ECS 120 Problem Set 5

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Problem 1 (a) Yes, this is a regular language.

Since the distinct number of decimal digits is finite, i.e. less than or equal to 10, we can construct an NFA to correspond to this. We need not know which digits appear infinitely often. We just need our NFA to have an arrow for each d in $\{0, 1, 2, ..., 9\}$ where d is a decimal digits that occurs infinitely often.

(b) No, this is not a regular language.

Assume for contradiction that L_b is regular.

Then, there exists some ρ such that, for all $s \in L_b, |s| \geq \rho$, there exists some $xyz = s, y \neq \varepsilon$ such that, for all $i \geq 0, xy^iz \in L_b$.

Now, this implies that there is some repeating pattern within the decimal representation of π. However, since π is irrational, there can be no such repeating pattern.

From this contraction, we see that L_b is not regular.

Problem 3 It helps to see some of the strings in the language.

$$L = \{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, abc, acb, bac, bbb, bca, \dots\}$$

We see that this language is quite a bit more complex than it appears on the face.

$$\begin{array}{lcl} S & \rightarrow & T \mid U \mid V \\ T & \rightarrow & aTbT \mid bTaT \mid abT \mid baT \mid cT \mid \varepsilon \\ U & \rightarrow & aUcU \mid cUaU \mid acU \mid caU \mid bU \mid \varepsilon \\ V & \rightarrow & bVcV \mid cVbV \mid bcV \mid cbV \mid aV \mid \varepsilon \end{array}$$

So, we can see that each production has an equal number of two different characters.

Problem 4 It helps to see some of the strings in the language.

$$\begin{aligned} L = \{&0 \neq 1, 1 \neq 0, \\ &0 \neq 00, 0 \neq 01, 0 \neq 10, 0 \neq 11, 1 \neq 00, 1 \neq 01, 1 \neq 10, 1 \neq 11, \\ &00 \neq 0, 01 \neq 0, 10 \neq 0, 11 \neq 0, 00 \neq 1, 01 \neq 1, 10 \neq 1, 11 \neq 1, \\ &00 \neq 01, 00 \neq 10, 00 \neq 11, 01 \neq 00, 01 \neq 10, \dots\} \end{aligned}$$

We can see that we'll have to look at at least two situation.

One where $|x| = |y|$ and one where $|x| \neq |y|$.

When $|x| = |y|$, we want to separate x and y into substrings $x = x_0x_1x_2$ and $y = y_0y_1y_2$ such that $|x_0| = |y_0|, |x_2| = |y_2|, x_1 \neq \varepsilon, y_1 \neq \epsilon, x_1 \neq y_1$.

We can easily construct a CFG for this. Let's start with each side.

$$
\begin{aligned}
X &\rightarrow BXB \mid 1 \\
Y &\rightarrow BYB \mid 0 \\
B &\rightarrow 0 \mid 1
\end{aligned}
$$

Now, we can connect both sides.

$$
E \rightarrow X{\neq}Y \mid Y{\neq}X
$$

For the other case, where $|x| \neq |y|$ we don't care which digits are in which position, just that one side has more digits than the other.

Let's start with each side.

$$
\begin{aligned}
T &\rightarrow BTB \mid U \\
B &\rightarrow 0 \mid 1 \\
U &\rightarrow 0{\neq}0 \mid 0{\neq}1 \mid 1{\neq}0 \mid 1{\neq}1
\end{aligned}
$$

Now, connect both sides, ensure that one has at least one more letter than the other.

$$
\begin{aligned}
N &\rightarrow VT \mid TV \\
V &\rightarrow 0V \mid 1V
\end{aligned}
$$

All together we end up with the following CFG.

$$
\begin{aligned}
S &\rightarrow E \mid N \\
E &\rightarrow X{\neq}Y \mid Y{\neq}X \\
N &\rightarrow TV \mid VT \\
X &\rightarrow BXB \mid 1 \\
Y &\rightarrow BYB \mid 0 \\
T &\rightarrow BTB \mid U \\
U &\rightarrow 0{\neq}0 \mid 0{\neq}1 \mid 1{\neq}0 \mid 1{\neq}1 \\
V &\rightarrow 0V \mid 1V \\
B &\rightarrow 0 \mid 1
\end{aligned}
$$