ECS 122A Homework 2

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1. (a) We can expand some of the polynomial

$$(n+2)^{10} = n^{10} + 20n^9 + \cdots + 1024$$

Looking at everything except the first term, we see that the greatest power is n^9. We can set a *little-oh* bound of n^{10}, because $\forall c > 0, \exists n_0 > 0$ such that $0 \leq 20n^9 + \cdots + 1024 < cn^{10}, \forall n \geq n_0$. That is:

$$(n+2)^{10} = n^{10} + o(n^{10})$$

Now, this is $\Theta(n^{10})$ because
$\exists c_1 > 0, c_2 > 0, n_0 > 0$ such that $0 \leq c_1 n^{10} \leq n^{10} + o(n^{10}) \leq c_2 n^{10},\ \forall n \geq n_0$
This simplifies to $0 \leq c_1 \leq 1 + \frac{o(n^{10})}{n^{10}} \leq c_2$.
Now, since $o(n^{10})$ is an upper bound we know the largest the term $\frac{o(n^{10})}{n^{10}}$ can be is 1. So, this further siplifies to $0 \leq c_1 \leq 2 \leq c_2$.
So this is $\Theta(n^{10})$ if we choose appropriate constants. For instance we can choose $c_1 = c_2 = 2$.
Thus, $(n+2)^{10} = \Theta(n^{10})$

(b) We can apply reasoning similar to the previous problem.
We can expand some of the polynomial

$$(n+a)^b = n^b + \binom{b}{1} n^{b-1}a + \cdots + a^b$$

Looking at everything except the first term, we see that the greatest power is n^{b-1}. We can set a *little-oh* bound of n^b, because $\forall c > 0, \exists n_0 > 0$ such that $0 \leq \binom{b}{1} n^{b-1}a + \cdots + a^b < cn^b, \forall n \geq n_0$. That is:

$$(n+a)^b = n^b + o(n^b)$$

Now, this is $\Theta(n^b)$ because
$\exists c_1 > 0, c_2 > 0, n_0 > 0$ such that $0 \leq c_1 n^b \leq n^b + o(n^b) \leq c_2 n^b,\ \forall n \geq n_0$
This simplifies to $0 \leq c_1 \leq 1 + \frac{o(n^b)}{n^b} \leq c_2$.
Now, since $o(n^b)$ is an upper bound we know the largest the term $\frac{o(n^b)}{n^b}$ can be is 1. So, this further siplifies to $0 \leq c_1 \leq 2 \leq c_2$.
So this is $\Theta(n^b)$ if we choose appropriate constants. For instance we can choose $c_1 = c_2 = 2$.
Thus, $(n+a)^b = \Theta(n^b)$