mjon238 Stats 326 Assignment 3

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Problem 1: ETS Modelling

First load the data

```
direct <- "Current Uni Stuff/Stats 326/Assignment 3/"
df <- read_csv("productivity.csv")%>%
  as_tsibble(index = Year)
```

```
## Rows: 44 Columns: 2
```

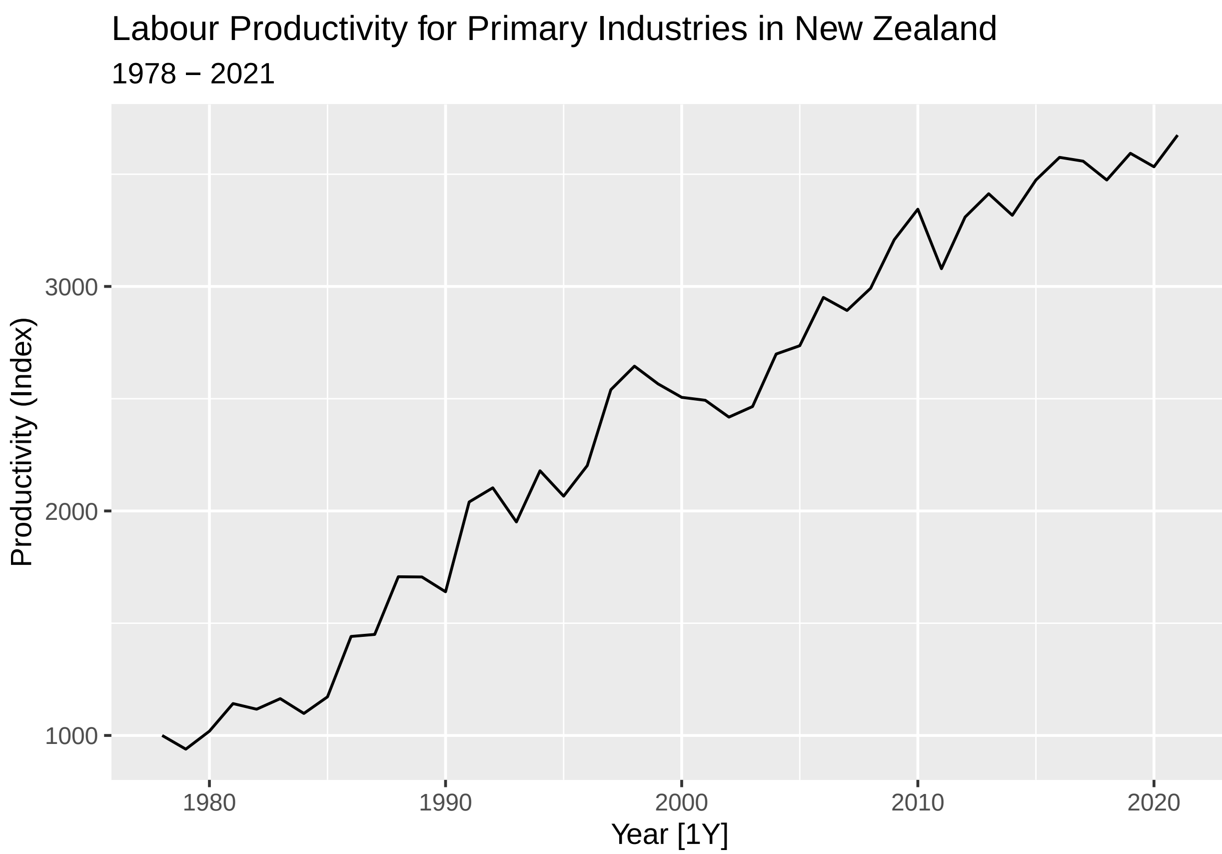
```
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (2): Year, Productivity
```

```
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

Part 1: Plot the Data

```
df%>%
  autoplot()+
  labs(title = "Labour Productivity for Primary Industries in New Zealand",
       subtitle = "1978 - 2021",
       y = "Productivity (Index)")
```

```
## Plot variable not specified, automatically selected `.vars = Productivity`
```



- There is a linear increasing trend in labour productivity for primary industries in New Zealand.
- The steepest increase is from 1978 to 1998, afterwards productivity continues to increase, however at a slightly slower rate.
- There are cyclical fluctuations in labour productivity, with a number of decreases, notably after 1998 and 2010.
- Productivity does pick-up after these cyclical decreases.

Part 2

1) Fit the Two Models

```
dfTrain <- df%>%
  filter(Year < 2017)

modelLinear <- dfTrain%>%
  model(Linear = ETS(Productivity ~ error("A") + trend("A") + season("N")))

modelDamped <- dfTrain%>%
  model(Linear = ETS(Productivity ~ error("A") + trend("Ad") + season("N")))
```

2) Intrepret Model Parameters and Compare AICc

```r
report(modelLinear)
```

```
## Series: Productivity
## Model: ETS(A,A,N)
##   Smoothing parameters:
##     alpha = 0.4889942
##     beta  = 0.0001000043
##
##   Initial states:
##      l[0]     b[0]
##  880.7263 68.67191
##
##   sigma^2:  19414.32
##
##      AIC     AICc      BIC
## 533.7355 535.5536 542.0533
```

```r
beta_star = 0.0001000043/0.4889942
beta_star
```

```
## [1] 0.0002045102
```

Parameters in Holts Linear Trend Method

- $\alpha = 0.489$, which means the level equation is an approximately 50/50 split of the previous observations level and the observations before.
- $\beta^* = 0.0002 \approx 0$, which means the slope is just the previous estimate of the slope ($b_t = b_{t-1}$).

```r
report(modelDamped)
```

```
## Series: Productivity
## Model: ETS(A,Ad,N)
##   Smoothing parameters:
##     alpha = 0.5939802
##     beta  = 0.0001001014
##     phi   = 0.98
##
##   Initial states:
##     l[0]     b[0]
##  879.452 86.63342
##
##   sigma^2:  20988.5
##
##      AIC     AICc      BIC
## 537.6455 540.2705 547.6269
```

```r
beta_star = 0.0001001014/0.5939802
beta_star
```

```
## [1] 0.0001685265
```

Parameters in Holts Linear Damped Trend Method

- $\alpha = 0.594$, which means the level equation has slightly more weighting to the previous observation than the older past.
- $\beta^* = 0.0002 \approx 0$, which means the slope is just the previous estimate of the slope ($b_t = \phi b_{t-1}$).
- $\phi = 0.98$, this is the maximum value ϕ can take, which indicates that the trend is approximately linear (as opposed to a decaying slope).

3) Compare AICc

Holts Linear Trend Method has an AICc of 535.55 and Holts Linear Damped Trend Method has an AICc of 540.27. Holts Linear Trend Method has a slightly better fit to the training data, but not by much.

Part 3

1) Create Forecasts

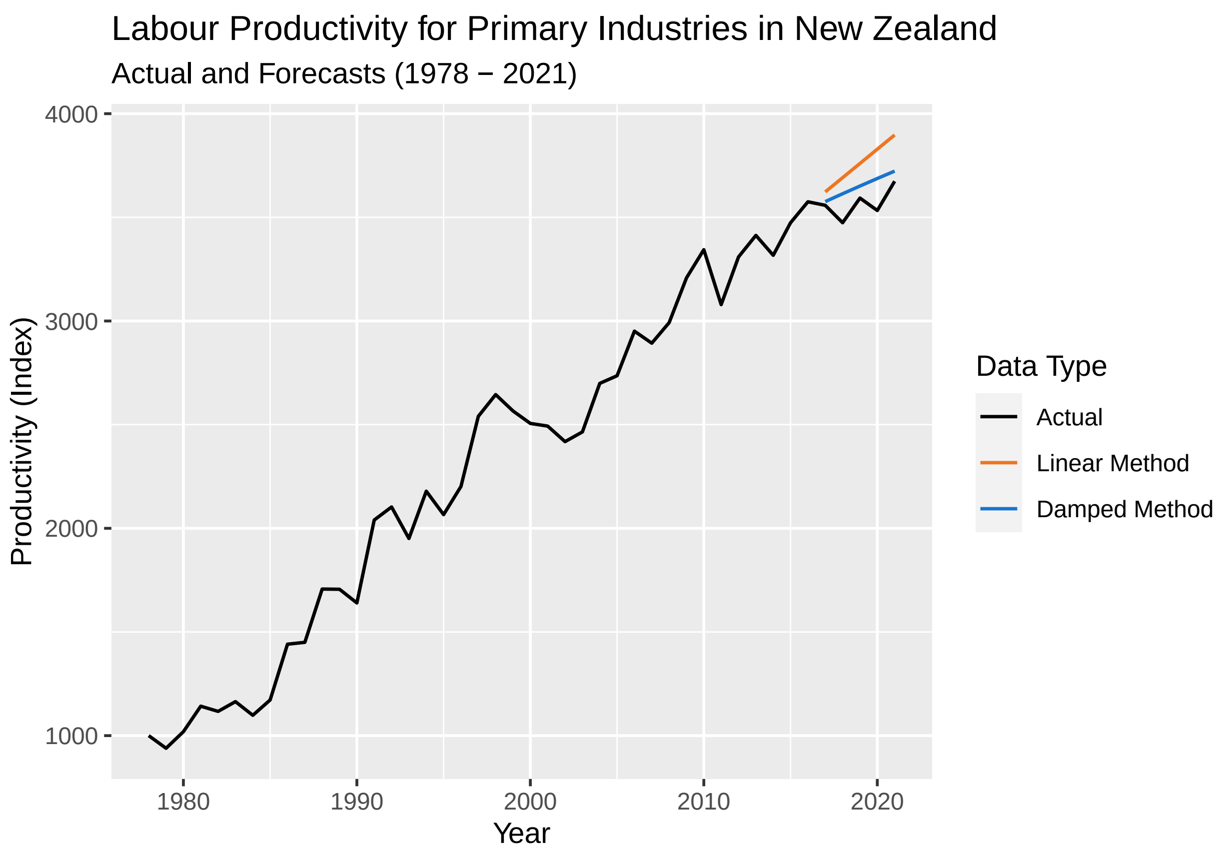
```r
fcLinear <- modelLinear%>%
  forecast(h = 5)

fcDamped <- modelDamped%>%
  forecast(h = 5)
```

2) Create Plot of Point Forecasts

```r
#Create data frame, overlay point forecasts on original data
forecasts <- data.frame(Year = c(1978:2021),
                   fcDamped = c(rep(NA,39), fcDamped$.mean),
                   fcLinear = c(rep(NA,39), fcLinear$.mean),
                   Actual = df$Productivity)%>%
  pivot_longer(cols = c(Actual, fcLinear, fcDamped),
               names_to = "Data Type",
               values_to = "Productivity")%>%
  mutate(`Data Type` = factor(`Data Type`, levels = c("Actual", "fcLinear", "fcDamped")))

#Create Plots
ggplot(aes(x = Year, y = Productivity, colour = `Data Type`),
       data = forecasts) +
  geom_line(size = 0.6)+
  labs(y = "Productivity (Index)",
       title = "Labour Productivity for Primary Industries in New Zealand",
       subtitle = "Actual and Forecasts (1978 - 2021)")+
  scale_color_manual(labels = c("Actual", "Linear Method", "Damped Method"),
                     values= c("black", "chocolate2", "dodgerblue3"))
```



3) Compute Measures of Accuracy

```
#Holts Linear Trend Method
accuracy(fcLinear, df)
```

```
## # A tibble: 1 x 10
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 Linear Test  -194.  208.  194. -5.43  5.43  1.52  1.31 -0.115
```

```
#Holts Linear Damped Trend Method
accuracy(fcDamped, df)
```

```
## # A tibble: 1 x 10
##   .model .type    ME  RMSE   MAE   MPE  MAPE  MASE RMSSE   ACF1
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>
## 1 Linear Test  -83.8  99.4  83.8 -2.37  2.37 0.659 0.628 -0.657
```

Holts Linear Damped Trend provides better forecasts. It has significantly lower MAE and RMSE. It is also fits the data more appropriately.

4) Prediction Intervals

```
fc22 <- modelDamped%>%
  forecast(h = 6)

fc22[6,]%>%
  hilo(level = 95)
```

```
## # A tsibble: 1 x 5 [1Y]
## # Key:       .model [1]
##   .model  Year   Productivity .mean                      `95%`
##   <chr>  <dbl>         <dist> <dbl>                     <hilo>
## 1 Linear  2022 N(3758, 58049) 3758. [3285.731, 4230.176]95
```

The 95% prediction interval for the year 2022 with the Holt Linear Damped Model is (3285.73, 4230.18).

The model estimates there is a 95% probability the New Zealand Labour Productivity for Primary Industries Index in 2022 will be between 3285.73 and 4230.18.

5) Discuss how you would reduce forecast uncertainty

The formula for the prediction interval in with additive errors is $y_{T+h|T} \pm z_{\alpha/2} \times \hat{\sigma_h}$. The primary way to reduce the prediction intervals would be to reduce the forecast standard deviation $(\hat{\sigma_h})$ and therefore the variance, σ_h^2.

The formula for the variance is as follows:

$$\sigma_h^2 = \sigma^2[1 + \alpha^2(h-1) + \frac{\beta\phi h}{(1-\phi)^2}\{2\alpha(1-\phi) + \beta\phi\} - \frac{\beta\phi(1-\phi^h)}{(1-\phi)^2}\{2\alpha(1-\phi)^2 + \beta\phi(1+2\phi-\phi^h)\}]$$

We can reduce σ_h^2 by:

- Reducing h, we can do this by increasing our training data set to include observations up-to 2021.
- Reducing α, this will increase weighting on the older past.
- Reducing $\beta = \beta^*\alpha$, this can be done by reducing alpha or reducing β^*, ie. have the slope change less often.

ϕ is a less clear variable.

Problem 2: ARIMA Modelling

Part 1: Determining an Appropriate ARIMA Model

1) Construct a KPSS Unit Root Test

Our null hypothesis is H0: The data is stationary.

Firstly, with no differencing:

```r
dfTrain%>%
  features(`Productivity`, unitroot_kpss)
```

```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1      1.06        0.01
```

We have a p-value of 0.01, therefore we reject the null hypothesis, the data is non-stationary.

Using differencing of order 1, with 1 lag and the same null hypothesis.

```r
dfTrain%>%
  features(difference(Productivity, lag = 1), unitroot_kpss)
```

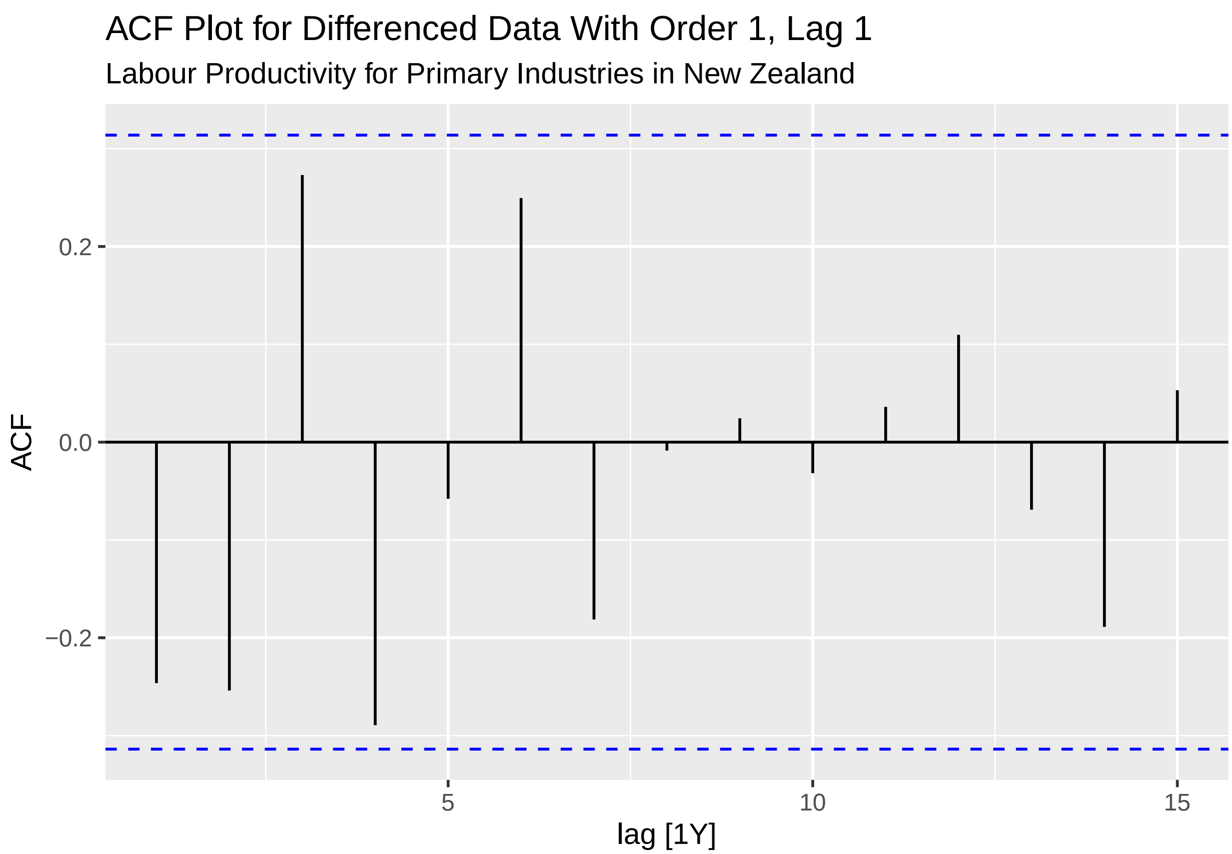
```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1    0.0520         0.1
```

We have a p-value of 0.1, therefore we accept the null hypothesis, the data is stationary.

2) Plotting ACF and PACF

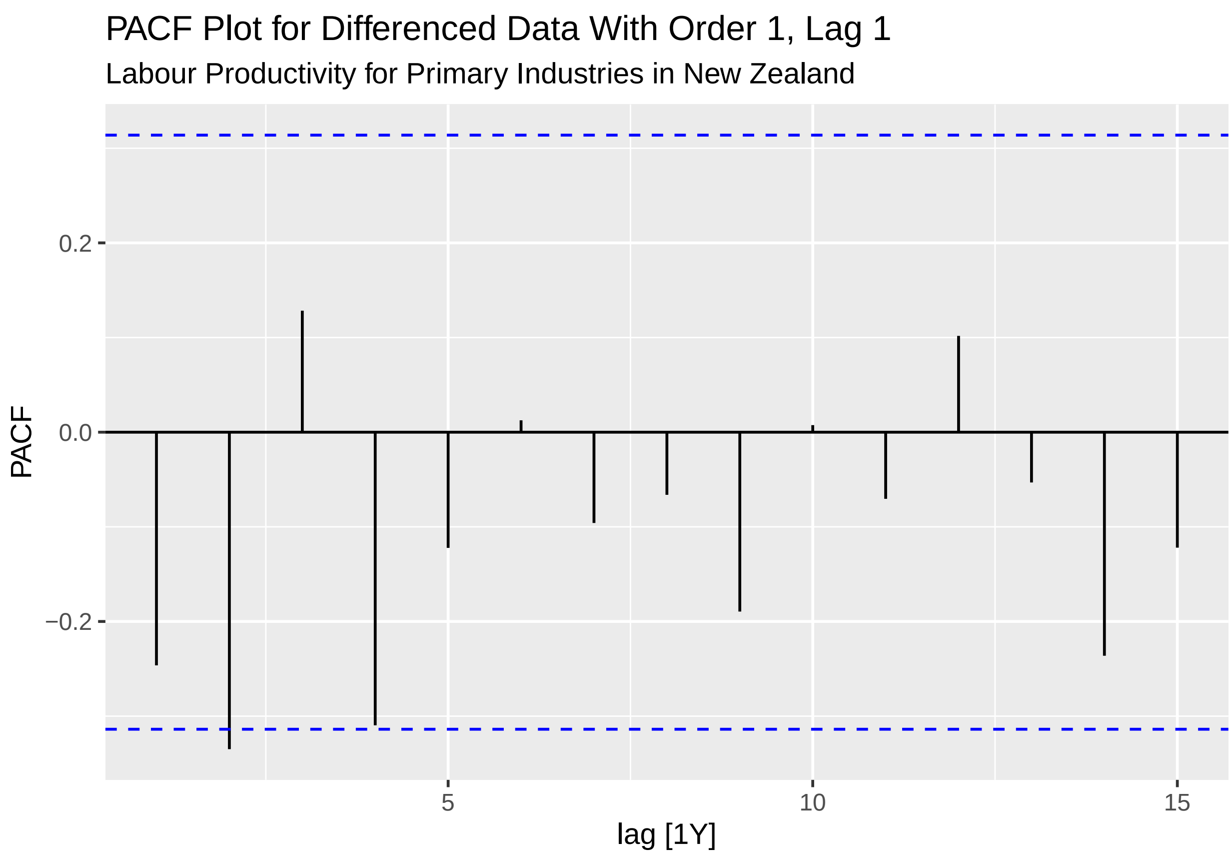
```r
dfTrain2 <- dfTrain%>%
  mutate(diff = difference(Productivity, lag  = 1))

dfTrain2%>%
  ACF(diff)%>%
  autoplot() +
  labs(title = "ACF Plot for Differenced Data With Order 1, Lag 1",
       subtitle = "Labour Productivity for Primary Industries in New Zealand",
       y = "ACF")
```



The ACF Plot tells us the differences series acts like a white-noise series, with no significant lags.

```
dfTrain2%>%
  PACF(diff)%>%
  autoplot() +
  labs(title = "PACF Plot for Differenced Data With Order 1, Lag 1",
       subtitle = "Labour Productivity for Primary Industries in New Zealand",
       y = "PACF")
```



The PACF Plot has a significant lag at lag 2, but the rest are insignificant.

3) Suggested ARIMA Model

Firstly the ACF Plot, has no significant lags, therefore our q-term will be 0. Secondly the PACF Plot, has a significant second lag, therefore our p-term will be 2. We are using difference data, so our d term is equal to 1.

Hence, I would fit an ARIMA (2,1,0).

In backshift notation this is: $(1 - \phi_1 B - \phi_2 B^2)(1 - B)y_t = c + \varepsilon_t$

where c is a constant and ε_t is a white noise series.

Part 2: ARIMA Model Fitting

```
#Fit Models
fitARIMA <- dfTrain%>%
  model(arima210 = ARIMA(Productivity ~ pdq(2,1,0)),
        stepwise = ARIMA(Productivity, stepwise = TRUE),
        search = ARIMA(Productivity, stepwise = FALSE))

fitARIMA
```

```
## # A mable: 1 x 3
```

```
##                   arima210                 stepwise                   search
##                    <model>                  <model>                  <model>
## 1 <ARIMA(2,1,0) w/ drift> <ARIMA(0,1,1) w/ drift> <ARIMA(0,1,1) w/ drift>
```

```
glance(fitARIMA)%>%
  arrange(AICc)%>%
  select(.model:BIC)
```

```
## # A tibble: 3 x 6
##   .model   sigma2 log_lik   AIC  AICc   BIC
##   <chr>     <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 stepwise 18873.   -240.  486.  487.  491.
## 2 search   18873.   -240.  486.  487.  491.
## 3 arima210 18401.   -239.  486.  487.  493.
```

The stepwise selected model and the non-stepwise (search) selected model are identical, ARIMA(0,1,1) with a drift. This model has the lowest AICc.

In backshift notation this is: $(1-B)y_t = c + (1+\theta_1 B)\varepsilon_t$

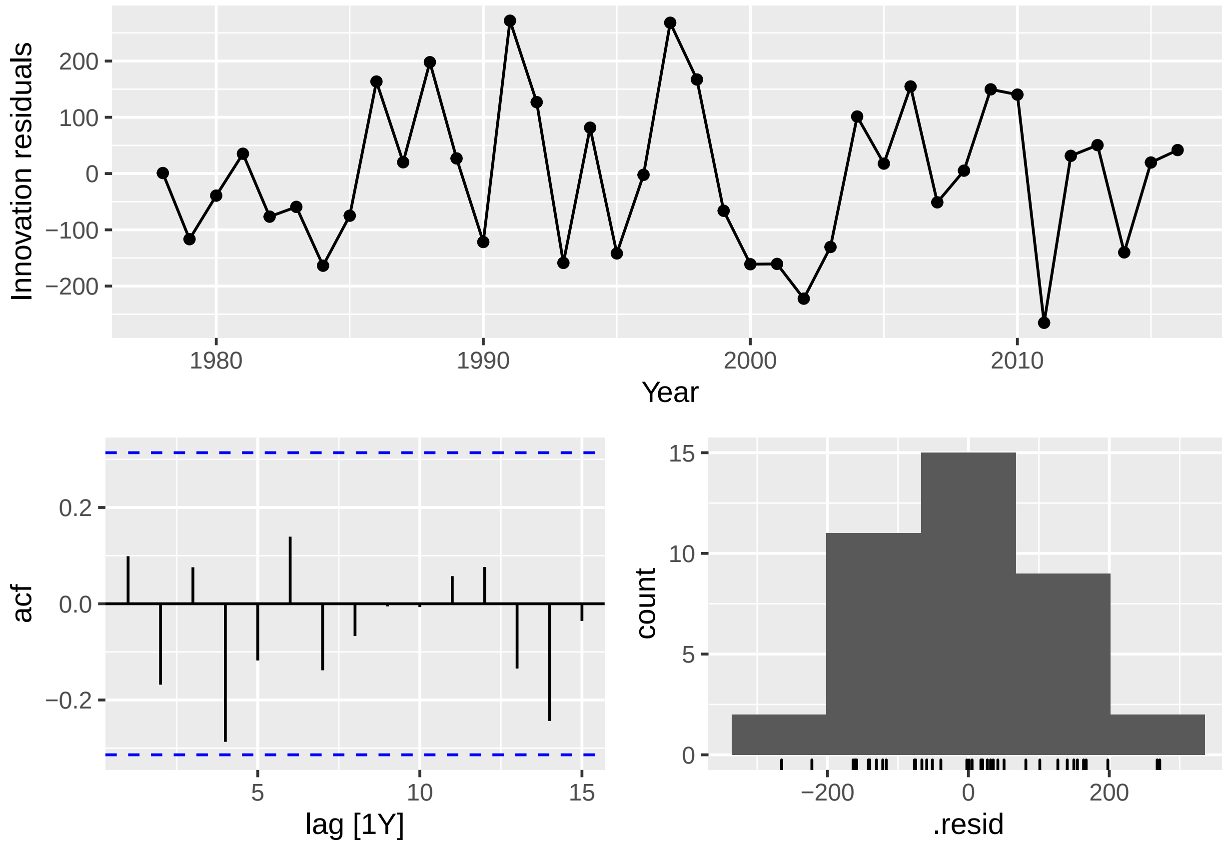
Where c is a constant and ε_t is a white noise series.

Part 3: Model Checking and Forecasting

1) Residuals

```
bestARIMA <- dfTrain%>%
  model(ARIMA(Productivity ~ pdq(0,1,1)))

bestARIMA%>%
  gg_tsresiduals()
```



The residuals look, overall, very good

- They are normally distributed.
- Have approximately zero mean and constant variance.
- They resemble a white-noise series.

I have no concerns about the model, we can accept the ARIMA(0,1,1) as our final model.

2) Forecasts

```
fc <- bestARIMA%>%
  forecast(h = 5)

fc
```

```
## # A fable: 5 x 4 [1Y]
## # Key:     .model [1]
##   .model                               Year   Productivity .mean
##   <chr>                               <dbl>         <dist> <dbl>
## 1 ARIMA(Productivity ~ pdq(0, 1, 1))  2017 N(3623, 18873) 3623.
## 2 ARIMA(Productivity ~ pdq(0, 1, 1))  2018 N(3692, 23805) 3692.
## 3 ARIMA(Productivity ~ pdq(0, 1, 1))  2019 N(3761, 28737) 3761.
## 4 ARIMA(Productivity ~ pdq(0, 1, 1))  2020 N(3830, 33669) 3830.
## 5 ARIMA(Productivity ~ pdq(0, 1, 1))  2021 N(3899, 38601) 3899.
```

3) Plotting

```
fc%>%
  autoplot(df, level = c(90, 99)) +
  labs(title = "5 Year Forecast Using an ARIMA(0,1,1) with Drift",
       subtitle = "Labour Productivity for Primary Industries in New Zealand",
       y = "Productivity (Index)")
```