- Governing equations from engineering first principles.

Consider a series LC circuit with

Inductor L

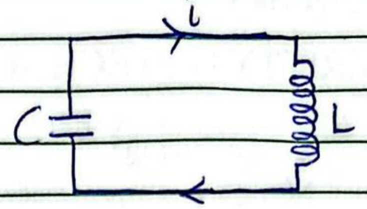
Capacitor C

No resistance (ideal lossless circuit)

Applying Kirchhoff's Voltage Law (KVL)

$$V_L + V_C = 0$$

- LC oscillator circuit



Substituting using voltage-current relationships

$$V_L = L\frac{di}{dt}$$

$$V_C = \frac{q(t)}{C} \quad \text{where } q = \text{charge on capacitor}$$

So,

$$L\frac{di}{dt} + \frac{q}{C} = 0$$

But $i(t) = \frac{dq}{dt}$

So, substituting current gives

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

This gives the governing second-order differential equation

$$\frac{d^2q(t)}{dt^2} + \frac{1}{LC}q(t) = 0$$

- System characteristics from mathematical analysis.

Natural frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

This gives a harmonic oscillator with the equation

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0$$

General solution,

$$q(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$i(t) = \frac{dq(t)}{dt} = -A\omega_0\sin(\omega_0 t) + B\omega_0\cos(\omega_0 t)$$