PHIL 112 Homework 3

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1. Explicate in terms of open and/or closed truth trees.

   (a) Quantificational validity
   An argument of **PL** is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion of the argument has a closed truth tree.

   (b) Quantificational equivalence
   Two sentences **P** and **Q** of **PL** are quantificationally equivalent if and only if the set $\{\neg(\mathbf{P} \equiv \mathbf{Q})\}$ has a closed truth tree.

2. Use the tree method to show whether:

   (a) is quantificationally true
   (b) is quantificationally valid
   (c) sentences are quantificationally equivalent
   (d) quantificational entailment holds

   (a) $[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$

1	$\neg[[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]]✓$	SM
2	$Fa \supset (\forall x)Fx✓$	1 $\neg \supset$ D
3	$\neg[(\exists x)Fx \supset (\forall x)Fx]✓$	1 $\neg \supset$ D
4	$(\exists x)Fx✓$	3 $\neg \supset$ D
5	$\neg(\forall x)Fx✓$	3 $\neg \supset$ D
6	$(\exists x)\neg Fx✓$	5 $\neg\forall$ D
7	Fb	4 $\exists$ D
8	Fc	6 $\exists$ D
9	$\neg Fa$ ○ $(\forall x)Fx$ ○	2 $\supset$ D

   Since this tree is not closed, the sentence is not quantificationally true.

(b) $\dfrac{(\forall x)[Nx \supset (\exists y)Rxy] \qquad \neg(\exists x)Rxx \wedge Na}{(\exists y)Ray}$

1	$(\forall x)[Nx \supset (\exists y)Rxy]$		SM
2	$\neg(\exists x)Rxx \wedge Na$ ✓		SM
3	$\neg(\exists y)Ray$ ✓		SM
4	$\neg(\exists x)Rxx$		2 $\wedge$ D
5	Na		2 $\wedge$ D
6	$(\forall y)\neg Ray$		3 $\neg\exists$ D
7	$Na \supset (\exists y)Ray$ ✓		1 $\forall$ D
8	$\neg Na$ $\times$	$(\exists y)Ray$ ✓	7 $\supset$ D
9		Rab	8 $\exists$ D
10		$\neg Rab$ $\times$	9 $\forall$ D

Since this tree is closed, the argument is quantificationally valid.

(c) $[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)$

1	$\neg[[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)]\checkmark$						SM
2	$(\forall x)Fx \supset Ga$		$\neg(\forall x)Fx \supset Ga\checkmark$				$1\ \neg \equiv$ D
3	$\neg(\exists x)(Fx \supset Ga)$		$(\exists x)(Fx \supset Ga)\checkmark$				$1\ \neg \equiv$ D
4			$Fb \supset Ga\checkmark$				$3\ \exists$ D
5			$\neg\neg(\forall x)Fx\checkmark$		Ga		$2 \supset$ D
6			$(\forall x)Fx$				$5\ \neg\neg$ D
7			Fb				$6\ \forall$ D
8			$\neg Fb$ $\times$	Ga $\circ$	$\neg Fb$ $\circ$	Ga $\circ$	$4 \supset$ D
9	$(\forall x)\neg(Fx \supset Ga)\checkmark$						$3\ \neg\exists$ D
10	$\neg(\forall x)Fx\checkmark$	Ga					$2 \supset$ D
11		$\neg(Fb \supset Ga)$					$9\ \forall$ D
12		Fb					$11\ \neg \supset$ D
13		$\neg Ga$ $\times$					$11\ \neg \supset$ D
14	$(\exists x)\neg Fx$						$10\ \neg\forall$ D
15	$\neg Fb$						$14\ \exists$ D
16	$\neg(Fb \supset Ga)$						$9\ \forall$ D
17	Fb						$11\ \neg \supset$ D
18	$\neg Ga$ $\times$						$11\ \neg \supset$ D

Since there is at least one open branch, this tree is not closed. Thus, the two sentences are not quantificationally equivalent.

(d) $\{(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)], Ha, a = g(a,b)\} \vDash (\exists y)Bg(y,y)$

3. Why does the rule *Existential Decomposition* require that the instantiating constant **a** be foreign to all preceding lines of the branch?

   By not requiring *Existential Decomposition* to introduce foreign constants we have opened up the possibility that the same constant can be reused in a conflicting predicate. So, we require foreign constants with *Existential Decomposition* in order to preserve truth, validity, equivalence, etc.