1. Mathematical Model for Wind Turbine Power Output

A. Power in the Wind

The **kinetic energy** of wind flowing through the rotor area:

$$P_{wind} = \frac{1}{2}\rho A v^3$$

Where:

- ρ: air density (≈ 1.225 kg/m³ at sea level)
- $A = \pi R^2$: rotor swept area (m²)
- v: wind speed (m/s)

B. Power Extracted by Turbine

$$P_{turbine} = C_p \cdot P_{wind} = \frac{1}{2}\rho A C_p v^3$$

Where:

- C_p: power coefficient (max is Betz limit ≈ 0.593)

C. Power Curve of a Real Turbine

Turbine output is limited by physical constraints:

$$P(v) = \begin{cases} 0, & v < v_{cut-in} \\ \frac{1}{2}\rho A C_p v^3, & v_{cut-in} \le v < v_{rated} \\ P_{rated}, & v_{rated} \le v < v_{cut-out} \\ 0, & v \ge v_{cut-out} \end{cases}$$

Where:

- v_{cut-in}: min wind speed to start generating (e.g., 3 m/s)
- v_{rated}: wind speed for max power (e.g., 12 m/s)
- $v_{cut-out}$: max speed before shutoff (e.g., 25 m/s)