

a) Mathematical equations.

- Definition of variables and parameters.

→  $N(t)$  = Population of bacteria at time  $t$

→  $r$  = intrinsic growth rate.

→  $K$  = Carrying capacity of environment (population)

→  $t$  = time

→  $N_0$  = initial population size.

- Mathematical equations

The logistic growth equation models population growth with a limiting factor

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

- Derivation from first principles

Start from idea that bacterial population change is proportional to existing population

$$\frac{dN}{dt} = rN$$

Exponential growth model. To account for limited resources, introduce a growth limiting factor based on current population.

$$\text{Net growth rate} = rN \left(1 - \frac{N}{K}\right)$$

Thus,

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$$

- Assumptions made.

→ Homogenous population, that is, no age or size structure

→ Constant environmental conditions

→ Growth rate  $r$  and carrying capacity  $K$  are constants.