MAT 168 Calculation 3

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3.1 We first start by rewriting as a dictionary, introducing the necessary slack variables, and ensure that it is degenerate.

$$\begin{aligned}
\zeta &= 0 + \ 10x_1 - 57\,x_2 - \quad 9x_3 - 24x_4 \\
x_5 &= 0 - 0.5x_1 + 5.5x_2 + 2.5x_3 - 9\ \ x_4 \\
x_6 &= 0 - 0.5x_1 + 1.5x_2 + 0.5x_3 - 1\ \ x_4 \\
x_7 &= 1 - \quad 1x_1
\end{aligned}$$

So, since $\bar{b}_5 = \bar{b}_6 = 0$, the dictionary is degenerate.

We use lexicographic perturbation to remove the degeneracy.

$$\begin{aligned}
\zeta &= 0 & & & & + 10\,x_1 - \ 57x_2 - 9\ \ x_3 - 24x_4 \\
x_5 &= 0 + \epsilon_1 & & & & - 0.5x_1 + 5.5x_2 + 2.5x_3 - \ \ 9x_4 \\
x_6 &= 0 & & + \epsilon_2 & & - 0.5x_1 + 1.5x_2 + 0.5x_3 - \ \ 1x_4 \\
x_7 &= 1 & & & + \epsilon_3 & - 1\ \ x_1
\end{aligned}$$

Now, we let can only let x_1 enter.

We look at the constraints and see:

- $x_5 \implies x_1 \leq 2\epsilon_1$
- $x_6 \implies x_1 \leq 2\epsilon_2$
- $x_7 \implies x_1 \leq 1 + \epsilon_3$

The more restrictive constraint is that $x_6 \leq 2\epsilon_2$, so set $x_6 = 2\epsilon_2$.

So we can let x_1 enter and x_6 leave.

$$x_1 = 2\epsilon_2 + 3x_2 + 1x_3 - 2x_4 - 2x_6$$

This gives a new value for ζ. $\zeta = 20\epsilon_2 - 27x_2 + 1x_3 - 44x_4 - 20x_6$

This gives a new value for x_5. $x_5 = \epsilon_1 - \epsilon_2 + 4x_2 + 2x_3 - 8x_4 + 1x_6$

This gives a new value for x_7. $x_7 = 1 - 2\epsilon_2 + \epsilon_3 - 3x_2 - 1x_3 + 2x_4 + 2x_6$

So we have a new dictionary:

$$\begin{aligned}
\zeta &= 0 + 20\epsilon_2 - 27x_2 + x_3 - 44x_4 - 20x_6\\
x_1 &= 0 + 2\epsilon_2 + 3x_2 + x_3 - 2x_4 - 2x_6\\
x_5 &= 0 + \epsilon_1 - \epsilon_2 + 4x_2 + 2x_3 - 8x_4 + x_6\\
x_7 &= 1 - 2\epsilon_2 + \epsilon_3 - 3x_2 - x_3 + 2x_4 + 2x_6
\end{aligned}$$

Now, we can continue optimizing since x_3 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_3 \geq 0$
- $x_5 \implies x_3 \geq 0$
- $x_7 \implies x_3 \leq 1 - 2\epsilon_2 + \epsilon_3$

The more restrictive constraint is that $x_7 \leq 1 - 2\epsilon_2 + \epsilon_3$, so set $x_7 = 1 - 2\epsilon_2 + \epsilon_3$.

So we can let x_3 enter and x_7 leave.

$$x_3 = 1 - 2\epsilon_2 + \epsilon_3 - 3x_2 + 2x_4 + 2x_6 - 1x_7$$

This gives a new value for ζ. $\zeta = 1 + 18\epsilon_2 + 1\epsilon_3 - 30x_2 - 42x_4 - 18x_6 - 1x_7$

This gives a new value for x_1. $x_1 = 1 + 1\epsilon_3 - 1x_7$

This gives a new value for x_5. $x_5 = 2 + 1\epsilon_1 - 5\epsilon_2 + 2\epsilon_3 - 2x_2 - 4x_4 + 5x_6 - 2x_7$

So we have a new dictionary:

$$\begin{aligned}
\zeta &= 1 + 18\epsilon_2 + \epsilon_3 - 30x_2 + x_4 - 44x_6 - 20x_7\\
x_1 &= 1 + \epsilon_3 - x_7\\
x_3 &= 1 - 2\epsilon_2 + \epsilon_3 - 3x_2 + 2x_4 + 2x_6 - x_7\\
x_5 &= 2 + \epsilon_1 - 5\epsilon_2 + 2\epsilon_3 - 2x_2 - 4x_4 + 5x_6 - 2x_7
\end{aligned}$$

Since we have no more optimizable variables (all variable coefficients of ζ are non-positive), we can no longer maximize ζ.

Then we have an optimal solution with $x_1 = x_3 = 1, x_2 = x_4 = x_6 = x_7 = 0, x_5 = 2$, and value 1.