ECS 120 Problem Set 2

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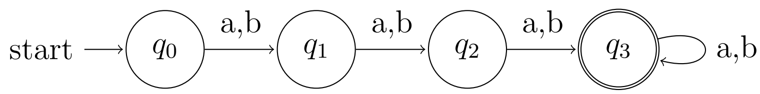


Problem 1 (a)

(b) The hint given provides some intuition. We have to realize what happens when we see a new binary digit. Given the current value p:

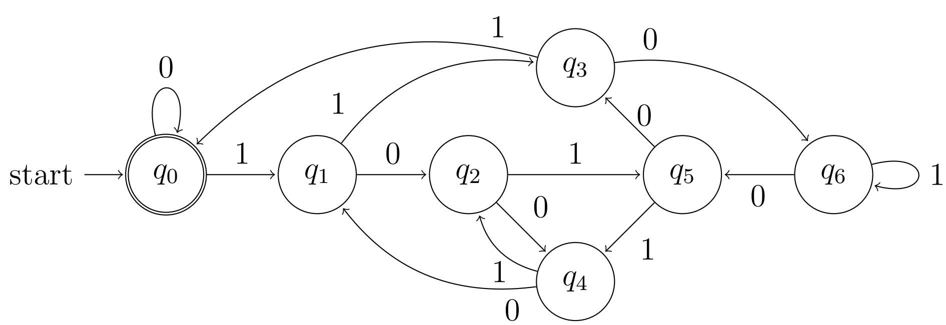
i. If the next digit is 0, then we have $2p$.

ii. If the next digit is 1, then we have $2p + 1$.

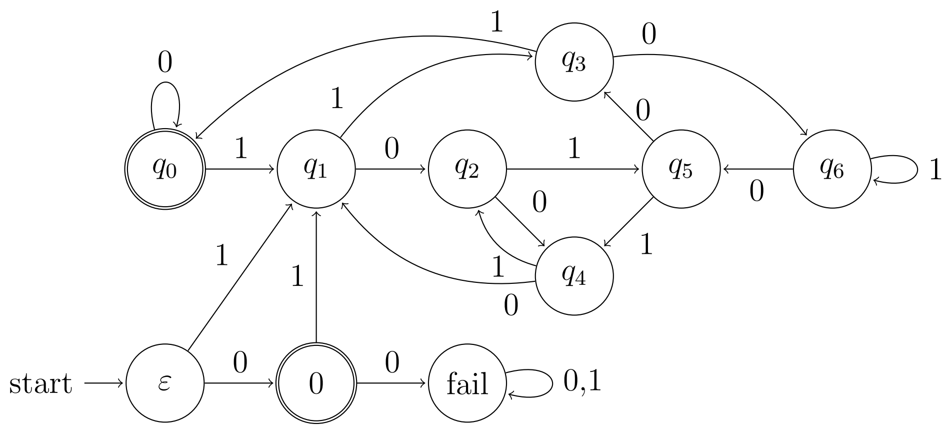
We can catalog the effect in a table.

$p \equiv n \pmod 7$	$2p \equiv n \pmod 7$	$2p + 1 \equiv n \pmod 7$
0	0	1
1	2	3
2	4	5
3	6	0
4	1	2
5	3	4
6	5	6

We can then map these directly to states, where 0 is the only accepting state.



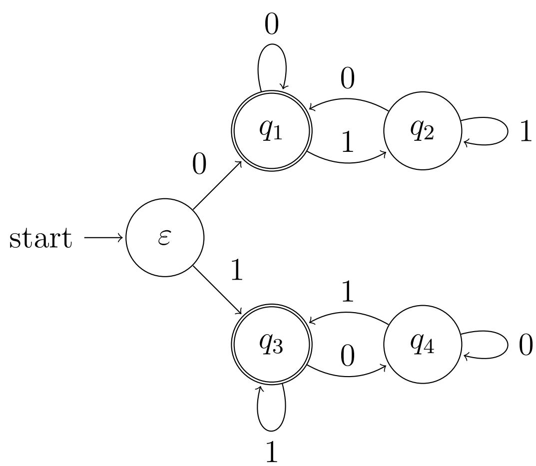
(c) We can extend the DFA presented above to ignore ε and leading zeros.



(d) For this, it is easiest to begin enumerating possibilities.

$$\{\varepsilon, 0, 1, 00, 11, 000, 010, 101, 111, 0000, \dots\}$$

What we find is that the string must contain the same starting and ending character.



Problem 2 Since Regular languages are equivalent to DFA's, we work with DFA's.

Given some DFA $M(L)$ we can construct a new DFA $M^R(L)$ that accepts L^R.

Let $L = (Q, \Sigma, \delta, q_0, F)$.

For each $q_i \in F$ we construct a new DFA $M_i = (Q, \Sigma, \delta_i, q_i, F_i)$ where

$$\delta_i ::= \forall q_j, q_k \in Q, \forall x \in \Sigma, \delta(q_j, x) = q_k \implies \delta_i(q_k, x) = q_j$$
$$F_i ::= q_0$$

Each of these DFA's will accept a reversed string from L. So, if we take the union of all of these DFA's, we have a single DFA which accepts all reversed strings from L, for DFA's are closed under union.