Definition of a metric

If X is a set, then a **metric** on X is a function $d : X \times X \to \mathbb{R}$ satisfying:

- $d(x, y) = 0 iff x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq d(x, z) + d(z, y)$

A pair (X,d) is called a **metric space**.

Give three examples of a metric space

- $\mathbb{R}^n$ with "usual (Euclidean) metric": $d_e(a, b) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)}$
- $\mathbb{R}^2$ with "Manhattan metric": $d_m(x, y) = |x_1 - y_1| + |x_2 - y_2|$
- $\mathbb{N}$ with the "discrete metric": $d_d(n, k) = \begin{cases} 0, & \text{if } n = k \\ 1, & \text{if } n \neq k \end{cases}$

Topology:

- $\emptyset$, X $\in \tau(d)$
- If U,V $\in \tau(d)$ then U $\cap$ V $\in \tau(d)$
- For any family $\{U_i | i \in I\} \subseteq \tau(d)$, $\bigcup_{i \in I} U_i \in \tau(d)$

T_0

$$\forall x \neq y, \exists U \in \tau \text{ s.t. } |U \cap \{x, y\}| = 1$$

T_1

$$\forall x \neq y, \exists U \in \tau | x \in U \text{ and } y \notin U$$

T_2

$$\forall x \neq y, \exists U, V \subseteq_{open} X | U \cap V = \emptyset, x \in U, y \in V$$

$Regular$

$$\forall F \subseteq_{closed} X \; and \; \forall x \notin F, \exists U, V \subseteq_{open} X | U \cap V = \emptyset, x \in U, F \in V$$

$Normal$

$$\forall A, B \subseteq_{closed} X\ w/\ A \cap B = \emptyset, \exists U, V \subseteq_{open} X | U \cap V = \emptyset, A \in U, B \in V$$

$Neighborhood$
If X is a space and $x \in X$ then A⊆X is called a **neighborhood** of x if

$$\exists U_{open} | x \in U \subseteq A$$

A set U⊆X is open iff (neighborhoods) $\forall x \in U$ there is a nhood V of x | V⊆U
$closure$
$\overline{\mathrm{A}}$ = $\{x \in X|$ every open $U \ni x$ meets A$\}$