MATH521
Numerical Analysis of Partial Differential Equations

Winter 2017/18, Term 2
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Homework Assignment 1: Model Answers

Please submit the following files as indicated below: source code | PDF file | image file | video file

Question 1 | **2 marks** | Consider the following PDE:

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_1 \partial x_2} + x_2 \frac{\partial^2 u}{\partial x_2^2} + \frac{1}{3} u^3 = 0$$

(a) Tick all that apply. This is a

◯ linear	☑ semi-linear	◯ fully nonlinear

and

☑ homogeneous	◯ inhomogeneous

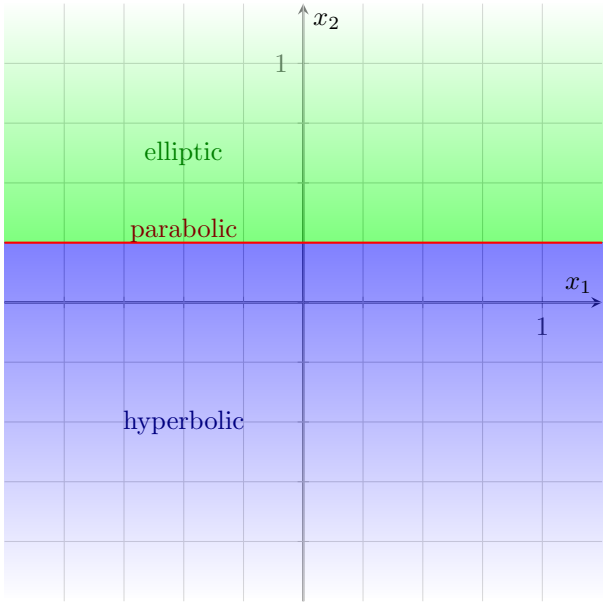
PDE of

◯ 1^{st} order	☑ 2^{nd} order	◯ 3^{rd} order

in

◯ 1 variable	☑ 2 variables	◯ 3 variables.

(b) Some PDEs are not uniformly elliptic, parabolic or hyperbolic, but their type may change within the domain. Determine the regions where the above PDE is elliptic, parabolic or hyperbolic and sketch these in the following coordinate system.



Show all working:
After comparing coefficients with the principal part

$$a_{11} \frac{\partial^2 u}{\partial x_1^2} + 2a_{12} \frac{\partial^2 u}{\partial x_1 \partial x_2} + a_{22} \frac{\partial^2 u}{\partial x_2^2}$$

of the PDE $(\star)$ in the notes we obtain $a_{11} = 1$, $a_{12} = 1/2$, $a_{22} = x_2$ and the discriminant is

$$a_{12}^2 - a_{11} a_{22} = \frac{1}{4} - x_2,$$

which is > 0 in the hyperbolic, $= 0$ in the parabolic and < 0 in the elliptic case.

Like I said in class, I didn't write the 2 in front of the a_{12}-term in the video. I also found out that my comment below this video where I point this out is strangely not visible to anyone but me. So if anyone forgot about the 2 as well, you won't have any marks deducted for that!

Question 2 | **3 marks** | In this first assignment we set up a core component for an implementation of the finite difference method, which we will build upon next week. I recommend to use GNU Octave / MATLAB for our first assignments, as you will have extra translation work to do if you prefer to use another programming language.

(a) Write a function `meshRectangle` which meshes a two-dimensional rectangular domain. The function should take two input variables

`x:` a 1×4 array, which defines the coordinates of the rectangle $[\texttt{x(1)}, \texttt{x(2)}] \times [\texttt{x(3)}, \texttt{x(4)}]$ (NB: this notation is a Cartesian product of two intervals)

`N`: a 1×2 array, which specifies that the domain is to be divided into `N(1)` subintervals in x_1-direction and `N(2)` subintervals in x_2-direction.

Furthermore, `meshRectangle` should return one output variable

`msh`: a structure with fields

`X1, X2`: both arrays of size $(\texttt{N(2)}+1) \times (\texttt{N(1)}+1)$ that contain the x_1 or x_2 components, respectively, of each grid point

`N`: a copy of the input variable of the same name

`h`: an array of size 1×2 which contains the width of the subintervals in x_1 and x_2-direction

(b) Complete and run the following program `hw1`:

```
% hw1.m
clear all; close all; clc;

% sample function
u = @(x1,x2) sin(2.*pi.*x1).*cos(6.*pi.*x2);

% mesh the rectange [0,1] x [2,3] with 20 / 60 subintervals in x1- / x2-direction, respectively
msh = meshRectangle([?,?,?,?],[?,?]);

% evaluate u on msh and draw a surface plot
surf(msh.X1,msh.X2,u(msh.X1,msh.X2));

% axis labels
???
```

Check that all details are correct, such as the exact number of subintervals in each direction and the orientation of the graph. Add labels to all three axes.

(c) Save the graph in a vector graphics format (recommended) or a high quality raster graphics format.

Hint: In GNU Octave / MATLAB, the commands `linspace`, `meshgrid`, `xlabel`, `ylabel` and `zlabel` may be helpful. Use the commands `help linspace`, `help meshgrid` etc. for more information and examples of use.

Submission instructions for this and all future assignments: Please upload your solutions on Canvas. You may annotate this document electronically with a stylus, or upload a scanned paper copy with your handwritten solutions. Additionally, for all computational questions, please include

- well-commented source code in its native file format, e.g. `hw1.m` and `meshRectangle.m`
- one (yes, *one*!) PDF file with a printout of all your code, e.g. `code.pdf`
- any extra files of graphs etc as instructed.

Please upload these files separately, not archived. I suggest not to include your name anywhere so that our marking will be fully blinded.

Your Learning Progress | What is the one most important thing that you have learnt from this assignment?

What is the most substantial new insight that you have gained from this course this week? Any *aha moment*?
