MAT 150A Homework 6

Hardy Jones
999397426
Professor Schilling
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1. The matrix is:

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

2. (a) To show that $\mathcal{O}_n \leq GL_n(\mathbb{R})$ is a subgroup, we need to show that it is closed, has the identity and has inverses.

   Proof. i. **Closure**
   Choose $A, B \in \mathcal{O}_n$.
   AB is in $\mathcal{O}_n$ if $A^T AB^T B = I_n$.

$$A^T AB^T B = (A^T A)(B^T B) = I_n I_n = I_n$$

   So $\mathcal{O}_n$ has closure.

   ii. **Identity**

$$I_n^T I_n = I_n I_n = I_n$$

   So $\mathcal{O}_n$ has the identity.

   iii. **Inverse**
   Choose $A \in \mathcal{O}_n$.
   Since $A^T A = I_n$ $A^T = A^{-1}$, so $\mathcal{O}_n$ has inverses.

   From these three, $\mathcal{O}_n \leq GL_n(\mathbb{R})$ is a subgroup. □

   (b) If we show that $\mathcal{SO}_n \leq \mathcal{O}_n$, then from above $\mathcal{SO}_n \leq GL_n(\mathbb{R})$.
   Again we need to show it's closed, has the identity and has inverses.

   Proof. i. **Closure**
   Choose $A, B \in \mathcal{SO}_n$.
   AB is in $\mathcal{SO}_n$ if $|AB| = 1$

$$|AB| = |A||B| = 1 \cdot 1 = 1$$

   So, $\mathcal{SO}_n$ is closed.

ii. **Identity**

$$|I_n| = 1$$

So, $\mathcal{SO}_n$ has the identity.

iii. **Inverse**
Choose $A \in \mathcal{SO}_n$.
Since $|A| \neq 0$, we know that A is invertible. So, $A^{-1} \in \mathcal{SO}_n$. Thus, $\mathcal{SO}_n$ has inverses.

From these three, $\mathcal{O}_n \leq \mathcal{O}_n$ is a subgroup. $\square$