STA 032 Homework 4

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§ 4.1

2 (1) $p_x = P(X = 1) = 0.20$

(2) $p_y = P(Y = 1) = 0.45$

(3) $p_z = P(X = 1 \cup Y = 1) = P(X = 1) + P(Y = 1) = 0.20 + 0.45 = 0.65$

(4) No, this is not possible. Each set is only one color, so X and Y are mutually exclusive.

(5) Yes, $p_z = p_x + p_y$.

(6) Yes.
If a red set is chosen,
then $X = 1$ and $Y = 0$ so $Z = 1 + 0 = 1 = X + Y$.
If a white set is chosen,
then $X = 0$ and $Y = 1$ so $Z = 0 + 1 = 1 = X + Y$.
If a blue set is chosen,
then $X = 0$ and $Y = 0$ so $Z = 0 + 0 = 0 = X + Y$.
These are the only possible choices, so by enumeration, $Z = X + Y$.

§ 4.2

8 We have $X \sim Bin(20, 0.2)$

(1) We want to find $P(X = 4)$. We can use Table A.1 and compute $f(4) - f(3) = 0.630 - 0.411 = 0.219$.
So the probability that exactly four contracts have overruns is 0.219.

(2) We want to find $P(X < 3)$.
Again, we use Table A.1 and find $f(2) = 0.206$
So the probability that fewer than three contracts have overruns is 0.206.

(3) We want to find $P(X = 0)$.
Again, we use Table A.1 and find $f(0) = 0.012$
So the probability that none of the contracts have overruns is 0.012.

(4) $\mu_X = 20(0.2) = 4$.
So the mean number of overruns is 4.

(5) $\sigma_X = \sqrt{20(0.2)(1 - 0.2)} = \sqrt{4(0.8)} = \sqrt{3.2} \approx 1.789$
So the standard deviation of the number of overruns is 1.79.

11 We have $A \sim Bin(100, 04.12)$ and $B \sim Bin(200, 0.05)$

(1) $\hat{p}_A = \frac{12}{100} = 0.12$ $\sigma_A = \sqrt{\frac{0.12(1-0.12)}{100}} \approx 0.03249$
So the estimated proportion of defective parts is 0.12 and uncertainty in this estimate is approximately 0.0325.

(2) $\hat{p}_B = \frac{10}{200} = 0.05$ $\sigma_B = \sqrt{\frac{0.05(1-0.05)}{200}} \approx 0.01541$
So the estimated proportion of defective parts is 0.05 and uncertainty in this estimate is approximately 0.0154.

(3) The estimated difference is $|\hat{p}_A - \hat{p}_B| = |0.12 - 0.05| = |0.07| = 0.07$. The uncertainty in this difference is $\sqrt{\sigma_A^2 + \sigma_B^2} = \sqrt{0.03249^2 + 0.01541^2} \approx 0.0360$.

20 We have $X \sim Bin(8, 0.8)$

(1) We want to find $P(X \leq 1)$.
We use Table A.1 and find $f(1) = 0.000$.
So the probability that no more than one policy holder in the sample has a smoke detector is 0.000.

(2) Yes, having exactly one policy holder in a sample size of 8 would be next to impossible.

(3) No, although the chances are small, it is still possible that the claim is true and the sample happened to choose mostly policy holders without a smoke detector.

(4) Again we turn to Table A.1 and find $f(6) = 0.497$.

(5) No, 6 in 8 has a probability of about 0.5. So it's a coin flip as to whether or not the sample would have six policy holders with smoke detectors.

§ 4.3 7 (1) We have $X \sim Poisson(4)$.
So we solve $P(X = 5) = e^{-4}\frac{4^5}{5!} \approx 0.15629$
Thus, the probability that 5 messages are given a minute is 0.156.

(2) We have $X \sim Poisson(4 \cdot 1.5) = Poisson(6)$.
So we solve $P(X = 9) = e^{-6}\frac{6^9}{9!} \approx 0.068838$
Thus, the probability that 9 messages are given in 1.5 minutes is 0.0688.

(3) We have $X \sim Poisson(4 \cdot 0.5) = Poisson(2)$.
So we solve

$$\begin{aligned} P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} + e^{-2}\frac{2^2}{2!} \\ &\approx 0.13533 + 0.27067 + 0.27067 \\ &\approx 0.67667 \end{aligned}$$

Thus, the probability that fewer than 3 messages are given in 30 seconds is 0.677.

8 (1) We have $X \sim Poisson(4)$.
So we solve $P(X = 3) = e^{-4}\frac{4^3}{3!} \approx 0.19536$
Thus, the probabiltiy that 3 cars arrive in a given second is 0.195.

(2) We have $X \sim Poisson(4 \cdot 3) = Poisson(12)$.
So we solve $P(X = 8) = e^{-12}\frac{12^8}{8!} \approx 0.065523$
Thus, the probabiltiy that 8 cars arrive in a three seconds is 0.0655.

(3) We have $X \sim Poisson(4 \cdot 2) = Poisson(8)$.
So we solve

$$
\begin{aligned}
P(X > 3) &= 1 - P(X \leq 3) \\
&= 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)) \\
&= 1 - \left(e^{-8}\frac{8^0}{0!} + e^{-8}\frac{8^1}{1!} + e^{-8}\frac{8^2}{2!} + e^{-8}\frac{8^3}{3!} \right) \\
&\approx 1 - (0.00033546 + 0.0026837 + 0.010735 + 0.028626) \\
&\approx 0.95761
\end{aligned}
$$

Thus, the probability that more than 3 cars arrive in 2 seconds is 0.958.

17 (1) Since we have two samples, we compute $\lambda_M = \frac{14+11}{2} = \frac{25}{2} = 12.5$
So we estimate Mom's cookies have a mean of 12.5 chips per cookie.

(2) Since we have two samples, we compute $\lambda_G = \frac{6+8}{2} = \frac{14}{2} = 7$
So we estimate Grandma's cookies have a mean of 7 chips per cookie.

(3) We solve $\sigma_{\lambda_M} = \sqrt{\frac{12.5}{2}} = \sqrt{6.25} = 2.5$.
So the uncertainty in our estimate of Mom's mean is 2.5 chips per cookie.

(4) We solve $\sigma_{\lambda_M} = \sqrt{\frac{7}{2}} = \sqrt{3.5} \approx 1.8708286933869707$.
So the uncertainty in our estimate of Grandma's mean is approximately 1.87 chips per cookie.

(5) We solve $\lambda_{M-G} = \lambda_M - \lambda_G = 12.5 - 7 = 5.5$
and $\sigma_{M-G} = \sqrt{\sigma_M^2 + \sigma_G^2} = \sqrt{6.25 + 3.5} = \sqrt{9.75} \approx 3.1225$.
So on average, we estimate that Mom's cookies have 5.5 more chips than Grandma's with an uncertainty of 3.12 chips per cookie.

§ 4.4 4 We have $X \sim Geom(0.4)$

(1) $P(X = 3) = 0.4(1 - 0.4)^{3-1} = 0.4(0.6)^2 = 0.144$.
So the probability that a car goes three days without encountering a red light at the intersection is 0.144.

(2)

$$
\begin{aligned}
P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \\
&= 0 + 0.4(1 - 0.4)^{1-1} + 0.4(1 - 0.4)^{2-1} + 0.4(1 - 0.4)^{3-1} \\
&= 0 + 0.4(0.6)^0 + 0.4(0.6)^1 + 0.4(0.6)^2 \\
&= 0 + 0.4 + 0.24 + 0.144 \\
&= 0.784
\end{aligned}
$$

So the probability that a car goes fewer than four days without encountering a red light at the intersection is 0.784.

(3) $\mu_X = \frac{1}{0.4} = 2.5$.
So the mean number of days a car goes without encountering a red light at the intersection is 2.5.

(4) $\sigma_X^2 = \frac{1-0.4}{0.4^2} = \frac{0.6}{0.4^2} = 3.75.$
So the variance is 3.75.

8 We have $X \sim Geom(0.01)$

(1) $\mu_X = \frac{1}{0.01} = 100.$
So the mean number of packages that will be filled before the process is stopped is 100.

(2) $\sigma_X^2 = \frac{1-0.01}{0.01^2} = \frac{0.99}{0.01^2} = 9900.$
So the variance is 9900.

(3) Now, we have $Y \sim NB(4, 0.01)$.
$\mu_Y = 4 \cdot \mu_X = 4 \cdot 100 = 400$
$\sigma_Y = 4 \cdot \mu_X = 4 \cdot 9900 = 39600$
So the mean number of packages that will be filled before the process is stopped is 400, with a variance of 39600.