STA 032 Homework 5

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Winter 2015

§ 4.5   8 We have $F \sim N(4.1, 0.36)$

(1) We can find

$$P(3.7 < F < 4.4) = P(F < 4.4) - P(F > 3.7)$$
$$= P\left(Z < \frac{4.4 - 4.1}{0.6}\right) - P\left(Z < \frac{3.7 - 4.1}{0.6}\right)$$
$$= P\left(Z < \frac{0.3}{0.6}\right) - P\left(Z < \frac{-0.4}{0.6}\right)$$
$$= P\left(Z < \frac{1}{2}\right) - P\left(Z < \frac{-2}{3}\right)$$
$$\approx 0.6915 - 0.2546$$
$$\approx 0.4369$$

So approximately 43.69% of the female cats are between 3.7 kg and 4.4 kg.

(2) We just need to find

$$P(Z > 0.5) = 1 - P(Z < 0.5) \approx 1 - 0.6915 \approx 0.3085$$

So approximately 30.85% of the female cats are heavier than 0.5 standard deviations above the mean.

(3) We need to first find the closest area to 0.8. The closest area is 0.8023. This area corresponds to a z-score of 0.85.
Now we just work backwards

$$0.85 = \frac{x - 4.1}{0.6}$$
$$0.85(0.6) + 4.1 = x$$
$$4.61 = x$$

So the female cat weight on the 80th percentile is 4.61kg.

(4) We can find

$$P(F > 4.5) = P\left(Z > \frac{4.5 - 4.1}{0.6}\right) \approx P(Z > 0.66) \approx P(Z < -0.66)$$
$$\approx 0.2546$$

So the probability that a female cat chosen at random weighs more than 4.5kg is approximately 25.46%.

(5) We assume that the sample is large enough that each choice is independent. We want to find

$$P(F > 4.5)P(F < 4.5)^5 \approx 0.2546(1 - 0.2546)^5 \approx 0.2546(0.7475)^5 \approx 0.05858$$

So the probability that exactly one of six females cats chosen at random weighs more than 4.5 kg is 5.86%

20 We have $X \sim N(200, 100)$

(1)

$$P(X \le 160) = P\left(Z < \frac{160 - 200}{10}\right) = P\left(Z < \frac{-40}{10}\right) = P(Z < -4)$$

Since the table only goes to -3.69 all we can say is that $P(Z < -4) < 0.0001$ So the probability of the strength being less than or equal to 160N is less than 0.01%.

(2) Yes, a strength of 160N would be unusually small.
Intuitively this makes sense, 160N is four standard deviations away from the the mean. 99.7% of the strengths are within three standard deviations, so it is natural to expect 160N to be unusually small.

(3) Yes, this would be enough evidence.
The process is producing adhesive with strength far below the mean. If a measurement is 160N, something is messing the process up.

21 We have $R_1 \sim N(100, 25)$ and $R_2 \sim N(120, 100)$.
It will help to compute $R_2 - R_1 \sim N(120 - 100, 100 + 25) = N(20, 125)$

(1) We need to solve $P(R_2 > R_1) = P(R_2 - R_1 > 0)$

$$P(R_2 - R_1 > 0) = P\left(Z > \frac{0 - 20}{\sqrt{125}}\right)$$
$$\approx P(Z > -1.79)$$
$$\approx P(Z < 1.79)$$
$$\approx 0.9633$$

So the probability that R_2 exceeds R_1 is approximately 96.33%.

2

(2) We need to solve $P(R_2 > R_1 + 30) = P(R_2 - R_1 > 30)$

$$P(R_2 - R_1 > 30) = P\left(Z > \frac{30 - 20}{\sqrt{125}}\right)$$
$$\approx P\left(Z > 0.89\right)$$
$$\approx P\left(Z < -0.89\right)$$
$$\approx 0.1867$$

So the probability that R_2 exceeds R_1 by 30 Ω is approximately 18.67%.

26   i. $k = 1$

$$\begin{aligned}
P(|X - \mu| \geq \sigma) &= P(X - \mu < -\sigma \cup X - \mu > \sigma) \\
&= P(X - \mu < -\sigma) + P(X - \mu > \sigma) \\
&= P(X - \mu < -\sigma) + P(X - \mu > \sigma) \\
&= P(X < \mu - \sigma) + P(X > \mu + \sigma) \\
&= P\left(Z < \frac{\mu - \sigma - \mu}{\sigma}\right) + P\left(Z > \frac{\mu + \sigma - \mu}{\sigma}\right) \\
&= P(Z < -1) + P(Z > 1) \\
&= P(Z < -1) + P(Z < -1) \\
&= 2P(Z < -1) \\
&= 2(0.1587) \\
&= 0.3174
\end{aligned}$$

$$\frac{1}{1^2} = 1$$

ii. $k = 2$

3

$$
\begin{aligned}
P(|X - \mu| \geq 2\sigma) &= P(X - \mu < -2\sigma \cup X - \mu > 2\sigma) \\
&= P(X - \mu < -2\sigma) + P(X - \mu > 2\sigma) \\
&= P(X - \mu < -2\sigma) + P(X - \mu > 2\sigma) \\
&= P(X < \mu - 2\sigma) + P(X > \mu + 2\sigma) \\
&= P\left(Z < \frac{\mu - 2\sigma - \mu}{\sigma}\right) + P\left(Z > \frac{\mu + 2\sigma - \mu}{\sigma}\right) \\
&= P(Z < -2) + P(Z > 2) \\
&= P(Z < -2) + P(Z < -2) \\
&= 2P(Z < -2) \\
&= 2(0.0228) \\
&= 0.0456
\end{aligned}
$$

$$
\frac{1}{2^2} = 0.25
$$

iii. $k = 3$

$$
\begin{aligned}
P(|X - \mu| \geq 3\sigma) &= P(X - \mu < -3\sigma \cup X - \mu > 3\sigma) \\
&= P(X - \mu < -3\sigma) + P(X - \mu > 3\sigma) \\
&= P(X - \mu < -3\sigma) + P(X - \mu > 3\sigma) \\
&= P(X < \mu - 3\sigma) + P(X > \mu + 3\sigma) \\
&= P\left(Z < \frac{\mu - 3\sigma - \mu}{\sigma}\right) + P\left(Z > \frac{\mu + 3\sigma - \mu}{\sigma}\right) \\
&= P(Z < -3) + P(Z > 3) \\
&= P(Z < -3) + P(Z < -3) \\
&= 2P(Z < -3) \\
&= 2(0.0013) \\
&= 0.0026
\end{aligned}
$$

$$
\frac{1}{3^2} = 0.\overline{1}
$$

So the actual probabilities are much smaller than Chebyshev's bound.

§ 4.7   2 We have $Req \sim Exp(\lambda)$ and $\mu_{Req} = 0.5$

(1)

$$
\lambda = \frac{1}{\mu_{Req}} = 2
$$

(2) We want to find

$$P(Req < x_{50}) = 0.5$$
$$1 - e^{-2x_{50}} =$$
$$0.5 = e^{-2x_{50}}$$
$$\ln 2 = 2x_{50}$$
$$\frac{\ln 2}{2} = x_{50}$$
$$0.3466 \approx$$

So the median time between requests is approximately 0.3466 seconds.

(3) The standard deviation is the same as the mean, 0.5 seconds.

(4) We want to find

$$P(Req < x_{80}) = 0.8$$
$$1 - e^{-2x_{80}} =$$
$$0.2 = e^{-2x_{80}}$$
$$1.6094 \approx 2x_{80}$$
$$\frac{1.6094}{2} \approx x_{80}$$
$$0.8047 \approx$$

So the 80th percentile time between requests is approximately 0.8047 seconds.

(5) We want to find

$$P(Req > 1) = e^{-2(1)} \approx 0.1353$$

So the probability of more than one second elapsing between requests is 13.53%.

(6) Since exponential distributions have no concept of memory, we have the same result as the previous question.

If two seconds have elapsed since the last request, the probability of another second elapsing before the next request is 13.53%.

4 We have $D \sim Exp(\lambda)$ and $\mu_D = 12$.

Thus, $\lambda = \frac{1}{12} = 0.08\overline{3}$

(1) We want to find

$$P(D > 15) = e^{-\frac{15}{12}} \approx 0.2865$$

So the probability that the distance between two flaws is greater than 15m is approximately 28.65%.

(2) We want to find

$$P(8 < D < 20) = P(D < 20) - P(D > 8) = 1 - e^{-\frac{20}{12}} - e^{-\frac{8}{12}} \approx 0.2977$$

So the probability that the distance between two flaws is between 8m and 20m is approximately 29.77%.

(3) We want to find

$$P(D < x_{50}) = 0.5$$
$$1 - e^{-\frac{x_{50}}{12}} =$$
$$0.5 = e^{-\frac{x_{50}}{12}}$$
$$\ln 2 = \frac{x_{50}}{12}$$
$$12 \ln 2 = x_{50}$$
$$8.3178 \approx$$

So the median distance between flaws is approximately 8.3178m.

(4) The standard deviation is the same as the mean 12m.

(5) We want to find

$$P(Req < x_{65}) = 0.65$$
$$1 - e^{-\frac{x_{65}}{12}} =$$
$$0.35 = e^{-\frac{x_{65}}{12}}$$
$$1.0498 \approx \frac{x_{65}}{12}$$
$$12.5979 \approx x_{65}$$

So the 65$^{\text{th}}$ percentile distance between flaws is approximately 12.5979m.

§ 4.8 1 We have $W \sim U(0, 15)$

(1) The mean wait time is $\frac{0+15}{2} = 7.5$ minutes.

(2) The standard deviation is $\frac{15-0}{\sqrt{12}} \approx 4.3301$ minutes.

(3) We want to find

$$P(5 < W < 11) = \frac{11 - 5}{15 - 0} = 0.4$$

So the probability that the wait time is between 5 and 11 minutes is 40%.

(4) We want to find

$$P(W < 5)^4 P(W > 5)^6 = \left(\frac{5 - 0}{15 - 0}\right)^4 \left(\frac{15 - 5}{15 - 0}\right)^6 = \left(\frac{1}{81}\right)\left(\frac{64}{729}\right)$$
$$\approx 0.00108382$$

So the probability that the wait time is less than 5 minutes on exactly 4 of 10 days is approximately 0.1084%.

2 (1)

   (2)

   (3)

   (4)