Linear, elliptic BVP w/o order-zero term

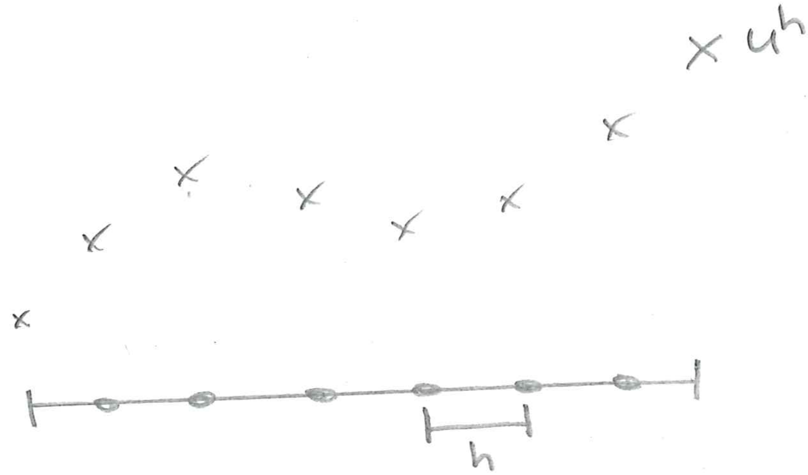
$$\begin{aligned} Lu &= f \quad \text{in } \Omega \\ u &= g \quad \text{on } \partial\Omega \end{aligned}$$

$$\Big\downarrow \text{ FDM, FEM}$$

Linear system

$$L^h u^h = f^h$$

↑ vector corresponding to f, g in the PDE



$$\underbrace{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}}_{L^h} \underbrace{\begin{pmatrix} \vdots \end{pmatrix}}_{u^h} = \underbrace{\begin{pmatrix} \vdots \end{pmatrix}}_{f^h}$$

If $f, g \geq 0$, then according to the maximum principle we also have $u \geq 0$. Therefore, we also want that all entries of the discrete solution vector u^h are ≥ 0 provided that all entries in the right hand side vector f^h are ≥ 0.
Under what assumption on the discrete matrix L^h can we guarantee that?

$$\underbrace{\begin{pmatrix} \vdots \end{pmatrix}}_{u^h} = \underbrace{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}}_{(L^h)^{-1}} \underbrace{\begin{pmatrix} \geq 0 \\ \geq 0 \\ \geq 0 \\ \geq 0 \\ \geq 0 \end{pmatrix}}_{f^h}$$

→ we need that all entries of $(L^h)^{-1}$ are ≥ 0

→ difficult / impossible to check in practice, but we'll soon derive conditions on L^h itself that will give us this