ECS 165A Homework 2

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1. (a) The keys are $\{A, B\}$ and $\{B, C\}$.

   We can see this by computing $\{A, B\}^+$ and $\{B, C\}^+$, seeing that no other combinations of attributes can be keys, and seeing that these are minimal.

   $$\{A, B\}^+ = \{A, B, C\}^+ = \{A, B, C, D\}^+ = \{A, B, C, D, E\}$$
   $$\{B, C\}^+ = \{B, C, E\}^+ = \{B, C, E, A\}^+ = \{B, C, E, A, D\}$$

   (b) This is a minimal basis.

   Proof.
   - Each right hand side is a singleton.
   - We have three FD's with 2 attributes on the left. We check to see if we can remove one of them by computing the closure of each in turn.
     - $AB \to C$
       $$\{A\}^+ = \{A\}$$
       $$\{B\}^+ = \{B, E\}$$
       Since neither of these closed over C we cannot remove either from the left.
     - $CE \to A$
       $$\{C\}^+ = \{C\}$$
       $$\{E\}^+ = \{E\}$$
       Since neither of these closed over A we cannot remove either from the left.
     - $AC \to D$
       $$\{A\}^+ = \{A\}$$
       $$\{C\}^+ = \{C\}$$
       Since neither of these closed over D we cannot remove either from the left.

     So, we cannot simplify the left sides of any of our FD's.
   - We check to see if we can remove any FD's from the set by computing the closure of the left with the rest of the set.
     - $AB \to C$
       $$\{A, B\}^+ = \{A, B, E\}$$
       Since this did not close over C, we cannot remove this FD.

- $B \to E$
$$\{B\}^+ = \{B\}$$
Since this did not close over E, we cannot remove this FD.
- $CE \to A$
$$\{C, E\}^+ = \{C, E\}$$
Since this did not close over A, we cannot remove this FD.
- $AC \to D$
$$\{A, C\}^+ = \{A, C\}$$
Since this did not close over D, we cannot remove this FD.

So, we cannot remove any FD's from the set.

Since we already have every right side being singleton, we cannot simplify the left sides of any of the FD's, and we cannot remove any FD's, this is a minimal basis. □

(c) We start by looking at which of the FD's given violate BCNF. Since we know the keys are $\{\{A, B\}, \{B, C\}\}$, we know that $AB \to C$ and anything that follows from it will not violate BCNF, as these are all superkeys.

This leaves the other FD's as violators of BCNF. These are $B \to E, CE \to A, AC \to D$.

We now check each FD that follows from the above. N.B. If a FD that follows contains a superkey, we needn't compute the closure, as it will not violate BCNF. We also needn't compute the closure of trivial FD's

- $B \to E$

FD	Closure	Violates BCNF
$BD \to E$	$\{B, D, E\}$	VIOLATES

- $CE \to A$

FD	Closure	Violates BCNF
$CDE \to A$	$\{A, C, D, E\}$	VIOLATES

- $AC \to D$

FD	Closure	Violates BCNF
$ACE \to D$	$\{A, C, D, E\}$	VIOLATES

So, we have listed all of the FD's that follow which violate BCNF and are not trivial.

(d) Given the violators above, we can decompose R to BCNF relations.

Start with $B \to E$.

$$R_1(A, B, C, D), R_2(B, E)$$

We have that R_2 is in BCNF as it is a 2-attribute relation with the key $\{B\}$.

For R_1, we still have the FD's $AB \to C, AC \to D$, and only one of our previous keys—$\{A, B\}$—still holds.

We check the closure of our FD's.

$$\{AB\}^+ = \{A, B, C, D\}$$

$$\{AC\}^+ = \{A, C, D\}$$

So, our old violator—$AC \to D$—still exists.

We decompose R_1 into

$$R_3(A, B, C), R_4(C, D)$$

We have that R_4 is in BCNF as it is a 2-attribute relation with the key $\{C\}$.

For R_3, we still have the FD $AB \to C$, and our key is $\{A, B\}$.

The closure can be computed to show this is in BCNF.

$$\{AB\}^+ = \{A, B, C\}$$

Thus, we have decomposed $R(A, B, C, D, E)$ into the BCNF relations:

$$R_2(B, E), R_3(A, B, C), R_4(C, D)$$

(e) Using the charts generated before:

- $B \to E$

FD	Closure	Violates BCNF
$BD \to E$	$\{B, D, E\}$	VIOLATES

- $CE \to A$

FD	Closure	Violates BCNF
$CDE \to A$	$\{A, C, D, E\}$	VIOLATES

- $AC \to D$

FD	Closure	Violates BCNF
$ACE \to D$	$\{A, C, D, E\}$	VIOLATES

We can see that the violators of 3NF are the ones whose right sides are not prime.

Since the FD $CE \to A$ and each of its followers that violates BCNF contain a prime right side (A), we have the rest as violators of 3NF.

That is:

- $B \to E$

FD	Closure	Violates 3NF
$BD \to E$	$\{B, D, E\}$	VIOLATES

- $AC \to D$

FD	Closure	Violates 3NF
$ACE \to D$	$\{A, C, D, E\}$	VIOLATES

(f) We decompose R by first creating relations of all the FD's:

$$R_1(A, B, C), R_2(B, E), R_3(A, C, E), R_4(A, C, D)$$

By definition these all have a minimal basis of the FD used to construct them. The keys of each relation is the left side of each FD.

Relation	Key
R_1	$\{A, B\}$
R_2	$\{B\}$
R_3	$\{C, E\}$
R_4	$\{A, C\}$

None of these relations is a subset of another, so we needn't remove any. Furthermore, R_1 is a superkey for R, so we needn't add any more relations.

Thus, we have decomposed R into 3NF.

(g) We start with all of the FD's that violated BCNF and promote them to MVD's

$$B \to E$$
$$BD \to E$$
$$CE \to A$$
$$CDE \to A$$
$$AC \to D$$
$$ACE \to D$$

becomes

$$B \twoheadrightarrow E$$
$$BD \twoheadrightarrow E$$
$$CE \twoheadrightarrow A$$
$$CDE \twoheadrightarrow A$$
$$AC \twoheadrightarrow D$$
$$ACE \twoheadrightarrow D$$

And, we complement these to obtain all of the violators of 4NF.

$$B \twoheadrightarrow E$$
$$B \twoheadrightarrow ACD$$
$$BD \twoheadrightarrow E$$
$$BD \twoheadrightarrow AC$$
$$CE \twoheadrightarrow A$$
$$CE \twoheadrightarrow BD$$
$$CDE \twoheadrightarrow A$$
$$CDE \twoheadrightarrow B$$
$$AC \twoheadrightarrow D$$
$$AC \twoheadrightarrow BE$$
$$ACE \twoheadrightarrow D$$
$$ACE \twoheadrightarrow B$$

(h) We can decompose R using the violators found above.

$$R_1(B,E), R_2(A,B,C,D)$$

Since R_1 is a 2-attribute schema, it is in 4NF with the key $\{B\}$, FD $B \to E$, and MVD $B \twoheadrightarrow E$.

R_2 has the key $\{A,B\}$, FD's $AB \to C, AC \to D$, and MVD's $AB \twoheadrightarrow C, AC \twoheadrightarrow D, AB \twoheadrightarrow D, AC \twoheadrightarrow B$.

The only violator is $AC \twoheadrightarrow D$ as $\{AC\}^+ = \{A,C,D\}$. The other MVD's either have a superkey on the left or their closure is $\{A,B,C,D\}$.

So, we decompose R_2 further.

$$R_3(A,C,D), R_4(A,B)$$

Since R_4 is a 2-attribute schema, it is in 4NF with the key $\{A,B\}$, and only the trivial FD's and MVD's.

R_3 has the key $\{A,C\}$, FD $AC \to D$, and MVD $AC \twoheadrightarrow D$.

Since R_3 has no violators, we have finished our decomposition.

That is:

$$R_1(B,E), R_3(A,C,D), R_4(A,B)$$

(i) We need to check the closure of the following sets:

$$\{B\}, \{C\}, \{D\}, \{B,C\}, \{B,D\}, \{C,D\}$$

We do not need to check $\{\}, \{B,C,D\}$ as they only have trivial FD.

$$\{B\}^+ = \{B,E\}$$

So, we have no new FD's.

$$\{C\}^+ = \{C\}$$

So, we have no new FD's.

$$\{D\}^+ = \{D\}$$

So, we have no new FD's.

$$\{B,C\}^+ = \{A,B,C,D,E\}$$

So, we have the FD's $BC \to D$.

$$\{B,D\}^+ = \{B,D,E\}$$

So, we have no new FD's.

$$\{C,D\}^+ = \{C,D\}$$

So, we have no new FD's.

So, the FD that holds in S is $BC \to D$.

From these we can promote to an MVD to get $BC \twoheadrightarrow D$

Thus, the dependencies that hold are $BC \to D, BC \twoheadrightarrow D$.

2. (a) We have the tableau

C	T	H	R	S	G
c_1	t_1	H	r_1	S	g_1
c_2	t_2	H	r_2	S	g_2

Using $HS \to R$, we infer $r_1 = r_2$

C	T	H	R	S	G
c_1	t_1	H	r_1	S	g_1
c_2	t_2	H	r_1	S	g_2

Using $HR \to C$, we infer $c_1 = c_2$

C	T	H	R	S	G
c_1	t_1	H	r_1	S	g_1
c_1	t_2	H	r_1	S	g_2

Using $C \to T$, we infer $t_1 = t_2$

C	T	H	R	S	G
c_1	t_1	H	r_1	S	g_1
c_1	t_1	H	r_1	S	g_2

Since both tuples agree in the T column, we conclude that $HS \to T$ holds.

(b) We have the tableau

C	T	H	R	S	G
c_1	T	H	R	s_1	g_1
c_2	T	H	R	s_2	g_2

Using $HR \to C$, we infer $c_1 = c_2$

C	T	H	R	S	G
c_1	T	H	R	s_1	g_1
c_1	T	H	R	s_2	g_2

Since we can apply no more FD's, we conclude that $THR \to G$ does not hold.

(c) We have the tableau

C	T	H	R	S	G
C	T	H	r_1	s_1	g_1
C	t_2	H	R	s_2	G
C	t_3	h_3	R	S	g_3

Using $C \to T$, we infer $T = t_2$

C	T	H	R	S	G
C	T	H	r_1	s_1	g_1
C	T	H	R	s_2	G
C	t_3	h_3	R	S	g_3

Using $C \to T$, we infer $T = t_3$

C	T	H	R	S	G
C	T	H	r_1	s_1	g_1
C	T	H	R	s_2	G
C	T	h_3	R	S	g_3

Using $HT \rightarrow R$, we infer $R = r_1$

C	T	H	R	S	G
C	T	H	R	s_1	g_1
C	T	H	R	s_2	G
C	T	h_3	R	S	g_3

Since we can apply no more FD's, we conclude that the decomposition is not a lossless join.