q1

May 17, 2023

[ ]:
```
import pandas as pd
import pandas_datareader as pdr
import numpy as np
import statsmodels.api as sm
```

1 1A

[ ]:
```
ff5_month = pdr.get_data_famafrench('F-F_Research_Data_5_Factors_2x3',␣
 ↪start='1963-07', end='2023-03')[0]
ff5_month = ff5_month
mom_month = pdr.get_data_famafrench('F-F_Momentum_Factor', start='1927-01',␣
 ↪end='2023-03')[0]
mom_month = mom_month

ff5_month = ff5_month.reset_index()
mom_month = mom_month.reset_index()
ff5mom_month = ff5_month.merge(mom_month, on='Date', how='left')
ff5mom_month = ff5mom_month.set_index('Date')
```

```
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/1568211615.py:1
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  ff5_month = pdr.get_data_famafrench('F-F_Research_Data_5_Factors_2x3',
start='1963-07', end='2023-03')[0]
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/1568211615.py:1
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  ff5_month = pdr.get_data_famafrench('F-F_Research_Data_5_Factors_2x3',
start='1963-07', end='2023-03')[0]
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/1568211615.py:3
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  mom_month = pdr.get_data_famafrench('F-F_Momentum_Factor', start='1927-01',
end='2023-03')[0]
```

```
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/1568211615.py:3
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  mom_month = pdr.get_data_famafrench('F-F_Momentum_Factor', start='1927-01',
end='2023-03')[0]
```

[ ]:
```
ff5_daily = pdr.get_data_famafrench('F-F_Research_Data_5_Factors_2x3_daily',␣
 ↪start='1963-07-01', end='2023-03-31')[0]
ff5_daily = ff5_daily
mom_daily = pdr.get_data_famafrench('F-F_Momentum_Factor_daily',␣
 ↪start='1926-11-03', end='2023-03-31')[0]
mom_daily = mom_daily

ff5_daily = ff5_daily.reset_index()
mom_daily = mom_daily.reset_index()
ff5mom_daily = ff5_daily.merge(mom_daily, on='Date', how='left')
ff5mom_daily = ff5mom_daily.set_index('Date')
```

```
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/3591460299.py:1
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  ff5_daily = pdr.get_data_famafrench('F-F_Research_Data_5_Factors_2x3_daily',
start='1963-07-01', end='2023-03-31')[0]
/var/folders/sg/4dp480wd1cjd288xvby34rpr0000gn/T/ipykernel_30429/3591460299.py:3
: FutureWarning: The argument 'date_parser' is deprecated and will be removed in
a future version. Please use 'date_format' instead, or read your data in as
'object' dtype and then call 'to_datetime'.
  mom_daily = pdr.get_data_famafrench('F-F_Momentum_Factor_daily',
start='1926-11-03', end='2023-03-31')[0]
```

[ ]:
```
ff5mom_month
```

[ ]:

Date	Mkt-RF	SMB	HML	RMW	CMA	RF	Mom
1963-07	-0.39	-0.41	-0.97	0.68	-1.18	0.27	0.90
1963-08	5.07	-0.80	1.80	0.36	-0.35	0.25	1.01
1963-09	-1.57	-0.52	0.13	-0.71	0.29	0.27	0.19
1963-10	2.53	-1.39	-0.10	2.80	-2.01	0.29	3.12
1963-11	-0.85	-0.88	1.75	-0.51	2.24	0.27	-0.74
…	…	…	…	…	…	…	…
2022-11	4.60	-2.67	1.38	6.01	3.11	0.29	-2.01
2022-12	-6.41	-0.16	1.32	0.09	4.19	0.33	4.52
2023-01	6.65	4.43	-4.05	-2.62	-4.53	0.35	-15.98
2023-02	-2.58	0.69	-0.78	0.90	-1.41	0.34	0.20
2023-03	2.51	-7.01	-9.01	1.92	-2.29	0.36	-2.52

```
[717 rows x 7 columns]
```

[ ]:
```
ff5mom_daily
```

[ ]:
```
            Mkt-RF   SMB   HML   RMW   CMA     RF  Mom
Date
1963-07-01   -0.67  0.02 -0.35  0.03  0.13  0.012   -0.21
1963-07-02    0.79 -0.28  0.28 -0.08 -0.21  0.012    0.42
1963-07-03    0.63 -0.18 -0.10  0.13 -0.25  0.012    0.41
1963-07-05    0.40  0.09 -0.28  0.07 -0.30  0.012    0.07
1963-07-08   -0.63  0.07 -0.20 -0.27  0.06  0.012   -0.45
…              …    …    …    …    …     …       …
2023-03-27    0.27  0.51  1.02 -0.28  0.35  0.016    0.82
2023-03-28   -0.17 -0.03  0.74  0.08  0.55  0.016    0.73
2023-03-29    1.39 -0.34 -0.50 -0.90 -0.54  0.016   -1.11
2023-03-30    0.51 -0.61 -0.59  0.20 -0.09  0.016   -0.39
2023-03-31    1.53  0.51 -0.77 -0.46 -0.76  0.016   -1.01

[15041 rows x 7 columns]
```

[ ]:
```
ff5mom_month.index = pd.to_datetime(ff5mom_month.index.to_timestamp())
ff5mom_month.index = ff5mom_month.index + pd.offsets.MonthEnd(0)
```

2 1B

[ ]:
```
# Calculate the rolling variance using a window of 22 trading days
rolling_variance_daily = ff5mom_daily.rolling(22, min_periods=22).var()
```

[ ]:
```
rolling_variance_daily
```

[ ]:
```
              Mkt-RF       SMB       HML       RMW       CMA            RF  \
Date
1963-07-01       NaN       NaN       NaN       NaN       NaN           NaN
1963-07-02       NaN       NaN       NaN       NaN       NaN           NaN
1963-07-03       NaN       NaN       NaN       NaN       NaN           NaN
1963-07-05       NaN       NaN       NaN       NaN       NaN           NaN
1963-07-08       NaN       NaN       NaN       NaN       NaN           NaN
…                …         …         …         …         …             …
2023-03-27  1.331266  0.424642  1.242579  0.226139  0.321225  4.935065e-07
2023-03-28  1.281044  0.427383  1.175384  0.225673  0.331312  3.463203e-07
2023-03-29  1.369642  0.422826  1.175816  0.272304  0.336809  1.818182e-07
2023-03-30  1.376558  0.394282  1.172678  0.253749  0.336463  0.000000e+00
2023-03-31  1.462451  0.402542  1.129082  0.267495  0.335966  0.000000e+00

                Mom
```

```
Date
1963-07-01       NaN
1963-07-02       NaN
1963-07-03       NaN
1963-07-05       NaN
1963-07-08       NaN
…                …
2023-03-27  0.758005
2023-03-28  0.716987
2023-03-29  0.776145
2023-03-30  0.775339
2023-03-31  0.694750

[15041 rows x 7 columns]
```

3 1C

[ ]:
```
monthly_data = rolling_variance_daily.resample('M').last()
```

[ ]:
```
monthly_data.columns
```

[ ]:
```
Index(['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA', 'RF', 'Mom   '], dtype='object')
```

[ ]:
```
for factor in monthly_data.columns:
    # Calculate the lagged variance
    df = pd.DataFrame()
    df['lagged_variance'] = monthly_data[factor]
    df['var'] = monthly_data[factor]

    df['lagged_variance'] = df['lagged_variance'].shift()
    # Drop any rows with missing values
    df = df.dropna()

    # Create the design matrix for the regression
    X = sm.add_constant(df['lagged_variance'])

    # Fit the regression model
    model = sm.OLS(df['var'], X)
    results = model.fit()

    # Print the regression results for each factor
    print(f"Results for {factor}:")
    print(results.summary())
    print()
```

```
Results for Mkt-RF:
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    var   R-squared:                       0.207
Model:                            OLS   Adj. R-squared:                  0.206
Method:                 Least Squares   F-statistic:                     186.5
Date:                Tue, 16 May 2023   Prob (F-statistic):           6.81e-38
Time:                        20:06:49   Log-Likelihood:                -1500.5
No. Observations:                 716   AIC:                             3005.
Df Residuals:                     714   BIC:                             3014.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
---
const            0.5725      0.081      7.025      0.000       0.412
0.732
lagged_variance  0.4551      0.033     13.657      0.000       0.390
0.520
==============================================================================
Omnibus:                     1211.995   Durbin-Watson:                   2.071
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           690407.053
Skew:                          10.557   Prob(JB):                         0.00
Kurtosis:                     153.653   Cond. No.                         2.80
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

Results for SMB:
                            OLS Regression Results
==============================================================================
Dep. Variable:                    var   R-squared:                       0.133
Model:                            OLS   Adj. R-squared:                  0.132
Method:                 Least Squares   F-statistic:                     109.7
Date:                Tue, 16 May 2023   Prob (F-statistic):           5.52e-24
Time:                        20:06:49   Log-Likelihood:                -457.08
No. Observations:                 716   AIC:                             918.2
Df Residuals:                     714   BIC:                             927.3
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025
0.975]
```

```
------------------------------------------------------------------------------
---
const               0.1830      0.020      9.216      0.000       0.144
0.222
lagged_variance     0.3649      0.035     10.476      0.000       0.297
0.433
==============================================================================
Omnibus:                     1312.204   Durbin-Watson:                   2.110
Prob(Omnibus):                  0.000   Jarque-Bera (JB):          1517406.276
Skew:                          12.164   Prob(JB):                         0.00
Kurtosis:                     227.212   Cond. No.                         2.25
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

Results for HML:
                            OLS Regression Results
==============================================================================
Dep. Variable:                    var   R-squared:                       0.563
Model:                            OLS   Adj. R-squared:                  0.562
Method:                 Least Squares   F-statistic:                     919.5
Date:                Tue, 16 May 2023   Prob (F-statistic):          1.94e-130
Time:                        20:06:49   Log-Likelihood:                -429.05
No. Observations:                 716   AIC:                             862.1
Df Residuals:                     714   BIC:                             871.3
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
===
                     coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
---
const               0.0839      0.018      4.556      0.000       0.048
0.120
lagged_variance     0.7509      0.025     30.323      0.000       0.702
0.800
==============================================================================
Omnibus:                      724.124   Durbin-Watson:                   2.472
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            63296.969
Skew:                           4.387   Prob(JB):                         0.00
Kurtosis:                      48.218   Cond. No.                         1.77
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
```

specified.

Results for RMW:

OLS Regression Results

Dep. Variable:	var	R-squared:	0.523
Model:	OLS	Adj. R-squared:	0.522
Method:	Least Squares	F-statistic:	782.5
Date:	Tue, 16 May 2023	Prob (F-statistic):	7.73e-117
Time:	20:06:49	Log-Likelihood:	104.16
No. Observations:	716	AIC:	-204.3
Df Residuals:	714	BIC:	-195.2
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0432	0.009	4.912	0.000	0.026	0.060
lagged_variance	0.7231	0.026	27.972	0.000	0.672	0.774

Omnibus:	721.121	Durbin-Watson:	2.191
Prob(Omnibus):	0.000	Jarque-Bera (JB):	120965.471
Skew:	4.091	Prob(JB):	0.00
Kurtosis:	66.149	Cond. No.	3.39

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Results for CMA:

OLS Regression Results

Dep. Variable:	var	R-squared:	0.472
Model:	OLS	Adj. R-squared:	0.471
Method:	Least Squares	F-statistic:	637.8
Date:	Tue, 16 May 2023	Prob (F-statistic):	4.75e-101
Time:	20:06:49	Log-Likelihood:	302.00
No. Observations:	716	AIC:	-600.0
Df Residuals:	714	BIC:	-590.9
Df Model:	1		
Covariance Type:	nonrobust		

```
===
                       coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
---
const                0.0441      0.007      6.252      0.000       0.030
0.058
lagged_variance      0.6871      0.027     25.254      0.000       0.634
0.741
==============================================================================
Omnibus:                      511.792   Durbin-Watson:                   2.347
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            27237.941
Skew:                           2.590   Prob(JB):                         0.00
Kurtosis:                      32.769   Cond. No.                         4.67
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

Results for RF:
                            OLS Regression Results
==============================================================================
Dep. Variable:                    var   R-squared:                       0.001
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.4358
Date:                Tue, 16 May 2023   Prob (F-statistic):              0.509
Time:                        20:06:49   Log-Likelihood:                 8493.8
No. Observations:                 716   AIC:                        -1.698e+04
Df Residuals:                     714   BIC:                        -1.697e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
===
                       coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
---
const            4.547e-07   6.62e-08      6.873      0.000    3.25e-07
5.85e-07
lagged_variance      0.0247      0.037      0.660      0.509      -0.049
0.098
==============================================================================
Omnibus:                      999.502   Durbin-Watson:                   2.010
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           173120.071
Skew:                           7.597   Prob(JB):                         0.00
Kurtosis:                      77.646   Cond. No.                     5.86e+05
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 5.86e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

Results for Mom   :
                            OLS Regression Results
==============================================================================
Dep. Variable:                    var   R-squared:                       0.397
Model:                            OLS   Adj. R-squared:                  0.396
Method:                 Least Squares   F-statistic:                     469.1
Date:                Tue, 16 May 2023   Prob (F-statistic):           2.37e-80
Time:                        20:06:49   Log-Likelihood:                -1021.7
No. Observations:                 716   AIC:                             2047.
Df Residuals:                     714   BIC:                             2056.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
===
                     coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
---
const              0.2160      0.041      5.227      0.000       0.135
0.297
lagged_variance    0.6296      0.029     21.659      0.000       0.573
0.687
==============================================================================
Omnibus:                      791.197   Durbin-Watson:                   2.333
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            98784.210
Skew:                           4.980   Prob(JB):                         0.00
Kurtosis:                      59.674   Cond. No.                         1.76
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

It seems to predict returns in all cases but RF, but that's a leftover and not actually a "factor" so yes, it predicts the returns in all cases.

4 1d

[ ]:
```
for factor in monthly_data.columns:
    # Calculate the lagged variance
    df = pd.DataFrame()
    df['var'] = monthly_data[factor]
    df['returns'] = ff5mom_month[factor]
    df['returns'] = df['returns'].shift(-1)
    # Drop any rows with missing values
    df = df.dropna()

    # Create the design matrix for the regression
    X = sm.add_constant(df['var'])

    # Fit the regression model
    model = sm.OLS(df['returns'], X)
    results = model.fit()

    # Print the regression results for each factor
    print(f"Results for {factor}:")
    print(results.summary())
    print()
```

```
                 var  returns
Date
1963-07-31  0.223726     5.07
1963-08-31  0.086949    -1.57
1963-09-30  0.154223     2.53
1963-10-31  0.171368    -0.85
1963-11-30  1.493974     1.83
…              …       …
2022-11-30  3.182266    -6.41
2022-12-31  1.833809     6.65
2023-01-31  1.169051    -2.58
2023-02-28  1.116542     2.51
2023-03-31  1.462451      NaN

[717 rows x 2 columns]
Results for Mkt-RF:
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                  0.005432
Date:                Tue, 16 May 2023   Prob (F-statistic):              0.941
Time:                        20:07:46   Log-Likelihood:                -2092.4
No. Observations:                 716   AIC:                             4189.
Df Residuals:                     714   BIC:                             4198.
```

```
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5636      0.186      3.026      0.003       0.198       0.929
var           -0.0056      0.076     -0.074      0.941      -0.155       0.144
==============================================================================
Omnibus:                       58.078   Durbin-Watson:                   1.919
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              118.925
Skew:                          -0.496   Prob(JB):                     1.50e-26
Kurtosis:                       4.732   Cond. No.                         2.80
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

                var  returns
Date
1963-07-31  0.020938    -0.80
1963-08-31  0.033653    -0.52
1963-09-30  0.052577    -1.39
1963-10-31  0.077462    -0.88
1963-11-30  0.207698    -2.10
…              …       …
2022-11-30  0.246766    -0.16
2022-12-31  0.171106     4.43
2023-01-31  0.209819     0.69
2023-02-28  0.349219    -7.01
2023-03-31  0.402542      NaN

[717 rows x 2 columns]
Results for SMB:
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.001
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.3619
Date:                Tue, 16 May 2023   Prob (F-statistic):              0.548
Time:                        20:07:46   Log-Likelihood:                -1809.2
No. Observations:                 716   AIC:                             3622.
Df Residuals:                     714   BIC:                             3632.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
const          0.1805      0.131      1.376      0.169      -0.077       0.438
var            0.1385      0.230      0.602      0.548      -0.314       0.591
==============================================================================
Omnibus:                       67.750   Durbin-Watson:                   1.848
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              289.454
Skew:                           0.310   Prob(JB):                     1.40e-63
Kurtosis:                       6.052   Cond. No.                         2.25
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

                var  returns
Date
1963-07-31  0.037883     1.80
1963-08-31  0.039540     0.13
1963-09-30  0.035055    -0.10
1963-10-31  0.116457     1.75
1963-11-30  0.079971    -0.02
…              …       …
2022-11-30  1.227538     1.32
2022-12-31  0.827769    -4.05
2023-01-31  0.451952    -0.78
2023-02-28  0.894674    -9.01
2023-03-31  1.129082      NaN

[717 rows x 2 columns]
Results for HML:
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.3543
Date:                Tue, 16 May 2023   Prob (F-statistic):              0.552
Time:                        20:07:46   Log-Likelihood:                -1800.2
No. Observations:                 716   AIC:                             3604.
Df Residuals:                     714   BIC:                             3613.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3257      0.125      2.607      0.009       0.080       0.571
var           -0.1000      0.168     -0.595      0.552      -0.430       0.230
==============================================================================
Omnibus:                       45.320   Durbin-Watson:                   1.639
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              166.529
```

```
Skew:                          0.121   Prob(JB):                     6.90e-37
Kurtosis:                      5.350   Cond. No.                         1.77
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

                var  returns
Date
1963-07-31  0.022160     0.36
1963-08-31  0.013295    -0.71
1963-09-30  0.028740     2.80
1963-10-31  0.061698    -0.51
1963-11-30  0.092893     0.03
…              …       …
2022-11-30  0.616556     0.09
2022-12-31  0.375447    -2.62
2023-01-31  0.296312     0.90
2023-02-28  0.265832     1.92
2023-03-31  0.267495      NaN

[717 rows x 2 columns]
Results for RMW:
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.019
Model:                            OLS   Adj. R-squared:                  0.017
Method:                 Least Squares   F-statistic:                     13.66
Date:                Tue, 16 May 2023   Prob (F-statistic):           0.000235
Time:                        20:07:46   Log-Likelihood:                -1580.9
No. Observations:                 716   AIC:                             3166.
Df Residuals:                     714   BIC:                             3175.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1248      0.092      1.349      0.178      -0.057       0.306
var            1.0053      0.272      3.697      0.000       0.471       1.539
==============================================================================
Omnibus:                      212.377   Durbin-Watson:                   1.731
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4347.068
Skew:                          -0.799   Prob(JB):                         0.00
Kurtosis:                      14.965   Cond. No.                         3.39
==============================================================================

Notes:
```

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Date	var	returns
1963-07-31	0.031652	-0.35
1963-08-31	0.023129	0.29
1963-09-30	0.028083	-2.01
1963-10-31	0.085728	2.24
1963-11-30	0.081876	-0.07
…	…	…
2022-11-30	0.890160	4.19
2022-12-31	0.437472	-4.53
2023-01-31	0.377593	-1.41
2023-02-28	0.580736	-2.29
2023-03-31	0.335966	NaN

[717 rows x 2 columns]
Results for CMA:

OLS Regression Results

Dep. Variable:	returns	R-squared:	0.023
Model:	OLS	Adj. R-squared:	0.022
Method:	Least Squares	F-statistic:	16.72
Date:	Tue, 16 May 2023	Prob (F-statistic):	4.81e-05
Time:	20:07:46	Log-Likelihood:	-1525.8
No. Observations:	716	AIC:	3056.
Df Residuals:	714	BIC:	3065.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.0906	0.091	1.001	0.317	-0.087	0.268
var	1.4290	0.349	4.090	0.000	0.743	2.115

Omnibus:	20.072	Durbin-Watson:	1.758
Prob(Omnibus):	0.000	Jarque-Bera (JB):	43.118
Skew:	-0.052	Prob(JB):	4.34e-10
Kurtosis:	4.198	Cond. No.	4.67

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Date	var	returns

```
1963-07-31  0.000000e+00     0.25
1963-08-31  0.000000e+00     0.27
1963-09-30  7.792208e-07     0.29
1963-10-31  0.000000e+00     0.27
1963-11-30  6.233766e-07     0.29
…                    …      …
2022-11-30  4.090909e-07     0.33
2022-12-31  1.818182e-07     0.35
2023-01-31  8.658009e-08     0.34
2023-02-28  1.233766e-07     0.36
2023-03-31  0.000000e+00      NaN

[717 rows x 2 columns]
Results for RF:
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.115
Model:                            OLS   Adj. R-squared:                  0.113
Method:                 Least Squares   F-statistic:                     92.40
Date:                Tue, 16 May 2023   Prob (F-statistic):           1.18e-20
Time:                        20:07:46   Log-Likelihood:                -26.991
No. Observations:                 716   AIC:                             57.98
Df Residuals:                     714   BIC:                             67.13
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3374      0.010     34.614      0.000       0.318       0.357
var         5.299e+04   5512.908      9.613      0.000    4.22e+04    6.38e+04
==============================================================================
Omnibus:                       35.788   Durbin-Watson:                   0.254
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.103
Skew:                           0.520   Prob(JB):                     1.19e-09
Kurtosis:                       3.544   Cond. No.                     5.86e+05
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 5.86e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

                var  returns
Date
1963-07-31  0.077050     1.01
1963-08-31  0.047702     0.19
1963-09-30  0.072686     3.12
```

```
1963-10-31  0.094405    -0.74
1963-11-30  0.639916     1.75
…              …       …
2022-11-30  5.081281     4.52
2022-12-31  1.727624   -15.98
2023-01-31  1.484575     0.20
2023-02-28  2.716785    -2.52
2023-03-31  0.694750      NaN

[717 rows x 2 columns]
Results for Mom   :
                            OLS Regression Results
==============================================================================
Dep. Variable:                returns   R-squared:                       0.033
Model:                            OLS   Adj. R-squared:                  0.031
Method:                 Least Squares   F-statistic:                     24.22
Date:                Tue, 16 May 2023   Prob (F-statistic):           1.07e-06
Time:                        20:07:46   Log-Likelihood:                -2035.6
No. Observations:                 716   AIC:                             4075.
Df Residuals:                     714   BIC:                             4084.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9536      0.170      5.598      0.000       0.619       1.288
var           -0.5896      0.120     -4.921      0.000      -0.825      -0.354
==============================================================================
Omnibus:                      160.844   Durbin-Watson:                   2.018
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1338.225
Skew:                          -0.756   Prob(JB):                    2.56e-291
Kurtosis:                       9.525   Cond. No.                         1.76
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Looks like the variance does not predict returns in all cases.

5 1e

[ ]:
```
# Calculate the long-run average monthly variances
sigi = monthly_data
ci = np.array(list(monthly_data.mean(axis=0)))
```

```
# Calculate the weights for each factor
wi = ci / sigi

# Calculate the excess returns on the volatility-managed portfolio
volatility_managed_returns = wi.shift() * ff5mom_month
```

[ ]:
```
volatility_managed_returns
```

[ ]:
```
               Mkt-RF        SMB        HML        RMW        CMA        RF  \
Date
1963-07-31        NaN        NaN        NaN        NaN        NaN       NaN
1963-08-31  23.747326 -10.981421  15.762410   2.515628  -1.545590       inf
1963-09-30 -18.921550  -4.441025   1.090686  -8.269661   1.752552       inf
1963-10-31  17.190736  -7.598421  -0.946349  15.085914 -10.004081  0.173271
1963-11-30  -5.197698  -3.265103   4.985029  -1.279979   3.652190       inf
…                 …          …          …          …          …         …
2022-11-30   1.527708  -2.902824   0.674295   1.561768   0.912653  0.742589
2022-12-31  -2.110785  -0.186353   0.356726   0.022603   0.657924  0.375562
2023-01-31   3.800056   7.441123  -1.623087  -1.080581  -1.447364  0.896228
2023-02-28  -2.312642   0.945160  -0.572530   0.470325  -0.521945  1.828306
2023-03-31   2.355705  -5.769276  -3.340842   1.118402  -0.551171  1.358494

                Mom
Date
1963-07-31      NaN
1963-08-31   7.617299
1963-09-30   2.314570
1963-10-31  24.943665
1963-11-30  -4.555054
…                 …
2022-11-30  -0.748517
2022-12-31   0.516916
2023-01-31  -5.375048
2023-02-28   0.078286
2023-03-31  -0.539014

[717 rows x 7 columns]
```

6 1f

[ ]:
```
cols = ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA', 'Mom   ']
```

[ ]:
```
volatility_managed_returns.index = pd.to_datetime(volatility_managed_returns.
  ↪index)

# Create a new index with only the year and month
```

```
new_index = volatility_managed_returns.index.strftime('%Y-%m')

# Assign the new index back to your dataframe or series
volatility_managed_returns.index = new_index

volatility_managed_returns = volatility_managed_returns.dropna(subset=cols)
```

[ ]:
```
volatility_managed_returns.index = pd.to_datetime(volatility_managed_returns.
 ↪index)
volatility_managed_returns.index = volatility_managed_returns.index + pd.
 ↪offsets.MonthEnd(0)
```

[ ]:
```
for column in cols:
    small_ff5 = ff5mom_month.loc[volatility_managed_returns[column].index, :]
    capm_model = sm.OLS(volatility_managed_returns[column], sm.
 ↪add_constant(small_ff5['Mkt-RF']))
    capm_results = capm_model.fit()
    print(capm_results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Mkt-RF   R-squared:                       0.447
Model:                            OLS   Adj. R-squared:                  0.446
Method:                 Least Squares   F-statistic:                     576.4
Date:                Tue, 16 May 2023   Prob (F-statistic):           7.74e-94
Time:                        20:09:15   Log-Likelihood:                -2538.3
No. Observations:                 716   AIC:                             5081.
Df Residuals:                     714   BIC:                             5090.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5149      0.316      1.629      0.104      -0.106       1.136
Mkt-RF         1.6750      0.070     24.009      0.000       1.538       1.812
==============================================================================
Omnibus:                      349.224   Durbin-Watson:                   1.917
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             8038.033
Skew:                           1.660   Prob(JB):                         0.00
Kurtosis:                      19.075   Cond. No.                         4.57
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
```

Dep. Variable:	SMB	R-squared:	0.044
Model:	OLS	Adj. R-squared:	0.042
Method:	Least Squares	F-statistic:	32.62
Date:	Tue, 16 May 2023	Prob (F-statistic):	1.64e-08
Time:	20:09:15	Log-Likelihood:	-2453.5
No. Observations:	716	AIC:	4911.
Df Residuals:	714	BIC:	4920.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.1602	0.281	0.571	0.568	-0.391	0.712
Mkt-RF	0.3540	0.062	5.712	0.000	0.232	0.476

Omnibus:	125.908	Durbin-Watson:	1.454
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2191.462
Skew:	0.080	Prob(JB):	0.00
Kurtosis:	11.569	Cond. No.	4.57

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	HML	R-squared:	0.021
Model:	OLS	Adj. R-squared:	0.020
Method:	Least Squares	F-statistic:	15.67
Date:	Tue, 16 May 2023	Prob (F-statistic):	8.31e-05
Time:	20:09:15	Log-Likelihood:	-2624.1
No. Observations:	716	AIC:	5252.
Df Residuals:	714	BIC:	5261.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.1358	0.356	3.187	0.001	0.436	1.835
Mkt-RF	-0.3113	0.079	-3.958	0.000	-0.466	-0.157

Omnibus:	147.796	Durbin-Watson:	1.569
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2900.400
Skew:	0.322	Prob(JB):	0.00
Kurtosis:	12.839	Cond. No.	4.57

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	RMW	R-squared:	0.001
Model:	OLS	Adj. R-squared:	-0.001
Method:	Least Squares	F-statistic:	0.4017
Date:	Tue, 16 May 2023	Prob (F-statistic):	0.526
Time:	20:09:15	Log-Likelihood:	-2192.6
No. Observations:	716	AIC:	4389.
Df Residuals:	714	BIC:	4398.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.8672	0.195	4.446	0.000	0.484	1.250
Mkt-RF	-0.0273	0.043	-0.634	0.526	-0.112	0.057

Omnibus:	166.975	Durbin-Watson:	1.765
Prob(Omnibus):	0.000	Jarque-Bera (JB):	1081.197
Skew:	0.876	Prob(JB):	1.66e-235
Kurtosis:	8.760	Cond. No.	4.57

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	CMA	R-squared:	0.033
Model:	OLS	Adj. R-squared:	0.031
Method:	Least Squares	F-statistic:	24.10
Date:	Tue, 16 May 2023	Prob (F-statistic):	1.13e-06
Time:	20:09:15	Log-Likelihood:	-2011.7
No. Observations:	716	AIC:	4027.
Df Residuals:	714	BIC:	4036.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.5499	0.152	3.630	0.000	0.253	0.847
Mkt-RF	-0.1641	0.033	-4.909	0.000	-0.230	-0.098

Omnibus:	100.033	Durbin-Watson:	1.812
Prob(Omnibus):	0.000	Jarque-Bera (JB):	338.162
Skew:	0.642	Prob(JB):	3.71e-74

```
Kurtosis:                       6.113   Cond. No.                         4.57
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    Mom   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.1765
Date:                Tue, 16 May 2023   Prob (F-statistic):              0.675
Time:                        20:09:15   Log-Likelihood:                -2902.5
No. Observations:                 716   AIC:                             5809.
Df Residuals:                     714   BIC:                             5818.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.0496      0.526      7.702      0.000       3.017       5.082
Mkt-RF         0.0487      0.116      0.420      0.675      -0.179       0.277
==============================================================================
Omnibus:                      236.345   Durbin-Watson:                   1.675
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1607.815
Skew:                           1.306   Prob(JB):                         0.00
Kurtosis:                       9.861   Cond. No.                         4.57
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[ ]:
```
for column in cols:
    small_ff5 = ff5mom_month.loc[volatility_managed_returns[column].index, :]
    capm_model = sm.OLS(volatility_managed_returns[column], sm.
 ↪add_constant(small_ff5[['Mkt-RF', 'SMB', 'HML']]))
    capm_results = capm_model.fit()
    print(capm_results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Mkt-RF   R-squared:                       0.447
Model:                            OLS   Adj. R-squared:                  0.445
Method:                 Least Squares   F-statistic:                     191.9
Date:                Tue, 16 May 2023   Prob (F-statistic):           3.54e-91
Time:                        20:06:49   Log-Likelihood:                -2538.1
No. Observations:                 716   AIC:                             5084.
```

```
Df Residuals:                      712   BIC:                             5102.
Df Model:                            3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.4892      0.319      1.533      0.126      -0.137       1.116
Mkt-RF         1.6876      0.074     22.664      0.000       1.541       1.834
SMB           -0.0120      0.108     -0.111      0.912      -0.224       0.200
HML            0.0732      0.108      0.680      0.496      -0.138       0.284
==============================================================================
Omnibus:                      349.746   Durbin-Watson:                   1.918
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             8063.402
Skew:                           1.663   Prob(JB):                         0.00
Kurtosis:                      19.100   Cond. No.                         4.80
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    SMB   R-squared:                       0.489
Model:                            OLS   Adj. R-squared:                  0.487
Method:                 Least Squares   F-statistic:                     226.8
Date:                Tue, 16 May 2023   Prob (F-statistic):          2.93e-103
Time:                        20:06:49   Log-Likelihood:                -2229.4
No. Observations:                 716   AIC:                             4467.
Df Residuals:                     712   BIC:                             4485.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0586      0.207     -0.283      0.778      -0.466       0.349
Mkt-RF         0.0335      0.048      0.692      0.489      -0.062       0.128
SMB            1.7435      0.070     24.832      0.000       1.606       1.881
HML            0.0462      0.070      0.661      0.509      -0.091       0.183
==============================================================================
Omnibus:                      177.647   Durbin-Watson:                   1.511
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             8110.178
Skew:                          -0.095   Prob(JB):                         0.00
Kurtosis:                      19.487   Cond. No.                         4.80
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

OLS Regression Results

Dep. Variable:	HML	R-squared:	0.389
Model:	OLS	Adj. R-squared:	0.386
Method:	Least Squares	F-statistic:	150.8
Date:	Tue, 16 May 2023	Prob (F-statistic):	1.16e-75
Time:	20:06:49	Log-Likelihood:	-2455.7
No. Observations:	716	AIC:	4919.
Df Residuals:	712	BIC:	4938.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.3970	0.284	1.395	0.163	-0.162	0.956
Mkt-RF	-0.0406	0.066	-0.612	0.541	-0.171	0.090
SMB	0.0412	0.096	0.428	0.669	-0.148	0.230
HML	1.9780	0.096	20.633	0.000	1.790	2.166

Omnibus:	176.018	Durbin-Watson:	1.662
Prob(Omnibus):	0.000	Jarque-Bera (JB):	6026.455
Skew:	0.312	Prob(JB):	0.00
Kurtosis:	17.199	Cond. No.	4.80

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	RMW	R-squared:	0.085
Model:	OLS	Adj. R-squared:	0.082
Method:	Least Squares	F-statistic:	22.15
Date:	Tue, 16 May 2023	Prob (F-statistic):	1.01e-13
Time:	20:06:49	Log-Likelihood:	-2160.8
No. Observations:	716	AIC:	4330.
Df Residuals:	712	BIC:	4348.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.0386	0.188	5.512	0.000	0.669	1.409
Mkt-RF	-0.0072	0.044	-0.163	0.870	-0.093	0.079
SMB	-0.3678	0.064	-5.765	0.000	-0.493	-0.243
HML	-0.3473	0.063	-5.469	0.000	-0.472	-0.223

Omnibus:	153.634	Durbin-Watson:	1.770

Prob(Omnibus):	0.000	Jarque-Bera (JB):	911.266
Skew:	0.818	Prob(JB):	1.32e-198
Kurtosis:	8.279	Cond. No.	4.80

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	CMA	R-squared:	0.244
Model:	OLS	Adj. R-squared:	0.241
Method:	Least Squares	F-statistic:	76.58
Date:	Tue, 16 May 2023	Prob (F-statistic):	6.11e-43
Time:	20:06:49	Log-Likelihood:	-1923.4
No. Observations:	716	AIC:	3855.
Df Residuals:	712	BIC:	3873.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.3121	0.135	2.307	0.021	0.047	0.578
Mkt-RF	-0.0727	0.032	-2.304	0.022	-0.135	-0.011
SMB	-0.0053	0.046	-0.116	0.907	-0.095	0.085
HML	0.6426	0.046	14.098	0.000	0.553	0.732

Omnibus:	91.892	Durbin-Watson:	1.907
Prob(Omnibus):	0.000	Jarque-Bera (JB):	399.995
Skew:	0.501	Prob(JB):	1.39e-87
Kurtosis:	6.522	Cond. No.	4.80

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	Mom	R-squared:	0.013
Model:	OLS	Adj. R-squared:	0.009
Method:	Least Squares	F-statistic:	3.078
Date:	Tue, 16 May 2023	Prob (F-statistic):	0.0270
Time:	20:06:49	Log-Likelihood:	-2898.0
No. Observations:	716	AIC:	5804.
Df Residuals:	712	BIC:	5822.
Df Model:	3		
Covariance Type:	nonrobust		

```
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.2503      0.528      8.056      0.000       3.214       5.286
Mkt-RF         0.0301      0.123      0.244      0.807      -0.212       0.272
SMB           -0.2482      0.179     -1.390      0.165      -0.599       0.102
HML           -0.4634      0.178     -2.606      0.009      -0.812      -0.114
==============================================================================
Omnibus:                      238.911   Durbin-Watson:                   1.674
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1632.139
Skew:                           1.321   Prob(JB):                         0.00
Kurtosis:                       9.909   Cond. No.                         4.80
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[ ]:
```
volatility_managed_returns
```

[ ]:
```
               Mkt-RF        SMB        HML        RMW        CMA        RF  \
Date
1963-08-31  23.747326 -10.981421  15.762410   2.515628  -1.545590       inf   \
1963-09-30 -18.921550  -4.441025   1.090686  -8.269661   1.752552       inf
1963-10-31  17.190736  -7.598421  -0.946349  15.085914 -10.004081  0.173271
1963-11-30  -5.197698  -3.265103   4.985029  -1.279979   3.652190       inf
1963-12-31   1.283602  -2.905950  -0.082965   0.050009  -0.119502  0.216589
…                 …          …          …          …          …         …
2022-11-30   1.527708  -2.902824   0.674295   1.561768   0.912653  0.742589
2022-12-31  -2.110785  -0.186353   0.356726   0.022603   0.657924  0.375562
2023-01-31   3.800056   7.441123  -1.623087  -1.080581  -1.447364  0.896228
2023-02-28  -2.312642   0.945160  -0.572530   0.470325  -0.521945  1.828306
2023-03-31   2.355705  -5.769276  -3.340842   1.118402  -0.551171  1.358494

                Mom
Date
1963-08-31   7.617299
1963-09-30   2.314570
1963-10-31  24.943665
1963-11-30  -4.555054
1963-12-31   1.589168
…                 …
2022-11-30  -0.748517
2022-12-31   0.516916
2023-01-31  -5.375048
2023-02-28   0.078286
2023-03-31  -0.539014
```

[716 rows x 7 columns]

[ ]:
```
for column in cols:
    small_ff5 = ff5mom_month.loc[volatility_managed_returns[column].index, :]
    capm_model = sm.OLS(volatility_managed_returns[column], sm.
 ↪add_constant(small_ff5))
    capm_results = capm_model.fit()
    print(capm_results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Mkt-RF   R-squared:                       0.467
Model:                            OLS   Adj. R-squared:                  0.462
Method:                 Least Squares   F-statistic:                     88.72
Date:                Tue, 16 May 2023   Prob (F-statistic):           1.63e-92
Time:                        20:06:49   Log-Likelihood:                -2524.7
No. Observations:                 716   AIC:                             5065.
Df Residuals:                     708   BIC:                             5102.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1675      0.535      0.313      0.754      -0.884       1.219
Mkt-RF         1.7723      0.078     22.590      0.000       1.618       1.926
SMB            0.0653      0.113      0.579      0.563      -0.156       0.287
HML            0.1688      0.150      1.126      0.260      -0.125       0.463
RMW            0.3018      0.153      1.971      0.049       0.001       0.603
CMA            0.0180      0.222      0.081      0.936      -0.418       0.454
RF            -0.2121      1.167     -0.182      0.856      -2.504       2.080
Mom            0.3542      0.077      4.576      0.000       0.202       0.506
==============================================================================
Omnibus:                      363.576   Durbin-Watson:                   1.937
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             8373.950
Skew:                           1.759   Prob(JB):                         0.00
Kurtosis:                      19.380   Cond. No.                         20.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    SMB   R-squared:                       0.499
Model:                            OLS   Adj. R-squared:                  0.495
Method:                 Least Squares   F-statistic:                     100.9
Date:                Tue, 16 May 2023   Prob (F-statistic):          4.98e-102
```

Time:	20:06:49	Log-Likelihood:	-2221.7
No. Observations:	716	AIC:	4459.
Df Residuals:	708	BIC:	4496.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.3255	0.351	-0.928	0.354	-1.014	0.363
Mkt-RF	0.0611	0.051	1.189	0.235	-0.040	0.162
SMB	1.8384	0.074	24.874	0.000	1.693	1.983
HML	-0.0086	0.098	-0.087	0.930	-0.201	0.184
RMW	0.3861	0.100	3.849	0.000	0.189	0.583
CMA	0.0881	0.146	0.605	0.545	-0.198	0.374
RF	0.2896	0.765	0.379	0.705	-1.212	1.791
Mom	0.0130	0.051	0.257	0.797	-0.087	0.113

Omnibus:	173.980	Durbin-Watson:	1.520
Prob(Omnibus):	0.000	Jarque-Bera (JB):	7580.081
Skew:	-0.030	Prob(JB):	0.00
Kurtosis:	18.940	Cond. No.	20.0

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	HML	R-squared:	0.445
Model:	OLS	Adj. R-squared:	0.439
Method:	Least Squares	F-statistic:	81.02
Date:	Tue, 16 May 2023	Prob (F-statistic):	3.34e-86
Time:	20:06:49	Log-Likelihood:	-2421.2
No. Observations:	716	AIC:	4858.
Df Residuals:	708	BIC:	4895.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.1215	0.463	2.421	0.016	0.212	2.031
Mkt-RF	-0.1000	0.068	-1.473	0.141	-0.233	0.033
SMB	-0.2325	0.098	-2.380	0.018	-0.424	-0.041
HML	2.2076	0.130	17.021	0.000	1.953	2.462
RMW	-1.1167	0.133	-8.424	0.000	-1.377	-0.856
CMA	-0.3148	0.192	-1.636	0.102	-0.693	0.063
RF	-0.9739	1.010	-0.964	0.335	-2.957	1.010
Mom	0.0960	0.067	1.433	0.152	-0.036	0.228

```
==============================================================================
Omnibus:                      185.235   Durbin-Watson:                   1.656
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6968.986
Skew:                           0.359   Prob(JB):                         0.00
Kurtosis:                      18.267   Cond. No.                         20.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    RMW   R-squared:                       0.413
Model:                            OLS   Adj. R-squared:                  0.407
Method:                 Least Squares   F-statistic:                     71.05
Date:                Tue, 16 May 2023   Prob (F-statistic):           1.24e-77
Time:                        20:06:49   Log-Likelihood:                -2002.3
No. Observations:                 716   AIC:                             4021.
Df Residuals:                     708   BIC:                             4057.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.2621      0.258     -1.016      0.310      -0.769       0.245
Mkt-RF         0.0343      0.038      0.907      0.365      -0.040       0.109
SMB           -0.0346      0.054     -0.636      0.525      -0.141       0.072
HML           -0.3523      0.072     -4.876      0.000      -0.494      -0.210
RMW            1.3814      0.074     18.710      0.000       1.236       1.526
CMA           -0.1779      0.107     -1.660      0.097      -0.388       0.032
RF             2.4195      0.563      4.299      0.000       1.315       3.524
Mom           -0.0100      0.037     -0.267      0.790      -0.083       0.063
==============================================================================
Omnibus:                      212.775   Durbin-Watson:                   1.890
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1761.313
Skew:                           1.090   Prob(JB):                         0.00
Kurtosis:                      10.368   Cond. No.                         20.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                    CMA   R-squared:                       0.516
Model:                            OLS   Adj. R-squared:                  0.511
Method:                 Least Squares   F-statistic:                     107.9
Date:                Tue, 16 May 2023   Prob (F-statistic):          3.34e-107
```

Time:	20:06:49	Log-Likelihood:	-1763.6
No. Observations:	716	AIC:	3543.
Df Residuals:	708	BIC:	3580.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.0084	0.185	-0.045	0.964	-0.371	0.355
Mkt-RF	0.0412	0.027	1.520	0.129	-0.012	0.094
SMB	-0.0716	0.039	-1.836	0.067	-0.148	0.005
HML	0.0668	0.052	1.291	0.197	-0.035	0.168
RMW	-0.3553	0.053	-6.716	0.000	-0.459	-0.251
CMA	1.3072	0.077	17.022	0.000	1.156	1.458
RF	0.5367	0.403	1.331	0.184	-0.255	1.328
Mom	-0.0553	0.027	-2.067	0.039	-0.108	-0.003

Omnibus:	161.069	Durbin-Watson:	1.873
Prob(Omnibus):	0.000	Jarque-Bera (JB):	863.984
Skew:	0.897	Prob(JB):	2.44e-188
Kurtosis:	8.073	Cond. No.	20.0

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS Regression Results

Dep. Variable:	Mom	R-squared:	0.319
Model:	OLS	Adj. R-squared:	0.312
Method:	Least Squares	F-statistic:	47.41
Date:	Tue, 16 May 2023	Prob (F-statistic):	3.35e-55
Time:	20:06:49	Log-Likelihood:	-2765.0
No. Observations:	716	AIC:	5546.
Df Residuals:	708	BIC:	5583.
Df Model:	7		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	1.1225	0.749	1.499	0.134	-0.348	2.593
Mkt-RF	0.3611	0.110	3.291	0.001	0.146	0.577
SMB	-0.2335	0.158	-1.480	0.139	-0.543	0.076
HML	0.4301	0.210	2.052	0.041	0.019	0.842
RMW	-0.0259	0.214	-0.121	0.904	-0.447	0.395
CMA	-0.6696	0.311	-2.153	0.032	-1.280	-0.059
RF	4.7949	1.633	2.936	0.003	1.589	8.001
Mom	1.8714	0.108	17.285	0.000	1.659	2.084

```
==============================================================================
Omnibus:                      353.503   Durbin-Watson:                   1.661
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4542.006
Skew:                           1.886   Prob(JB):                         0.00
Kurtosis:                      14.748   Cond. No.                         20.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

The reason the strategies still produce alpha is that maybe the increased risk in the future is not properly priced in time.

Volatility is persistent. Low volatility still performs well in the future, the alpha is attained by identifying these low volatility opportunities early, thus giving us alpha in the future.