Mathematical Model for Fluid Flow in Pipes

We focus on **steady, incompressible, internal flow** through circular pipes. The governing equations include:

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1. Variables and Parameters

Symbol	Description	Unit
P	Pressure	Pa
v	Flow velocity	m/s
ρ	Fluid density	kg/m³
μ	Dynamic viscosity	Pa·s
D	Pipe diameter	m
L	Pipe length	m
f	Darcy friction factor	–
ΔP	Pressure drop	Pa
g	Gravitational acceleration	9.81 m/s²
Q	Volumetric flow rate	m³/s

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2. Bernoulli's Equation (Simplified)

For **horizontal flow** without pumps or elevation difference:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_f$$

Where h_f is the **head loss** due to friction.

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3. Darcy–Weisbach Equation

Relates pressure loss to friction:

$$\Delta P = f \cdot \frac{L}{D} \cdot \frac{\rho v^2}{2}$$

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4. Friction Factor f

Depends on **Reynolds number**:

- Laminar ($Re < 2000$):

$$f = \frac{64}{Re}$$

- Turbulent: Use empirical approximations like **Swamee-Jain** equation:

$$f = 0.25\left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}$$

Where:

- ε: roughness of pipe wall

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5. Reynolds Number

$$Re = \frac{\rho v D}{\mu}$$

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6. Volumetric Flow Rate and Velocity

$$Q = v \cdot A = v \cdot \frac{\pi D^2}{4} \Rightarrow v = \frac{4Q}{\pi D^2}$$