STA 032 Homework 3

Hardy Jones
999397426
Professor Melcon
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§ 2.4 5 (1)

x	$f(x)$
1	0.7
2	0.15
3	0.1
4	0.03
5	0.02

(2)
$$P(X \leq 2) = P(X = 1) + P(X = 2) = 0.7 + 0.15 = 0.85$$

(3)
$$P(X > 3) = P(X = 4) + P(X = 5) = 0.03 + 0.02 = 0.05$$

(4)
$$\begin{aligned}
\mu_X &= x_1 f(x_1) + x_2 f(x_2) + x_3 f(x_3) + x_4 f(x_4) + x_5 f(x_5) \\
&= 1(0.7) + 2(0.15) + 3(0.1) + 4(0.03) + 5(0.02) \\
&= 0.7 + 0.3 + 0.3 + 0.12 + 0.1 \\
&= 1.52
\end{aligned}$$

(5)
$$\begin{aligned}
\sigma_X &= \sqrt{x_1^2 f(x_1) + x_2^2 f(x_2) + x_3^2 f(x_3) + x_4^2 f(x_4) + x_5^2 f(x_5) - \mu_X^2} \\
&= \sqrt{1^2(0.7) + 2^2(0.15) + 3^2(0.1) + 4^2(0.03) + 5^2(0.02) - 1.52^2} \\
&= \sqrt{1(0.7) + 4(0.15) + 9(0.1) + 16(0.03) + 25(0.02) - 2.3104} \\
&= \sqrt{0.8696} \\
&\approx 0.93
\end{aligned}$$

8 (1) We want to find
$$P(X \leq 2) = F(2) = 0.83$$

(2) We want to find
$$P(X > 3) = F(4) - F(3) = 1.00 - 0.95 = 0.05$$

(3) We want to find

$$P(X = 1) = F(1) - F(0) = 0.72 - 0.41 = 0.31$$

(4) We want to find

$$P(X = 0) = F(0) = 0.41$$

(5) If we look at the probability of each number of error, we can answer this.

$$\begin{aligned}
&P(X = 0) = 0.41 \\
&P(X = 1) = 0.31 \\
&P(X = 2) = F(2) - F(1) = 0.83 - 0.72 = 0.11 \\
&P(X = 3) = F(3) - F(2) = 0.95 - 0.83 = 0.12 \\
&P(X = 4) = 0.05
\end{aligned}$$

Since $P(X = 0)$ has the largest probability, it is most probable that 0 errors will be detected.

15 (1)

$$\begin{aligned}
\mu_t &= \int_{-\infty}^{\infty} t f(t)\ \mathrm{d}t \\
&= 0.1 \int_{0}^{\infty} t e^{-0.1t}\ \mathrm{d}t
\end{aligned}$$

Using the tabular method for integration by parts

u	dv
t	$e^{-0.1t}$
1	$-10e^{-0.1t}$
0	$100e^{-0.1t}$

So we have

$$\begin{aligned}
\mu_t &= 0.1 \left(t(-10e^{-0.1t}) - 1(100e^{-0.1t}) \right) \Big|_0^{\infty} \\
&= 0.1 \left(-(10t + 100)(e^{-0.1t}) \right) \Big|_0^{\infty} \\
&= -\frac{t + 10}{e^{0.1t}} \Big|_0^{\infty} \\
&= 0 - (-10) \\
&= 10
\end{aligned}$$

(2)

$$\begin{aligned}
\sigma_t &= \sqrt{\int_0^\infty t^2 f(t)\ \mathrm{d}t - \mu_t^2} \\
&= \sqrt{0.1 \int_0^\infty t^2 e^{-0.1t}\ \mathrm{d}t - 10^2} \\
&= \sqrt{0.1 \int_0^\infty t^2 e^{-0.1t}\ \mathrm{d}t - 100}
\end{aligned}$$

Using the tabular method for integration by parts

u	$\mathrm{d}v$
t^2	$e^{-0.1t}$
$2t$	$-10e^{-0.1t}$
2	$100e^{-0.1t}$
0	$-1000e^{-0.1t}$

So we have

$$\begin{aligned}
\sigma_t &= \sqrt{0.1\left(t^2(-10e^{-0.1t}) - 2t(100e^{-0.1t}) + 2(-1000e^{-0.1t})\right)\Big|_0^\infty - 100} \\
&= \sqrt{0.1\left((-10t^2 - 200t - 2000)e^{-0.1t}\right)\Big|_0^\infty - 100} \\
&= \sqrt{\frac{-t^2 - 20t - 200}{e^{0.1t}}\Big|_0^\infty - 100} \\
&= \sqrt{(0 - (-200)) - 100} \\
&= \sqrt{100} \\
&= 10
\end{aligned}$$

(3)

$$\begin{aligned}
F(x) &= \int_0^x f(t)\ \mathrm{d}t \\
&= 0.1 \int_0^x e^{-0.1t}\ \mathrm{d}t \\
&= -e^{-0.1t}\Big|_0^x \\
&= -\frac{1}{e^{0.1t}}\Big|_0^x \\
&= -\frac{1}{e^{0.1x}} - (-1) \\
&= 1 - \frac{1}{e^{0.1x}}
\end{aligned}$$

(4) We want to find $P(X < 12) = F(11)$, since we are dealing with discrete random variables.

$$F(11) = 1 - \frac{1}{e^{0.1(11)}} \approx 0.6671$$

So the probability that the lifetime will be less than 12 months is approximately 66.71%.

24 (1)

(2)

(3)

(4)

(5)

(6)

(7)

§ 2.5 8 (1)

(2)

(3)

(4)

(5)

10 (1)

(2)