ECS 120 Problem Set 1

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Problem 1 We can gain some insight by looking at the first few n-digit palindromes.

n	elements	$\lvert n\rvert$
1	1,2,3,4,5,6,7,8,9	9
2	11,22,33,44,55,66,77,88,99	9
3	101,202,303,404,505,606,707,808,909 111,212,313,414,515,616,717,818,919 ⋮ 191,292,393,494,595,696,797,898,999	90
4	1001,2002,3003,4004,5005,6006,7007,8008,9009 1111,2112,3113,4114,5115,6116,7117,8118,9119 ⋮ 1991,2992,3993,4994,5995,6996,7997,8998,9999	90

If we continue in this way we see that the number of palindromes increases by 10 for every 2 digits added.

We can see a recurrence relation here:

$$\begin{aligned} D_1 &= 9 \\ D_2 &= 9 \\ D_n &= D_{n-2} \cdot 10, \text{ for n} > 2 \end{aligned}$$

From this we can manipulate it to a closed form:

$$D_n = 9 \cdot 10^{\lfloor \frac{n-1}{2} \rfloor}$$

We can prove this by induction.

Proof. Base Cases:

$n = 1$

$$D_1 = 9 \cdot 10^{\lfloor \frac{1-1}{2} \rfloor} = 9 \cdot 10^0 = 9$$

$n = 2$

$$D_2 = 9 \cdot 10^{\lfloor \frac{2-1}{2} \rfloor} = 9 \cdot 10^0 = 9$$

Inductive Hypothesis:

$$D_n = 9 \cdot 10^{\lfloor \frac{n-1}{2} \rfloor}$$

Inductive Case:

$$\begin{aligned}
D_{n+1} &= D_{n-1} \cdot 10 \\
&= 9 \cdot 10^{\lfloor \frac{(n-1)-1}{2} \rfloor} \cdot 10 \\
&= 9 \cdot 10^{\lfloor \frac{n-2}{2} \rfloor} \cdot 10 \\
&= 9 \cdot 10^{\lfloor \frac{n}{2} - \frac{2}{2} \rfloor} \cdot 10 \\
&= 9 \cdot 10^{\lfloor \frac{n}{2} - 1 \rfloor} \cdot 10 \\
&= 9 \cdot 10^{\lfloor \frac{n}{2} - 1 + 1 \rfloor} \\
&= 9 \cdot 10^{\lfloor \frac{n}{2} \rfloor}
\end{aligned}$$

Thus, we have proved that our formula is correct.

$\square$

Now, we can calculate $D_{20} = 9 \cdot 10^{\lfloor \frac{20-1}{2} \rfloor} = 9 \cdot 10^9 = 9000000000$

Problem 2 Let's begin by enumerating some of the first few strings in lexographic order.

w_1	ϵ
w_2	0
w_3	1
w_4	00
w_5	01
w_6	10
w_7	11
w_8	000

Interestingly, this looks like the binary representation of each n in w_n without the leading digit. In the case of $n = 1$, removing the leading (only) digit leaves the empty string.

So we have a mapping: $n \in \mathbb{N} \to w_n$.

This means that $w_{1234567}$ can be easily computed. We just find the binary representation for 1234567_{10} and remove the first digit.

$$1234567_{10} = 100101101011010000111_2$$

Removing the first digit, we end up with: 00101101011010000111