ECS 120 Problem Set 3

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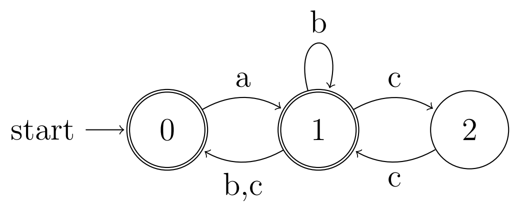


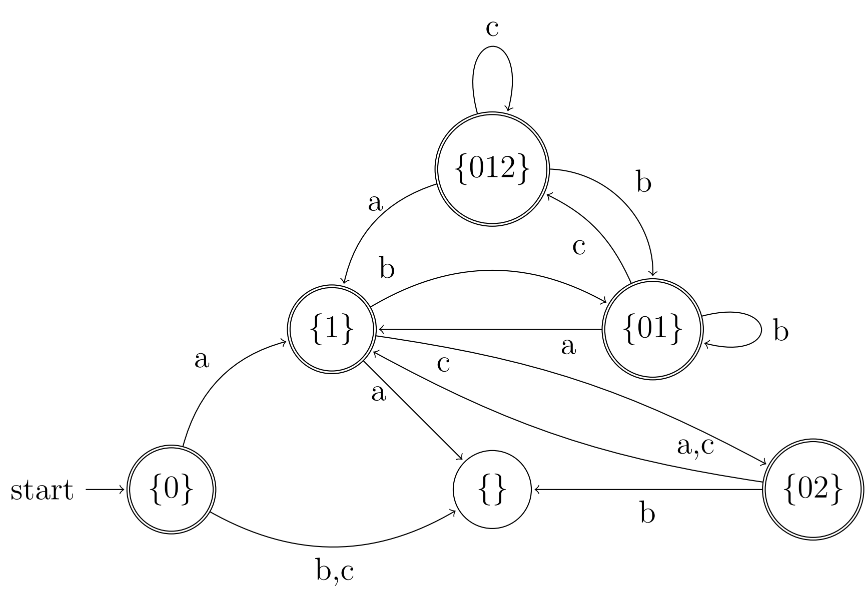
Problem 1

We need to start by enumerating all sets of states, and their transitions for each input.

State	a	b	c
{0}	{1}	{}	{}
{1}	{}	{0,1}	{0,2}
{0,1}	{1}	{0,1}	{0,1,2}
{0,2}	{1}	{}	{1}
{0,1,2}	{1}	{0,1}	{0,1,2}

Now we need to generate new final states. These are any state sets that contain the original final states. In this case, all of the states in the first column are final states.

Finally, we can generate our DFA.



Problem 2 We have ε-arrows from 1 to 2, from 2 to 5, and from 3 to 1.

From 1 to 2, we can get to state 4 following the path "εa". So, we can eliminate this ε-arrow by creating a new path from 1 to 4 via a.

From 1 to 2, we can also get to state 5 through an ε-arrow to state 2 following the path "$\varepsilon\varepsilon$". Since state 5 is a final state, we can eliminate these ε-arrows by making state 1 a final state.

We can get from 2 to 5 via an ε-arrow and state 5 is a final state. So, we can eliminate this ε-arrow by making state 2 a final state.

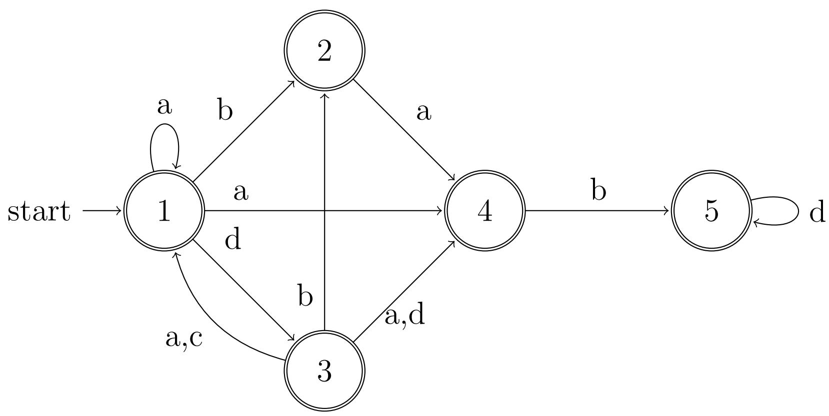
From 3 to 1, we can get to 1 again following the path "εa". So, we can eliminate this ε-arrow by creating a new path from 3 to 1 via a.

From 3 to 1, we can get to 1 again following the path "εc". So, we can eliminate this ε-arrow by creating a new path from 3 to 1 via c.

From 3 to 1, we can get to state 4 through an ε-arrow to state 2 following the path "$\varepsilon\varepsilon$a". So, we can eliminate this ε-arrow by creating a new path from 3 to 4 via a.

Finally, from 3 to 1, we can also get to state 5 through an ε-arrow to state 2 and another ε-arrow to state 5 following the path "$\varepsilon\varepsilon\varepsilon$". Since state 5 is a final state, we can eliminate these ε-arrows by making state 3 a final state.

Our ε-arrow-free NFA is:



Problem 3 *Proof.* If L_1, L_2, L_3 are DFA-acceptable languages, then the intersection of these languages is also acceptable. We can reformulate the definition of **maj**:

$\mathbf{maj}(L_1, L_2, L_3) =$

$\{x \in \Sigma^* | (x \in L_1 \wedge x \in L_2) \vee (x \in L_1 \wedge x \in L_3) \vee (x \in L_2 \wedge x \in L_3) \vee (x \in L_1 \wedge x \in L_2 \wedge x \in L_3)\}$

Or put more succinctly:

$\mathbf{maj}(L_1, L_2, L_3) =$

$\{x \in \Sigma^* | x \in (L_1 \cap L_2) \vee x \in (L_1 \cap L_3) \vee x \in (L_2 \cap L_3) \vee x \in (L_1 \cap L_2 \cap L_3)\}$

But we know that each of these intersections is DFA-acceptable, so $\mathbf{maj}(L_1, L_2, L_3)$ is DFA-acceptable. $\square$