Outline of objectives and project significance

In magnetic resonance imaging (MRI), insight into different imaging modalities can be gained through the simulation of the MR signal from first principles. This can be done by solving partial differential equations which govern how the magnetization changes through time during an MRI scan, called the Bloch-Torrey (BT) equations(1). By considering only the magnetization orthogonal to the constant external magnetic field B_0, the Bloch-Torrey equations simplify to

$$\frac{\partial}{\partial t}M(\vec{r},t) = D\Delta M(\vec{r},t) - \Gamma(\vec{r})M(\vec{r},t)$$

where $M = M_x + iM_y$ is the complex magnetization, D is the diffusion constant, $\Gamma(\vec{r}) = R_2(\vec{r}) + i\omega(\vec{r})$ is the complex decay rate, $R_2(\vec{r})$ is the relaxation rate, and $\omega(\vec{r})$ is the resonance frequency, proportional to the local magnetic field.

In the brain's white matter these simulations are challenging to perform due to structures such as blood vessels and capillaries having different magnetic properties than the surrounding tissue, causing $R_2(\vec{r})$ and $\omega(\vec{r})$ to be only piecewise smooth with sharp changes at blood-tissue boundaries.

In my research thus far I have dealt with this issue by solving the BT equation on a very fine grid. However, since I additionally must fit the BT solution to observed data, requiring the BT equation to be solved many times, this process is becoming increasingly costly in both time and computational resources.

I propose to further investigate methods of solving the BT equation which address both the discontinuity of the data and the need for having extremely fast solutions. In particular, I aim to minimize computational resource use while maintaining a suitably accurate solution.

Literature review

Eigenfunction decomposition of the BT equation has been studied(2), but unfortunately is only effective for suitably smooth magnetic field distributions $\omega(\vec{r})$, as otherwise a prohibitively large set of basis vectors must be computed. This also rules out Fourier decomposition methods.

Finite element methods have been used effectively in order to produce accurate solutions of complex geometries(3). However, these methods are generally only fast and effective for single large simulations with known geometries, whereas I need to generate random meshes programmatically for each of a large number of BT solutions, and this process would be difficult and the result may be slow.

Currently, the best trade-off of accuracy vs. speed which I have encountered are splitting methods(4,5) based on the equivalence of the BT equation and the imaginary-time Schrödinger equation(6). These methods are fast and simple to implement, but still require a very fine grid on which to compute the solution.

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