Definition of a metric

If X is a set, then a **metric** on X is a function $d : X \times X \to \mathbb{R}$ satisfying:

- $d(x, y) = 0 iff x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq d(x, z) + d(z, y)$

A pair (X,d) is called a **metric space**.
Give three examples of a metric space

- $\mathbb{R}^n$ with "usual (Euclidean) metric": $d_e(a, b) = \sqrt{\sum_{i=1}^{n}(a_i - b_i)}$
- $\mathbb{R}^2$ with "Manhattan metric": $d_m(x, y) = |x_1 - y_1| + |x_2 - y_2|$
- $\mathbb{N}$ with the "discrete metric": $d_d(n, k) = \begin{cases} 0, & \text{if } n = k \\ 1, & \text{if } n \neq k \end{cases}$