# Effect of Electron Beam Polarization Asymmetry on Parity Experiments

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July 16, 2021

**Abstract**

Future planned parity experiments MOLLER and SOLID require beam polarization at the 0.4% uncertainty level, requiring a careful look at many effects considered negligible for most experiments. In this document I look at the effect of a beam polarization asymmetry on the physics result. Parity violation (PV) experiments normalize their measured asymmetries to the beam polarization to give the PV physics asymmetry i.e. in the absence of backgrounds, $A^{PV} = A_{meas}/P$. This assumes that the beam polarization is equal in both helicity states and that only the sign flips. Furthermore, the Compton, Møller and to a lesser extent Mott polarimeters rely on this same assumption. I demonstrate that this assumption introduces a small but potentially significant error into the Møller and Compton results but that the change in the measured parity violating (PV) asymmetry $A^{PV}_{meas}$ result is negligible since the error includes a factor of $A^{PV}$ which is typically of order ppm or less. **This means that there is no serendipitous cancellation of the error in the polarization by a similar term in the measured parity asymmetry $A^{PV}_{meas}$. Therefore, it is recommended that we at least place an upper limit on this potential difference.**

## 1 Error in Absolute Polarization

Møller and Compton polarimeters, which are set up to optimally measure longitudinal beam polarization, assume that both beam helicity states have equal polarization and that only a sign flip occurs allowing an accurate polarization by forming an asymmetry independent where common factors of current and cross section cancel. The spin-dependent cross section for longitudinal polarization of target and beam can be written as

$$\sigma_{\pm} = \sigma_0(1 \pm A_{zz}), \tag{1}$$

where $\sigma_0$ is the unpolarized cross section and $A_{zz}$ is the longitudinal spin-dependent fraction of the cross section. The measured longitudinal spin-dependent scattering rate will depend on beam and target polarization, $P_{beam}$ and $P_{targ}$, so that the scattering rate measured in a given helicity state $R_{\pm}$ is given by

$$R_{\pm} = R_0^{pol}(1 \pm P_{targ} P_{beam} A_{zz}),$$

where $R_0^{pol}$ is the average rate over both helicity states. Now we can see that the beam polarization can be measured from the scattering asymmetry as follows:

$$A_{meas}^{pol} = \frac{R_+ - R_-}{R_+ + R_-} = P_{targ} P_{beam} A_{zz}. \tag{2}$$

Now assume that instead of a single $P_{beam}$ we have a helicity dependent polarization

$$P_{beam}^{\pm} = P_{beam}(1 \pm \Delta).$$

Equation 2 now becomes

$$\begin{aligned} \tilde{A}_{meas}^{pol} &= \frac{R_+ - R_-}{R_+ + R_-} \\ &= \frac{(1 + P_{targ} P_{beam} A_{zz}(1 + \Delta)) - (1 - P_{targ} P_{beam} A_{zz}(1 - \Delta))}{(1 + P_{targ} P_{beam} A_{zz}(1 + \Delta)) + (1 - P_{targ} P_{beam} A_{zz}(1 - \Delta))} \\ &= \frac{P_{targ} P_{beam} A_{zz}}{1 + P_{targ} P_{beam} A_{zz} \Delta} \\ &\approx P_{targ} P_{beam} A_{zz}(1 - P_{targ} P_{beam} A_{zz} \Delta). \end{aligned} \tag{3}$$

So $\tilde{A}_{meas}^{pol} \approx A_{meas}^{pol}(1 - P_{targ} P_{beam} A_{zz} \Delta)$ and the relative error comes in at leading order as $P_{targ} P_{beam} A_{zz} \Delta$.[1] For Møller polarimetry $A_{zz} \approx 7/9$ and $P_{targ} \approx 0.08$, so $\Delta$ =1% gives rise to an error in beam polarization of 0.06%. For Compton polarimeter where $A_{zz}$ is energy dependent but can be as high as 30% at 11 GeV and $P_{targ} \approx P_{beam} \approx 1.0$, the polarization error is as large as 0.3% for $\Delta$ =1%.

# 2 Error in $A^{PV}$

Although the error in beam polarization from the $\Delta$ term may not be completely negligible, I show here that it largely cancels in the scattering asymmetries typically used in PV experiments. The spin dependent scattering cross section for longitudinally polarized electrons on an unpolarized target is given as

$$\sigma = \sigma_0(1 \pm A^{PV}), \tag{4}$$

where the spin-dependent fraction $A^{PV}$ is parity violating. This translates into a helicity dependent scattering rate which depends on the beam polarization given as

$$R_{\pm}^{PV} = R_0^{PV}(1 \pm P_{beam} A^{PV}). \tag{5}$$

[1]This result disagrees with the calculation in [1] by a factor of $P_{beam}$.

Therefore, the measured PV asymmetry assuming no backgrounds is related to $A^{PV}$ as

$$\begin{aligned} A^{PV}_{meas} &= \frac{R^{PV}_{+} - R^{PV}_{-}}{R^{PV}_{+} + R^{PV}_{-}} \\ &= A^{PV} P_{beam}. \end{aligned} \tag{6}$$

Now if you perform the experiment with the polarization different for each helicity i.e. $P^{\pm}_{beam} = P_{beam}(1 \pm \Delta)$, you will measure a slightly different asymmetry

$$\begin{aligned} \tilde{A}^{PV}_{meas} &= \frac{P_{beam} A^{PV}}{1 + P_{beam} A^{PV} \Delta} \\ &\approx A^{PV} P_{beam}(1 - P_{beam} A^{PV} \Delta) \end{aligned} \tag{7}$$

Therefore, the final result is

$$A^{PV}_{meas} = \tilde{A}^{PV}_{meas}(1 + P_{beam} A^{PV} \Delta).$$

Thus I conclude that if the beam polarization is the average polarization of the two helicity states, the error largely cancels in the physics asymmetry we care about. For parity experiments with asymmetries typically in the ppm range or less, this correction is completely negligible. Perhaps, it is surprising to find that even in the worst case with $\Delta = 1$ and $P_{beam} = 0.5$, i.e. for right helicity the beam is 100% polarized and 0% polarized for left helicity, the correction is still less than ppm level (relative) for typical PV asymmetries.

# 3 Conclusion

The impact of having different polarization for each beam helicity state has the possibility of impacting the Møller and Compton polarimetry measurements. A difference term $\Delta = 1\%$ in the polarization between right and left helicities would have a negligible impact on the Møller result at the 0.06% level. Near the Compton edge where the analyzing power $A_{zz}$ is greatest, this would change the Compton result at the 0.3% level. On the other hand, the relative change from such a polarization difference in the PV asymmetry carries a factor of $A^{PV}$ making even the largest $\Delta$ difference negligible. As a result, there is no cancellation of the error arising in the polarimetry measurement and an upper limit on the size of the $\Delta$ term should be determined by measurement. The most likely place for such a difference to arise would be in the source laser. Fortunately, given the care with which the polarization states are set up, a $\Delta$ term as large as seems highly unlikely.

I will attempt to calculate conservative upper limit for the polarization difference from the laser given previous experience. A $\Delta = 1\%$ difference term assumed in the text above, means the circular polarization of the laser differs by 2% between the two states which implies a huge difference in the linear polarization of at least 20%! Differences this large would likely have noticeable effects on the helicity-correlated beam parameters like intensity, spot size and position and would almost certainly be corrected by the standard source setup techniques. If we conservatively estimate an upper limit of a 5% difference in linear polarization between the two states and go slightly off 100% circular polarization where one state is 99.5% circularly polarized and the other is 99.9% polarized, this gives a $\Delta$ of only 0.2% which would have a negligible impact on both Compton and Møller measurements. For Compton, this would be a 0.055% error at the Compton edge and less than that for the integrated spectrum. For Møller, this would imply a negligible error of 0.01%.

A more complex model involves a helicity dependence to the polarization of the electron beam off the photocathode even with perfectly symmetric laser states. This might arise from a helicity-dependence in the excitation from the P-level state to the conduction band in GaAs, for example, where the degeneracy of the $p_{-3/2}$ and $p_{+3/2}$ orbitals is broken. One could imagine in principle that an applied magnetic field could break this degeneracy. This is more difficult to detect since it requires a polarimeter that doesn't rely on a helicity-dependent polarization. In principle, the Mott polarimeter would be a good candidate for something like this, but in practice, even the Mott utilizes the helicity flip to cancel systematics from detector acceptance and target density.

Following the calculation in [1] the Mott polarimeter measures an up/down asymmetry from scattering 5 MeV beam electrons of nuclei in Au foil. The Mott scattering cross section has a term that is sensitive to the component of the incident electron spin transverse to the scattering plane defined by the two symmetric up and down detectors. In principle, if one had perfect knowledge, a measurement of the difference in counts between up and down detectors can provide the polarization independently for each helicity state. However, in practice, the detector acceptance is not perfectly symmetric and the efficiency of the detectors is not exactly the same and so the counts from the two helicities are combined to cancel these systematics. The number of scattered electrons into the up and down detectors for + and − beam helicities is given by

$$\begin{aligned} N_u^{\pm} &= i^{\pm}\rho^{\pm} Q_u \Delta\Omega_u \left(1 \pm PS(E_{beam}, \theta)\right) \\ N_d^{\pm} &= i^{\pm}\rho^{\pm} Q_d \Delta\Omega_d \left(1 \mp PS(E_{beam}, \theta)\right), \end{aligned} \tag{8}$$

where $i$ is the beam current, $\rho$ is the target density, $Q$ is the detector efficiency, $\Omega$ is the detector solid angle acceptance, $P$ is the beam polarization and $S$ is the so-called Sherman function which is the Mott analyzing power as a function of beam energy and scattering angle. In principle the average scattering angle also need not be exactly equal, so one could add an additional subscript $\theta_{u(d)}$ but this is a small correction. Once the Sherman function is known, final polarization is obtained from the following super-ratio which cancels most systematic differences:

$$\epsilon = \frac{\sqrt{N_u^+ N_d^-} - \sqrt{N_u^- N_d^+}}{\sqrt{N_u^+ N_d^-} + \sqrt{N_u^- N_d^+}} = PS(E_{beam}, \theta).$$

Obtaining a polarization from a single helicity would look like the following:

$$A^{\pm} = \frac{N_u^{\pm} - N_d^{\pm}}{N_u^{\pm} + N_d^{\pm}}$$

If changes in the laser can be ruled out by direct measurement then the Mott might be useful in ruling out effects from the photocathode. For example, if the new injector vacuum window actually ends up having negligible birefringence, then the laser could be set up with virtually perfect symmetry upon insertion of the IHWP. With the only change on the laser being a flip of sign, the Mott could then an asymmetry could be formed using HWP IN and OUT that cancels systematics to set an upper limit on the polarization difference between the two helicity states. For example the difference between the following two asymmetries could be used to set an upper limit:

$$A^{\pm} = \frac{(N^{\pm}_{u,In} + N^{\pm}_{d,Out}) - (N^{\pm}_{d,In} + N^{\pm}_{u,Out})}{(N^{\pm}_{u,In} + N^{\pm}_{d,Out}) + (N^{\pm}_{d,In} + N^{\pm}_{u,Out})}.$$

Another idea would be to set the polarization of the beam to 0 and use the measured rates to calibrate the acceptance times efficiency for the up and down detectors. Once again, an injector vacuum window with negligible birefringence would allow one to project a 100% linearly polarized laser on the photocathode. With calibrated efficiencies, one could then go back to fully polarized beam and measure the polarization separately for both helicity states.

# References

[1] Kurt Aulenbacher, Eugene Chudakov, David Gaskell, Joseph Grames, and Kent D. Paschke. Precision electron beam polarimetry for next generation nuclear physics experiments. *International Journal of Modern Physics E*, 27(7):1830004–1159, January 2018.