


# Elliptic Maximum Principle

Here are our in-class activities for the elliptic maximum principle:

## Steady-state temperature distribution in a pot

[metalpot.pdf \(https://canvas.ubc.ca/courses/2337/files/539191/download?wrap=1\)](https://canvas.ubc.ca/courses/2337/files/539191/download?wrap=1)   
[\(https://canvas.ubc.ca/courses/2337/files/539191/download?wrap=1\)](https://canvas.ubc.ca/courses/2337/files/539191/download?wrap=1)

We assume that the temperature  $T$  inside the metal volume obeys Poisson's equation at steady state. On different parts of the pot's surface we impose different temperature values as boundary conditions.

**1st case: no external heat source.** Now we can use the maximum principle and the minimum principle to derive that the temperature  $T(x)$  everywhere inside the metal volume remains bounded between the smallest and highest temperatures on the boundary.

**2nd case: external heat source, e.g. due to microwave radiation.** Now the maximum principle is no longer applicable, but the minimum principle still is. It still cannot get colder inside the metal than on the pot's surface, but it may get hotter inside than on the boundary.

## Discrete maximum principle

[discreteMaximumPrinciple.pdf \(https://canvas.ubc.ca/courses/2337/files/539190/download?wrap=1\)](https://canvas.ubc.ca/courses/2337/files/539190/download?wrap=1)  
 [\(https://canvas.ubc.ca/courses/2337/files/539190/download?wrap=1\)](https://canvas.ubc.ca/courses/2337/files/539190/download?wrap=1)

If we discretise a linear, elliptic boundary value problem, e.g. Poisson's equation with Dirichlet boundary conditions, with finite differences or finite elements, we obtain a big linear system.

It is not automatic that the discrete solution of this linear system will satisfy a maximum principle even if the analytical solution does. E.g. even if the analytical solution is guaranteed to be nonnegative, a poor discretisation scheme could give us negative values in a numerical solution.

In this exercise we found out that any 'good' discretisation scheme produces a discrete differential operator, the inverse of which only has nonnegative entries. Then the numerical solution will indeed satisfy the same maximum principle as the analytical solution.

We'll soon learn about a more practical criterion than looking at the signs of the inverse operator!



