MAT 108 HW 2

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§1.2

6 (a) This statement is true since both sides of the bi-conditional have the same truth values.
Triangles have three sides and squares have four sides.

(b) This statement is true since both sides of the bi-conditional have the same truth values.
$7 + 5 = 12$ and $1 + 1 = 2$.

(d) This statement is true since both sides of the bi-conditional have the same truth values.
parallelograms have four sides and 27 is not prime..

10 (a) (f has a relative minimum at $x_0 \wedge f$ is differentiable at x_0) $\implies f'(x_0) = 0$

(b) n is prime $\implies$ ($n = 2 \vee n$ is odd)

(c) R is irreflexive $\implies$ (R is symmetric $\wedge$ R is transitive)

(d) det $\mathbf{B} = 0 \implies$ ($\mathbf{B}$ is square $\wedge$ not invertible)

(f) $(2n < 4 \vee n > 4) \implies 2 < n - 6$

(g) $6 \geq n - 3 \implies (n > 4 \vee n > 10)$

(h) x is Cauchy $\implies$ x is convergent.

15 (a) (f has a relative minimum at $x_0 \wedge f$ is differentiable at x_0) $\implies f'(x_0) = 0$

- Converse:
  $f'(x_0) = 0 \implies$ (f has a relative minimum at $x_0 \wedge f$ is differentiable at x_0)
  This sentence is false since $f'(x_0) = 0$ can be true, but f may have a relative maximum at x_0.
- Contrapositive:
  $(\sim f'(x_0) = 0) \implies (\sim$ (f has a relative minimum at $x_0 \wedge f$ is differentiable at x_0))
  $f'(x_0) \neq 0 \implies (\sim f$ has a relative minimum at $x_0 \vee \sim f$ is differentiable at x_0)
  $f'(x_0) \neq 0 \implies$ (f does not have a relative minimum at $x_0 \vee f$ is not differentiable at x_0)
  Since the original statement is true, and the contrapositive has the same truth value, this sentence is also true.

(b) n is prime $\implies$ ($n = 2 \vee n$ is odd)

- Converse:
  $(n = 2 \vee n \text{ is odd}) \implies n \text{ is prime}$
  This sentence is false. If $n = 9$ then n is odd, but n is not prime.
- Contrapositive:
  $(\sim (n = 2 \vee n \text{ is odd})) \implies (\sim n \text{ is prime})$
  $(\sim n = 2 \vee \sim n \text{ is odd}) \implies n \text{ is not prime}$
  $(n \neq 2 \vee n \text{ is not odd}) \implies n \text{ is not prime}$
  Since the original statement is true, and the contrapositive has the same truth value, this sentence is also true.

(f) $(2n < 4 \vee n > 4) \implies 2 < n - 6$

- Converse:
  $2 < n - 6 \implies (2n < 4 \vee n > 4)$
  This sentence is true.
  We can see this by simplifying both sides a bit:
  $8 < n \implies (n < 2 \vee 4 < n)$
  If n is greater than 8, it is also greater than 4. So if the antecedent is true, the whole sentence is true.
- Contrapositive:
  $(\sim 2 < n - 6) \implies (\sim (2n < 4 \vee n > 4))$
  $2 \geq n - 6 \implies (\sim 2n < 4 \wedge \sim n > 4)$
  $2 \geq n - 6 \implies (2n \geq 4 \wedge n \leq 4)$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $8 \geq n \implies (n \geq 2 \wedge 4 \geq n)$
  If n is 0, then the antecedent is true, but 0 is not greater than or equal to 2 so the consequent is false.

(g) $6 \geq n - 3 \implies (n > 4 \vee n > 10)$

- Converse:
  $(n > 4 \vee n > 10) \implies 6 \geq n - 3$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $(n > 4 \vee n > 10) \implies 9 \geq n$
  If n is 100, then the antecedent is true, but 100 is not less than or equal to 9 so the consequent is false.
- Contrapositive:
  $(\sim (n > 4 \vee n > 10)) \implies (\sim 6 \geq n - 3)$
  $(\sim n > 4 \wedge \sim n > 10) \implies (6 < n - 3)$
  $(n \leq 4 \wedge n \leq 10) \implies (6 < n - 3)$
  This sentence is false.
  We can see this by simplifying both sides a bit:
  $(n \leq 4 \wedge n \leq 10) \implies (9 < n)$
  If n is 0, then the antecedent is true, but 0 is not greater than 9 so the consequent is false.

16 (a) We can use a truth table to enumerate all possibilities.

P	Q	$P \implies Q$	$(P \implies Q) \implies Q$	$[(P \implies Q) \implies Q] \implies P$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

Since the truth value of final column is neither all true nor all false, this is neither a tautology nor a contradiction.

(b) We can use a truth table to enumerate all possibilities.

P	Q	$P \vee Q$	$P \wedge (P \vee Q)$	$P \iff P \wedge (P \vee Q)$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

Since the truth value of final column is all true, this is a tautology.

(c) We can use a truth table to enumerate all possibilities.

P	Q	$P \implies Q$	$\sim Q$	$P \wedge \sim Q$	$P \implies Q \iff P \wedge \sim Q$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

Since the truth value of final column is all false, this is a contradiction.

§1.3 1 (b)

(d)

(e)

(i)

(l)

5

6

8 (c)

(d)

(e)

(f)

(k)

(l)

9 (a)

(c)

(g)

10 (i)

(j)

(k)

11 (b)

12 (a)

(b)

(c)

(d)