Dynamics and Relativity

Jonathan Gai

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Contents

Lecture 1: Introduction

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1 Basic Concepts

Books

1. Classical Mechanics - Douglas (more examples)
2. Classical Mechanics - Tom Kibblet (more chatty)
3. Lecture Notes - David Tong

1.1 Newtonian Mechanics

A *particle* is an object of insignificant size. For now, its only attribute is its position.

For large objects, we take the center of mass to define the position and treat them like a particle.

To describe the position, we pick a *reference frame*: a choice of origin and 3 coordinate axes. With respect to this frame, a particle sweep out a *trajectory* $\mathbf{x}(t)$. (sometimes, we may write $\mathbf{r}(t)$).

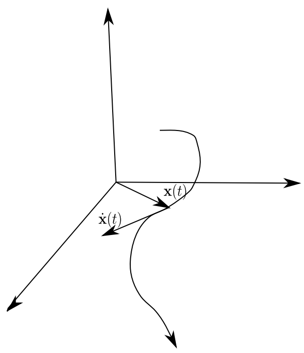

Figure 1: position and acceleration of a particle

Given two vector functions $\mathbf{f}(t)$ and $\mathbf{g}(t)$, $\frac{d}{dt}(\mathbf{f}\cdot\mathbf{g}) = \frac{d\mathbf{g}}{t}\cdot\mathbf{g}+\mathbf{f}\cdot\frac{d\mathbf{g}}{dt}$ and $\frac{d}{dt}(\mathbf{f}\times\mathbf{g}) = \frac{d\mathbf{g}}{t}\times\mathbf{g}+\mathbf{f}\times\frac{d\mathbf{g}}{dt}$.

1.2 Newtonian Laws of Motion

The framework of Newtonian mechanics rely on these axioms, known as *Newton's Laws*:

Definition 1.1 (Newton's Laws). The following are true (for inertial frames):

- **N1**: Left alone, a particle moves with constant velocity.
- **N2**: The rate of change of momentum is proportional to the force.
- **N3**: Every action has an equal and opposite reaction.

1.3 Inertial Frames and The First Law

For many reference frames, **N1** isn't true! It only holds for frames that are not themselves accelerating. Such frames are called *inertial frames*:

Definition 1.2 (Inertial Frame). In an inertial frame, $\ddot{\mathbf{x}} = 0$ when left alone.

A better framing of the 1st law is (N1' inertial frames exist).

For most purposes, this room approximates an inertial frame.

1.4 Galilean Relativity

Inertial frames are not unique. Given an inertial frame S, in which a particle has coordinates $\mathbf{x}$, we can construct another inertial frame S' in which the coordinates of the particle are given by $\mathbf{x}'$.

1. Translations: $\mathbf{x}' = \mathbf{x} + \mathbf{a}$, where a is a constant.
2. Rotations: $\mathbf{x}' = R\mathbf{x}$, where R is a 3×3 matrix with $R^T R = I$.
3. (Galilean) Boost: $\mathbf{x}' = \mathbf{x} + \mathbf{v}t$.

For each of these, if there is no force on a particle. $\ddot{\mathbf{x}} = 0 \implies \ddot{\mathbf{x}}' = 0$

The *Galilean principle of relativity* tells us that the laws of physics are the same

1. At every point in space.
2. No matter which direction you face.
3. No matter what constant velocity you move at.
4. At all moments in time.

The above are experimentally tested facts.

There is no such thing as "absolutely stationary", but notice that acceleration is absolute. You don't have to accelerate relative to something.