PHIL 112 Homework 1

Hardy Jones
999397426
Dr. Landry
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1. Specify the atomic formulas of PL.

   Every expression of PL that is either a sentence letter of PL, or an n-place predicate of PL followed by n individual terms of PL is an atomic formula of PL.

2. Give the recursive definition of 'formula of PL'.

   (a) Every atomic formula **P** is a formula of PL.

   (b) If **P** is a formula of PL, then so is $\neg$**P**.

   (c) If **P** and **Q** are formulae of PL, then so are **P**$\wedge$**Q**, **P**$\vee$**Q**, **P**$\supset$**Q**, and **P**$\equiv$**Q**.

   (d) If **P** is a formula of PL that contains at least one occurrence of **x** and no **x**-quantifier, then $\forall$**xP** and $\exists$**xP** are formulae of PL.

   (e) Nothing else is a formula of PL unless it can be made from the previous rules.

3. Indicate which of the following are formulas of PL, and which of those are sentences of PL.

   (a) Quantified Formula of PL.
   Not a Sentence of PL since the subformula has a quantified x.
   $\underline{(\forall x)}$[Fxa $\supset$ $(\forall x)$Gax]

   (b) Truth-functionally compound Formula of PL and Sentence of PL.
   $(\forall z)$Fza $\underline{\supset}$ $\neg(\exists z)$Gaz

   (c) Truth-functionally compound Formula of PL and Sentence of PL.
   $\underline{\supset}(\forall y)$Gyy

   (d) Truth-functionally compound Formula of PL.
   Not a sentence of PL for the subformula has at least one free variable.
   Faz $\underline{\supset}$ $(\forall x)$Fxa

   (e) Not a formula of PL for there is no x in Fab.
   $\neg(\exists x)$Fab

4. List all the sub-formulas of each of the following:

formula	subformulae
$(\forall x)[(\exists y)Fxy \supset Gax]$	$(\forall x)[(\exists y)Fxy \supset Gax]$
	$(\exists y)Fxy \supset Gax$
	$(\exists y)Fxy$
	Fxy
	Gax
$\neg Fab \equiv (\forall x)\neg Fxb$	$\neg Fab \equiv (\forall x)\neg Fxb$
	$\neg Fab$
	Fab
	$(\forall x)\neg Fxb$
	$\neg Fxb$
	Fxb

5. Symbolize English sentences a-d in PL, and give English readings for e-h

   (a) $(\forall x)(Tx \supset Ux)$

   (b) $(\forall x)(Tx \supset \neg Ux)$

   (c) $(\exists x)(Tx \wedge \neg Ux)$

   (d) $(\exists x)(Tx \wedge Ux)$

   (e) Sarah likes all brown toads.

   (f) Some brown toads like Sarah but not all brown toads like Sarah.

   (g) Sarah likes all toads if and only if they are brown.

   (h) Sarah does not like any toads and no toads like Sarah.

6. Symbolize a-d in PL and give English readings of e-h.

   (a) $Ccs \wedge (\exists x)((Fx \wedge Gx) \wedge Csx)$

   (b) $\neg(\exists x)[Fx \wedge (\forall y)(Ty \supset Lxy)]$

   (c) $(\forall x)[(\forall y)[(Fx \wedge Ty) \supset \neg Lxy]]$

   (d) $(\forall x)[(Fx \wedge Lsx) \supset (\forall y)(Ty \supset Lyx)]$

   (e) No frog likes any toad.

   (f) There exists at least one frog that is liked by all toads.

   (g) All green frogs like all brown toads.

   (h) Every toad that is liked by at least one frog is liked by Sarah.

7. Indicate which of the listed expressions are substitution instances of:
   '$(\exists x)\neg(\forall w)\neg Mwx$'

   (a) Substitution

   (b) Substitution

(c) Not a substitution

(d) Substitution

8. Using the symbolization key given below, symbolize English sentences a-c in PLE, and give English readings of d-f.

   (a) $Pa \wedge (\exists y)[(f(a) = y) \wedge Py]$

   (b) $(\forall x)(\exists y)[[f(x) = y] \supset Gyx]$

   (c) $(\forall x)[[Px \wedge (\exists y)[(f(x) = y) \wedge Py]] \supset x = a]$

   (d) No positive integer equals its successor.

   (e) All primes are the successor of some positive integer.

   (f) The successor function is injective.