# mjon238 Stats 326 Assignment 3 Q1

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## Problem 1: ETS Modelling

First load the data

```
direct <- "Current Uni Stuff/Stats 326/Assignment 3/"
df <- read_csv("productivity.csv")%>%
  as_tsibble(index = Year)
```

```
## Rows: 44 Columns: 2
```

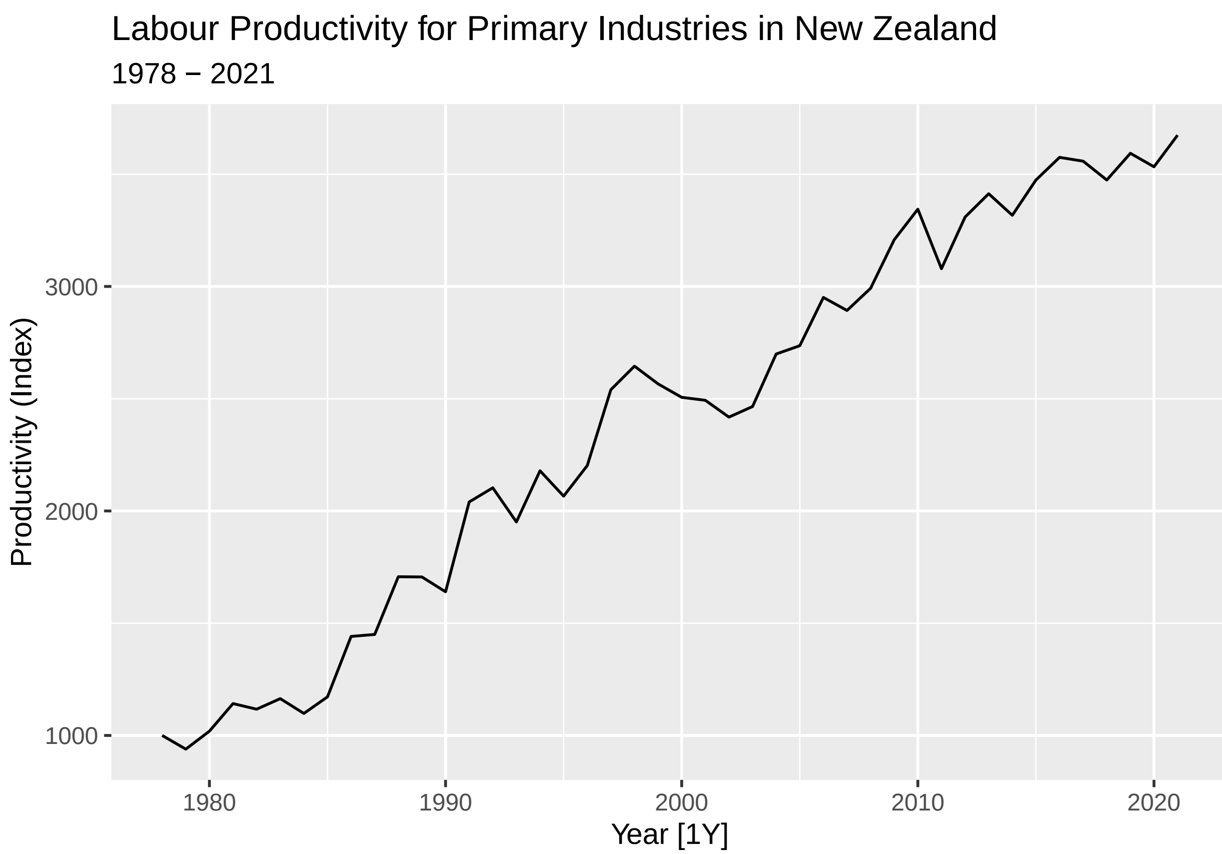
```
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (2): Year, Productivity
```

```
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

### Part 1: Plot the Data

```
df%>%
  autoplot()+
  labs(title = "Labour Productivity for Primary Industries in New Zealand",
       subtitle = "1978 - 2021",
       y = "Productivity (Index)")
```

```
## Plot variable not specified, automatically selected `.vars = Productivity`
```




- There is a linear increasing trend in labour productivity for primary industries in New Zealand.
- The steepest increase is from 1978 to 1998, afterwards productivity continues to increase, however at a slightly slower rate.
- There are cyclical fluctuation sin labour productivity, with a number of decreases, notably after 1998 and 2010.
- Productivity does pick-up after these cyclical decreases.

## Part 2

### 1) Fit the Two Models

```
dfTrain <- df%>%
  filter(Year < 2017)

modelLinear <- dfTrain%>%
  model(Linear = ETS(Productivity ~ error("A") + trend("A") + season("N")))

modelDamped <- dfTrain%>%
  model(Linear = ETS(Productivity ~ error("A") + trend("Ad") + season("N")))
```

### 2) Intrepret Model Parameters and Compare AICc

```r
report(modelLinear)
```

```
## Series: Productivity
## Model: ETS(A,A,N)
##   Smoothing parameters:
##     alpha = 0.4889942
##     beta  = 0.0001000043
##
##   Initial states:
##      l[0]     b[0]
##  880.7263 68.67191
##
##   sigma^2:  19414.32
##
##      AIC     AICc      BIC
## 533.7355 535.5536 542.0533
```

```r
beta_star = 0.0001000043/0.4889942
beta_star
```

```
## [1] 0.0002045102
```

Parameters in Holts Linear Trend Method

- $\alpha = 0.489$, which means the level equation is an approximately 50/50 split of the previous observations level and the observations before.
- $\beta^* = 0.0002 \approx 0$, which means the slope is just the previous estimate of the slope ($b_t = b_{t-1}$).

```r
report(modelDamped)
```

```
## Series: Productivity
## Model: ETS(A,Ad,N)
##   Smoothing parameters:
##     alpha = 0.5939802
##     beta  = 0.0001001014
##     phi   = 0.98
##
##   Initial states:
##     l[0]     b[0]
##  879.452 86.63342
##
##   sigma^2:  20988.5
##
##      AIC     AICc      BIC
## 537.6455 540.2705 547.6269
```

```r
beta_star = 0.0001001014/0.5939802
beta_star
```

```
## [1] 0.0001685265
```

Parameters in Holts Linear Damped Trend Method

- $\alpha = 0.594$, which means the level equation has slightly more weighting to the previous observation than the older past.
- $\beta^* = 0.0002 \approx 0$, which means the slope is just the previous estimate of the slope ($b_t = \phi b_{t-1}$).
- $\phi = 0.98$, this is the maximum value $\phi$ can take, which indicates that the trend is approximately linear (as opposed to a decaying slope).

### 3) Compare AICc

Holts Linear Trend Method has an AICc of 535.55 and Holts Linear Damped Trend Method has an AICc of 540.27. Holts Linear Trend Method has a slightly better fit to the training data, but not by much.

## Part 3

### 1) Create Forecasts

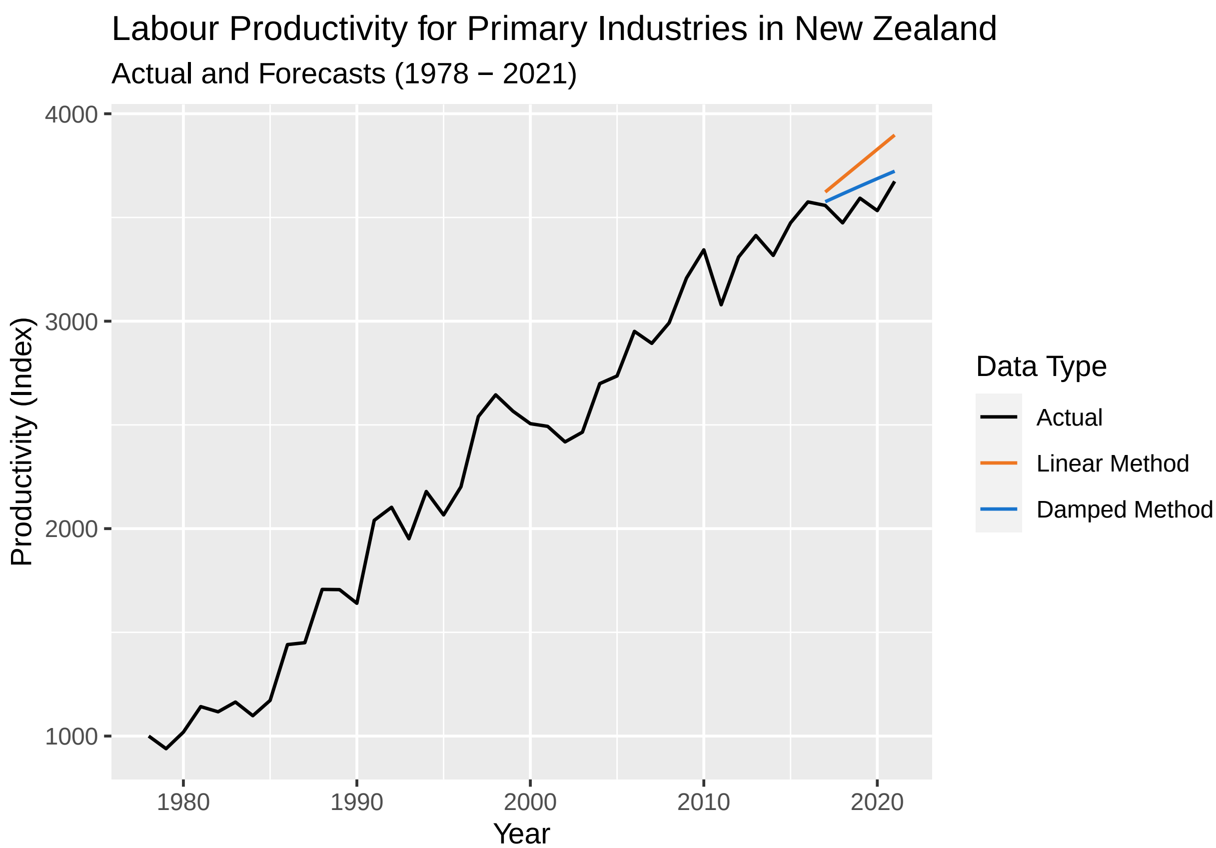
```r
fcLinear <- modelLinear%>%
  forecast(h = 5)

fcDamped <- modelDamped%>%
  forecast(h = 5)
```

### 2) Create Plot of Point Forecasts

```r
#Create data frame, overlay point forecasts on original data
forecasts <- data.frame(Year = c(1978:2021),
                   fcDamped = c(rep(NA,39), fcDamped$.mean),
                   fcLinear = c(rep(NA,39), fcLinear$.mean),
                   Actual = df$Productivity)%>%
  pivot_longer(cols = c(Actual, fcLinear, fcDamped),
               names_to = "Data Type",
               values_to = "Productivity")%>%
  mutate(`Data Type` = factor(`Data Type`, levels = c("Actual", "fcLinear", "fcDamped")))

#Create Plots
ggplot(aes(x = Year, y = Productivity, colour = `Data Type`),
       data = forecasts) +
  geom_line(size = 0.6)+
  labs(y = "Productivity (Index)",
       title = "Labour Productivity for Primary Industries in New Zealand",
       subtitle = "Actual and Forecasts (1978 - 2021)")+
  scale_color_manual(labels = c("Actual", "Linear Method", "Damped Method"),
                     values= c("black", "chocolate2", "dodgerblue3"))
```




**3) Compute Measures of Accuracy**

```r
#Holts Linear Trend Method
accuracy(fcLinear, df)
```

```
## # A tibble: 1 x 10
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 Linear Test  -194.  208.  194. -5.43  5.43  1.52  1.31 -0.115
```

```r
#Holts Linear Damped Trend Method
accuracy(fcDamped, df)
```

```
## # A tibble: 1 x 10
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 Linear Test  -83.8  99.4  83.8 -2.37  2.37 0.659 0.628 -0.657
```

Holts Linear Damped Trend provides better forecasts. It has significantly lower MAE and RMSE. It is also fits the data more appropriately.

### 4) Prediction Intervals

```
fc22 <- modelDamped%>%
  forecast(h = 6)

fc22[6,]%>%
  hilo(level = 95)
```

```
## # A tsibble: 1 x 5 [1Y]
## # Key:       .model [1]
##   .model  Year  Productivity .mean                     `95%`
##   <chr>  <dbl>        <dist> <dbl>                    <hilo>
## 1 Linear  2022 N(3758, 58049) 3758. [3285.731, 4230.176]95
```

The 95% prediction interval for the year 2022 with the Holt Linear Damped Model is (3285.73, 4230.18).

The model estimates there is a 95% probability the New Zealand Labour Productivity for Primary Industries Index in 2022 will be between 3285.73 and 4230.18.

### 5) Discuss how you would reduce forecast uncertainty

The formula for the prediction interval in with additive errors is $y_{T+h|T} \pm z_{\alpha/2} \times \hat{\sigma_h}$. The primary way to reduce the prediction intervals would be to reduce the forecast standard deviation ($\hat{\sigma_h}$) and therefore the variance, $\sigma_h^2$.

The formula for the variance is as follows:

$$\sigma_h^2 = \sigma^2[1 + \alpha^2(h-1) + \frac{\beta\phi h}{(1-\phi)^2}\{2\alpha(1-\phi) + \beta\phi\} - \frac{\beta\phi(1-\phi^h)}{(1-\phi)^2}\{2\alpha(1-\phi)^2 + \beta\phi(1+2\phi-\phi^h)\}]$$

We can reduce $\sigma_h^2$ by:

- Reducing h, we can do this by increasing our training data set to include observations up-to 2021.
- Reducing $\alpha$, this will increase weighting on the older past.
- Reducing $\beta = \beta^*\alpha$, this can be done by reducing alpha or reducing $\beta^*$, ie. have the slope change less often.

$\phi$ is a less clear variable.

# Problem 2: ARIMA Modelling

## Part 1: Determining an Appropriate ARIMA Model

### 1) Construct a KPSS Unit Root Test

Our null hypothesis is H0: The data is stationary.

Firstly, with no differencing:

```
dfTrain%>%
  features(`Productivity`, unitroot_kpss)
```

```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1      1.06        0.01
```

We have a p-value of 0.01, therefore we reject the null hypothesis, the data is non-stationary.

Using differencing of order 1, with 1 lag and the same null hypothesis.

```
dfTrain%>%
  features(difference(Productivity, lag = 1), unitroot_kpss)
```

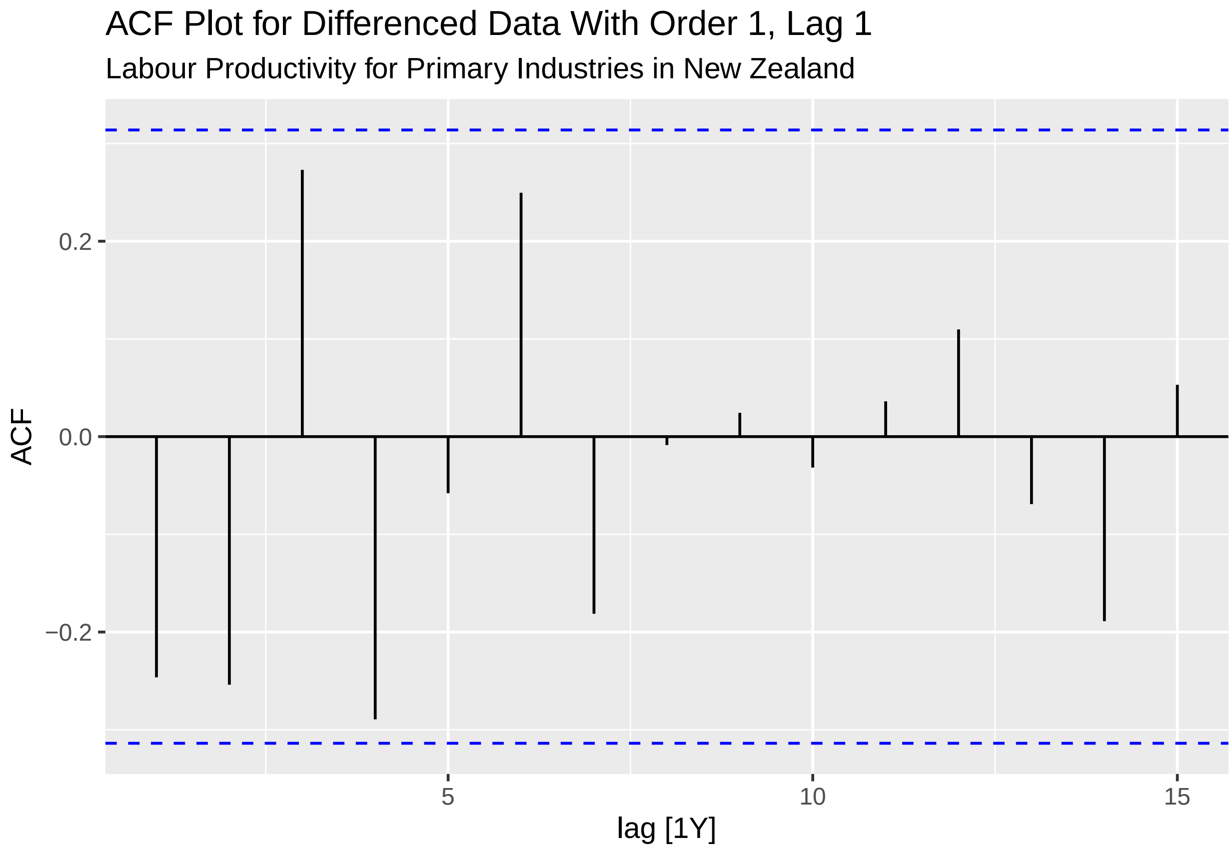
```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1    0.0520         0.1
```

We have a p-value of 0.1, therefore we accept the null hypothesis, the data is stationary.

### 2) Ploting ACF and PACF

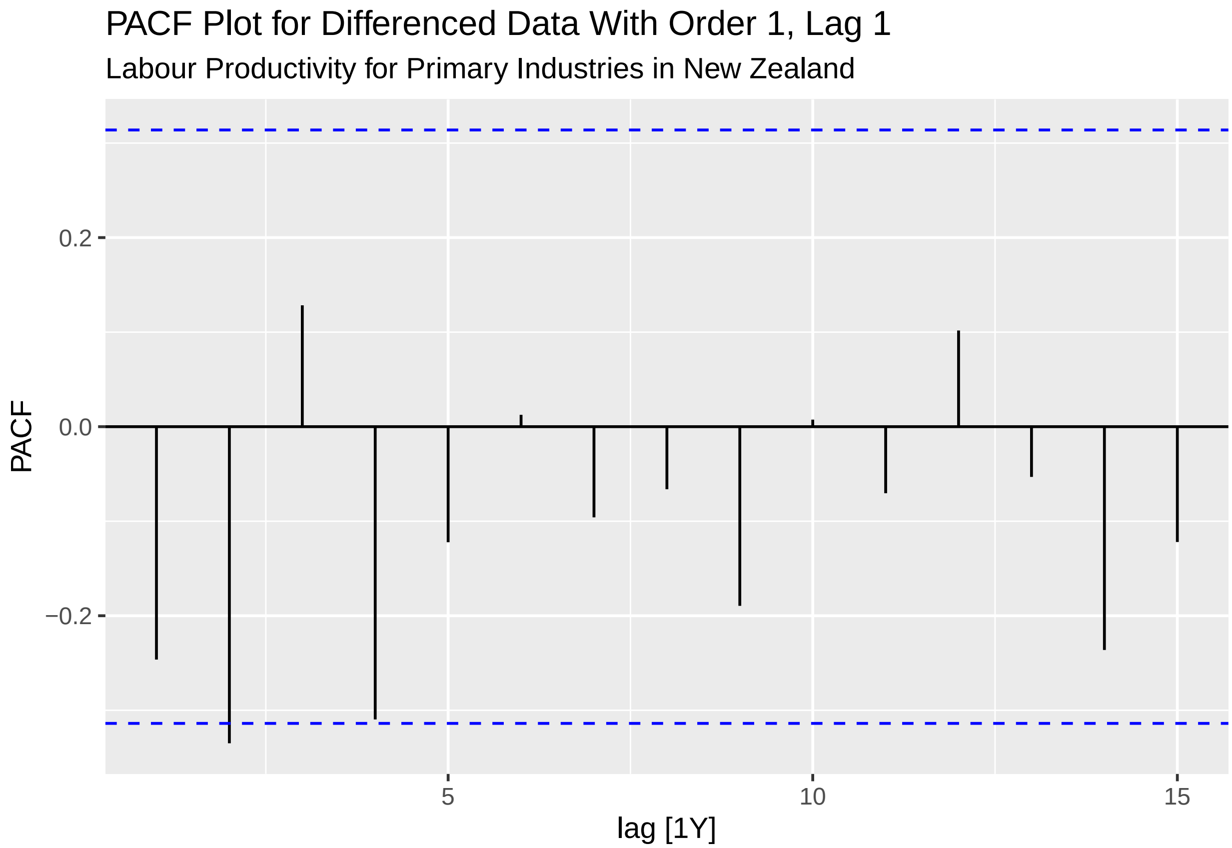
```
dfTrain2 <- dfTrain%>%
  mutate(diff = difference(Productivity, lag  = 1))

dfTrain2%>%
  ACF(diff)%>%
  autoplot() +
  labs(title = "ACF Plot for Differenced Data With Order 1, Lag 1",
       subtitle = "Labour Productivity for Primary Industries in New Zealand",
       y = "ACF")
```




The ACF Plot is good, the differences series acts like a white-noise series.

```
dfTrain2%>%
  PACF(diff)%>%
  autoplot() +
  labs(title = "PACF Plot for Differenced Data With Order 1, Lag 1",
       subtitle = "Labour Productivity for Primary Industries in New Zealand",
       y = "PACF")
```




The second lag is approximately a white noise series, there is one observation outside of the “blue-line” interval, however it is only marginally outside.