ECS 122A Homework 6

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1. We need to state our variables, constants and constraints.

   - Variables: $p_i \in \{0, 1\}$ for $i \in [1, n]$
     Where $p_i = 1$ when $p_i \in P'$ and $p_i = 0$ when $p_i \notin P'$ We want:

     $$min \sum_{i=1}^{n} p_i$$

   - Constraints:
     We need to look at cases where the issue passed and where the issue failed.
     For passed issues:

     $$\begin{aligned} v_{11}(p_1 - 1) + v_{21}(p_2 - 1) + \cdots + v_{n1}(p_n - 1) &\geq 1 \\ v_{12}(p_1 - 1) + v_{22}(p_2 - 1) + \cdots + v_{n2}(p_n - 1) &\geq 1 \\ &\vdots \\ v_{1m}(p_1 - 1) + v_{2m}(p_2 - 1) + \cdots + v_{nm}(p_n - 1) &\geq 1 \end{aligned}$$

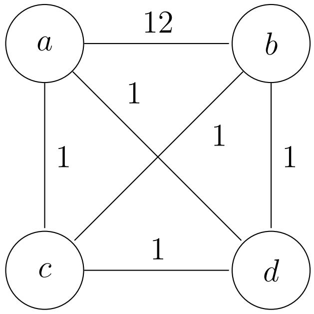
     For failed issues:

     $$\begin{aligned} v_{11}(p_1 + 1) + v_{21}(p_2 + 1) + \cdots + v_{n1}(p_n + 1) &\leq -1 \\ v_{12}(p_1 + 1) + v_{22}(p_2 + 1) + \cdots + v_{n2}(p_n + 1) &\leq -1 \\ &\vdots \\ v_{1m}(p_1 + 1) + v_{2m}(p_2 + 1) + \cdots + v_{nm}(p_n + 1) &\leq -1 \end{aligned}$$

     Where v_{ij} represents how person p_i voted on issue I_j. That is:
     $v_{ij} = 1$ if p_i voted yes.
     $v_{ij} = 0$ if p_i abstained.
     $v_{ij} = -1$ if p_i voted no.

   We get back from our 01-IP formulation a set of p values P'. We need to check that $|P'| = k$. If so, then we have found an optimal solution.

2. (a) Using the following graph:



You can construct a tour starting at node a using the greedy approach: a, c, d, b, a. This tour has a cost of $1 + 1 + 1 + 12 = 15$. Whereas an optimal tour is a, d, b, c, a with a cost of $1 + 1 + 1 + 1 = 4$.

So the greedy approach did not find an optimal tour.

(b) We can use the above graph and change the edge between a and b to have a value of

$$k + \text{cost of optimal tour} - \text{cost of greedy tour excluding edge } k$$

. Now, the greedy approach will always find a tour of cost

$$k + \text{cost of optimal tour}$$

. This tour is exactly k greater than the optimal tour.

3. (a) One benefit of linear programming is that you can describe any problem in **P** and solve it, even if you do not explicitly know an algorithm for the problem. One drawback of linear programming is that you have no control over the runtime of the algorithm; you may be able to solve the problem, but the LP solver may take $O(n^5)$ time when an $O(n)$ solution is possible.

(b) One benefit of 01-integer programming is that you can solve problems in **NP** now, even without an explicit algorithm for the original problem. One drawback of 01-integer programming is that the solver now does not run in polynomial time for sure.

(c) One benefit of linear programming relaxation is that you can take a problem in **NP** and find "some" local optimal solution in polynomial time. One drawback of linear programming relaxation is that you have to interpret the results of the output in order to use the solution; this interpretation may not be trivial.