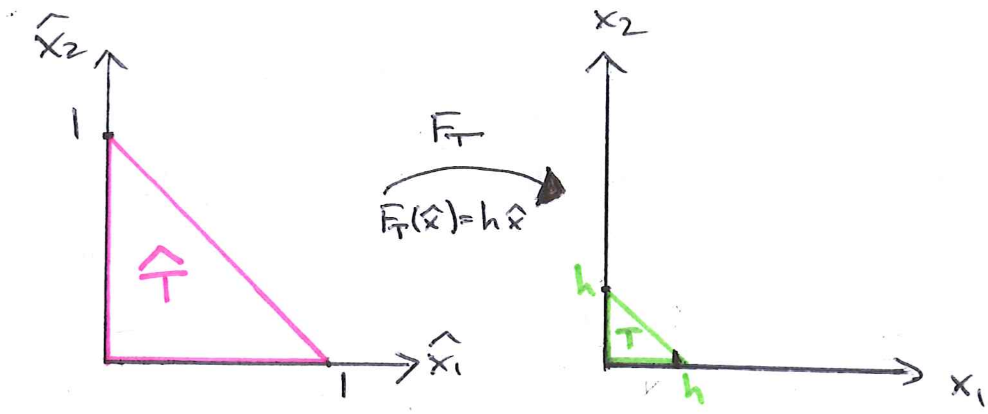


$w(x) = \hat{w}(\hat{x})$

$\|w\|_{L^2(T)} = \underline{h} \; \|\hat{w}\|_{L^2(\hat{T})}$

$\|\nabla w\|_{L^2(T)} = \underline{\qquad} \; \|\hat{\nabla}\hat{w}\|_{L^2(\hat{T})}$

$\|\nabla^2 w\|_{L^2(T)} = \underline{\frac{1}{h}} \; \|\hat{\nabla}^2\hat{w}\|_{L^2(\hat{T})}$

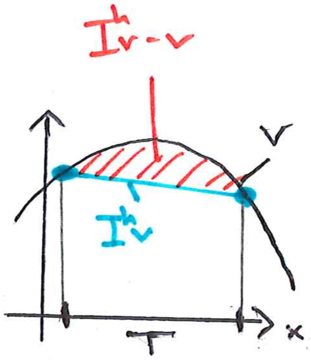
$\|\nabla^i w\|_{L^2(T)} = \underline{h^{1-i}} \; \|\hat{\nabla}^i\hat{w}\|_{L^2(\hat{T})}$

$\rightarrow \boxed{F(\hat{v}) = \|I^h\hat{v} - \hat{v}\|_{L^2(\hat{T})}}$

$\boxed{F(\hat{v}) = \|\nabla(I^h\hat{v} - \hat{v})\|_{L^2(\hat{T})}}$

Apply the Bramble-Hilbert lemma on $\hat{T}$:

$$\|I^h\hat{v} - \hat{v}\|_{L^2(\hat{T})} \leq c\,\|\hat{\nabla}^2\hat{v}\|_{L^2(\hat{T})}$$



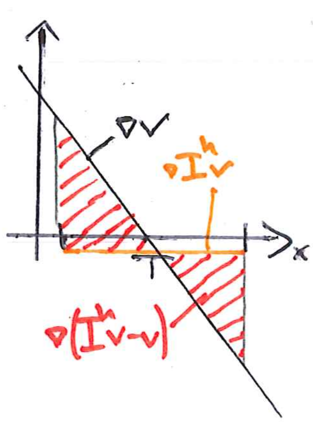
$$\frac{1}{h}\|I^h v - v\|_{L^2(T)} \leq c\,h\,\|\nabla^2 v\|_{L^2(T)}$$

$$\boxed{\|I^h v - v\|_{L^2(T)} \leq c\,h^2\|\nabla^2 v\|_{L^2(T)}}$$

L^2 interpolation error $\leq O(h^2)$ if $v \in H^2(T)$

$$\boxed{\|\nabla(I^h v - v)\|_{L^2(T)} \leq c\,h\,\|\nabla^2 v\|_{L^2(T)}}$$

H^1 interpolation error $\leq O(h)$ if $v \in H^2(T)$



i.e. "it is more difficult to approximate the gradients of v than the function values of v"

The above is only true for the particular transformation

$$F_T(\hat{x}) = A_T\hat{x} + b_T$$

with $A_T = \begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}$. General transformations, that also have a shear/rotation/reflection component, need some technicalities $\longrightarrow$ Thm 2.3.14