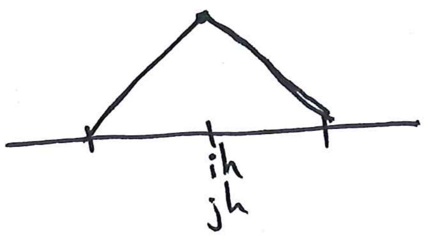
$$\|u^h\|_{L^2} = \sqrt{\int_\Omega (u^h)^2 dx} = \; ? \; \ldots \quad \vec{u}^h$$

$$\int_\Omega (u^h)^2 dx = \int_0^1 \Big(\sum_{i=1}^{N-1} u_i^h \phi_i^h\Big)\Big(\sum_{j=1}^{N-1} u_j^h \phi_j^h\Big) dx$$

$$= \sum_{i,j} u_i^h u_j^h \underbrace{\int_0^1 \phi_i^h \phi_j^h dx}_{= m_{ij}^h \text{ (mass matrix)}}$$



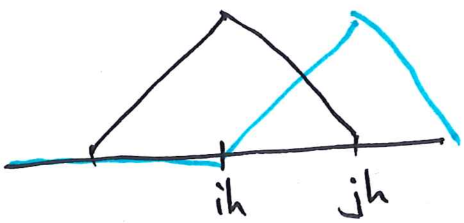
<u>if $j=i$</u>

$$\int_0^1 (\phi_i^h)^2 dx = \int_{(i-1)h}^{ih} (\phi_i^h)^2 dx + \int_{ih}^{(i+1)h} (\phi_i^h)^2 dx$$

exact for quadratics $\rightarrow$ Simpson's rule

$$\stackrel{\text{Simpson's rule}}{=} \frac{h}{6}\Big(1\cdot 0^2 + 4\cdot\big(\tfrac{1}{2}\big)^2 + 1\cdot 1^2\Big) + \frac{h}{6}\Big(1\cdot 1^2 + 4\cdot\big(\tfrac{1}{2}\big)^2 + 1\cdot 0^2\Big)$$

$$= \frac{2h}{3}$$



<u>if $j=i\pm 1$</u>

$$\int_0^1 \phi_i^h \phi_{i+1}^h dx = \int_{ih}^{(i+1)h} \phi_i^h \phi_{i+1}^h dx$$

$$= \frac{h}{6}\Big(1\cdot 1\cdot 0 + 4\cdot\tfrac{1}{2}\cdot\tfrac{1}{2} + 1\cdot 0\cdot 1\Big)$$

$$= \frac{h}{6}$$

$$= \int_0^1 \phi_i^h \phi_{i-1}^h dx$$

$$M^h = \frac{h}{6}\begin{pmatrix} 4 & 1 & & & \\ 1 & 4 & 1 & & \\ & 1 & 4 & 1 & \\ & & & \ddots & \\ & & & 1 & 4 \end{pmatrix}$$

<u>otherwise</u>

$$\int_0^1 \phi_i^h \phi_j^h dx = 0 \quad \text{(no overlap)}$$

$$\|u^h\|_{L^2} = \sqrt{(\vec{u}^h)^T M^h (\vec{u}^h)}$$