MAT 168 Calculation 1

Hardy Jones
999397426
Professor Köppe
Spring 2015

2.1 We first start by rewriting as a dictionary:

$$\begin{array}{rrl} \text{maximize} & \zeta & = 0 + 6x_1 + 8x_2 + 5x_3 + 9x_4 \\ \text{subject to} & x_5 & = 5 - 2x_1 - \ \ x_2 - \ \ x_3 - 3x_4 \\ & x_6 & = 3 - \ \ x_1 - 3x_2 - \ \ x_3 - 2x_4 \end{array}$$

Then we can begin by entering with the largest variable, x_4.

We look at the constraints and see:

- $x_5 \implies x_4 \leq \frac{5}{3}$
- $x_6 \implies x_4 \leq \frac{3}{2}$

The more restrictive constraint is that $x_4 \leq \frac{3}{2}$, so set $x_4 = \frac{3}{2}$.

So we can let x_4 enter and x_6 exit.

$$x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_6$$

This gives a new value for x_5.

$$x_5 = \frac{1}{2} - \frac{1}{2}x_1 + \frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{3}{2}x_6$$

This gives a new value for ζ.

$$\zeta = \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}x_6$$

So we have a new dictionary:

$$
\begin{array}{lrl}
\text{maximize} & \zeta & = \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}x_6 \\
\text{subject to} & x_5 & = \frac{1}{2} - \frac{1}{2}x_1 + \frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{3}{2}x_6 \\
& x_4 & = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_6
\end{array}
$$

Now, we can continue optimizing since x_1 has a positive coefficient.

We look at the constraints and see:

- $x_5 \implies x_1 \leq 1$
- $x_4 \implies x_1 \leq 3$

The more restrictive constraint is that $x_1 \leq 1$, so set $x_1 = 1$.

So we can let x_1 enter and x_5 exit.

$$x_1 = 1 + 7x_2 + x_3 + 3x_6 - 2x_5$$

This gives a new value for x_4.

$$x_4 = 1 - 5x_2 - x_3 - 2x_6 + x_5$$

This gives a new value for ζ.

$$\zeta = 15 + 5x_2 + 2x_3 - 3x_5$$

So we have a new dictionary:

$$
\begin{array}{lrl}
\text{maximize} & \zeta & = 15 + 5x_2 + 2x_3 \qquad - 3x_5 \\
\text{subject to} & x_1 & = 1 + 7x_2 + x_3 + 3x_6 - 2x_5 \\
& x_4 & = 1 - 5x_2 - x_3 - 2x_6 + x_5
\end{array}
$$

Now, we can continue optimizing since x_2 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_2 \geq 0$
- $x_4 \implies x_2 \leq \frac{1}{5}$

The more restrictive constraint is that $x_2 \le \frac{1}{5}$, so set $x_2 = \frac{1}{5}$.

So we can let x_2 enter and x_4 exit.

$$x_2 = \frac{1}{5} - \frac{1}{5}x_4 - \frac{1}{5}x_3 - \frac{2}{5}x_6 + \frac{1}{5}x_5$$

This gives a new value for x_1.

$$x_1 = \frac{12}{5} - \frac{7}{5}x_4 - \frac{2}{5}x_3 + \frac{1}{5}x_6 - \frac{3}{5}x_5$$

This gives a new value for ζ.

$$\zeta = 16 - x_4 + x_3 - 2x_6 - 2x_5$$

So we have a new dictionary:

$$\begin{array}{rrl}
\text{maximize} & \zeta & = 16 - x_4 + x_3 - 2x_6 - 2x_5 \\
\text{subject to} & x_1 & = \frac{12}{5} - \frac{7}{5}x_4 - \frac{2}{5}x_3 + \frac{1}{5}x_6 - \frac{3}{5}x_5 \\
 & x_2 & = \frac{1}{5} - \frac{1}{5}x_4 - \frac{1}{5}x_3 - \frac{2}{5}x_6 + \frac{1}{5}x_5
\end{array}$$

Now, we can continue optimizing since x_3 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_3 \le 6$
- $x_2 \implies x_3 \le 1$

The more restrictive constraint is that $x_3 \le 1$, so set $x_3 = 1$.

So we can let x_3 enter and x_2 exit.

$$x_3 = 1 - x_4 - 5x_2 - 2x_6 + x_5$$

This gives a new value for x_1.

$$x_1 = 2 - x_4 + 2x_2 + x_6 - x_5$$

This gives a new value for ζ.

$$\zeta = 17 - 2x_4 - 5x_2 - 4x_6 - x_5$$

So we have a new dictionary:

$$\begin{array}{rl} \text{maximize} & \zeta \;\; = 17 - 2x_4 - 5x_2 - 4x_6 - x_5 \\ \text{subject to} & x_1 = 2 \;\; - \;\; x_4 + 2x_2 + \;\; x_6 - x_5 \\ & x_3 = 1 \;\; - \;\; x_4 - 5x_2 - 2x_6 + x_5 \end{array}$$

Since we have no more optimizable variables (all variable coefficients of ζ are non-positive), we can no longer maximize ζ.

Then we have an optimal solution with $x_1 = 2, x_3 = 1, x_2 = x_4 = x_5 = x_6 = 0$, and value 17.

2.2 We first start by rewriting as a dictionary:

$$\begin{array}{rl} \text{maximize} & \zeta \;\; = 0 + 2x_1 + \;\; x_2 \\ \text{subject to} & x_3 = 4 - 2x_1 - \;\; x_2 \\ & x_4 = 3 - 2x_1 - 3x_2 \\ & x_5 = 5 - 4x_1 - \;\; x_2 \\ & x_6 = 1 - \;\; x_1 - 5x_2 \end{array}$$

Then we can begin by entering with the largest variable, x_1.

We look at the constraints and see:

- $x_3 \implies x_1 \leq 2$
- $x_4 \implies x_1 \leq \frac{3}{2}$
- $x_5 \implies x_1 \leq \frac{5}{4}$
- $x_6 \implies x_1 \leq 1$

The more restrictive constraint is that $x_1 \leq 1$, so set $x_1 = 1$.

So we can let x_1 enter and x_6 exit.

$$x_1 = 1 + 5x_2 - x_6$$

This gives a new value for x_3.

$$x_3 = 2 - 9x_2 + 2x_6$$

This gives a new value for x_4.

$$x_4 = 1 + 7x_2 + 2x_6$$

This gives a new value for x_5.

$$x_5 = 1 + 19x_2 + 4x_6$$

This gives a new value for ζ.

$$\zeta = 2 - 9x_2 - 2x_6$$

So we have a new dictionary:

$$\begin{array}{rrl} \text{maximize} & \zeta & = 2 - 9\ x_2 - 2x_6 \\ \text{subject to} & x_3 & = 2 - 9\ x_2 + 2x_6 \\ & x_4 & = 1 + 7\ x_2 + 2x_6 \\ & x_5 & = 1 + 19x_2 + 4x_6 \\ & x_1 & = 1 + 5\ x_2 - \ \ x_6 \end{array}$$

Since we have no more optimizable variables (all variable coefficients of ζ are non-positive), we can no longer maximize ζ.

Then we have an optimal solution with $x_1 = 1, x_2 = x_3 = x_4 = x_5 = x_6 = 0$, and value 2.

2.3