MATH521
Numerical Analysis of Partial Differential Equations

Winter 2017/18, Term 2
Lecturer: Timm Treskatis

Course Outline and Policies

What surface does a soap film span inside a bent wire frame? What does the electromagnetic field inside a microwave look like? How does air flow around a car? How does the shockwave of a supersonic aircraft propagate in space? How does chemotaxis work? How do rain clouds develop over the Pacific?

Partial differential equations (PDEs) model a vast range of problems from physics, chemistry, biology, engineering, meteorology, statistics, mathematical finance and many more disciplines. Virtually all real-life problems are too complex to be solved analytically and require numerical techniques such as *finite differences*, *finite elements* or *finite volumes*. Given their mathematical superiority, we will devote most of our attention to the analysis and implementation of finite-element methods, including some of the latest advances in the field.

This course is designed to foster development of analytical, computational and professional skills. Not only will you learn how to solve PDEs numerically and how to assess the quality of your results, but you will also apply these skills in your own mini research project, you will gain effective communication skills to present your results in oral and written form and you will find out how to peer-review someone else's work. If you have an interest or even consider a future career in computational, applied or industrial mathematics, then this course is for you!

One of last year's homework problems: a numerical solution of the Allen-Cahn equation, which models phase-separation processes. The results can be interpreted as a salad dressing that gradually separates into blobs of oil and vinegar.

Lectures Tuesdays & Thursdays from 4 January to 5 April (except 20 and 22 February), 9:30 - 11:00 am in Room 203, Mathematics Building

Lecturer Timm Treskatis is a Postdoctoral Research and Teaching Fellow in the Mathematics Department. He obtained his PhD from the University of Canterbury in Christchurch, New Zealand. Timm has worked in both academic and industrial environments and is looking forward to sharing his nine-year experience in PDEs, optimisation and applications with the participants of this course.

Course Website `https://blogs.ubc.ca/timm/teaching/math521/`

Course Content

1. Classification of PDEs
   a) Basic Properties
   b) Second-Order PDEs
   c) Conservation Equations
2. Second-Order Elliptic Equations
   a) Characteristic Features
   b) Finite Differences for Poisson's Equation
   c) Finite Elements for Poisson's Equation

3. Second-Order Parabolic Equations
   a) Characteristic Features
   b) Finite Elements for the Heat Equation
4. Second-Order Hyperbolic Equations
   a) Characteristic Features
   b) Finite Elements for the Wave Equation
5. Conservation Equations
   a) Characteristic Features
   b) Finite Elements for the Advection-Diffusion Equation

Learning Outcomes

At the successful completion of this course, you will be able to

(LO1) derive characteristic features of solutions to a given PDE problem

(LO2) select and implement a suitable numerical method that preserves these features for first-order or second-order PDE problems in one or two spatial dimensions

(LO3) describe the fundamental notions of *consistency*, *stability* and *convergence* of a numerical scheme

(LO4) calculate *a priori* and *a posteriori* error estimates for some elliptic model problems

(LO5) identify how the notions and techniques from the numerical analysis of PDEs are applied in fields of interest to the class

(LO6) follow a goal-oriented approach to written and oral forms of communication in academia, to convey scientific findings in an effective, interesting and captivating manner

It is my hope that you will develop a keen interest in the topics of this course, grasp their relevance to your prospective career and experience a strong sense of accomplishment: you will have written your own code for use in future projects and you will have created many colourful graphs and animations!

Prerequisites or Preparation for this Course

Prior to the start of this course, you should already be able to

1. apply the rules of multivariable calculus, such as the divergence theorem, Green's identity or the Cauchy-Schwarz inequality
2. implement, e.g. in GNU Octave or MATLAB
   - Runge-Kutta methods for integrating ordinary differential equations
   - Newton's method for solving nonlinear systems of equations
3. undastend the Nuw Zullund eccint ☺

If you have worked with Lebesgue spaces, Sobolev spaces and weak derivatives before, that will make the first few weeks of this course easier for you.

Additionally, if you consider doing research in the fields of scientific computing or the numerical analysis of PDEs, then I strongly recommend you also take courses on functional analysis and the theory of PDEs!

Logistics

Format I have designed this course to follow *study–action–feedback* or *action–study–feedback* patterns with some elements of *blended learning* and the *flipped-classroom approach*:

- **Study** You will learn about new concepts and techniques through short video clips that will be available on the course website and also more traditional lectures.
- **Action** In class, you will be working in teams on various active-learning challenges to discover, practice or apply new knowledge. The homework assignments and your mini research project offer further, individual learning opportunities.
- **Feedback** Your work for this course will be evaluated to help you identify your strengths and areas for further improvement. Some of this feedback will be purely formative for your own benefit only, some will be used to calculate your grade for this course. (For all the details, please refer to the next section.)

Participation Aristotle was already convinced that *what we have to learn to do, we learn by doing.* To put you in the position to participate actively in class and benefit from the classroom learning activities, it is vital that you arrive prepared. This means that you should have watched the video clip(s) for that day, reviewed any material from previous classes and that you keep on track with your homework assignments.

Students who take this course typically come from a diverse range of mathematical and applied disciplines. We can all benefit and learn about different approaches to problem-solving, find out about alternative perspectives and various applications we may not have thought of before. I look forward to learning from you, too.

Punctuality Classes generally begin and end on time.

Canvas This course has a page on Canvas (`https://canvas.ubc.ca`). Here you will find all assignments, an overview of your grades, any notices, a discussion forum and more.

You are strongly encouraged to use the platform to start or engage in discussions on the course content, your project or any other pertinent questions.

Class Reps My objective is to make this course as useful to you as somehow possible. To that end, it is important for me to know whether the course is too easy, too difficult, too theoretical, too applied, too time-consuming, etc. I cannot act upon things of which I am unaware.

During the first lecture, we will appoint one or two class reps. Their role is to act as a liaison between the lecturer and the class. Please talk to them immediately about any concerns, and also things that are going well with the course. They will then confidentially pass your feedback on to me.

Assessment, Evaluation and Grading

Attainment of the learning objectives is assessed by means of a small individual research project (50% of your final grade), learning objectives (LO1)-(LO4) are additionally evaluated in weekly homework assignments (also worth 50%).

Homework Assignments (50%)	
Assessment Type	Twelve analytical and/or computational problem sets. Assignments are due on Thursdays from Week 2 to Week 13 and should be submitted by file upload on Canvas. Problems will be marked with the following symbols, indicating what files are required: PDF file (a scanned copy of handwritten solutions is fine, typesetting is not needed) executable source code (e.g. M files, if you are using GNU Octave or MATLAB) image file video file
Evaluation Criteria	Timely submission, correctness and completeness of solutions. Assignments submitted up to 24 hours late will be marked out of 50%, assignments received more than 24 hours late will not be marked.

Grading	Up to five marks for each of the twelve assignments, with a maximum of 50 marks overall. Only your best ten of these twelve grades will be counted, so that you could skip two complete assignments and still reach the maximum score. This gives you some flexibility when things get busy.

Research Project: Proposal (5%)	
Assessment Type	During the first few weeks, you will choose a topic related to this course for your own mini research project. Most students pick a PDE problem which is part of their work for a Master's or PhD project, but you should also feel free to choose anything else of interest to you. This could be a specific problem from an area of application, a concept of theoretical nature, a computational question or a study of new developments in the fields. You will submit a one-page proposal composed of: **1**. a succinct working title **2**. an outline of the objectives and the significance of the project (100-250 words) **3**. a short review of any pertinent literature (at most 150 words) **4**. at least three different references in this review, correctly cited **5**. a bibliography with a consistent citation style I strongly recommended to let me know about your ideas prior to submitting a proposal. Proposals are due in PDF format by Thursday 25 January 2018.
Evaluation Criteria	Timely submission and complete coverage of the above five items
Grading	Up to five marks

Research Project: Primer Talk with Peer Review (5%)	
Assessment Type	In Week 6, you will deliver a 2-3 minute primer talk on the subject you are investigating for your research project. You will also participate in formative peer review of all other talks, by answering a few guiding questions on a feedback form. I will videotape your talk and make the recording available to you (and only you), so that you can evaluate yourself as well.
Evaluation Criteria	Participation
Grading	Five marks

Research Project: Oral Presentation (15%)	
Assessment Type	In Week 13, we will host a conference workshop for interested members of the department and the wider community. You will give an oral presentation on your project and answer questions from the audience. Depending on enrolment numbers, this will be around five to ten minutes for your talk plus one to three minutes for Q&A. You will also assume the role of chairperson for another student's presentation, giving a short introduction, inviting questions from the audience and ensuring that the session adheres to time. You will receive a video of your oral presentation as well.

Evaluation Criteria	**Content Selection and Organisation** (1) Does the talk begin with a succinct opening to introduce the topic? (2) Does the presentation cover all the main highlights of the project, while unnecessary details are left out? (3) Does the conclusion re-emphasise a significant take-home message? **Attainment of Purpose** (4) Is the presentation tailored to the audience's knowledge levels? (5) Does the talk employ illustrative analogies from everyday life or relate to existing knowledge? **Supporting Material and Q&A** (6) Does the speaker employ appropriately selected* visual aids (slide show, screen cast, blackboard / whiteboard / flip chart, props) to illustrate ideas, data or results? (7) Do they answer a question from the lecturer concisely and competently? *Please do not (mis-)use slides for bullet points, text, complicated formulas or as a teleprompter for your talk. They are best used only whenever you think spoken words alone wouldn't suffice, e.g. for figures, images, videos or animations. If you wish to present calculations, a large system of equations or a proof, then doing that live on the board is often a much more effective choice.
Grading	Up to two marks for each of the seven criteria plus one participation mark for chairing.

Research Project: Paper or Poster (20%)	
Assessment Type	You may choose between a succinct and concise paper (with a strict upper limit of six pages) or a poster to communicate the results of your project in written form. This written work should be targeted towards an audience that is knowledgeable in the numerical analysis of PDEs, such as your classmates who have also attended this course. PDF files of papers or posters along with any source code files are due by Thursday 12 April 2018.
Evaluation Criteria	**Structure** (1) Are key points like a 'big idea', the novelty or significance of this work, main results or own contributions featured in a prominent position? (2) Is the content organised in a logical fashion? (3) Are the main conclusions succinctly summarised at the end? **Mathematics** (4) Does the work address questions of well-posedness, characteristic problem features and convergence? (5) Are all conclusions supported with sound evidence, e.g. citations, proofs, data or numerical studies? (6) Are mathematical arguments presented accurately? (7) Is the technical terminology from the numerical analysis of PDEs used and is it used correctly? **Understandability** (8) Is the content independently meaningful for the target audience without need for further explanation? (9) Is all notation defined when first used and are figures fully labelled? (10) Are there no major grammatical or typographical errors?
Grading	Up to two marks for each of the ten criteria

Research Project: Peer Review of a Paper or Poster (5%)

Assessment Type	You will write an anonymous review on another student's paper or poster, due by Thursday 19 April 2018. Canvas will assign someone's work to you for review. Open their submission and use the provided text box for general comments to write a short report. As scientific journals usually require you to submit a review in plain text, I suggest you also use this text box on Canvas instead of the document annotation function. **1.** Give a brief synopsis of the objectives, the methodology and the significance of the work in your own words to show that you have read the work properly. **2.** What is your overall impression? **3.** Comment on anything that you think deserves special praise or that requires attention. Is there anything you disagree with? Anything you did not understand? You may refer to the rubric for the paper or poster for guiding questions so that you know what you could look out for. **4.** Remember to be specific (refer to line numbers, equation numbers, paragraphs or cite what you are commenting on) and constructive (state why you think some aspect is particularly effective or make a suggestion on how it could be improved). **5.** Final comment: What did you find most interesting? Or do you have any additional idea for future research or possible applications?
Evaluation Criteria	Timely submission and complete coverage of the above five points
Grading	Up to five marks

Literature

You do not need to purchase a textbook. There will be lecture notes tailored to this course.

Here is a small selection of the relevant literature that you may want to have a look at:

Numerical Analysis of PDEs

[1] Stig Larsson and Vidar Thomée: *Partial Differential Equations with Numerical Methods*. Springer, 2009.

[2] Susanne Brenner and Scott Ridgway: *The Mathematical Theory of Finite Element Methods*. Springer, 2007.

[3] Kenneth Eriksson et al: *Computational Differential Equations*. Cambridge University Press, 1996.

[4] Philippe G Ciarlet et al: *Series: Handbook of Numerical Analysis*. North-Holland, 1981–2017.

Theory of PDEs

[5] Lawrence C Evans: *Partial Differential Equations*. American Mathematical Society, 2010.

Scientific Communication

[6] Michael Alley: *The Craft of Scientific Presentations*. Springer, 2013.

[7] Ed Neal and Doug Dollar (Eds): *Academic Writing: Individual and Collaborative Strategies for Success*. New Forums, 2013.

Academic Integrity

The academic enterprise is founded on honesty, civility, and integrity. As members of this enterprise, all students are expected to know, understand, and follow the codes of conduct regarding academic integrity. At the most basic level, this means submitting only original work done by you and acknowledging all sources of information or ideas and attributing them to others as required. This also means you should not cheat, copy, or mislead others about what is your work. Violations of academic integrity (i.e., misconduct) lead to the breakdown of the academic enterprise, and therefore serious consequences arise and harsh sanctions are imposed. For example, incidences of plagiarism or cheating may result in a mark of zero on the assignment or exam and more serious consequences may apply if the matter is referred to the President's Advisory Committee on Student Discipline. Careful records are kept in order to monitor and prevent recurrences.

A more detailed description of academic integrity, including the University's policies and procedures, may be found in the Academic Calendar at `http://calendar.ubc.ca/vancouver/index.cfm?tree=3,54,111,0`.