PHIL 112 Homework 4

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1. Define
   (a) Theorem in *PD*
   A sentence **P** of *PL* is a theorem in *PD* if and only if **P** is derivable in *PD* from the empty set.
   (b) Equivalence in *PD*
   Sentences **P** and **Q** of *PL* are equivalent in *PD* if and only if **Q** is derivable in *PD* from {**P**} and **P** is derivable in *PD* from {**Q**}.
2. Construct derivations that show each of the following:
   (a) $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*

1. ¬(∀x)(∃y)Lxy	**Assum**
2. (∃y)(∀x)Lxy	**Assum**
3. (∃x)¬(∃y)Lxy	**QN 1**
4. (∃x)(∀y)¬Lxy	**QN 3**
5. (∀y)¬Lay	**Assum** /∃ **Elim**
6. (∀y)¬Lay	**Reit:** 5
7. (∀y)¬Lay	∃ **Elim:** 5, 6
8. (∀x)Lxb	**Assum** /∃ **Elim**
9. (∀x)Lxb	**Reit:** 8
10. (∀x)Lxb	∃ **Elim:** 8, 9
11. ¬Lab	∀ **Elim:** 7
12. Lab	∀ **Elim:** 10

   From lines 11 and 12 we have $\neg Lab \land Lab$.
   Thus $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*.

(b) $\{((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)\}$ is a theorem in PD

1. $(\exists x)Fx \vee (\exists x)Gx$	**Assum**
2. $(\exists x)Fx$	**Assum**
3. Fa	**Assum**
4. $Fa \vee Ga$	$\vee$ **Intro:** 3
5. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 4
6. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 2, 3–5
7. $(\exists x)Gx$	**Assum**
8. Gb	**Assum**
9. $Fb \vee Gb$	$\vee$ **Intro:** 8
10. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 9
11. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 7, 8–10
12. $(\exists x)(Fx \vee Gx)$	$\vee$ **Elim:** 1, 2–6,7–11,
13. $((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)$	$\rightarrow$ **Intro:** 1–12

3. Symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

1. $(\forall x)(\forall y)[(Fx \wedge Sy) \rightarrow Bxy]$	**Assum**
2. $(Fh \vee Ff) \wedge Ss$	**Assum**
3. $Fh \vee Ff$	$\wedge$ **Elim:** 2
4. Ss	$\wedge$ **Elim:** 2
5. $(\forall y)[(Fx \wedge Sy) \rightarrow Bhy]$	$\forall$ **Elim:** 1
6. $(Fh \wedge Ss) \rightarrow Bhs$	$\forall$ **Elim:** 5
7. $\neg(Fh \wedge Ss) \vee Bhs$	**Impl:** 5
8. $Fh \vee Ff$	**Assum**
9. Fh	**Assum**
10. $(\exists x)Fx$	$\exists$ **Intro:** 9
11. Ff	**Assum**
12. $(\exists x)Fx$	$\exists$ **Intro:** 11
13. $(\exists x)Fx$	$\vee$ **Elim:** 8, 9–10, 11–12
14. $(\exists x)Fx$	$\exists$ **Intro:** 13
15. Fh	**Assum**
16. Fh	**Reit:** 15
17. Fh	$\exists$ **Elim:** 14, 15–16
18. $Fh \wedge Ss$	$\wedge$ **Intro:** 4,17
19. Bhs	$\rightarrow$ **Elim:** 6, 18
20. $Bhs \vee Bfs$	$\vee$ **Intro:** 19
21. $(\exists x)(Bhx \vee Bfx)$	$\exists$ **Intro:** 20

4. Show that the following argument is valid.

$$\begin{array}{l} (\forall x)[(\exists y)(Byb \wedge Lxyb) \rightarrow Fx] \\ (\exists x)(Cxb \wedge Lxab) \\ \hline (\forall x)(Cxb \rightarrow \neg Fx) \rightarrow \neg Bab \end{array}$$

1. $(\forall x)[(\exists y)(Byb \wedge Lxyb) \rightarrow Fx]$	**Assum**
2. $(\exists x)(Cxb \wedge Lxab)$	**Assum**
3. $(\forall x)(Cxb \rightarrow \neg Fx)$	**Assum** / $\rightarrow$ **Intro**
4. $Ccb \wedge Lcab$	**Assum** /$\exists$ **Elim**
5. Ccb	$\wedge$**Elim 4**
6. $Lcab$	$\wedge$**Elim 4**
7. $Ccb \rightarrow \neg Fc$	$\forall$ **Elim:** 3
8. $\neg Fc$	**MP** 5, 7
9. $(\exists y)(Byb \wedge Lcyb) \rightarrow Fc$	$\forall$ **Elim:** 1
10. $\neg(\exists y)(Byb \wedge Lcyb)$	**MT** 8, 9
11. $(\forall y)\neg(Byb \wedge Lcyb)$	**QN** 10
12. $\neg(Bab \wedge Lcab)$	$\forall$ **Elim:** 11
13. $\neg Bab \vee Lcab$	**DeM** 12
14. $\neg Bab$	**DS** 6, 13
15. $\neg Bab$	$\exists$ **Elim:** 4, 5,14
16. $(\forall x)(Cxb \rightarrow \neg Fx) \rightarrow \neg Bab$	$\rightarrow$ **Intro** 3, 4-15

Thus the argument is valid.