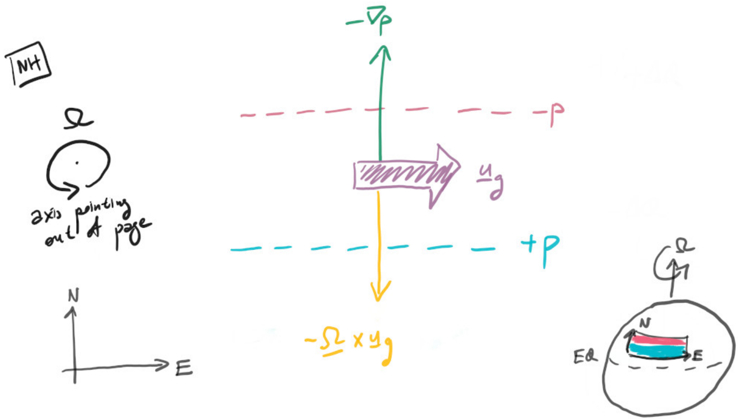


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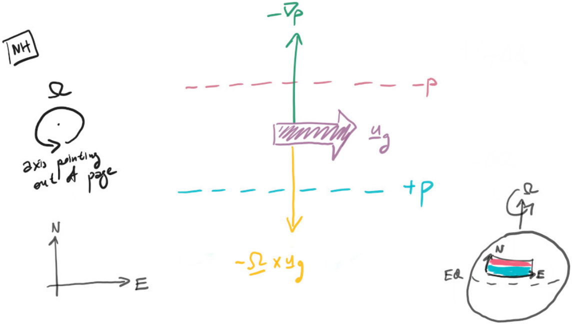
- for **force balance**, Coriolis has to be opposite of $-\nabla p$

# Geostrophic flow: rationalisation




- or, really, **because** of force balance, $\boldsymbol{u}_g$ has to be to the right of $-\nabla p$ in NH
- $\boldsymbol{\Omega} \sim +\boldsymbol{e}_z$ (Earth NH), $-\nabla p \sim \boldsymbol{e}_y$, so

$$2\boldsymbol{\Omega} \times \boldsymbol{u} = -\frac{1}{\rho}\nabla p \quad \Rightarrow \quad \boldsymbol{e}_z \times \boldsymbol{u}_g \sim \boldsymbol{e}_y \quad \text{or} \quad \boldsymbol{u}_g \times \boldsymbol{e}_z \sim -\boldsymbol{e}_y,$$

so $\boldsymbol{u} \sim +\boldsymbol{e}_x$ only possibility, i.e. to the E (right of N is E)

# Geostrophic flow: rationalisation

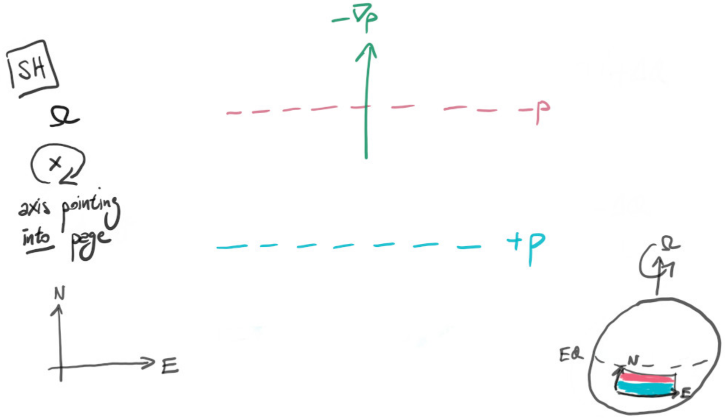
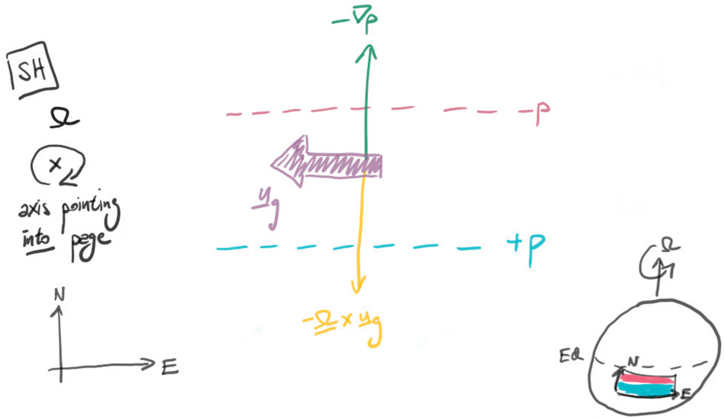



**Figure:** Geostrophic balance and resulting geostrophic flow $\boldsymbol{u}_g$ in Southern Hemisphere. Note $\boldsymbol{u}_g$ is **along** (rather than **across**) isobars.

- in SH top-down view, rotation is **clockwise**

# Geostrophic flow: rationalisation

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- $\boldsymbol{u}_g$ to the **left** of $-\nabla p$ (same arguments as but $\boldsymbol{\Omega} \to -\boldsymbol{\Omega}$)

# Geostrophic flow

Suppose hydrostatic as well as geostrophic balance:

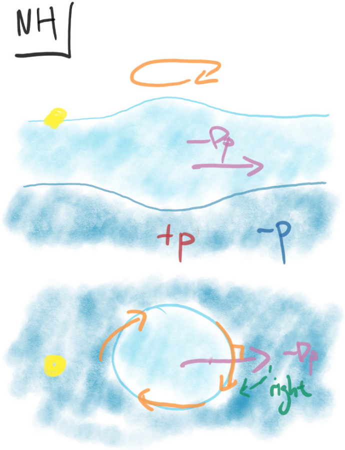


**Figure:** Schematic for an anti-cyclonic (warm core) eddy.

- bulge ($+\Delta h$) so $+\Delta p$ in the centre

  $\rightarrow -\nabla p$ points **away** from region
- geostrophic current $\boldsymbol{u}_g$ to the **right** of $-\nabla p$ (since NH)

  $\Rightarrow$ **clockwise** around bulge
- **opposite** sense to planet rotation $f$ (in NH), **anti-cyclonic** eddy (in NH)

  $\rightarrow$ other direction in SH (since $f < 0$)

# Geostrophic flow

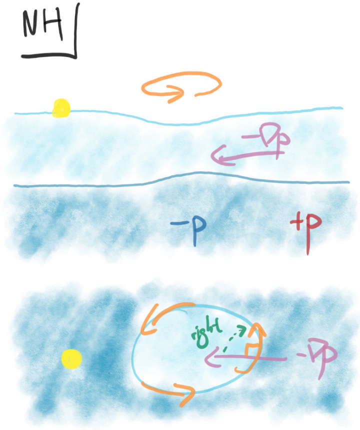


**Figure:** Schematic for a cyclonic (cold core) eddy.

- depression ($-\Delta h$) so $-\Delta p$ in the centre

  $\rightarrow -\nabla p$ points **into** region
- geostrophic current $\boldsymbol{u}_g$ to the **right** of $-\nabla p$ (since NH)

  $\Rightarrow$ **anti**-clockwise around depression
- **same** sense as rotation of planet $f$ (in NH), **cyclonic** eddy (in NH)

  $\rightarrow$ other direction in SH (since $f < 0$)

## Geostrophic flow

Above was for NH, for SH:

- pressure and $-\nabla p$ still related to hydrostatic balance
- deflection is to the **left** (cf. opposite rotation sense)
- $\boldsymbol{u}_g$ is now

  $\rightarrow$ anti-clockwise around bulge

  $\rightarrow$ clockwise around depression
- BUT!

  $\rightarrow$ bulges are still anti-cyclonic

  $\rightarrow$ depressions are still cyclonic

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- BUT!

  $\rightarrow$ bulges are still anti-cyclonic

  $\rightarrow$ depressions are still cyclonic

- cf. atmosphere, low pressures $\leftrightarrow$ depressions $\leftrightarrow$ cyclonic

  $\rightarrow$ in atmosphere low pressures are convergence zones, related to unsettled weather (and vice-versa in high pressures) (relation to Ekman up/downwelling next Lec.)

# Geostrophic flow from SSH

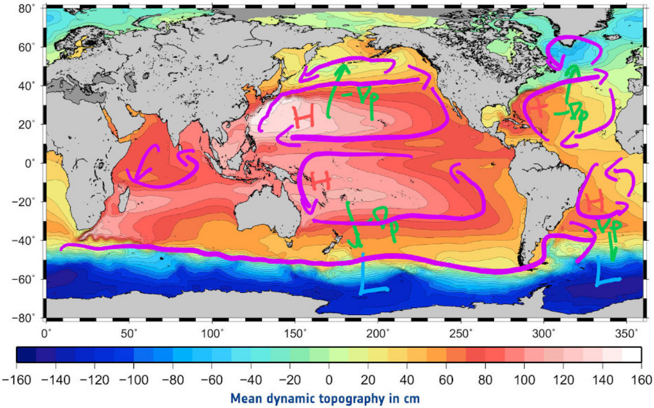


**Figure:** Time-mean global SSH (also called mean dynamic topography, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via hydrostatic balance

  → flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

# Geostrophic flow from SSH

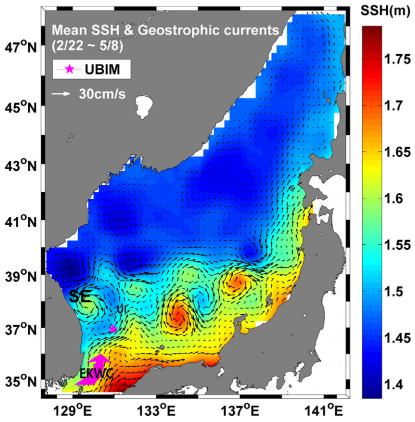


**Figure:** SSH and inferred currents from AVISO satellite altimeter data. Figure 4 of Son *et al.* (2014), *Biogeosciences*.

- observed SSH from AVISO (see Lec. 20) + inferred geostrophic velocities near Japan

  → anti-cyclonic around positive SSH anomalies (clockwise in NH)
- note current is strongest where gradients are large

  (the arrow lengths)

# Summary

**geostrophic flow goes to the right of pressure gradient in NH**

- flip this in SH (because rotation is "reversed")

  $\rightarrow$ again, Coriolis effect is frame dependent, i.e. depend on your point of view, a pseudo-force

  $\rightarrow$ Coriolis force does **no work** (cf. Lec 5 + 6; see assignment)

- anti-cyclonic eddies $\sim$ bulges in SSH (in SH and NH)

  $\rightarrow$ rotation same sense as planet, $\nabla \times \boldsymbol{u} \sim f$

  $\rightarrow$ clockwise in NH (because $f > 0$), reverse in SH

- cyclonic eddies $\sim$ depressions in SSH (in SH and NH)

  $\rightarrow$ rotation opposite sense as planet, $\nabla \times \boldsymbol{u} \sim -f$

  $\rightarrow$ anti-clockwise in NH (because $f > 0$), reverse in SH

# Summary

Turns out the deflection aspect is also (largely) true for wind forced flows (see next Lec.):

**Ekman transport is to the right of the wind in NH** (flip in SH)

$\rightarrow$ wind drives flow at surface in direction of wind...

$\rightarrow$ but flow needs to turn 90° (or $\pm\pi/2$) at depth?

- Ekman spirals