

Numerical Analysis of Partial Differential Equations

Winter 2017/18, Term 2 Timm Treskatis

## Finite Difference Discretisation of the Poisson-Dirichlet Problem on a Rectangular Domain

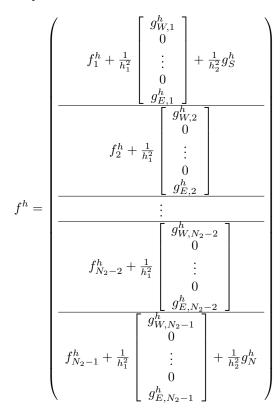
$$\begin{split} -\Delta u &= f &\quad \text{in } \Omega &\qquad \qquad u &= g &\quad \text{on } \partial \Omega \\ \Leftrightarrow \left( -\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} \right) u &= f &\quad \text{in } \Omega &\qquad \qquad u &= g &\quad \text{on } \partial \Omega \\ \stackrel{\text{FDM}}{\leadsto} \left( L_1^h + L_2^h \right) u^h &= f^h &\qquad \qquad \end{split}$$

## Discrete Laplacian

$L_1^h = \frac{1}{h_1^2}$	$ \begin{pmatrix} 2 & -1 \\ -1 & 2 & -1 \\ & & \ddots \\ & & -1 \end{pmatrix} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	21 :	$2 -1$ $\cdot$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	-1 ··. -1 :	$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$
$L_2^h = \frac{1}{h_2^2}$	2 -1 -2 -1	2		−1 ··.	-1	2 -1	··. 2 ··1	-1 ··.	

Note that these matrices are composed of  $(N_2-1)\times(N_2-1)$  blocks, and each block is  $(N_1-1)\times(N_1-1)$ .

## Discrete Source Term with Boundary Data



where  $f_j^h$  is the vector of length  $N_1 - 1$  with the source term f evaluated on the j-th row of interior grid points,  $g_N^h, g_S^h$  are vectors of length  $N_1 - 1$  with the boundary values on the top or bottom and  $g_W^h, g_E^h$  are vectors of length  $N_2 - 1$  with the boundary values on the left or right.