REFERENCES

Each reference is followed by a note highlighting a contribution of that publication that is relevant to this book. These notes are by no means comprehensive: in most cases the references include other significant contributions too. Papers listed by authors such as Cauchy, Chebyshev, Gauss, Jacobi, and Weierstrass can also be found in their collected works.

Among mathematicians of the 19th century, it is hard not to be struck by the remarkable creativity of Jacobi (1804–1851), who in his short life made key early contributions to barycentric interpolation [1825], orthogonal polynomials and Gauss quadrature [1826], and Padé approximation and rational interpolation [1846], as well as innumerable topics outside the scope of this book.

As of May 2012, the Mathematics Genealogy Project lists 8605 adademic descendents of Pafnuty Lvovich Chebyshev. For example, one chain runs Chebyshev–Lyapunov–Steklov–Smirnov–Sobolev–V. I. Lebedev, and another runs Chebyshev–Markov–Voronoy–Sierpinsky–Mazurkiewicz–Zygmund–Stein–C. Fefferman.

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H. A. Priestley, *Introduction to Complex Analysis,* 2nd ed., Oxford U. Press, 2003. [Well known introductory complex analysis textbook first published in 1985.]

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P. Rabinowitz, Rough and ready error estimates in Gaussian integration of analytic functions, *Comm. ACM* 12 (1969), 268–270. [Derives tight bounds on accuracy of Gaussian quadrature by simple arguments.]

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L. F. Richardson, The deferred approach to the limit. I—single lattice. *Phil. Trans. Roy. Soc. A* (1927), 299–349. [Systematic discussion of Richardson extrapolation, emphasizing discretizations with $O(h^2)$ error behavior.]

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G. A. Watson, *Approximation Theory and Numerical Methods,* Wiley, 1980. [Textbook with special attention to L_1 and L_p norms.]

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K. WEIERSTRASS, Über continuierliche Functionen eines reellen Arguments, die für keinen Werth des letzteren einen bestimmten Differentialquotienten besitzen, *Königliche Akademie der Wissenschaften,* 1872. [Weierstrass's publication of an example (which he had lectured on a decade earlier) of a continuous, nowhere-differentiable function.]

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H. WERNER, On the rational Tschebyscheff operator, *Math. Z.* 86 (1964), 317–326. [Shows that the operator mapping a real function $f \in C[-1,1]$ to its best real rational approximation of type (m,n) is continuous if and only if f is itself rational of type (m,n) or its best approximation has defect 0 ("nondegenerate").]

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P. WYNN, On a device for computing the $e_m(S_n)$ transformation, *Math. Comp.* 10 (1956), 91–96. [Wynn's first of many papers about the epsilon algorithm for acceleration of convergence of sequences.]

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