

PHIL 112 Homework 1

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1. Specify the atomic formulas of PL.

Every expression of PL that is either a sentence letter of PL, or an n -place predicate of PL followed by n individual terms of PL is an atomic formula of PL.

2. Give the recursive definition of 'formula of PL'.

- (a) Every atomic formula \mathbf{P} is a formula of PL.
- (b) If \mathbf{P} is a formula of PL, then so is $\neg\mathbf{P}$.
- (c) If \mathbf{P} and \mathbf{Q} are formulae of PL, then so are $\mathbf{P}\wedge\mathbf{Q}$, $\mathbf{P}\vee\mathbf{Q}$, $\mathbf{P}\supset\mathbf{Q}$, and $\mathbf{P}\equiv\mathbf{Q}$.
- (d) If \mathbf{P} is a formula of PL that contains at least one occurrence of \mathbf{x} and no \mathbf{x} -quantifier, then $\forall\mathbf{x}\mathbf{P}$ and $\exists\mathbf{x}\mathbf{P}$ are formulae of PL.
- (e) Nothing else is a formula of PL unless it can be made from the previous rules.

3. Indicate which of the following are formulas of PL, and which of those are sentences of PL.

- (a) Quantified Formula of PL.

Not a Sentence of PL since the subformula has a quantified \mathbf{x} .

$(\forall\mathbf{x})[\mathbf{F}\mathbf{x}\mathbf{a} \supset (\forall\mathbf{x})\mathbf{G}\mathbf{x}\mathbf{a}]$

- (b) Truth-functionally compound Formula of PL and Sentence of PL.

$(\forall\mathbf{z})\mathbf{F}\mathbf{z}\mathbf{a} \supseteq \neg(\exists\mathbf{z})\mathbf{G}\mathbf{z}\mathbf{a}$

- (c) Truth-functionally compound Formula of PL and Sentence of PL.

$\supset(\forall\mathbf{y})\mathbf{G}\mathbf{y}\mathbf{y}$

- (d) Truth-functionally compound Formula of PL.

Not a sentence of PL for the subformula has at least one free variable.

$\mathbf{F}\mathbf{a}\mathbf{z} \supseteq (\forall\mathbf{x})\mathbf{F}\mathbf{x}\mathbf{a}$

- (e) Not a formula of PL for there is no \mathbf{x} in $\mathbf{F}\mathbf{a}\mathbf{b}$.

$\neg(\exists\mathbf{x})\mathbf{F}\mathbf{a}\mathbf{b}$