MAT 108 HW 2

Hardy Jones
999397426
Professor Bandyopadhyay
Spring 2015

§1.2

6 (a) This statement is true since both sides of the bi-conditional have the same truth values.
Triangles have three sides and squares have four sides.

(b) This statement is true since both sides of the bi-conditional have the same truth values.
$7 + 5 = 12$ and $1 + 1 = 2$.

(d) This statement is true since both sides of the bi-conditional have the same truth values.
parallelograms have four sides and 27 is not prime..

10 (a) $(f$ has a relative minimum at $x_0 \wedge f$ is differentiable at $x_0) \implies f'(x_0) = 0$

(b) n is prime $\implies (n = 2 \vee n$ is odd$)$

(c) R is irreflexive $\implies (R$ is symmetric $\wedge R$ is transitive$)$

(d) $\det \mathbf{B} = 0 \implies (\mathbf{B}$ is square $\wedge$ not invertible$)$

(f) $(2n < 4 \vee n > 4) \implies 2 < n - 6$

(g) $6 \geq n - 3 \implies (n > 4 \vee n > 10)$

(h) x is Cauchy $\implies x$ is convergent.

15 (a) $(f$ has a relative minimum at $x_0 \wedge f$ is differentiable at $x_0) \implies f'(x_0) = 0$

- Converse:
  $f'(x_0) = 0 \implies (f$ has a relative minimum at $x_0 \wedge f$ is differentiable at $x_0)$
  This sentence is false since $f'(x_0) = 0$ can be true, but f may have a relative maximum at x_0.
- Contrapositive:
  $(\sim f'(x_0) = 0) \implies (\sim (f$ has a relative minimum at $x_0 \wedge f$ is differentiable at $x_0))$
  $f'(x_0) \neq 0 \implies (\sim f$ has a relative minimum at $x_0 \vee \sim f$ is differentiable at $x_0)$
  $f'(x_0) \neq 0 \implies (f$ does not have a relative minimum at $x_0 \vee f$ is not differentiable at $x_0)$
  Since the original statement is true, and the contrapositive has the same truth value, this sentence is also true.

(b) n is prime $\implies (n = 2 \vee n$ is odd$)$

- Converse:
  $(n = 2 \vee n \text{ is odd}) \implies n \text{ is prime}$
  This sentence is false. If $n = 9$ then n is odd, but n is not prime.
- Contrapositive:
  $(\sim (n = 2 \vee n \text{ is odd})) \implies (\sim n \text{ is prime})$
  $(\sim n = 2 \vee \sim n \text{ is odd}) \implies n \text{ is not prime}$
  $(n \neq 2 \vee n \text{ is not odd}) \implies n \text{ is not prime}$
  Since the original statement is true, and the contrapositive has the same truth value, this sentence is also true.

(f) $(2n < 4 \vee n > 4) \implies 2 < n - 6$

- Converse:
  $2 < n - 6 \implies (2n < 4 \vee n > 4)$
  This sentence is true.
  We can see this by simplifying both sides a bit:
  $8 < n \implies (n < 2 \vee 4 < n)$
  If n is greater than 8, it is also greater than 4. So if the antecedent is true, the whole sentence is true.
- Contrapositive:
  $(\sim 2 < n - 6) \implies (\sim (2n < 4 \vee n > 4))$
  $2 \geq n - 6 \implies (\sim 2n < 4 \wedge \sim n > 4)$
  $2 \geq n - 6 \implies (2n \geq 4 \wedge n \leq 4)$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $8 \geq n \implies (n \geq 2 \wedge 4 \geq n)$
  If n is 0, then the antecedent is true, but 0 is not greater than or equal to 2 so the consequent is false.

(g) $6 \geq n - 3 \implies (n > 4 \vee n > 10)$

- Converse:
  $(n > 4 \vee n > 10) \implies 6 \geq n - 3$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $(n > 4 \vee n > 10) \implies 9 \geq n$
  If n is 100, then the antecedent is true, but 100 is not less than or equal to 9 so the consequent is false.
- Contrapositive:
  $(\sim (n > 4 \vee n > 10)) \implies (\sim 6 \geq n - 3)$
  $(\sim n > 4 \wedge \sim n > 10) \implies (6 < n - 3)$
  $(n \leq 4 \wedge n \leq 10) \implies (6 < n - 3)$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $(n \leq 4 \wedge n \leq 10) \implies (9 < n)$
  If n is 0, then the antecedent is true, but 0 is not greater than 9 so the consequent is false.

16 (a) We can use a truth table to enumerate all possibilities.

P	Q	$P \implies Q$	$(P \implies Q) \implies Q$	$[(P \implies Q) \implies Q] \implies P$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

Since the truth value of final column is neither all true nor all false, this is neither a tautology nor a contradiction.

(b) We can use a truth table to enumerate all possibilities.

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$	$P \iff P \wedge (P \vee Q)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

Since the truth value of final column is all true, this is a tautology.

(c) We can use a truth table to enumerate all possibilities.

P	Q	$P \implies Q$	$\sim Q$	$P \wedge \sim Q$	$P \implies Q \iff P \wedge \sim Q$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

Since the truth value of final column is all false, this is a contradiction.

§1.3 1 (b) $(\forall x)(x \text{ is precious} \implies x \text{ is not beautiful})$

(d) $\sim (\exists x)(x \text{ is a right triangle} \wedge x \text{ is isosceles})$

(e) $(\forall x)(x \text{ is not isosceles} \implies x \text{ is a right triangle})$

(i) $(\forall x)(x \in \mathbb{Z} \implies (x > -4 \vee x < 6))$

(l) $(\forall x)(\forall y)(x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge x < y \implies (\exists z)(x < z < y))$

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- All people dislike all taxes:
  $(\forall x)(x \text{ is a person} \implies (\forall y)(y \text{ is a tax} \implies x \text{ dislikes } y))$
- All people dislike some taxes:
  $(\forall x)(x \text{ is a person} \implies (\exists y)(y \text{ is a tax} \wedge x \text{ dislikes } y))$
- Some people dislike all taxes:
  $(\exists x)(x \text{ is a person} \wedge (\forall y)(y \text{ is a tax} \implies x \text{ dislikes } y))$
- Some people dislike some taxes:
  $(\exists x)(x \text{ is a person} \wedge (\exists y)(y \text{ is a tax} \wedge x \text{ dislikes } y))$

6 (a) This statement is true in all four universes T, U, V, W, using the elements $17, 6, 24, 2$ respectively.
In T, both the antecedent and the consequent are true so the statement is true. In the other three, the antecedent is false so the statement is true.

(b) This statement is true only in universe T. The other three have no elements which satisfy both parts of the conjunct.

(c) This statement is true in universes T, U, V.

In T, both the antecedent and the consequent are true so the statement is true. In U and V, the antecedent is false so the statement is true. In W, one of the elements–namely 7–has a true antecedent but a false consequent so the statement is false.

(d) This statement is true only in universe T. The other three have no elements which satisfy both parts of the conjunct.

8 (c) This is false. We can see this by simplifying a bit.
$(\exists x)(2x + 3 = 6x + 7)$
$(\exists x)(2x = 6x + 4)$
$(\exists x)(-4x = 4)$
$(\exists x)(x = -1)$
Since the universe is $\mathbb{N}$, it does not contain negative numbers. So, there are no elements which satisfy this sentence.

(d) We can see this by graphing the two equations.
In Figure 1 we see that the two lines intersect, so there exists some real number such that $3^x = x^2$.
Thus, the sentence is true.

(e) We can see this by graphing the two equations.
In Figure 2 we see that the two do not intersect, so there is no real number such that $3^x = x$.
Thus, the sentence is false.

(f) This sentence is true.
We can see this by simplifying the equation a bit.

$$\begin{aligned} 3(2-x) &= 5 + 8(1-x) \\ 6 - 3x &= 5 + 8 - 8x \\ 6 - 3x &= 13 - 8x \\ 5x &= 7 \\ x &= \frac{7}{5} \end{aligned}$$

So there is exactly one value that satisfies the sentence, in specific $\frac{7}{5}$.
Thus, the sentence is true.

(k) This sentence is false.
If we let $x = -20$ then we have the value

$$(-20)^3 + 17(-20)^2 + 6(-20) + 100 = -1220$$

and this is less than 0.
So the sentence is false.

(l) This sentence is true.
For any two real numbers we can construct a new number between the two such that $w = \frac{x+y}{2}$.

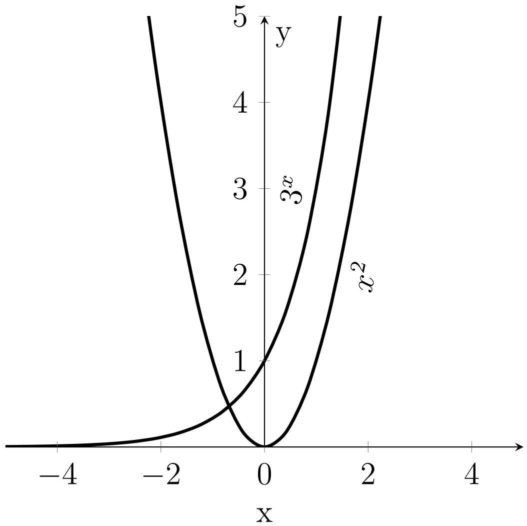

Figure 1: 8 (d)

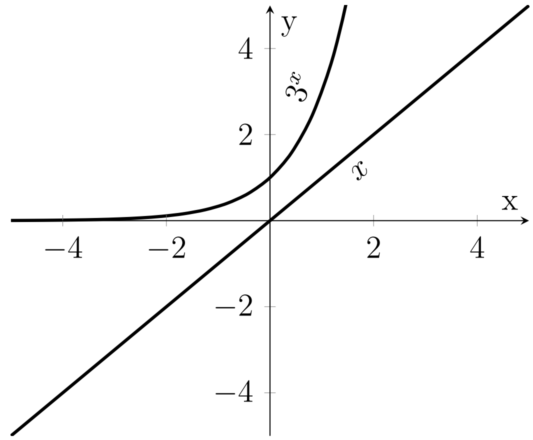

Figure 2: 8 (e)

9 (a) All naturals are greater than or equal to 1.

(c) All naturals that are prime and not equal to 2 are odd.

(g) For any odd natural x, x^2 is also odd.

10 (i) This sentence is false. If we were given some unique x then we would have to have: $x = 3^2 = 9$ and also $x = 4^2 = 16$, but this is false.

(j) This sentence is true since $x = y^2$ is a function.

(k) This sentence is false since the x and y aren't unique.

11 (b) The converse of Theorem 1.3.2 (a) is:
If $A(x)$ is an open sentence with variable x, then $(\exists x)A(x) \implies (\exists! x)A(x)$
We want to prove this statement false.

Proof. Let $A(x)$ be the open sentence $x > 1$ in the universe $\mathbb{N}$.
Then we have the antecedent true, as every element in $\mathbb{N} \setminus \{1\}$ is greater than 1.
However, the consequent is not true since $\mathbb{N} \setminus \{1\}$ contains more than one element.
Thus this theorem is not true for all open sentences. □

12 (a) This sentence does not have to be true. We can have $a_n = b_n$, but $a_0 \neq b_0$ and then the two polynomials are not equal.

(b) This sentence does not have to be true. The reason is the same as the previous argument. All of the individual coefficients could be the same except for one and the polynomials would be different.

(c) This sentence does not have to be true. This sentence is a recapitulation of the previous sentence. So the answer is the same.

(d) This sentence must be true. There must be at least one coefficient that is not equal between the two polynomials.