ECS 120 Problem Set 2

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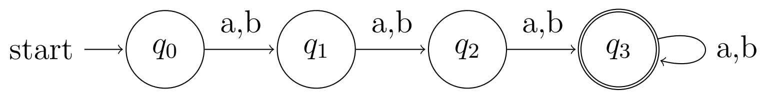
Spring 2014

Problem 1 (a)



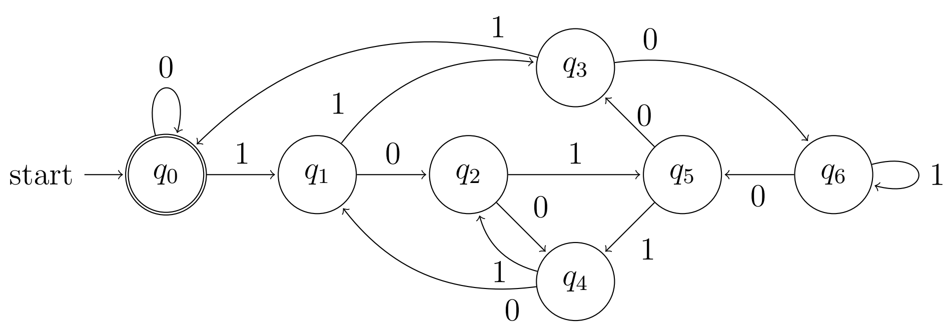
(b) The hint given provides some intuition. We have to realize what happens when we see a new binary digit. Given the current value p:

i. If the next digit is 0, then we have $2p$.
ii. If the next digit is 1, then we have $2p + 1$.

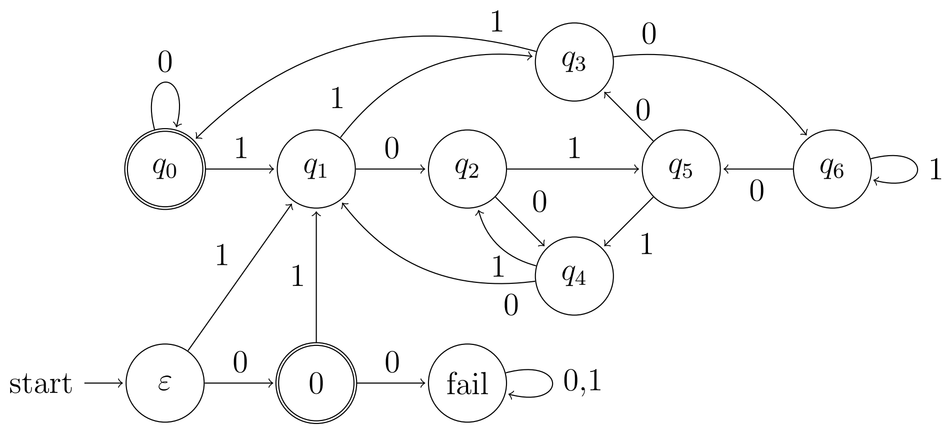
We can catalog the effect in a table.

$p \equiv n \pmod 7$	$2p \equiv n \pmod 7$	$2p + 1 \equiv n \pmod 7$
0	0	1
1	2	3
2	4	5
3	6	0
4	1	2
5	3	4
6	5	6

We can then map these directly to states, where 0 is the only accepting state.



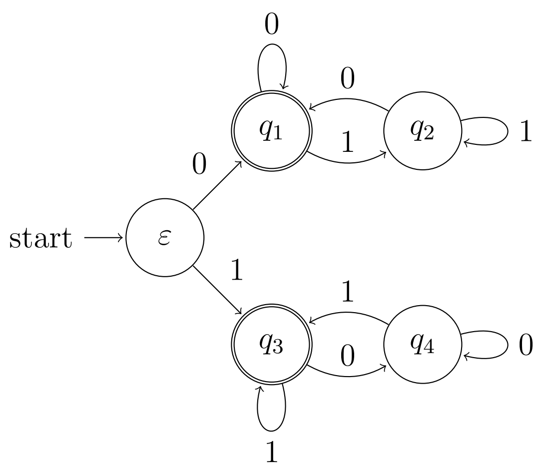
(c) We can extend the DFA presented above to ignore ε and leading zeros.



(d) For this, it is easiest to begin enumerating possibilities.

$$\{\varepsilon, 0, 1, 00, 11, 000, 010, 101, 111, 0000, \dots\}$$

What we find is that the string must contain the same starting and ending character.



Problem 2 Since Regular languages are equivalent to DFA's, we work with DFA's.

Given some DFA $M(L)$ we can construct a new DFA $M^R(L)$ that accepts L^R.

Let $L = (Q, \Sigma, \delta, q_0, F)$.

For each $q_i \in F$ we construct a new DFA $M_i = (Q, \Sigma, \delta_i, q_i, F_i)$ where

$$\delta_i ::= \forall q_j, q_k \in Q, \forall x \in \Sigma, \delta(q_j, x) = q_k \implies \delta_i(q_k, x) = q_j$$
$$F_i ::= q_0$$

Each of these DFA's will accept a reversed string from L. So, if we take the union of all of these DFA's, we have a single DFA which accepts all reversed strings from L, for DFA's are closed under union.

Problem 3 (a) True. We have two cases:

i. The DFA already has an odd number of states, in which case the statement is true.

ii. The DFA has an even number of states.
In this case, we can add another state to our DFA which is unreachable, and does not change the strings the DFA accepts. Thus we have an odd number of states in our new DFA, and it still accepts the same language as before.

Thus, Every DFA-acceptable language can be accepted by a DFA with an odd number of states.

(b) True. For any DFA where q_0 is visited multiple times, we introduce a new state q' and new transition function δ' such that

$$\delta(q_0, x) = q_0 \implies \delta'(q_0, x) = q'$$
$$\delta(q_0, x) = q_1 \implies \delta'(q', x) = q_1$$

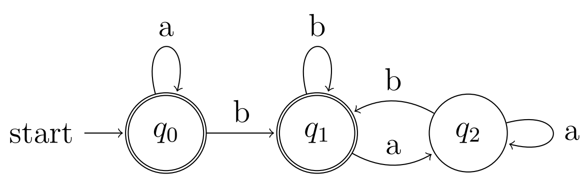
Now the start state is only visited once, and the DFA still accepts the same language.

Thus, Every DFA-acceptable language can be accepted by a DFA whose start state is never visited twice.

(c) False. Consider an infinite language $\{1^i | \ i \in \mathbb{N}\}$. The DFA for this language would have to have a state for each element in the language. In other words we create a mapping from $\mathbb{N}$ to the states in DFA. Since $\mathbb{N}$ is countably infinite, our DFA would have to have an infinite number of states. However, it would no longer be a DFA at this point.

Thus, it is not the case that every DFA-acceptable language can be accepted by a DFA no state of which is ever visited more than once.

(d) False. Assume $\delta^*(q_0, a) = q_0$, then we have this DFA:



So, while this DFA can determine if string starts and ends with b, or if it contains all a's it also accepts strings such as ab.

Thus, the assumption was false.