

# MAT 167 HW 1

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§ 1.4 2

$$\begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

7 (a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$$

9 Assuming  $A$  has as many pivots as rows, we have the following results.

(a)  $a_{11}$

(b)  $l_{i1} = \frac{a_{i1}}{a_{11}}$

(c)  $a_{ij} - a_{1j} \left( \frac{a_{i1}}{a_{11}} \right)$

(d)  $a_{22} - a_{12} \left( \frac{a_{21}}{a_{11}} \right)$

10 (a) True.

(b) False.

$AB$  may not even have three rows.

For example, let  $A$  be a  $1 \times 3$  matrix and  $B$  be a  $3 \times 3$  matrix.

Then  $AB$  is a  $1 \times 3$  matrix, so it has no third row.

(c) True.

- (d) False.  
Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Then

$$\begin{aligned} (AB)^2 &= \left( \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right)^2 \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \\ &= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \end{aligned}$$

But

$$\begin{aligned} A^2 B^2 &= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}^2 \\ &= \begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \end{aligned}$$

So  $(AB)^2 \neq A^2 B^2$ .

- 12 • Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \end{aligned}$$

- *Proof.* For any lower triangular matrices  $A, B$  with dimension  $n \times n$ , each entry  $ab_{ij}$  in  $AB$  it is computed by:

$$\sum_{k=1}^n a_{ik} b_{kj}$$

If  $i < k$ ,  $a_{ik} = 0$ .

If  $k < j$ ,  $b_{kj} = 0$ .

Each entry above the main diagonal has one of either  $i < k$  or  $k < j$ .  
 So for each entry above the main diagonal of  $AB$ , we have a sum of products  
 where at least one of the factors is 0.  
 So, each entry above the main diagonal is 0.  
 Thus, the product of any two lower triangular matrices is lower triangular.  $\square$

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