Implicit Midpoint Rule vs Backward Euler for a Shock Wave

The major numerical difficulty of solving hyperbolic equations, or 'almost hyperbolic' (e.g. advection-dominated) problems is that they admit discontinuous or almost discontinuous solutions, respectively. Since our meshes always have a finite spacing, discontinuities are extremely difficult to capture.

We are generally confronted with two issues that play a dominant role for solutions containing shocks or steep gradients:

- numerical dissipation: artificial smoothing, loss of energy
- numerical dispersion: artificial oscillations, loss of monotonicity

Here are two extreme cases with numerical solutions of the wave equation using a discontinuous initial condition:

1. **Implicit Midpoint Rule (Crank-Nicolson) (https://canvas.ubc.ca/courses/2337/files/1150119/download?wrap=1)**
    - no numerical dissipation
    - strong numerical dispersion
2. **Backward Euler Method (https://canvas.ubc.ca/courses/2337/files/1150118/download?wrap=1)**
    - strong numerical dissipation
    - no (particularly obvious) numerical dispersion