MAT 25 Homework 5

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1. 2.2.1

   (a) $\lim \frac{1}{6n^2+1} = 0$
   We need to show

$$\begin{aligned} \frac{1}{6n^2+1} &< \epsilon \\ \frac{1}{\epsilon} &< 6n^2+1 \\ \frac{1}{\epsilon} - 1 &< 6n^2 \\ \frac{1-\epsilon}{\epsilon} &< 6n^2 \\ \frac{1-\epsilon}{6\epsilon} &< n^2 \\ \sqrt{\frac{1-\epsilon}{6\epsilon}} &< n \end{aligned}$$

   Let $\epsilon > 0$. Choose $N \in \mathbb{N} | N > \sqrt{\frac{1-\epsilon}{6\epsilon}}$.
   Let $n \geq N$. So, $n \geq N > \sqrt{\frac{1-\epsilon}{6\epsilon}} \implies \frac{1}{6n^2+1} < \epsilon$
   Thus $|a_n - 0| < \epsilon$.

   (b) $\lim \frac{3n+1}{2n+5} = \frac{3}{2}$
   We need to show

$$\begin{aligned} \frac{3n+1}{2n+5} &< \epsilon \\ 3n+1 &< 2n\epsilon + 5\epsilon \\ 1 - 5\epsilon &< (2\epsilon - 3)n \\ \frac{1-5\epsilon}{2\epsilon-3} &< n \end{aligned}$$

   Let $\epsilon > 0$. Choose $N \in \mathbb{N} | N > \frac{1-5\epsilon}{2\epsilon-3}$.
   Let $n \geq N$. So, $n \geq N > \frac{1-5\epsilon}{2\epsilon-3} \implies \frac{3n+1}{2n+5} < \epsilon$.
   Thus $|a_n - \frac{3}{2}| < \epsilon$.

(c) $\lim \frac{2}{\sqrt{n+3}} = 0$

We need to show

$$\begin{aligned} \frac{2}{\sqrt{n+3}} &< \epsilon \\ \frac{2}{\epsilon} &< \sqrt{n+3} \\ \frac{4}{\epsilon^2} &< n+3 \\ \frac{4}{\epsilon^2} - 3 &< n \end{aligned}$$

Let $\epsilon > 0$. Choose $N \in \mathbb{N} | N > \frac{4}{\epsilon^2} - 3$.

Let $n \geq N$. So, $n \geq N > \frac{4}{\epsilon^2} - 3 \implies \frac{2}{\sqrt{n+3}} < \epsilon$.

Thus $|a_n - 0| < \epsilon$.

2. 2.2.5

(a) $a_n = \left\lfloor \frac{1}{n} \right\rfloor$

It is easy to see that after the first element in the sequence, all values are 0.

$\lim a_n = 0$

Proof. Let $\epsilon > 0$. Choose $N > 1$.

Let $n \geq N$. So, $n \geq N > 1 \implies \left\lfloor \frac{1}{n} \right\rfloor = 0 < \epsilon$.

Thus, $|a_n - 0| < \epsilon$. □

(b) $a_n = \left\lfloor \frac{10+n}{2n} \right\rfloor$

Again we see that after some elements all values are 0.

$\lim a_n = 0$

Proof. Let $\epsilon > 0$. Choose $N > 10$.

Let $n \geq N$. So, $n \geq N > 10 \implies \left\lfloor \frac{10+n}{2n} \right\rfloor = 0 < \epsilon$.

Thus, $|a_n - 0| < \epsilon$. □

3. 2.2.7

(a) A sequence (a_n) diverges to ∞ if, for every positive number ϵ, there exists an $N \in \mathbb{N}$ such that whenever $n \geq N$ it follows that $|a_n| > \epsilon$

$\lim \sqrt{n} = \infty$

Proof. Let $\epsilon > 0$.

We want to show

$$\begin{aligned} \sqrt{n} &> \epsilon \\ n &> \epsilon^2 \end{aligned}$$

Choose $N > \epsilon^2$.
Let $n \geq N$. So, $n \geq N > \epsilon^2 \implies \sqrt{n} > \epsilon$.
Thus, $|a_n| > \epsilon$. $\square$