MAT 25 Homework 4

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1. 1.4.10 Show that the set of all finite subsets of $\mathbb{N}$ is a countable set.

   Proof. In order to prove this, we simply need to construct an isomorphism from $\mathbb{N}$ to the set of all finite subsets of $\mathbb{N}$, hereafter referred to as S.

   Let's take a look at some elements of S. We have:

   $$S = \{\{\}, \{1\}, \{2\}, \{1,2\}, \{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}, ...\}$$

   What we see is that we can arbitrarily number these sets:

   $$\begin{array}{lcccccccc} \mathbb{N}: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & ... \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & ... \\ S: & \{\} & \{1\} & \{2\} & \{1,2\} & \{3\} & \{1,3\} & \{2,3\} & \{1,2,3\} & ... \end{array}$$

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