PHIL 112 Homework 4

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1. Define

   (a) Theorem in *PD*

   A sentence **P** of *PL* is a theorem in *PD* if and only if **P** is derivable in *PD* from the empty set.

   (b) Equivalence in *PD*

   Sentences **P** and **Q** of *PL* are equivalent in *PD* if and only if **Q** is derivable in *PD* from {**P**} and **P** is derivable in *PD* from {**Q**}.

2. Construct derivations that show each of the following:

   (a) $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*

1. ¬(∀x)(∃y)Lxy	**Assum**
2. (∃y)(∀x)Lxy	**Assum**
3. (∃x)¬(∃y)Lxy	**QN 1**
4. (∃x)(∀y)¬Lxy	**QN 3**
5. (∀y)¬Lay	**Assum** /∃ **Elim**
6. (∀y)¬Lay	**Reit:** 5
7. (∀y)¬Lay	∃ **Elim:** 5, 6
8. (∀x)Lxb	**Assum** /∃ **Elim**
9. (∀x)Lxb	**Reit:** 8
10. (∀x)Lxb	∃ **Elim:** 8, 9
11. ¬Lab	∀ **Elim:** 7
12. Lab	∀ **Elim:** 10

   From lines 11 and 12 we have $\neg Lab \wedge Lab$.

   Thus $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*.

(b) $\{((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)\}$ is a theorem in *PD*

1. $(\exists x)Fx \vee (\exists x)Gx$	**Assum**
2. $(\exists x)Fx$	**Assum**
3. Fa	**Assum**
4. $Fa \vee Ga$	$\vee$ **Intro:** 3
5. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 4
6. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 2, 3–5
7. $(\exists x)Gx$	**Assum**
8. Gb	**Assum**
9. $Fb \vee Gb$	$\vee$ **Intro:** 8
10. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 9
11. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 7, 8–10
12. $(\exists x)(Fx \vee Gx)$	$\vee$ **Elim:** 1, 2–6,7–11,
13. $((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)$	$\rightarrow$ **Intro:** 1–12

3. Symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

1. $(\forall x)(\forall y)[(Fx \wedge Sy) \rightarrow Bxy]$	**Assum**
2. $(Fh \vee Ff) \wedge Ss$	**Assum**
3. $Fh \vee Ff$	$\wedge$ **Elim:** 2
4. Ss	$\wedge$ **Elim:** 2
5. $(\forall y)[(Fx \wedge Sy) \rightarrow Bhy]$	$\forall$ **Elim:** 1
6. $(Fh \wedge Ss) \rightarrow Bhs$	$\forall$ **Elim:** 5
7. $\neg(Fh \wedge Ss) \vee Bhs$	**Impl:** 5
8. $Fh \vee Ff$	**Assum**
9. Fh	**Assum**
10. $(\exists x)Fx$	$\exists$ **Intro:** 9
11. Ff	**Assum**
12. $(\exists x)Fx$	$\exists$ **Intro:** 11
13. $(\exists x)Fx$	$\vee$ **Elim:** 8, 9–10, 11–12
14. $(\exists x)Fx$	$\exists$ **Intro:** 13
15. Fh	**Assum**
16. Fh	**Reit:** 15
17. Fh	$\exists$ **Elim:** 14, 15–16
18. $Fh \wedge Ss$	$\wedge$ **Intro:** 4,17
19. Bhs	$\rightarrow$ **Elim:** 6, 18
20. $Bhs \vee Bfs$	$\vee$ **Intro:** 19
21. $(\exists x)(Bhx \vee Bfx)$	$\exists$ **Intro:** 20

4. Show that the following argument is valid.

$$\begin{array}{l} (\forall x)[(\exists y)(Byb \wedge Lxyb) \rightarrow Fx] \\ (\exists x)(Cxb \wedge Lxab) \\ \hline (\forall x)(Cxb \rightarrow \neg Fx) \rightarrow \neg Bab \end{array}$$

1. $(\forall x)[(\exists y)(Byb \wedge Lxyb) \to Fx]$	**Assum**
2. $(\exists x)(Cxb \wedge Lxab)$	**Assum**
3. $(\forall x)(Cxb \to \neg Fx)$	**Assum** / $\to$ **Intro**
4. $Ccb \wedge Lcab$	**Assum** /$\exists$ **Elim**
5. Ccb	$\wedge$**Elim 4**
6. $Lcab$	$\wedge$**Elim 4**
7. $Ccb \to \neg Fc$	$\forall$ **Elim:** 3
8. $\neg Fc$	**MP** 5, 7
9. $(\exists y)(Byb \wedge Lcyb) \to Fc$	$\forall$ **Elim:** 1
10. $\neg(\exists y)(Byb \wedge Lcyb)$	**MT** 8, 9
11. $(\forall y)\neg(Byb \wedge Lcyb)$	**QN** 10
12. $\neg(Bab \wedge Lcab)$	$\forall$ **Elim:** 11
13. $\neg Bab \vee Lcab$	**DeM** 12
14. $\neg Bab$	**DS** 6, 13
15. $\neg Bab$	$\exists$ **Elim:** 4, 5,14
16. $(\forall x)(Cxb \to \neg Fx) \to \neg Bab$	$\to$ **Intro** 3, 4-15

Thus the argument is valid.

5. Suppose that a set is inconsistent in PD. Is an argument that has the sentences in the set as premises valid in PD?

   Yes, the argument would still be valid. Validity only states that the conclusion must be derivable from the premises. Since a conclusion can still be derived, it can still be valid.

6. Show in PDE that $\{a = b, b = c\} \vdash c = a$

1. $a = b$	**Assum**
2. $b = c$	**Assum**
3. $a = c$	1, 2 =**Elim**
4. $c = b$	1, 3 =**Elim**
5. $c = a$	1, 4 =**Elim**

7. Complete the following definition in PDE:

   $(\forall x)[Rf(x)g(x) \equiv Rg(x)f(x)]$

. $(\forall x)(\forall y)(Ryx \to Rxy)$	**Assum**
. $Rg(a)f(a)$	**Assum** / $\equiv$ **Intro**
. $(\forall y)(Ryf(a) \to Rf(a)y)$	$\forall$ **Elim:** 1
. $Rg(a)f(a) \to Rf(a)g(a)$	$\forall$ **Elim:** 1
. $Rf(a)g(a)$	**MP** 2, 4
. $Rf(a)g(a)$	**Assum** / $\equiv$ **Intro**
. $(\forall y)(Ryg(a) \to Rf(a)y)$	$\forall$ **Elim:** 1
. $Rf(a)g(a) \to Rg(a)f(a)$	$\forall$ **Elim:** 1
. $Rg(a)f(a)$	**MP** 6, 8
. $Rf(a)g(a) \equiv Rg(a)f(a)$	$\equiv$ **Intro**
. $(\forall x)[Rf(x)g(x) \equiv Rg(x)f(x)]$	$\forall$ **Intro:** 10