ECS 170 Homework 2

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1. Given a set of admissible heuristics $\mathcal{H} = \{h_1, h_2, ..., h_n\}$ one can define a new heuristic h_{max} such that for any node n:

$$h_{max}(n) = \max_i h_i(n)$$

   (a) Show that h_{max} is an admissible heuristic.

   Proof. Assume h_{max} is not an admissible heuristic.
   We know h_{max} is some heuristic from the set $h_1, h_2, ..., h_n$. This means some heuristic in $\mathcal{H}$ is not admissible.
   This is a contradiction since $\mathcal{H}$ contains only admissible heuristics, thus our assumption was incorrect.
   Therefore, h_{max} is an admissible heuristic. □

   (b) Show that h_{max} dominates all other h_i

   Proof. Assume h_{max} does not dominate all other h_i.
   Then there must be some heuristic h_j in $\mathcal{H}$ such that $h_{max}(n) < h_j(n)$. But we know that $\forall i, 1 \leq i \leq n, h_{max}(n) \geq h_i(n)$.
   This is a contradiction, thus our assumption was incorrect.
   Therefore, h_{max} dominates all other h_i. □

2. Recall the "number of misplaced tiles" heuristic for the 8-puzzle problem. Show that this heuristic is admissible.

Proof. The rule for the 8-puzzle problem is:

- A tile can move from one position to another if the two positions are horizontally or vertically adjacent and the new position is blank.

If we decide to 'relax' the rule for this problem we can get rid of the two restrictions that "the two positions must be adjacent" and "the new position must be blank". By removing these two restrictions, we get our "number of misplaced tiles" heuristic.

We can show this is admissible by induction.

In the base case, no tiles are out of place, so the heuristic gives the optimal cost of 0 (which is not an overestimation).

For the inductive case, if n tiles are out of place, then at least n moves must be performed in order to reach the goal state. Again, this is not an overestimation.

Thus, this heuristic is admissible. □