

MAT 167 HW 1

Hardy Jones
999397426
Professor Cheer
Spring 2015

§ 1.4 2

$$\begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

7 (a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$$

9 Assuming A has as many pivots as rows, we have the following results.

(a) a_{11}

(b) $l_{i1} = \frac{a_{i1}}{a_{11}}$

(c) $a_{ij} - a_{1j} \left(\frac{a_{i1}}{a_{11}} \right)$

(d) $a_{22} - a_{12} \left(\frac{a_{21}}{a_{11}} \right)$

10 (a) True.

(b) False.

AB may not even have three rows.

For example, let A be a 1×3 matrix and B be a 3×3 matrix.

Then AB is a 1×3 matrix, so it has no third row.

(c) True.

(d) False.

Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Then

$$\begin{aligned} (AB)^2 &= \left(\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right)^2 \\ &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}^2 \\ &= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \end{aligned}$$

But

$$\begin{aligned} A^2 B^2 &= \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}^2 \\ &= \begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \end{aligned}$$

So $(AB)^2 \neq A^2 B^2$.

12 • Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \end{aligned}$$

- *Proof.* For any lower triangular matrices A, B with dimension $n \times n$, each entry ab_{ij} in AB it is computed by:

$$\sum_{k=1}^n a_{ik} b_{kj}$$

If $i < k$, $a_{ik} = 0$.

If $k < j$, $b_{kj} = 0$.

Each entry above the main diagonal has one of either $i < k$ or $k < j$.
 So for each entry above the main diagonal of AB , we have a sum of products where at least one of the factors is 0.
 So, each entry above the main diagonal is 0.
 Thus, the product of any two lower triangular matrices is lower triangular. \square

13 (a) Let

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \left(\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} \right)^2 \\ &= \frac{1}{3} \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= -I \end{aligned}$$

(b) Let

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq 0$$

$$B^2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

(c) Let

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$CD = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(d) Let

$$E = F = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$EF = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

24

42

46

§ 1.5	11
	12
	18
	22
	28
	33
	42
§ 1.6	2
	4
	5
	10
	17
	21
	40
	49