ECS 120 Problem Set 4

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Professor Rogaway
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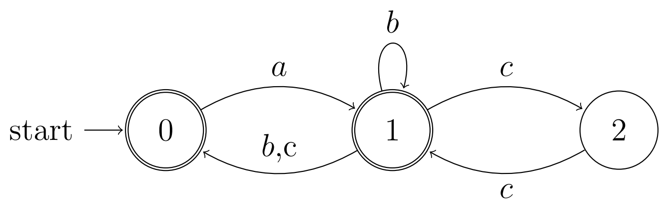
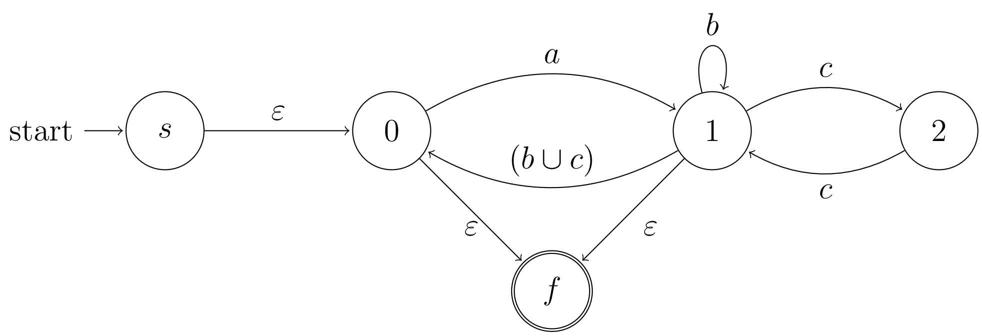
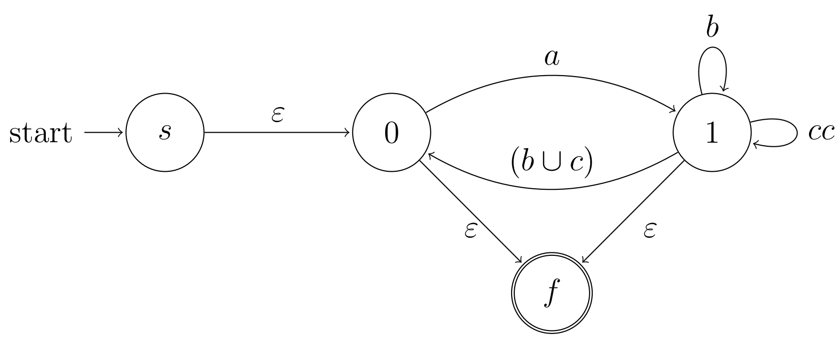


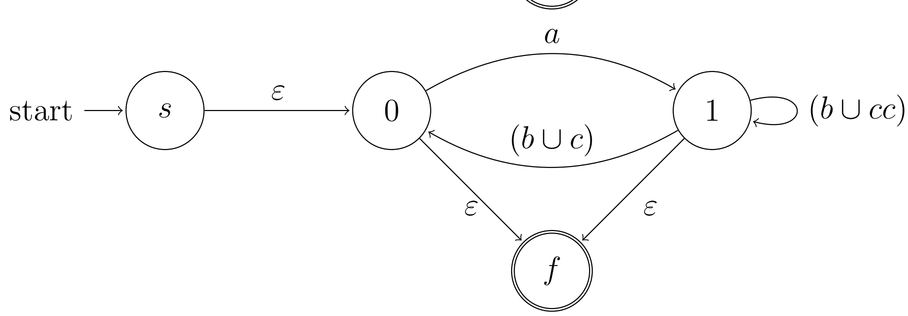
Problem 1 (a)

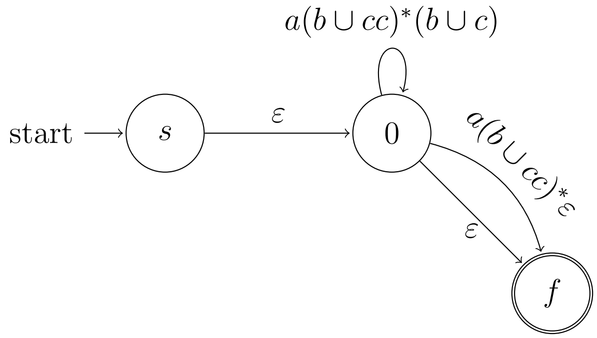
We start by creating new initial and final states. We also convert the two arrows from 1 to 0 to a regular expression.

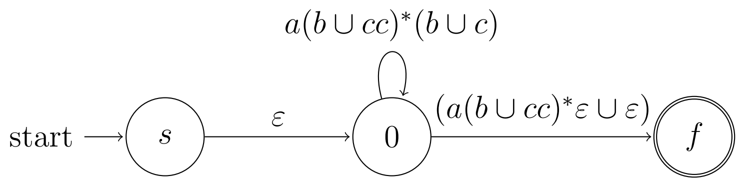


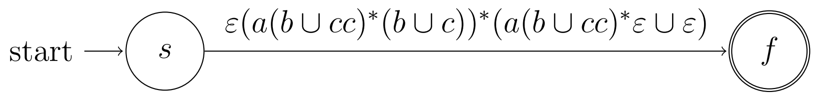
Now we start eliminating states.







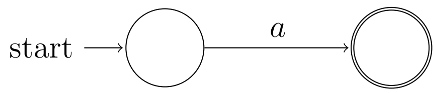


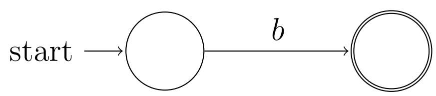


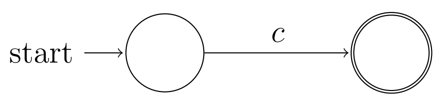
Thus, our regular expression is:

$$\varepsilon(a(b \cup cc)^*(b \cup c))^*(a(b \cup cc)^*\varepsilon \cup \varepsilon)$$

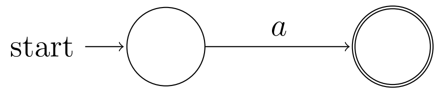
(b) Let's start by creating a state for each atom of the regular expression $(ab^* \cup c)^*$.

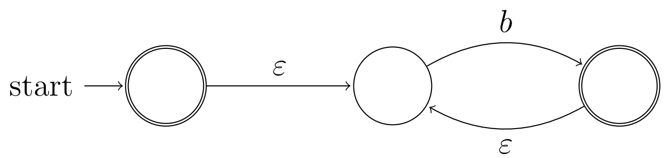


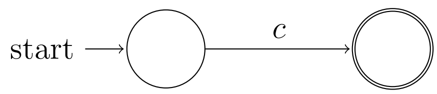




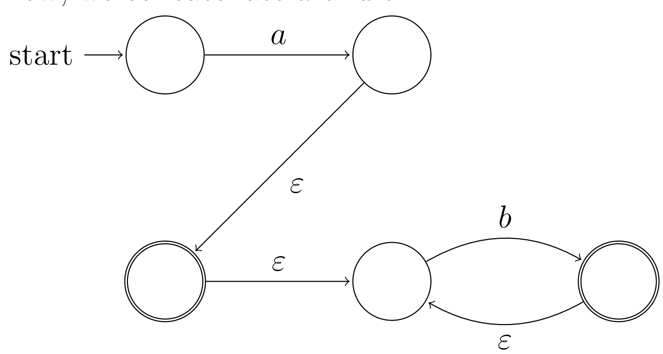
Now we make the Kleene Star of b.

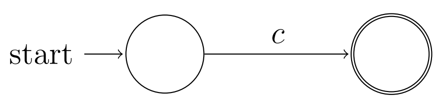




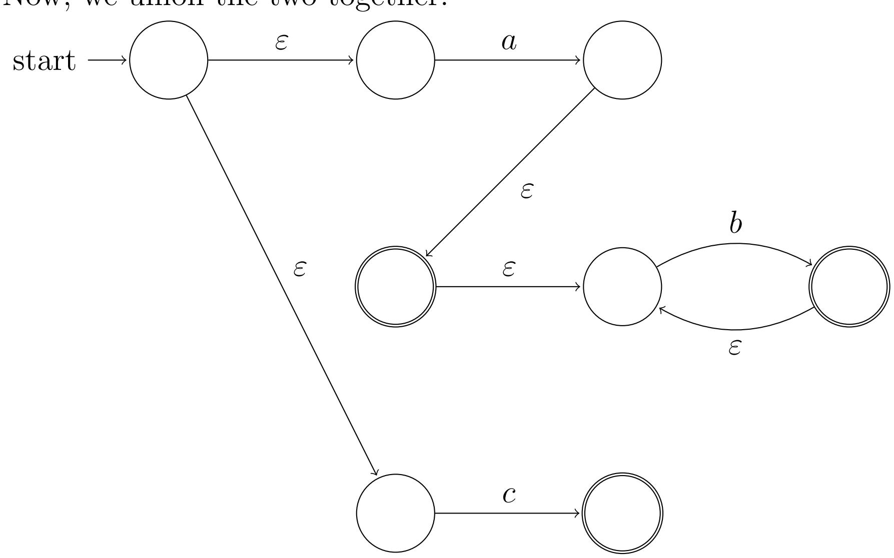


Now, we concatenate a and b^*.

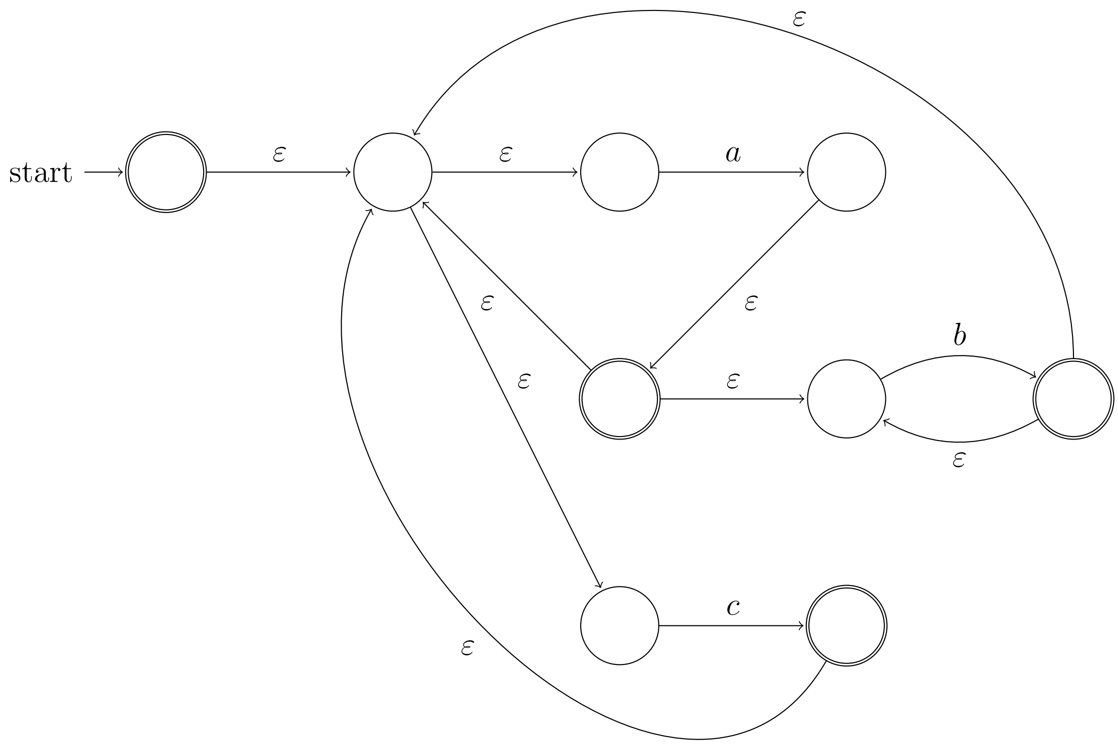




Now, we union the two together.



Finally, we construct the Kleene star of this.



(c) We start by converting α from a regular expression to an NFA. For every atom, we have two states, an initial state, and a final state, connected by an arrow for the atom. This gives us $2c$ states.

For every composition operator, we only add ε-arrows to our states. So, we still have $2c$ states.

For every star operator, we add one state and some ε-arrows to our states. So, we now have $2c + s$ states.

For every union operator, we add one state and some ε-arrows to our states. So, we now have $2c + s + u$ states.

At this point, we have constructed an NFA for the regular expression.

Now, we just need to convert it to a DFA. We end up with 2^{2c+s+u} states in this DFA.

So M has 2^{2c+s+u} states.

Problem 3 (a) $L = \{x \in a, b : x \text{ is not a palindrome}\}$

Assume L is regular, then the pumping lemma should hold.

So we choose $s = a^i b a^I$, where $x = a^i b, y = a^I, z = \varepsilon$, for some integer values of $i, I \geq 0$.

Since the pumping lemma should hold, we should be able to choose any values for i and I. Let's choose $i = mI$, for some integer $m > 1$. Clearly s is not a palindrome.

Now, we should be able to pump y any number of times, and have our new string in L, since it was assumed to be regular. However, we see that after pumping y m-times, we have a palindrome. So our pumped string is not in the the language L. So, the pumping lemma does not hold for L.

Thus, L is not regular.

Problem 6 (a) False. Let $L = \{a\}^*\{b\}^*$, and $L' = \{a^n b^n | n \geq 0\}$. $L \cup L' = L$, so their union is regular, but L' is not.

(b) False. Let $L = \{a^n b^n | n \geq 0\}$. This is not regular. So, by the contrapositive, L^* should not be regular. However, $L^* = \{ab, ba\}^*$