MAT 167 Homework 2

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Winter 2014

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Write down the 3 by 3 finite-difference matrix equation ($h = \frac{1}{4}$) for

$$-\frac{d^2u}{dx^2} + u = x, \qquad u(0) = u(1) = 0$$

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Following from the text

$$\frac{d^2u}{dx^2} \approx \frac{\Delta^2 u}{\Delta x^2} = \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

So we have

$$-\frac{u(x+h) - 2u(x) + u(x-h)}{h^2} + u = x$$

Letting $x = jh$, and realizing that u must be a function of x

$$\begin{aligned}
-\frac{u(jh+h) - 2u(jh) + u(jh-h)}{h^2} + u(jh) &= jh \\
-\frac{u((j+1)h) - 2u(jh) + u((j-1)h)}{h^2} + u(jh) &= jh \\
-u((j+1)h) + 2u(jh) - u((j-1)h) + u(jh)h^2 &= jh^3
\end{aligned}$$

We take another cue from the text and use the notation $u(jh) = u_j$

$$\begin{aligned}
-u_{j+1} + 2u_j - u_{j-1} + u_j h^2 &= jh^3 \\
-u_{j+1} + u_j h^2 + 2u_j - u_{j-1} &= jh^3 \\
-u_{j+1} + (h^2 + 2)u_j - u_{j-1} &= jh^3
\end{aligned}$$

Since we have $h = \frac{1}{4}$

$$\begin{aligned}
-u_{j+1} + ((\tfrac{1}{4})^2 + 2)u_j - u_{j-1} &= j(\tfrac{1}{4})^3 \\
-u_{j+1} + (\tfrac{1}{16} + 2)u_j - u_{j-1} &= \tfrac{j}{64} - u_{j+1} + \tfrac{33}{16}u_j - u_{j-1} &&= \tfrac{j}{64}
\end{aligned}$$

Now we check each of $j = 1, 2, 3$, remembering $u(0) = u_0 = u(1) = u_4 = 0$
$j = 1$

$$\begin{aligned} -u_{1+1} + \tfrac{33}{16}u_1 - u_{1-1} &= \tfrac{1}{64} \\ -u_2 + \tfrac{33}{16}u_1 - u_0 &= \tfrac{1}{64} \\ -u_2 + \tfrac{33}{16}u_1 &= \tfrac{1}{64} \end{aligned}$$

$j = 2$

$$\begin{aligned} -u_{2+1} + \tfrac{33}{16}u_2 - u_{2-1} &= \tfrac{2}{64} \\ -u_3 + \tfrac{33}{16}u_2 - u_1 &= \tfrac{2}{64} \end{aligned}$$

$j = 3$

$$\begin{aligned} -u_{3+1} + \tfrac{33}{16}u_3 - u_{3-1} &= \tfrac{3}{64} \\ -u_4 + \tfrac{33}{16}u_3 - u_2 &= \tfrac{3}{64} \\ \tfrac{33}{16}u_3 - u_2 &= \tfrac{3}{64} \end{aligned}$$

So we have

$$\begin{aligned} -u_2 + \tfrac{33}{16}u_1 &= \tfrac{1}{64} \\ -u_3 + \tfrac{33}{16}u_2 - u_1 &= \tfrac{2}{64} \\ \tfrac{33}{16}u_3 - u_2 &= \tfrac{3}{64} \end{aligned}$$

Rearranging

$$\begin{aligned} \tfrac{33}{16}u_1 - u_2 &= \tfrac{1}{64} \\ -u_1 + \tfrac{33}{16}u_2 - u_3 &= \tfrac{2}{64} \\ -u_2 + \tfrac{33}{16}u_3 &= \tfrac{3}{64} \end{aligned}$$

$$\begin{bmatrix} \frac{33}{16} & -1 & 0 \\ -1 & \frac{33}{16} & -1 \\ 0 & -1 & \frac{33}{16} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{64} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

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For the same matrix

$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

compare the right-hand sides of $Hx = b$ when the solutions are $x_1 = (1, 1, 1)$ and $x_2 = (0, 6, -3.6)$

$$Hx_1 = \begin{bmatrix} 1.833 \\ 1.083 \\ 0.783 \end{bmatrix}, Hx_2 = \begin{bmatrix} 1.800 \\ 1.100 \\ 0.780 \end{bmatrix}$$

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Compare the pivots in direct elimination to those with partial pivoting for

$$A = \begin{bmatrix} .001 & 0 \\ 1 & 1000 \end{bmatrix}$$

So the pivots are .001 and 1000 with direct elimination. After rescaling, the pivots are 1 and 1000.

With partial pivoting, we swap the rows initially.

$$A = \begin{bmatrix} 1 & 1000 \\ .001 & 0 \end{bmatrix}$$

Then eliminate.

$$A = \begin{bmatrix} 1 & 1000 \\ .001 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1000 \\ 0 & -1 \end{bmatrix}$$

So the pivots are 1 and -1 with partial pivoting. These are much better pivots, as the pivots are similar in magnitude.

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2.2

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2.3

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