MAT 125A HW 6

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6.2 11 (a) Let $h_n = f_n + g_n$

Proof. For any $\epsilon > 0$, since f_n, g_n are uniformly convergent, we know that there exists some $M, N \in \mathbb{N}$ such that for all $m, n \in \mathbb{N}$, with $m \geq M, n \geq N$, we have $|f_m(x) - f(x)| < \frac{\epsilon}{2}, |g_n(x) - g(x)| < \frac{\epsilon}{2}$
Choose $P = max(M, N)$.
Then we have for all $p \geq P \in \mathbb{N}$.

$$\begin{aligned}
|h_p(x) - h(x)| &= |(f_p + g_p)(x) - (f + g)(x)| \\
&= |f_p(x) + g_p(x) - f(x) - g(x)| \\
&= |f_p(x) - f(x) + g_p(x) - g(x)| \\
&\leq |f_p(x) - f(x)| + |g_p(x) - g(x)| \\
&< \frac{\epsilon}{2} + \frac{\epsilon}{2} \\
&= \epsilon
\end{aligned}$$

So h_n is uniformly convergent. □

(b) *Proof.* Let $f_n(x) = x, g_n(x) = \frac{1}{n}$.
So $\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} x = x$, and $\lim_{n\to\infty} g_n(x) = \lim_{n\to\infty} \frac{1}{n} = 0$.
Then, for any $\epsilon > 0$, choose $M = 1$, then for all $m > 1 \in \mathbb{N}$

$$|f_m(x) - f(x)| = |x - x| = 0 < \epsilon$$

Also, for any $\epsilon > 0$, choose $N = 1$, then for all $n > 1 \in \mathbb{N}$

$$|g_n(x) - f(x)| = |x - x| = 0 < \epsilon$$

□

(c)

15 (a)

(b)

16 (a)

(b)

		(c)
6.3	2	(a)
		(b)
	5	
6.4	1	(a)
		(b)
	2	(a)
		(b)
		(c)
		(d)
	5	
	6	
	7	(a)
		(b)