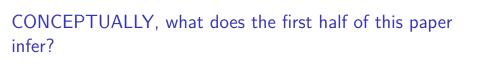
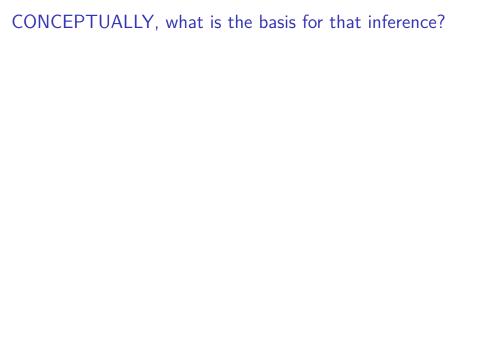
### Lecture 18 - State-space modeling: Gatenby and

Vincent 2003









#### Modeling density-independent growth

Unlimited growth of a population over discrete time steps:

$$N_{t+1} = N_t + RN_t$$

Change in population over a single time step:

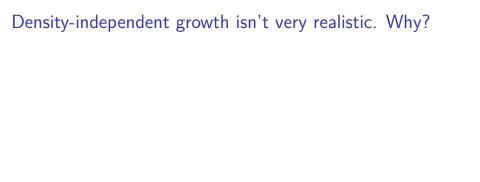
$$N_{t+1} - N_t = RN_t$$

Change in population over any time step:

$$\frac{\Delta N}{\Delta t} = R_N N$$

Considering a very small change in time gives us the continuous differential equation:

$$\frac{dN}{dt} = RN$$



## Density-dependent growth adds realism to density-independent growth

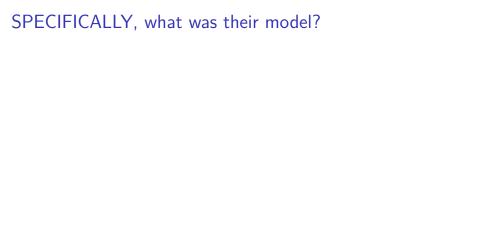
$$\frac{dN}{dt} = R(1 - \alpha N)N$$

 $\alpha$  is a coefficient describing competition with individuals from the same population.

The inverse of this is carrying capacity  $\frac{1}{K}$ .

So the equation above can also be written as:

$$\frac{dN}{dt} = R(1 - \frac{N}{K})N$$



#### State variables vs. parameters

**State variables** are quantities of individuals, matter, or energy that are simulated through time.

**Parameters** are constants or values that describe change in state variables or relationships amongst state variables.

# What are the the state variables and parameters in Gatenby and Vincent?

## What are the the state variables and parameters in Gatenby and Vincent?

- State variables
  - $\triangleright$   $N_N$ : normal cells
  - $\triangleright$   $N_T$ : tumor cells
- Parameters
  - ► R<sub>N</sub>: growth rate for normal cells; [time^{-1}]
  - $ightharpoonup K_N$ : carrying capacity for normal cells; [cells]
  - α<sub>NT</sub>: effect of tumor cells on normal cells; [cells cell<sup>-</sup>{-1}]
  - ▶ R<sub>T</sub>: growth rate for tumor cells; [time^{-1}]
  - ► K<sub>T</sub>: carrying capacity for normal cells; [cells]
  - α<sub>TN</sub>: effect of normal cells on tumor cells; [cells cell<sup>^</sup>{-1}]

SPECIFICALLY, what model behavior was the basis for the authors' conceptual inferences?



talk about how to simulate models with Python and  $\ensuremath{\mathtt{R}}$ 

recreate Gatenby & Vincent 2003 and extend to other examples