PHIL 112 Homework 3

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1. Explicate in terms of open and/or closed truth trees.

   (a) Quantificational validity
   An argument of **PL** is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion of the argument has a closed truth tree.

   (b) Quantificational equivalence
   Two sentences **P** and **Q** of **PL** are quantificationally equivalent if and only if the set $\{\neg(\mathbf{P} \equiv \mathbf{Q})\}$ has a closed truth tree.

2. Use the tree method to show whether:

   (a) is quantificationally true
   (b) is quantificationally valid
   (c) sentences are quantificationally equivalent
   (d) quantificational entailment holds

   (a) $[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$

1	$\neg[[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]]$✓	SM
2	$Fa \supset (\forall x)Fx$✓	$1 \neg \supset$ D
3	$\neg[(\exists x)Fx \supset (\forall x)Fx]$✓	$1 \neg \supset$ D
4	$(\exists x)Fx$✓	$3 \neg \supset$ D
5	$\neg(\forall x)Fx$✓	$3 \neg \supset$ D
6	$(\exists x)\neg Fx$✓	$5 \neg\forall$ D
7	Fb	$4 \exists$ D
8	Fc	$6 \exists$ D
9	$\neg Fa$ ∘ $(\forall x)Fx$ ∘	$2 \supset$ D

Since this tree is not closed, the sentence is not quantificationally true.

(b) $$\frac{\begin{array}{l}(\forall x)[Nx \supset (\exists y)Rxy] \\ \neg(\exists x)Rxx \wedge Na\end{array}}{(\exists y)Ray}$$

1	$(\forall x)[Nx \supset (\exists y)Rxy]$		SM
2	$\neg(\exists x)Rxx \wedge Na\checkmark$		SM
3	$\neg(\exists y)Ray\checkmark$		SM
4	$\neg(\exists x)Rxx$		$2 \wedge$ D
5	Na		$2 \wedge$ D
6	$(\forall y)\neg Ray$		$3 \neg\exists$ D
7	$Na \supset (\exists y)Ray\checkmark$		$1 \forall$ D
8	$\neg Na$ $\times$	$(\exists y)Ray\checkmark$	$7 \supset$ D
9		Rab	$8 \exists$ D
10		$\neg Rab$ $\times$	$9 \forall$ D

Since this tree is closed, the argument is quantificationally valid.

One interpretation is:
UD: The natural numbers
Nx: x is positive
Rxy: x is less than y
a: 1

(c) $[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)$

1	$\neg[[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)]$✓						SM
2	$(\forall x)Fx \supset Ga$		$\neg(\forall x)Fx \supset Ga$✓				$1\ \neg \equiv$ D
3	$\neg(\exists x)(Fx \supset Ga)$		$(\exists x)(Fx \supset Ga)$✓				$1\ \neg \equiv$ D
4			$Fb \supset Ga$✓				$3\ \exists$ D
5			$\neg\neg(\forall x)Fx$✓		Ga		$2 \supset$ D
6			$(\forall x)Fx$				$5\ \neg\neg$ D
7			Fb				$6\ \forall$ D
8			$\neg Fb$ ×	Ga ○	$\neg Fb$ ○	Ga ○	$4 \supset$ D
9	$(\forall x)\neg(Fx \supset Ga)$✓						$3\ \neg\exists$ D
10	$\neg(\forall x)Fx$✓	Ga					$2 \supset$ D
11		$\neg(Fb \supset Ga)$					$9\ \forall$ D
12		Fb					$11\ \neg \supset$ D
13		$\neg Ga$ ×					$11\ \neg \supset$ D
14	$(\exists x)\neg Fx$						$10\ \neg\forall$ D
15	$\neg Fb$						$14\ \exists$ D
16	$\neg(Fb \supset Ga)$						$9\ \forall$ D
17	Fb						$11\ \neg \supset$ D
18	$\neg Ga$ ×						$11\ \neg \supset$ D

Since there is at least one open branch, this tree is not closed. Thus, the two sentences are not quantificationally equivalent.

(d) $\{(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)], Ha, a = g(a,b)\} \vDash (\exists y)Bg(y,y)$

1	$(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)]$		SM
2	Ha		SM
3	$a = g(a,b)$		SM
4	$\neg(\exists y)Bg(y,y)$✓		SM
5	$(\forall y)\neg Bg(y,y)$		$4\ \neg\exists$ D
6	$(\exists y)Hg(a,y) \supset Bg(a,a)$✓		$1\ \forall$ D
7	$\neg(\exists y)Hg(a,y)$✓	$Bg(a,a)$	$6 \supset$ D
8		$\neg Bg(a,a)$ ×	$5\ \forall$ D
9	$(\forall y)\neg Hg(a,y)$		$7\ \neg\exists$ D
10	$\neg Hg(a,b)$		$9\ \forall$ D
11	$\neg Ha$ ×		$3 =$ D

Since this tree is closed, the quantificational entailment holds.

One interpretation is:

UD: Natural numbers
Hx: x is positive.
g(x, y): Minimum of x and y.
Bx: x is divisible by 1.
a: 1
b: 2

3. Why does the rule *Existential Decomposition* require that the instantiating constant **a** be foreign to all preceding lines of the branch?

   By not requiring *Existential Decomposition* to introduce foreign constants we have opened up the possibility that the same constant can be reused in a conflicting predicate. So, we require foreign constants with *Existential Decomposition* in order to preserve truth, validity, equivalence, etc.