FAST SOLUTIONS TO THE BLOCH-TORREY PARTIAL DIFFERENTIAL EQUATION

Simulation of Cerebral Magnetization

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THE BLOCH-TORREY EQUATION

- The Bloch-Torrey equation is a fundamental equation in MRI physics which describes how the *complex* transverse magnetization changes through time

$$\frac{dM}{dt} = D \cdot \nabla^2 M - (R_2 + i\delta\omega) \cdot M$$

- Where

$$M := M_x + iM_y$$

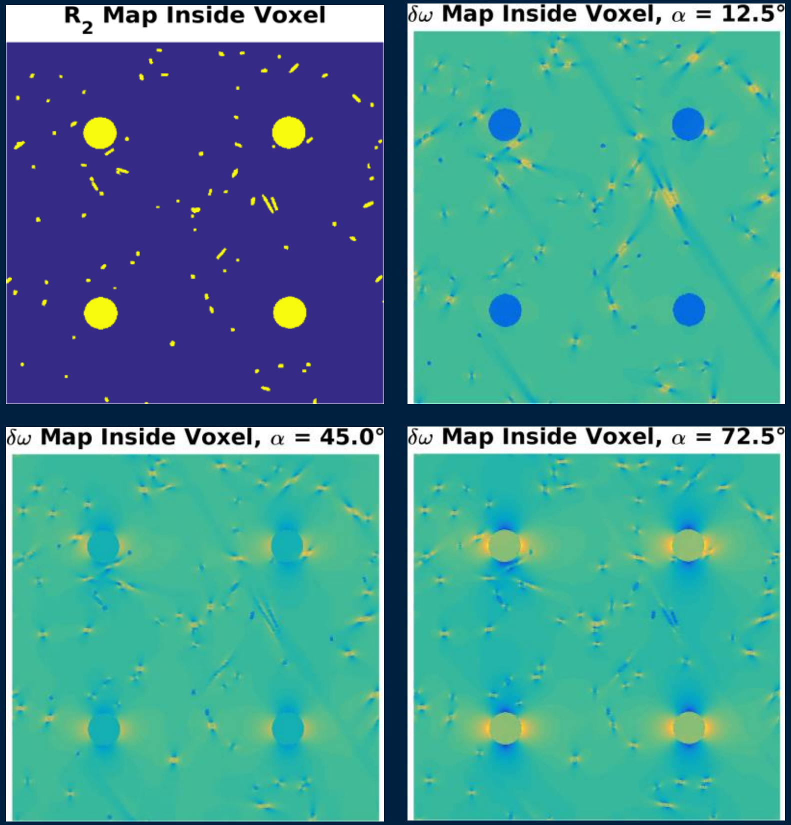
- Therefore, this problem must be solved in complex variables

GEOMETRY

$$\frac{dM}{dt} = D \cdot \nabla^2 M - (R_2 + i\delta\omega) \cdot M$$

- Equation is solved in a finely discretized 3D box, called a “voxel”

- R_2 is piecewise constant, and $\delta\omega$ is piecewise smooth
    - Problem is very discontinuous!



FAST SOLUTIONS

$$\frac{dM}{dt} = D \cdot \nabla^2 M - (R_2 + i\delta\omega) \cdot M$$

- In addition to solving the equation, the resulting signal (integral of *M*) must be fit to observed data

- Involves thousands of system solves

FAST SOLUTIONS

$$\frac{dM}{dt} = D \cdot \nabla^2 M - (R_2 + i\delta\omega) \cdot M$$

- Currently, the best trade off of accuracy vs. speed is to use (relatively crude) "splitting methods"

- Essentially, based on the approximation

$$e^{(A+B)t} \approx e^{\frac{At}{2}} e^{Bt} e^{\frac{At}{2}}$$

- For linear operators *A* and *B*
  - We take $A = -(R_2 + i\delta\omega)$ and $B = D\nabla^2$, and use the fact that the action of the exponential of $D\nabla^2 t$ (i.e. the solution of the heat equation) is the convolution with a Gaussian kernel, and can be performed efficiently via the FFT