MAT 67 Homework 5

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Fall 2013

1. Let V be a vector space over $\mathbb{F}$, and suppose that the list $(v_1, v_2, ..., v_n)$ of vectors spans V, where each $v_i \in V$. Prove that the list

$$(v_1 - v_2, v_2 - v_3, v_3 - v_4, ..., v_{n-2} - v_{n-1}, v_{n-1} - v_n, v_n)$$

also spans V.

Proof. Each $v_j \in (v_1, v_2, ..., v_n)$ can be constructed from our new list.

$$\begin{aligned}
v_1 &= (v_1 - v_2) + (v_2 - v_3) + (v_3 - v_4) + ... + (v_{n-2} - v_{n-1}) + (v_{n-1} - v_n) + v_n \\
&= (v_1 - \cancel{v_2}) + (\cancel{v_2} - \cancel{v_3}) + (\cancel{v_3} - \cancel{v_4}) + ... + (\cancel{v_{n-2}} - \cancel{v_{n-1}}) + (\cancel{v_{n-1}} - \cancel{v_n}) + \cancel{v_n} \\
&= v_! \\
v_2 &= 0(v_1 - v_2) + (v_2 - v_3) + (v_3 - v_4) + ... + (v_{n-2} - v_{n-1}) + (v_{n-1} - v_n) + v_n \\
&= (v_2 - \cancel{v_3}) + (\cancel{v_3} - \cancel{v_4}) + ... + (\cancel{v_{n-2}} - \cancel{v_{n-1}}) + (\cancel{v_{n-1}} - \cancel{v_n}) + \cancel{v_n} \\
&= v_2 \\
&\vdots \\
v_n &= 0(v_1 - v_2) + 0(v_2 - v_3) + 0(v_3 - v_4) + ... + 0(v_{n-2} - v_{n-1}) + 0(v_{n-1} - v_n) + v_n \\
&= v_n
\end{aligned}$$

Since we see that we can generate each one of these, we can generate the entire list $(v_!, v_2, ..., v_n)$, which spans V. So, $span(v_1 - v_2, v_2 - v_3, ..., v_{n-1} - v_n, v_n) = V$ □

2. Let V be a nite-dimensional vector space over $\mathbb{F}$ with $dim(V) = n$ for some $n \in \mathbb{Z}^+$. Prove that there are n one-dimensional subspaces $U_1, U_2, ..., U_n$ of V such that

$$V = U_1 \oplus U_2 \oplus \quad \oplus U_n$$

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