

# MAT 167 Homework 2

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## 1.7

### 4

Write down the 3 by 3 finite-difference matrix equation ( $h = \frac{1}{4}$ ) for

$$-\frac{d^2u}{dx^2} + u = x, \quad u(0) = u(1) = 0$$

Following from the text

$$\frac{d^2u}{dx^2} \approx \frac{\Delta^2 u}{\Delta x^2} = \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

So we have

$$-\frac{u(x+h) - 2u(x) + u(x-h)}{h^2} + u = x$$

Letting  $x = jh$ , and realizing that  $u$  must be a function of  $x$

$$\begin{aligned} -\frac{u(jh+h) - 2u(jh) + u(jh-h)}{h^2} + u(jh) &= jh \\ -\frac{u((j+1)h) - 2u(jh) + u((j-1)h)}{h^2} + u(jh) &= jh \\ -u((j+1)h) + 2u(jh) - u((j-1)h) + u(jh)h^2 &= jh^3 \end{aligned}$$

We take another cue from the text and use the notation  $u(jh) = u_j$

$$\begin{aligned} -u_{j+1} + 2u_j - u_{j-1} + u_j h^2 &= jh^3 \\ -u_{j+1} + u_j h^2 + 2u_j - u_{j-1} &= jh^3 \\ -u_{j+1} + (h^2 + 2)u_j - u_{j-1} &= jh^3 \end{aligned}$$

Since we have  $h = \frac{1}{4}$

$$\begin{aligned} -u_{j+1} + ((\frac{1}{4})^2 + 2)u_j - u_{j-1} &= j(\frac{1}{4})^3 \\ -u_{j+1} + (\frac{1}{16} + 2)u_j - u_{j-1} &= \frac{j}{64} - u_{j+1} + \frac{33}{16}u_j - u_{j-1} &= \frac{j}{64} \end{aligned}$$

Now we check each of  $j = 1, 2, 3$ , remembering  $u(0) = u_0 = u(1) = u_4 = 0$   
 $j = 1$

$$\begin{aligned} -u_{1+1} + \frac{33}{16}u_1 - u_{1-1} &= \frac{1}{64} \\ -u_2 + \frac{33}{16}u_1 - u_0 &= \frac{1}{64} \\ -u_2 + \frac{33}{16}u_1 &= \frac{1}{64} \end{aligned}$$

$j = 2$

$$\begin{aligned} -u_{2+1} + \frac{33}{16}u_2 - u_{2-1} &= \frac{2}{64} \\ -u_3 + \frac{33}{16}u_2 - u_1 &= \frac{2}{64} \end{aligned}$$

$j = 3$

$$\begin{aligned} -u_{3+1} + \frac{33}{16}u_3 - u_{3-1} &= \frac{3}{64} \\ -u_4 + \frac{33}{16}u_3 - u_2 &= \frac{3}{64} \\ \frac{33}{16}u_3 - u_2 &= \frac{3}{64} \end{aligned}$$

So we have

$$\begin{aligned} -u_2 + \frac{33}{16}u_1 &= \frac{1}{64} \\ -u_3 + \frac{33}{16}u_2 - u_1 &= \frac{2}{64} \\ \frac{33}{16}u_3 - u_2 &= \frac{3}{64} \end{aligned}$$

Rearranging

$$\begin{aligned} \frac{33}{16}u_1 - u_2 &= \frac{1}{64} \\ -u_1 + \frac{33}{16}u_2 - u_3 &= \frac{2}{64} \\ -u_2 + \frac{33}{16}u_3 &= \frac{3}{64} \end{aligned}$$

$$\begin{bmatrix} \frac{33}{16} & -1 & 0 \\ -1 & \frac{33}{16} & -1 \\ 0 & -1 & \frac{33}{16} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{64} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

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**2.1**

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**2.2**

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**2.3**

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