PHIL 112 Homework 4

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1. Define

   (a) Theorem in *PD*

   A sentence **P** of *PL* is a theorem in *PD* if and only if **P** is derivable in *PD* from the empty set.

   (b) Equivalence in *PD*

   Sentences **P** and **Q** of *PL* are equivalent in *PD* if and only if **Q** is derivable in *PD* from {**P**} and **P** is derivable in *PD* from {**Q**}.

2. Construct derivations that show each of the following:

   (a) $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*

1. ¬(∀x)(∃y)Lxy	**Assum**
2. (∃y)(∀x)Lxy	**Assum**
3. (∃x)¬(∃y)Lxy	**QN 1**
4. (∃x)(∀y)¬Lxy	**QN 3**
5. (∀y)¬Lay	**Assum ∃ Elim**
6. (∀y)¬Lay	**Reit:** 5
7. (∀y)¬Lay	∃ **Elim:** 5, 6
8. (∀x)Lxb	**Assum ∃ Elim**
9. (∀x)Lxb	**Reit:** 8
10. (∀x)Lxb	∃ **Elim:** 8, 9
11. ¬Lab	∀ **Elim:** 7
12. Lab	∀ **Elim:** 10

   From lines 11 and 12 we have $\neg Lab \wedge Lab$.

   Thus $\{\neg(\forall x)(\exists y)Lxy, (\exists y)(\forall x)Lxy\}$ is inconsistent in *PD*.

   (b) $\{((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)\}$ is a theorem in *PD*

1. $(\exists x)Fx \vee (\exists x)Gx$	**Assum**
2. $(\exists x)Fx$	**Assum**
3. Fa	**Assum**
4. $Fa \vee Ga$	$\vee$ **Intro:** 3
5. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 4
6. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 2, 3–5
7. $(\exists x)Gx$	**Assum**
8. Gb	**Assum**
9. $Fb \vee Gb$	$\vee$ **Intro:** 8
10. $(\exists x)(Fx \vee Gx)$	$\exists$ **Intro:** 9
11. $(\exists x)(Fx \vee Gx)$	$\exists$ **Elim:** 7, 8–10
12. $(\exists x)(Fx \vee Gx)$	$\vee$ **Elim:** 1, 2–6,7–11,
13. $((\exists x)Fx \vee (\exists x)Gx) \rightarrow (\exists x)(Fx \vee Gx)$	$\rightarrow$ **Intro:** 1–12

3. Symbolize Casino Slim's reasoning and construct a derivation in *PD+* showing that the symbolized argument is valid in *PD+*.

1. $(\forall x)(\forall y)[(Fx \wedge Sy) \rightarrow Bxy]$	**Assum**
2. $(Fh \vee Ff) \wedge Ss$	**Assum**
3. $Fh \vee Ff$	$\wedge$ **Elim:** 2
4. Ss	$\wedge$ **Elim:** 2
5. $(\forall y)[(Fx \wedge Sy) \rightarrow Bhy]$	$\forall$ **Elim:** 1
6. $(Fh \wedge Ss) \rightarrow Bhs$	$\forall$ **Elim:** 5
7. $\neg(Fh \wedge Ss) \vee Bhs$	**Impl:** 5
8. $Fh \vee Ff$	**Assum**
9. Fh	**Assum**
10. $(\exists x)Fx$	$\exists$ **Intro:** 9
11. Ff	**Assum**
12. $(\exists x)Fx$	$\exists$ **Intro:** 11
13. $(\exists x)Fx$	$\vee$ **Elim:** 8, 9–10, 11–12
14. $(\exists x)Fx$	$\exists$ **Intro:** 13
15. Fh	**Assum**
16. Fh	**Reit:** 15
17. Fh	$\exists$ **Elim:** 14, 15–16
18. $Fh \wedge Ss$	$\wedge$ **Intro:** 4,17
19. Bhs	$\rightarrow$ **Elim:** 6, 18
20. $Bhs \vee Bfs$	$\vee$ **Intro:** 19
21. $(\exists x)(Bhx \vee Bfx)$	$\exists$ **Intro:** 20