Implementation of Algebraic Flux Correction Schemes

Today we have seen how we can recognise

- global mass conservation,
- zeroth-order consistency (i.e. consistency for constant solutions)
- positivity preservation / the local extremum diminishing property

as simple properties of the matrices in our discrete system. We start with a high-order discretisation scheme (i.e. at least second order accurate) which satisfies the first two, but not the last property. Then we add discrete diffusion to this system to obtain a low-order scheme that also satisfies the last property, while still not violating the first two ones. Finally, we add as much antidiffusion to the low-order scheme, in a way that all three properties still hold. To this end, we

1. decompose the antidiffusion into nodal fluxes (to maintain mass conservation even if the fluxes are modified),
2. try to always add 100% of each nodal flux for higher-order accuracy, but if this flux violated the maximum principle
   - by creating a new local max/min at this node
   - by making existing local max/min grow

   then we'd only add a lesser percentage of this flux, namely only such an amount for which these violations no longer occur. This procedure is called "flux limiting".

Note that since these flux limiting factors, the percentages, depend on the solution (since they test whether or not the solution would satisfy the maximum principle), we obtain a nonlinear scheme. This allows us to overcome the Godunov barrier and achieve better than first-order accuracy despite the presence of advection.

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For more details on finite element schemes for conservation laws and algebraic flux correction, I recommend Dmitri Kuzmin's free **CFD textbook** **(http://www.mathematik.uni-dortmund.de/~kuzmin/cfdbook.html)**.