PHIL 112 Homework 1

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1. Specify the atomic formulas of PL.

   Every expression of PL that is either a sentence letter of PL, or an n-place predicate of PL followed by n individual terms of PL is an atomic formula of PL.

2. Give the recursive definition of 'formula of PL'.

   (a) Every atomic formula $\mathbf{P}$ is a formula of PL.
   (b) If $\mathbf{P}$ is a formula of PL, then so is $\neg\mathbf{P}$.
   (c) If $\mathbf{P}$ and $\mathbf{Q}$ are formulae of PL, then so are $\mathbf{P}\wedge\mathbf{Q}$, $\mathbf{P}\vee\mathbf{Q}$, $\mathbf{P}\supset\mathbf{Q}$, and $\mathbf{P}\equiv\mathbf{Q}$.
   (d) If $\mathbf{P}$ is a formula of PL that contains at least one occurrence of $\mathbf{x}$ and no $\mathbf{x}$-quantifier, then $\forall\mathbf{xP}$ and $\exists\mathbf{xP}$ are formulae of PL.
   (e) Nothing else is a formula of PL unless it can be made from the previous rules.

3. Indicate which of the following are formulas of PL, and which of those are sentences of PL.

   (a) Quantified Formula of PL.
   Not a Sentence of PL since the subformula has a quantified x.
   $\underline{(\forall x)}[Fxa \supset (\forall x)Gax]$
   (b) Truth-functionally compound Formula of PL and Sentence of PL.
   $(\forall z)Fza \underline{\supset} \neg(\exists z)Gaz$
   (c) Truth-functionally compound Formula of PL and Sentence of PL.
   $\underline{\neg}(\forall y)Gyy$
   (d) Truth-functionally compound Formula of PL.
   Not a sentence of PL for the subformula has at least one free variable.
   $Faz \underline{\supset} (\forall x)Fxa$
   (e) Not a formula of PL for there is no x in Fab.
   $\neg(\exists x)Fab$

4. List all the sub-formulas of each of the following:

formula	subformulae
$(\forall x)[(\exists y)Fxy \supset Gax]$	$(\forall x)[(\exists y)Fxy \supset Gax]$
	$(\exists y)Fxy \supset Gax$
	$(\exists y)Fxy$
	Fxy
	Gax
$\neg Fab \equiv (\forall x) \neg Fxb$	$\neg Fab \equiv (\forall x) \neg Fxb$
	$\neg Fab$
	Fab
	$(\forall x) \neg Fxb$
	$\neg Fxb$
	Fxb