# PHIL 112 Homework 3

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Winter 2014

1. Explicate in terms of open and/or closed truth trees.

   (a) Quantificational validity
   An argument of **PL** is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion of the argument has a closed truth tree.

   (b) Quantificational equivalence
   Two sentences **P** and **Q** of **PL** are quantificationally equivalent if and only if the set $\{\neg(\mathbf{P} \equiv \mathbf{Q})\}$ has a closed truth tree.

2. Use the tree method to show whether:

   (a) is quantificationally true
   (b) is quantificationally valid
   (c) sentences are quantificationally equivalent
   (d) quantificational entailment holds

   (a) $[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$

| | | |
|---|---|---|
| 1 | $\neg[[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]]\checkmark$ | SM |
| 2 | $Fa \supset (\forall x)Fx\checkmark$ | $1 \neg\supset$ D |
| 3 | $\neg[(\exists x)Fx \supset (\forall x)Fx]\checkmark$ | $1 \neg\supset$ D |
| 4 | $(\exists x)Fx\checkmark$ | $3 \neg\supset$ D |
| 5 | $\neg(\forall x)Fx\checkmark$ | $3 \neg\supset$ D |
| 6 | $(\exists x)\neg Fx\checkmark$ | $5 \neg\forall$ D |
| 7 | $Fb$ | $4 \exists$ D |
| 8 | $Fc$ | $6 \exists$ D |
| 9 | $\neg Fa$ $\circ$     $(\forall x)Fx$ $\circ$ | $2 \supset$ D |

Since this tree is not closed, the sentence is not quantificationally true.

(b) $\dfrac{\begin{array}{l}(\forall x)[Nx \supset (\exists y)Rxy] \\ \neg(\exists x)Rxx \wedge Na\end{array}}{(\exists y)Ray}$

| | | | |
|---|---|---|---|
| 1 | $(\forall x)[Nx \supset (\exists y)Rxy]$ | | SM |
| 2 | $\neg(\exists x)Rxx \wedge Na\checkmark$ | | SM |
| 3 | $\neg(\exists y)Ray\checkmark$ | | SM |
| 4 | $\neg(\exists x)Rxx$ | | $2 \wedge$ D |
| 5 | $Na$ | | $2 \wedge$ D |
| 6 | $(\forall y)\neg Ray$ | | $3 \neg\exists$ D |
| 7 | $Na \supset (\exists y)Ray\checkmark$ | | $1 \forall$ D |
| 8 | $\neg Na$ $\times$ | $(\exists y)Ray\checkmark$ | $7 \supset$ D |
| 9 | | $Rab$ | $8 \exists$ D |
| 10 | | $\neg Rab$ $\times$ | $9 \forall$ D |

Since this tree is closed, the argument is quantificationally valid.

(c) $[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)$

(d) $\{(\forall x)[(\exists y)Hg(x, y) \supset Bg(x, x)], Ha, a = g(a, b)\} \vDash (\exists y)Bg(y, y)$

3. Why does the rule *Existential Decomposition* require that the instantiating constant **a** be foreign to all preceding lines of the branch?

   By not requiring *Existential Decomposition* to introduce foreign constants we have opened up the possibility that the same constant can be reused in a conflicting predicate. So, we require foreign constants with *Existential Decomposition* in order to preserve truth, validity, equivalence, etc.