## PHIL 112 Homework 3

Hardy Jones 999397426 Dr. Landry Winter 2014

1. Explicate in terms of open and/or closed truth trees.

(a) Quantificational validity

An argument of **PL** is quantificationally valid if and only if the set consisting of the premises and the negation of the conclusion of the argument has a closed truth tree.

(b) Quantificational equivalence

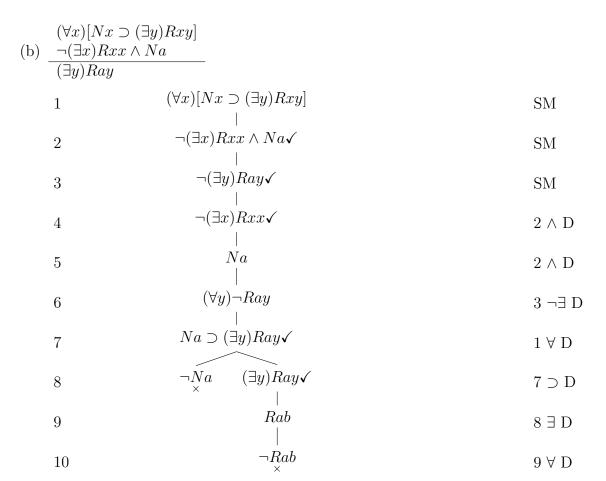
Two sentences **P** and **Q** of **PL** are quantificationally equivalent if and only if the set  $\{\neg(\mathbf{P} \equiv \mathbf{Q})\}$  has a closed truth tree.

2. Use the tree method to show whether:

- (a) is quantificationally true
- (b) is quantificationally valid
- (c) sentences are quantificationally equivalent
- (d) quantificational entailment holds

(a) 
$$[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]$$
 $1 \quad \neg [[Fa \supset (\forall x)Fx] \supset [(\exists x)Fx \supset (\forall x)Fx]] \checkmark$ 
 $2 \quad Fa \supset (\forall x)Fx \checkmark$ 
 $1 \quad \neg \supset D$ 
 $3 \quad \neg [(\exists x)Fx \supset (\forall x)Fx] \checkmark$ 
 $1 \quad \neg \supset D$ 
 $4 \quad (\exists x)Fx \checkmark$ 
 $5 \quad \neg (\forall x)Fx \checkmark$ 
 $6 \quad (\exists x) \neg Fx \checkmark$ 
 $7 \quad Fb$ 
 $1 \quad \neg \supset D$ 
 $1 \quad \neg \supset D$ 

Since this tree is not closed, the sentence is not quantificationally true.



Since this tree is closed, the argument is quantificationally valid.

One interpretation is:

UD: The natural numbers

Nx: x is positive Rxy: x is less than y

a: 1

(c) 
$$[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)$$

1  $\neg [[(\forall x)Fx \supset Ga] \equiv (\exists x)(Fx \supset Ga)] \checkmark$  SM

2  $(\forall x)Fx \supset Ga$   $\neg (\forall x)Fx \supset Ga \checkmark$  1  $\neg \equiv D$ 

3  $\neg (\exists x)(Fx \supset Ga) \checkmark$   $(\exists x)(Fx \supset Ga) \checkmark$  1  $\neg \equiv D$ 

4  $Fb \supset Ga \checkmark$  3  $\exists D$ 

5  $\neg \neg (\forall x)Fx \checkmark$   $Ga$  2  $\supset D$ 

6  $(\forall x)Fx \supset Ga)$   $(\forall x)Fx \checkmark$   $Ga$  4  $\supset D$ 

8  $\neg \neg (\forall x)Fx \checkmark$   $Ga$  4  $\supset D$ 

9  $(\forall x)\neg (Fx \supset Ga)$  3  $\neg \exists D$ 

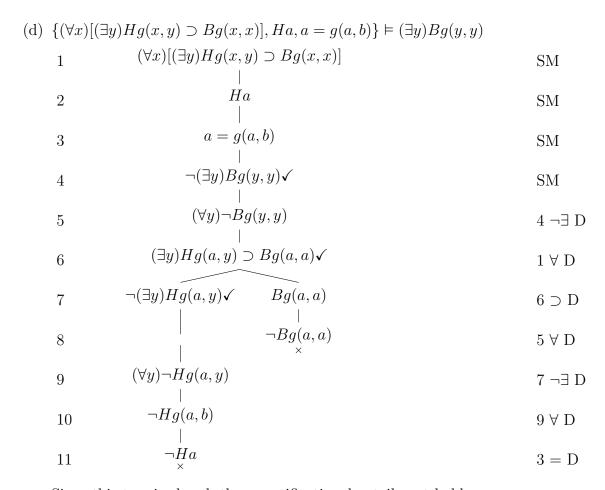
10  $\neg (\forall x)Fx \checkmark$   $Ga$  2  $\supset D$ 

11  $\neg (Fb \supset Ga) \checkmark$  9  $\forall D$ 

12  $\neg (Fb \supset Ga) \checkmark$  9  $\forall D$ 

13  $\neg (Fb \supset Ga) \checkmark$  11  $\neg (Fb \supset Ca)$  12  $\neg (Fb \supset Ca) \checkmark$  11  $\neg (Fb \supset Ca)$  12  $\neg (Fb \supset Ca) \checkmark$  10  $\neg (Fb \supset Ca) \checkmark$  10  $\neg (Fb \supset Ca) \checkmark$  11  $\neg (Fb \supset Ca) \checkmark$  11  $\neg (Fb \supset Ca) \checkmark$  12  $\neg (Fb \supset Ca) \checkmark$  11  $\neg ($ 

Since there is at least one open branch, this tree is not closed. Thus, the two sentences are not quantificationally equivalent.



Since this tree is closed, the quantificational entailment holds.

One interpretation is:

UD: Natural numbers

Hx: x is positive.

g(x, y): Minimum of x and y.

Bx: x is divisible by 1.

a: 1

b: 2

3. Why does the rule *Existential Decomposition* require that the instantiating constant **a** be foreign to all preceding lines of the branch?

By not requiring *Existential Decomposition* to introduce foreign constants we have opened up the possibility that the same constant can be reused in a conflicting predicate. So, we require foreign constants with *Existential Decomposition* in order to preserve truth, validity, equivalence, etc.