ECS 154A Homework 1

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1. Determine if the following are equivalent or not.

   (a)

   $$WX\overline{Y} + WZ + \overline{W}Y + \overline{WX} = (W + \overline{X} + Y)(\overline{W} + X + Z)(\overline{W} + \overline{Y} + Z)$$

   Equivalent

   (b)

   $$\overline{X} + X\overline{Y}Z = (\overline{X} + Z)(W + \overline{X} + \overline{Y})(\overline{W} + \overline{X} + \overline{Y})$$

   Equivalent

   (c)

   $$\overline{W}XYZ + W\overline{X}YZ + WX\overline{Y}Z + WXY\overline{Z} = (W + Y)(\overline{W} + \overline{Y})(\overline{X} + Z)$$

   Not equivalent. Take the case when $W = 1$ and $X = Y = Z = 0$. The LHS is obviously 0, but the RHS is 1.

   (d)

   $$Z(X + \overline{W}) = Z\overline{W} + XYZ$$

   Not equivalent. Take the case when $Y = 1$ and $W = X = Z = 0$. The LHS is 1, but the RHS is 0.

   (e)

   $$W + \overline{W}Z + Y + \overline{XY} = W + \overline{X} + Y + Z$$

   Equivalent.

2. Use algebraic manipulation to find the minimum sum of products expression.

$$\begin{aligned}
&\overline{(\overline{X}+Y)(W+Y+Z+\overline{W})}+\overline{X}Z+(W+\overline{Y}Z)(\overline{Y}Z+\overline{W}) && \\
&\overline{(\overline{X}+Y)(W+Y+Z+\overline{W})}+\overline{X}Z+(\overline{Y}Z+W)(\overline{Y}Z+\overline{W}) && \text{(Commutativity)}\\
&\overline{(\overline{X}+Y)(W+Y+Z+\overline{W})}+\overline{X}Z+\overline{Y}Z && \text{(Combining)}\\
&\overline{(\overline{X}+Y)(W+Y+Z+\overline{W})}+\overline{X}Z+\overline{Y}Z && \text{(Distributivity)}\\
&\overline{(\overline{X}+Y)}+\overline{(W+Y+Z+\overline{W})}+\overline{X}Z+\overline{Y}Z && \text{(De Morgan's)}\\
&(\overline{\overline{X}}+\overline{Y})\ \overline{(W+Y+Z+\overline{W})}+\overline{X}Z+\overline{Y}Z && \text{(De Morgan's)}\\
&X\overline{Y}+\overline{(W+Y+Z+\overline{W})}+\overline{X}Z+\overline{Y}Z && \text{(Involution)}\\
&X\overline{Y}+(\overline{W}\ \overline{Y}\ \overline{Z}\ \overline{\overline{W}})+\overline{X}Z+\overline{Y}Z && \text{(De Morgan's)}\\
&X\overline{Y}+(\overline{W}\ \overline{Y}\ \overline{Z}\ W)+\overline{X}Z+\overline{Y}Z && \text{(Involution)}\\
&X\overline{Y}+(\overline{Y}\ \overline{Z}W\overline{W})+\overline{X}Z+\overline{Y}Z && \text{(Commutativity)}\\
&X\overline{Y}+(\overline{Y}\ \overline{Z}\ 0)+\overline{X}Z+\overline{Y}Z && \text{(Complementation)}\\
&X\overline{Y}+\overline{X}Z+\overline{Y}Z && \text{(Nilpotent)}
\end{aligned}$$

3. Use algebraic manipulation to find the minimum product of sums expression.

$$\overline{(\overline{X}Z + \overline{W})(\overline{X}Z + \overline{Y}) + \overline{W}X\overline{Y}}$$

$$\overline{(\overline{X}Z + \overline{W})(\overline{X}Z + \overline{Y})}\ \overline{\overline{W}X\overline{Y}} \qquad \text{(De Morgan's)}$$

$$\overline{(\overline{X}Z + \overline{W})(\overline{X}Z + \overline{Y})}\left(\overline{\overline{W}} + \overline{X} + \overline{\overline{Y}}\right) \qquad \text{(De Morgan's)}$$

$$\overline{(\overline{X}Z + \overline{W})(\overline{X}Z + \overline{Y})}\left(W + \overline{X} + \overline{\overline{Y}}\right) \qquad \text{(Involution)}$$

$$\overline{(\overline{X}Z + \overline{W})(\overline{X}Z + \overline{Y})}(W + \overline{X} + Y) \qquad \text{(Involution)}$$

$$\left(\overline{(\overline{X}Z + \overline{W})} + \overline{(\overline{X}Z + \overline{Y})}\right)(W + \overline{X} + Y) \qquad \text{(De Morgan's)}$$

$$\left(\overline{\overline{X}Z}\ \overline{\overline{W}} + \overline{(\overline{X}Z + \overline{Y})}\right)(W + \overline{X} + Y) \qquad \text{(De Morgan's)}$$

$$\left(\overline{\overline{X}Z}\ W + \overline{(\overline{X}Z + \overline{Y})}\right)(W + \overline{X} + Y) \qquad \text{(Involution)}$$

$$\left(\overline{\overline{X}Z}\ W + \overline{\overline{X}Z}\ \overline{\overline{Y}}\right)(W + \overline{X} + Y) \qquad \text{(De Morgan's)}$$

$$\left(\overline{\overline{X}Z}\ W + \overline{\overline{X}Z}\ Y\right)(W + \overline{X} + Y) \qquad \text{(Involution)}$$

$$\left(\overline{\overline{X}Z}(W + Y)\right)(W + \overline{X} + Y) \qquad \text{(Distributivity)}$$

$$(\overline{\overline{X}} + \overline{Z})(W + Y)(W + \overline{X} + Y) \qquad \text{(De Morgan's)}$$

$$(X + \overline{Z})(W + Y)(W + \overline{X} + Y) \qquad \text{(Involution)}$$

$$(X + \overline{Z})(W + Y)(W + Y + \overline{X}) \qquad \text{(Commutativity)}$$

$$(X + \overline{Z})(W + Y) \qquad \text{(Covering)}$$