

# ECS 122A Homework 1

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Spring 2014

1. The entries of  $BB^T$  describe the edges in the graph.

For each  $i, j$  in  $BB^T$

- if  $i = j$  then it describes how many edges are at node  $i$ .
- if  $i \neq j$  then it describes if there is an edge between node  $i$  and node  $j$ .

2. (a) *Proof.* Every edge  $e$  in  $G$  connects two vertices  $v_i, v_j$ . So each vertex  $v_i$  adds 1 to the total sum for every edge  $e$  it has. This means that each edge  $e$  is counted exactly twice in the sum.

Thus,

$$\sum_{v \in V} d(v) = 2|E|$$

□

- (b) *Proof.* Assume not, that is assume there is an odd number of vertices with odd degree. Call this set of vertices  $v_o$ , and the rest of the vertices  $v_e$

We can see that  $\sum_{v \in v_e} d(v)$  is even, and that  $\sum_{v \in v_o} d(v)$  is odd. Also we have,  $\sum_{v \in (v_e \cup v_o)} d(v)$  is odd. N.B.  $v_e \cup v_o = V$ . but we know that  $\sum_{v \in V} d(v) = 2|E|$ , so we have a contradiction.

So our assumption was wrong.

Thus, there must be an even number of vertices with odd degree.

□

- (c) No, there is no similar statement. We can have a simple directed graph from  $A$  to  $B$ , but, there is only one *indegree*, which is odd.

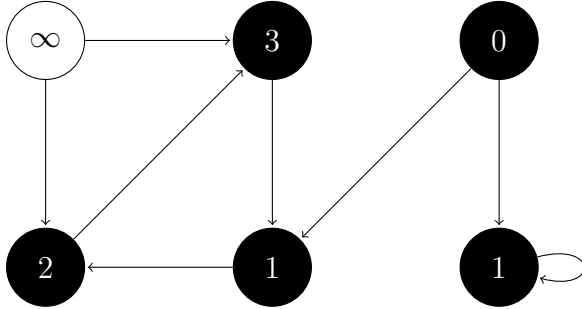
4. After running BFS on the graph, we have the following list as a result:

$$[\infty, 3, 0, 2, 1, 1]$$

where each entry corresponds to the list:

$$[1, 2, 3, 4, 5, 6]$$

As a graph, the enumerated vertices are:



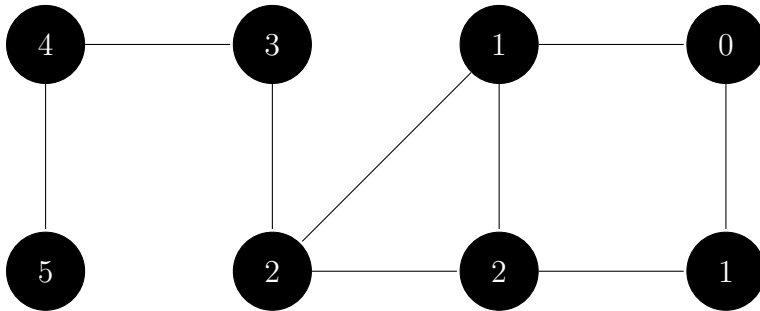
5. After running BFS on the graph, we have the following list as a result:

$$[4, 3, 1, 0, 5, 2, 2, 1]$$

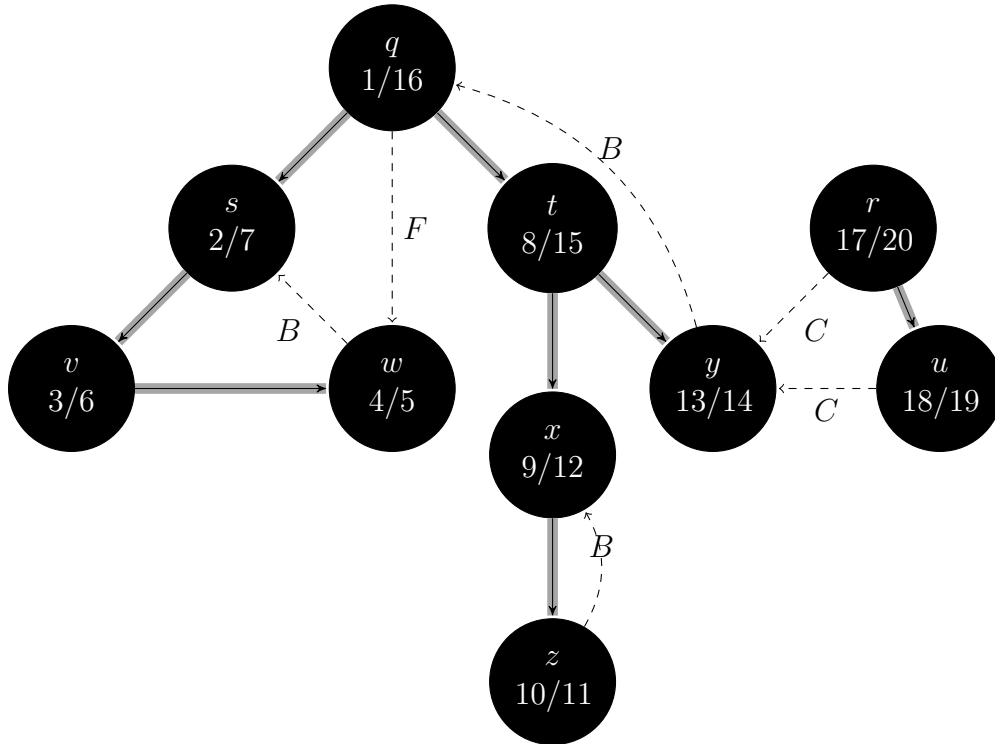
where each entry corresponds to the list:

$$[r, s, t, u, v, w, x, y]$$

As a graph, the enumerated vertices are:



6. After running DFS on the graph, assuming everything in alphabetical order, we have the following graph, with the convention of (discovery time / finish time):



8. After a topological sort we have the following:

