Kane's Method and Kinematic Loops

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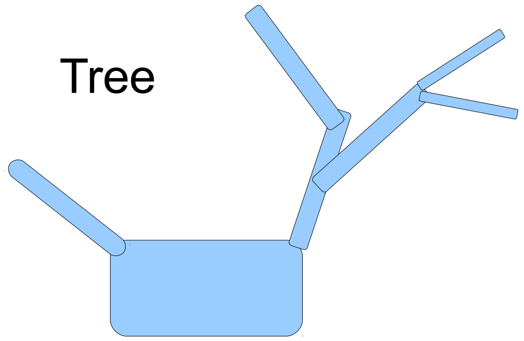
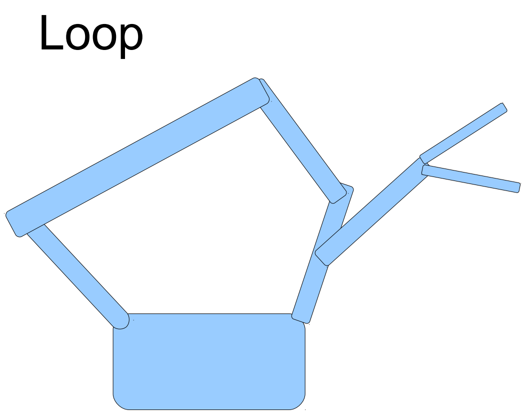
Trees and Loops

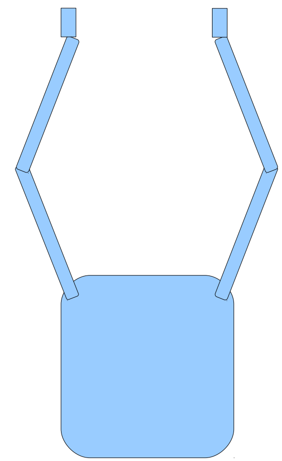
- Our previous work has been limited to “tree” topologies
  - Representative of large class of interesting problems
  - Easy to assign one generalized speed to each DOF
- Kane's method also works on systems with kinematic loops
  - Loops arise in multi-armed robotic grapplers
  - Loops introduce more constraints

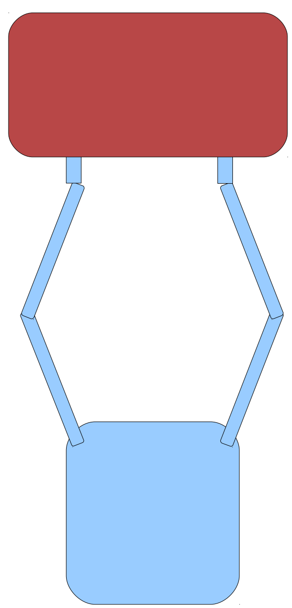




Example: A Robotic Grappler

- For illustration, let's consider a fairly realistic example problem
- A spacecraft servicing robot has two arms
  - 3DOF shoulder joint
  - 1DOF elbow joint
  - 3DOF wrist joint
- By itself, the grappler is a tree
- When it grabs a target with both arms, a loop is formed

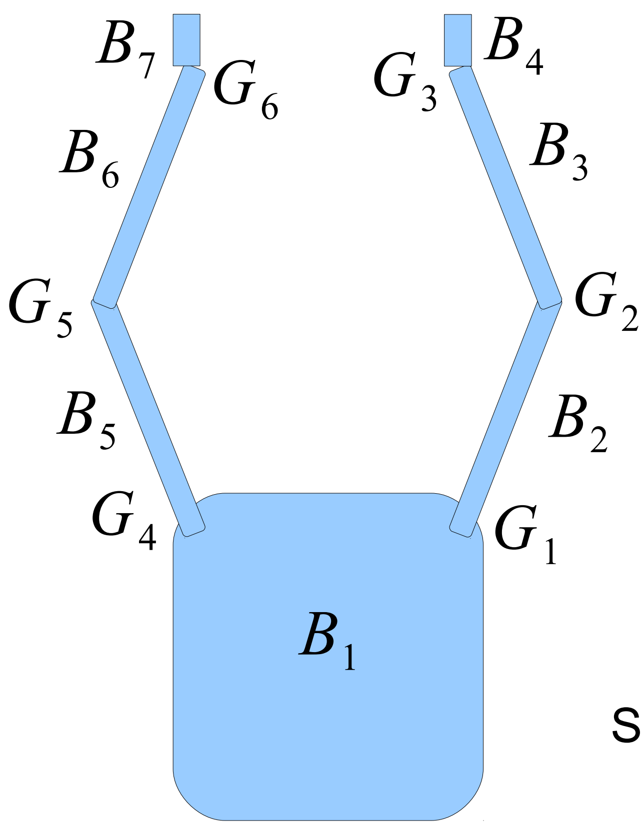




Constraints and DOFs

- Before capture, the grappler has 6+7+7=20 DOF, and the target has 6
  - Total of 26 DOF in system
- Capturing with one arm introduces 6 constraints
  - Reduces DOF count to 20
- Grappling with the second arm introduces 6 more constraints
- Fourteen DOFs remain in the grappler+target system
  - Six DOF in main body position, attitude
  - Eight DOF in arms, target

Generalized Speeds Before Capture



$$u = \{\omega_1 \;\; \sigma_1 \;\; \sigma_2 \;\; \sigma_3 \;\; \sigma_4 \;\; \sigma_5 \;\; \sigma_6 \;\; v_1\}^T$$

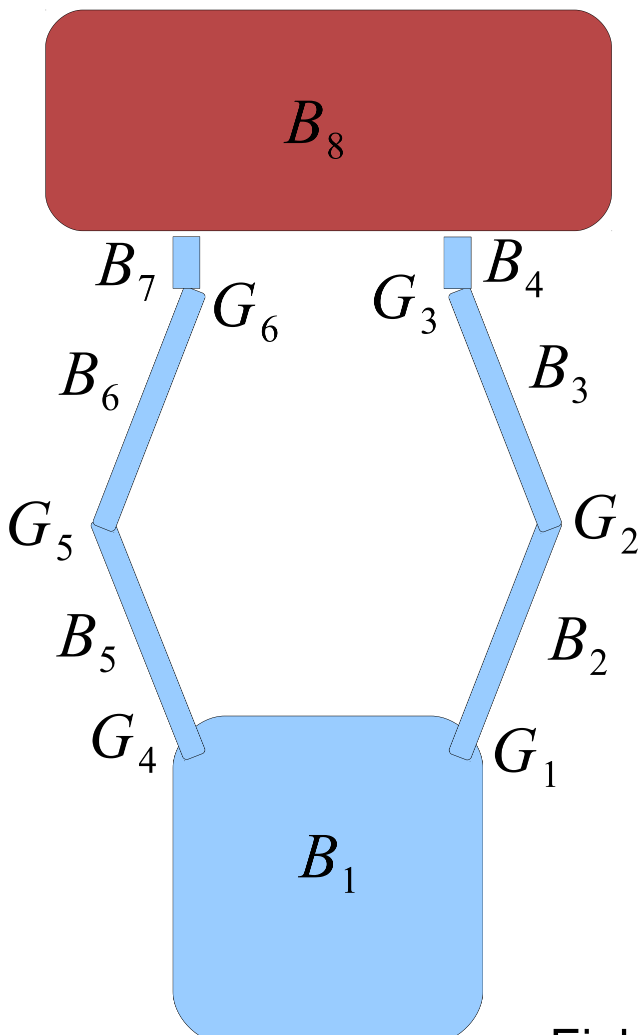
$$\underbrace{\{\omega\}_{BC}}_{21\times 1} = \underbrace{\Omega}_{21\times 20} \; \underbrace{u}_{20\times 1}$$

$$\underbrace{\{v\}_{BC}}_{21\times 1} = \underbrace{V}_{21\times 20} \; \underbrace{u}_{20\times 1}$$

Before capture, these generalized speeds are independent, and span all DOFs.

Seven bodies, 20 DOF, 20 generalized speeds

After Capture, These Gen Speeds are Not Independent



After capture, we gain one body but lose a net 6 DOF. So these generalized speeds are no longer independent.

We note this by renaming them as $\tilde{u}$, and seek a new, independent set of u's

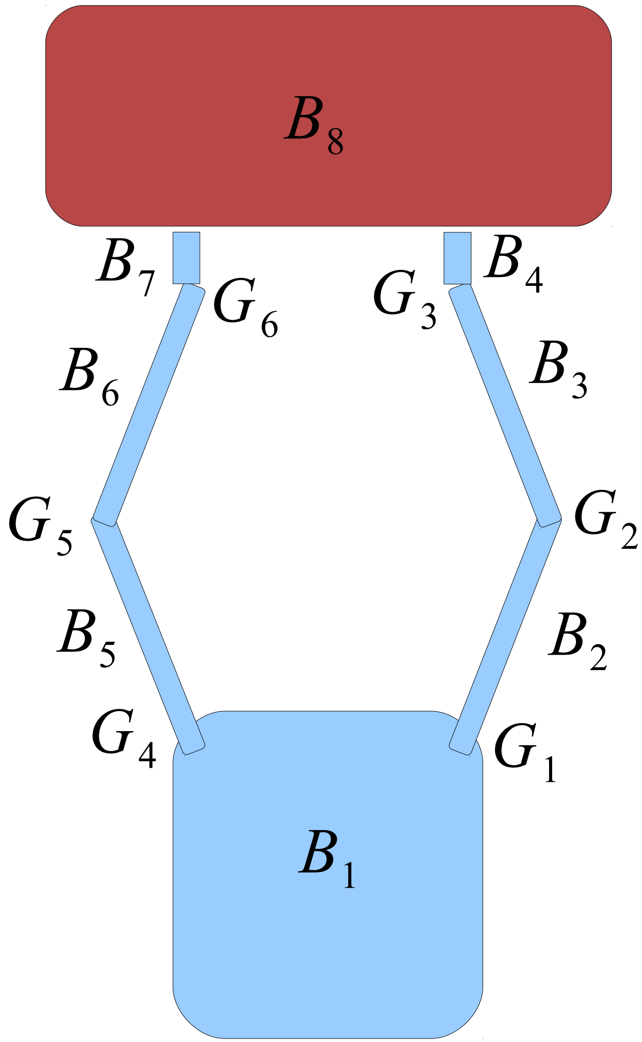
$$\tilde{u} = \{\omega_1 \quad \sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4 \quad \sigma_5 \quad \sigma_6 \quad v_1\}^T$$

$$\underbrace{\{\omega\}_{AC}}_{24\times 1} = \underbrace{\tilde{\Omega}}_{24\times 20} \underbrace{\tilde{u}}_{20\times 1}$$

$$\underbrace{\{v\}_{AC}}_{24\times 1} = \underbrace{\tilde{V}}_{24\times 20} \underbrace{\tilde{u}}_{20\times 1}$$

Eight bodies, 14 DOF, 20 generalized speeds

Seeking New Gen Speeds from Old



Let $\tilde{u} = \Phi u$ so that the elements of the new u are independent and span the permissible motion space.

$$\underbrace{\{\omega\}_{AC}}_{24\times 1} = \underbrace{\tilde{\Omega}}_{24\times 20} \underbrace{\tilde{u}}_{20\times 1} = \underbrace{\tilde{\Omega}}_{24\times 20} (\underbrace{\Phi}_{20\times 14} \underbrace{u}_{14\times 1}) = (\underbrace{\tilde{\Omega}}_{24\times 20} \underbrace{\Phi}_{20\times 14}) \underbrace{u}_{14\times 1} = \underbrace{\Omega}_{24\times 14} \underbrace{u}_{14\times 1}$$

$$\underbrace{\{v\}_{AC}}_{24\times 1} = \underbrace{\tilde{V}}_{24\times 20} \underbrace{\tilde{u}}_{20\times 1} = \underbrace{\tilde{V}}_{24\times 20} (\underbrace{\Phi}_{20\times 14} \underbrace{u}_{14\times 1}) = (\underbrace{\tilde{V}}_{24\times 20} \underbrace{\Phi}_{20\times 14}) \underbrace{u}_{14\times 1} = \underbrace{V}_{24\times 14} \underbrace{u}_{14\times 1}$$

The possibilities are endless.
We will look at two.

Scheme 1: Pick Any Fourteen

- Any 14 of the old generalized speeds are independent
- The other 6 old generalized speeds may be expressed in terms of the independent set, from the 6 added constraint equations

$$\underbrace{\tilde{u}}_{20\times 1} = \begin{bmatrix} U_{14\times 14} \\ C_{6\times 14} \end{bmatrix} \underbrace{u}_{14\times 1}$$

Scheme 2: “Modal” Decomposition

- Find fourteen independent motions that the system can undergo. Make these the new u's.
- Express the old u's in terms of these new u's.
- For example:
  - Translation and rotation of main body
    - Yield basic 6 DOF
  - Move arms to move target in pure translation
    - X, Y, Z translation yield three independent modes.
  - Move arms to cause single-axis rotation of target
    - X, Y, Z rotations yield three more independent modes.
  - Holding target still, wiggle elbows
    - Right and left elbows may be moved independently, yielding remaining two required modes.

Conclusion

- Solving a kinematic loop is simply a problem of reducing the existing set of generalized speeds to a new independent set, with the appropriate mapping.
- Partial velocity matrices absorb the mapping from old gen-speeds to new.
- The rest of the formulation of equations of motion proceeds as for the tree topology.