$[Email, Password, FirstName, LastName]$

$$UserCredential == Email \times Password$$
$$FullName == FirstName \times LastName$$
$$Applicant == UserCredential \times FullName$$
$$PToken == \mathbb{N} \times Applicant$$

function 42 **leftassoc** $(\_ \otimes \_)$

Next we use an axdef/gendef/freetype/== definition to define the semantics.

$$\_ \otimes \_ : \mathbb{P}\, Applicant \times Applicant \to \mathbb{P}\, Applicant$$

$$\forall\, a : Applicant;\ as : \mathbb{P}\, Applicant \bullet \exists\, p0, p1 : Password;\ n0, n1 : FullName;\ e : Email \mid a = (e \mapsto p0$$

HomestayInitial

$$Applicants : \mathbb{P}\, Applicant$$
$$Emails : \mathbb{P}\, Email$$
$$Passwords : \mathbb{P}\, Password$$
$$Valid, Invalid : \mathbb{P}\, PToken$$
$$Names : \mathbb{P}(FirstName \times LastName)$$

$$Applicants = \varnothing$$
$$Emails = \varnothing$$
$$Passwords = \varnothing$$
$$Valid = Invalid = \varnothing$$
$$Names = \varnothing$$

HomestayDatabase

$$HomestayInitial$$

$$\mathrm{dom}\, Applicants = Emails \times Passwords$$
$$\mathrm{ran}\, Applicants = Names$$
$$Valid \cap Invalid = \varnothing$$

CreateAccount

$\Delta HomestayDatabase$
$E? : Email$
$FN? : FirstName$
$LN? : LastName$
$P? : Password$
$Name : FullName$
$User : UserCredential$

$E? \notin Emails$
$Name = FN? \mapsto LN?$
$User = E? \mapsto P?$
$Emails' = Emails \cup \{E?\}$
$Passwords' = Passwords \cup \{P?\}$
$Names' = Names \cup \{Name\}$
$Applicants' = Applicants \cup \{User \mapsto Name\}$

Login

$\Xi HomestayDatabase$
$E? : Email$
$P? : Password$

$E? \in Emails$
$P? \in Passwords$
$E? \mapsto P? \in \mathrm{dom}\ Applicants$

ForgotPassword

$\Delta HomestayDatabase$
$E? : Email$
$App : Applicant$
$T : PToken$
$ID : \mathbb{N}$
$P : Password$

$E? \in Emails$
$\{App\} = \{E? \mapsto P\} \lhd Applicants$
$ID = \#\ Valid + \#\ Invalid + 1$
$T = ID \mapsto App$
$Valid' = Valid \cup \{T\}$

ResetPassword

$\Delta HomestayDatabase$
$P?, P : Password$
$T? : PToken$
$ID : \mathbb{N}$
$User : UserCredential$
$E : Email$
$Name : FullName$
$App : Applicant$

$T? \in Valid$
$ID \mapsto ((E \mapsto P) \mapsto Name) = T?$
$Valid' = Valid \setminus \{T?\}$
$Invalid' = Invalid \cup \{T?\}$
$App = (E \mapsto P?) \mapsto Name$
$Applicants' = Applicants \otimes App$