

MAT 67 Homework 5

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1. Let V be a vector space over \mathbb{F} , and suppose that the list (v_1, v_2, \dots, v_n) of vectors spans V , where each $v_i \in V$. Prove that the list

$$(v_1 - v_2, v_2 - v_3, v_3 - v_4, \dots, v_{n-2} - v_{n-1}, v_{n-1} - v_n, v_n)$$

also spans V .

Proof. Each $v_j \in (v_1, v_2, \dots, v_n)$ can be constructed from our new list.

$$\begin{aligned} v_1 &= (v_1 - v_2) + (v_2 - v_3) + (v_3 - v_4) + \dots + (v_{n-2} - v_{n-1}) + (v_{n-1} - v_n) + v_n \\ &= (v_1 - \cancel{v_2}) + (\cancel{v_2} - \cancel{v_3}) + (\cancel{v_3} - \cancel{v_4}) + \dots + (\cancel{v_{n-2}} - \cancel{v_{n-1}}) + (\cancel{v_{n-1}} - \cancel{v_n}) + \cancel{v_n} \\ &= v_1 \\ v_2 &= 0(v_1 - v_2) + (v_2 - v_3) + (v_3 - v_4) + \dots + (v_{n-2} - v_{n-1}) + (v_{n-1} - v_n) + v_n \\ &= (v_2 - \cancel{v_3}) + (\cancel{v_3} - \cancel{v_4}) + \dots + (\cancel{v_{n-2}} - \cancel{v_{n-1}}) + (\cancel{v_{n-1}} - \cancel{v_n}) + \cancel{v_n} \\ &= v_2 \\ &\vdots \\ v_n &= 0(v_1 - v_2) + 0(v_2 - v_3) + 0(v_3 - v_4) + \dots + 0(v_{n-2} - v_{n-1}) + 0(v_{n-1} - v_n) + v_n \\ &= v_n \end{aligned}$$

Since we see that we can generate each one of these...

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