

Unicycle Control using DOB

Advanced Control Methods Term Project

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December 15th, 2023

Abstract

One of the control methods taught in the advanced control methods class is the disturbance observer or DOB. DOB makes the inner loop dynamics behave like the nominal plant, eliminating the effects of model uncertainty. It also has an added bonus of disturbance rejection making it a very versatile controller. This project implemented a DOB inner loop to a unicycle, which was previously controlled using a PD controller. The PD controller was designed using the linearized plant dynamics. The DOB improved the controller's performance on the real, non-linear plant dynamics by stabilizing the plant behaviour while under control.

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1 Introduction

This project stems from a personal passion for all things related to transportation. Trains, cars, aircraft, the author of this project has had an irrational obsession with things that move. Therefore, it was decided to choose the topic of this project within the realm of vehicle control. Of all wheeled vehicles, the dynamics and control of the unicycle was assumed to be the simplest to analyze.



Figure 1: Typical unicycle

A unicycle is a one wheeled vehicle that typically consists of a wheel, a frame, a pedal and a saddle. The pedals are used to propel the vehicle forward, while the direction is controlled by the rider moving his/her center of mass around, creating torque. It is an inherently unstable vehicle and can only be stabilized while moving forward at a certain speed[1].

The other crucial part of this project is the disturbance observer or DOB. The disturbance observer is an inner loop controller that is designed to compensate the uncertainty in the plant and the external disturbances into the plant[2]. It does this by essentially using singular perturbation to quickly converge the behaviour of the actual plant to the ideal nominal plant. This means a controller designed using

the nominal plant dynamics will perform as theoretically predicted despite the uncertainties in the plant modelling and the external disturbances.

This project is a continuation of the research done by Xincheng Cao et al[1]. They derived the dynamics of the unicycle, linearized it around an equilibrium point and created a PD controller to control the pitch, roll and yaw of the vehicle. This project aims to create a DOB inner loop controller that will make the actual non-linear plant behave like the linear nominal plant dynamics wise.

2 Dynamics and Modelling

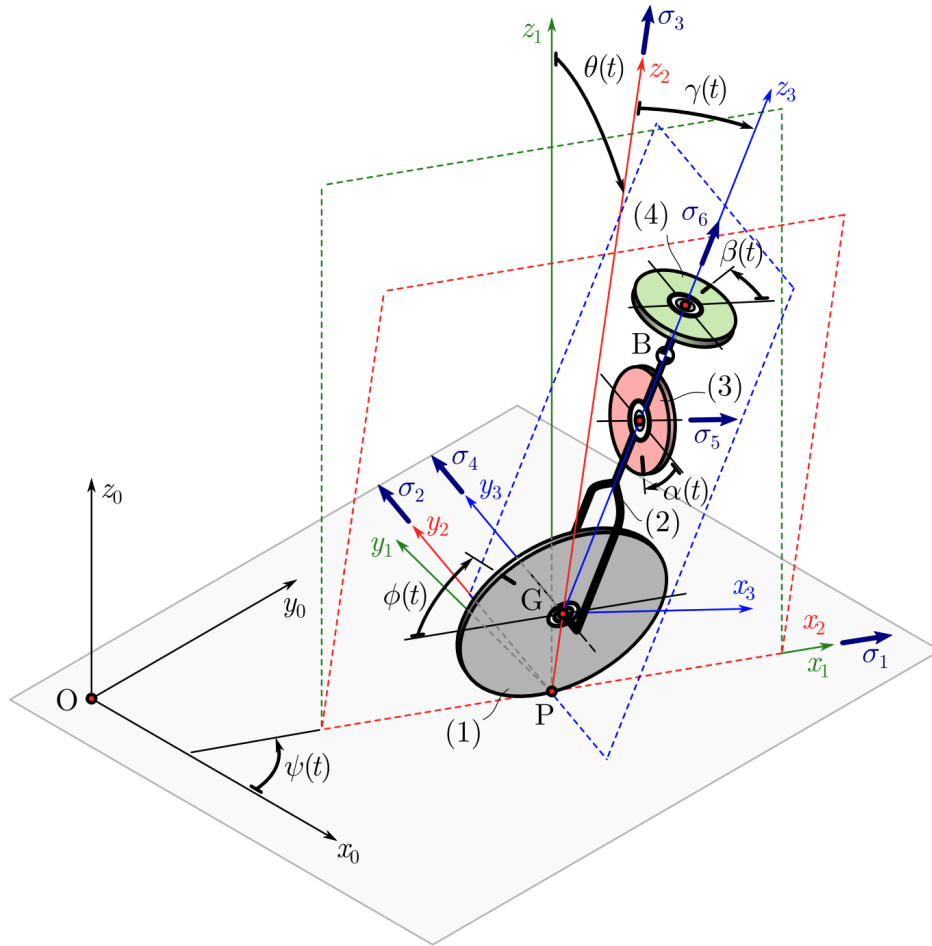


Figure 2: Model of the unicycle

This project directly utilized the unicycle dynamics and model from the paper by Xincheng Cao[1]. A summary of the dynamics and model are provided below, however readers interested in the details

should read the referenced paper.

2.1 Unicycle Model

The unicycle was modelled as a wheel, frame and two control wheels. The wheel and the two control wheels are placed such that their axis are all orthogonal to each other. The red wheel is the balancing wheel, denoted as b , and the green wheel is the steering wheel, denoted as s . The model, frames and variables can all be seen on figure 2. On the model O is the center of the world frame 0, while G is the center of the body frames 1,2 and 3. It is important to note that the two control wheels in green and red are both located on the center of mass of the body B ; they are only separated on the illustration for better readability.

The model utilizes $z-x-y$ Euler angles, with each angle denoted as ψ, θ, γ respectively. This means frame 1 is frame 0 rotated on the z -axis by ψ , frame 2 is frame 1 rotated on the x -axis by θ and frame 3 is frame 2 rotated on the y -axis by γ .

The location of the wheel center G is denoted as x_G, y_G . The rotation angle of the wheel about the y_2 -axis is denoted as ϕ , while the angle of rotation of the balancing and steering wheels are α and β respectively.

The unicycle parameters are described in table 1.

Parameter	Description
m_w	mass of rolling wheel
R	radius of rolling wheel
m	mass of body
h	distance between wheel center point and the center of mass of the body
J_x, J_y, J_z	principal mass moment of inertias of the body
m_b	mass of balancing wheel
r_b	radius of balancing wheel
m_s	mass of steering wheel
r_s	radius of steering wheel

Table 1: Unicycle system parameters

The unicycle receives three inputs: an internal driving torque M_w between the rolling wheel and the body, the lateral balancing torque M_b between the balancing flywheel and the body, and the steering torque M_s applied between the steering flywheel and the body.

2.2 Unicycle Dynamics

The dynamic equations were constructed using the Appellian approach. This approach necessitates the use of pseudovelocities, six of them for this model. All of the pseudovelocities are derived from angular velocities.

The main wheel's angular velocity of the body in the 2-frame is the following

$$\omega = \begin{bmatrix} \dot{\theta} \\ \dot{\gamma} + \dot{\psi} \sin(\theta) \\ \dot{\psi} \cos(\theta) \end{bmatrix}_2 \quad (1)$$

The angular velocity of the balancing wheel in the 3-frame is

$$\omega_b = \begin{bmatrix} \dot{\alpha} + \dot{\theta} \cos(\gamma) - \dot{\psi} \cos(\theta) \sin(\gamma) \\ \dot{\gamma} + \dot{\psi} \sin(\theta) \\ \dot{\theta} \sin(\gamma) + \dot{\psi} \cos(\theta) \cos(\gamma) \end{bmatrix}_3 \quad (2)$$

The angular velocity of the steering wheel in the 3-frame is

$$\omega_s = \begin{bmatrix} \dot{\theta} \cos(\gamma) - \dot{\psi} \cos(\theta) \sin(\gamma) \\ \dot{\gamma} + \dot{\psi} \sin(\theta) \\ \dot{\beta} + \dot{\theta} \sin(\gamma) + \dot{\psi} \cos(\theta) \cos(\gamma) \end{bmatrix}_3 \quad (3)$$

The six pseudovelocities are then chosen as the three components of ω , the second component of ω_b , the first component of ω_b and the third component of ω_s . In other words,

$$\begin{aligned} \sigma_1 &:= \dot{\theta} \\ \sigma_2 &:= \dot{\phi} + \dot{\psi} \sin(\theta) \\ \sigma_3 &:= \dot{\psi} \cos(\theta) \\ \sigma_4 &:= \dot{\gamma} + \dot{\psi} \sin(\theta) \\ \sigma_5 &:= \dot{\alpha} + \dot{\theta} \cos(\gamma) - \dot{\psi} \cos(\theta) \sin(\gamma) \\ \sigma_6 &:= \dot{\beta} + \dot{\theta} \sin(\gamma) + \dot{\psi} \cos(\theta) \cos(\gamma) \end{aligned} \quad (4)$$

With the pseudovelocities set, the dynamics of the rest of the variables $\theta, \gamma, \psi, x_G, y_G, \phi, \alpha, \beta$ can be

written as the following.

$$\begin{aligned}
\dot{\theta} &= \sigma_1 \\
\dot{\gamma} &= \sigma_4 - \sigma_3 \tan(\theta) \\
\dot{\psi} &= \frac{\sigma_3}{\cos(\theta)} \\
\dot{x}_G &= \sigma_2 R \cos(\psi) + \sigma_1 R \sin(\psi) \cos(\theta) \\
\dot{y}_G &= \sigma_2 R \sin(\psi) - \sigma_1 R \cos(\psi) \cos(\theta) \\
\dot{\psi} &= \sigma_2 - \sigma_3 \tan(\theta) \\
\dot{\alpha} &= \sigma_5 - \sigma_1 \cos(\gamma) + \sigma_3 \sin(\gamma) \\
\dot{\beta} &= \sigma_5 - \sigma_1 \sin(\gamma) - \sigma_3 \cos(\gamma)
\end{aligned} \tag{5}$$

The dynamics of the pseudovelocities are considerably more complex. In fact it consists of thousands of terms and therefore won't be written in its entirety here. Instead, the Matlab program for this report includes the code used to generate f_1 to f_4 , which is in itself is also from [1].

$$\begin{aligned}
\dot{\sigma}_1 &= f_1(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \theta, \gamma, M_b, M_s) \\
\dot{\sigma}_2 &= f_2(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \theta, \gamma, M_w) \\
\dot{\sigma}_3 &= f_3(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \theta, \gamma, M_b, M_s) \\
\dot{\sigma}_4 &= f_4(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \theta, \gamma, M_w) \\
\dot{\sigma}_5 &= \frac{2M_b}{m_b r_b^2} \\
\dot{\sigma}_6 &= \frac{2M_s}{m_s r_s^2}
\end{aligned} \tag{6}$$

In conclusion, the unicycle is a highly non-linear system with 14 state variables and 3 inputs. As in

$$X := [\sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \theta \ \gamma \ \psi \ x_G \ y_G \ \phi \ \alpha \ \beta]^T \tag{7}$$

$$u := [M_w \ M_b \ M_s]^T \tag{8}$$

2.3 Linearized Dynamics

Clearly designing a controller using the actual plant dynamics is highly impractical due to the complexity. Instead the dynamics can be linearized around an equilibrium point. In fact there exists an equilibrium point for zero input $u^* = [0\ 0\ 0]^T$,

$$X^* = [0 \ \frac{v^*}{R} \ 0 \ 0 \ \omega_b^* \ \omega_s^* \ 0 \ 0 \ 0 \ v^*t \ 0 \ \frac{v^*t}{R} \ \omega_b^*t \ \omega_s^*t]^T \quad (9)$$

The variables that are to be controlled here are the three body Euler angles, meaning $y = [\theta \ \gamma \ \psi]$.

For simplicity and from the fact there is no good reason to be spinning the control wheels at a constant angular speed, the control wheel angular velocity will be set to zero as in $\omega_b^* = \omega_s^* = 0$. Around this equilibrium point the dynamics can be linearized as

$$\begin{aligned} \dot{\tilde{X}} &= A\tilde{X} + Bu \\ y &= CX \end{aligned} \quad (10)$$

where

$$A = \begin{bmatrix} 0 & 0 & A_{13} & 0 & 0 & 0 & A_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{28} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{48} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

$$B = \begin{bmatrix} 0 & B_{12} & 0 \\ B_{21} & 0 & 0 \\ 0 & 0 & B_{33} \\ B_{41} & 0 & 0 \\ 0 & \frac{2}{m_b r_b^2} & 0 \\ 0 & 0 & \frac{2}{m_s r_s^2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (13)$$

The definition of the variables such as A_{13} can be found in the appendix of the reference[1].

The parameters of the system will need to be set to a real number in order to execute a simulation.

They were set to be the same as the original reference paper values given in the table below.

Parameter	Value	Parameter	Value
m_w	2kg	R	0.15m
m	3kg	h	0.3m
J_x	$0.1kg\,m^2$	J_y	$0.1kg\,m^2$
J_z	$0.02kg\,m^2$	m_b	1kg
r_b	0.15m	m_s	1kg
r_s	0.15m	v^*	3m/s

Table 2: Unicycle system parameter values

3 Controller Design

The overall control scheme will closely follow any controller that utilizes the DOB. Figure 3 shows what that control scheme looks like. An inner loop DOB will work to make sure the response y to the nominal input u_n will be the same as the nominal plant. In this case, the nominal plant dynamics will be the linearized plant dynamics while the actual plant will follow the real, non-linear plant dynamics. Then an outer loop controller, designed using the nominal plant dynamics, will give the nominal input u_n based on y feedback. In this paper, this will be a PD controller.

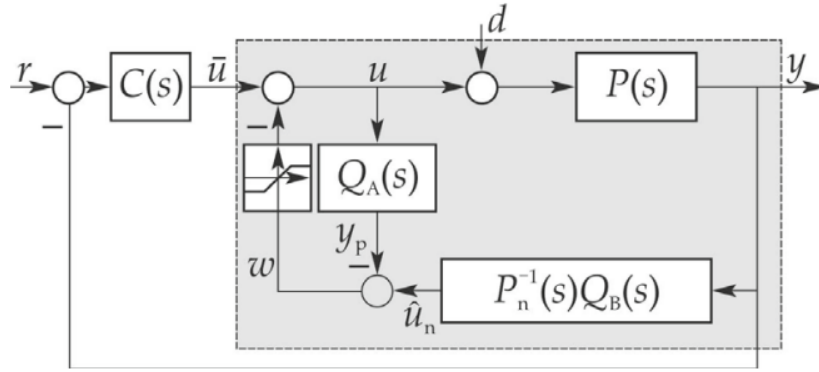


Figure 3: DOB Block Diagram[2]

3.1 Nominal Plant Controller

The nominal plant controller is a PD feedback controller similar to the one used in the unicycle paper[1]. This means that the control law is the following.

$$\begin{aligned}
 M_w &= p_w \gamma + d_w \dot{\gamma} \\
 M_b &= p_b \theta + d_b \dot{\theta} \\
 M_s &= p_s \psi + d_s \dot{\psi}
 \end{aligned} \tag{14}$$

Since $\dot{\theta} = \sigma_1, \dot{\gamma} = \sigma_4, \dot{\psi} = \sigma_3$, this PD controller is ultimately also a state feedback controller in the form of $u_n = KX$. Therefore by analyzing the eigenvalues of $A + BK$ the stability of the controlled nominal system can be determined.

An important characteristic of this system should be noted here. The linearized system is in fact not

controllable; the rank of the controllability matrix is only 11. The system is however, stabilizable since using an appropriate PD controller the eigenvalues of the controllability matrix can be set to either be 0 or have negative real parts.

3.2 The Disturbance Observer

The controller above has been designed based on a linearized nominal plant dynamics. The real plant dynamics, as discussed in section 2, is highly non-linear. Therefore, a disturbance observer will now be employed to make sure the controller designed for the nominal linearized plant works correctly on the actual plant. An additional benefit of the DOB is that it will allow for disturbance rejection.

Before starting any DOB design, the state space will have to be reorganized into the normal form. Here, the normal form will be defined as the following. This normal form is based on the linearized dynamics, with the states being split into $X_n \in \mathbb{R}^6, Z_n \in \mathbb{R}^8$.

$$\begin{aligned}
y_1 &= x_1 = \theta \\
\dot{x}_1 &= \sigma_1 = x_2 \\
\dot{x}_2 &= A_{17}x_1 + A_{13}x_6 + B_{12}u_2 \\
y_2 &= x_3 = \gamma \\
\dot{x}_3 &= \sigma_4 = x_4 \\
\dot{x}_4 &= A_{48}x_3 + B_{41}u_1 \\
y_3 &= x_5 = \psi \\
\dot{x}_5 &= \sigma_3 = x_6 \\
\dot{x}_6 &= A_{31}x_2 + B_{33}u_3 \\
Z_n &= [\sigma_2 \ \sigma_5 \ \sigma_6 \ x_G \ y_G \ \phi \ \alpha \ \beta]^T \\
\dot{Z}_n &= h(X_n, Z_n, u)
\end{aligned} \tag{15}$$

An important point to note about this DOB implementation is that it is MIMO. Creating a MIMO DOB is possible as demonstrated by Back et al[3]. However, this paper did not directly utilize the method outlined in that paper due to assumption 3 at section 2. In short, assumption 3 requires finding

the range of values the dynamics of the real plant can have. This assumption is similar to finding the maximum value of β_l . With the complexity of the actual plant, this was deemed an impractical approach.

Instead, the fact that the Q-filter for the MIMO DOB estimated each y independently was observed. This implied that creating 3 DOBs that were designed similarly to a SISO DOB could be utilized. This possibility was further reinforced by the fact that each input was used only once in the dynamics and used separately from each other. In terms of the notations used in [3], this meant \bar{G} was already nearly diagonal, requiring only one elementary row swap to be fully diagonal. Specifically in this case

$$\bar{G} = \begin{bmatrix} 0 & B_{12} & 0 \\ B_{41} & 0 & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \quad (16)$$

With all this in mind, a DOB similar to a SISO DOB described in the DOB tutorial was made[2]. The overall structure of the DOB is the same as any DOB with a Q-filter and an inverse nominal plant dynamics block, as shown in figure 3. The Q-filter was designed to be exactly the same as the one described in the MIMO DOB paper[3], meaning

$$\begin{aligned} A_{a\tau} &= \text{blockdiag} \left\{ \begin{bmatrix} 0 & 1 \\ \frac{-a_{10}}{\tau^2} & \frac{-a_{11}}{\tau} \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ \frac{-a_{20}}{\tau^2} & \frac{-a_{21}}{\tau} \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ \frac{-a_{30}}{\tau^2} & \frac{-a_{31}}{\tau} \end{bmatrix} \right\} \\ B_{a\tau} &= \text{blockdiag} \left\{ \begin{bmatrix} 0 \\ \frac{a_{10}}{\tau^2} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{a_{20}}{\tau^2} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{a_{30}}{\tau^2} \end{bmatrix} \right\} \end{aligned} \quad (17)$$

With these gains the Q-filter dynamics can be written as $\dot{p} = A_{a\tau}p + B_{a\tau}u$ and $\dot{q} = A_{a\tau}q + B_{a\tau}y$ for Q_A and Q_B respectively.

Knowing that the Q-filter acts like a high gain observer, this will mean that $q(t) \rightarrow X_n(t)$, and it will do so quite quickly if τ is reduced. Like all Q-filters there is a stability requirement for a_{i0}, a_{i1} , the specifics of which are written in the MIMO DOB paper[3]. As each individual normal form has a relative degree of two, the requirement that the polynomial $s + a_{i1}$ be Hurwitz just means $a_{i1} > 0$. As for a_{i0} , for the sake of simplicity only the fact that the stability requirement can be met by reducing a_{i0} was noted.

Next the inverse nominal plant dynamics block P_n^{-1} was designed. Here careful attention was paid to the structure of the system dynamics. From the DOB tutorial, it is clear that this block stems from the

last differential equation in the normal form, as in the equation $\dot{x}_\nu = f(x, z) + \beta_l(u + d)$. Since this system has 3 inputs and outputs, there are three of these equations as well. Here the unique characteristics of this system greatly simplifies this block. The first characteristic is the fact that \bar{G} is nearly diagonal, which means the differential equations can be reorganized about a single u_i . The second characteristic is that the zero dynamics have no affect on the dynamics of the rest of the state variables X_n . This means that the inverse nominal plant dynamics are the following.

$$\begin{aligned}\nabla_1 &= \frac{1}{B_{12}} \left(\frac{-a_{20}}{\tau^2} q_3 + \frac{-a_{21}}{\tau} q_4 + \frac{a_{20}}{\tau^2} y_2 - A_{48} q_3 \right) \\ \nabla_2 &= \frac{1}{B_{41}} \left(\frac{-a_{10}}{\tau^2} q_1 + \frac{-a_{11}}{\tau} q_2 + \frac{a_{10}}{\tau^2} y_1 - A_{17} q_1 - A_{13} q_6 \right) \\ \nabla_3 &= \frac{1}{B_{33}} \left(\frac{-a_{30}}{\tau^2} q_5 + \frac{-a_{31}}{\tau} q_6 + \frac{a_{30}}{\tau^2} y_3 - A_{31} q_2 \right)\end{aligned}\tag{18}$$

Combining all of the above blocks, the final input into the real plant becomes the following.

$$\begin{aligned}u_1 &= u_{n1} - \nabla_1 + p_1 \\ u_2 &= u_{n2} - \nabla_2 + p_3 \\ u_3 &= u_{n3} - \nabla_3 + p_5\end{aligned}\tag{19}$$

Finally, a saturation block is added to create robust transient behaviour and to avoid peaking of the output variables.

4 Simulation

The simulation of this system was done using Matlab and Simulink. The overall Simulink system looks like figure 4. The system parameter values were the ones outlined in table 2.

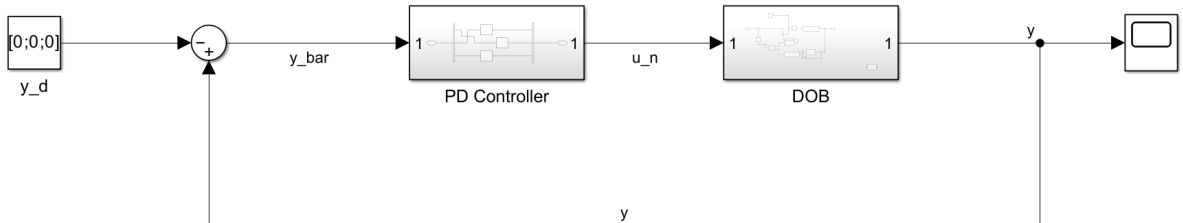


Figure 4: Overall Simulink System

The PD controller was implemented as figure 5.

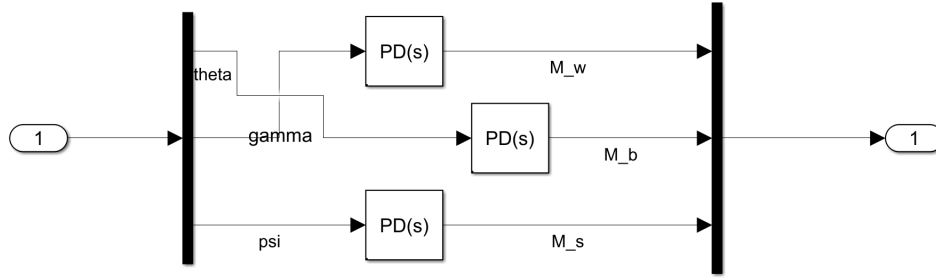


Figure 5: PD Controller in Simulink

The PD controller gain was set as the following values.

$$\begin{aligned}
 p_w &= 15 \quad d_w = 5 \\
 p_b &= 30 \quad d_b = 10 \\
 p_s &= 15 \quad d_s = 5
 \end{aligned} \tag{20}$$

With those gains, the overall controlled nominal system has the eigenvalues -131.8, -9.07, -2.66, -0.54, -1.77, -38.4 with the other 8 being zero. A further inspection using the just the normal form and just observing the dynamics of X_n confirms that these are the eigenvalues for X_n . Therefore, under the nominal plant dynamics X_n will converge to zero and the zero dynamics will remain neutral.

The DOB was implemented as figure 6.

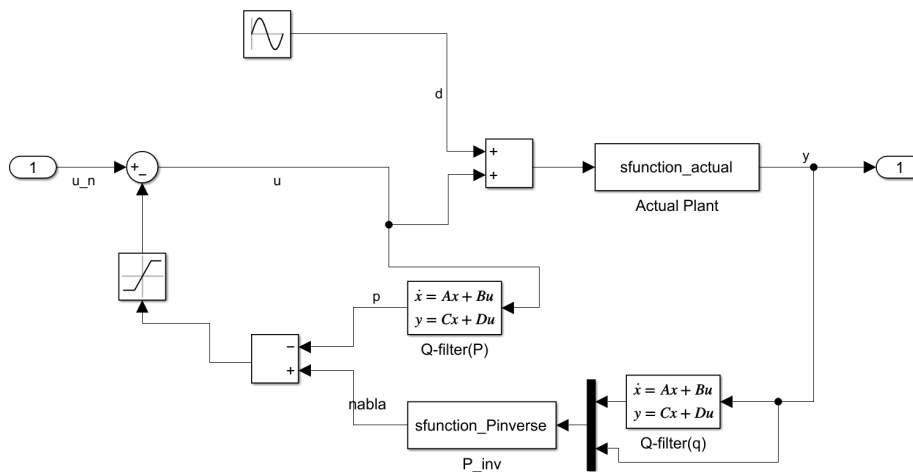


Figure 6: DOB in Simulink

Because the conditions the DOB gains should meet was not specifically calculated, it was instead

found using a trial and error method. This was especially more critical as the values intended to control, Y , consisted of Euler angles and their value could not realistically exceed $\pi/2 = 1.571$. The system dynamics described in equation 6 contain cosines in the denominator and therefore if any of the Euler angles were to hit $\pi/2$, the system dynamics would likely diverge. After much trial and error, the DOB gains were set to the following values.

$$\begin{aligned}\tau &= 0.001 \\ a_{10} &= a_{20} = a_{30} = 0.01 \\ a_{11} &= a_{21} = a_{31} = 20\end{aligned}\tag{21}$$

The saturation block had a magnitude limit of 200 and a sinusoidal disturbance of amplitude 1 and frequency 100 was added for all 3 inputs.

With these values the system with DOB was tested using an initial value of $Y = [1 \ 1 \ 1]^T$. It was then compared to a controller without a DOB. Figure 7 depicts the response of the system with the DOB while figure 8 depicts the response of the system without one.

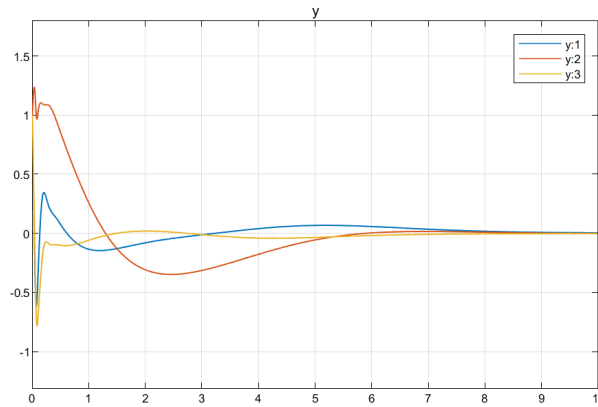


Figure 7: System with DOB Response

The overall performance of the PD controller was good despite the disturbance and the variation in plant dynamics at the initial Y value. However, the DOB had done a better job reducing the peak values, especially for y_3 . A smaller peak is generally better since a higher peak means leaning closer to the floor.

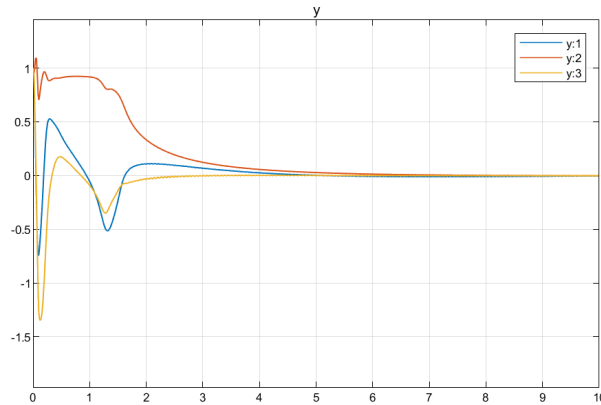


Figure 8: System without DOB Response

5 Conclusion

The objective given to the students for the final project was to apply the principles learned in class. This final project aimed to implement the DOB into a realistic and complex system and chose the unicycle as the candidate system. This system was previously controlled using a PD controller which had been designed with the linearized plant dynamics. A DOB was therefore implemented to make the system resistant to external disturbance and to compensate for the behaviour difference between the linearized dynamics and the real one. The DOB was successfully implemented and its performance was confirmed. However, the improvements in performance were marginal due to the already great performance of the PD controller. Having derivative terms is not ideal, and finding a slightly worse controller that does not utilize it would present a better use case for the DOB here. Also the DOB coefficients were not specifically proven to be stable, and there could be more optimal gains. All of these should be considered for future research.

References

- [1] Xincheng Cao et al. “Autonomous unicycle: modeling, dynamics, and control”. In: *Multibody System Dynamics* (2023), pp. 1–34.
- [2] Hyungbo Shim et al. “Yet another tutorial of disturbance observer: robust stabilization and recovery of nominal performance”. In: *Control Theory and Technology* (2016).
- [3] Juhoon Back et al. “An Inner-Loop Controller Guaranteeing Robust Transient Performance for Uncertain MIMO Nonlinear Systems”. In: *IEEE Transactions on Automatic Control* (2009).