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The Victorian Lock-down Criteria Philomaths Technical Note - TN8

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1 Introduction

On August 2, the Victorian Government declared [1] a state of disaster and tightened lock-down conditions in the state of NSW, Australia. The Government published a four step road map [2] for emerging from this lock-down, together with quantitative criteria for the decision as to when to emerge from each stage of the lock-down. Currently the state is at step 3, which has onerous lock-down provisions. The criteria for moving from this step are as follows:

- Daily average number of cases in the last 14 days is less than 5 (statewide) AND
- Less than 5 cases with an unknown source in the last 14 days (statewide total)

As at October 8, 2020, the status [3] in regard to these criteria is a 14 day average of 10.1 cases and over the past 14 days a total of 10 cases with an unknown source.

The first criterion is based on a relatively large sample of data, i.e a daily average of 5 cases over fourteen days represents about 70 samples. However, the second criterion is based on a very small number of samples. This paper analyses the statistical properties of the second criterion, which we will refer to as the *sources criterion*. A full analysis would use a complex stochastic model of the epidemic, but we believe that considerable insight can be gained by using a much simpler approach.

2 Stationary Error Analysis

Cases with an unknown source are of particular concern to those managing an epidemic using a suppression or elimination strategy. Cases with an unknown source provides an indication that there may be other individuals infected with the disease who are not under management (either quarantine or self-isolation), but who are freely moving around the community. We will call such individuals, *Free Infected Individuals (FIIs)*. The issue with a FII is that the person can cause a chain of infection that can infect many other individuals before the outbreak has been detected.

If we assume that cases with an unknown source are detected by random testing, and each detection is independent of other detections, then the number of detections in a particular period could be modeled by a Poisson distribution

$$P(k \text{ events in interval } T) = \frac{(rt)e^{-(rT)}}{k!} \tag{1}$$

where r is the number of detections per day. For the second lock-down criterion, the value of T is 14 days, and the lock-down sources criterion is satisfied if the number of detections in that period is less than 5.

From this, we can gain an idea of the probability of an error of a single application of the lock-down formula. Suppose the mean of 14 day sum of unknown sources count has dropped to 4. If the actual fourteen day count is 5 or greater, the sources criterion will not be satisfied, and so the test would incorrectly leave Melbourne in lock-down. The probability of this happening is

$$P_e = \Sigma_{k=5}^{\infty} \frac{(rT)e^{-(rT)}}{k!} \tag{2}$$

As an example, if rT is 4, then $P_e = 0.37$, i.e. over a 37% error, so the test has a high error rate. This is somewhat mitigated by the fact it is applied on a daily basis. The effect of an error, on any single day, will be delay lock-down by one day. In the next section we will look at the performance of the sources criterion when applied daily.

In this simple analysis, the probability of a single test detecting a FII is

$$p_d = \frac{n_f}{N} \tag{3}$$

where n_f is the number of FIIS and N is the total population.

If there are n_t tests per day, then the average number of detections in a fourteen day period will be given by

$$n_u = 14 p_d n_t = 14 n_t \frac{n_f}{N} \tag{4}$$

From this equation, we can develop an estimator for n_f based on the counts of n_u, i.e.

$$\widehat{n_f} = \frac{N}{14 n_t} E(n_u) \tag{5}$$

Assuming that the distribution of n_u is Poisson, then the distribution of $\widehat{n_f}$ will also be Poisson and its variance will equal its mean.

From this equation and the sources criterion, we gain a rough idea of the desired upper limit to the number of FIIs. The first step of the lock-down occurred on September 13, 2020. At that time the number of daily tests were in the vicinity of 15,000 [4]. The population of Victoria is about 7 million, and the cut-off for n_u is 5. Substituting these numbers into equation 5 yields 167 FIIs.

Equation 4, explicitly demonstrates the proportionality of n_u to n_t, which puts the Victorian government in the unenviable position of making it harder to leave the lock-down if they increase the testing rate, n_t.

3 Time Varying Analysis

We can predict course of the epidemic using the SIR model [5],

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t) I(t) \\ \frac{dI(t)}{dt} = \beta S(t) I(t) - \alpha I(t) \\ \frac{dR(t)}{dt} = \alpha I(t) \end{cases}$$

where $S(t)$, $I(t)$, and $R(t)$ are the number of susceptible, infected and recovered individuals, at time t. These are a deterministic set of equations, whereas the actual progress of the epidemic will be stochastic, so using a deterministic model actually reduces the amount of variation. The initial conditions will be denoted as S_0 I_0, and R_0.

The initial conditions assume that all detected individuals have been removed from the system, so I_0 only includes the FIIs. The initial condition for I_0 will be around 300. The total cases in Victoria is of the order of 20 thousand, so with a population of the order of 7 million, to a very good approximation we can treat $S(t)$ as a constant in the second equation of the SIR model, so that equation becomes

$$\frac{dI(t)}{dt} = \beta S_0 I(t) - \alpha I(t) \tag{6}$$

We have that

$$\mathcal{R}_L = \frac{\beta S_0}{\alpha} \tag{7}$$

where $\mathcal{R}_{\mathcal{L}}$ is the reproduction number during the lock-down. The value of $\mathcal{R}_{\mathcal{L}}$ will be estimated from the actual case numbers, so effectively takes into account the process of free infected individuals being detected and placed into quarantine and isolation, so the evolution of $I(t)$ will continue to represent only FIIs.

Substituting equation 7 into equation 3 gives

$$\frac{dI(t)}{dt} = \alpha(\mathcal{R}_L - 1)I(t) \tag{8}$$

The solution this equation is

$$I(t) = I_0 e^{\alpha(\mathcal{R}_L - 1)t} \tag{9}$$

This equation allows us to simulate the progress of the epidemic in terms of the number of FIIs (I(t)). We can predict the daily mean number of detections, n_u, using the equation 4, where n_f is set equal to $I(t)$. We model the actual number of detections as a Poisson distribution, with a mean arrival rate per day equal to n_u. We calculate the 14 day total of the number of detections and compare this with the lock-down sources criterion of 5. We carry out many independent runs, each time recording the time the lockdown was exited, i.e. how many days it takes to exit the lock-down.

In this simulation, we used a value of $\mathcal{R}_L$ of 0.69. This was derived by averaging the $\mathcal{R}$ estimated by the epistim R package run on the Victoria case data for the period of the September. In order to give sufficient time to eliminate artifacts from the startup of the two weekly filter, we set the start day and the value of I_0 such that the first day the value of $I(t)$ was less than 167 was on day 0. The value of 167 is the number we calculated above for the number of FIIS that would yield a fourteen day count of 5. In other words, the simulation was set up so that with perfect knowledge, the lock-down would be exited at day 0.

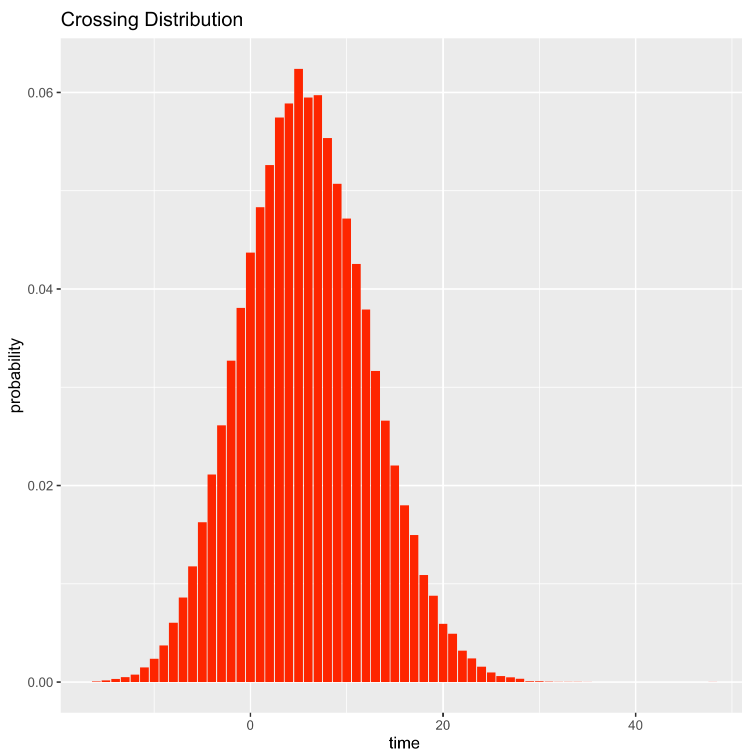
Figure 3 shows the probability distribution for the day the lock-down is lifted. This was based on 100,000 samples. Any exits prior to day zero are errors, in the sense that an exit is being made prior to the source criterion being satisfied. It can be seen there is a large spread. The standard deviation is 6.5 days, and there is a 5% probability that the lock-down could continue for 17 days more than necessary, and a 1% probability that the lock-down could continue for 21 days more than necessary. The delay is partly due to the averaging filter, but the lengthy delays are caused by the random fluctuations inherent in basing a decision on such a small number of counts.

The above calculations are not a prediction of the distribution of lock-down exit times, a much more complex model would be needed to make such a prediction. However, it is clear that because of the extremely low number of detections used in the lock-down sources criterion, there is a high probabilty that the lockdown could continue for a much longer period than is necessary.

As well, this longer period does not necessarily imply a greater chance of avoiding a third lockdown. The fundamental problem for the suppression model is that serious breaches in compliance are most likely highly connected, whereas the modeling and detection processes are not. It only takes non-compliance in a high-rise apartment or a close knit church community to create potentially unmanageable outbreak. The current level of testing is does not necessarily pick up this problem.

For example, the lock-down sources criterion will be passed when there are only four cases of unknown source in a fortnight. Suppose that one quarter of the FIIs at that time are in a highly connected group, assuming the testing is done randomly, the probability that all four detected cases do not come from that group is $(\frac{3}{4})^4$ is 0.32, so there is a 32% chance this highly connected group is not detected. One way to find such a group early is by a very large increase in the testing rate, but this would make it much harder to pass the lock-down sources criterion.

Figure 1: CDF of Threshold Crossing



4 Discussion

In summary, the current lock-down sources criterion is ill-conceived. In particular

- The sources criterion discourages increasing the testing rate.
- The sources criterion is likely to unnecessarily extend the period of the lock-down
- The current testing sources criterion and testing regime is not a particularly effective tool for detecting highly connected outbreak.

The above analysis is based on the random fluctuations on the unknown sources detections. An analysis by Chang et al [6] indicate high level of compliance are needed for successful control of an epidemic. If the criterion is highly sensitive to random fluctuations, it will also be highly sensitive to detections caused by deliberate non-compliance. As the lock-down continues, non-compliance is likely to increase. Slavish adherence to the sources-criterion may mean that the lock-down continues till a vaccine has been distributed.

More sophisticated modeling could be used to verify these assertions. However, we think that most people with experience in modeling random processes would agree it is extra-ordinary that the fate of 5 million people is based four or less detections. The sources-criterion is *ill-conceived*. Give both criteria have to be satisfied, then the criteria are ill-conceived. Surely there are other lines of evidence that can be fused with human judgment to end the the Victorian lock-down as soon as possible.

A Resources

The resources for this technical note are available for access at https://github.com/philomaths-org/covid-19. The conditional_victoria folder contains the pdf for this paper and relevant code. You can access resources for earlier versions of this note on Github by clicking on the tag corresponding to the earlier technical note's version number.

References

[1] Wikipedia, "Covid-19 pandemic in australia." https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Australia.

[2] V. Government, "Coronavirus (covid-19) roadmap for reopening."

[3] D. of Health and H. S. Victoria, "Coronavirus (covid-19) case averages for easing restrictions," October.

[4] C. L. website, "Vic positive test rate."

[5] M. Martcheva, *An introduction to mathematical epidemiology.* Springer, 2015.

[6] S. L. Chang, N. Harding, C. Zachreson, O. M. Cliff, and M. Prokopenko, "Modelling transmission and control of the covid-19 pandemic in australia," *arXiv preprint arXiv:2003.10218*, 2020.