

ECS 122A Homework 1

Hardy Jones
999397426
Professor Bai
Spring 2014

1. The entries of BB^T describe the edges in the graph.

For each i, j in BB^T

- if $i = j$ then it describes how many edges are at node i .
- if $i \neq j$ then it describes if there is an edge between node i and node j .

2. (a) *Proof.* Every edge e in G connects two vertices v_i, v_j . So each vertex v_i adds 1 to the total sum for every edge e it has. This means that each edge e is counted exactly twice in the sum.

Thus,

$$\sum_{v \in V} d(v) = 2|E|$$

□

- (b) *Proof.* Assume not, that is assume there is an odd number of vertices with odd degree. Call this set of vertices v_o , and the rest of the vertices v_e

We can see that $\sum_{v \in v_e} d(v)$ is even, and that $\sum_{v \in v_o} d(v)$ is odd. Also we have, $\sum_{v \in (v_e \cup v_o)} d(v)$ is odd. N.B. $v_e \cup v_o = V$. but we know that $\sum_{v \in V} d(v) = 2|E|$, so we have a contradiction.

So our assumption was wrong.

Thus, there must be an even number of vertices with odd degree.

□

- (c) No, there is no similar statement. We can have a simple directed graph from A to B , but, there is only one *indegree*, which is odd.

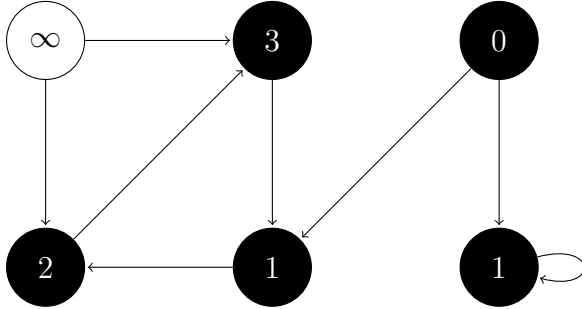
4. After running BFS on the graph, we have the following list as a result:

$$[\infty, 3, 0, 2, 1, 1]$$

where each entry corresponds to the list:

$$[1, 2, 3, 4, 5, 6]$$

As a graph, the enumerated vertices are:



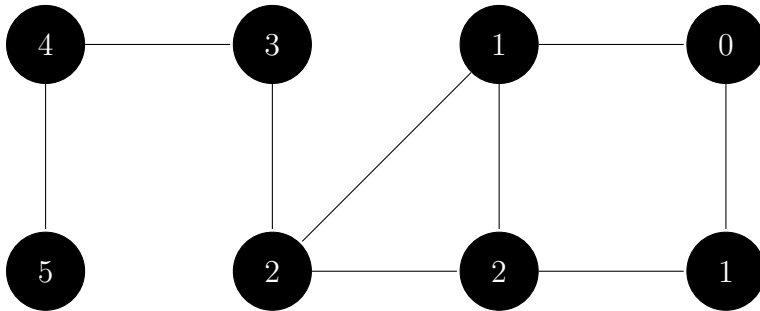
5. After running BFS on the graph, we have the following list as a result:

$$[4, 3, 1, 0, 5, 2, 2, 1]$$

where each entry corresponds to the list:

$$[r, s, t, u, v, w, x, y]$$

As a graph, the enumerated vertices are:



6. After running DFS on the graph, assuming everything in alphabetical order, we have the following graph, with the convention of (discovery time / finish time):

