MAT 108 HW 5

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§2.2 13 (c) $A \times B$

$\{(\varnothing, (\varnothing, \{\varnothing\})), (\varnothing, \{\varnothing\}), (\varnothing, (\{\varnothing\}, \varnothing)), (\{\varnothing\}, (\varnothing, \{\varnothing\})), (\{\varnothing\}, \{\varnothing\}), (\{\varnothing\}, (\{\varnothing\}, \varnothing)), (\{\varnothing, \{\varnothing\}\}, (\varnothing, \{\varnothing\})), (\{\varnothing, \{\varnothing\}\}, \{\varnothing\}), (\{\varnothing, \{\varnothing\}\}, (\{\varnothing\}, \varnothing))\}$

$B \times A$

$\{((\varnothing, \{\varnothing\}), \varnothing), ((\varnothing, \{\varnothing\}), \{\varnothing\}), ((\varnothing, \{\varnothing\}), \{\varnothing, \{\varnothing\}\}), (\{\varnothing\}, \varnothing), (\{\varnothing\}, \{\varnothing\}), (\{\varnothing\}, \{\varnothing, \{\varnothing\}\}), ((\{\varnothing\}, \varnothing), \varnothing), ((\{\varnothing\}, \varnothing), \{\varnothing\}), ((\{\varnothing\}, \varnothing), \{\varnothing, \{\varnothing\}\})\}$

(d) $A \times B$

$\{((2, 4), (4, 1)), ((2, 4), (2, 3)), ((3, 1), (4, 1)), ((3, 1), (2, 3))\}$

$B \times A$

$\{((4, 1), (2, 4)), ((4, 1), (3, 1)), ((2, 3), (2, 4)), ((2, 3), (3, 1))\}$

17 We're asked to show that $(a, b) = (x, y)$ iff $a = x$ and $b = y$.

Proof.

$$
\begin{aligned}
(a,b) = (x,y) &\iff \{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\} \\
&\iff (\{\{a\},\{a,b\}\} \subseteq \{\{x\},\{x,y\}\}) \\
&\qquad \wedge (\{\{x\},\{x,y\}\} \subseteq \{\{a\},\{a,b\}\}) \\
&\iff (\{a\} \in \{\{x\},\{x,y\}\}) \\
&\qquad \wedge (\{a,b\} \in \{\{x\},\{x,y\}\}) \\
&\qquad \wedge (\{x\} \in \{\{a\},\{a,b\}\}) \\
&\qquad \wedge (\{x,y\} \in \{\{a\},\{a,b\}\}) \\
&\iff (\{a\} = \{x\}) \wedge (\{a,b\} = \{x,y\}) \\
&\iff (a = x) \wedge (b = y)
\end{aligned}
$$

Since we have connected both sides with a series of bi-conditional statements, we have proven that:
$(a, b) = (x, y)$ iff $a = x$ and $b = y$. □

18 (a) *Proof.*

$$A \Delta B = (A - B) \cup (B - A) = (B - A) \cup (A - B) = B \Delta A$$

□

(b) This proof is a bit longer than the others.

Proof.

$$\begin{aligned}
A\Delta B &= (A-B)\cup(B-A) \\
&= \{x|\,(x\in A\wedge x\notin B)\vee(x\in B\wedge x\notin A)\} \\
&= \{x|\,[(x\in A\wedge x\notin B)\vee x\in B]\wedge[(x\in A\wedge x\notin B)\vee x\notin A]\} \\
&= \{x|\,(x\in A\vee x\in B)\wedge(x\notin B\vee x\in B) \\
&\qquad \wedge(x\in A\vee x\notin A)\wedge(x\notin B\vee x\notin A)\} \\
&= \{x|\,(x\in A\vee x\in B)\wedge(x\notin B\vee x\notin A)\} \\
&= \{x|\,(x\in A\vee x\in B)\wedge(x\notin A\vee x\notin B)\} \\
&= \{x|\,(x\in A\vee x\in B)\wedge\sim(x\in A\wedge x\in B)\} \\
&= \{x|\,(x\in A\cup B)\wedge\sim(x\in A\cap B)\} \\
&= \{x|\,(x\in A\cup B)\wedge(x\notin A\cap B)\} \\
&= (A\cup B)-(A\cap B)
\end{aligned}$$

□

(c) *Proof.*

$$A\Delta A=(A-A)\cup(A-A)=\varnothing\cup\varnothing=\varnothing$$

□

(d) *Proof.*

$$A\Delta\varnothing=(A-\varnothing)\cup(\varnothing-A)=A\cup\varnothing=A$$

□

§2.3 1 (f)

$$\bigcup_{i=1}^{10}A_i=\{1,2,\ldots,19\},\bigcap_{i=1}^{10}A_i=\varnothing$$

(h)

$$\bigcup_{r\in(0,\infty)}A_r=[-\pi,\infty),\quad\bigcap_{r\in(0,\infty)}A_r=[-\pi,0)$$

(j)

$$\bigcup_{i=1}^{\infty}M_i=\mathbb{Z},\bigcap_{i=1}^{\infty}M_i=\{0\}$$

12 Let $A_n=(0,\frac{1}{n})$.

Then for any $m,n\in\mathbb{N}$

$$M_m\cap M_n=\begin{cases}\left(0,\frac{1}{m}\right), & \text{if } m<n\\ \left(0,\frac{1}{n}\right), & \text{otherwise}\end{cases}$$

But, $\bigcap_{i=1}^{\infty}M_i=\varnothing$

15 (e) *Proof.* Choose an arbitrary $x \in \bigcup_{i=1}^{k} A_i$.

Then there exists some $l \in \mathbb{N}$ such that $l \leq k$ and $x \in A_l$.

Now, since $l \leq k, l \leq m$, so $A_l \subseteq \bigcup_{i=1}^{m} A_i$, and $x \in \bigcup_{i=1}^{m} A_i$.

Since the choice of x was arbitrary, this works for all $x \in \bigcup_{i=1}^{k} A_i$.

Then every x contained in $\bigcup_{i=1}^{k} A_i$ is also in $\bigcup_{i=1}^{m} A_i$.

Thus $\bigcup_{i=1}^{k} A_i \subseteq \bigcup_{i=1}^{m} A_i$ □

(f) *Proof.* Choose an arbitrary $x \in \bigcap_{i=1}^{m} A_i$.

Then for all $l \in \{1, 2, \ldots, k, k+1, \ldots, m\}, x \in A_l$.

This implies that for all $l \in \{1, 2, \ldots, k\}, x \in A_l$.

Which means that $x \in \bigcap_{i=1}^{k} A_i$.

Since the choice of x was arbitrary, this works for all $x \in \bigcap_{i=1}^{m} A_i$.

Then every x contained in $\bigcap_{i=1}^{m} A_i$ is also in $\bigcap_{i=1}^{k} A_i$.

Thus, $\bigcap_{i=1}^{m} A_i \subseteq \bigcap_{i=1}^{k} A_i$. □

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17 (c)

(d)

§2.4 6 (i)

(k)

7 (l)

(m)

8 (h)

12 (b)