ECS 160 Homework 2

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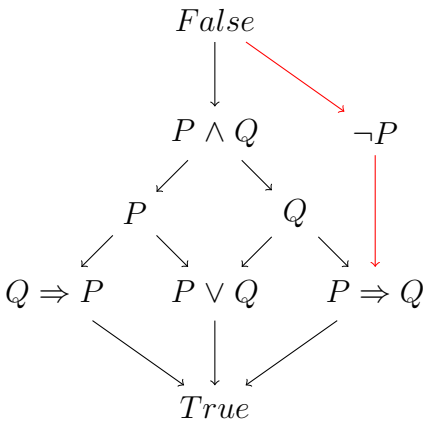
1. (a) You cannot say much about two predicates not connected by an arrow. The only thing you can say is that neither is strong with relative to the other.

   (b) The strongest thing that $\neg P$ can imply is $P \Rightarrow Q$. This is because, when $\neg P$ is True, P is False. So, $P \Rightarrow Q$ is True.



2.

$$\begin{aligned}\mathbb{P}(\mathbb{P}(\{1\})) &= \mathbb{P}(\{\{\}, \{1\}\}) \\ &= \{\{\}, \{\{\}\}, \{\{1\}\}, \{\{\}, \{1\}\}\}\end{aligned}$$

3.

$[People]$

$$\begin{array}{|l} knows : People \leftrightarrow People \\ livesAlone : \mathbb{P}\, People \\ \hline \forall\, x : People \bullet (\exists\, y : People \bullet x \mapsto y \in knows \land y \in livesAlone) \end{array}$$

4.

$$PrimeNumbers == \{n, p : \mathbb{N} \mid p > 1 \land 1 < n < p \land (p \bmod n \neq 0) \bullet p\}$$

5.

$[Student]$

$size : \mathbb{N}$

$Category ::= c1 \mid c2 \mid c3$

Class

$enrolled, tested : \mathbb{P}\, Student$

$\#\, enrolled \leq size$

$tested \subseteq enrolled$

CanBeTested

$\Xi Class$

$s? : Student$

$c? : Category$

$testedIn : Student \leftrightarrow Category$

$s? \in enrolled$

$s? \mapsto c? \notin testedIn$

$testedIn = testedIn \cup \{s? \mapsto c?\}$

CanBeDeTested

$\Xi Class$

$s? : Student$

$c? : Category$

$testedIn : Student \leftrightarrow Category$

$s? \in enrolled$

$s? \mapsto c? \in testedIn$

$testedIn = testedIn \setminus \{s? \mapsto c?\}$

6.

LeaveSetOk

$\Delta Class$

$students? : \mathbb{P}\, Student$

$students? \cap enrolled = students?$

$students? \cap tested = \{\}$

$enrolled' = enrolled \setminus students?$

7.

$[Char]$
$Text == \operatorname{seq} Char$

$maxsize : \mathbb{N}$
$Printing : \mathbb{P}\, Char$

Editor

$left, right, text : Text$

$text = left \frown right$

Init

$left, right, text : Text$

$left = \{\} right = \{\} text = \{\}$

Insert

$\Delta Editor$
$ch? : Char$

$\# left + \# right < maxsize$
$left' = \{\# left + 1 \mapsto ch?\} \cup left$

Forward

$\Delta Editor$
$c : Char$

$\# right > 0$
$1 \mapsto c \in right$
$left' = left \cup \{\# left + 1 \mapsto c\}$
$right' = \{n : \mathbb{N};\ char : Char \mid n \mapsto char \in right \land n > 1 \bullet n + 1 \mapsto char\}$

8.

$$\mathbf{relation}(\_in\_)$$

$$[X]$$

$$\_in\_ : (\operatorname{seq} X \leftrightarrow \operatorname{seq} X)$$

$$\begin{aligned}
&\forall\, u, v : \operatorname{seq} X \\
&\bullet\ (u\ in\ v \Leftrightarrow \\
&\qquad \exists\, x, y : \operatorname{seq} X \mid x \frown u \frown y = v)
\end{aligned}$$

9. $\mathbb{P}(X)$ has 2^n members

10.

$$xs == \{n : \mathbb{Z} \mid n \neq -1\} \cup \{1\}$$

11. This set is exactly $\mathbb{Z}$. Since for every element of $\mathbb{Z}$, we can find two elements of $\mathbb{Z}$ that add to our chosen element.

13.

$$R2R1 == \{1 \mapsto 3, 2 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1\}$$
$$R1R1R1 == \{1 \mapsto 4, 2 \mapsto 2, 2 \mapsto 3, 3 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1\}$$

14.

$$[CHAR]$$

$$blank : \mathbb{P}\, CHAR$$

$$TEXT == \operatorname{seq} CHAR$$
$$SPACE == \operatorname{seq} blank$$
$$WORD == \operatorname{seq}(CHAR \setminus blank)$$

$$words : TEXT \to \operatorname{seq} WORD$$

$$\begin{aligned}
&\forall\, s : SPACE;\ w : WORD;\ l, r : TEXT \\
&\bullet\ words\langle\rangle = \langle\rangle \\
&\land\ words\ s = \langle\rangle \\
&\land\ words\ w = \langle w\rangle \\
&\land\ words(s \frown r) = words\ r \\
&\land\ words(r \frown s) = words\ r \\
&\land\ words(l \frown s \frown r) = words\ l \frown words\ r
\end{aligned}$$