ECS 122A Homework 1

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1. The entries of BB^T are describe the edges in the graph.

   For each i, j in BB^T

   - if $i = j$ then it describes how many edges are at node i.
   - if $i \neq j$ then it describes if there is an edge between node i and node j.

2. (a) *Proof.* Every edge e in G connects two vertices v_i, v_j. So each vertex v_i adds 1 to the total sum for every edge e it has. This means that each edge e is counted exactly twice in the sum.

   Thus,

   $$\sum_{v \in V} d(v) = 2|E|$$

   □

   (b) *Proof.* Assume not, that is assume there is an odd number of vertices with odd degree. Call this set of vertices v_o, and the rest of the vertices v_e

   We can see that $\sum_{v \in v_e} d(v)$ is even, and that $\sum_{v \in v_o} d(v)$ is odd. Also we have, $\sum_{v \in (v_e \cup v_o)} d(v)$ is odd. N.B. $v_e \cup v_o = V$. but we know that $\sum_{v \in V} d(v) = 2|E|$, so we have a contradiction.

   So our assumption was wrong.

   Thus, there must be an even number of vertices with odd degree. □

   (c) No, there is no similar statement. We can have a simple directed graph from A to B, but, there is only one *indegree*, which is odd.

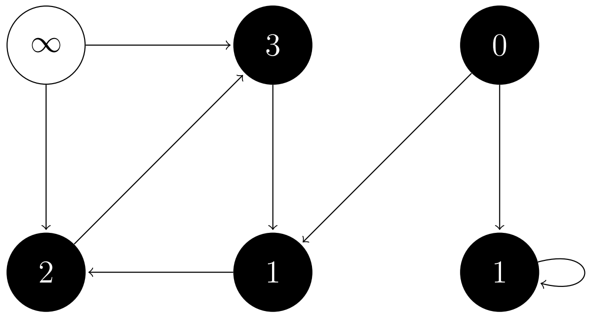
3. After running BFS on the graph, we have the following list as a result:

   $$[\infty, 3, 0, 2, 1, 1]$$

   where each entry corresponds to the list:

   $$[1, 2, 3, 4, 5, 6]$$

   As a graph, the enumerated vertices are:



4. After running BFS on the graph, we have the following list as a result:

$$[4, 3, 1, 0, 5, 2, 2, 1]$$

where each entry corresponds to the list:

$$[r, s, t, u, v, w, x, y]$$

As a graph, the enumerated vertices are: