ECS 122A Sections **001** Midterm Exam Solution

1. (a) False, $H(n) = \ln n + O(1) \in \Theta(\log n)$

   (b) True, observe that $\log(c\sqrt{n}) = \log c + \log(\sqrt{n})$.

   (c) False, it takes $\Theta(n^{\log 7})$.

   (d) False. Dynamic programming (not greedy algorithm)

2. (a) $T(n) = 2 \cdot T(n-2) = 2^k T(n-2k) = \Theta(2^{n/2})$, where assumes that $n = 2k$.

   (b) $T(n) = 3 \cdot T(\frac{n}{4}) + n + 1 = \Theta(n)$ by the case 3 of the master theorem.

   (c) $T(n) = 16 \cdot T(\frac{n}{4}) + n^2 = \Theta(n^2 \log n)$ by the case 2 of the master theorem.

3. (a) $\sum_{i=0}^{k} \frac{1}{2^i} = \frac{1 - \frac{1}{2^{k+1}}}{1 - \frac{1}{2}} = 2 - \frac{1}{2^k}$, geometric sum

   (b) $\Omega(g(n)) = \{f(n) : \exists\, c, n_0 \text{ such that } f(n) \geq cg(n) \geq 0, \forall n \geq n_0\}$

   (c) Counting sort, $O(n)$

   (d) The optimal solution to the problem contains optimal solutions to subproblems. For example, the matrix-chain multiplication.

4. (a) The codes are

char	frequency	code
h	21	1
g	13	01
f	8	001
e	5	0001
d	3	00001
c	2	000001
b	1	0000001
a	1	0000000

   (b) In general, the code for the ith character, in a sequence of n characters where the frequencies are the first n Fibonacci numbers, is $n-1$ 0's when $i = 1$, and $0^{(n-i)}1$ for $2 \leq i \leq n$.

5. (a) For example, for the change of $N = 55$ cents,

		total
greedy:	$50 + 5 \cdot 1$	6 coins
optimal:	$25 + 3 \cdot 10$	4 coins

   (b) The smallest number of coins required $= \min\{\textsc{Change}(N), 1 + \textsc{Change}(N - 50)$. This assumes that $\textsc{Change}(N)$ returns ∞ for $N < 0$.

   (c) The optimal solution has to either have 0 or 1 half-dollars in it. The solution finds the best of options and returns the minimum of those.

   Running time $= 2 \cdot O(N) = O(N)$.