MAT 168 HW 1

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1.1 The first thing we should do is try to understand the domain. The steel company wants to know how many hours to allocate to each job. Since the description does not mention that each job must be a full hour–or some minimum fraction of an hour–we assume that hours can be real valued. Since the hours are what the problem explicitly asks for, these can be our decision variables.

Call these

- $x_1 :=$ Band
- $x_2 :=$ Coil

Next we look at the other information in the statement.

- Each product has a different rate, but these are constant values not constrained by anything else in the process.

  We have

  - $r_1 = 200\frac{ton}{hour}$
  - $r_2 = 140\frac{ton}{hour}$
- Each product has a different profit, but these are constant values not constrained by anything else in the process.

  We have

  - $p_1 = 25\frac{dollar}{ton}$
  - $p_2 = 30\frac{dollar}{ton}$
- Each product has a maximum constraint on the weight that can be produced. Since the problem does not state otherwise, we assume this can be a real value.
- The objective is to maximum profit.

  We can find the profit of either product by taking the expression

$$p_i \cdot r_i \cdot x_i, \text{ for } i \in \{1, 2\}$$

With this information, we can now formalize the problem.

$$
\begin{array}{rrrl}
\text{maximize} & 5000x_1 + 4500x_2 & & \\
\text{subject to} & 200x_1 & & \leq 6000 \\
& & 140x_2 & \leq 4000 \\
& x_1 + & x_2 & \leq 40 \\
& & x_1, x_2 & \geq 0 \\
& & x_1, x_2 & \in \mathbb{R}
\end{array}
$$

It turns out what we have modeled is an instance of the fractional knapsack problem. We can see it better if we simplify the first two constraints to

$$\left.\begin{array}{rl} x_1 & \leq 30 \\ x_2 & \leq \frac{200}{7} \end{array}\right\}, \text{ respectively.}$$

Here, our "knapsack" is the number of hours to schedule. The maximum "weight" of the "knapsack" is the 40 hours available for the next week. The maximum amount of each "item" we want to take is the first two constraints. Finally, we have the amount of profit per "weight" (in this case hours).

Importantly, the decision variables in the problem are positive real values. The variables being real values allows us to formalize the model as a fractional knapsack rather than 0-1 knapsack or some other version.

Since we already know that fractional knapsack has optimal substructure and satisfies the greedy choice property, we can use a greedy approach to solve this.

Let's catalog the steps.

- We have a system of constraints

  $$
  \begin{array}{rrl}
  200x_1 & & \leq 6000 \\
  & 140x_2 & \leq 4000 \\
  x_1 + & x_2 & \leq 40 \\
  x_1 & & \geq 0 \\
  & x_2 & \geq 0
  \end{array}
  $$

  Bands are worth more from the profit perspective, so we take as many as possible. Using the first constraint, this gives us a direct number for x_1.

  Namely $200x_1 \leq 6000 \implies x_1 \leq 30$.

  This choice does not violate any other constraints, so we choose the maximum possible value x_1 can take which is 30.

- Our new system is

  $$
  \begin{array}{rl}
  140x_2 & \leq 4000 \\
  x_2 & \leq 10 \\
  x_2 & \geq 0
  \end{array}
  $$

So we take the maximum value possible for x_2 which is 10.

The profit can now be computed and we end up with the following result:

With a choice of 30 hours making Bands and 10 hours making Coils, the company can make an optimally maximized profit of $\$192,000$.

1.2 We use as our decision variables the number of tickets of each flight combination, say $x_i, i \in \{1, 2, \ldots, 9\}$.

Now we look at the rest of the information:

- Each flight has a maximum constraint on the number of passengers since the plane can only hold 30 people.
- Each flight has only so many seats available for each fare, say $s_i, i \in \{1, 2, \ldots, 9\}$.
- Each flight fare has a ticket price associated with it, say $p_i, i \in \{1, 2, \ldots, 9\}$.
- Our objective function is to maximize revenue for each flight.

So we can formalize this model as:

$$
\begin{array}{lll}
\text{maximize} & \displaystyle\sum_{i=1}^{9} p_i x_i & \\
\text{subject to} & \forall i \in \{1, 2, \ldots, 9\}, & x_i \leq s_i \\
 & \forall j \in \{0, 1, 2\}, & \displaystyle\sum_{i=3j+1}^{3j+3} x_i \leq 30 \\
 & \forall i \in \{1, 2, \ldots, 9\}, & x_i \geq 0 \\
 & \forall i \in \{1, 2, \ldots, 9\}, & x_i \in \mathbb{Z}
\end{array}
$$

We model this in ZIMPL in order to arrive at a solution.

```
set Flight := {"Ithaca_to_Newark", "Newark_to_Boston", "Ithaca_to_Boston"};
set Fare := {"Y", "B", "M"};

param prices[Fare * Flight] :=
    |"Ithaca_to_Newark", "Newark_to_Boston", "Ithaca_to_Boston"|
|"Y"|              300,              160,              360|
|"B"|              220,              130,              280|
|"M"|              100,               80,              140|;

param seating[Fare * Flight] :=
    |"Ithaca_to_Newark", "Newark_to_Boston", "Ithaca_to_Boston"|
|"Y"|                4,                8,                3|
|"B"|                8,               13,               10|
|"M"|               22,               20,               18|;

var passenger[Fare * Flight] integer >= 0;

maximize revenue: sum <fare, flight> in Fare * Flight:
    prices[fare, flight] * passenger[fare, flight];

subto seating_limit: forall <fare, flight> in Fare * Flight:
    passenger[fare, flight] <= seating[fare, flight];
subto flight_limit: forall <flight> in Flight:
    (sum <fare> in Fare: passenger[fare, flight]) <= 30;
```

Now we can use scip to find the solution:

```
SCIP> read hw1_airline.zpl

read problem <hw1_airline.zpl>
==============


base directory for ZIMPL parsing: </home/joneshf/school/ucd/mat168>

original problem has 9 variables (0 bin, 9 int, 0 impl, 0 cont) and 12 constraints
SCIP> opt

...

SCIP Status        : problem is solved [optimal solution found]
Solving Time (sec) : 0.00
Solving Nodes      : 1
Primal Bound       : +1.47100000000000e+04 (2 solutions)
Dual Bound         : +1.47100000000000e+04
Gap                : 0.00 %

SCIP> disp solution

objective value:                                14710
passenger$Y$Ithaca to Newark                        4   (obj:300)
passenger$Y$Newark to Boston                        8   (obj:160)
passenger$Y$Ithaca to Boston                        3   (obj:360)
passenger$B$Ithaca to Newark                        8   (obj:220)
passenger$B$Newark to Boston                       13   (obj:130)
passenger$B$Ithaca to Boston                       10   (obj:280)
passenger$M$Ithaca to Newark                       18   (obj:100)
passenger$M$Newark to Boston                        9   (obj:80)
passenger$M$Ithaca to Boston                       17   (obj:140)
```

So we see that the optimal revenue is $14,710, assuming the appropriate number of tickets are sold.

2-2 (a)

(b)

(c)

(d)

4-2 (a)

(b)