STA 032 Homework 5

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§ 4.5 8 We have $F \sim N(4.1, 0.36)$

(1) We can find

$$\begin{aligned} P(3.7 < F < 4.4) &= P(F < 4.4) - P(F > 3.7) \\ &= P\left(Z < \frac{4.4 - 4.1}{0.6}\right) - P\left(Z < \frac{3.7 - 4.1}{0.6}\right) \\ &= P\left(Z < \frac{0.3}{0.6}\right) - P\left(Z < \frac{-0.4}{0.6}\right) \\ &= P\left(Z < \frac{1}{2}\right) - P\left(Z < \frac{-2}{3}\right) \\ &\approx 0.6915 - 0.2546 \\ &\approx 0.4369 \end{aligned}$$

So approximately 43.69% of the female cats are between 3.7 kg and 4.4 kg.

(2) We just need to find

$$P(Z > 0.5) = 1 - P(Z < 0.5) \approx 1 - 0.6915 \approx 0.3085$$

So approximately 30.85% of the female cats are heavier than 0.5 standard deviations above the mean.

(3) We need to first find the closest area to 0.8. The closest area is 0.8023. This area corresponds to a z-score of 0.85.
Now we just work backwards

$$\begin{aligned} 0.85 &= \frac{x - 4.1}{0.6} \\ 0.85(0.6) + 4.1 &= x \\ 4.61 &= x \end{aligned}$$

So the female cat weight on the 80^{th} percentile is 4.61kg.

(4) We can find

$$P(F > 4.5) = P\left(Z > \frac{4.5 - 4.1}{0.6}\right) \approx P(Z > 0.66) \approx P(Z < -0.66)$$
$$\approx 0.2546$$

So the probability that a female cat chosen at random weighs more than 4.5kg is approximately 25.46%.

(5) We assume that the sample is large enough that each choice is independent. We want to find

$$P(F > 4.5)P(F < 4.5)^5 \approx 0.2546(1 - 0.2546)^5 \approx 0.2546(0.7475)^5 \approx 0.05858$$

So the probability that exactly one of six females cats chosen at random weighs more than 4.5 kg is 5.86%

20 We have $X \sim N(200, 100)$

(1)

$$P(X \leq 160) = P\left(Z < \frac{160 - 200}{10}\right) = P\left(Z < \frac{-40}{10}\right) = P(Z < -4)$$

Since the table only goes to -3.69 all we can say is that $P(Z < -4) < 0.0001$ So the probability of the strength being less than or equal to 160N is less than 0.01%.

(2) Yes, a strength of 160N would be unusually small.
Intuitively this makes sense, 160N is four standard deviations away from the the mean. 99.7% of the strengths are within three standard deviations, so it is natural to expect 160N to be unusually small.

(3) Yes, this would be enough evidence.
The process is producing adhesive with strength far below the mean. If a measurement is 160N, something is messing the process up.

21 We have $R_1 \sim N(100, 25)$ and $R_2 \sim N(120, 100)$.
It will help to compute $R_2 - R_1 \sim N(120 - 100, 100 + 25) = N(20, 125)$

(1) We need to solve $P(R_2 > R_1) = P(R_2 - R_1 > 0)$

$$\begin{aligned} P(R_2 - R_1 > 0) &= P\left(Z > \frac{0 - 20}{\sqrt{125}}\right) \\ &\approx P\left(Z > -1.79\right) \\ &\approx P\left(Z < 1.79\right) \\ &\approx 0.9633 \end{aligned}$$

So the probability that R_2 exceeds R_1 is approximately 96.33%.

(2) We need to solve $P(R_2 > R_1 + 30) = P(R_2 - R_1 > 30)$

$$\begin{aligned}
P(R_2 - R_1 > 30) &= P\left(Z > \frac{30 - 20}{\sqrt{125}}\right) \\
&\approx P\left(Z > 0.89\right) \\
&\approx P\left(Z < -0.89\right) \\
&\approx 0.1867
\end{aligned}$$

So the probability that R_2 exceeds R_1 by 30 Ω is approximately 18.67%.

26 i. $k = 1$

$$\begin{aligned}
P(|X-\mu| \geq \sigma) &= P(X-\mu < -\sigma \cup X-\mu > \sigma) \\
&= P(X-\mu < -\sigma) + P(X-\mu > \sigma) \\
&= P(X-\mu < -\sigma) + P(X-\mu > \sigma) \\
&= P(X < \mu-\sigma) + P(X > \mu+\sigma) \\
&= P\left(Z < \frac{\mu-\sigma-\mu}{\sigma}\right) + P\left(Z > \frac{\mu+\sigma-\mu}{\sigma}\right) \\
&= P(Z < -1) + P(Z > 1) \\
&= P(Z < -1) + P(Z < -1) \\
&= 2P(Z < -1) \\
&= 2(0.1587) \\
&= 0.3174
\end{aligned}$$

$$\frac{1}{1^2} = 1$$

ii. $k = 2$

$$
\begin{aligned}
P(|X-\mu| \geq 2\sigma) &= P(X-\mu < -2\sigma \cup X-\mu > 2\sigma) \\
&= P(X-\mu < -2\sigma) + P(X-\mu > 2\sigma) \\
&= P(X-\mu < -2\sigma) + P(X-\mu > 2\sigma) \\
&= P(X < \mu-2\sigma) + P(X > \mu+2\sigma) \\
&= P\left(Z < \frac{\mu-2\sigma-\mu}{\sigma}\right) + P\left(Z > \frac{\mu+2\sigma-\mu}{\sigma}\right) \\
&= P(Z < -2) + P(Z > 2) \\
&= P(Z < -2) + P(Z < -2) \\
&= 2P(Z < -2) \\
&= 2(0.0228) \\
&= 0.0456
\end{aligned}
$$

$$\frac{1}{2^2} = 0.25$$

iii. $k = 3$

$$
\begin{aligned}
P(|X-\mu| \geq 3\sigma) &= P(X-\mu < -3\sigma \cup X-\mu > 3\sigma) \\
&= P(X-\mu < -3\sigma) + P(X-\mu > 3\sigma) \\
&= P(X-\mu < -3\sigma) + P(X-\mu > 3\sigma) \\
&= P(X < \mu-3\sigma) + P(X > \mu+3\sigma) \\
&= P\left(Z < \frac{\mu-3\sigma-\mu}{\sigma}\right) + P\left(Z > \frac{\mu+3\sigma-\mu}{\sigma}\right) \\
&= P(Z < -3) + P(Z > 3) \\
&= P(Z < -3) + P(Z < -3) \\
&= 2P(Z < -3) \\
&= 2(0.0013) \\
&= 0.0026
\end{aligned}
$$

$$\frac{1}{3^2} = 0.\overline{1}$$

So the actual probabilities are much smaller than Chebyshev's bound.

§ 4.7 2 (1)

(2)

(3)

(4)
(5)
(6)

4 (1)
(2)
(3)
(4)
(5)

§ 4.8 1 (1)
(2)
(3)
(4)

2 (1)
(2)
(3)
(4)