5.2.10 Remark (Algebraic Flux Correction) An implementation of algebraic flux correction now works as follows:

1. Start with a high-order scheme
$$M\dot{u} + Cu = 0$$
(e.g. central differencing, linear finite elements).

2. Add discrete diffusion to this scheme so that the three design criteria are met (e.g. mass lumping, turning C into a Z-matrix).

   This results in a low-order scheme
$$\tilde{M}\dot{u} + \tilde{C}u = 0$$

   ⊕ positivity preserving / LED for all nonnegative initial conditions

   ⊖ strongly dissipative

   ⊖ inaccurate (at most 1^{st} order)

3. Add as much discrete antidiffusion as possible back to the low-order scheme to obtain a blended scheme which should ideally agree with the high-order scheme, while still preserving the monotonicity of the low-order scheme.

   ⊕ still positivity preserving /LED for this particular initial condition

   ⊕ reduced dissipation

   ⊕ 2^{nd} order accurate wherever possible (e.g. where solution is smooth)

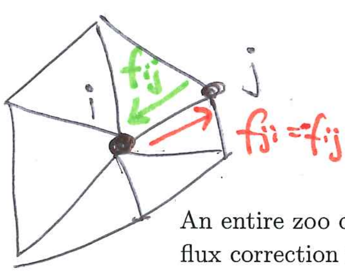
HO - scheme:

$$\tilde{M}\dot{u} + \tilde{C}u \underbrace{- (\tilde{M} - M)\dot{u} - (\tilde{C} - C)u}_{\text{antidiffusion}} = 0$$

$$\tilde{M}\dot{u} + \tilde{C}u - \overbrace{Du} = 0 \xrightarrow{i\text{th row}} (\tilde{M}\dot{u} + \tilde{C}u)_i = \sum_j d_{ij}u_j$$

$$\overset{\sum_j d_{ij}=0}{=} \sum_{j\neq i} d_{ij}(u_i - u_j)$$

$$= \underbrace{\sum_{j\neq i} \underbrace{f_{ij}}_{\text{antidiffusive nodal fluxes } j\to i}}_{\text{net flux into node } i}$$



An entire zoo of flux limiters can be found in the literature. A particularly well-suited limiter for algebraic flux correction in this very general form is the so-called ZALESAK limiter.

↓ replace f_{ij} with limited fluxes $\overline{f_{ij}} = \alpha_{ij}(u)\, f_{ij}$

$$\sum_{j\neq i} \overline{f_{ij}}$$

Coefficients $\alpha_{ij}(u) \in [0,1]$ are chosen as large as possible, but so that local maxima don't grow + local minima don't decrease
⇒ "wiggles" are suppressed in a conservative + consistent fashion.