Calculating Heating by the Electron Beam In an Fe Foil

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This technical note TargetHeating.tex, TargetHeating.pdf and the accompanying code FeFoil-Heating.C can be found in the following Github repository:
https://github.com/jonesdc76/MollerPolarimetry/tree/master/TargetPolarization

1 Solving the Heat Equation for Conditions Specific to the Hall A Møller Polarimeter

To calculate the heating of the Møller polarimeter iron foil we start with the heat equation. Given the geometry of the Møller foil where we have a circular10 μm thick foil with a beam heat source located at the center, we can assume this has no azimuthal or z-dependence and we are left with only a radial dependence:

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \nabla^2 T + \rho \alpha B_{flux} - \frac{2\sigma\epsilon}{\Delta z}\left(T^4 - T_0^4\right). \tag{1}$$

- $T(r,t)$ is the foil temperature in Kelvin,
- κ is the temperature dependent thermal conductivity of Fe which is approximately 0.8 W/(K cm) at room temperature (see Fig. 1),
- $\rho = 7.87$ g/cm^3 is the density of Fe,
- $\sigma = 5.67 \times 10^{-12}$ W/(K^4 cm^2) is the Stefan-Boltzmann constant,
- ϵ is the foil emissivity which depends on the polish and structure of the surface ranging from 0 (perfect polish) to 1 (perfect blackbody). Given the polish of the foil, something like 0.1 can be assumed.
- $T_0 = 294$ K, is the ambient temperature of the target ladder holding the foil at its boundary,
- $\Delta z = 10$ μm is the thickness of the foil,
- α is the collision stopping power for electrons in Fe. It is a function of electron energy and is 2.043 (MeV cm^2)/g=3.273$\times 10^{-13}$(J cm^2)/g for a 10 GeV electron using ESTAR. The ESTAR data along with a 5-degree polynomial fit used to calculate α as a function of energy is shown in Fig. 2. Care should be exercised when extrapolating outside the 1-10 GeV range.

- $C_p = 0.45$ J/(g K) is the specific heat of Fe and,
- $B_{flux} = \frac{d^3N_e}{dsdt}$ is the flux density of the beam in $e^-/(\mathrm{cm}^2\ \mathrm{s})$.

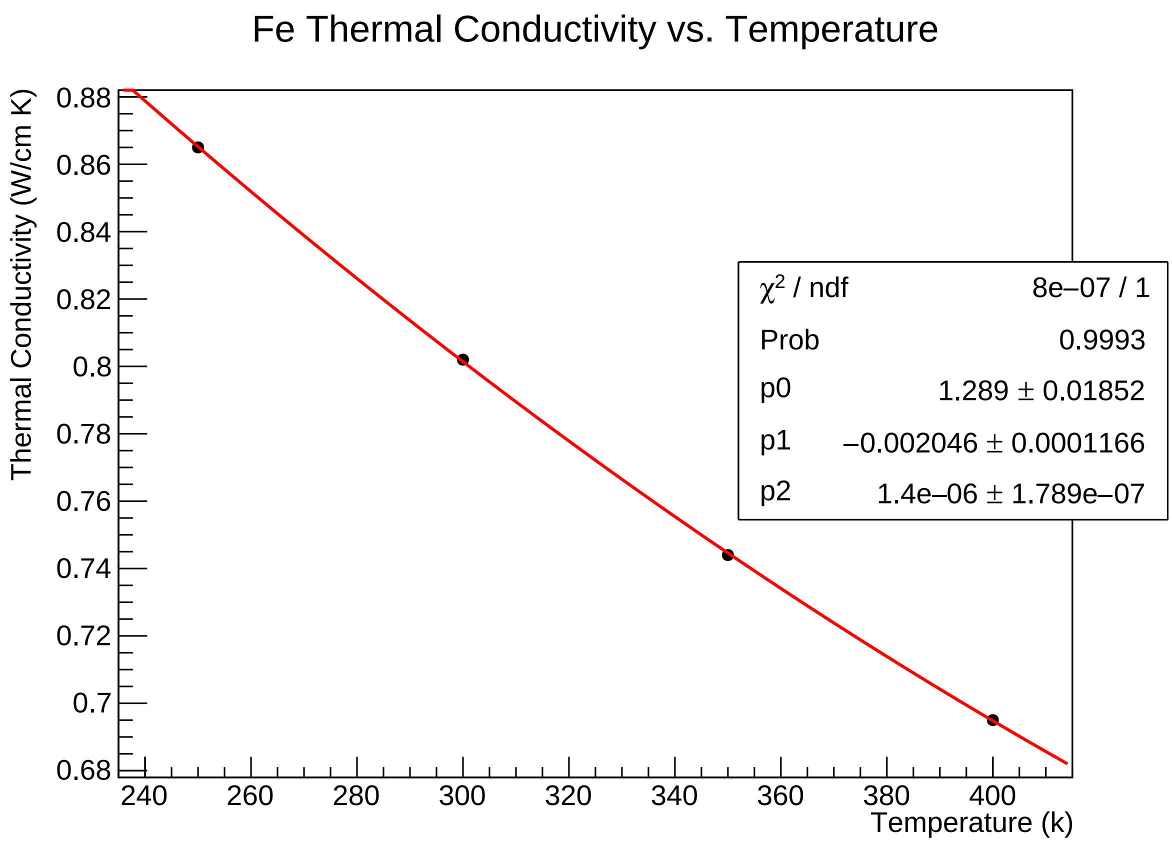

Figure 1: Fe thermal conductivityκ as a function of temperature. Data are from https://www.efunda.com/materials/elements/TC_Table.cfm?Element_ID=Fe and are fit to a 2nd degree polynomial.

In principle T and B_{flux} are functions of position and time. However, we are interested in the temperature of the steady state which is presumably reached quite rapidly when the beam turns on. Setting $\frac{\partial T}{\partial t} = 0$ simplifies Eq. 1. The expected heat load on a 10 μm thick Fe foil in the electron beam is about 12 mW/μA. If the temperature increase with beam inside the beam flux is of 30 degrees Celsius or less, over a beam radius of 1 mm, then the radiated energy in this circular area is 0.13 mW or about 1% of the heat load. In this case, we can safely neglect the radiative cooling term. If we end up with a temperature increase greater than 30 degrees, then we will have to revisit this assumption. Under these assumptions, Eq. 1 simplifies to

$$\kappa \nabla^2 T = -\rho\alpha B_{flux} \tag{2}$$

$$\frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\rho\alpha B_{flux} \tag{3}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\rho\alpha}{\kappa} r B_{flux}. \tag{4}$$

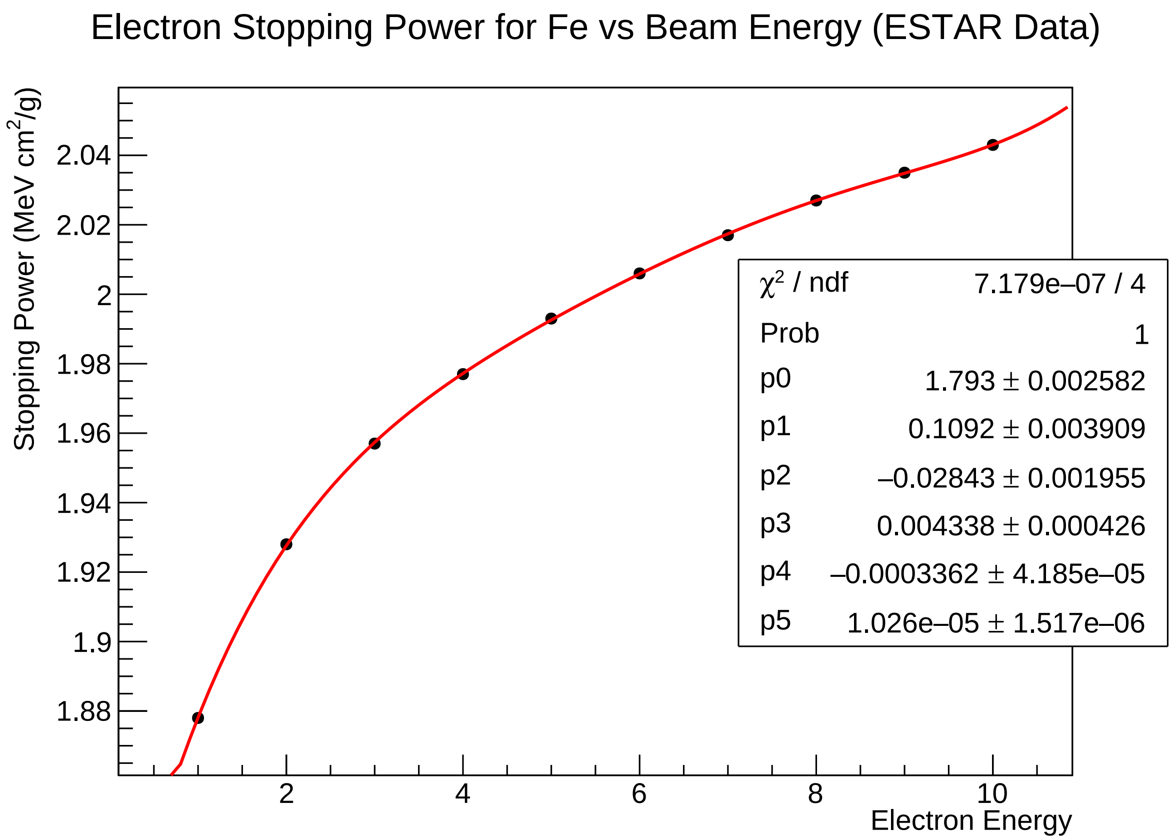

Figure 2: Stopping power for electrons as a function of energy in Fe. Data are from ESTAR and are fit to a 5-degree polynomial.

1.1 Solving for a heating from a Gaussian profile beam spot

The Hall A Møller polarimeter, does not typically take rastered beam, and it is thus reasonable to assume a Gaussian beam flux profile of radius r_b. Therefore, the Gaussian profiled electron flux B_{flux} from a beam current I in Amperes with a 1 σ radius of r_b becomes

$$B_{flux} = \frac{I}{1.6 \times 10^{-19}\,(2\pi r_b^2)} e^{-r^2/2r_b^2}. \tag{5}$$

Inserting this density profile for the electron beam heat source into Eq. 4 gives

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\gamma r e^{-r^2/2r_b^2}, \tag{6}$$

where $\gamma \equiv \frac{I\rho\alpha}{1.6\times10^{-19}\kappa\left(2\pi r_b^2\right)}$. Integrating both sides of Eq. 6 w.r.t. r gives

$$r\frac{\partial T}{\partial r} = r_b^2\gamma e^{-r^2/2r_b^2} + C, \tag{7}$$

$$\frac{\partial T}{\partial r} = \frac{r_b^2\gamma}{r} e^{-r^2/2r_b^2} + \frac{C}{r} \tag{8}$$

where C is a constant of integration to be determined from boundary conditions in the steady state. To determine C, the total heat load from the beam is given by $I\alpha\rho\Delta z/1.6 \times 10^{-19} =$

16.1Δz W/(μA cm). The heat flow through the boundary is the product of the conductivity κ, the cross sectional area of the foil along the foil perimeter $2\pi R_{foil}\Delta z$ and the temperature slope $\partial T/\partial r$, where length units are in cm. The perimeter of the foil at R_{foil} is assumed to be kept fixed at room temperature. The heat flow at the boundary has to equal the beam heat load in the steady state, so

$$(\kappa 2\pi R_{foil}\Delta z)\,\frac{\partial T}{\partial r}|_{r=R_{foil}} \approx -16.1\Delta z\left(\frac{\mathrm{W}}{\mu\mathrm{A\ cm}}\right) \approx \frac{(\kappa 2\pi R_{foil}\Delta z)\,C}{R_{foil}},$$

where the first term on the left side of Eq. 8) is not included since it is negligible at the boundary of the foil R_{foil}. The negative sign comes from the direction of heat flow towards higher radius making the temperature decrease with increasing r.

$$C \approx -\frac{16.1}{2\pi\kappa} = -3.20\left(\frac{\mathrm{K}}{\mu\mathrm{A}}\right),$$

where the temperature dependent κ for Fe has been used (see Fig. 1). Now to find the temperature difference between the outside perimeter of the foil at $r = R_{foil}$ and some $r < R_{foil}$ integrate both sides from R_{foil} to r yielding

$$\Delta T = \int_{R_{foil}}^{r}\left(\frac{r_b^2\gamma}{r'}e^{-r'^2/2r_b^2} + \frac{C}{r'}\right)dr'. \tag{9}$$

This can easily be integrated numerically as shown in Figures 3 and 4.

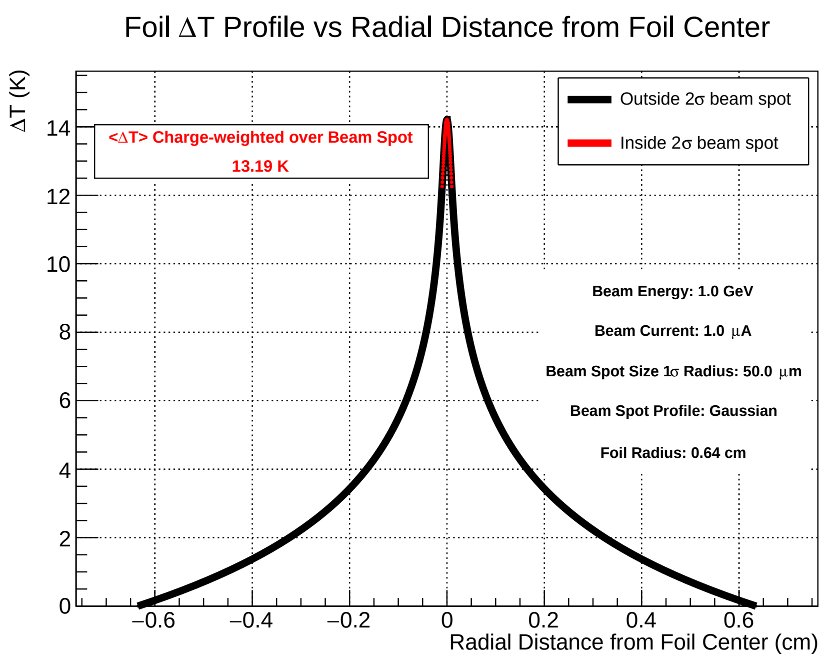

Figure 3: Fe foil ΔT profile from integrating Eq. 9 with beam spot size, and energy given.

1.2 Solving for heating from a uniform circular distribution

In the case where a beam is rastered, the charge distribution can be considered to be uniform. Let's solve for the case of a uniform circular raster pattern of radius r_{rast} centered on the foil. In

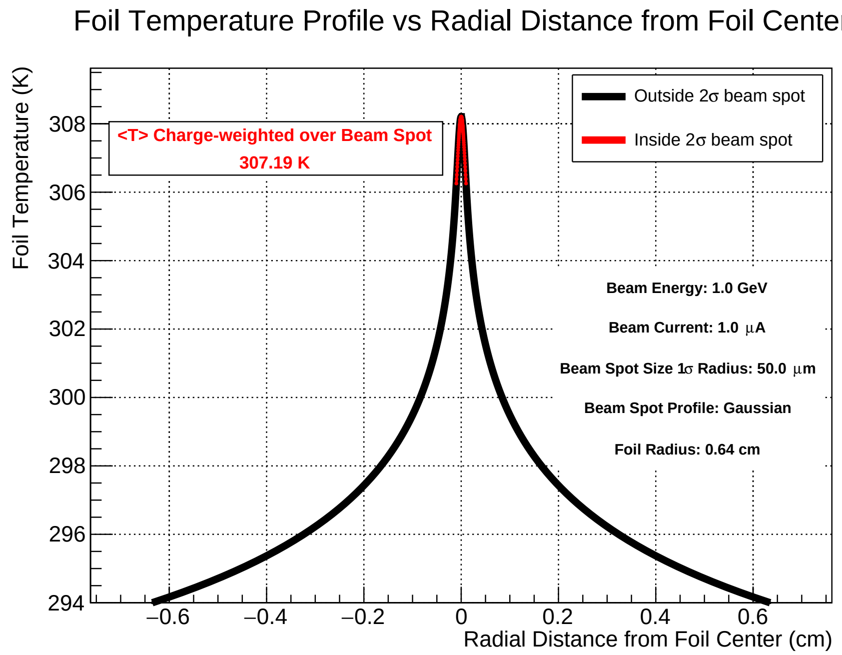

Figure 4: Fe foil temperature profile from integrating Eq. 9 with beam spot size, and energy given.

this case, the electron flux density B is given by

$$B = \frac{I\Theta(r_{rast} - r)}{1.6 \times 10^{-19}\pi r_{rast}^2}, \tag{10}$$

where $\Theta(r_{rast} - r)$ is the Heaviside function which is unity for $r < r_{rast}$ and zero for $r > r_{rast}$. Inserting Eq. 10 into Eq. 4 yields

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\rho\alpha}{\kappa}\frac{I\Theta(r_{rast} - r)}{1.6 \times 10^{-19}\pi r_{rast}^2}r \tag{11}$$

$$= -\gamma\Theta(r_{rast} - r)r, \tag{12}$$

where $\gamma \equiv \frac{\rho\alpha I}{1.602\times10^{-19}\kappa\pi r_{rast}^2}$. Integrating both sides with respect to r gives

$$r\frac{\partial T}{\partial r} = \begin{cases} -\frac{\gamma r^2}{2} + C, & r < r_{rast} \\ -\frac{\gamma r_{rast}^2}{2} + C, & r \geq r_{rast}. \end{cases}$$

This becomes

$$\frac{\partial T}{\partial r} = \begin{cases} -\frac{\gamma r^2}{2r} + \frac{C}{r}, & r < r_{rast} \\ -\frac{\gamma r_{rast}^2}{2r} + \frac{C}{r}, & r \geq r_{rast}. \end{cases}$$

Similar to before, the heat flow through the foil thickness at $r \geq r_{rast}$ has to equal the beam heat load in the steady state, so let's solve at $r = r_{rast}$:

$$(\kappa 2\pi r_{rast}\Delta z)\frac{\partial T}{\partial r}|_{r=R_{foil}} \approx \frac{\rho\alpha I}{1.602 \times 10^{-19}}\Delta z\left(\frac{\text{W}}{\mu\text{A cm}}\right) \approx \left(\frac{-\gamma r_{rast}^2}{2} + C\right)2\pi\kappa\Delta z.$$

Solving gives $C = 0$, so we now have

$$\frac{\partial T}{\partial r} = \begin{cases} -\frac{\gamma r^2}{2r}, & r < r_{rast} \\ -\frac{\gamma r_{rast}^2}{2r}, & r \geq r_{rast}. \end{cases} \tag{13}$$

Integrating both sides with respect to r in reverse direction from $r = R_{foil}$ to $r \leq r_{rast}$ gives ΔT

$$\Delta T = \begin{cases} -\frac{\gamma r_{rast}^2}{2} \int_{R_{foil}}^{r_{rast}} \frac{dr}{r} - \frac{\gamma}{2} \int_{r_{rast}}^{r} r' dr', & r < r_{rast} \\ , -\frac{\gamma r_{rast}^2}{2} \int_{R_{foil}}^{r} \frac{dr'}{r'}, & r \geq r_{rast} \end{cases},$$

which can be piecewise solved analytically yielding

$$\Delta T = \begin{cases} \frac{\gamma r_{rast}^2}{2} \ln\left(\frac{R_{foil}}{r_{rast}}\right) + \frac{\gamma}{4}\left(r_{rast}^2 - r^2\right), & r < r_{rast} \\ \frac{\gamma r_{rast}^2}{2} \ln\left(\frac{R_{foil}}{r}\right), & r \geq r_{rast} \end{cases}. \tag{14}$$

Figures 5 and 6 give plots of ΔT and T respectively for a uniformly rastered beam for the parameters given on the plots.

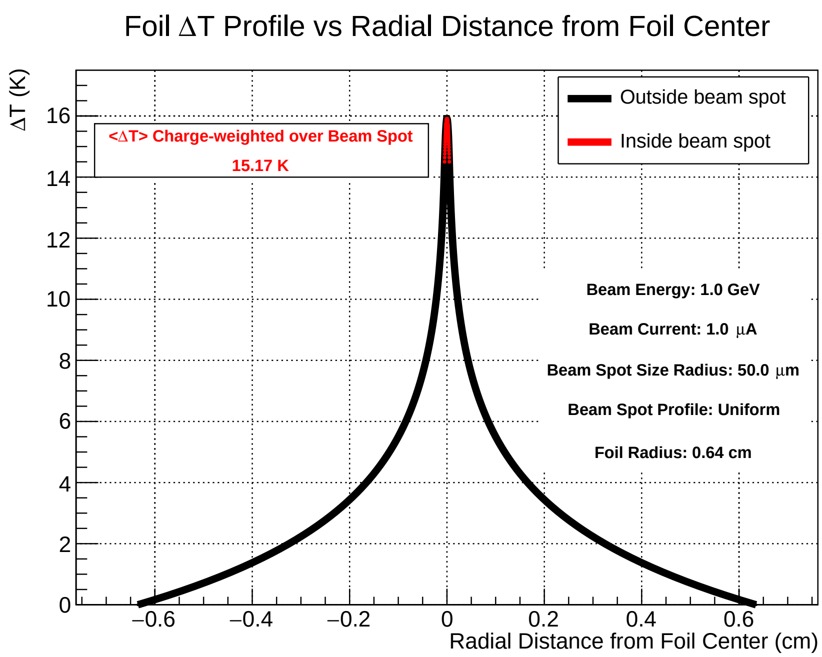

Figure 5: Fe foil ΔT profile from using Eq. 14 with beam spot size, and energy given.

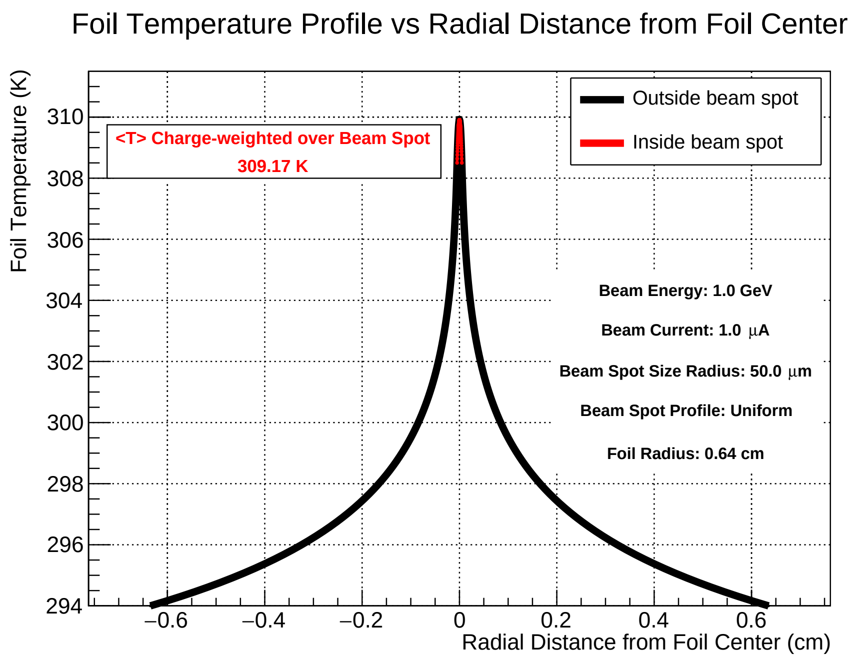

Figure 6: Fe foil temperature profile from integrating Eq. 14 with beam spot size, and energy given.

2 C++/ROOT Code for Numerically Integrating Eq. 9

The following ROOT macro uses Eq. 9 to calculate the foil heating for a circular Fe foil in a Gaussian profile electron beam.

```
#include "TF1.h"
#include <iostream>
#include "TGraph.h"
#include "TLegend.h"
#include "TAxis.h"
#include "TPad.h"
#include "TCanvas.h"
#include "TStyle.h"
#include "TPaveText.h"
#include "TString.h"

///////////////////
//Donald C. Jones//
//Nov. 2021      //
///////////////////

//////////////////////////////////////////////////////////////////////
//FeFoilHeating() calculates and graphs the temperature difference
//in a thin circular Fe foil between its edge held at a fixed
//temperature T0 and inside a circular Gaussian-distributed
//or uniformly rastered electron beam.
//
//
//Arguments:
//  beam_cur: beam current in Amperes
//  beam_r:   1 sigma beam spot size radius in cm
//  beam_E:   beam energy in GeV
//  T0:       ambient (Hall) temperature in Kelvin taken as foil
//            boundary temperature
//  uniform: uniform charge distribution? Otherwise, Gaussian assumed.
//
//Returns the foil temperature difference in degrees K between T0
//at the foil edge and the temperature at the 1-sigma beam
//radius r_beam.
//
//NOTE: it is helpful to recall that for a 2D circular Gaussian
//distribution the volume between r=0 and the n-sigma points
//are as follows:
//1sigma = 39.35%, 2sigma = 86.47%, 3sigma = 98.89%, 4sigma = 99.97%
//Therefore, the temperature should be averaged over at least 3 sigma.
/////////////////////////////////////////////////////////////////////////

double FeFoilHeating(double beam_cur = 1e-6, double beam_r=5e-3, double beam_E = 11,
   double T0 = 294, double foil_r = 0.635, bool uniform = 0){
  gStyle->SetStatY(0.7);
```

```
gStyle->SetStatH(0.2);
gStyle->SetOptFit(1111);
gStyle->SetTitleW(0.9);

bool save_plots = 1;
const double rho = 7.87;//density of Fe
const double sigma = 5.670e-12;//Stefan Boltzman constant W/(cm^2 K^4)
const double Cp = 0.45;//Fe specific heat capacity in J/(g K)
const double echarge = 1.602e-19;//Coulombs per electron
const double PI = 3.1415927;//pi obviously



//Use ESTAR data to estimate energy loss as a function of electron energy
//-------------------------------------------------------------------------------
TCanvas *c = new TCanvas("c","c",0,0,800,600);
double beam_en[10]={1,2,3,4,5,6,7,8,9,10};//beam energy in GeV
double stop_en[10]={1.878,1.928,1.957,1.977,1.993, //collision stopping power
         2.006,2.017,2.027,2.035,2.043};//in (MeV cm^2/g) using ESTAR
TGraph *grStop = new TGraph(10,beam_en,stop_en);
grStop->SetTitle("Electron Stopping Power for Fe vs Beam Energy (ESTAR Data)");
grStop->SetMarkerStyle(8);
grStop->Draw("ap");
grStop->GetXaxis()->SetTitle("Electron Energy");
grStop->GetYaxis()->SetTitle("Stopping Power (MeV cm^{2}/g)");
gPad->Update();
TF1 *fStop = new TF1("fStop","pol5",0,1);//use fit to give continuous function
grStop->Fit(fStop);
double alpha = echarge*fStop->Eval(beam_E)*1e6;//Collision stopping power in
   (Jcm^2/g)
cout<<"Stopping power "<<alpha<<" (J cm^2/g)"<<endl;
if(save_plots)
  c->SaveAs("StoppingPower.pdf");



//Calculate the energy dependent thermal conductivity of Fe using data either from
//https://www.efunda.com/materials/elements/TC_Table.cfm?Element_ID=Fe
//or
//https://www.engineeringtoolbox.com/thermal-conductivity-metals-d_858.html
//-------------------------------------------------------------------------------
bool data_efunda = 1;
TCanvas *ct = new TCanvas("ct","ct",0,0,800,600);
double temp[4] = {250,300,350,400};
double cond[4] = {0.865,0.802,0.744,0.695};//www.efunda.com
TGraph *grC = new TGraph(4,temp,cond);
grC->SetTitle("Fe Thermal Conductivity vs. Temperature");
grC->SetMarkerStyle(8);
grC->Draw("ap");
grC->GetXaxis()->SetTitle("Temperature (k)");
```

```
grC->GetYaxis()->SetTitle("Thermal Conductivity (W/cm K)");
TF1 *fCond = new TF1("fCond","pol2",0,1);
grC->Fit(fCond);
gPad->Update();
if(!data_efunda)//www.engineeringtoolbox.com
  fCond = new TF1("fCond","0.835-0.001102*(x-273)",0,1);
double slope = uniform ? 19.5 : 17;
double guessTemp = T0+slope*beam_cur/1e-6;//starting guess for final foil
    temperature
double kappa = fCond->Eval(guessTemp);
cout<<"Conductivity at "<<guessTemp<<" K is "<<kappa<<endl;
if(save_plots)
  ct->SaveAs("FeThermalCond.pdf");



//Integral of f(r) gives delta T. Create the integrand f(r)
//-----------------------------------------------------------------------------------
double gam = beam_cur/echarge*rho*alpha/kappa/PI/pow(beam_r,2)/(uniform ? 1.0 :
    2.0);
double C = -beam_cur/echarge*alpha*rho/2.0/PI/kappa;
TF1 *f = new TF1("f",Form("%e/x*exp(-x*x/%e)+%e/x",
           beam_r*beam_r*gam,2*beam_r*beam_r,C),0,foil_r);



//Improve thermal conductivity estimate using the calculated temperature.
//Temperature at 1.3*beam_r is a good estimate of the average temperature
//weighted by a Gaussian beam spot charge distribution. For a uniform distribution
//0.7*beam_r is a good estimate.
//------------------------------------------------------------------------------------
double r_est = (uniform ? 0.7 : 1.3)*beam_r;
if(uniform)
  guessTemp =
      gam*pow(beam_r,2)/2.0*log(foil_r/beam_r)+gam/4*(pow(beam_r,2)-pow(r_est,2))+T0;
else
  guessTemp = f->Integral(foil_r, r_est)+T0;
kappa = fCond->Eval(guessTemp);
gam = beam_cur/echarge*rho*alpha/kappa/PI/pow(beam_r,2)/(uniform ? 1.0 : 2.0);
C = -beam_cur/echarge*alpha*rho/2.0/PI/kappa;

cout<<"Conductivity re-calculated at "<<guessTemp<<" K is "<<kappa<<endl;
f = new
    TF1("f",Form("%e/x*exp(-x*x/%e)+%e/x",beam_r*beam_r*gam,2*beam_r*beam_r,C),0,foil_r);



//Graph resulting temperature profile by integrating f(r)dr. Make points red inside
//beam spot radius (2 sigma if Gaussian).
//------------------------------------------------------------------------------------
```

```
const int N=1000;
double r[N], T[N], dT[N],ri[N],Ti[N], dTi[N];
int n=0, ni=0;
double rp = foil_r;
double red_zone = uniform ? beam_r : 2*beam_r;
for(int i=0;i<N/2;++i){
  r[i]=rp;
  if(uniform){
    if(rp<red_zone)
 dT[i] = gam*pow(beam_r,2)/2.0*log(foil_r/beam_r)+gam/4.0*(pow(beam_r,2)-pow(rp,2));
    else
 dT[i] = gam*pow(beam_r,2)/2.0*log(foil_r/rp);
  }else{
    dT[i] = f->Integral(foil_r,rp);
  }
  T[i] = dT[i]+T0;
  if(rp<red_zone){
    ri[ni]=rp;
    Ti[ni]=T[i];
    dTi[ni]=dT[i];
    ++ni;
  }
  rp*=0.95;
  ++n;
  if(rp<0.00001)break;
}
for(int i=0;i<n;++i){
  r[i+n]=-r[n-i-1];
  dT[i+n] = dT[n-i-1];
  T[i+n] = T[n-i-1];
}
for(int i=0;i<ni;++i){
  ri[i+ni]=-ri[ni-i-1];
  dTi[i+ni] = dTi[ni-i-1];
  Ti[i+ni] = Ti[ni-i-1];
}
TCanvas *c1 = new TCanvas("c1","c1",0,0,800,600);
TGraph *grdT = new TGraph(2*n,r,dT);
grdT->SetMarkerStyle(8);
grdT->SetLineWidth(6);
grdT->SetMarkerSize(0.3);
grdT->Draw("acp");
grdT->SetTitle(Form("Foil #DeltaT Profile vs Radial Distance from Foil Center"));
grdT->GetXaxis()->SetTitle("Radial Distance from Foil Center (cm)");
grdT->GetYaxis()->SetTitle("#DeltaT (K)");
TGraph *gridT = new TGraph(2*ni,ri,dTi);
gridT->SetMarkerStyle(8);
gridT->SetMarkerColor(kRed);
gridT->SetLineColor(kRed);
gridT->SetLineWidth(6);
```

```
gridT->SetMarkerSize(0.4);
gridT->Draw("samep");
TPaveText *pt = new TPaveText(0.6,0.3,0.89,0.6,"ndc");
pt->SetFillColor(0);
pt->SetShadowColor(0);
pt->SetBorderSize(0);
pt->AddText(Form("Beam Energy: %0.1f GeV",beam_E));
pt->AddText(Form("Beam Current: %0.1f #muA", beam_cur*1e6));
TString str = Form("Beam Spot Size 1#sigma Radius: %0.1f #mum",beam_r*1e4);
if(uniform)
  str = Form("Beam Spot Size Radius: %0.1f #mum",beam_r*1e4);
pt->AddText(str.Data());
pt->AddText((char*)(uniform ? "Beam Spot Profile: Uniform" : "Beam Spot Profile:
    Gaussian"));
pt->AddText(Form("Foil Radius: %0.2f cm",foil_r));
pt->Draw();
TLegend *lg = new TLegend(0.62,0.76,0.89,0.89);
if(uniform){
  lg->AddEntry(grdT,"Outside beam spot","lp");
  lg->AddEntry(gridT,"Inside beam spot","lp");
}else{
  lg->AddEntry(grdT,"Outside 2#sigma beam spot","lp");
  lg->AddEntry(gridT,"Inside 2#sigma beam spot","lp");
}
lg->Draw();
TCanvas *c2 = new TCanvas("c2","c2",0,0,800,600);
TGraph *gr = new TGraph(2*n,r,T);
gr->SetMarkerStyle(8);
gr->SetLineWidth(6);
gr->SetMarkerSize(0.3);
gr->Draw("acp");
gr->SetTitle(Form("Foil Temperature Profile vs Radial Distance from Foil Center"));
gr->GetYaxis()->SetTitle("Foil Temperature (K)");
gr->GetXaxis()->SetTitle("Radial Distance from Foil Center (cm)");
gr->GetYaxis()->SetRangeUser(T0,T0+grdT->GetYaxis()->GetXmax());
TGraph *gri = new TGraph(2*ni,ri,Ti);
gri->SetMarkerStyle(8);
gri->SetMarkerColor(kRed);
gri->SetLineColor(kRed);
gri->SetLineWidth(2);
gri->SetMarkerSize(0.4);
gri->Draw("samecp");
lg->Draw();
pt->Draw();


//Integrate f(r) weighted by the beam charge distribution to find the average delta
    T
//-------------------------------------------------------------------------------------
gStyle->SetOptFit(0);
```

```
TF1 *fGaus = new TF1("fGaus","[0]*exp(-x*x/(2*[1]*[1]))+[2]",-2*beam_r,2*beam_r);
fGaus->SetParameters((guessTemp-T0)/2.,beam_r,T0);cout<<(guessTemp-T0)/2.<<endl;
fGaus->SetLineWidth(2);
fGaus->SetLineColor(kRed);
gr->Fit(fGaus,"r");
if(uniform){
  fGaus->SetRange(-beam_r,beam_r);
  //fGaus->FixParameter(0,fGaus->GetParameter(0));
  fGaus->FixParameter(2,fGaus->GetParameter(2));
  gr->Fit(fGaus,"r");
}
TString fstr = Form("x*exp(-x*x/2./%e)/%e",beam_r*beam_r,beam_r*beam_r);
if(uniform)fstr = Form("2*x/%e",beam_r*beam_r);
TString func = Form("(%e*exp(-x*x/(2*%e))+%e)*%s",
          fGaus->GetParameter(0),pow(fGaus->GetParameter(1),2),
          fGaus->GetParameter(2), fstr.Data());
TF1 *fAvgT = new TF1("fAvgT",func.Data(),0,1);
fAvgT->SetNpx(1000);
 //fAvgT->Draw();
if(uniform)
  cout<<"dT at 0.7 beam radius is "<<fGaus->Eval(beam_r*0.7)<<endl;
else
  cout<<"dT at 1.3 sigma is "<<f->Integral(foil_r,beam_r*1.3)<<endl;



//Return average temperature, weighted by the beam spot charge distribution.
//------------------------------------------------------------------------------------
c1->SetGrid();
c1->cd();
double avg = fAvgT->Integral(0, (uniform ? 1.0 : 10.0 ) * beam_r);
TPaveText *pt1 = new TPaveText(0.12,0.74,0.48,0.82,"ndc");
pt1->SetFillColor(0);
pt1->SetShadowColor(0);
//pt1->SetBorderSize(0);
pt1->SetTextColor(kRed);
pt1->AddText(Form("<#DeltaT> Charge-weighted over Beam Spot"));
pt1->AddText(Form("%0.2f K",avg-T0));
pt1->Draw();
gPad->Update();
if(save_plots)
  c1->SaveAs(Form("FoilHeatingdT%s.pdf",(char*)(uniform ? "Uniform":"")));
c2->SetGrid();
c2->cd();
TPaveText *pt2 = new TPaveText(0.12,0.74,0.48,0.82,"ndc");
pt2->SetFillColor(0);
pt2->SetShadowColor(0);
//pt2->SetBorderSize(0);
pt2->SetTextColor(kRed);
pt2->AddText(Form("<T> Charge-weighted over Beam Spot"));
```

```
  pt2->AddText(Form("%0.2f K",avg));
  pt2->Draw();
  if(save_plots)
    c2->SaveAs(Form("FoilHeatingT%s.pdf",(char*)(uniform ? "Uniform":"")));
  cout<<"Total correction to magnetization for Fe: "<<-0.0238*(avg-T0)<<"
      emu/g"<<endl;
  return avg;
}
```