MAT 167 Homework 1

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Winter 2014

1.4

10

True or false? Give a specic counterexample when false.

1. If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.

   True

2. If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.

   False

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}, AB = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3. If rows 1 and 3 of A are the same, so are rows 1 and 3 of AB.

   True

4. $(AB)^2 = A^2B^2$

   False

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, (AB)^2 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} A^2B^2 = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

12

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, AB = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

This is the case because each entry in AB is the dot product of a row from A and a column from B. Given two n x n matricies A and B, each A_{ij} where $i + j > 2i$ is 0. Similarly, each B_{ij} where $i + j > 2i$ is 0.

When we multiply these two matricies together, we are always multiplying the zero-upper-entries of A by the nonzero-entries of B. This always gives back a lower triangular matrix.

17

$$(A+B)^2 = (A+B)(B+A) = A(A+B) + B(A+B) = A^2 + 2AB + B^2$$

42

True of False?

1. If A^2 is defined then A is necessarily square.

   True

2. If AB and BA are dened then A and B are square.

   False,

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}, AB = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}, BA = \begin{pmatrix} 0 & 0 \\ 6 & 6 \end{pmatrix}$$

3. If AB and BA are dened then AB and BA are square.

   True.

4. If $AB = B$ then $A =$ I.

   False,

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, AB = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = B, A \neq I$$

52

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52