STA 032 Homework 8

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§ 6.1

5 We have $n = 80, \overline{X} = 4.5$ and $s = 2.7$.

i. We have our hypotheses:

$$H_0 : \mu \geq 5.4$$
$$H_A : \mu < 5.4$$

ii. The test statistic is:

$$Z_s = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{4.5 - 5.4}{\frac{2.7}{\sqrt{80}}} = -2.98$$

iii. The corresponding p-value is: 0.001 40

iv. Since the p-value is so small, we reject the null hypothesis.

(1) See part iii above.

(2) I am convinced the mean number of sick days is less than 5.4 after allowing telecommuting. The probability that the sample was from a population where the mean number of sick days was greater than 5.4 is exceedingly small. It is much more likely that the sample was taken from a population with a mean number of sick days less than 5.4.

8 We have $n = 100, \overline{X} = 25$ and $s = 60$.

i. We have our hypotheses:

$$H_0 : \mu = 0$$
$$H_A : \mu \neq 0$$

ii. The test statistic is:

$$Z_s = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{25 - 0}{\frac{60}{\sqrt{100}}} = 4.17$$

iii. The corresponding p-value is: 1

iv. Since the p-value is large, we assume the null hypothesis is true.

(1) See part iii above.

(2) I am convinced that the laser is properly calibrated. Since the probability that the sample was from a population with 0 mean error is effectively 1, it makes sense that the laser is properly calibrated.

§ 6.2 9 (1) The null hypothesis should be:

$$H_0 : \mu \leq 8 \text{ years}$$

If H_0 is rejected, then it is known that the batteries will have a mean lifetime of more than 8 years and so can be installed in pacemakers.
If H_0 is not rejected, then the batteries might have a mean lifetime less than 8 years and so cannot be installed in pacemakers.

(2) The null hypothesis should be:

$$H_0 : \mu \leq 60\,000 \text{ miles}$$

If H_0 is rejected, then it is known that the tires will have a mean lifetime more than 60 000 miles and so the new material can be used to make tires.
If H_0 is not rejected, then the tires might have a mean lifetime less than 60 000 miles and so the new material cannot be used to make tires.

(3) The null hypothesis should be:

$$H_0 : \mu = 10 \text{ mL}\,\text{s}^{-1}$$

If H_0 is rejected, then it is known that the flow rate will be 10 $\text{mL}\,\text{s}^{-1}$ and so the flowmeter should be recalibrated.
If H_0 is not rejected, then the flow rate might not be 10 $\text{mL}\,\text{s}^{-1}$ and so the flowmeter should not be recalibrated.

14 i. Greater than 0.05.

We can compute the necessary values from the information given.
With the interval from 1.2 to 2.0, we can compute $\overline{X} = \frac{1.2+2.0}{2} = 1.6$.
We also know that

$$1.2 = 1.6 - 1.96\frac{s}{\sqrt{n}}$$

$$\begin{aligned} 1.96\frac{s}{\sqrt{n}} &= 0.4 \\ \frac{s}{\sqrt{n}} &= \frac{0.4}{1.96} \\ \frac{s}{\sqrt{n}} &= 0.204 \end{aligned}$$

So, we can compute a z-score

$$Z = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.6 - 1.4}{0.204} = 0.98$$

The corresponding p-value is 0.8365, which much greater than 0.05.

§ 6.3 7 We want the hypotheses:

$$H_0 : \mu \leq 0.7$$
$$H_A : \mu > 0.7$$

So $n = 150$ and $p = 0.7$.
Thus, $\hat{p} \sim N\left(150, \frac{0.7(1-0.7)}{150}\right) = N(150, 0.001\,40)$.
This lets us compute $\sigma_{\hat{p}} = \sqrt{0.001\,40} = 0.0374$, and an observed $\hat{p} = \frac{110}{150} = 0.733$.
So we have a z-score of $Z = \frac{0.733-0.7}{0.0374} = 0.8913$.
The corresponding p-value is $1 - 0.8133 = 0.1867$.
Since this value is much higher than 0.05, we do not reject the null hypothesis. We cannot conclude that more than 70% of the households in the city have high-speed internet access.

8 We want the hypotheses:

$$H_0 : \mu \geq 0.08$$
$$H_A : \mu < 0.08$$

So $n = 300$ and $p = 0.08$.
Thus, $\hat{p} \sim N\left(300, \frac{0.08(1-0.08)}{300}\right) = N(300, 0.000\,245)$.
This lets us compute $\sigma_{\hat{p}} = \sqrt{0.000\,245} = 0.0157$, and an observed $\hat{p} = \frac{12}{300} = 0.0400$.
So we have a z-score of $Z = \frac{0.0400-0.08}{0.0157} = -2.55$.
The corresponding p-value is 0.0054.
Since this value is very small, we reject the null hypothesis. We can conclude that less than 8% of the produced parts are defective.

§ 6.4 3 We have $n = 8, \overline{X} = 6.5$ and $s = 1.9$.

i. We have our hypotheses:

$$H_0 : \mu \leq 5$$
$$H_A : \mu > 5$$

ii. The test statistic is:

$$t = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{6.5 - 5}{\frac{1.9}{\sqrt{8}}} = 2.23$$

iii. The corresponding p-value is: $0.025 < P < 0.050$

iv. Since the p-value is less than 0.050, we reject the null hypothesis.

(1) See part i above.

(2) See part iii above.

(3) Since we reject the null hypothesis, the alternative hypothesis states that the mean flow rate is more than 5 gpm.
So, the pumps should be put into service.

4 We have $n = 10, \overline{X} = 23.2$ and $s = 0.2$.

i. We have our hypotheses:

$$\begin{aligned} H_0 &: \mu = 23 \\ H_A &: \mu \neq 23 \end{aligned}$$

ii. The test statistic is:

$$t = \frac{\overline{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{23.2 - 23}{\frac{0.2}{\sqrt{10}}} = 3.16$$

iii. The corresponding p-value is: $0.01 < P < 0.005$

iv. Since the p-value is less than 0.050, we reject the null hypothesis.

(1) See part i above.

(2) See part iii above.

(3) Since we reject the null hypothesis, the alternative hypothesis states that the mean percentage of potassium by weight is not 23%.
So, the process should be recalibrated.

§ 6.5 7 (1) We have $n_X = 80, \mu_X = 7.79$ and $s_X = 1.06$.
We have $n_Y = 80, \mu_Y = 7.64$ and $s_Y = 1.31$.
From this we compute

$$\overline{X} - \overline{Y} \sim N\left(\mu_X - \mu_Y, \frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}\right) = N\left(0.15, 0.03548\right)$$

A. We have our hypotheses:

$$\begin{aligned} H_0 &: \mu_X - \mu_Y \leq 0 \\ H_A &: \mu_X - \mu_Y > 0 \end{aligned}$$

B. The test statistic is:

$$z = \frac{\left(\overline{X} - \overline{Y}\right) - 0}{\sigma} = \frac{(7.79 - 7.64) - 0}{\sqrt{0.03548}} = 0.796$$

C. The corresponding p-value is: 0.2119

D. Since the p-value is not small, we do not reject the null hypothesis.

From the above, we cannot conclude that the mean score on one-tailed hypothesis test questions is higher than the mean score on two-tailed hypothesis test questions.

(2) We have $n_X = 80, \mu_X = 7.79$ and $s_X = 1.06$.
We have $n_Y = 80, \mu_Y = 7.64$ and $s_Y = 1.31$.
From this we compute

$$\overline{X} - \overline{Y} \sim N\left(\mu_X - \mu_Y, \frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}\right) = N(0.15, 0.03548)$$

A. We have our hypotheses:

$$H_0 : \mu_X - \mu_Y = 0$$
$$H_A : \mu_X - \mu_Y \neq 0$$

B. The test statistic is:

$$z = \frac{\left(\overline{X} - \overline{Y}\right) - 0}{\sigma} = \frac{(7.79 - 7.64) - 0}{\sqrt{0.03548}} = 0.796$$

C. The corresponding p-value is: 0.4238

D. Since the p-value is not small, we do not reject the null hypothesis.

From the above, we cannot conclude that the mean score on one-tailed hypothesis test questions differs from the mean score on two-tailed hypothesis test questions.

§ 6.6 12 We have $n_X = 546$ and $p_X = 87$.
We have $n_Y = 508$ and $p_Y = 74$.
And we know $X \sim Bin(546, 87)$ and $Y \sim Bin(508, 74)$.
From this we compute

$$\hat{p}_X = \frac{X}{n_X} = \frac{87}{546} = 0.15933$$
$$\hat{p}_Y = \frac{Y}{n_Y} = \frac{74}{508} = 0.14566$$
$$\hat{p} = \frac{X + Y}{n_X + n_Y} = \frac{87 + 74}{546 + 508} = 0.15274$$

i. We have our hypotheses:

$$H_0 : p_X - p_Y = 0$$
$$H_A : p_X - p_Y \neq 0$$

ii. The test statistic is:

$$z = \frac{\hat{p}_X - \hat{p}_Y}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_X}+\frac{1}{n_Y}\right)}} = \frac{0.15933 - 0.14566}{\sqrt{0.15274(1-0.15274)\left(\frac{1}{546}+\frac{1}{508}\right)}} = 0.0221$$

iii. The corresponding p-value is: 0.5871

iv. Since the p-value is not small, we do not reject the null hypothesis.

From the above, we do not reject the null hypothesis. So we cannot conclude that the proportion of boys who are overweight differs from the proportion of girls who are overweight.

§ 6.7 13 We have $n_X = 12, \mu_X = 27.3$ and $s_X = 5.2$.
We have $n_Y = 14, \mu_Y = 32.7$ and $s_Y = 4.1$.
From this we compute

$$\nu = \left\lfloor \frac{\left(\frac{s_X^2}{n_X}+\frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X-1}+\frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y-1}} \right\rfloor = \left\lfloor \frac{\left(\frac{5.2^2}{12}+\frac{4.1^2}{14}\right)^2}{\frac{\left(\frac{5.2^2}{12}\right)^2}{12-1}+\frac{\left(\frac{4.1^2}{14}\right)^2}{14-1}} \right\rfloor = \lfloor 20.84085 \rfloor = 20$$

i. We have our hypotheses:

$$H_0 : \mu_X - \mu_Y \geq 0$$
$$H_A : \mu_X - \mu_Y < 0$$

ii. The test statistic is:

$$t_{20} = \frac{\left(\overline{X}-\overline{Y}\right) - 0}{\sqrt{\frac{s_X^2}{n_X}+\frac{s_Y^2}{n_Y}}} = \frac{(27.3-32.7)-0}{\sqrt{\frac{5.2^2}{12}+\frac{4.1^2}{14}}} = -2.91$$

iii. The corresponding p-value is: $0.001 < P < 0.005$

iv. Since the p-value is small, we reject the null hypothesis.

From the above, we reject the null hypothesis. So we conclude that the mean time to sleep is less for the new drug than the old drug.

§ 6.12 4 (1) Since μ satisfies H_0, H_0 should not be rejected. This is a type I error.
(2) Since μ satisfies H_0, H_0 should not be rejected. The decision is correct.
(3) Since μ does not satisfy H_0, H_0 should be rejected. This is a type II error.
(4) Since μ does not satisfy H_0, H_0 should be rejected. The decision is correct.

5 (1) Since the claim is true, H_0 should not be rejected. This is a type I error.
(2) Since the claim is false, H_0 should be rejected. This decision is correct.
(3) Since the claim is true, H_0 should not be rejected. This decision is correct.
(4) Since the claim is false, H_0 should be rejected. This is a type II error.