

ECS 154A Homework 1

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1. Determine if the following are equivalent or not.

(a)

$$WX\bar{Y} + WZ + \bar{W}Y + \bar{W}\bar{X} = (W + \bar{X} + Y)(\bar{W} + X + Z)(\bar{W} + \bar{Y} + Z)$$

Equivalent

(b)

$$\bar{X} + X\bar{Y}Z = (\bar{X} + Z)(W + \bar{X} + \bar{Y})(\bar{W} + \bar{X} + \bar{Y})$$

Equivalent

(c)

$$\bar{W}XYZ + W\bar{X}YZ + WX\bar{Y}Z + WXY\bar{Z} = (W + Y)(\bar{W} + \bar{Y})(\bar{X} + Z)$$

Not equivalent. Take the case when $W = 1$ and $X = Y = Z = 0$. The LHS is obviously 0, but the RHS is 1.

(d)

$$Z(X + \bar{W}) = Z\bar{W} + XYZ$$

Not equivalent. Take the case when $Y = 1$ and $W = X = Z = 0$. The LHS is 1, but the RHS is 0.

(e)

$$W + \bar{W}Z + Y + \bar{X}\bar{Y} = W + \bar{X} + Y + Z$$

Equivalent.

2. Use algebraic manipulation to find the minimum sum of products expression.

$$\begin{aligned}
& \overline{(\overline{X} + Y)(W + Y + Z + \overline{W})} + \overline{X}Z + (W + \overline{Y}Z)(\overline{Y}Z + \overline{W}) \\
& \overline{(\overline{X} + Y)(W + Y + Z + \overline{W})} + \overline{X}Z + (\overline{Y}Z + W)(\overline{Y}Z + \overline{W}) & \text{(Commutativity)} \\
& \overline{(\overline{X} + Y)(W + Y + Z + \overline{W})} + \overline{X}Z + \overline{Y}Z & \text{(Combining)} \\
& \overline{(\overline{X} + Y)(W + Y + Z + \overline{W})} + \overline{X}Z + \overline{Y}Z & \text{(Distributivity)} \\
& \overline{(\overline{X} + Y)} + \overline{(W + Y + Z + \overline{W})} + \overline{X}Z + \overline{Y}Z & \text{(De Morgan's)} \\
& (\overline{\overline{X}} + \overline{Y}) \overline{(W + Y + Z + \overline{W})} + \overline{X}Z + \overline{Y}Z & \text{(De Morgan's)} \\
& X\overline{Y} + \overline{(W + Y + Z + \overline{W})} + \overline{X}Z + \overline{Y}Z & \text{(Involution)} \\
& X\overline{Y} + (\overline{W} \overline{Y} \overline{Z} \overline{\overline{W}}) + \overline{X}Z + \overline{Y}Z & \text{(De Morgan's)} \\
& X\overline{Y} + (\overline{W} \overline{Y} \overline{Z} W) + \overline{X}Z + \overline{Y}Z & \text{(Involution)} \\
& X\overline{Y} + (\overline{Y} \overline{Z} W \overline{W}) + \overline{X}Z + \overline{Y}Z & \text{(Commutativity)} \\
& X\overline{Y} + (\overline{Y} \overline{Z} 0) + \overline{X}Z + \overline{Y}Z & \text{(Complementation)} \\
& X\overline{Y} + \overline{X}Z + \overline{Y}Z & \text{(Nilpotent)}
\end{aligned}$$

3. Use algebraic manipulation to find the minimum product of sums expression.

$$\begin{aligned}
 & \overline{(\overline{XZ} + \overline{W})(\overline{XZ} + \overline{Y}) + \overline{W}X\overline{Y}} \\
 & \overline{(\overline{XZ} + \overline{W})(\overline{XZ} + \overline{Y})} \overline{\overline{W}X\overline{Y}} && \text{(De Morgan's)} \\
 & \overline{(\overline{XZ} + \overline{W})(\overline{XZ} + \overline{Y})} (\overline{\overline{W}} + \overline{\overline{X}} + \overline{\overline{Y}}) && \text{(De Morgan's)} \\
 & \overline{(\overline{XZ} + \overline{W})(\overline{XZ} + \overline{Y})} (W + \overline{X} + \overline{Y}) && \text{(Involution)} \\
 & \overline{(\overline{XZ} + \overline{W})(\overline{XZ} + \overline{Y})} (W + \overline{X} + Y) && \text{(Involution)} \\
 & \overline{(\overline{XZ} + \overline{W}) + (\overline{XZ} + \overline{Y})} (W + \overline{X} + Y) && \text{(De Morgan's)} \\
 & (\overline{\overline{XZ} \overline{W}} + \overline{\overline{XZ} + \overline{Y}}) (W + \overline{X} + Y) && \text{(De Morgan's)} \\
 & (\overline{\overline{XZ}} \overline{W} + \overline{\overline{XZ} + \overline{Y}}) (W + \overline{X} + Y) && \text{(Involution)} \\
 & (\overline{\overline{XZ}} \overline{W} + \overline{\overline{XZ}} \overline{\overline{Y}}) (W + \overline{X} + Y) && \text{(De Morgan's)} \\
 & (\overline{\overline{XZ}} \overline{W} + \overline{\overline{XZ}} \overline{Y}) (W + \overline{X} + Y) && \text{(Involution)} \\
 & (\overline{\overline{XZ}} (W + Y)) (W + \overline{X} + Y) && \text{(Distributivity)} \\
 & (\overline{\overline{X}} + \overline{\overline{Z}}) (W + Y) (W + \overline{X} + Y) && \text{(De Morgan's)} \\
 & (X + \overline{Z}) (W + Y) (W + \overline{X} + Y) && \text{(Involution)} \\
 & (X + \overline{Z}) (W + Y) (W + Y + \overline{X}) && \text{(Commutativity)} \\
 & (X + \overline{Z}) (W + Y) && \text{(Covering)}
 \end{aligned}$$

4. Minimize the circuit.

