MATH521
Numerical Analysis of Partial Differential Equations

Winter 2017/18, Term 2
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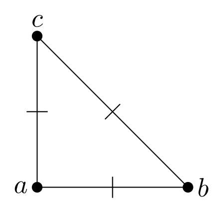
Homework Assignment 6

Please submit the following files as indicated below: source code PDF file image file video file

Question 1 | **2 marks** | Given three points $a, b, c \in \mathbb{R}^2$ that are not collinear (not all on one line) and that are sorted in anticlockwise order, we define

$$T = \Delta(a, b, c) \qquad \text{(the triangle with these vertices)}$$
$$P = P_2(T)$$
$$L = \left\{ p \mapsto p(a), \quad p \mapsto p(b), \quad p \mapsto p(c), \quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{a+b}{2}\right), \quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{b+c}{2}\right), \quad p \mapsto \frac{\partial p}{\partial n}\left(\frac{c+a}{2}\right) \right\} \subset P^*$$



(a) Show that prescribed data for

$$p(a), \quad p(b), \quad p(c), \quad \frac{\partial p}{\partial n}\left(\frac{a+b}{2}\right), \quad \frac{\partial p}{\partial n}\left(\frac{b+c}{2}\right) \quad \text{and} \quad \frac{\partial p}{\partial n}\left(\frac{c+a}{2}\right)$$

uniquely determines any $p \in P$. You don't have to show that such p always exists.

(b) Now let Ω^h be a domain with a regular triangulation $\mathcal{T}^h$ such that

$$\bar{\Omega}^h = \bigcup_{T \in \mathcal{T}^h} T.$$

Is the space

$$V^h = \left\{ v^h : \bar{\Omega}^h \to \mathbb{R} \;\middle|\; v^h\big|_T \in P_2(T), v^h \text{ is continuous in all vertices}, \frac{\partial v^h}{\partial n} \text{ is continuous in all edge midpoints} \right\}$$

H^1-conforming, i.e. is $V^h \subset H^1(\Omega^h)$?

Hint: Check if there may be any jumps of v^h across triangle edges.

Question 2 | **3 marks** | We will now complete our finite-element solver for the linear elasticity problem

$$\begin{aligned} -c\Delta u + au &= f \quad \text{in } \Omega \\ u &= g \quad \text{on } \partial\Omega. \end{aligned} \tag{1}$$

(a) Remove lines 1-10 from `discretiseLinearElasticity.m` and uncomment the sections of code that are currently commented out. Complete the missing commands, including the subfunction `assembleStiffness`. Also inspect the `assembleLoad` subfunction.

(b) Write a script `hw6.m` which

- solves the linear elasticity problem on Ω^h, which you may choose from `kiwi.mat`, `maple.mat`, `pi.mat`, `ubc.mat`. You may also select your own data for $f(x_1, x_2)$, $g(x_1, x_2)$, a and c.

  Hint: You have to set `GammaD = @(x1,x2) true(size(x1))`. For debugging, you might want to use `video10.mat` and check the sparsity patterns of the various matrices.

- calculates the L^2, H^1 and energy norms

$$\|u^h\|_{L^2} = \sqrt{\int_{\Omega^h} |u^h|^2 \, \mathrm{d}x}$$

$$\|u^h\|_{H^1} = \sqrt{\|u^h\|_{L^2}^2 + \|\nabla u^h\|_{L^2}^2} = \sqrt{\int_{\Omega^h} |u^h|^2 \, \mathrm{d}x + \int_{\Omega^h} |\nabla u^h|^2 \, \mathrm{d}x}$$

$$\|u^h\|_B = \sqrt{B(u^h, u^h)} = \sqrt{c \int_{\Omega^h} |\nabla u^h|^2 \, \mathrm{d}x + a \int_{\Omega^h} |u^h|^2 \, \mathrm{d}x}$$

  of the solution, where B is the bilinear form corresponding to the elliptic operator

- creates undistorted plots of the mesh, the force f and the solution u^h

(c) What problem do you solve numerically when you set `GammaD = @(x1,x2) false(size(x1))`? Analyse the code to infer its weak formulation:

Your Learning Progress | What is the one most important thing that you have learnt from this assignment?

Any new discoveries or achievements towards the objectives of your course project?

What is the most substantial new insight that you have gained from this course this week? Any *aha moment*?