MAT 167 Homework 3

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Winter 2014

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Find the dimension and basis for the four fundamental subspaces for

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. C(A)

   dimension: 2

   basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$

   C(U)

   dimension: 2

   basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

2. N(A)

   dimension: 2

   basis:9 $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   N(U)

   dimension: 2

   basis: $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. $C(A^T)$

   dimension: 2

   basis: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   $C(U^T)$

   dimension: 2

   basis: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

4. $N(A^T)$

   dimension: 1

   basis: $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

   $N(U^T)$

   dimension: 1

   basis: $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

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Why is there no matrix whose row space and nullspace both contain (1,1,1)?

   The only vector in common between the row space and null space is the zero vector, since these two spaces are orthogonal by definition.

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Find the rank of A and write the matrix as uv^T

1.
$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}$$

   rank: 1

$$A = uv^T = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$$

rank: 1

$$A = uv^T = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -2 \end{bmatrix}$$

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2.5

1

2

3

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5

10

2.6

6

14

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