Sage 480 Final Project

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1 Introduction

This paper is an overview of the basics of linear optimization. This paper will be some notes from a previous class for the use by my group in reviewing linear optimization

2 Basics of Linear Programs

2.1 Linear Program in Tableaux format

The Primal Linear program is of the form,

$$\text{max: } c^T x$$

$$\text{subject to: } Ax \leq b$$

$$x \geq 0$$

Consider the given function and constraints,

$$\text{max: } 3x_1 + 2x_2 - 4x_3 = z$$

$$\text{subject to:}$$

$$x_1 + 4x_2 \leq 5$$

$$2x_1 + 4x_2 - 2x_3 \leq 6$$

$$x_1 + x_2 - 2x_3 \leq 2$$

Where the coefficents of the objective function is c^T. The coefficents of the constraint functions is matrix A and the inequality values is matrix b.

Use these matrices to create a Tableaux of the form

$$\begin{bmatrix} A & I & b \\ c & 0 & 0 \end{bmatrix}$$

So the tableaux form of our example is:

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 0 & 0 & 5 \\ 2 & 4 & -2 & 0 & 1 & 0 & 6 \\ 1 & 1 & -2 & 0 & 0 & 1 & 2 \\ 3 & 2 & -4 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.2 Dual to the Primal

Let me first introduce some theories that will be useful in effiecently solving our linear optimization problem.

1. Weak-Duality Theorem: Let P be a primal problem and D be its dual. If x if P-feasible and y is D-feasible, then $c^T x^T y$ If we find a pair $(\bar{x}, \bar{y})$ such that $\bar{x}$ is P-feasible and $\bar{y}$ is D-feasible and $c^T \bar{x} = b^T \bar{y}$, then $\bar{x}$ is P-optimal and $\bar{y}$ is D-optimal.

2. Corollary: if P is unbounded then D is infeasible. Likewise, if D is unbounded then P is infeasible.

3. Strong Duality Theorem: if either P or D have a finite optimal value, then so does the other, and in fact the optimal values coinside.

The dual to the primial initial tableaux,

$$\text{min: } b^T y$$

$$\text{subject to: } A^T \geq c$$

$$y \geq 0$$

Referring back to our example, the dual is:

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 & 3 \\ 4 & 4 & 1 & 0 & 1 & 0 & 2 \\ 0 & -2 & -2 & 0 & 0 & 1 & -4 \\ 5 & 6 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In order for a primal linear program to be dual feasible the objective function must have all non-positive coefficents. That is $c^T \leq 0$. Therefore our example is not dual feasible.