PHIL 112 Homework 3

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1. Explicate in terms of open and/or closed truth trees.
   (a) Quantificational validity
   An argument of **PL** is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion of the argument has a closed truth tree.
   (b) Quantificational equivalence
   Two sentences **P** and **Q** of **PL** are quantificationally equivalent if and only if the set $\{\neg(\mathbf{P} \equiv \mathbf{Q})\}$ has a closed truth tree.

2. Use the tree method to show whether:
   (a) is quantificationally true
   (b) is quantificationally valid
   (c) sentences are quantificationally equivalent
   (d) quantificational entailment holds

   (a) $[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$

1	$\neg[[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]]\checkmark$	SM
2	$Fa \supset (\forall x)Fx\checkmark$	$1 \neg\supset$ D
3	$\neg[(\exists x)Fx \supset (\forall x)Fx]\checkmark$	$1 \neg\supset$ D
4	$(\exists x)Fx\checkmark$	$3 \neg\supset$ D
5	$\neg(\forall x)Fx\checkmark$	$3 \neg\supset$ D
6	$(\exists x)\neg Fx\checkmark$	$5 \neg\forall$ D
7	Fb	$4 \exists$ D
8	Fc	$6 \exists$ D
9	$\neg Fa$ ○ | $(\forall x)Fx$ ○	$2 \supset$ D

Since this tree is not closed, the sentence is not quantificationally true.

(b) $\dfrac{\begin{array}{l}(\forall x)[Nx \supset (\exists y)Rxy] \\ \neg(\exists x)Rxx \wedge Na\end{array}}{(\exists y)Ray}$

1	$(\forall x)[Nx \supset (\exists y)Rxy]$		SM
2	$\neg(\exists x)Rxx \wedge Na$ ✓		SM
3	$\neg(\exists y)Ray$ ✓		SM
4	$\neg(\exists x)Rxx$ ✓		$2 \wedge$ D
5	Na		$2 \wedge$ D
6	$(\forall y)\neg Ray$		$3 \neg\exists$ D
7	$Na \supset (\exists y)Ray$ ✓		$1 \forall$ D
8	$\neg Na$ ×	$(\exists y)Ray$ ✓	$7 \supset$ D
9		Rab	$8 \exists$ D
10		$\neg Rab$ ×	$9 \forall$ D

Since this tree is closed, the argument is quantificationally valid.

One interpretation is:
UD: The natural numbers
Nx: x is positive
Rxy: x is less than y
a: 1

(c) $[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)$

1	$\neg[[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)]$ ✓		SM
2	$(\forall x)Fx \supset Ga$	$\neg(\forall x)Fx \supset Ga$ ✓	$1 \neg \equiv$ D
3	$\neg(\exists x)(Fx \supset Ga)$ ✓	$(\exists x)(Fx \supset Ga)$ ✓	$1 \neg \equiv$ D
4		$Fb \supset Ga$ ✓	$3 \exists$ D
5		$\neg\neg(\forall x)Fx$ ✓ $\quad$ Ga	$2 \supset$ D
6		$(\forall x)Fx$	$5 \neg\neg$ D
7		Fb	$6 \forall$ D
8		$\neg Fb$ × $\quad$ Ga ○ $\quad$ $\neg Fb$ ○ $\quad$ Ga ○	$4 \supset$ D
9	$(\forall x)\neg(Fx \supset Ga)$		$3 \neg\exists$ D
10	$\neg(\forall x)Fx$ ✓ $\quad$ Ga		$2 \supset$ D
11	$\quad$ $\neg(Fb \supset Ga)$ ✓		$9 \forall$ D
12	$\quad$ Fb		$11 \neg \supset$ D
13	$\quad$ $\neg Ga$ ×		$11 \neg \supset$ D
14	$(\exists x)\neg Fx$ ✓		$10 \neg\forall$ D
15	$\neg Fb$		$14 \exists$ D
16	$\neg(Fb \supset Ga)$ ✓		$9 \forall$ D
17	Fb		$11 \neg \supset$ D
18	$\neg Ga$ ×		$11 \neg \supset$ D

Since there is at least one open branch, this tree is not closed. Thus, the two sentences are not quantificationally equivalent.

(d) $\{(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)], Ha, a = g(a,b)\} \vDash (\exists y)Bg(y,y)$

1	$(\forall x)[(\exists y)Hg(x,y) \supset Bg(x,x)]$		SM
2	Ha		SM
3	$a = g(a,b)$		SM
4	$\neg(\exists y)Bg(y,y)\checkmark$		SM
5	$(\forall y)\neg Bg(y,y)$		$4\ \neg\exists$ D
6	$(\exists y)Hg(a,y) \supset Bg(a,a)\checkmark$		$1\ \forall$ D
7	$\neg(\exists y)Hg(a,y)\checkmark$	$Bg(a,a)$	$6 \supset$ D
8		$\neg Bg(a,a)$ $\times$	$5\ \forall$ D
9	$(\forall y)\neg Hg(a,y)$		$7\ \neg\exists$ D
10	$\neg Hg(a,b)$		$9\ \forall$ D
11	$\neg Ha$ $\times$		$3 =$ D

Since this tree is closed, the quantificational entailment holds.

One interpretation is:

UD: Natural numbers
Hx: x is positive.
g(x, y): Minimum of x and y.
Bx: x is divisible by 1.
a: 1
b: 2

3. Why does the rule *Existential Decomposition* require that the instantiating constant **a** be foreign to all preceding lines of the branch?

   By not requiring *Existential Decomposition* to introduce foreign constants we have opened up the possibility that the same constant can be reused in a conflicting predicate. So, we require foreign constants with *Existential Decomposition* in order to preserve truth, validity, equivalence, etc.