MAT 167 HW 1

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$$\begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

7 (a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$$

9 Assuming A has as many pivots as rows, we have the following results.

(a) a_{11}

(b) $l_{i1} = \frac{a_{i1}}{a_{11}}$

(c) $a_{ij} - a_{1j} \left(\frac{a_{i1}}{a_{11}} \right)$

(d) $a_{22} - a_{12} \left(\frac{a_{21}}{a_{11}} \right)$

10 (a) True.

(b) False.
AB may not even have three rows.
For example, let A be a 1×3 matrix and B be a 3×3 matrix.
Then AB is a 1×3 matrix, so it has no third row.

(c) True.

(d) False.
Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Then

$$\begin{aligned} (AB)^2 &= \left(\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right)^2 \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \\ &= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \end{aligned}$$

But

$$\begin{aligned} A^2B^2 &= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}^2 \\ &= \begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \end{aligned}$$

So $(AB)^2 \neq A^2B^2$.

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- Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \end{aligned}$$

- *Proof.* For any lower triangular matrices A, B with dimension $n \times n$, each entry ab_{ij} in AB it is computed by:

$$\sum_{k=1}^{n} a_{ik} b_{kj}$$

If $i < k, a_{ik} = 0$.
If $k < j, b_{kj} = 0$.

Each entry above the main diagonal has one of either $i < k$ or $k < j$.
So for each entry above the main diagonal of AB, we have a sum of products where at least one of the factors is 0.
So, each entry above the main diagonal is 0.
Thus, the product of any two lower triangular matrices is lower triangular. $\square$

13 (a) Let

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \left(\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} \right)^2 \\ &= \frac{1}{3} \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= -I \end{aligned}$$

(b) Let

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq 0$$

$$B^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

(c) Let

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$CD = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(d) Let

$$E = F = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$EF = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

24 We want

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$
\begin{aligned}
M &= E_{32}E_{31}E_{21} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}
\end{aligned}
$$

$$
MA = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}
$$

42 (a) True.
Since matrix multiplication is only defined for matrices A, B with dimension $m \times n, n \times p$, respectively, $A^2 = A * A$ must have $m = n = p$. That is A must be a square matrix.

(b) False.
We can choose A, B with dimension $m \times n, n \times m$, respectively, where $m \neq n$. So A and B are not square.
Then AB is defined, as well as BA, yet A and B are not square.

(c) True.
We can choose A, B with dimension $m \times n, n \times m$, respectively.
Then AB is defined, as well as BA.
These two products have dimension $m \times m$ and $n \times n$ respectively, so AB and BA are square.

(d) False.
Let $B = 0$.
Then $A0 = 0$ for all appropriate matrices, but A is not necessarily I.

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