MAT 108 HW 5

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Spring 2015

§2.2 13 (c) $A \times B$

$\{(\varnothing, (\varnothing, \{\varnothing\})), (\varnothing, \{\varnothing\}), (\varnothing, (\{\varnothing\}, \varnothing)), (\{\varnothing\}, (\varnothing, \{\varnothing\})), (\{\varnothing\}, \{\varnothing\}), (\{\varnothing\}, (\{\varnothing\}, \varnothing)), (\{\varnothing, \{\varnothing\}\}, (\varnothing, \{\varnothing\})), (\{\varnothing, \{\varnothing\}\}, \{\varnothing\}), (\{\varnothing, \{\varnothing\}\}, (\{\varnothing\}, \varnothing))\}$

$B \times A$

$\{((\varnothing, \{\varnothing\}), \varnothing), ((\varnothing, \{\varnothing\}), \{\varnothing\}), ((\varnothing, \{\varnothing\}), \{\varnothing, \{\varnothing\}\}), (\{\varnothing\}, \varnothing), (\{\varnothing\}, \{\varnothing\}), (\{\varnothing\}, \{\varnothing, \{\varnothing\}\}), ((\{\varnothing\}, \varnothing), \varnothing), ((\{\varnothing\}, \varnothing), \{\varnothing\}), ((\{\varnothing\}, \varnothing), \{\varnothing, \{\varnothing\}\})\}$

(d) $A \times B$

$\{((2, 4), (4, 1)), ((2, 4), (2, 3)), ((3, 1), (4, 1)), ((3, 1), (2, 3))\}$

$B \times A$

$\{((4, 1), (2, 4)), ((4, 1), (3, 1)), ((2, 3), (2, 4)), ((2, 3), (3, 1))\}$

17 We're asked to show that $(a, b) = (x, y)$ iff $a = x$ and $b = y$.

Proof.

$$\begin{aligned}
(a,b) = (x,y) &\iff \{\{a\},\{a,b\}\} = \{\{x\},\{x,y\}\} \\
&\iff (\{\{a\},\{a,b\}\} \subseteq \{\{x\},\{x,y\}\}) \\
&\quad \wedge (\{\{x\},\{x,y\}\} \subseteq \{\{a\},\{a,b\}\}) \\
&\iff (\{a\} \in \{\{x\},\{x,y\}\}) \\
&\quad \wedge (\{a,b\} \in \{\{x\},\{x,y\}\}) \\
&\quad \wedge (\{x\} \in \{\{a\},\{a,b\}\}) \\
&\quad \wedge (\{x,y\} \in \{\{a\},\{a,b\}\}) \\
&\iff (\{a\} = \{x\}) \wedge (\{a,b\} = \{x,y\}) \\
&\iff (a = x) \wedge (b = y)
\end{aligned}$$

Since we have connected both sides with a series of bi-conditional statements, we have proven that:
$(a, b) = (x, y)$ iff $a = x$ and $b = y$. □

18 (a) *Proof.*

$$A \Delta B = (A - B) \cup (B - A) = (B - A) \cup (A - B) = B \Delta A$$

□

(b) This proof is a bit longer than the others.

Proof.

$$
\begin{aligned}
A\Delta B &= (A - B) \cup (B - A) \\
&= \{x|\, (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\} \\
&= \{x|\, [(x \in A \wedge x \notin B) \vee x \in B] \wedge [(x \in A \wedge x \notin B) \vee x \notin A]\} \\
&= \{x|\, (x \in A \vee x \in B) \wedge (x \notin B \vee x \in B) \\
&\qquad \wedge (x \in A \vee x \notin A) \wedge (x \notin B \vee x \notin A)\} \\
&= \{x|\, (x \in A \vee x \in B) \wedge (x \notin B \vee x \notin A)\} \\
&= \{x|\, (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)\} \\
&= \{x|\, (x \in A \vee x \in B) \wedge \sim (x \in A \wedge x \in B)\} \\
&= \{x|\, (x \in A \cup B) \wedge \sim (x \in A \cap B)\} \\
&= \{x|\, (x \in A \cup B) \wedge (x \notin A \cap B)\} \\
&= (A \cup B) - (A \cap B)
\end{aligned}
$$

□

(c) *Proof.*

$$A\Delta A = (A - A) \cup (A - A) = \varnothing \cup \varnothing = \varnothing$$

□

(d) *Proof.*

$$A\Delta\varnothing = (A - \varnothing) \cup (\varnothing - A) = A \cup \varnothing = A$$

□

§2.3 1 (f)

(h)

(j)

12

15 (e)

(f)

16

17 (c)

(d)

§2.4 6 (i)

(k)

7 (l)

(m)

8 (h)

12 (b)