

ECS 120 Problem Set 1

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Problem 1 We can gain some insight by looking at the first few n -digit palindromes.

n	elements	$ n $
1	1,2,3,4,5,6,7,8,9	9
2	11,22,33,44,55,66,77,88,99	9
3	101,202,303,404,505,606,707,808,909	90
	111,212,313,414,515,616,717,818,919	
	\vdots	
	191,292,393,494,595,696,797,898,999	
4	1001,2002,3003,4004,5005,6006,7007,8008,9009	90
	1111,2112,3113,4114,5115,6116,7117,8118,9119	
	\vdots	
	1991,2992,3993,4994,5995,6996,7997,8998,9999	

If we continue in this way we see that the number of palindromes increases by 10 for every 2 digits added.

We can see a recurrence relation here:

$$D_1 = 9$$

$$D_2 = 9$$

$$D_n = D_{n-2} \cdot 10, \text{ for } n > 2$$

From this we can manipulate it to a closed form:

$$D_n = 9 \cdot 10^{\lfloor \frac{n-1}{2} \rfloor}$$

We can prove this by induction.

Proof. Base Cases:

$$n = 1$$

$$D_1 = 9 \cdot 10^{\lfloor \frac{1-1}{2} \rfloor} = 9 \cdot 10^0 = 9$$

$$n = 2$$

$$D_2 = 9 \cdot 10^{\lfloor \frac{2-1}{2} \rfloor} = 9 \cdot 10^0 = 9$$

Inductive Hypothesis:

$$D_n = 9 \cdot 10^{\lfloor \frac{n-1}{2} \rfloor}$$

Inductive Case:

$$\begin{aligned} D_{n+1} &= D_{n-1} \cdot 10 \\ &= 9 \cdot 10^{\lfloor \frac{(n-1)-1}{2} \rfloor} \cdot 10 \\ &= 9 \cdot 10^{\lfloor \frac{n-2}{2} \rfloor} \cdot 10 \\ &= 9 \cdot 10^{\lfloor \frac{n}{2} - \frac{2}{2} \rfloor} \cdot 10 \\ &= 9 \cdot 10^{\lfloor \frac{n}{2} - 1 \rfloor} \cdot 10 \\ &= 9 \cdot 10^{\lfloor \frac{n}{2} - 1 + 1 \rfloor} \\ &= 9 \cdot 10^{\lfloor \frac{n}{2} \rfloor} \end{aligned}$$

Thus, we have proved that our formula is correct.

□

Now, we can calculate $D_{20} = 9 \cdot 10^{\lfloor \frac{20-1}{2} \rfloor} = 9 \cdot 10^9 = 9000000000$