MAT 168 Calculation 2

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2.8 We first start by rewriting as a dictionary, introducing the necessary slack variables:

$$
\begin{aligned}
\zeta &= \quad\;\; 3x_1 + 2x_2 \\
x_3 &= 1 - x_1 + 2x_2 \\
x_4 &= 2 - x_1 + x_2 \\
x_5 &= 6 - 2x_1 + x_2 \\
x_6 &= 5 - x_1 \\
x_7 &= 16 - 2x_1 - x_2 \\
x_8 &= 12 - x_1 - x_2 \\
x_9 &= 21 - x_1 - 2x_2 \\
x_{10} &= 10 \qquad\; - x_2
\end{aligned}
$$

Then we can begin by entering with the first non-basic variable, x_1.

We look at the constraints and see:

- $x_3 \implies x_1 \leq 1$
- $x_4 \implies x_1 \leq 2$
- $x_5 \implies x_1 \leq 3$
- $x_6 \implies x_1 \leq 5$
- $x_7 \implies x_1 \leq 8$
- $x_8 \implies x_1 \leq 12$
- $x_9 \implies x_1 \leq 21$
- $x_{10} \implies x_1 \geq 0$

The more restrictive constraint is that $x_1 \leq 1$, so set $x_1 = 1$.

So we can let x_1 enter and x_3 leave.

$$x_1 = 1 + 2x_2 - x_3$$

This gives a new value for ζ. $\zeta = 3 + 8x_2 - 3x_3$

This gives a new value for x_4. $x_4 = 1 - 1x_2 + 1x_3$

This gives a new value for x_5. $x_5 = 4 - 3x_2 + 2x_3$

This gives a new value for x_6. $x_6 = 4 - 2x_2 + 1x_3$

This gives a new value for x_7. $x_7 = 14 - 5x_2 + 2x_3$

This gives a new value for x_8. $x_8 = 11 - 3x_2 + 1x_3$

This gives a new value for x_9. $x_9 = 20 - 4x_2 + 1x_3$

This gives a new value for x_{10}. $x_{10} = 10 - 1x_2$

So we have a new dictionary:

$$\begin{aligned}
\zeta &= \ \ 3 + 8x_2 - 3x_3 \\
x_1 &= \ \ 3 + 2x_2 - 1x_3 \\
x_4 &= \ \ 1 - 1x_2 + 1x_3 \\
x_5 &= \ \ 4 - 3x_2 + 2x_3 \\
x_6 &= \ \ 4 - 2x_2 + 1x_3 \\
x_7 &= 14 - 5x_2 + 2x_3 \\
x_8 &= 11 - 3x_2 + 1x_3 \\
x_9 &= 20 - 4x_2 + 1x_3 \\
x_{10} &= 10 - 1x_2
\end{aligned}$$

Now, we can continue optimizing since x_1 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_1 \geq 0$
- $x_4 \implies x_1 \leq 1$
- $x_5 \implies x_1 \leq \frac{4}{3}$
- $x_6 \implies x_1 \leq 2$
- $x_7 \implies x_1 \leq \frac{14}{5}$
- $x_8 \implies x_1 \leq \frac{11}{3}$
- $x_9 \implies x_1 \leq 5$
- $x_{10} \implies x_1 \leq 10$

The more restrictive constraint is that $x_4 \leq 1$, so set $x_4 = 1$.

So we can let x_2 enter and x_4 leave.

$$x_2 = 1 + x_3 - x_4$$

This gives a new value for ζ. $\zeta = 11 + 5x_3 - 8x_4$

This gives a new value for x_1. $x_1 = 3 + 1x_3 - 2x_4$

This gives a new value for x_5. $x_5 = 1 - 1x_3 + 3x_4$

This gives a new value for x_6. $x_6 = 2 - 1x_3 + 2x_4$

This gives a new value for x_7. $x_7 = 9 - 3x_3 + 5x_4$

This gives a new value for x_8. $x_8 = 8 - 2x_3 + 3x_4$

This gives a new value for x_9. $x_9 = 16 - 3x_3 + 4x_4$

This gives a new value for x_{10}. $x_{10} = 9 - 1x_3 + 1x_4$

So we have a new dictionary:

$$\begin{aligned}
\zeta &= 11 + 5x_3 - 8x_4 \\
x_1 &= 3 + 1x_3 - 2x_4 \\
x_2 &= 1 + 1x_3 - 1x_4 \\
x_5 &= 1 - 1x_3 + 3x_4 \\
x_6 &= 2 - 1x_3 + 2x_4 \\
x_7 &= 9 - 3x_3 + 5x_4 \\
x_8 &= 8 - 2x_3 + 3x_4 \\
x_9 &= 16 - 3x_3 + 4x_4 \\
x_{10} &= 9 - 1x_3 + 1x_4
\end{aligned}$$

Now, we can continue optimizing since x_3 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_1 \geq 0$
- $x_2 \implies x_1 \geq 0$
- $x_5 \implies x_1 \leq 1$
- $x_6 \implies x_1 \leq 2$
- $x_7 \implies x_1 \leq 3$
- $x_8 \implies x_1 \leq 4$
- $x_9 \implies x_1 \leq \frac{16}{3}$
- $x_{10} \implies x_1 \leq 9$

The more restrictive constraint is that $x_5 \leq 1$, so set $x_5 = 1$.

So we can let x_3 enter and x_5 leave.

$$x_3 = 1 + 3x_4 - 1x_5$$

This gives a new value for ζ. $\zeta = 16 + 7x_4 - 5x_5$

This gives a new value for x_1. $x_1 = 4 + 1x_4 - 1x_5$

This gives a new value for x_2. $x_2 = 2 + 2x_4 - 1x_5$

This gives a new value for x_6. $x_6 = 1 - 1x_4 + 1x_5$

This gives a new value for x_7. $x_7 = 6 - 4x_4 + 3x_5$

This gives a new value for x_8. $x_8 = 6 - 3x_4 + 2x_5$

This gives a new value for x_9. $x_9 = 13 - 5x_4 + 3x_5$

This gives a new value for x_{10}. $x_{10} = 8 - 2x_4 + 1x_5$

So we have a new dictionary:

$$\begin{aligned}
\zeta &= 16 + 7x_4 - 5x_5 \\
x_1 &= 4 + 1x_4 - 1x_5 \\
x_2 &= 2 + 2x_4 - 1x_5 \\
x_3 &= 1 - 1x_4 + 1x_5 \\
x_6 &= 1 - 1x_4 + 1x_5 \\
x_7 &= 6 - 4x_4 + 3x_5 \\
x_8 &= 6 - 3x_4 + 2x_5 \\
x_9 &= 13 - 5x_4 + 3x_5 \\
x_{10} &= 8 - 2x_4 + 1x_5
\end{aligned}$$

Now, we can continue optimizing since x_4 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_1 \geq 0$
- $x_2 \implies x_1 \geq 0$
- $x_3 \implies x_1 \geq 0$
- $x_6 \implies x_1 \leq 1$
- $x_7 \implies x_1 \leq \frac{6}{4}$
- $x_8 \implies x_1 \leq 2$
- $x_9 \implies x_1 \leq \frac{13}{5}$

- $x_{10} \implies x_1 \leq 4$

The more restrictive constraint is that $x_6 \leq 1$, so set $x_6 = 1$.

So we can let x_4 enter and x_6 leave.

$$x_4 = 1 + 1x_5 - 1x_6$$

This gives a new value for ζ. $\zeta = 23 + 2x_5 - 7x_6$

This gives a new value for x_1. $x_1 = 5 + 0x_5 - 1x_6$

This gives a new value for x_2. $x_2 = 4 + 1x_5 - 2x_6$

This gives a new value for x_3. $x_3 = 4 + 2x_5 - 3x_6$

This gives a new value for x_7. $x_7 = 2 - 1x_5 + 4x_6$

This gives a new value for x_8. $x_8 = 3 - 1x_5 + 3x_6$

This gives a new value for x_9. $x_9 = 8 - 2x_5 + 5x_6$

This gives a new value for x_{10}. $x_{10} = 6 - 1x_5 + 2x_6$

$$\begin{aligned}
\zeta &= 23 + 2x_5 - 7x_6 \\
x_1 &= \ \ 5 + 0x_5 - 1x_6 \\
x_2 &= \ \ 4 + 1x_5 - 2x_6 \\
x_3 &= \ \ 4 + 2x_5 - 3x_6 \\
x_4 &= \ \ 1 + 1x_5 - 1x_6 \\
x_7 &= \ \ 2 - 1x_5 + 4x_6 \\
x_8 &= \ \ 3 - 1x_5 + 3x_6 \\
x_9 &= \ \ 8 - 2x_5 + 5x_6 \\
x_{10} &= \ \ 6 - 1x_5 + 2x_6
\end{aligned}$$

Now, we can continue optimizing since x_5 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_1 \geq 0$
- $x_2 \implies x_1 \geq 0$
- $x_3 \implies x_1 \geq 0$
- $x_4 \implies x_1 \geq 0$
- $x_7 \implies x_1 \leq 2$
- $x_8 \implies x_1 \leq 3$
- $x_9 \implies x_1 \leq 4$

- $x_{10} \implies x_1 \leq 6$

The more restrictive constraint is that $x_7 \leq 2$, so set $x_7 = 2$.
So we can let x_5 enter and x_7 leave.

$$x_5 = 2 + 4x_6 - 1x_7$$

This gives a new value for ζ. $\zeta = 27 + 1x_6 - 2x_7$
This gives a new value for x_1. $x_1 = 5 - 1x_6 - 0x_7$
This gives a new value for x_2. $x_2 = 6 + 2x_6 - 1x_7$
This gives a new value for x_3. $x_3 = 6 + 5x_6 - 2x_7$
This gives a new value for x_4. $x_4 = 3 + 3x_6 - 1x_7$
This gives a new value for x_8. $x_8 = 1 - 1x_6 + 1x_7$
This gives a new value for x_9. $x_9 = 4 - 3x_6 + 2x_7$
This gives a new value for x_{10}. $x_{10} = 4 - 2x_6 + 1x_7$

$$\begin{aligned}
\zeta &= 27 + 1x_6 - 2x_7 \\
x_1 &= \ \ 5 - 1x_6 - 0x_7 \\
x_2 &= \ \ 6 + 2x_6 - 1x_7 \\
x_3 &= \ \ 6 + 5x_6 - 2x_7 \\
x_4 &= \ \ 3 + 3x_6 - 1x_7 \\
x_5 &= \ \ 2 + 4x_6 - 1x_7 \\
x_8 &= \ \ 1 - 1x_6 + 1x_7 \\
x_9 &= \ \ 4 - 3x_6 + 2x_7 \\
x_{10} &= \ \ 4 - 2x_6 + 1x_7
\end{aligned}$$

Now, we can continue optimizing since x_6 has a positive coefficient.
We look at the constraints and see:

- $x_1 \implies x_1 \geq 0$
- $x_2 \implies x_1 \geq 0$
- $x_3 \implies x_1 \geq 0$
- $x_4 \implies x_1 \geq 0$
- $x_5 \implies x_1 \geq 0$
- $x_8 \implies x_1 \leq 1$
- $x_9 \implies x_1 \leq \frac{4}{3}$

- $x_{10} \implies x_1 \leq 2$

The more restrictive constraint is that $x_8 \leq 1$, so set $x_8 = 1$.

So we can let x_6 enter and x_8 leave.

$$x_6 = 1 + 1x_7 - 1x_8$$

This gives a new value for ζ. $\zeta = 28 - 1x_7 - 1x_8$

This gives a new value for x_1. $x_1 = 4 - 1x_7 + 1x_8$

This gives a new value for x_2. $x_2 = 8 + 1x_7 - 2x_8$

This gives a new value for x_3. $x_3 = 13 + 3x_7 - 5x_8$

This gives a new value for x_4. $x_4 = 6 + 2x_7 - 3x_8$

This gives a new value for x_5. $x_5 = 6 - 3x_7 - 4x_8$

This gives a new value for x_9. $x_9 = 1 - 1x_7 + 3x_8$

This gives a new value for x_{10}. $x_{10} = 2 - 1x_7 + 2x_8$

$$\begin{aligned}
\zeta &= 28 - 1x_7 - 1x_8 \\
x_1 &= 4 - 1x_7 + 1x_8 \\
x_2 &= 8 + 1x_7 - 2x_8 \\
x_3 &= 13 + 3x_7 - 5x_8 \\
x_4 &= 6 + 2x_7 - 3x_8 \\
x_5 &= 6 - 3x_7 - 4x_8 \\
x_6 &= 1 + 1x_7 - 1x_8 \\
x_9 &= 1 - 1x_7 + 3x_8 \\
x_{10} &= 2 - 1x_7 + 2x_8
\end{aligned}$$

Since we have no more optimizable variables (all variable coefficients of ζ are non-positive), we can no longer maximize ζ.

Then we have an optimal solution with $x_1 = 4, x_2 = 8, x_3 = 13, x_4 = x_5 = 6, x_6 = x_9 = 1, x_{10} = 2, x_7 = x_8 = 0$, and value 28.

2.16