1. **Question 1**:

   Proof: I negate this statement and show the new one is true, which implies the original one is false. The negated statement is as follows.

   $$(\forall m \in N)(\forall n \in N)(3m + 5n \neq 12)$$

   I delineate every case of m and n.

   If $m = 1$ then $5n \neq 9$, which is true as $5|9$ is false.

   If $m = 2$ then $5n \neq 6$, which is true as $5|6$ is false.

   If $m \geq 3$ then $12 - 3m \leq 3$. $5|12 - 3m$ is false which is equivalent to $5n \neq 12 - 3m$.

   In every case of m and n I showed $3m + 5n \neq 12$ is true. Thus, the negated statement is true which is equivalent to saying the original statement, $(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$, is false. Q.E.D.

2. **Question 2**:

   Proof: The statement can be expressed as below.

   $$(\forall a \in Z)(\exists q \in Z)[(a-2) + (a-1) + (a) + (a+1) + (a+2) = 5q]$$

   $(a-2) + (a-1) + (a) + (a+1) + (a+2) = 5q$ is equivalent to $5a = 5q$. Then the statement is the same as saying

   $$(\forall a \in Z)(\exists q \in Z)[a = q]$$

   which is true because I can find at least one q for every possible a. Q.E.D.

3. **Question 3**:

   Proof: I show every possible case of n and prove this statement is true.

   $n^2 + n + 1 = n(n+1) + 1$ (By algebra.)

   For every integer $n > 0$, $n(n+1)$ yields even number and adding 1 to it leads to odd number.

   For every integer $n \leq -2$, $n(n+1)$ yields even number and adding 1 to it leads to odd number.

   For integer $n = \{-1, 0\}$, $n = -1$ yields 1 and $n = 0$ yields 1.

   For every case of integer, $n^2 + n + 1$ yields odd number. Q.E.D.

4. **Question 4**:

   Proof: I logically express the statement as below.

   $$(\forall k \in N)(\exists n \in Z)[2k - 1 \Rightarrow (4n + 1|2k - 1 \vee 4n + 3|2k - 1)]$$

   In order to prove this statement is true, either $4n + 1|2k - 1$ or $4n + 3|2k - 1$ has to be true given $2k - 1$ is true.

   Suppose $2k - 1$ is true. Then for all natural number k, we can always find an integer $2n$ to equal either $k - 1$ or $k - 2$. (e.g. if $k = 3$, then $k - 1 = 2$, $k - 2 = 1$. Then, $n = 1$ makes $2n = 2$ which equals $k - 1 = 2$.)

Because $2n$ always equals either $k-1$ or $k-2$ for all k, then either $4n+1|2k-1$ or $4n+3|2k-1$ always becomes true: i.e. if $2n=k-1$ then $4n+1$ becomes $2k-1$, which makes $4n+1|2k-1$ true. if $2n=k-2$ then $4n+3$ becomes $2k-1$, which makes $4n+3|2k-1$ true.

Consequence is true for every natural number as I can find an adequate integer n. Thus, the implication is true. Q.E.D.

5. **Question 5**:

   Proof: I consider every possible case to prove the statement.

   Every integer n is one of $3q, 3q+1, 3q+2$ where $q \in Z$.

   If $n=3q$, then $3|n$. n is divisible by 3.

   If $n=3q+1$, then $n+2=3q+3$. $n+2$ is divisible by 3.

   If $n=3q+2$, then $n+4=3q+6$. $n+4$ is divisible by 3.

   For every possible n, I showed that at least $n, n+2, n+4$ is divisible by 3. Q.E.D.

6. **Question 6**:

   Proof: For all natural number n, I pick three numbers where each is 2 from the next: e.g. {1,3,5}, {2,4,6}. Under this setting, every possible set can be expressed as $(\forall x \in N)(\{x, x+2, x+4\})$.

   If $x>3$, then at least one element in every set is divisible by 3. Thus, there are no prime triple sets in $x>3$.

   If $x \leq 3$, then the remaining sets are {1, 3, 5}, {2, 4, 6}, {3, 5, 7}. The only prime triple among these sets is {3, 5, 7}. Q.E.D.

7. **Question 7**:

   Proof: I use mathematical induction to prove the statement.

   Let $A(n)$ be $\sum_{i=1}^{n} 2^i = 2^{n+1}-2$.

   $A(1)$ is $2 = 2^{1+1}-2$ which is true. Initial step is finished.

   We move onto the Induction step. I must show $(\forall n \in N)(A(n) \Rightarrow A(n+1))$ is true.

   $\sum_{i=1}^{n} 2^i = 2^{n+1}-2$, by the induction hypothesis
   $\sum_{i=1}^{n} 2^i + 2^{n+1} = 2^{n+1}-2+2^{n+1}$, by algebra
   $\sum_{i=1}^{n+1} 2^i = 2 \cdot 2^{n+1}-2 = 2^{n+2}-2$, by rearrangement
   Given $A(n)$ holds I showed $A(n+1)$, which is $\sum_{i=1}^{n+1} 2^i = 2^{n+2}-2$, is true. By the principle of mathematical induction, the statement is true. Q.E.D.

8. **Question 8**:

   Proof: I can logically express the statement as below.

   $$(\forall M>0)(\lim_{n\to\infty} a_n \to L \Rightarrow \lim_{n\to\infty} Ma_n \to ML)$$

   To show this implication is true, I look at the consequent first and see if it can be derived from antecedent.

   The consequent part is equivalent to saying

$$(\forall \epsilon > 0)(\exists n \in N)(\forall m \geq n)[|Ma_m - ML| < \epsilon]$$

Because $M > 0$, $|Ma_m - ML| = |M||a_m - L| = M|a_m - L|$. Then I can simplify the consequent part as below.

$$(\forall \epsilon > 0)(\exists n \in N)(\forall m \geq n)[|a_m - L| < \frac{\epsilon}{M}]$$

If antecedent is true, it means for every $\epsilon > 0$ we can find an n such that $|a_m - L| < \epsilon$. Let $\epsilon' = \frac{\epsilon}{M}$. ϵ' is one of a kind of ϵ. Given the antecedent is true, then finding n such that $|a_m - L| < \epsilon'$ is also true.

As a result, if antecedent is true then consequent is true. Q.E.D.

9. **Question 9**:

   Proof: Let the interval A_n be $(-\frac{1}{n}, \frac{1}{n})$. Then, $(\forall n \in N)(A_n \supset A_{n+1})$ is true.

   Also, $\bigcap_{n=1}^{\infty} A_n = \emptyset$ is true because the interval $A_\infty = (-\frac{1}{\infty}, \frac{1}{\infty}) = (0, 0) = \{x | 0 < x < 0\}$ is an empty set. Because A_∞ is an empty set, it must be that $\bigcap_{n=1}^{\infty} A_n = \emptyset$.

   Thus, the interval $A_n = (-\frac{1}{n}, \frac{1}{n})$ satisfies both properties. Q.E.D.

10. **Question 10**:

    Proof: Let the interval A_n be $[1, 1 + \frac{1}{n}]$. Then, $(\forall n \in N)(A_n \supset A_{n+1})$ is true.

    Also, $\bigcap_{n=1}^{\infty} A_n = 1$ is true because the interval $A_\infty = [1, 1 + \frac{1}{\infty}] = [1, 1] = \{x | 1 \leq x \leq 1\}$ is a set with one element.

    Thus, the interval $A_n = [1, 1 + \frac{1}{n}]$ satisfies both properties. Q.E.D.