MAT 168 Calculation 1

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2.1 We first start by rewriting as a dictionary:

$$\begin{aligned}
\text{maximize} \quad \zeta &= 0 + 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
\text{subject to} \quad x_5 &= 5 - 2x_1 - x_2 - x_3 - 3x_4 \\
x_6 &= 3 - x_1 - 3x_2 - x_3 - 2x_4
\end{aligned}$$

Then we can begin by entering with the largest variable, x_4.

We look at the constraints and see:

- $x_5 \implies x_4 \leq \frac{5}{3}$
- $x_6 \implies x_4 \leq \frac{3}{2}$

The more restrictive constraint is that $x_4 \leq \frac{3}{2}$, so set $x_4 = \frac{3}{2}$.

So we can let x_4 enter and x_6 exit.

$$x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_6$$

This gives a new value for x_5.

$$x_5 = \frac{1}{2} - \frac{1}{2}x_1 + \frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{3}{2}x_6$$

This gives a new value for ζ.

$$\zeta = \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}x_6$$

So we have a new dictionary:

$$\text{maximize} \quad \zeta \;=\; \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}x_6$$
$$\text{subject to} \quad x_5 = \frac{1}{2} - \frac{1}{2}x_1 + \frac{7}{2}x_2 + \frac{1}{2}x_3 + \frac{3}{2}x_6$$
$$x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_6$$

Now, we can continue optimizing since x_1 has a positive coefficient.
We look at the constraints and see:

- $x_5 \implies x_1 \leq 1$
- $x_4 \implies x_1 \leq 3$

The more restrictive constraint is that $x_1 \leq 1$, so set $x_1 = 1$.
So we can let x_1 enter and x_5 exit.

$$x_1 = 1 + 7x_2 + x_3 + 3x_6 - 2x_5$$

This gives a new value for x_4.

$$x_4 = 1 - 5x_2 - x_3 - 2x_6 + x_5$$

This gives a new value for ζ.

$$\zeta = 15 + 5x_2 + 2x_3 - 3x_5$$

So we have a new dictionary:

$$\text{maximize} \quad \zeta \;=\; 15 + 5x_2 + 2x_3 \qquad\quad - 3x_5$$
$$\text{subject to} \quad x_1 = 1 \;+\; 7x_2 + \;\; x_3 + 3x_6 - 2x_5$$
$$x_4 = 1 \;-\; 5x_2 - \;\; x_3 - 2x_6 + \;\; x_5$$

Now, we can continue optimizing since x_2 has a positive coefficient.
We look at the constraints and see:

- $x_1 \implies x_2 \geq 0$
- $x_4 \implies x_2 \leq \frac{1}{5}$

The more restrictive constraint is that $x_2 \leq \frac{1}{5}$, so set $x_2 = \frac{1}{5}$.

So we can let x_2 enter and x_4 exit.

$$x_2 = \frac{1}{5} - \frac{1}{5}x_4 - \frac{1}{5}x_3 - \frac{2}{5}x_6 + \frac{1}{5}x_5$$

This gives a new value for x_1.

$$x_1 = \frac{12}{5} - \frac{7}{5}x_4 - \frac{2}{5}x_3 + \frac{1}{5}x_6 - \frac{3}{5}x_5$$

This gives a new value for ζ.

$$\zeta = 16 - x_4 + x_3 - 2x_6 - 2x_5$$

So we have a new dictionary:

$$
\begin{aligned}
\text{maximize} \quad \zeta &= 16 - x_4 + x_3 - 2\,x_6 - 2x_5 \\
\text{subject to} \quad x_1 &= \frac{12}{5} - \frac{7}{5}x_4 - \frac{2}{5}x_3 + \frac{1}{5}x_6 - \frac{3}{5}x_5 \\
x_2 &= \frac{1}{5} - \frac{1}{5}x_4 - \frac{1}{5}x_3 - \frac{2}{5}x_6 + \frac{1}{5}x_5
\end{aligned}
$$

Now, we can continue optimizing since x_3 has a positive coefficient.

We look at the constraints and see:

- $x_1 \implies x_3 \leq 6$
- $x_2 \implies x_3 \leq 1$

The more restrictive constraint is that $x_3 \leq 1$, so set $x_3 = 1$.

So we can let x_3 enter and x_2 exit.

$$x_3 = 1 - x_4 - 5x_2 - 2x_6 + x_5$$

This gives a new value for x_1.

$$x_1 = 2 - x_4 + 2x_2 + x_6 - x_5$$

This gives a new value for ζ.

$$\zeta = 17 - 2x_4 - 5x_2 - 4x_6 - x_5$$

So we have a new dictionary:

$$
\begin{aligned}
\text{maximize} \quad & \zeta = 17 - 2x_4 - 5x_2 - 4x_6 - x_5 \\
\text{subject to} \quad & x_1 = 2 - x_4 + 2x_2 + x_6 - x_5 \\
& x_3 = 1 - x_4 - 5x_2 - 2x_6 + x_5
\end{aligned}
$$

Since we have no more optimizable variables (all variable coefficients of ζ are non-positive), we can no longer maximize ζ.

Then we have an optimal solution with $x_1 = 2, x_3 = 1, x_2 = x_4 = x_5 = x_6 = 0$, and value 17.

2.2

2.3