

STA 032 Homework 7

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- § 5.2 1 We have $n = 70$ independent Bernoulli trials with $X = 28$ successes.
- (1) $\frac{28}{70} = 0.4 = 40\%$ of the sampled automobiles had emission levels that exceeded the standard.
- (2) Taking $p = 0.4$ we have a $X \sim \text{Bin}(70, 0.4)$.
We take $\tilde{n} = 70 + 4 = 74$, $\tilde{p} = \frac{28+2}{74} = 0.405$.
A 95% confidence interval can now be found.

$$\begin{aligned}\tilde{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} &\approx 0.405 \pm z_{0.025} \sqrt{\frac{0.405(1-0.405)}{74}} \\ &\approx 0.405 \pm 1.96 \sqrt{0.00326} \\ &\approx 0.405 \pm 0.112\end{aligned}$$

So the interval is (0.293, 0.517).

- (3) Using the values calculated before, a 98% confidence interval can be found.

$$\begin{aligned}\tilde{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} &\approx 0.405 \pm z_{0.01} \sqrt{\frac{0.405(1-0.405)}{74}} \\ &\approx 0.405 \pm 2.33 \sqrt{0.00326} \\ &\approx 0.405 \pm 0.133\end{aligned}$$

So the interval is (0.272, 0.538).

- (4) Since we're going to be solving more than one of these similar questions, let's find a closed form to calculate this easily.
For each problem we need to know α, \tilde{p} and the range r .
We want to solve the following equation for n :

$$\begin{aligned}
r &= z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \\
&= z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} \\
\frac{r}{z_{\frac{\alpha}{2}}} &= \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} \\
\left(\frac{r}{z_{\frac{\alpha}{2}}}\right)^2 &= \frac{\tilde{p}(1-\tilde{p})}{n+4} \\
n+4 &= \frac{\tilde{p}(1-\tilde{p})}{\left(\frac{r}{z_{\frac{\alpha}{2}}}\right)^2} \\
n &= \frac{\tilde{p}(1-\tilde{p})}{\left(\frac{r}{z_{\frac{\alpha}{2}}}\right)^2} - 4 \\
n &= z_{\frac{\alpha}{2}}^2 \frac{\tilde{p}(1-\tilde{p})}{r^2} - 4
\end{aligned} \tag{1}$$

Now with equation 1 we can solve with plug and chug.

$$n = 1.96^2 \frac{0.405(1-0.405)}{0.10^2} - 4 = 88.573$$

So 89 samples are needed for the proportion to exceed the standard to within ± 0.10 with 95% confidence.

(5) Using equation 1 we can solve with plug and chug.

$$n = 2.33^2 \frac{0.405(1-0.405)}{0.10^2} - 4 = 126.823$$

So 127 samples are needed for the proportion to exceed the standard to within ± 0.10 with 98% confidence.

(6) We would do well to also find a closed form for this question.

We need the \tilde{n}, \tilde{p} and the upper bound u .

Then we can solve the following equation for z_{α} :

$$\begin{aligned}
u &= \tilde{p} + z_\alpha \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \\
u - \tilde{p} &= z_\alpha \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \\
\frac{u - \tilde{p}}{\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}} &= z_\alpha \\
z_\alpha &= \frac{u - \tilde{p}}{\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}} \\
z_\alpha &= (u - \tilde{p}) \sqrt{\frac{\tilde{n}}{\tilde{p}(1-\tilde{p})}} \tag{2}
\end{aligned}$$

Now, using equation 2 we can solve for an upper bound of 0.50:

$$\begin{aligned}
z_\alpha &= (0.50 - 0.405) \sqrt{\frac{74}{0.405(1 - 0.405)}} \\
&= 1.66
\end{aligned}$$

So the z score corresponds to $0.9515 = 95.15\%$.

Thus we can say with 95.15% confidence that less than half of the vehicles in the state exceed the standard.

4 We have $n = 444$ independent Bernoulli trials with $X = 170$ successes.

$\frac{170}{444} = 0.382882$ of the sampled smokers used the patch.

So we have $X \sim \text{Bin}(444, 0.383)$.

We also calculate $\tilde{n} = 444 + 4 = 448$ and $\tilde{p} = \frac{170+2}{448} = 0.384$

(1) A 95% confidence interval can be found:

$$\begin{aligned}
\tilde{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} &= 0.384 \pm 1.96 \sqrt{\frac{0.384(1-0.384)}{448}} \\
&= 0.384 \pm 0.0450
\end{aligned}$$

So the interval is $(0.339, 0.429)$.

(2) A 98% confidence interval can be found:

$$\begin{aligned}\tilde{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} &= 0.384 \pm 2.58 \sqrt{\frac{0.384(1-0.384)}{448}} \\ &= 0.384 \pm 0.0592\end{aligned}$$

So the interval is (0.325, 0.443).

- (3) Using equation 2 we can solve for an upper bound of 0.40:

$$\begin{aligned}z_{\alpha} &= (0.40 - 0.384) \sqrt{\frac{448}{0.384(1-0.384)}} \\ &= 0.70\end{aligned}$$

So the z score corresponds to 0.7580 = 75.80%.

Thus we can say with 75.80% confidence that proportion is less than 0.40.

- (4) Using equation 1 we can solve with plug and chug.

$$n = 1.96^2 \frac{0.384(1-0.384)}{0.03^2} - 4 = 1005.675$$

So 1006 samples are needed for a 95% confidence to specify the proportion within ± 0.03 .

- (5) Using equation 1 we can solve with plug and chug.

$$n = 2.58^2 \frac{0.384(1-0.384)}{0.03^2} - 4 = 1749.479$$

So 1750 samples are needed for a 99% confidence to specify the proportion within ± 0.03 .

§ 5.3 10 We have $n = 15, \bar{X} = 13, s = 2$.

We compute

$$13 \pm 2.977 \frac{2}{\sqrt{15}} = (11.463, 14.537)$$

So we can say with 99% confidence that the mean track length is in the interval (11.463 μ m, 14.537 μ m).

11 We have $n = 6, \bar{X} = 2.03, s = 0.090$.

We compute

$$2.03 \pm 2.015 \frac{0.090}{\sqrt{6}} = (1.956, 2.104)$$

So we can say with 90% confidence that the mean deflection caused by a 160kN load is in the interval (1.956mm, 2.104mm).

- § 5.4 3 We have $n_X = 1559, \bar{X} = 30.4, \sigma_X = 0.6$
and $n_Y = 1924, \bar{Y} = 31.1, \sigma_Y = 0.2$
We can find a 99% confidence interval as follows:

$$\bar{X} - \bar{Y} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}} = 30.4 - 31.1 \pm 2.576 \sqrt{\frac{0.6^2}{1559} + \frac{0.2^2}{1924}} = (-0.741, -0.659)$$

So we can say with 99% confidence that men have between -0.741 and -0.659 lower BMI than women.

- 4 We have $n_X = 296, \bar{X} = 54.1, \sigma_X = 4.4$
and $n_Y = 296, \bar{Y} = 72.7, \sigma_Y = 4.7$
We can find a 99% confidence interval as follows:

$$\bar{X} - \bar{Y} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}} = 54.1 - 72.7 \pm 1.960 \sqrt{\frac{4.4^2}{296} + \frac{4.7^2}{296}} = (-19.333, -14.867)$$

So we can say with 95% confidence that 27 week-old hens lay eggs that weight between -19.333g and -14.867g lower than 59 week-old hens.

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