

ECS 122A Homework 1

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1. The entries of BB^T describe the edges in the graph.

For each i, j in BB^T

- if $i = j$ then it describes how many edges are at node i .
- if $i \neq j$ then it describes if there is an edge between node i and node j .

2. (a) *Proof.* Every edge e in G connects two vertices v_i, v_j . So each vertex v_i adds 1 to the total sum for every edge e it has. This means that each edge e is counted exactly twice in the sum.

Thus,

$$\sum_{v \in V} d(v) = 2|E|$$

□

- (b) *Proof.* Assume not, that is assume there is an odd number of vertices with odd degree. Call this set of vertices v_o , and the rest of the vertices v_e

We can see that $\sum_{v \in v_e} d(v)$ is even, and that $\sum_{v \in v_o} d(v)$ is odd. Also we have, $\sum_{v \in (v_e \cup v_o)} d(v)$ is odd. N.B. $v_e \cup v_o = V$. but we know that $\sum_{v \in V} d(v) = 2|E|$, so we have a contradiction.

So our assumption was wrong.

Thus, there must be an even number of vertices with odd degree.

□

- (c) No, there is no similar statement. We can have a simple directed graph from A to B , but, there is only one *indegree*, which is odd.