Application of DNN - Image segmentation

Numerical Methods for Deep Learning

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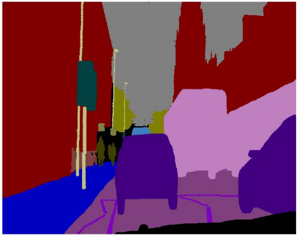
Image segmentation

The problem: Given an image $\mathbf{Y} : R^2 \to R$ compute n probability maps

$$\mathbf{P}_j : R^2 \to [0,1], \quad \sum \mathbf{P}_j = 1$$



$\mathbf{P}_j$ is the probability of a pixel to belong to class j

Image segmentation

Problem setup.

- Map the original image into n “images”.
- Define the cross entropy for each pixel
- Optimize over network parameters

Simplest example, a single layer and linear classification

Image segmentation - single layer NN

Problem setup.

$$\mathbf{Y}_2 = \sigma(\mathbf{K}\mathbf{Y}_1 + \mathbf{B}\mathbf{b}).$$

Use convolutions for **K**.
For color images with n classes we have

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{K}_{23} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \mathbf{K}_{n1} & \mathbf{K}_{n2} & \mathbf{K}_{n3} \end{pmatrix}.$$

Then we have to classify

Image segmentation - single layer NN

```
Y     = reshape(Y,n1,n2,np)
for i=1:np
    Wi = reshape(W(:,i),1,1,np)
    WY(:,:,i) = sum(Wi.*Y,3)
end

% Now use the exponent to obtain the map
C = exp(WY)./sum(exp(WY),3)
```

Image segmentation - matrix notation

Note that we can write the objective function using kroneker products

$$\mathbf{W}_i = \mathbf{w}_i^\top \otimes \mathbf{I}_2 \otimes \mathbf{I}_1$$

And

$$\mathbf{W} = \begin{pmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_n \end{pmatrix}$$

Finally, define the sum over the third dimension as

$$\mathbf{S} = \mathbf{e}^\top \otimes \mathbf{I}_2 \otimes \mathbf{I}_1$$

To obtain

$$\mathbf{C} = \frac{\exp(\mathbf{WY})}{\mathbf{S}\exp(\mathbf{WY})}.$$

Image segmentation - single layer NN

Put it all together

The network

$$\mathbf{Y}_2 = \sigma(\mathbf{K}\mathbf{Y}_1 + \mathbf{B}\mathbf{b}).$$

The class

$$\mathbf{C} = \frac{\exp(\mathbf{W}\mathbf{Y}_2)}{\mathbf{S}\exp(\mathbf{W}\mathbf{Y}_2)}.$$

And minimizing the cross entropy

$$\min -\mathbf{C}_{\rm obs}^{\top} \log \mathbf{C}(\mathbf{K}, \mathbf{b}, \mathbf{W})$$