Morphisms

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Problem 2

In Example 3.3 we have seen how to construct a category from a set endowed with a relation, provided this latter is reflexive and transitive. For what types of relations is the corresponding category a groupoid (cf. Example 4.6)?

Solution. The construction in Example 3.3 requires reflectivity and transitivity for the requirement for identity and composition of morphisms. For the category to be a groupoid, we need every morphism to be isomorphic. In particular, we need $(a, b) \in \mathrm{Hom}_{\mathsf{C}}(a, b) \implies (b, a) \in \mathrm{Hom}_{\mathsf{C}}(b, a)$. In the context of the relation, we have $a \; R \; b \implies b \; R \; a$. That is, the relation is an equivalence relation. ■