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Effect of Social Distancing Non-compliance on an Epidemic's Trajectory and Hospital Utilization.

Dr. Chris Drane
Philomaths Pty. Limited
chris@philomaths.org

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1 Introduction

A recent study [1] shows that rigorous compliance can be important to the effectiveness of a social distancing strategy in controlling an epidemic.

This paper looks at the effects on an epidemic's trajectory of non-compliance with social distancing guidelines. It first examines the effect of non-compliance amongst the general population and then examines the effects of non-compliance by a small, vulnerable subset of the population. It then examines the effect of non-compliance on the Intensive Care Unit (ICU) utilization.

2 Social Distancing

During the Covid-19 pandemic, social distancing measures have been used by many countries to slow or squash the progress of the pandemic. These social distancing measures can include requiring everyone to;

- maintain a minimum distance when in public.
- only move about for essential purposes such as work, shopping, exercise, compassionate reasons.

- a strict limit on the number of people who can meet at once in public spaces.
- requiring at risk individuals (such as those over 70 years old) to self isolate.

There can also be the closing of businesses that are not essential e.g. cafes, restaurants, pubs, pools, and gyms. Many countries enforce these restrictions with large fines and even prison sentences. As well, contact tracing and early detection can also reduce the spread a disease.

We can gain insight into the effect of non-compliance by using the Kermack and McKendrick SIR model [2] of epidemic progression. This model uses three simultaneous non-linear differential equations:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t)I(t) \\ \frac{dI(t)}{dt} = \beta S(t)I(t) - \alpha I(t) \\ \frac{dR(t)}{dt} = \alpha I(t) \end{cases}$$

where $S(t)$, $I(t)$, and $R(t)$ are the number of susceptible, infected and recovered individuals, at time t. The parameter β is the per capita rate of infection and α is the per capita rate that individuals are removed from the pool of the infected.

The critical parameter for these equations is R_0, the basic reproduction number, given by

$$R_0 = \frac{\beta S_0}{\alpha} \tag{1}$$

where S_0 is the fraction of susceptible individuals at $t = 0$.

Suppose that social distancing and other methods are used to reduce the reproduction number, and if everyone complied with these measures the reproduction number would be R_c. Assume that only a fraction of the population, p_c comply with these measures, and the fraction that do not comply increase the reproduction number by δR. So the mean reproduction number would be

$$R_m = p_c R_c + (1 - p_c)(R_c + \delta R) \tag{2}$$

or

$$R_m = p_c R_c + (1 - p_c)\delta R. \tag{3}$$

In the early stages of the epidemic, the growth in the fraction of infections, $I(t)$ is an exponential with a growth rate of r. The SIR model implicitly specifies the generational interval as having an exponential distribution, so the relationship between the R_m and r is given by equation 3.1 of Wallinga and Lipsitch [3]

$$R_m = 1 + rT_c \tag{4}$$

where T_c is the mean generation interval, so we have

$$r = \frac{R_m - 1}{T_c} \tag{5}$$

The doubling time, τ_d is given by

$$\frac{log(2)}{log(1+r)} \tag{6}$$

Now the exponential trajectory of I(t) will have the form e^{kt} where k is a constant. The relationship between k and the growth rate of the exponential, r is given by

$$k = ln(1+r). \tag{7}$$

For small r this can be approximated by

$$k \approx r \tag{8}$$

so

$$k \approx \frac{(R_m - 1)}{T_c} \tag{9}$$

Consider the situation outlined in Chang et. al. [1] where the policy makers are endeavoring to suppress the epidemic. In that case R_c will be less than one. The Chang paper explores an scenario, shown vividly in Figure 2c [1], where p_c is 0.7 (in Chang's paper 'SD compliance=0.7'), and that causes a diverging trajectory for the epidemic. In our terms, this means that the reproduction number corresponding to Changs scenario is R_m, and the value of R_m in this case is greater than 1. From the 'SD compliance=0.7' line in Chang's Figure 2d, we can estimate the value of r in this case as approximately 0.02 cases per day. Chang assumes the value of T_c is 6.4 days, so using equation 4 we can estimate that R_m is 1.13. Even though this value is close to one, it will still result in an exponential rise in cases, so non-compliance has a very heavy cost.

Using this analysis, we can also understand the case where policy makers are not trying to suppress the epidemic but prepared to accept a value of R_c

above one. We can see the effect of not all citizens complying by looking at the ratio of exponentials for the cases of $R = R_m$ and $R = R_c$.

$$\frac{e^{k_m t}}{e^{k_c t}} = e^{(k_m - k_c)t} \tag{10}$$

or using equations 4 and 3

$$\frac{e^{k_m t}}{e^{k_c t}} \approx e^{(1-p_c)\delta R t}. \tag{11}$$

Given that $(1 - p_c)\delta R > 0$, it can be seen that the infections in the population with non-compliant individual grows exponentially faster then the fully compliant population. This demonstrates why only a small percentage of non-compliant individuals can cause social distancing measures to have little effect. This results in the need to heavily police the social distancing if a voluntary regime does not result in high enough compliance.

This analysis can be extended to generation intervals with distributions other than exponential. Wallinga and Lipsitch [3] show that for an arbitrary generation interval distribution the relationship between R and r is given by

$$R_m = \frac{1}{M(-r)} \tag{12}$$

where $M(z)$ is the moment generating function of the distribution.

Now consider the case of where a fraction of the population, p_v, is asked to isolate, and assume that p_{cv} of the vulnerable population comply with that requirement. Assume further that if a vulnerable person isolates then they have no chance of infection. Under these assumptions, the compliant vulnerable fraction of the population, $p_v p_{cv}$ is not part of the Susceptible population, effectively they are in another 'country'. Accordingly, in analyzing the trajectory of the population, the compliant vulnerable fraction can be ignored. The non-compliant vulnerable section of the population, $p_v p_{cv}$, will add to the pool of susceptible individuals, but assuming that their behaviour is similar to the rest of the population, their reproduction number will be R_m. This means that their participation does not change the exponential trajectory, i.e. it is still $e^{k_m t}$, they just increase the pool of susceptible individuals. In other words, non-compliance by the vulnerable part of the population only has an additive effect not an exponential effect.

This means, from the viewpoint of the trajectory of an epidemic, that a government does not need to be nearly as strict with enforcing isolation of an vulnerable subset of the population, because non-compliance does not cause an exponential acceleration of the epidemic. However, it does have an effect on the ICU utilization, and that is discussed in the next section.

3 ICU Utilization

Assume that only the vulnerable members of the population will require ICU care if they contract the disease, and the probability of a vulnerable person needing ICU care is p_{icu}. An important consideration is to ensure that there are sufficient ICU beds during the course of the epidemic. Suppose there are N people in the population and the number of ICU beds is N_{icu} and due to social distancing constraints, the value of R is R_m.

In the absence of isolating the vulnerable, the constraint of ensuring sufficient ICU beds can be written as

$$I(t)p_v p_{icu} \leq N_{icu} \tag{13}$$

Denoting the maximum value of $I(t)$ as I_{max}, we have that the number of beds need is

$$N_{icu} = I_{max} p_v p_{icu} \tag{14}$$

Using equation 2.6 of Martcheva [2], the maximum value of I(t) will be

$$I_{max} = \frac{\alpha}{\beta}(log(\frac{\alpha}{\beta}) - 1 - log(S_0)) + S_0 + I_0 \tag{15}$$

or

$$I_{max} = \frac{\alpha}{\beta}(log(\frac{\alpha}{\beta S_0}) - 1) + S_0 + I_0 \tag{16}$$

so using equation 1

$$I_{max} = \frac{S_0}{R}(log(\frac{1}{R}) - 1) + S_0 + I_0 \tag{17}$$

As check on this, if $R = 1$, then as expected, $I_{max} = I_0$, i.e the epidemic does not grow beyond the initial number.

Now consider a situation where the population is social distancing to give an R of R_m, and that a fraction p_v are vulnerable, and that R_m is significantly above 1. In this case, if the population is large, we can neglect I_0 and have that $S_0 = N$.

$$N_{icu} = p_v p_{icu} \frac{N}{R}(log(\frac{1}{R}) - 1). \tag{18}$$

On the other hand if all the vulnerable members of the population self-isolate, under the assumptions only vulnerable people need hospitalization, $S_0 = N(1 - p_v)$ and there will be no need for ICU beds, i.e. $N_{icu} = 0$.

More realistically, suppose that only p_{cv} of the vulnerable comply with the self-isolation requirement, and the ones that do not comply behave like the rest of the population, so the R remains at R_m. The number in the population will be $S_0 = N(1 - p_v p_{cv})$, and the number in the population who are vulnerable will be $Np_v(1 - p_{cv})$. Accordingly, N_{icu} becomes

$$N'_{icu} = \frac{p_v(1 - p_{cv})}{(1 - p_v p_{cv})} p_{icu} \frac{N(1 - p_v p_{cv})}{R}(log(\frac{1}{R}) - 1). \tag{19}$$

So if ratio of of N'_{icu} to N_{icu} is

$$\frac{\frac{p_v(1-p_{cv})}{(1-p_v p_{cv})} p_{icu} \frac{N(1-p_v p_{cv})}{R}(log(\frac{1}{R}) - 1)}{p_v p_{icu} \frac{N}{R}(log(\frac{1}{R}) - 1)} \tag{20}$$

which simplifies to

$$(1 - p_{cv}). \tag{21}$$

As an example, if 80% of the vulnerable popululation complying with the self-isolation request then approximately 20% of the ICU beds will be needed compared to the case where there is no self-isolation.

Equation 21 is the intuitive result one would expect, but note that it only applies in the case where R is large and I_0 is much smaller than S_0.

4 Conclusion

The effect of non-compliance by the self-isolated vulnerable population does not have the same exponential effect that can result from non-compliance with social distancing by the general population. As well, the effect of non-compliance by those self-isolating only has a proportionate effect on the number of required hospital beds.

Actual policy decisions would benefit from far more complicated analysis that is provided here, but the analysis might provide useful insights.

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References

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