

Coherence and Wave Packets in Neutrino Oscillations

Carlo Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica,
Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy

Abstract

General arguments in favor of the necessity of a wave packet description of neutrino oscillations are presented, drawing from analogies with other wave phenomena. We present a wave packet description of neutrino oscillations in stationary beams using the density matrix formalism. Recent claims of the necessity of an equal energy of different massive neutrinos are refuted.

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1 Introduction

The physics of massive and mixed neutrinos is one of the hot topics in today's research in high energy physics, following the discovery of oscillations of atmospheric and solar neutrinos (see the reviews in Refs. [12, 41, 52, 13] and references therein, as well as the references in [40]).

Neutrino oscillations have been proposed in the late 50's by Pontecorvo [66, 67]. The oscillations are generated by the interference of different massive neutrinos, which are produced and detected coherently because of their very small mass difference.

In 1962 Maki, Nakagawa and Sakata [59] considered for the first time a model with mixing of different neutrino flavors. In 1967 Pontecorvo proposed the possibility of solar neutrino oscillations [68], before the discovery in 1968 of the solar neutrino problem in the Homestake experiment [21]. In 1969 Pontecorvo and Gribov considered $\nu_e \rightarrow \nu_\mu$ oscillations as a possible explanation of the solar neutrino problem [42].

The theory of neutrino oscillations in the plane-wave approximation was developed in the middle 70's by Eliezer and Swift [26], Fritzsche and Minkowski [27], Bilenky and Pontecorvo [14], and beautifully reviewed by Bilenky and Pontecorvo in Ref. [15].

In 1976 Nussinov [63] for the first time considered the wave packet nature of propagating neutrinos and inferred the existence of a coherence length, beyond which the interference of different massive neutrinos is not observable. This is due to the different group velocities of different massive neutrinos, that causes a separation of their wave packets. In 1996 Kiers, Nussinov and Weiss [53] first pointed out the importance of the detection process for the coherence of neutrino oscillations and discussed some implications for the wave packet approach.

In 1981 Kayser [51] presented the first detailed discussion of the quantum mechanical problems of neutrino oscillations, pointing out the necessity of a wave packet treatment.

Wave packet models of neutrino oscillations have been later developed in the framework of quantum mechanics [37, 38, 24, 34, 22, 23, 33] and in the framework of quantum field theory [36, 39, 54, 18, 10, 32] (see also Ref. [55] and the reviews in Refs. [74, 11]).

In spite of the well-known fact that in quantum theory localized particles are described by wave packets and in spite of the success of the wave packet treatment of neutrino oscillations, some authors have presented arguments against such approach. In Section 2 we briefly review well-known evidences of the wave packet nature of light, which, by analogy, imply a wave packet nature of neutrinos. In Sections 3 and 4 we present objections to arguments against a wave packet description of neutrinos. In Section 4 we present also a wave packet derivation of neutrino oscillations in the density matrix formalism and we discuss some implications for the case of stationary beams.

Another debated problem in the theory of neutrino oscillations is the determination of the energies of massive neutrinos. In Section 5 we present objections to recently proposed arguments in favor of the equality of the energies of different massive neutrinos.

2 No source of waves vibrates indefinitely

The title of this section is taken from the beginning sentence of Section 11.11 of Ref. [49], which concludes with the following paragraph:

In light sources, the radiating atoms emit wave trains of finite length. Usually, because of collisions or damping arising from other causes, these packets are very short. According to the theorem mentioned above¹, the consequence is that the spectrum lines will not be very narrow but will have an appreciable width $\Delta\lambda$. A measurement of this width will yield the effective “lifetime” of the electromagnetic oscillators in the atoms and the average length of the wave packets. A low-pressure discharge through the vapor of mercury containing the single isotope ^{198}Hg yields very sharp spectral lines, of width about 0.005 \AA . Taking the wavelength of one of the brightest lines, 5461 \AA , we may estimate that there are roughly 10^6 waves in a packet and that the packets themselves are some 50 cm long.

The broadening of optical lines due to the finite lifetime of atomic transitions is known as *natural linewidth*.

It is interesting to note that the broadening of optical lines was well known to experimental physicists in the nineteenth century and explained by classical models before the advent of quantum theory (see Ref. [17] and references therein). In 1895 Michelson [60] listed among the hypotheses formulated before that time to account for line broadening

3. The exponential diminution in amplitude of the vibrations due to communication of energy to the surrounding medium or to other causes.

As explained in [17],

We consider an emitting atom which we shall proceed to remove to infinity and reduce the “temperature” to the point where, classically at least, no translational motion exists. Now from the classical picture of a vibrating electron or the simple picture of a pair of energy levels between which our radiation transition takes place, we should expect these conditions to yield a spectral line of a single frequency. We, of course, do not obtain this result, but, rather, we obtain the familiar natural line shape which is attributable to Michelson’s Cause 3.

A classical derivation of the natural linewidth can be found in Section 17.7 of Ref. [48]. Let us emphasize that the natural linewidth of atomic lines has been *observed experimentally* (see Section 21.4 of Ref. [72])!

Another important cause of line broadening known in 1895 was [60]

4. The change in wavelength due to the Doppler effect of the component of the velocity of the vibrating atom in the line of sight.

The Doppler line broadening due to the thermal motion of atoms in a medium was calculated by Rayleigh in 1889 [69]. This important effect, that must be always taken into account in calculating the spectral shape of monochromatic beams, does not concern us here, because it does not generate a coherent broadening. It is simply due to the different motion of different atoms, whose radiation is incoherent.

To these (and other) causes of line broadening Michelson added in 1895 [60]

¹“The largest the number N of waves in the group, the smaller the spread $\Delta\lambda$, and in fact theory shows that $\Delta\lambda/\lambda_0$ is approximately equal to $1/N$.” (Section 11.11 of Ref. [49].)

5. The limitation of the number of regular vibrations by more or less abrupt changes of phase amplitude or plane of vibration caused by collisions.

This important coherent effect has been called with several names, among which *collision broadening*, *pressure broadening* and *interruption broadening*, it has been studied in depth by many authors (see, for example, Refs.[17, 58]) and it has been *observed experimentally* (see Section 21.5 of Ref. [72])!

In quantum theory, the fact that “no source of waves vibrates indefinitely” implies that all particles are produced as wave packets, whose size is determined by the finite lifetime of the parent particle, or by its finite mean free path if the production process occurs in a medium. Since also no wave detector vibrates indefinitely, it is clear that all particles are also detected as wave packets.

Usually, at least in high energy physics, the wave packet character of particles is not important and they can be well approximated by plane waves. The phenomenon of neutrino oscillations is an exception, as shown in 1981 by Kayser [51], because the localization of the source and detector requires a wave packet treatment.

The wave packet approach to neutrino oscillations implemented in Refs. [37, 38, 24, 34, 22, 33] in the framework of quantum mechanics and in Refs. [36, 39, 54, 18, 10, 32] in the framework of quantum field theory allows to derive the neutrino oscillation probability from first principles in a consistent framework.

In the light of these considerations, it seems rather surprising that some physicists do not agree with a wave packet description of neutrinos and put forward arguments in favor of a unique value of neutrino energy. A tentative explanation of this approach stems from the common education and practice in physics of working with energy eigenstates, forgetting that they are only approximations of the states of physical systems in the real world². In the following Sections we present a critical discussion of recently advocated approaches to neutrino oscillations in which neutrinos are not described by wave packets with some spread in momentum and energy.

3 Bound states

Bound states of atomic electrons are usually calculated with different degrees of approximation taking into account the static potential generated by the nucleus and other electrons and neglecting the coupling of the electron with the electromagnetic radiation field. In this approximation, bound states are stationary and have definite energy. In spite of the obvious fact that in such approximation electrons cannot jump from one bound state to another, the energy difference between two stationary bound states is sometimes associated without any further explanation with the energy of the photon emitted or absorbed in the transition of an electron from one bound state to the other.

However, in order to describe the transitions between bound states it is necessary to take into account the coupling of the electron with the time-dependent electromagnetic radiation field. Since the bound states are not eigenstates of the full Hamiltonian, in reality they are not stationary and they do not have definite energy. The lack of a definite energy is a necessary requirement for the existence of transitions: since the

²I would like to thank I. Pesando for an illuminating discussion about this point.

Schrödinger equation implies that the time evolution of energy eigenstates evolve by a phase, orthogonality between different energy eigenstates is conserved in time, excluding transitions.

The fact that unstable systems do not have a definite energy is well-known to high energy physicists from their experience with resonances (hadronic excited states) and highly unstable particles (Z and W bosons), which do not have a definite mass (energy in the rest frame). Their mass width is inversely proportional to the decay rate.

From these considerations it follows that the quantum field theoretical model of neutrino oscillations in the process

$$n \rightarrow p + e^- + \bar{\nu}_e \xrightarrow{\bar{\nu}_e \rightarrow \bar{\nu}_e} \bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^- \quad (3.1)$$

presented in Ref. [44], in which a neutrino is assumed to be produced by a nuclear β -decay between two stationary bound states with definite energy and detected through scattering with an electron in a stationary atomic state with definite energy, is unrealistic, not to mention the description of the final-state electrons by plane waves, which is in contradiction with the fact that the electrons interact with the surrounding medium and with the necessity to observe the final lepton in the detection process, as already noted in [32]. Of course, once the matters of principle are clear, one can consider the assumptions in Ref. [44] as approximations acceptable in some cases. One must also notice that Ref. [44] is very interesting for the useful theorem proved in the appendix, which has been used in the quantum field theoretical calculation of neutrino oscillations in the wave packet approach in Refs. [39, 10].

Developing the technique proposed in Ref. [44], the authors of Refs. [45, 43] considered neutrino oscillations in the process

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \xrightarrow{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} \bar{\nu}_e + p \rightarrow n + e^+ \quad (3.2)$$

with the muon correctly described by a wave packet, but the proton was still described by an unrealistic stationary bound state with definite energy. Furthermore, all final particles were described by plane waves, in contradiction with the fact that the positrons and neutron interact with the surrounding medium and the necessity to observe the final positron in the detection process in order to measure oscillations. Localized interactions of a particle with a detector obviously imply that the particle cannot be described by an unlocalized plane wave and a wave packet description is necessary.

4 Stationary beams

The author of the very interesting Ref. [70] wrote:

... many wavepacket discussions for the coherence properties of particle beams are unnecessary since they deal with stationary sources; and when the problem is stationary, essentially all information is in the energy spectrum.

Indeed, as proved in Ref. [53],

... under very general assumptions it is not possible to distinguish experimentally neutrinos produced in some region of space as wave packets from those

produced in the same region of space as plane waves with the same energy distribution.

It is not clear if the purpose of Ref. [70] was to show that neutrinos are not described by wave packets, but it is certain that it has been interpreted in this way by some physicists. However, Ref. [70] certainly does not provide any help for the derivation of neutrino oscillations in the framework of quantum field theory. Even in the framework of quantum mechanics Ref. [70] does not go beyond the well-known discussion of neutrino oscillations assuming equal energies for the different massive neutrino components [57, 46], which has been shown to be unrealistic in Refs. [35, 31] and is further criticized in the Section 5.

An interpretation of Ref. [70] as a proof that neutrinos are not described by wave packets seems to stem from a confusion between microscopic and macroscopic stationarity, as already remarked in [18, 35]. Since most neutrino sources emit neutrinos at a constant rate over macroscopic intervals of time, they can be considered macroscopically stationary. However, the microscopic processes of neutrino production (nuclear or particle decay, etc.) are certainly not stationary and the single neutrino does not know if it will be part of a stationary beam or not. Because of the localization in space and time of the production and detection processes, a description of neutrino oscillations in terms of wave packets is inescapable.

Of course, after the matters of principle are settled, neutrino oscillations are derived from first principles in a consistent theoretical framework, and an estimation of the coherence length has shown that it is much longer than the source-detector distance in a neutrino oscillation experiment, the analysis of the experimental data can be performed using the standard oscillation probability obtained in the plane wave approximation. This turns out to be the case in all present-day neutrino oscillation experiments.

In the following part of this section we present a wave packet derivation of neutrino oscillations in the density matrix formalism, which is suitable for the description of a stationary beam, as emphasized in Ref. [70]. Our calculation follows a method similar to the one presented in Ref. [38].

In a quantum-mechanical wave packet treatment of neutrino oscillations a neutrino produced at the origin of the space-time coordinates by a weak interaction process with definite flavor α ($\alpha = e, \mu, \tau$) and propagating along the x axis is described by the state

$$|\nu_\alpha(x, t)\rangle = \sum_k U_{\alpha k}^* \psi_k(x, t) |\nu_k\rangle, \quad (4.1)$$

where U is the mixing matrix, $|\nu_k\rangle$ is the state of a neutrino with mass m_k and $\psi_k(x, t)$ is its wave function. Approximating the momentum distribution of the massive neutrino ν_k with a gaussian,

$$\psi_k(p) = \left(2\pi\sigma_p^{\text{P}2}\right)^{-1/4} \exp\left[-\frac{(p-p_k)^2}{4\sigma_p^{\text{P}2}}\right], \quad (4.2)$$

where p_k is the average momentum and σ_p^{P} is the momentum uncertainty determined by the production process, the wave function is

$$\psi_k(x, t) = \int \frac{dp}{\sqrt{2\pi}} \psi_k(p) e^{ipx - iE_k(p)t}, \quad (4.3)$$

with the energy $E_k(p)$ given by³

$$E_k(p) = \sqrt{p^2 + m_k^2}. \quad (4.4)$$

If the gaussian momentum distribution (4.2) is sharply peaked around the mean momentum p_k (*i.e.* if $\sigma_p^P \ll E_k^2(p_k)/m_k$), the energy $E_k(p)$ can be approximated by

$$E_k(p) \simeq E_k + v_k (p - p_k), \quad (4.5)$$

where

$$E_k \equiv E_k(p_k) = \sqrt{p_k^2 + m_k^2} \quad (4.6)$$

is the average energy and

$$v_k \equiv \left. \frac{\partial E_k(p)}{\partial p} \right|_{p=p_k} = \frac{p_k}{E_k}. \quad (4.7)$$

Under this approximation, the integration over p in Eq. (4.3) is gaussian and leads to the wave packet in coordinate space

$$\psi_k(x, t) = \left(2\pi\sigma_x^{P^2}\right)^{-1/4} \exp \left[-iE_k t + ip_k x - \frac{(x - v_k t)^2}{4\sigma_x^{P^2}} \right], \quad (4.8)$$

where

$$\sigma_x^P = \frac{1}{2\sigma_p^P} \quad (4.9)$$

is the spatial width of the wave packet. From Eq. (4.8) one can see that v_k is the group velocity of the wave packet of the massive neutrino ν_k .

The pure state in Eq. (4.1) can be written in the form of a density matrix operator as

$$\hat{\rho}_\alpha(x, t) = |\nu_\alpha(x, t)\rangle \langle \nu_\alpha(x, t)|. \quad (4.10)$$

Using Eq. (4.8), we have

$$\begin{aligned} \hat{\rho}_\alpha(x, t) = \frac{1}{\sqrt{2\pi\sigma_x^{P^2}}} \sum_{k,j} U_{\alpha k}^* U_{\alpha j} \exp \left[-i(E_k - E_j)t + i(p_k - p_j)x \right. \\ \left. - \frac{(x - v_k t)^2}{4\sigma_x^{P^2}} - \frac{(x - v_j t)^2}{4\sigma_x^{P^2}} \right] |\nu_k\rangle \langle \nu_j|. \end{aligned} \quad (4.11)$$

This space and time dependent density matrix operator describes neutrino oscillations in space and time. Although in laboratory experiments it is possible to measure neutrino oscillations in time through the measurement of both the production and detection processes (see Ref. [64])⁴, in all existing neutrino oscillation experiments only the source-detector distance is known. In this case the relevant density matrix operator $\hat{\rho}_\alpha(x)$ is

³We do not make unnecessary assumptions about the values of the energies and momenta of different massive neutrinos. The claim that different massive neutrinos must have the same energy has been confuted in Refs. [35, 31] and is further discussed in Section 5.

⁴Let us also mention the possibility of time-dependent flavor oscillations of neutrinos in the primordial cosmological plasma (see Ref. [23] and references therein).

given by the time average of $\hat{\rho}_\alpha(x, t)$. Since the integral over time is gaussian, one easily obtains⁵

$$\hat{\rho}_\alpha(x) = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} \exp \left\{ -i \left[\frac{v_k + v_j}{v_k^2 + v_j^2} (E_k - E_j) - (p_k - p_j) \right] x - \frac{(v_k - v_j)^2 x^2}{4 (v_k^2 + v_j^2) \sigma_x^{\text{P}2}} - \frac{(E_k - E_j)^2}{4 (v_k^2 + v_j^2) \sigma_p^{\text{P}2}} \right\} |\nu_k\rangle \langle \nu_j|. \quad (4.12)$$

Since this density matrix operator is time independent, it is suitable for the description of a stationary beam in neutrino oscillation experiments, as noticed in Ref. [70].

In order to obtain the oscillation probability, it is convenient to simplify Eq. (4.12) for the realistic case of extremely relativistic neutrinos (see the discussion in Section 2 of Ref. [32]). In general, the average massive neutrino energies E_k can be written as

$$E_k \simeq E + \xi_{\text{P}} \frac{m_k^2}{2E}, \quad (4.13)$$

where E is the neutrino energy in the limit of zero mass and ξ_{P} is a dimensionless quantity that depends from the characteristics of the production process⁶. From Eq. (4.4), the corresponding momentum in the relativistic approximation is

$$p_k \simeq E - (1 - \xi_{\text{P}}) \frac{m_k^2}{2E}. \quad (4.14)$$

The generality of these relations can be understood by noting that, since the neutrino mass m_k enters quadratically in the energy-momentum dispersion relation (4.4), its the first order contribution to the energy and momentum must be proportional to m_k^2 and inversely proportional to E , which is the only available quantity with dimension of energy.

Using Eqs. (4.13), (4.14) and the relativistic approximation

$$v_k \simeq 1 - \frac{m_k^2}{2E_k^2}, \quad (4.15)$$

we obtain

$$\hat{\rho}_\alpha(x) = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} \exp \left[-i \frac{\Delta m_{kj}^2 x}{2E} - \left(\frac{\Delta m_{kj}^2 x}{4\sqrt{2}E^2 \sigma_x^{\text{P}}} \right)^2 - \left(\xi_{\text{P}} \frac{\Delta m_{kj}^2}{4\sqrt{2}E \sigma_p^{\text{P}}} \right)^2 \right] |\nu_k\rangle \langle \nu_j|, \quad (4.16)$$

with $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$.

⁵The density matrix operator in Eq. (4.12) is normalized by $\sum_\alpha \hat{\rho}_\alpha(x) = \hat{I}$, where \hat{I} is the identity operator.

⁶ The quantity ξ_{P} cannot be calculated in a quantum mechanical framework, but its value can be estimated from energy-momentum conservation in the production process. The calculation of the value of ξ_{P} requires a quantum field theoretical treatment (see Ref. [32]).

In analogy with the production process, we describe the process of detection of a neutrino with flavor β at the coordinate $x = L$ with the operator

$$\hat{\mathcal{O}}_\beta(x - L) = \sum_{k,j} U_{\beta k}^* U_{\beta j} \times \exp \left[-i \frac{\Delta m_{kj}^2 (x - L)}{2E} - \left(\frac{\Delta m_{kj}^2 (x - L)}{4\sqrt{2}E^2 \sigma_x^D} \right)^2 - \left(\xi_D \frac{\Delta m_{kj}^2}{4\sqrt{2}E \sigma_p^D} \right)^2 \right] |\nu_k\rangle \langle \nu_j|, \quad (4.17)$$

where ξ_D is a dimensionless quantity that depends from the characteristics of the detection process (see footnote 6), σ_p^D is the momentum uncertainty of the detection process and $\sigma_x^D = 1/2\sigma_p^D$.

The probability of $\nu_\alpha \rightarrow \nu_\beta$ transitions is given by

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \text{Tr} \left(\hat{\rho}_\alpha(x) \hat{\mathcal{O}}_\beta(x - L) \right) = \int dx \sum_k \langle \nu_k | \hat{\rho}_\alpha(x) \hat{\mathcal{O}}_\beta(x - L) | \nu_k \rangle, \quad (4.18)$$

which leads to

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} U_{\beta k} U_{\beta j}^* \exp \left[-2\pi i \frac{L}{L_{kj}^{\text{osc}}} - \left(\frac{L}{L_{kj}^{\text{coh}}} \right)^2 - 2\pi^2 \xi^2 \left(\frac{\sigma_x}{L_{kj}^{\text{osc}}} \right)^2 \right], \quad (4.19)$$

with the oscillation and coherence lengths

$$L_{kj}^{\text{osc}} = \frac{4\pi E}{\Delta m_{kj}^2}, \quad (4.20)$$

$$L_{kj}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{kj}^2|} \sigma_x, \quad (4.21)$$

and

$$\sigma_x^2 = \sigma_x^{\text{P}^2} + \sigma_x^{\text{D}^2}, \quad (4.22)$$

$$\xi^2 \sigma_x^2 = \xi_{\text{P}}^2 \sigma_x^{\text{P}^2} + \xi_{\text{D}}^2 \sigma_x^{\text{D}^2}. \quad (4.23)$$

This result was already obtained⁷ in a quantum mechanical framework in Ref. [34], where the wave packet treatment with pure states presented several years before in Ref. [37] was extended in order to take into account the coherence properties of the detection process, whose importance was first recognized in Ref. [53]. Expressions for the $\nu_\alpha \rightarrow \nu_\beta$ transition probability similar to Eq. (4.19) have been also obtained with wave packet treatments of neutrino oscillations in a quantum field theoretical framework in Refs. [36, 39, 18, 10, 32].

It is important to remark that the wave packet treatment of neutrino oscillations confirms the standard value in Eq. (4.20) for the oscillation length L_{kj}^{osc} . The coherence length L_{kj}^{coh} in Eq. (4.21) is the distance beyond which the interference of the massive neutrinos ν_k and ν_j is suppressed because the separation of their wave packets when

⁷The only improvement in the expressions above with respect to the analogous ones in Ref. [34] is the introduction of ξ_D , that takes into account the properties of the detection process.

they arrive at the detector is so large that they cannot be absorbed coherently. As shown by Eqs. (4.21) and (4.22) the coherence lengths L_{kj}^{coh} are proportional to the total coherence size, that is dominated by the largest between the coherence sizes σ_x^{P} and σ_x^{D} of the production and detection process. The last term in the exponential of Eq. (4.19) implies that the interference of the massive neutrinos ν_k and ν_j is observable only if the localization of the production and detection processes is smaller than the oscillation length.

The localization term is important for the distinction of neutrino oscillation experiments from experiments on the measurement of neutrino masses. As first shown by Kayser in Ref. [51], neutrino oscillations are suppressed in experiments able to measure the value of a neutrino mass, because the measurement of a neutrino mass implies that only the corresponding massive neutrino is produced or detected.

Kayser's [51] argument goes as follows. Since a neutrino mass is measured from energy-momentum conservation in a process in which a neutrino is produced or detected, from the energy-momentum dispersion relation (4.4), the uncertainty of the mass determination is

$$\delta m_k^2 = \sqrt{(2E_k \delta E_k)^2 + (2p_k \delta p_k)^2} \simeq 2\sqrt{2}E\sigma_p, \quad (4.24)$$

where the approximation holds for realistic extremely relativistic neutrinos and σ_p is the momentum uncertainty. If the mass of ν_k is measured with an accuracy better than Δm_{kj}^2 , *i.e.*

$$\delta m_k^2 < |\Delta m_{kj}^2| \iff \frac{|\Delta m_{kj}^2|}{2\sqrt{2}E\sigma_p} > 1, \quad (4.25)$$

the neutrino ν_j is not produced or detected⁸ and the interference of ν_k and ν_j is not observed.

The localization term in the oscillation probability (4.19) automatically implements Kayser's mechanism. Indeed, it can be written as

$$2\pi^2 \xi^2 \left(\frac{\sigma_x}{L_{kj}^{\text{osc}}} \right)^2 = \xi^2 \left(\frac{\Delta m_{kj}^2}{4\sqrt{2}E\sigma_p} \right)^2, \quad (4.26)$$

with the momentum uncertainty

$$\frac{1}{\sigma_p^2} = 4\sigma_x^2 = \frac{1}{\sigma_p^{\text{P}^2}} + \frac{1}{\sigma_p^{\text{D}^2}}. \quad (4.27)$$

If the condition (4.25) for neutrino mass measurement is satisfied, the localization term (4.26) suppresses⁹ the interference of ν_k and ν_j .

⁸The smallness of σ_p required to satisfy the condition (4.25) implies that ν_k and ν_j cannot be produced or detected in the same process, because their momentum difference is larger than the momentum uncertainty, or their energy difference is larger than the energy uncertainty (for extremely relativistic neutrinos the energy uncertainty is practically equal to the momentum uncertainty σ_p). In this case only one of the two massive neutrinos is produced or detected.

⁹This argument assumes that ξ is not too small. As discussed in Ref. [32], in realistic cases ξ is a number of order one, but, at least in principle, there could be some cases in which ξ is very small, or even zero. In these cases Kayser's mechanism cannot be implemented in the simplified quantum mechanical framework presented here. However, it has been shown in Ref. [10] that a wave packet quantum field theoretical derivation of the neutrino oscillation probability produces additional terms, independent from ξ or similar quantities, which suppress oscillations when the condition (4.25) is satisfied. I would like to thank M. Beuthe for a illuminating discussion about this problem.

In the above derivation of neutrino oscillations the wave-packet description of massive neutrinos is crucial in order to allow the integration over time of the time-dependent density matrix operator $\hat{\rho}_\alpha(x, t)$ in Eq. (4.11), which leads to the time-independent density matrix operator $\hat{\rho}_\alpha(x)$ in Eq. (4.12). Of course, as emphasized in Ref. [70],

A single, given density matrix can arise in different ways, especially when incoherence is involved.

As already noticed in Ref. [53] the density matrix operator $\hat{\rho}_\alpha(x)$ can also be generated through an appropriate incoherent average over the energy spectrum. The same is obviously true for the oscillation probability (4.19).

In order to illustrate this point, let us consider the simplest case of two-neutrino mixing. The wave packet oscillation probability (4.19) becomes

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \frac{1}{2} \sin^2 2\vartheta \left\{ 1 - \cos \left(\frac{\Delta m^2 L}{2E} \right) \exp \left[- \left(\frac{L}{L^{\text{coh}}} \right)^2 - 2\pi^2 \xi^2 \left(\frac{\sigma_x}{L^{\text{osc}}} \right)^2 \right] \right\}, \quad (4.28)$$

with the oscillation and coherence lengths L^{osc} and L^{coh} given, respectively, by Eqs. (4.20) and (4.21) with $\Delta m_{kj}^2 = \Delta m^2$ (Δm^2 is the squared-mass difference and ϑ is the mixing angle; see the reviews in Refs. [12, 41, 52, 13]). An incoherent average of the probability in the plane-wave approximation over a gaussian energy spectrum with width σ_E reads

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{\text{incoh}}(L) = \frac{1}{2} \sin^2 2\vartheta \left\{ 1 - \int \frac{dE'}{\sqrt{2\pi}\sigma_E} \cos \left(\frac{\Delta m^2 L}{2E'} \right) \exp \left[- \left(\frac{(E - E')^2}{2\sigma_E^2} \right)^2 \right] \right\}. \quad (4.29)$$

Figure 1 shows the values of the probabilities (4.29) and (4.28) as functions of the source-detector distance L for $\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\vartheta = 1$, $E = 10 \text{ GeV}$, and $\sigma_E = \sigma_p = 1 \text{ GeV}$. One can see that it is hard to distinguish the dashed line obtained with Eq. (4.29) from the solid line obtained with Eq. (4.28).

This result does not mean that wave packet effects are irrelevant¹⁰. It means that in practice one can calculate with reasonable approximation the decoherence of oscillations due to wave packet effects either taking into account the momentum spread in the calculation of the amplitude, as done in the derivation of Eq. (4.28), or averaging the probability over the same energy spread, as done in the derivation of Eq. (4.29)¹¹. As emphasized in Ref. [53], it also means that if one does not have control on both the production and detection processes, one cannot know if oscillations are suppressed because of incoherent averaging over energy of different microscopic processes or because of decoherence due to the separation of wave packets. However, having good control of both the production and detection processes, it may be possible in the far future to reduce the causes of incoherent broadening of the energy spectrum and prove experimentally that oscillations can

¹⁰Let us clarify here that with “wave packet effects” we mean effects due to the momentum and energy uncertainty of a single process associated with the interruption of the emitted wave train caused by decay (natural linewidth) or collisions (collision broadening) discussed in Section 2. These effects are usually not taken into account in the calculation of neutrino energy spectra (see, for example, Ref. [8]).

¹¹One must only be careful to notice that the energy distribution in the incoherent average of the probability must be normalized to one, whereas the squared modulus of the momentum distribution of the wave packets must be normalized to one.

be suppressed because of wave packet effects (in practice using an approach similar to the one adopted for the measurement of the natural linewidth and collision broadening of atomic lines discussed in Section 2).

Let us finally mention that the fact that coherent and incoherent stationary beams are indistinguishable without a theoretical analysis is well known in optics (see Section 7.5.8 of Ref. [16]), in neutron interferometry [20, 50] and in general stationary particle beams [9].

5 Energy of massive neutrinos

In the recent Ref. [56] the claim that different massive neutrinos must have the same energy [57, 46], which was refuted in Refs. [35, 31], has been renewed. If such claim were correct, it would mean that, at least in a quantum mechanical framework, a wave packet description of neutrinos is not necessary. Indeed, in this case one can construct a time-independent density matrix operator from the plane wave state

$$|\tilde{\nu}_\alpha(x, t)\rangle = \sum_k U_{\alpha k}^* e^{ip_k x - iEt} |\nu_k\rangle, \quad (5.1)$$

with $p_k = \sqrt{E^2 - m_k^2}$:

$$\hat{\rho}_\alpha(x) = |\tilde{\nu}_\alpha(x, t)\rangle \langle \tilde{\nu}_\alpha(x, t)| = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} \exp[i(p_k - p_j)x]. \quad (5.2)$$

Using this density matrix operator one can easily derive the standard oscillation probability without need of neutrino wave packets. Therefore, in this section we consider recent claims in favor of an equal energy of different massive neutrinos and show that they are faulty.

The author of Ref. [56] wrote:

...states with different ENERGIES ARE NEVER COHERENT in any realistic experiment. States of the same energy and different momenta can be coherent, but may not be.

The reason offered by the author of Ref. [56] is

The usual detector is a nucleon, which changes its state after absorbing a neutrino and emitting a charged lepton, and is initially either in an energy eigenstate or in a statistical mixture in thermal equilibrium with its surroundings. No neutrino detector has ever been prepared in a coherent mixture of energy eigenstates and no such detector has been proposed for future experiments.

First, one must note that in terminology of Ref. [56] “detector” is not a macroscopic device, but a nucleon hit by a neutrino. Then, the claim in Ref. [56] is easily proved to be wrong using the arguments presented in Section 2, where we have shown that all kinds of waves, classical or quantistic, can be emitted or absorbed only as wave packets, *i.e.* as coherent superpositions of plane waves with different frequency (energy). If the claim

in Ref. [56] were correct, photons, as well as other particles, could never be emitted or absorbed. The claim that a nucleon should be initially in an energy eigenstate has been shown to be incorrect in Section 3. Furthermore, it is a mystery why all the claims on energy in Ref. [56], if they were true, do not apply also to momentum.

Also the author of Ref. [70] claimed that different massive neutrinos have the same energy:

Energy or momentum? In mixing problems, where we have to deal with linear combinations of particles of different mass, the question comes up as to whether one should deal with states of the same energy or the same momentum. Since as stated above, for stationary conditions we are to perform the calculation as an incoherent sum over energies, we have given the answer “energy”. Evidently, for stationary problems it is most natural to use stationary wave functions $\sim e^{-iEt}$.

The meaning of this paragraph seems to be that different massive neutrinos are supposed to manage to have equal energy in order to satisfy the calculational needs of some theoretical physicists. Therefore, we cannot consider it a proof of anything.

A simple way to show the absurdity of the claims that massive neutrinos must have equal energies is to consider the transformation properties of energy and momentum from one inertial frame to another, following the discussion presented in Ref. [31].

If the arguments presented in Ref. [56] were correct, in order to be produced and detected coherently massive neutrinos should necessarily have exactly equal energies in the inertial reference frames of both the production and detection processes (which presumably coincide with the rest frames of the initial nucleons in the production and detection processes, in the approach of Ref. [56]; let us skip the useless discussion of the determination of the relevant inertial reference frame in neutrino-electron scattering). Such a requirement would imply that the production and detection processes have to be at rest in exactly the same inertial system, because, as shown in Ref. [31] the equal-energy requirement is not Lorentz invariant.

Therefore if the arguments presented in Ref. [56] were correct it would mean that in practice neutrino oscillations are not observable, since it is practically impossible to suppress random thermal motion in the source and detector. Certainly, oscillations of solar and atmospheric neutrinos would be impossible. Let me remind that in this discussion we are concerned with the oscillatory terms in the flavor transition probability (4.19), due to the interference of different massive neutrinos, which require coherence. If these terms are suppressed, it is still possible to measure the distance-independent and energy-independent flavor-changing probability

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2, \quad (5.3)$$

or the distance-independent flavor-changing probability calculated in Ref. [65] if matter effects are important [73, 61, 62]. According to the results of solar neutrino experiments [19, 47, 6, 1, 28, 2, 3], the solar neutrino problem is very likely due to neutrino oscillations in matter, in which the distance-independent flavor-changing probability calculated in Ref. [65] is relevant. However, recently the KamLAND experiment [25] observed indications in favor of distance and energy dependent oscillations in vacuum of reactor

antineutrinos due to the same mass-squared difference responsible of solar neutrino oscillations. Distance and energy dependent flavor-changing transitions have been also observed in the Super-Kamiokande [29], Soudan 2 [5] and MACRO [7] atmospheric neutrino experiments and in the long-baseline experiment K2K [4]. These experimental evidences show without any doubt that the arguments presented in Ref. [56] in favor of an equal energy for different massive neutrinos are wrong.

Indeed, if we consider for example atmospheric neutrinos produced by decays in the atmosphere of highly energetic pions and muons, the rest frames of the source and detector are very different and different massive neutrinos cannot have the same energy in both frames. If the arguments presented in Ref. [56] were correct different massive neutrinos could not be either produced or detected coherently, in contradiction with the observed oscillations. The same reasoning applies to reactor and accelerator neutrinos, remembering that according to the arguments presented in Ref. [56] coherence is possible only if the source and detector rest frames coincide *exactly* (which, by the way, is an absurd concept, if velocity is a real-valued quantity).

The only rigorous and correct calculation which could be erroneously interpreted¹² as a proof in favor of the equal energy assumption has been presented in Ref. [10] (see also Section 2.5 of Ref. [11]). The argument can be illustrated with the density matrix model of neutrino oscillations presented in Section 4. Instead of writing the density matrix operator (4.10) in the form (4.11) using the wave packets (4.8) in which the integration over the momentum p in Eq. (4.3) has been already performed, one could write the density matrix operator (4.10) using the wave functions (4.3) as

$$\hat{\rho}_\alpha(x, t) = \sum_{k,j} U_{\alpha k}^* U_{\alpha j} \int \frac{dp dp'}{2\pi} \psi_k(p) \psi_j(p') e^{i(p-p')x - i(E_k(p) - E_j(p'))t} |\nu_k\rangle \langle \nu_j|. \quad (5.4)$$

As in Section 4, the time-independent density matrix operator that describes a stationary beam is given by the average over time of $\hat{\rho}_\alpha(x, t)$. The integral over t of $\exp[-i(E_k(p) - E_j(p'))t]$ yields a $\delta(E_k(p) - E_j(p'))$, which means that “interference occurs only between wave packet components with the same energy” [11] (see Refs.[30, 71] for similar results in electron interferometry and kaon oscillations). This means that the time average of $\hat{\rho}_\alpha(x, t)$ is equivalent to an appropriate incoherent average over the contributions of the single-energy components of the wave packets. This is the reason why coherent and incoherent stationary beams cannot be distinguished without a theoretical analysis, as discussed in Section 4.

Let us emphasize, however, that this argument does not mean that different massive neutrinos have the same energy, or that wave packets are unnecessary. The wave packet nature of massive neutrinos is necessary for the existence of components with the same energy that produce the observable interference, and the average energies of the wave packets of different massive neutrino are in general different.

¹²Indeed, I made such mistake in the first version of this paper appeared in the **hep-ph** electronic archive. I am deeply indebted to M. Beuthe for his comments that helped to correct this mistake and my wrong criticisms of Refs. [10, 11].

6 Conclusions

Starting from the well-known fact that “no source of waves vibrates indefinitely”, we have argued that neutrinos, as all other particles, are naturally described by wave packets. This is in agreement with the well known fact that in quantum theory localized particles are described by wave packets. Even when neutrinos are produced or detected through interactions with particles in bound states, they are described by wave packets, because bound states in interaction with an external field do not have a definite energy.

Since the production and detection processes in neutrino oscillation experiments are localized in space-time, a consistent description of neutrino oscillations requires a wave packet treatment. In particular, the wave packet character of massive neutrinos is crucial for the derivation of neutrino oscillations in space, because the group velocity establishes a connection between space and time and allows the time average of the space and time dependent probability (or density matrix).

Finally, we have shown that the claimed arguments in favor of an equal energy of massive neutrinos (which could allow a quantum mechanical derivation of neutrino oscillations without wave packets) are in contradiction with well known physical laws and phenomena.

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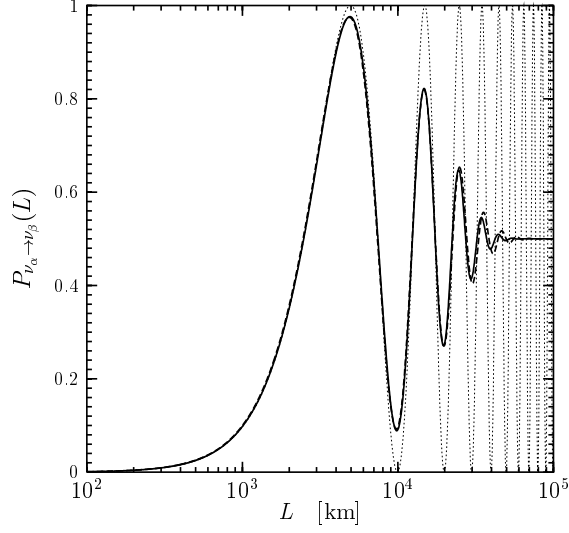


Figure 1: Oscillation probability as a function of distance L for $\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\vartheta = 1$, and $E = 10 \text{ GeV}$. Dotted line: Unsuppressed and unaveraged oscillation probability. Dashed line: Oscillation probability (4.29) averaged over a gaussian energy spectrum with width $\sigma_E = 1 \text{ GeV}$. Solid line: Oscillation probability (4.28) suppressed by decoherence due to a momentum uncertainty $\sigma_p = 1 \text{ GeV}$.