

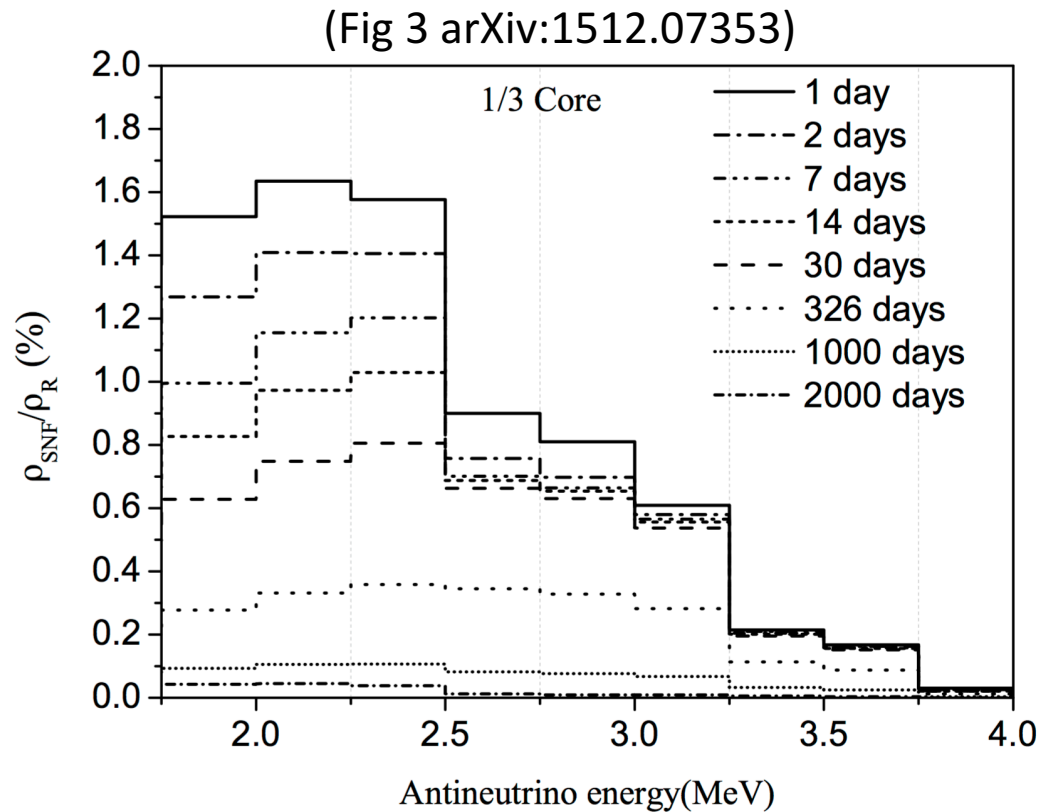
Studying Spent Nuclear Fuel Contributions to P15A

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Calculated SNF fractional spectrum by X. B. Ma *et al.*



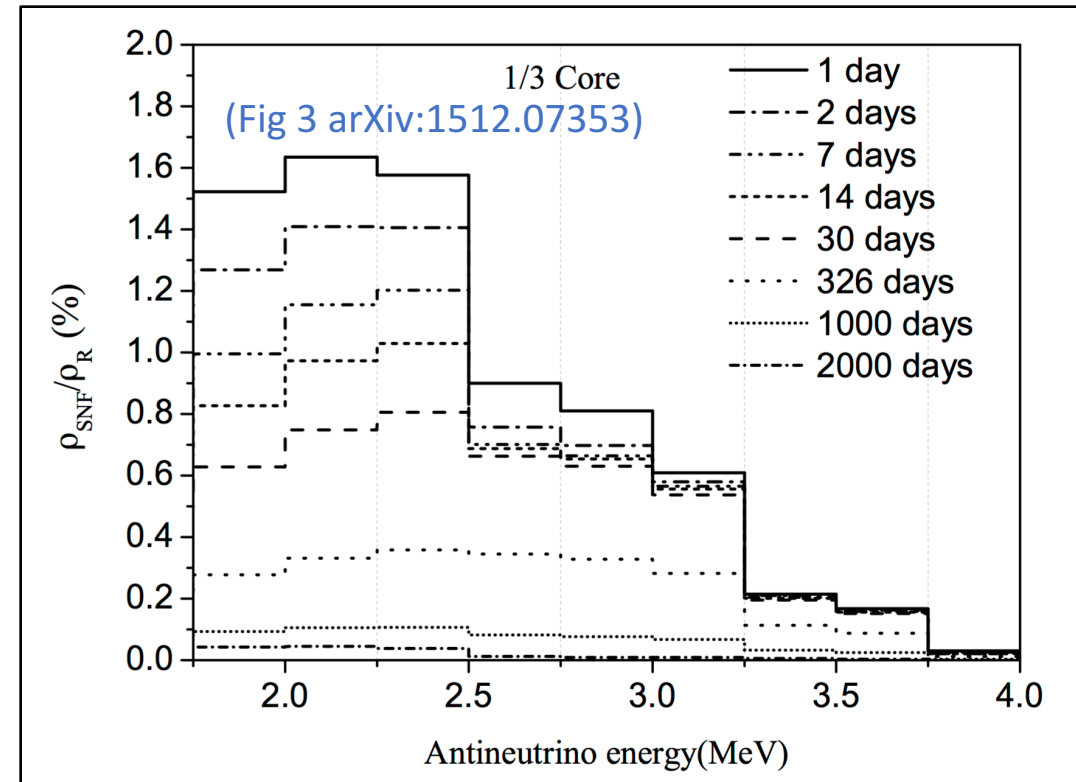
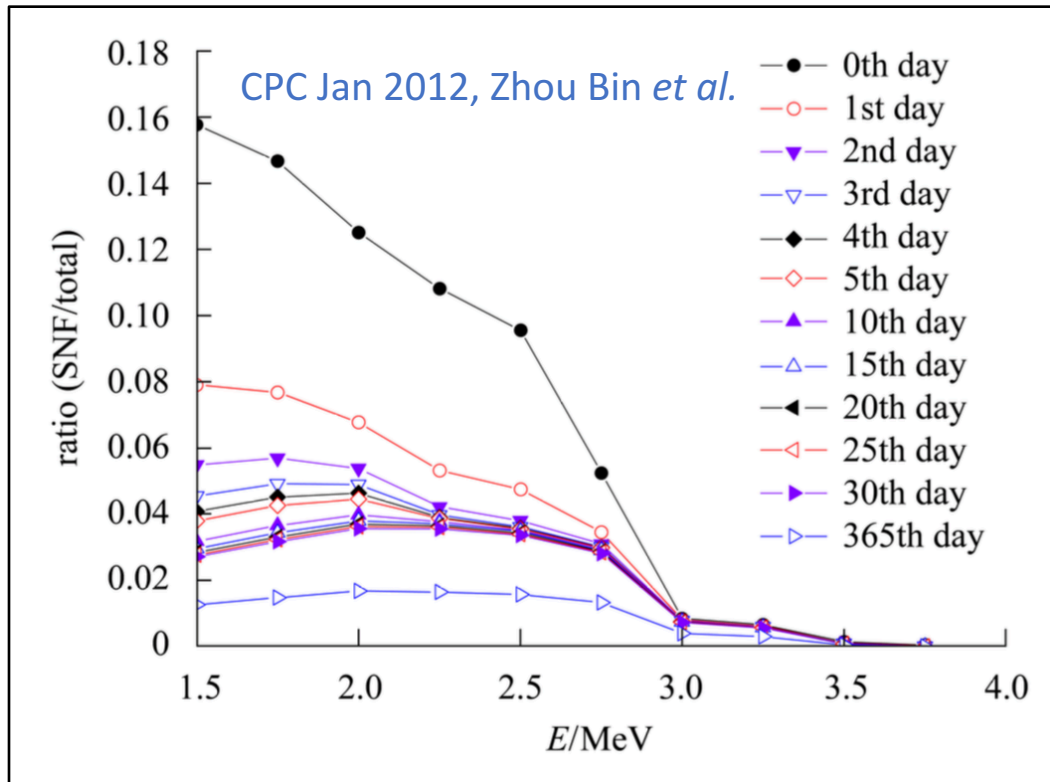
Assumptions of spectra

- Refueling: 1/3 core per year
- Burnup: 45 GWd/tU
- Close to Daya Bay conditions
“Neutron flux vary with fuel burnup, and the geometry obtained from the nuclear power plant is used to make results more accurately.”
- Isotopes limited to $t_{1/2} > 10\text{hr}$ and energy larger than IBD threshold 1.8 MeV

Note: For my analysis I picked the values for this spectrum from the plot.

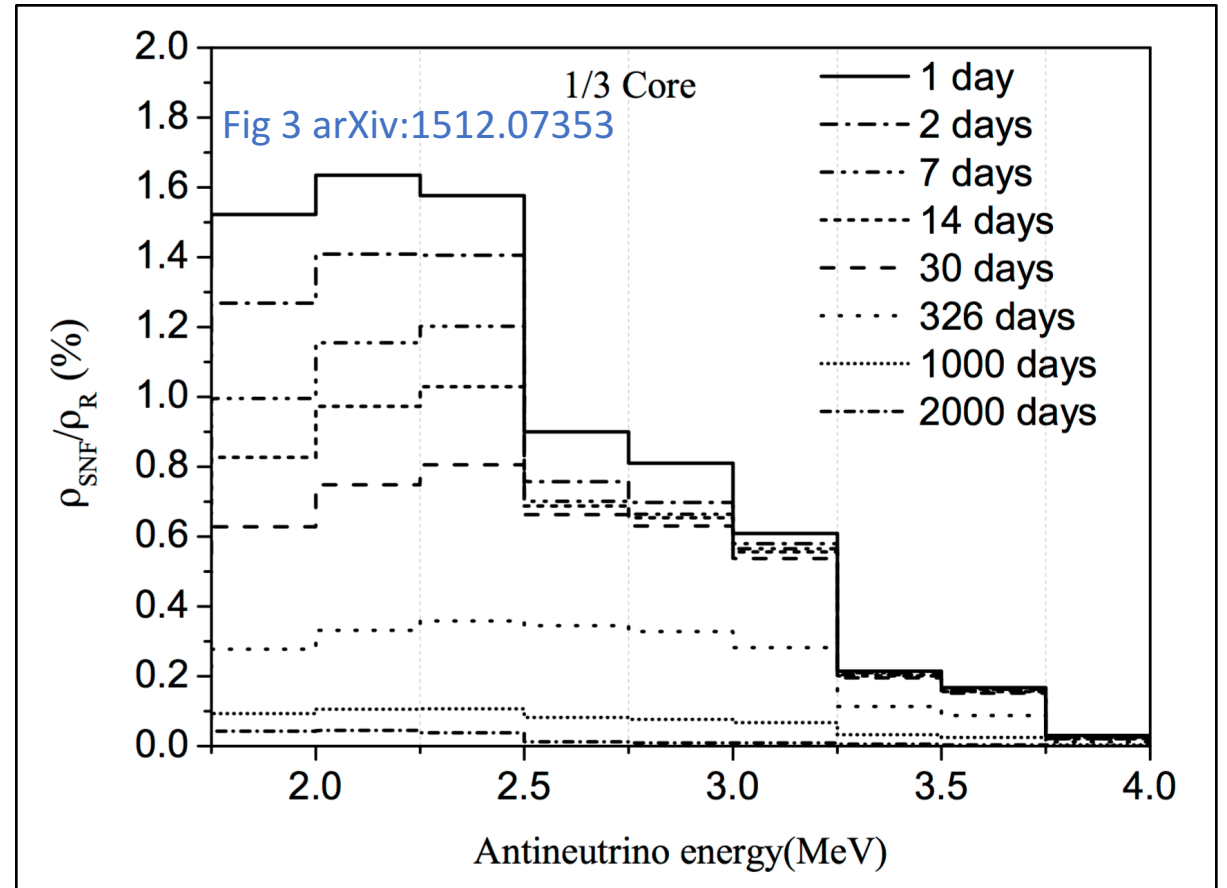
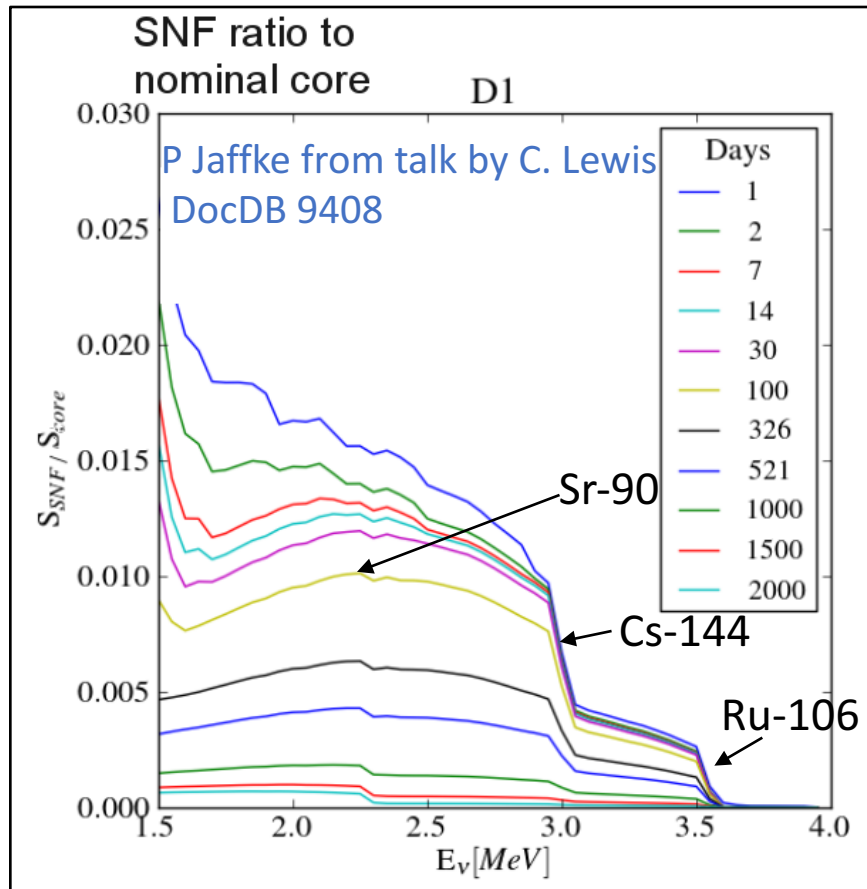
Discrepancy with previous Zhou Bin et al. analysis

- 2012 analysis by Zhou Bin et al. assumes 1 GW PWR reactor burning for 11 months then turned off: calculation includes neutrinos from full core
- Recent calculation assumes 45 GWd/tU burnup then 1/3 core removed as SNF.
- Claim: spectral shape averaged over Daya Bay data (which dataset?) agrees at 27% level.



Decent agreement with previous analysis of P. Jaffke

- C. Lewis used curves from Jaffke/Huber SNF simulation.
- Used 100% uncertainty for shape and 50% for rate analysis.



Defining the task

- Take calculated SNF results and use them to calculate the fractional contribution to the Daya Bay detector spectrum as a function of time.
- Results must be weighted to reflect reactor power fluctuations as well as distance between reactors and detectors $L_{r,d}$.
- For a single reactor r the fractional SNF flux spectrum can be given by

$$S_d^{SNF}(E, t) = \frac{S_d^{SNF}}{S_d^{On}} = \frac{\overbrace{\sum_r^6}^{\text{Sum over reactors}} \overbrace{\sum_{i=1}^N}^{\text{Sum over refuels}} \overbrace{\frac{\delta_r(t) P_{sur}(E, L_r)}{L_{r,d}^2}}^{\text{Reactor off scale factor}} \overbrace{[\alpha_r \rho^{SNF}(E, t - t_i)]}^{\substack{\text{Survival probability} \\ \text{(optional)}}} \overbrace{[P_r^{max} S^{on}(1GW)]}^{\text{Reactor r Full Power Flux}}}{\underbrace{\sum_r^6 \frac{P_{sur}(E, L_r)}{L_{r,d}^2} [P_r(t) S^{on}(1GW)]}_{\text{Weighted flux at detector d}}}$$

% SNF flux for reactor r
Reactor r Full Power Flux

$$S_d^{SNF}(E, t) = \frac{S_d^{SNF}}{S_d^{On}} = \frac{\sum_r^6 \sum_{i=1}^N \frac{\delta_r(t) P_{sur}(E, L_r)}{L_{r,d}^2} [\alpha_r \rho^{SNF}(E, t - t_i)] [P_r^{max} S^{on}(1GW)]}{\sum_r^6 \frac{P_{sur}(E, L_r)}{L_{r,d}^2} [P_r(t) S^{on}(1GW)]}$$

$\delta_r = \begin{cases} 2, & \text{reactor on} \\ 1, & \text{reactor off} \end{cases}$

Scale factor if more SNF burnup per batch than assumed in model

Simulated fractional spectrum (Fig 3 arXiv:1512.07353)

Rated reactor power output

Reactor flux spectrum

Reactor to detector distances

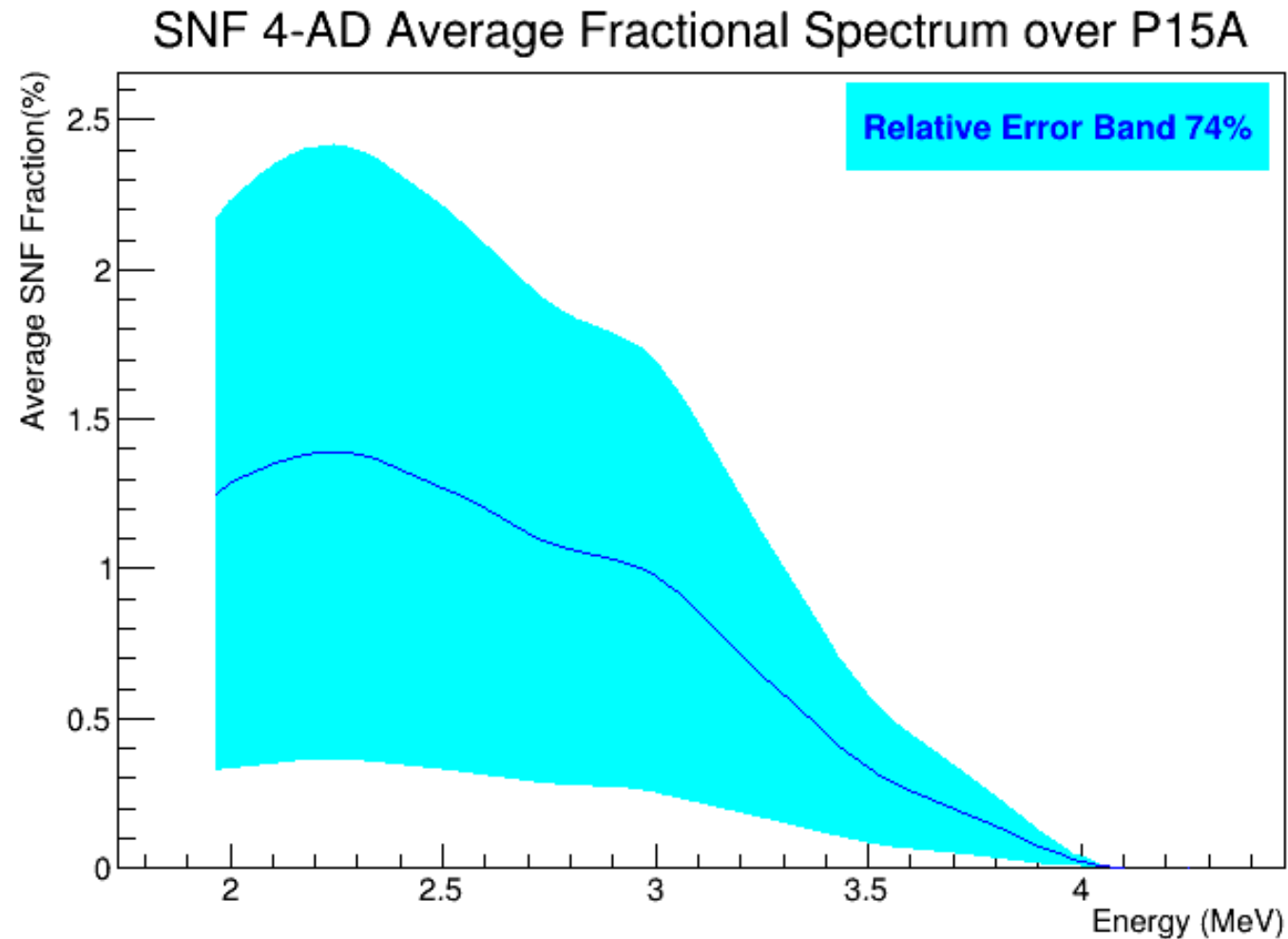
But since rated thermal power is constant (29 GW) for all reactors we can move $P_r^{max} \equiv P^{max}$ to denominator

Which then simplifies to

$$S_d^{SNF}(E, t) = \frac{\sum_r^6 \frac{\delta_r(t) \alpha_r P_{sur}(E, L_r)}{L_{r,d}^2} \sum_{i=1}^N \rho^{SNF}(E, t - t_i)}{\sum_r^6 \frac{P_{sur}(E, L_r)}{L_{r,d}^2} \left(\frac{P_r(t)}{P^{max}} \right)}$$

Fractional thermal power output

Average SNF contribution to spectrum over P15A



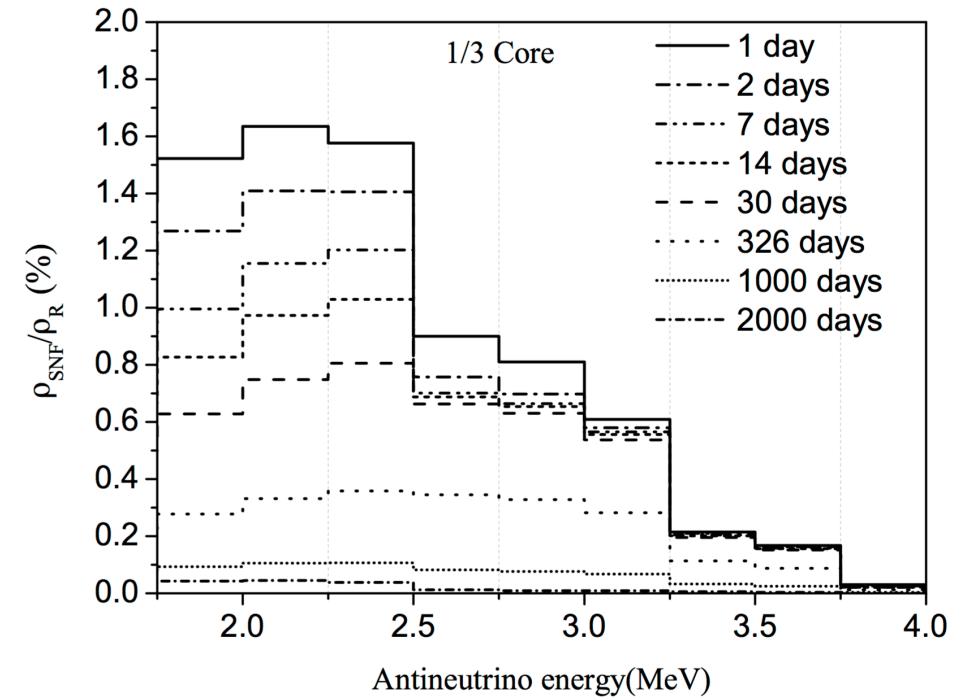
Parameterization of reference SNF spectra

- Use quadruple exponential decay for each energy bin.

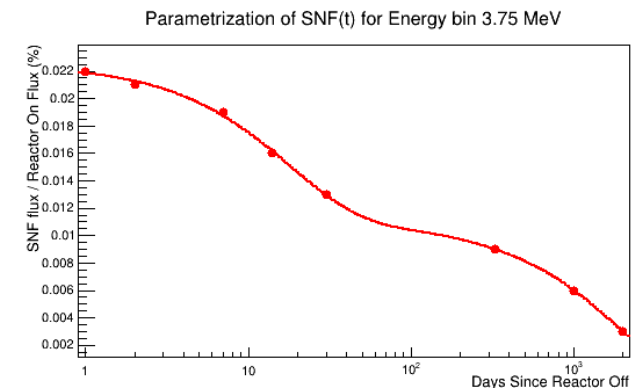
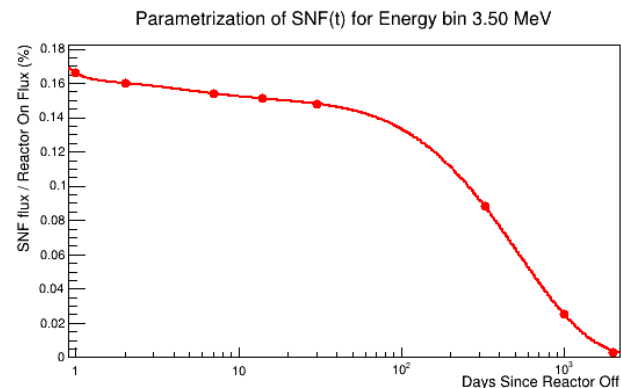
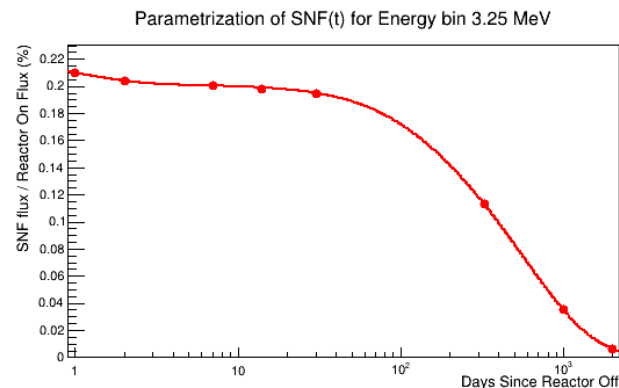
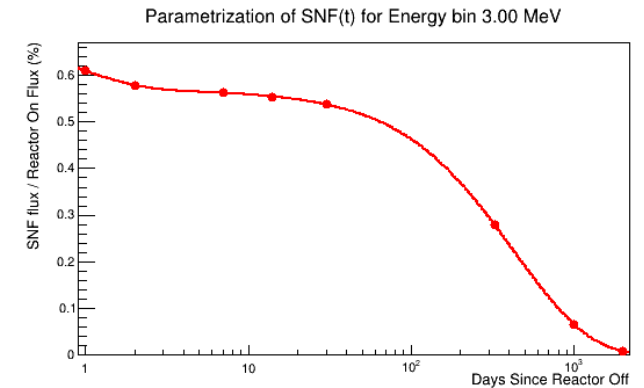
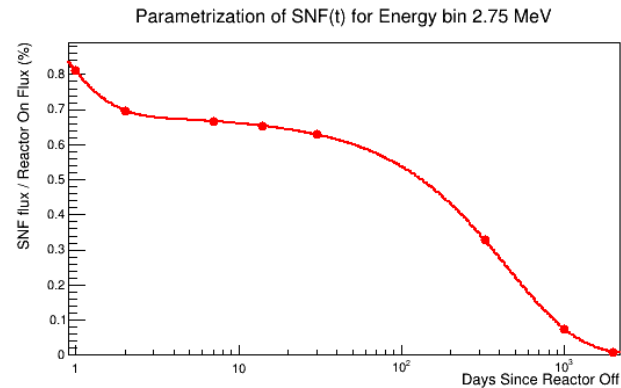
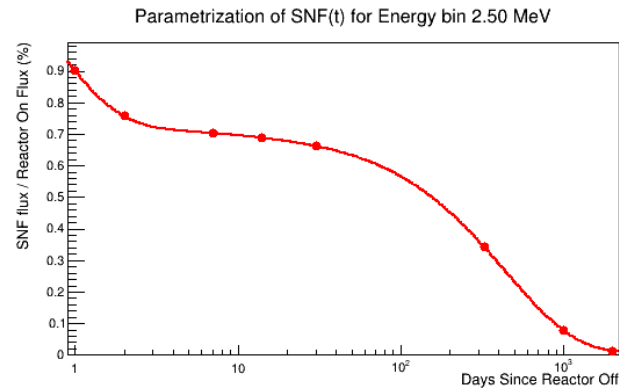
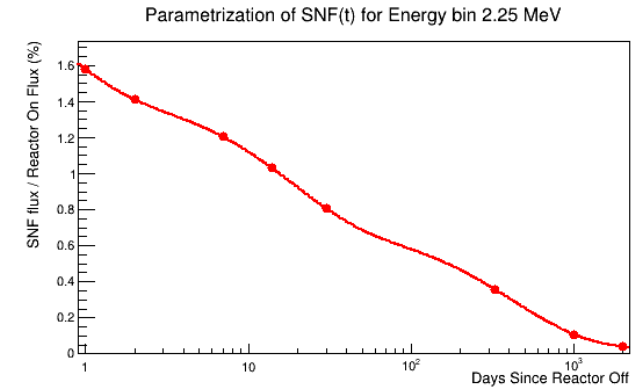
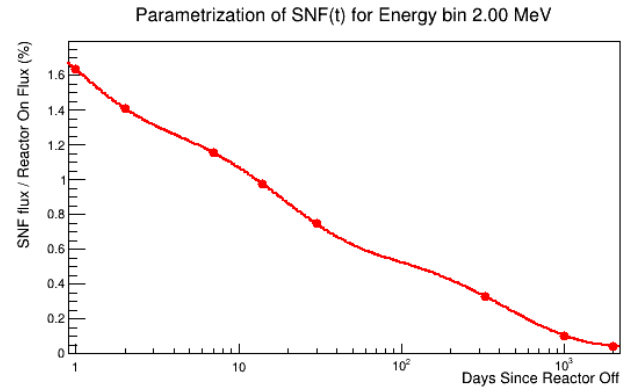
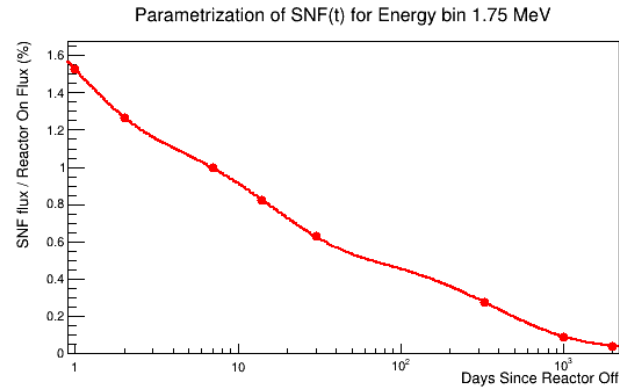
$$\rho^{SNF(\%)} = \sum_{i=1}^4 a_i e^{-b_i t},$$

where a's and b's are fit parameters.

- Reasonable model for interpolation of intermediate times
- Allows extrapolation to 0 and beyond 6 years.
- No parameterization in energy. Retain stepwise function of paper.



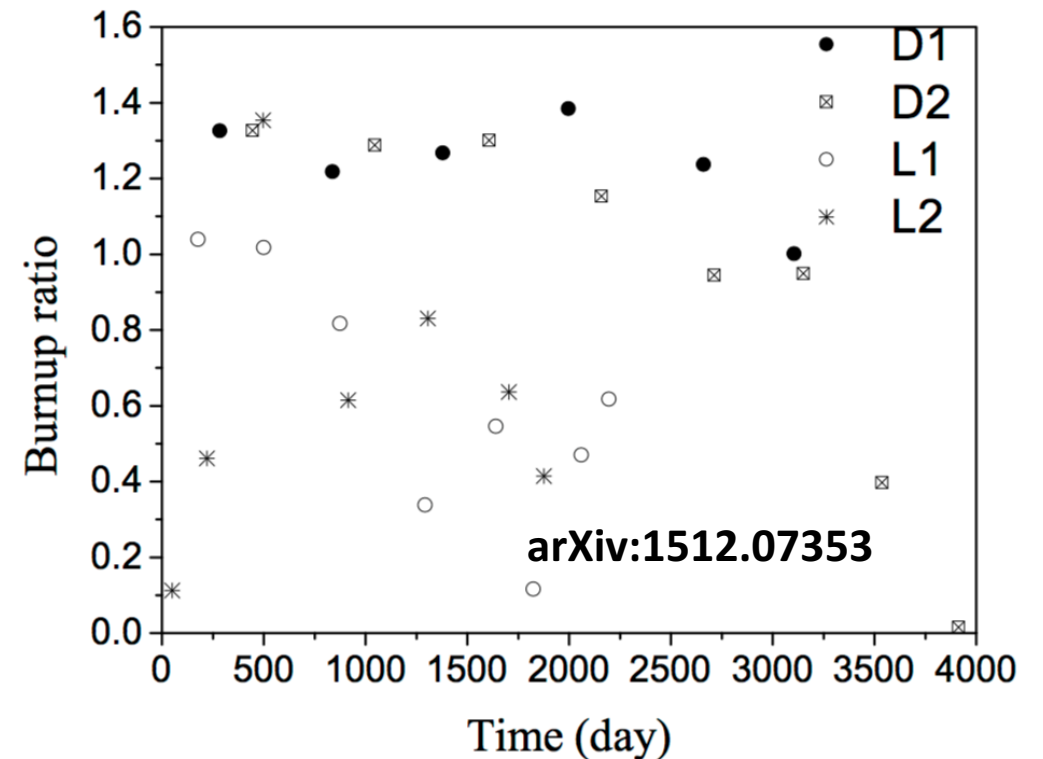
Parameterization of reference SNF spectra



Dealing with burnup factor α_r

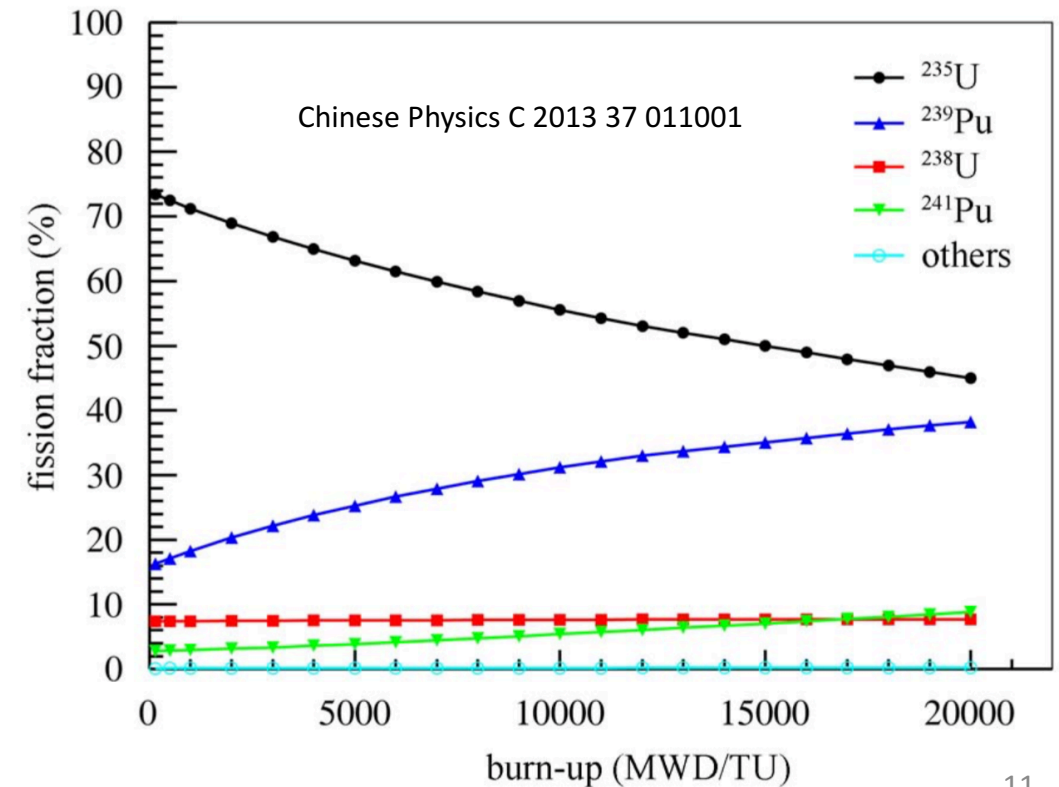
- SNF neutrino flux \propto burnup : burnup depends on how long fuel is in reactor
- Assumed burnup 45 GWd/tU
- Scale each reactor using “burnup ratio” to account for more or less burnup
- More information than I have about reactor fuel.
- Assumes LA3 and LA4 have burnup ratio of 1 due to lack of knowledge.

$$\alpha(m) = \sum_i \text{Burnup}(m, i) / (\text{Burnup}_{\text{average}} \times N/3)$$



Assumptions in determining reactor burnup α_r

- Using published model of Daya Bay reactors we expect about 20 GWd/tU per cycle.
- Assuming refueling cycle with the oldest 1/3 of fuel being replaced each cycle gives burnup for Daya Bay reactors of 60 GWd/tU.
- Must scale calculated SNF spectra by 1.33 for Daya Bay reactors.
- Roughly consistent with burnup calculations in paper



Conclusions of burnup study

- Precise calculation of SNF requires detailed burnup history of reactors.
- Unsure how to include burnup information in publication for Ling Ao 1 and 2
- Move forward with the following simplifying assumptions:
 1. Burnup per cycle for Daya Bay 20 GWd/tU: scale calculated spectra by 1.333
 2. Burnup unknown for Ling Ao reactors so assumed same as calculation in paper of 15 GWd/tU per cycle with 1/3 changeout per cycle: scale calculated spectra by 1.000.
 3. Burnup vs time plot appears to show this is typically an overestimate for the period of time shown. What time period this covers is not given in the text. Assume 50% random error on each refuel and keep in mind no scaling of calculation is likely an overestimate.
- This factor and its uncertainty (unlike point to point uncertainty in spectrum shape) scales entire spectrum in same direction– will not average down as one integrates over spectrum
- Averages down over multiple reactors (and over refuels if varying in time).

Reactor off scaling

- When reactor ON all fuel is burning— no SNF in reactor, only in pools
- When reactor OFF all fuel contains SNF depending on how long in reactor.
- On average the fuel is 2 cycles old (1/3 is 1 cycle old, 1/3 is two cycles old, and 1/3 is 3 cycles old)
- This means the 1/3 oldest fuel rods being replaced are giving off roughly the same number of neutrinos as the 2/3 unchanged fuel rods during the time the reactor is off.
- Scale SNF spectra by factor of 2 during reactor off periods to account for SNF in all fuel rods (same as Fengpeng and Weili slide7 DocDB 8939)
- Include no extra uncertainty on this factor to avoid double counting error already included in α_r (burnup assumed proportional to neutrino flux)

Converting to IDB SNF spectrum

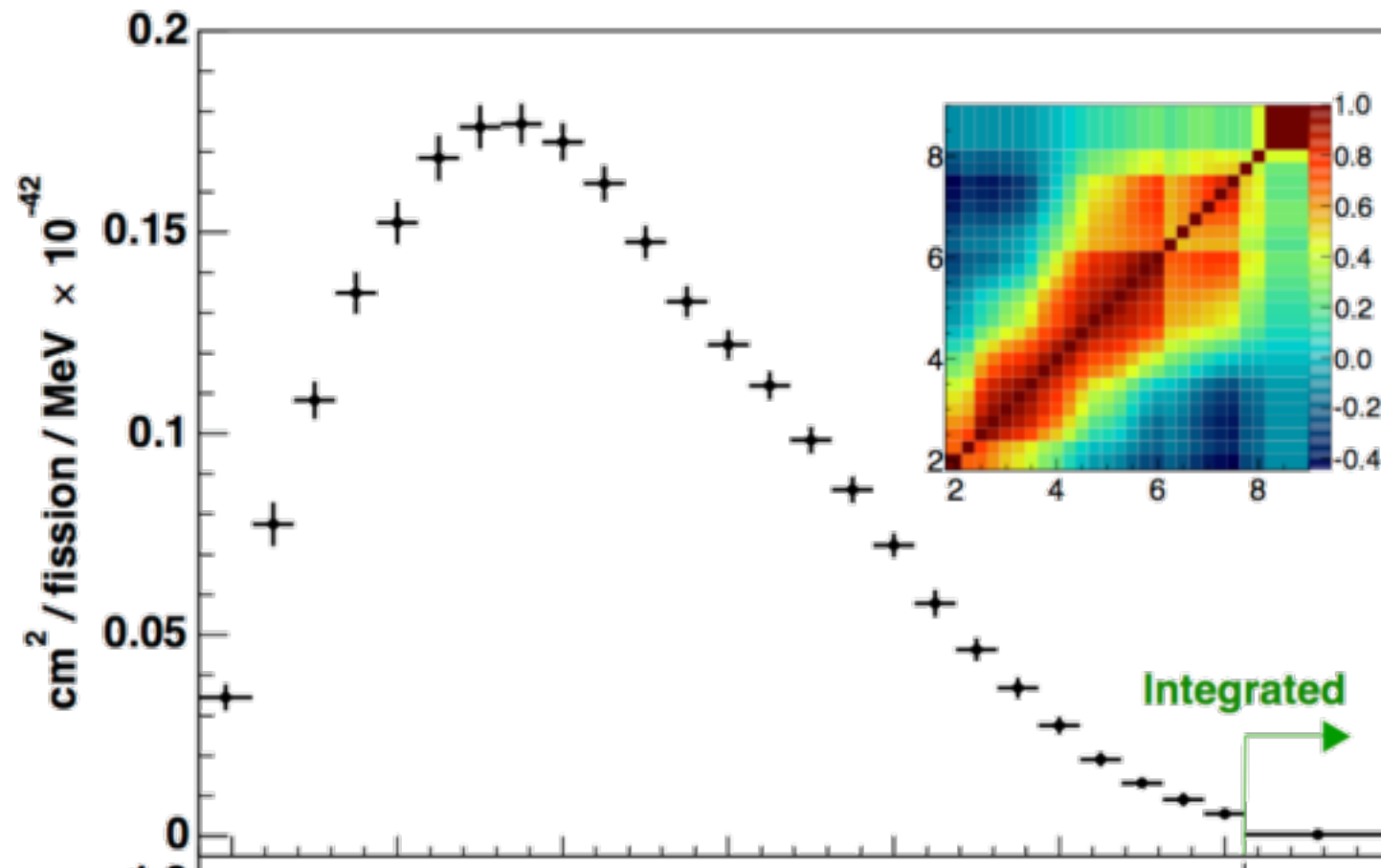
- To see what fraction shows up as a signal in our detector we need to convolve the SNF flux spectrum with our detector response.
- For a given energy range centered on E with width dE this looks like

$$S_{IBD}^{SNF(\%)}(E, t) = \frac{\int_{E-dE}^{E+dE} S^{SNF(\%)}(E', t) S^{DB}(E) dE'}{\int_{E-dE}^{E+dE} S^{DB}(E') dE'},$$

where S^{DB} is the measured Daya Bay detector spectrum.

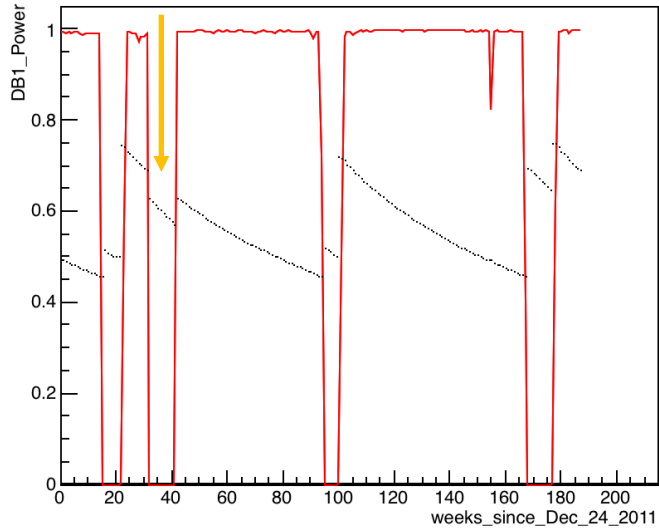
- S^{DB} For I chose to use a published Daya Bay near hall spectrum from “Measurement of the Reactor Antineutrino Flux and Spectrum” <http://journals.aps.org/prl/pdf/10.1103/PhysRevLett.116.061801>
- Only shape is important for this since looking at fraction.

IBD Spectrum taken from “Measurement of the reactor antineutrino flux and spectrum at Daya Bay”

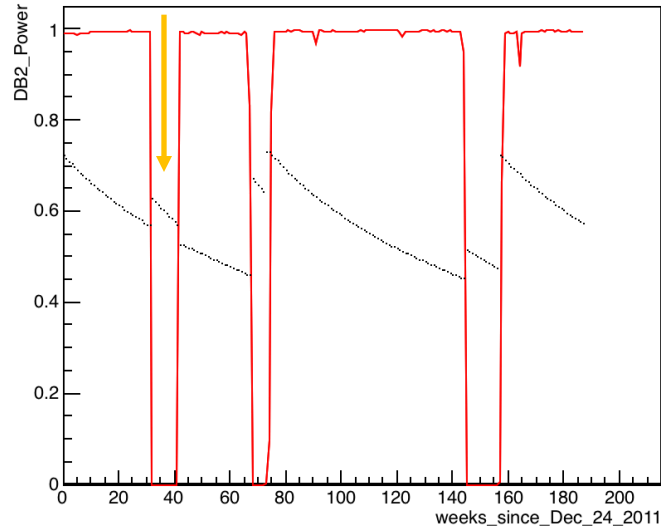


Reactor power/refueling history

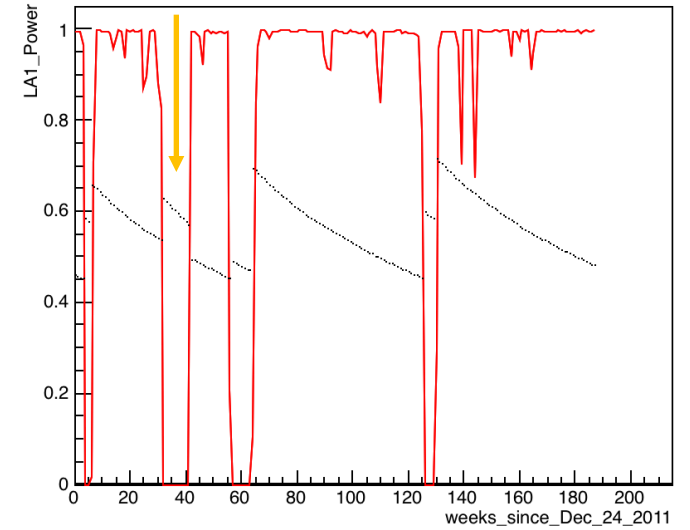
DB1_Power:weeks_since_Dec_24_2011



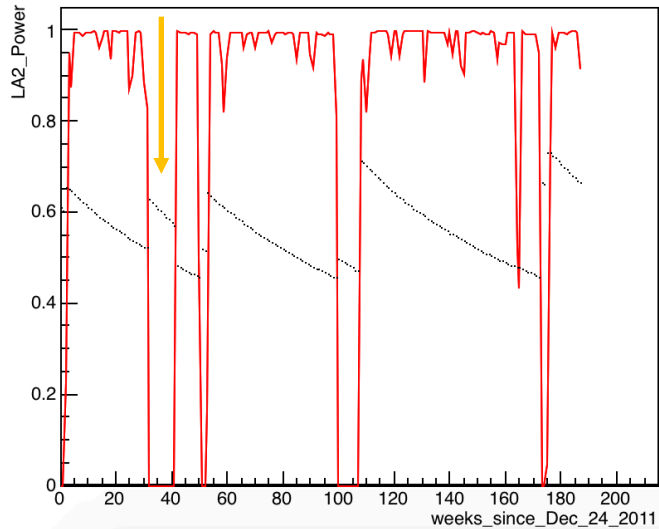
DB2_Power:weeks_since_Dec_24_2011



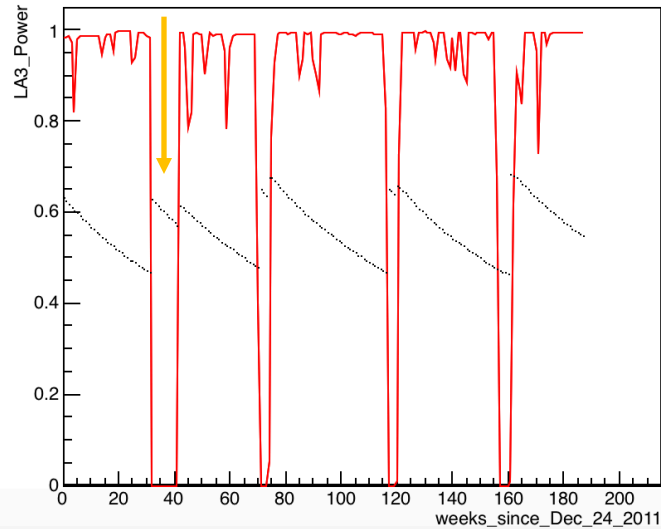
LA1_Power:weeks_since_Dec_24_2011



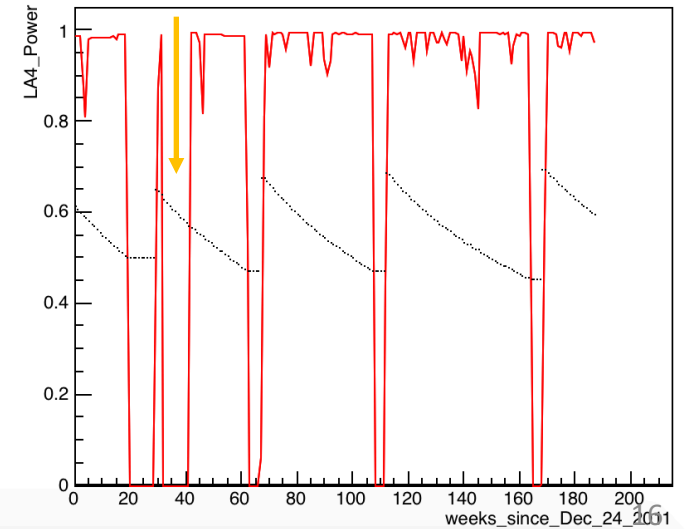
LA2_Power:weeks_since_Dec_24_2011



LA3_Power:weeks_since_Dec_24_2011



LA4_Power:weeks_since_Dec_24_2011



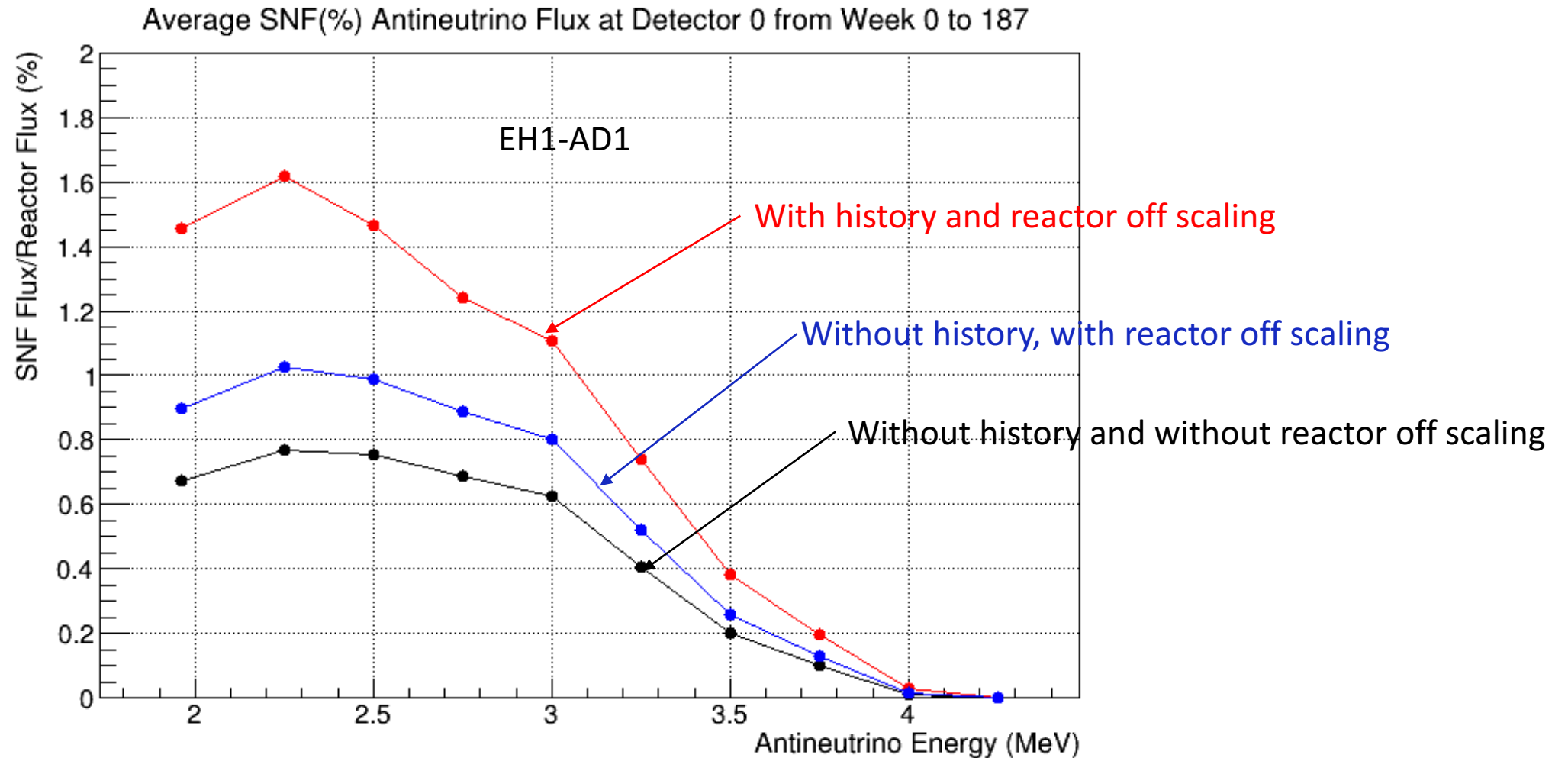
Reactor history (data taken from IAEA PRIS)

- Daya Bay reactors 1 and 2 running since early 1993 and 1994 resp.
- Ling Ao reactors 1 and 2 reactors running since 2002 and 2003.
- Ling Ao reactors 3 and 4 running since Sept 2010 and Aug 2011.
- P15A starts Dec 24,2011.
- Newly removed SNF typically removed to nearby pools for a few years. Onsite storage provided for 5-10 years.
- Stored SNF history has a potentially large impact.
- I assume 5 cycles (7.5 yrs) of 18 months prior to P15A for Daya Bay reactors and for Ling Ao reactors 7 cycles of 12 months before P15A.
- I assume 1 cycle before P15A for Ling Ao 3 and 0 for Ling Ao 4.
- Don't trust extrapolation much beyond calculation.

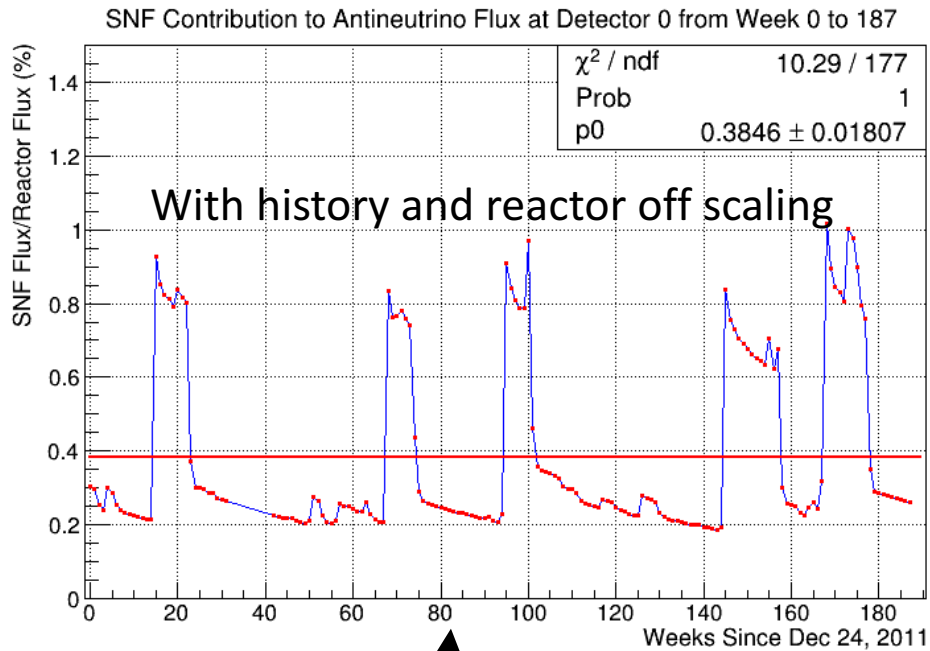
Further assumptions

- No oscillation: same detector response spectrum regardless of baseline.
- All weeks weighted equally: not weighted by power or number of events.
- SNF at same location as reactors: expected to be true up to a few tens of meters.

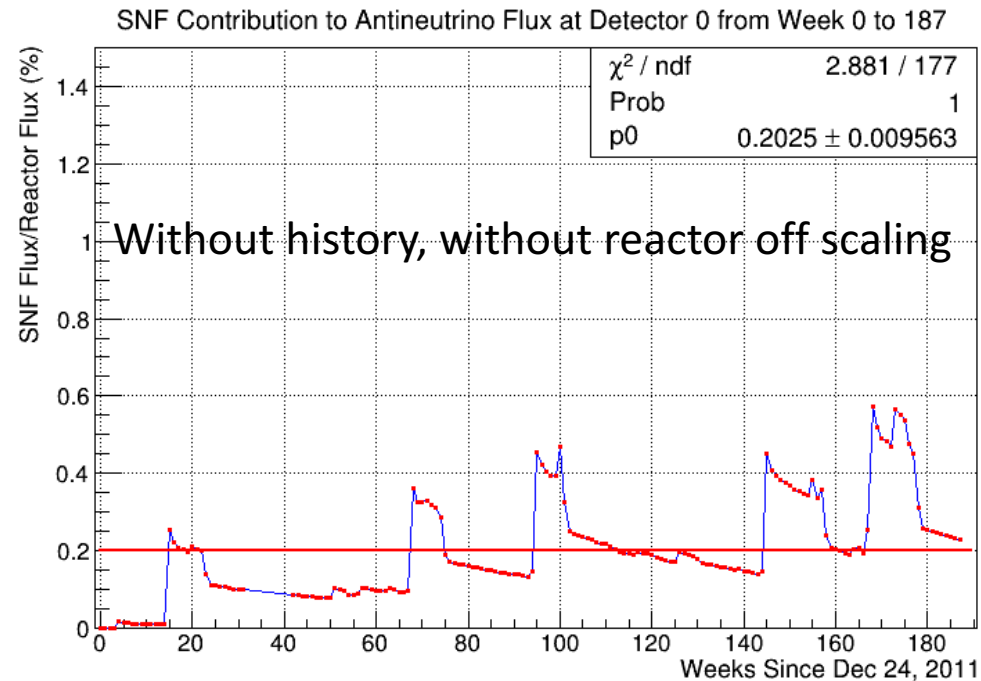
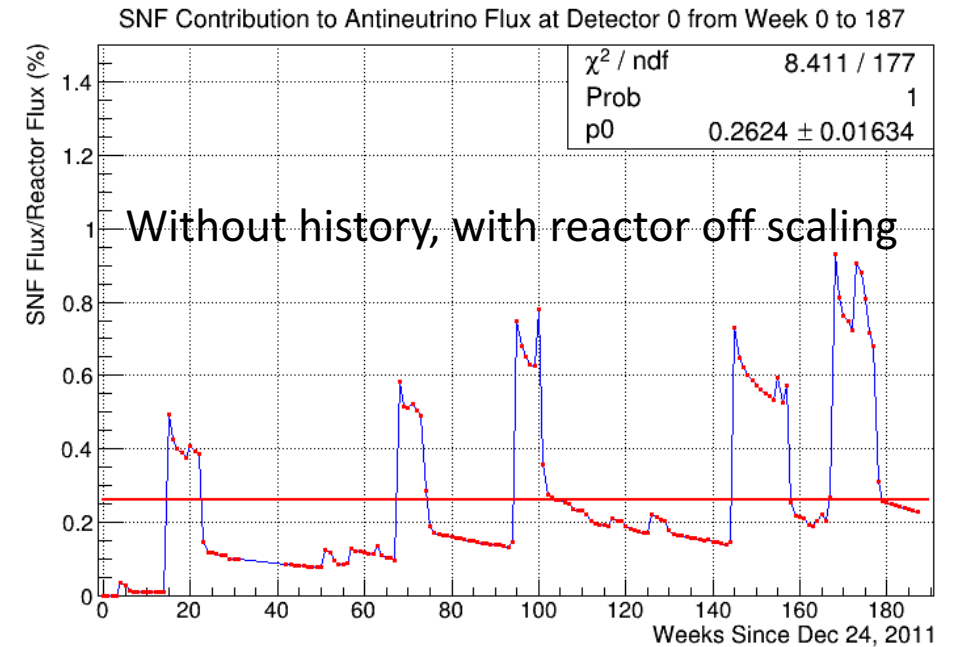
Results: Average SNF fractional IBD spectrum



Percent SNF versus time



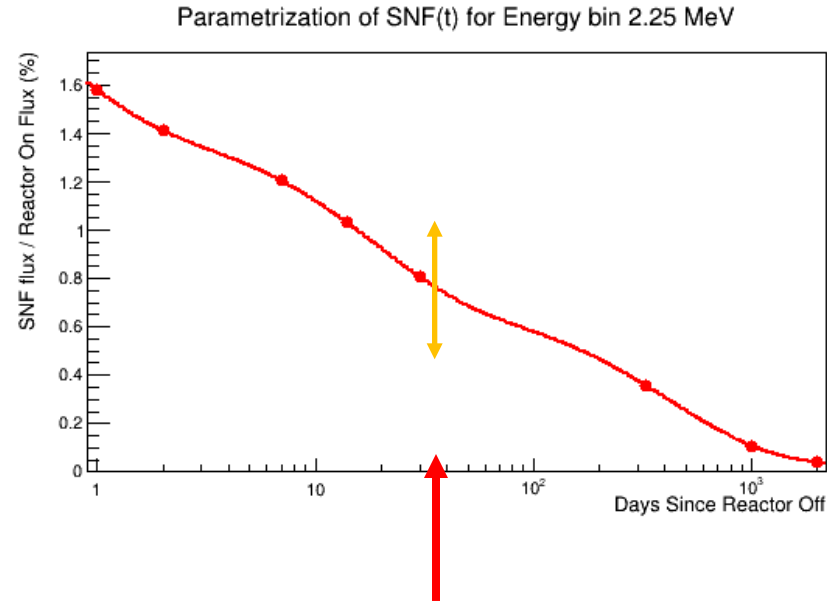
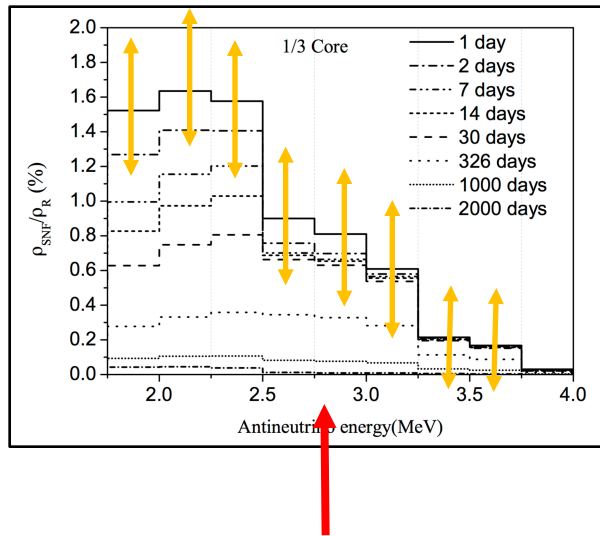
Best estimate of SNF for AD1 contribution is $\sim 0.38\%$



Error Determination

- Sources of error
 1. Baseline: ± 30 meters
 2. Burnup (α_r): $\pm 40\%$
 3. SNF spectrum shape: $\pm 70\%$ point to point in energy not time.
- Baseline error not included because comparatively small and complicated to do properly:
 - Comparatively small: uncertainty $< 10\%$ averaged down over multiple reactors
 - Complicated: to correctly incorporate baseline need to have absolute coordinates of reactors and detectors not just distances. Also need to scale SNF (eg. scale SNF fraction by 4 if storage pool located half way between reactor and detector)
- Burnup factor (α_r) uncertainty of 40% equally scales full spectrum but averages down to ~20-25% over reactors (see backup for justification of 25%).
- No error added for reactor off scale factor δ_r —already included in burnup error
- Parameterization error included in SNF spectral shape uncertainty

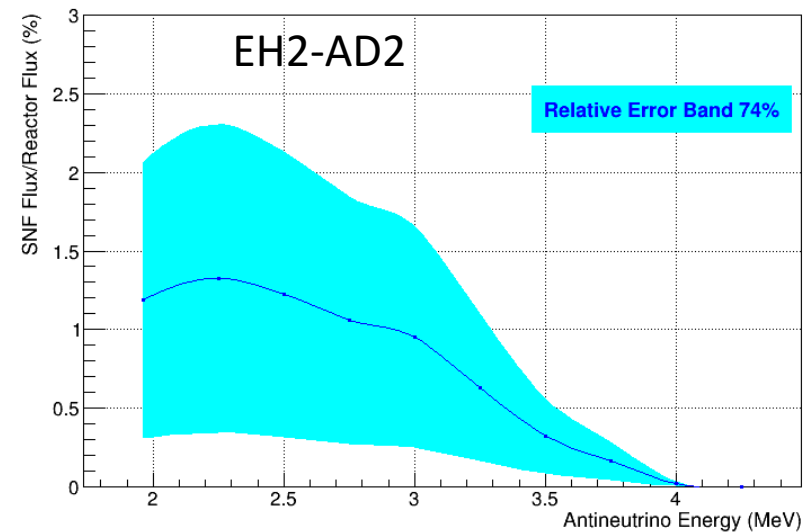
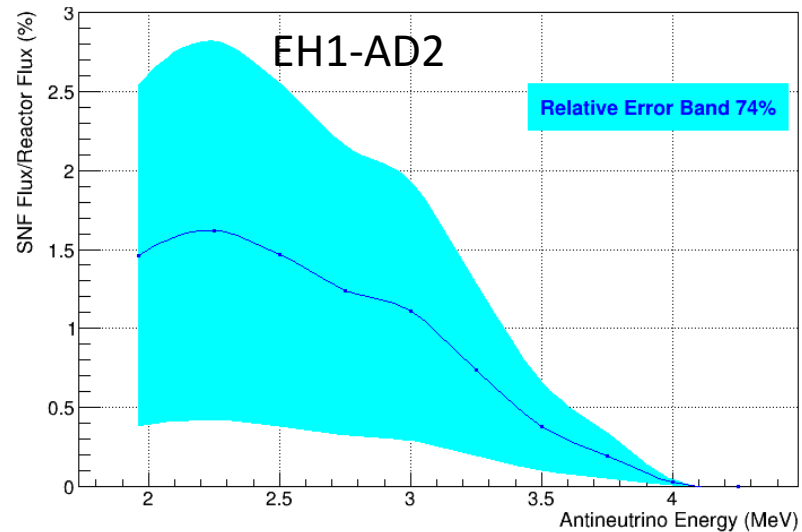
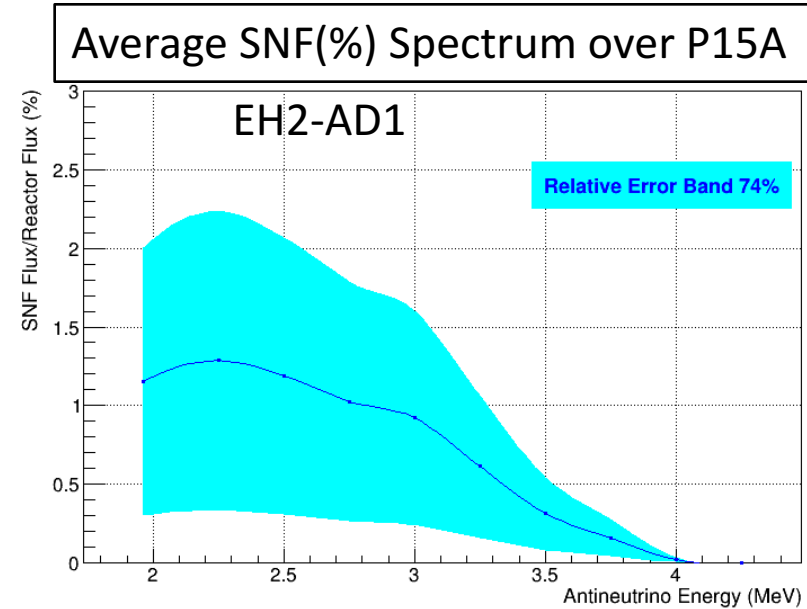
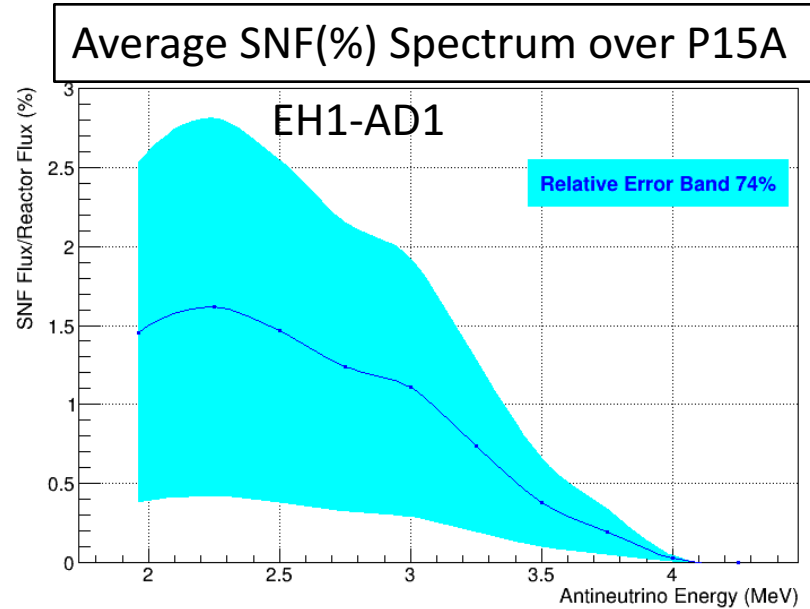
Error Determination cont.



- SNF spectrum shape: random $\pm 70\%$ point to point error in energy averages down to $\sim \pm 35\%$ over spectrum integral.
- Previous analysis by C. Lewis used 100% on shape analysis and 50% on the rate from which I infer she meant 100% point to point on the spectrum and a factor of 2 averaging down when integrating flux over all energies.
- Why the smaller uncertainty? 1. New SNF calculation since then and 2. Error in burnup factor now a separate scale uncertainty.

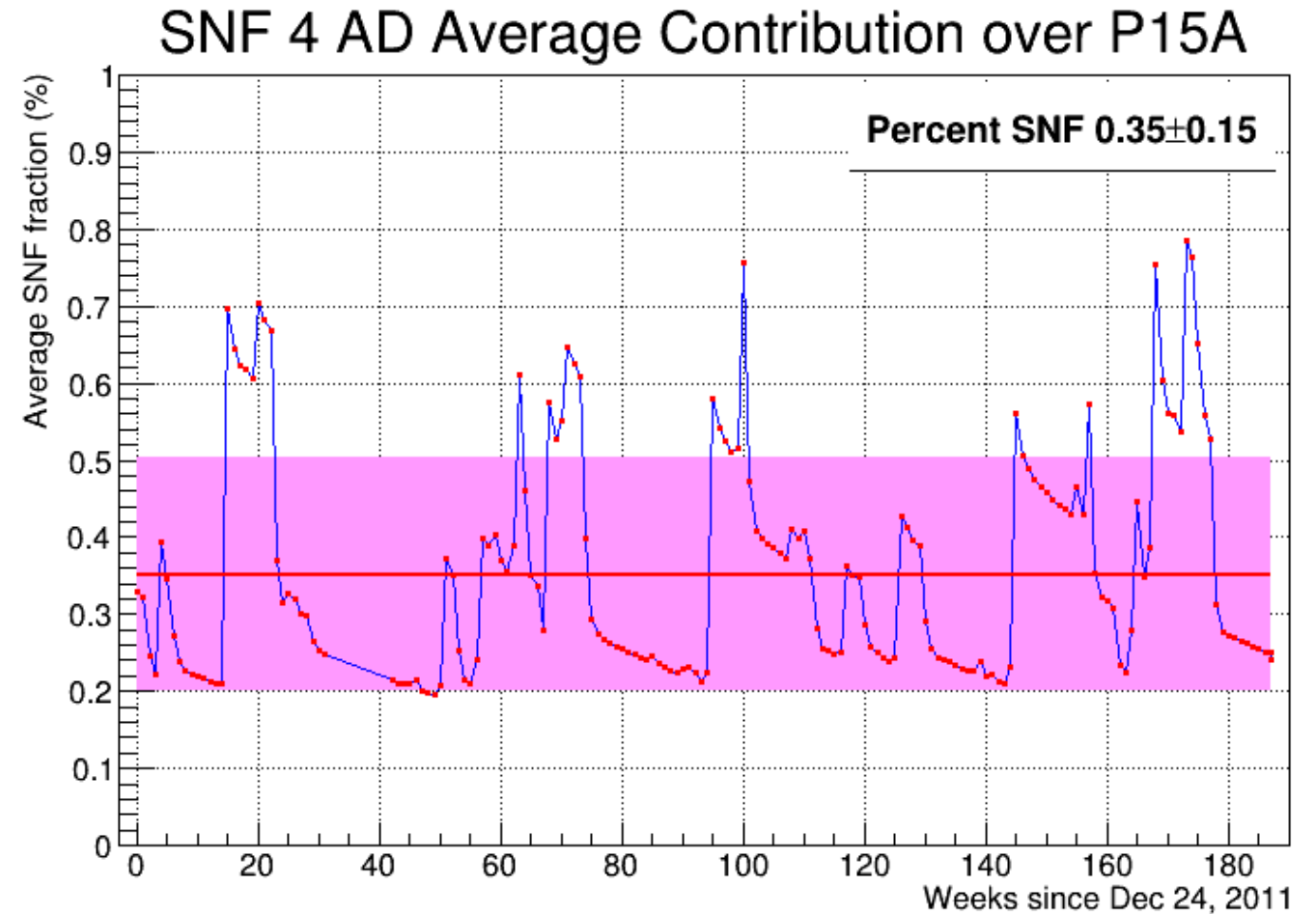
74% uncertainty on spectrum

(quadrature sum of 25% burnup and 70% spectrum shape)

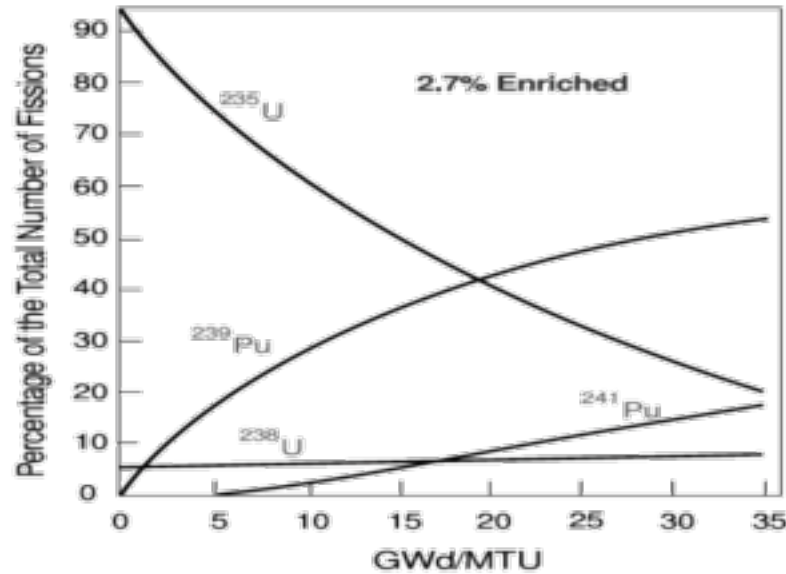


4-AD-Average SNF fraction over P15A: $0.34 \pm 0.14\%$

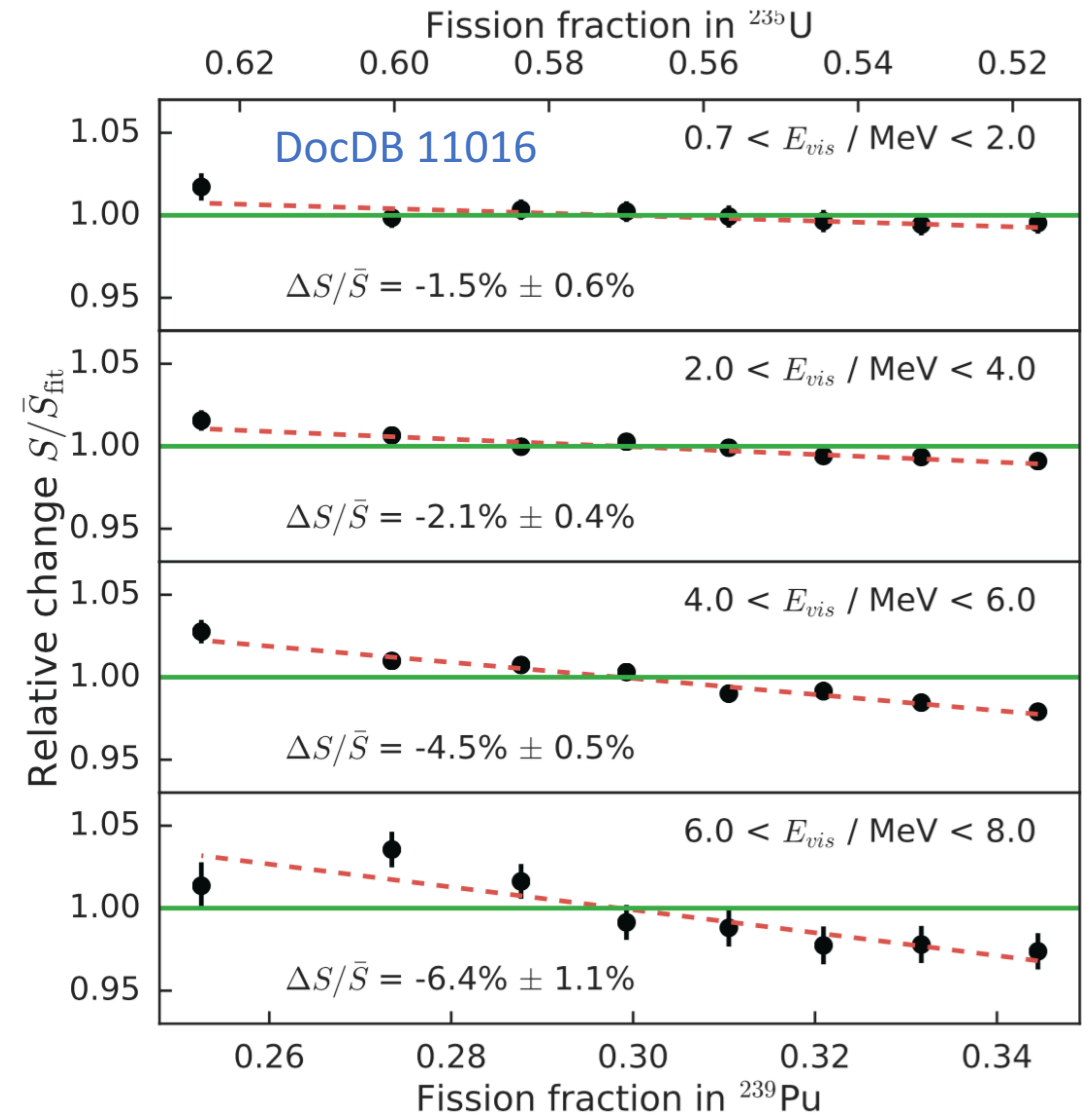
43% relative uncertainty on rate
(quadrature sum of 25% burnup
and 35% integrated spectrum)



Will this affect fuel evolution results?



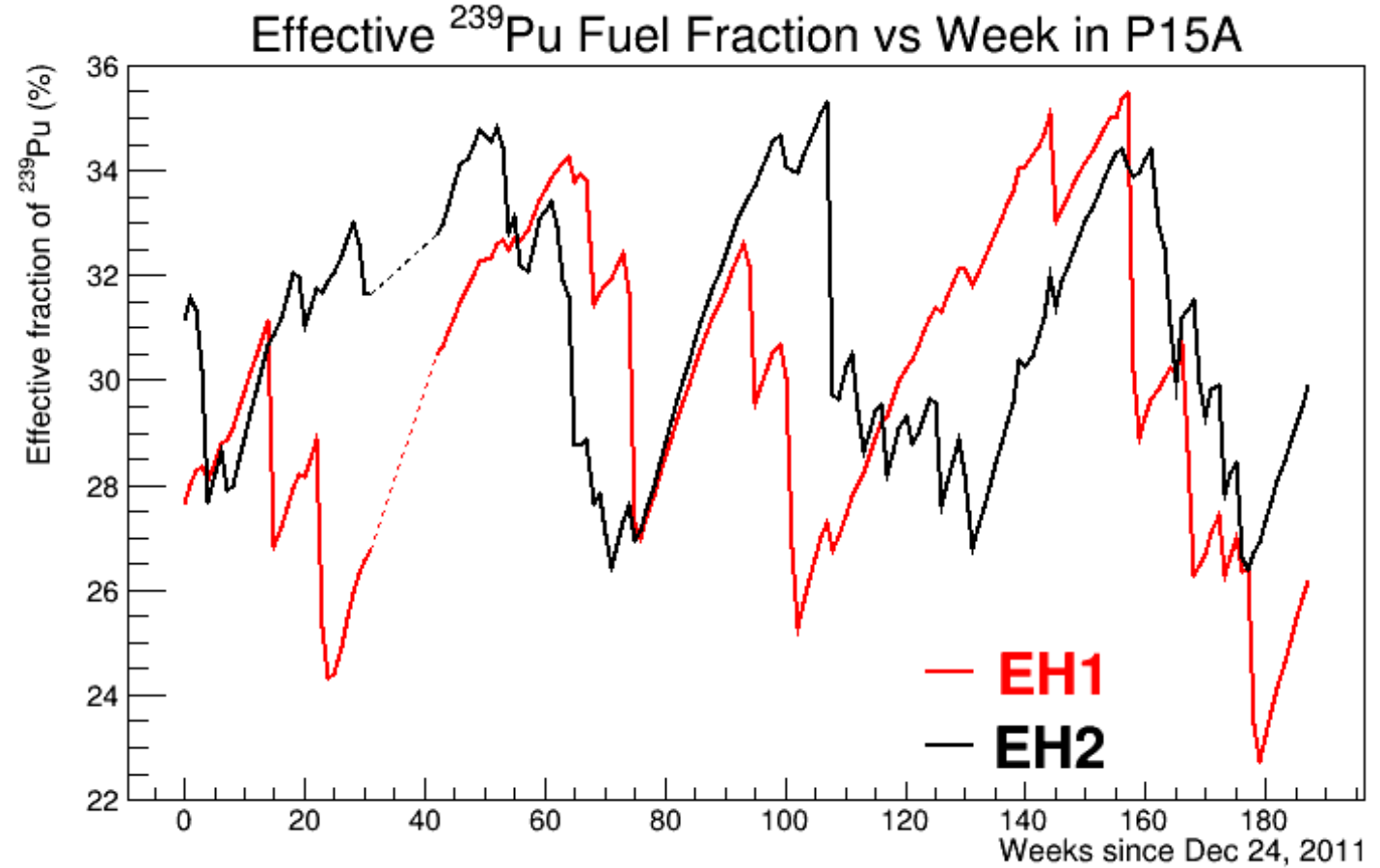
- Absolute neutrino flux goes down ~10% over a fuel cycle
- Plots to the left show Daya Bay detector sensitivity to fuel composition / burnup
- Does SNF decay compromise findings?



Pu-239 effective fuel fraction over P15A

Effective fuel fraction at a given detector is sum of all reactor fractions weighted by reactor power, survival probability and inverse baseline squared.

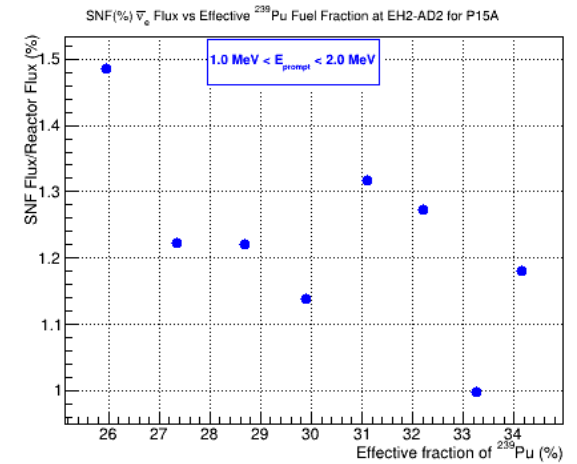
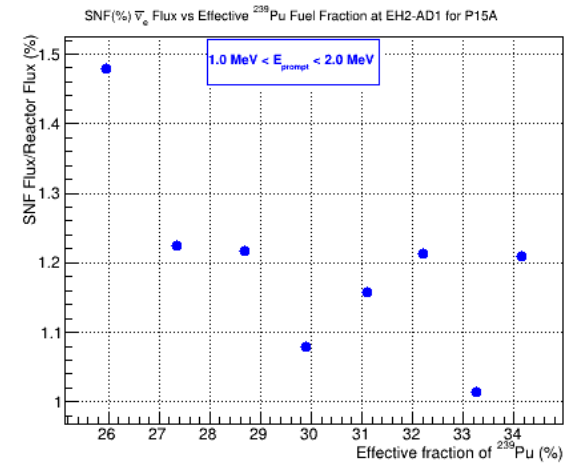
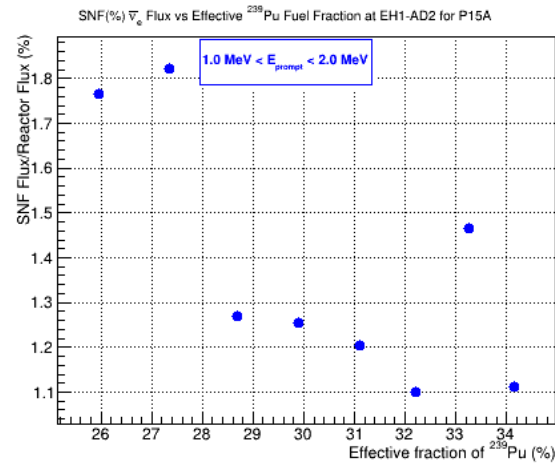
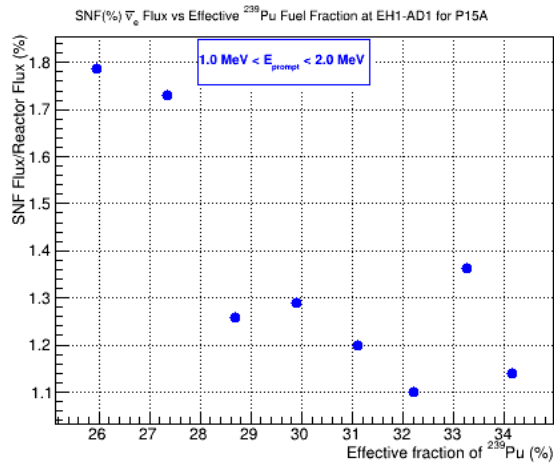
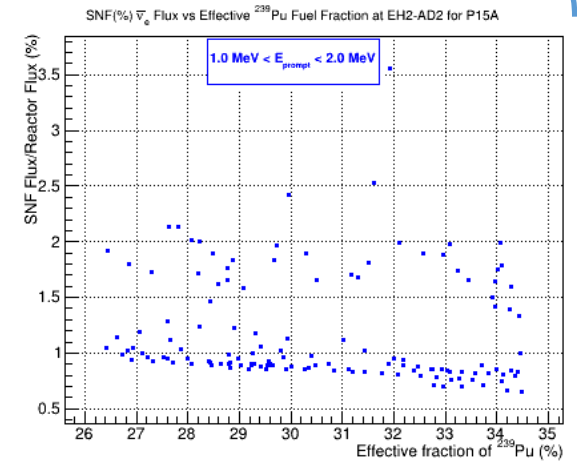
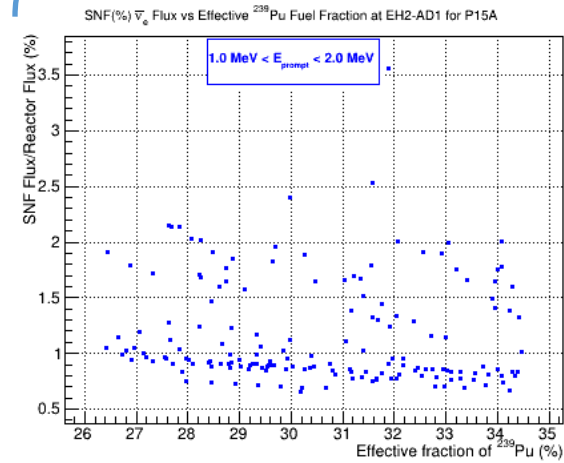
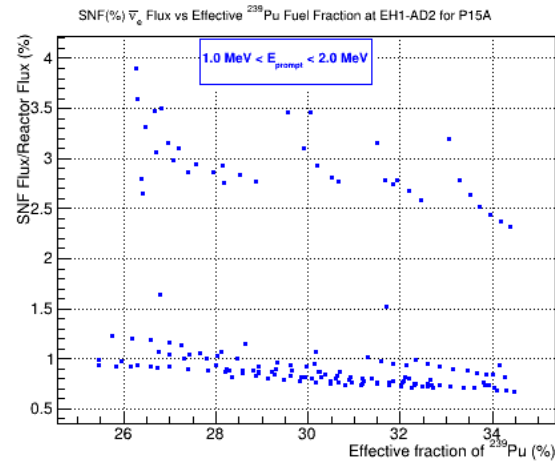
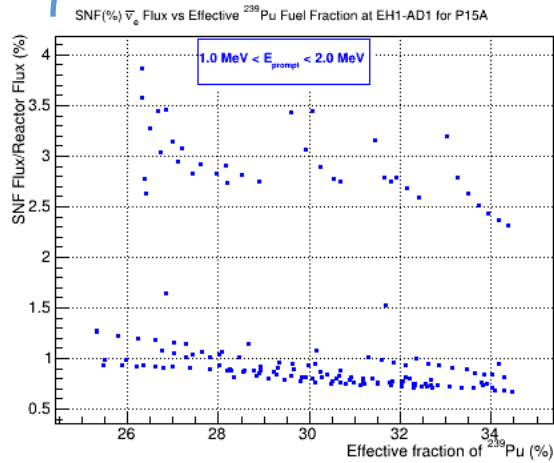
$$F_{239,d} = \frac{\sum_r \frac{\bar{p}_r^{sur} P_r(t) f_r^{239}}{L_{r,d}^2}}{\sum_r \frac{\bar{p}_r^{sur} P_r(t)}{L_{r,d}^2}}$$



$\sim 0.5\%$ trend in energy range $1 < E_{vis} < 2 \text{ MeV}$

0.7% in EH1

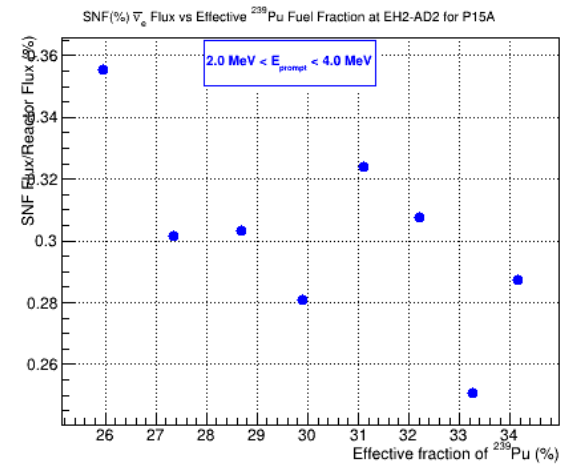
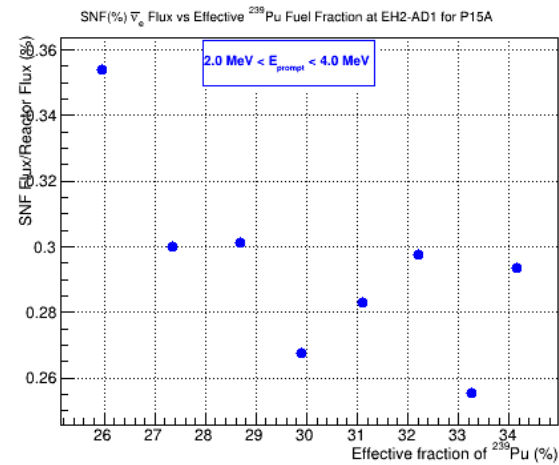
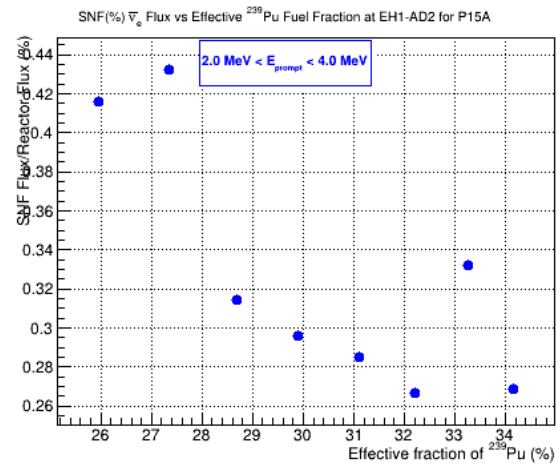
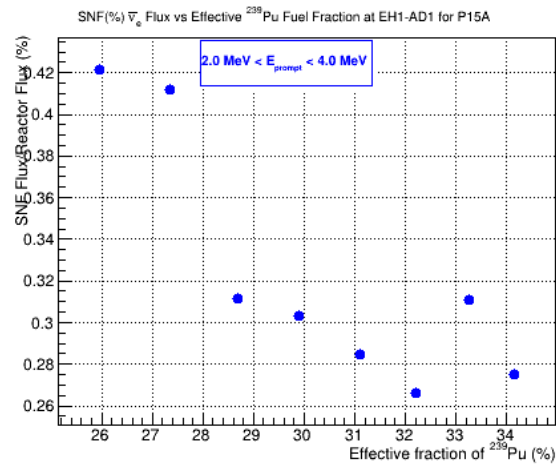
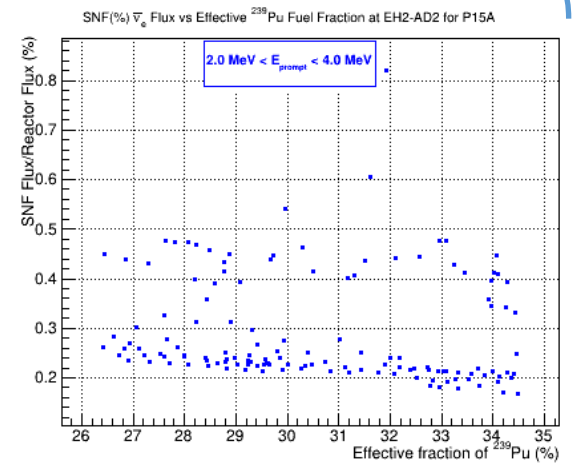
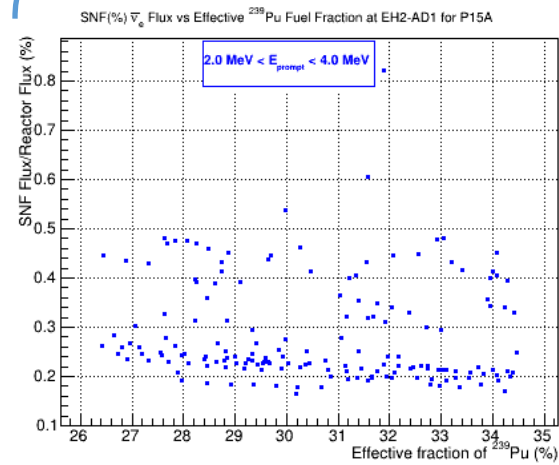
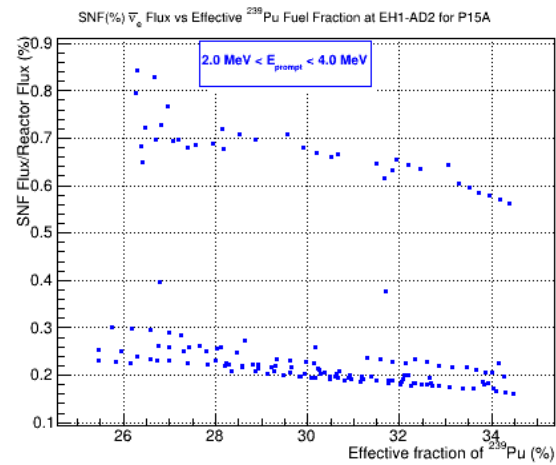
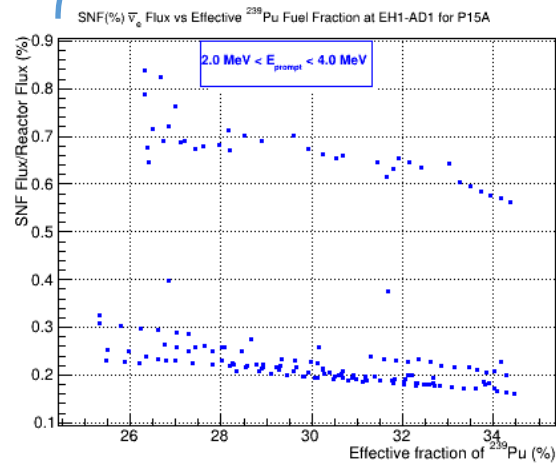
0.2% in EH2



$\sim 0.1\%$ trend in energy range $2 < E_{vis} < 4 \text{ MeV}$

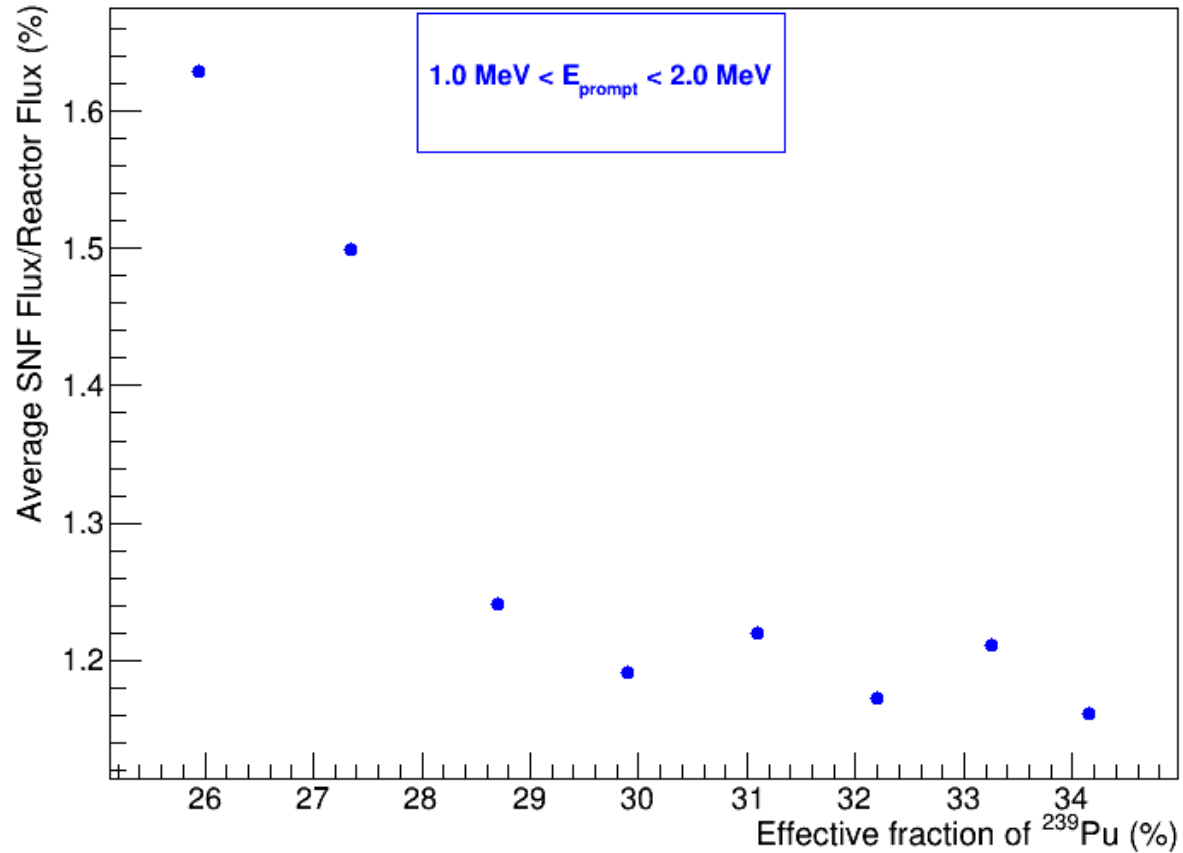
0.15% in EH1

0.07% in EH2

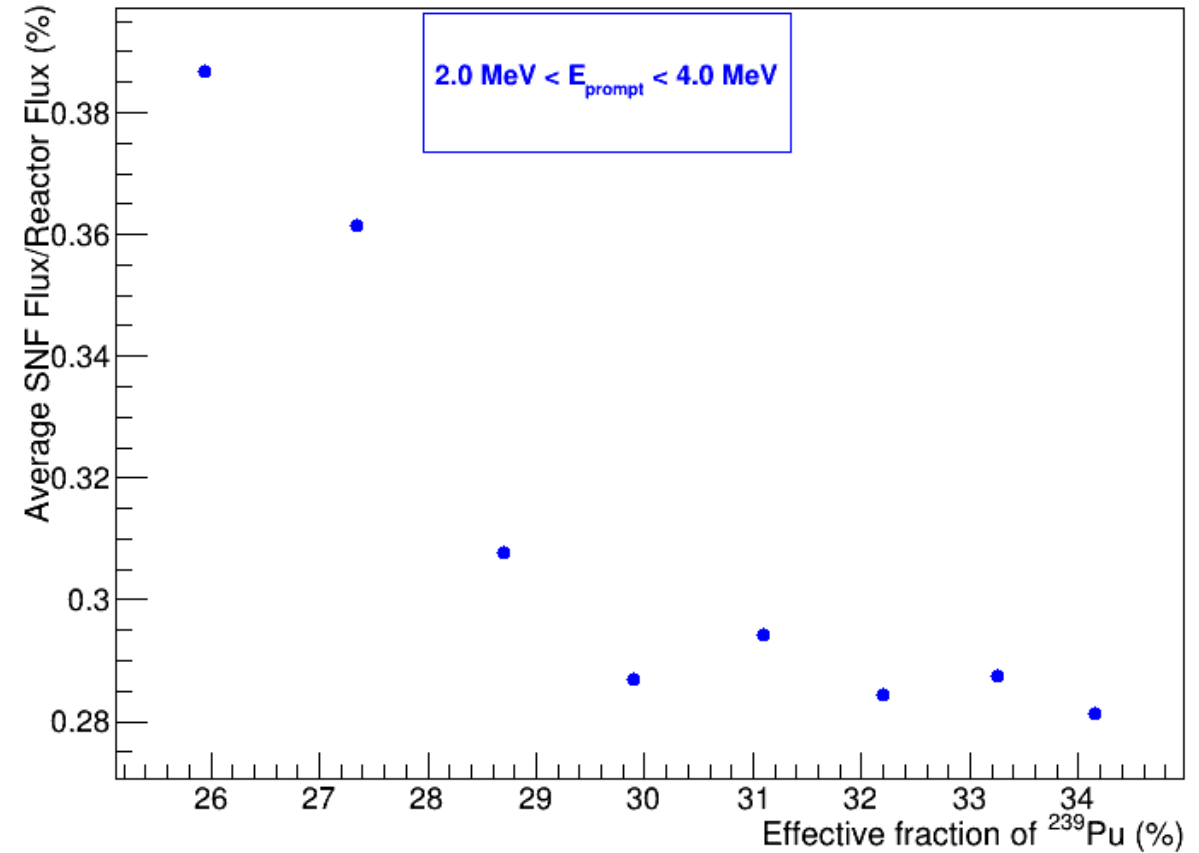


4 Near Hall AD Average

4-AD Average SNF(%) Flux vs Effective ^{239}Pu Fuel Fraction for P15A

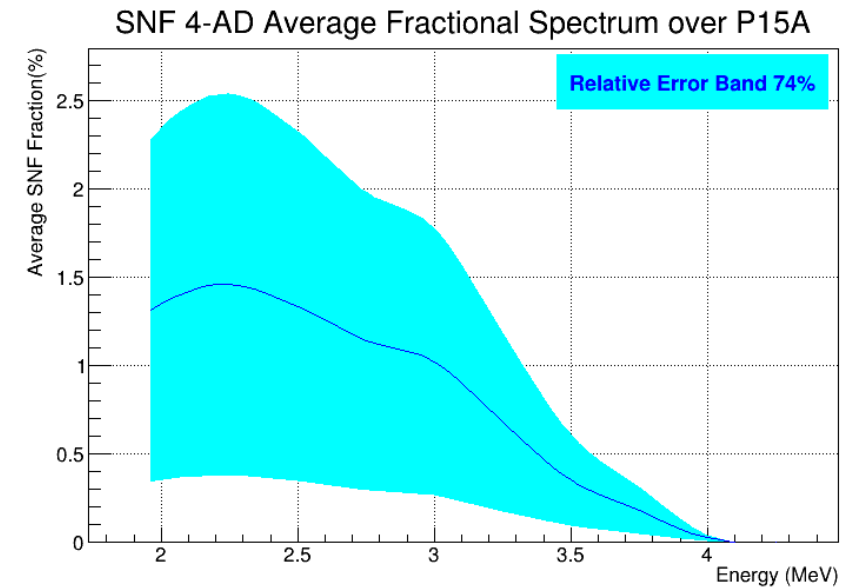
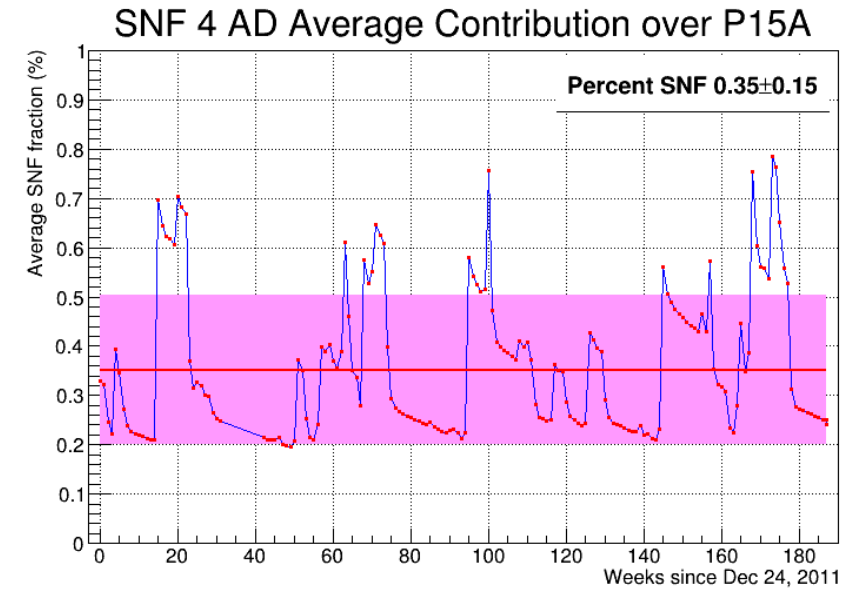


4-AD Average SNF(%) Flux vs Effective ^{239}Pu Fuel Fraction for P15A



Conclusion

- SNF contribution to P15A data set calculated using new calculation of SNF spectrum
- SNF expected to contribute $0.35 \pm 0.15\%$ of the IBD events
- The average percent SNF spectrum shape has been calculated with an expected $\sim 75\%$ uncertainty.
- SNF is not expected to compromise findings of fuel evolution analysis.



Backups

C++ class SNF

- Reads in spectra and parameters from configuration file
- Calculates SNF spectra by the week for any detector
- Calculates total SNF fractions
- Builds graphs for displaying results
- Typical script shown

```
{
    SNF snf;
    snf.Configure("SNF.config");
    snf.LoadReactorPowerTree("WeeklyReactorData.root","tree");
    snf.ParameterizeSNFvsT(0);

    if(0){
        for(int icore=0; icore<nCore;++icore)
            snf.FindRefuelTimes(icore);
        //Add refuels prior to P15A data at expected rate for each reactor
        if(add_old_SNF){
            //Add 7 refuels (~10 yr) for Daya Bay 1,2 and Ling Ao 1,2
            for(int i=0; i<7;++i){
                snf.InsertRefuelTime(0);
                snf.InsertRefuelTime(1);
                snf.InsertRefuelTime(2);
                snf.InsertRefuelTime(3);
            }
            //Add 1 refuels (~1 yr) for Ling Ao 3
            snf.InsertRefuelTime(4);
        }
    }
    //Now do some stuff!!!!!!
    //For example....
    int det = 0, week = 15;
    snf.GetRelSNFSpectrumPlot(det, week) -> Draw("alp");
}
```

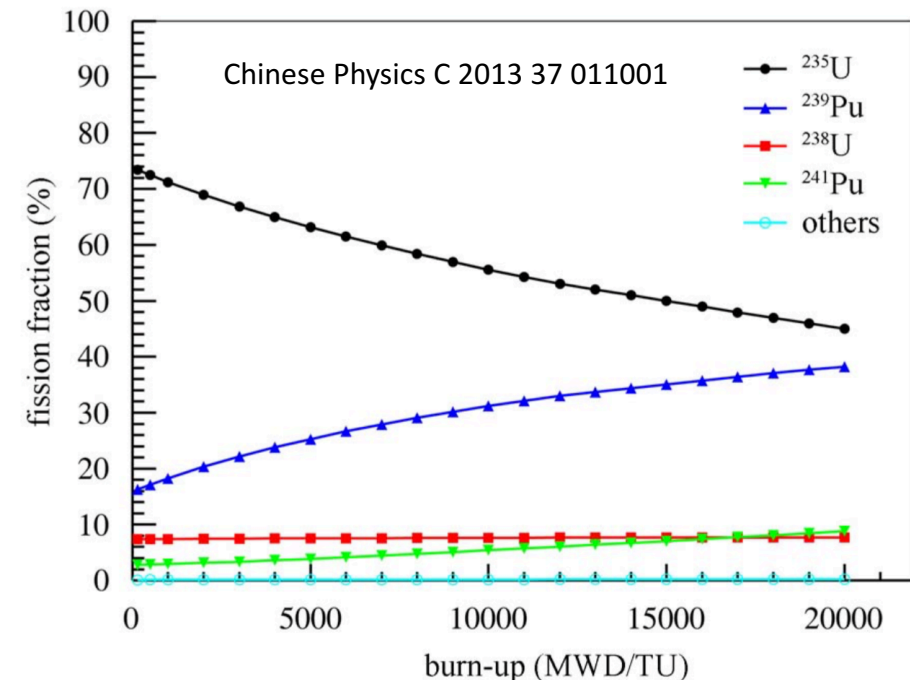
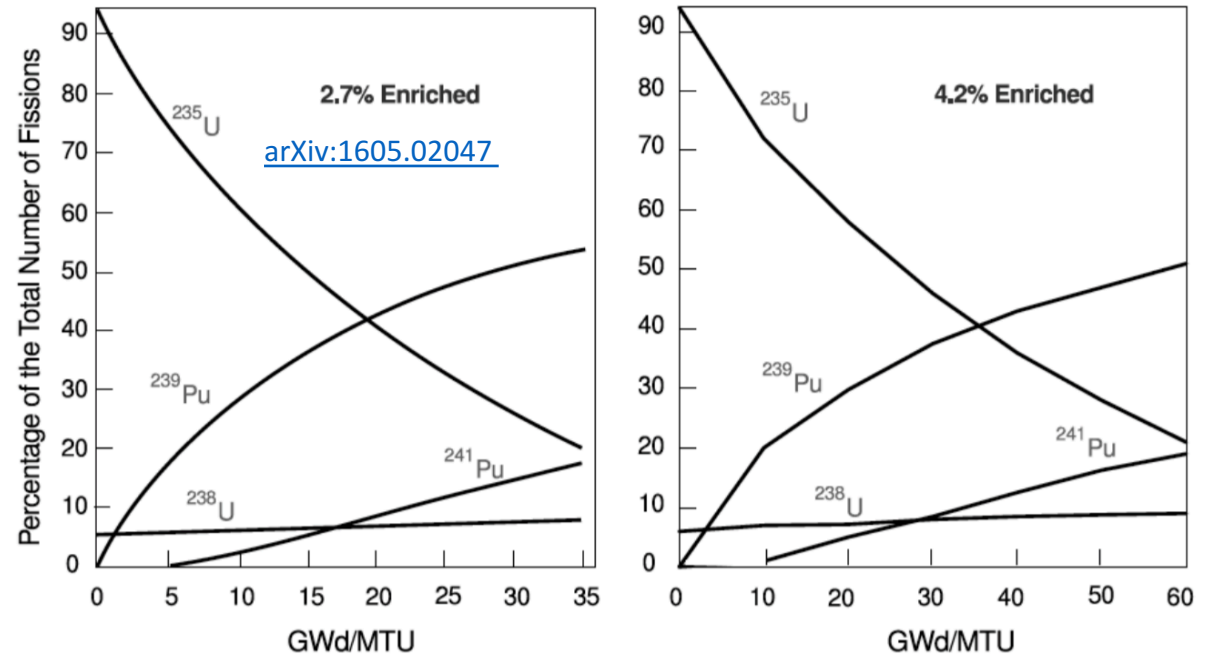

Table of isotopes included in earlier calculation meeting these criteria (CPC vol. 36 No. 1, Jan 2012,1-5 Zhou Bin *et al.*)

Table 1. Isotopes from fission fragments with
 $E_d \geq 1.8$ MeV, $T_{1/2} \geq 10$ h.

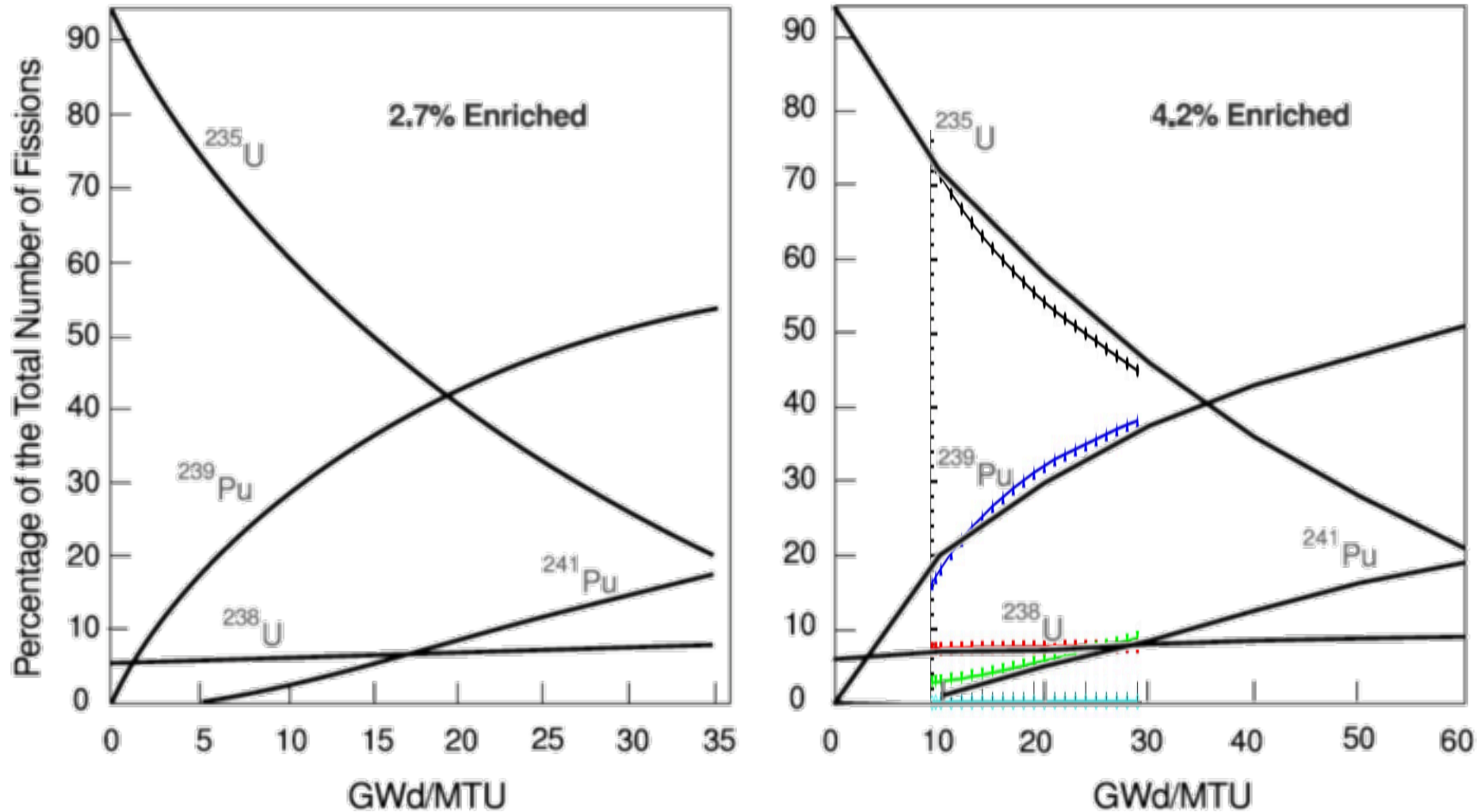
M	$T_{1/2}$	E_0/MeV	D	$T_{1/2}$	E_0/MeV
^{90}Sr	28.78 a	0.546	Y	64.1 h	2.282
^{91}Sr	9.63 h	2.699	Y	58.51 d	1.544
^{93}Y	10.18 h	2.874	Zr	1.53×10^6 a	0.091
^{97}Zr	16.9 h	2.658	Nb	72.1 min	1.934
^{106}Ru	373.6 d	0.039	Rh	29.8 s	3.541
^{112}Pd	21.03 h	0.288	Ag	3.13 h	3.956
^{125}Sn	9.64 d	2.364	Sb	2.758 a	0.767
$^{131\text{m}}\text{Te}$	30 h	0.182	Te	25 min	2.233
^{132}Te	3.204 d	0.493	I	2.295 h	3.577
^{159}Sm	9.4 h	0.722	Eu	15.19 d	2.451
^{140}Ba	12.75 d	1.047	La	1.678 d	3.762
^{144}Ce	284.9 d	0.319	Pr	17.28 min	2.997

Burnup Models

- Must know enrichment to calculate burnup for given energy output.
- Model for Daya Bay suggests 20 GWd/tU per cycle
- Fits reasonably well with 4.2% enrichment curves
- 2008 information from this website claims Daya Bay uses 4.45% enriched uranium oxide and Ling Ao 3.7%
<http://home.pacific.net.hk/~nuclear/info0211.htm>



Daya Bay model overlayed on 4.2% enriched model.
Hints that Daya Bay less than 4.2%?



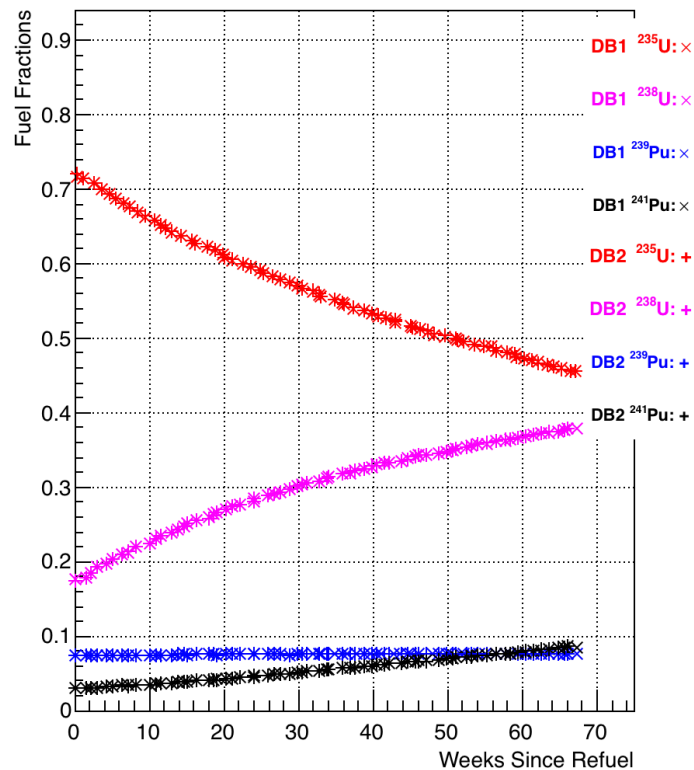
Data from P15A suggest Daya Bay 1 and 2 as well as Ling Ao 1 and 2 are very similar:

- Cycle length ~18 months
- Enrichment likely around 4%

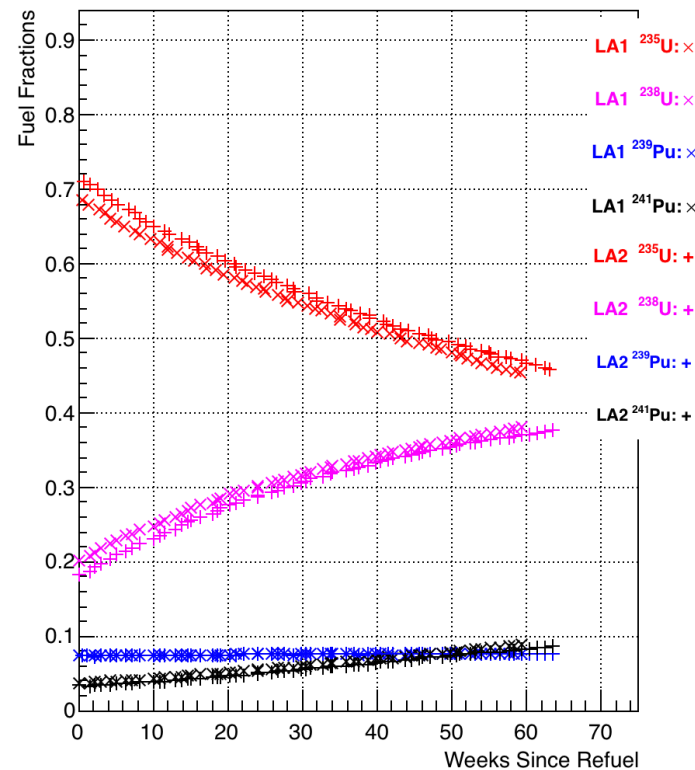
Data from P15A suggest Ling Ao 3 and 4 have shorter cycles suggesting lower enrichment:

- Cycle length ~12 months

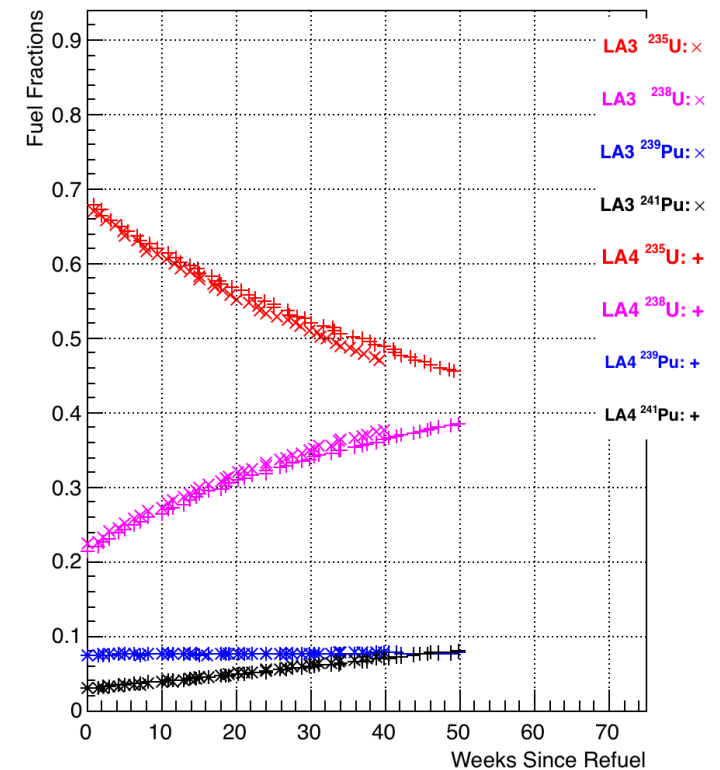
Fuel Fractions Daya Bay Reactors



Fuel Fractions Ling Ao 1,2 Reactors

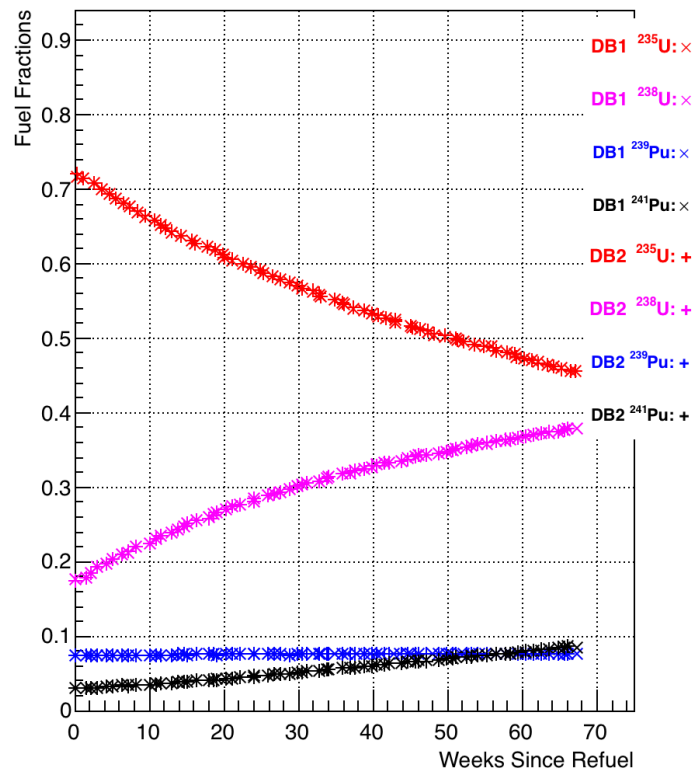


Fuel Fractions Ling Ao 3,4 Reactors

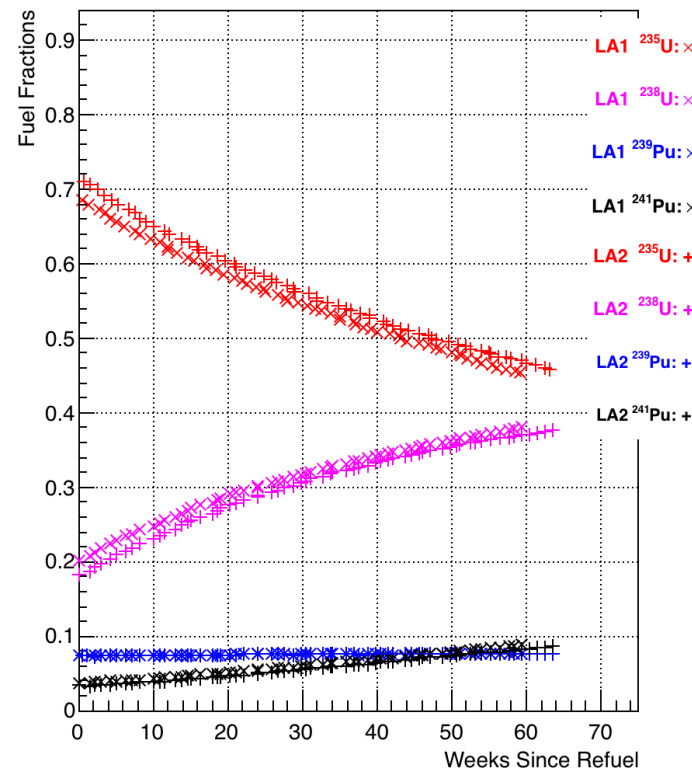


- It is difficult to get >70% U-235 and <20% Pu-239 with only 1/3 fuel changout.
- The following investor website suggests Daya Bay changes about 40% of the fuel each time: <https://www.hknuclear.com/dayabay/plant/nuclearisland/reactor/pages/nuclear.aspx>
- Fortunately it doesn't matter—any given fractional changeout at a set frequency gives the same amount of SNF.

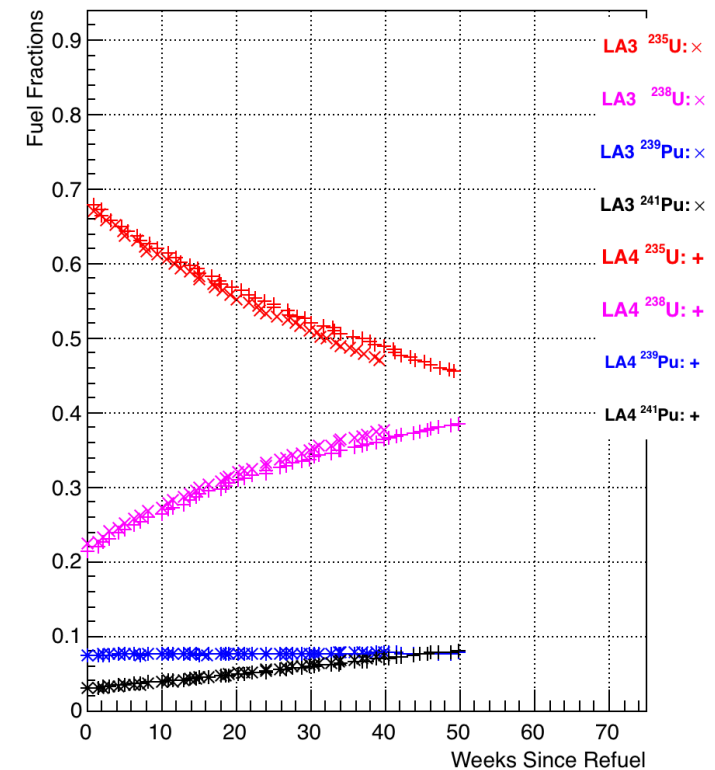
Fuel Fractions Daya Bay Reactors



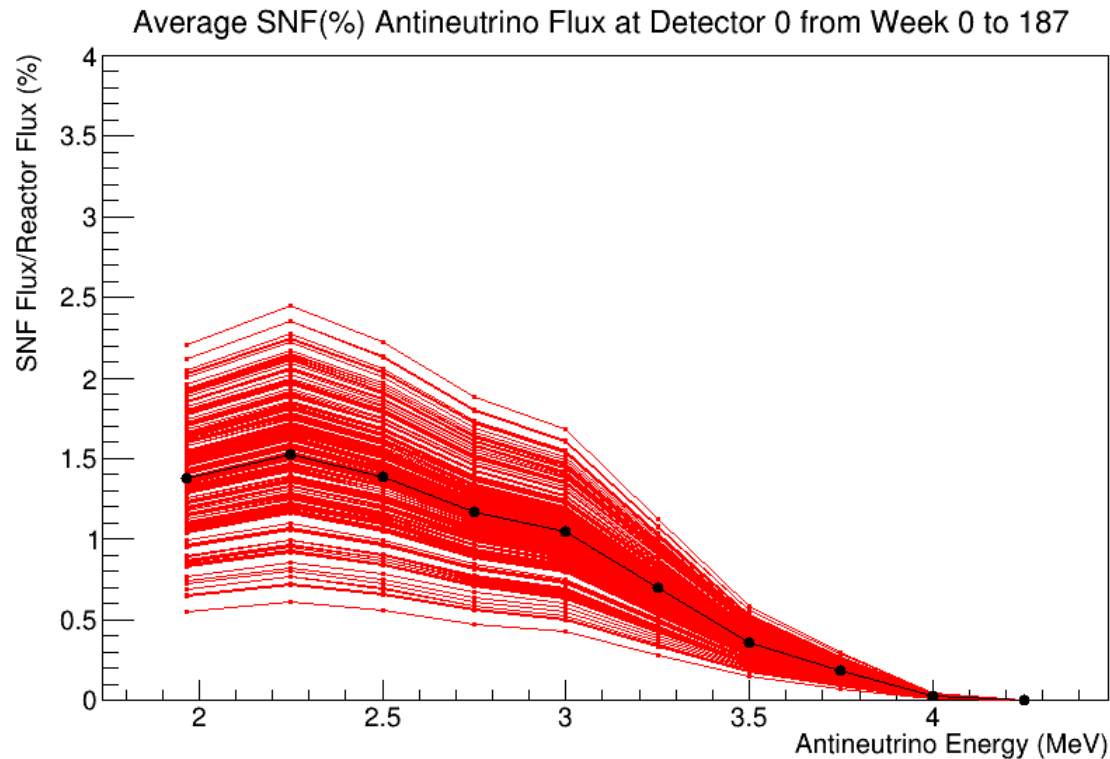
Fuel Fractions Ling Ao 1,2 Reactors



Fuel Fractions Ling Ao 3,4 Reactors



EH1-AD1 SNF spectra with error in burnup α_r



- 225 trials with burnup ratio randomly scaled by $\pm 40\%$
- Spectra averaged over P15A
- RMS of results for each bin calculated
- RMS/Calculated SNF for each energy bin almost identical at $\pm 24\%$
- So 40% uncertainty in α_r scale factor averages down over reactors to $< 25\%$
- Other ADs range from 18% to 25%