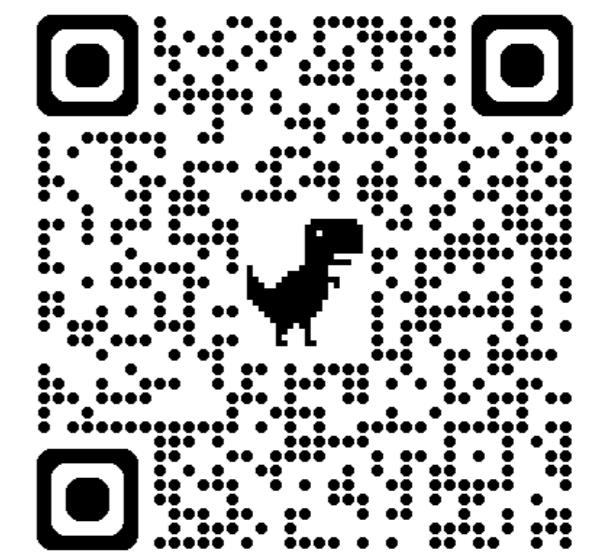


Periodic Skill Discovery

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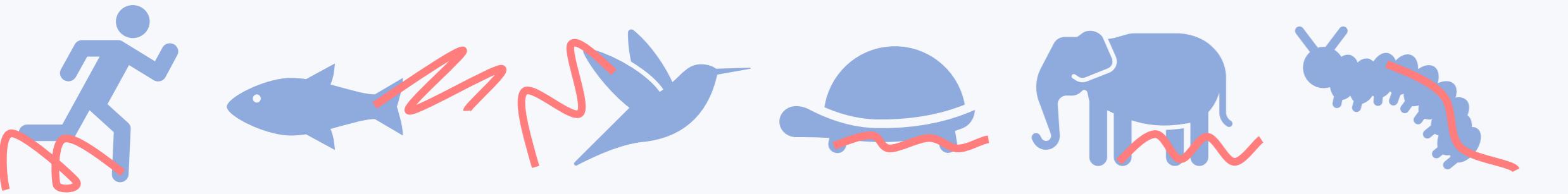


Project Page
(Demos & Code)

Why Periodic Skills Matter?

A fundamental observation in nature is that nearly all forms of locomotion are inherently periodic.

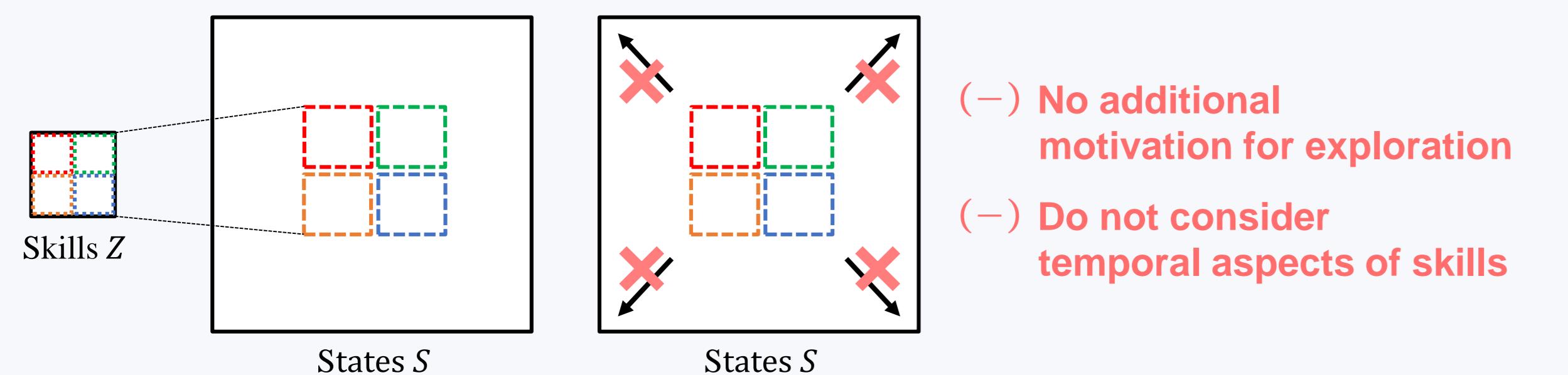
However, existing unsupervised skill discovery methods have rarely addressed the role of periodicity.



Prior works often fail to learn multi-timescale behaviors

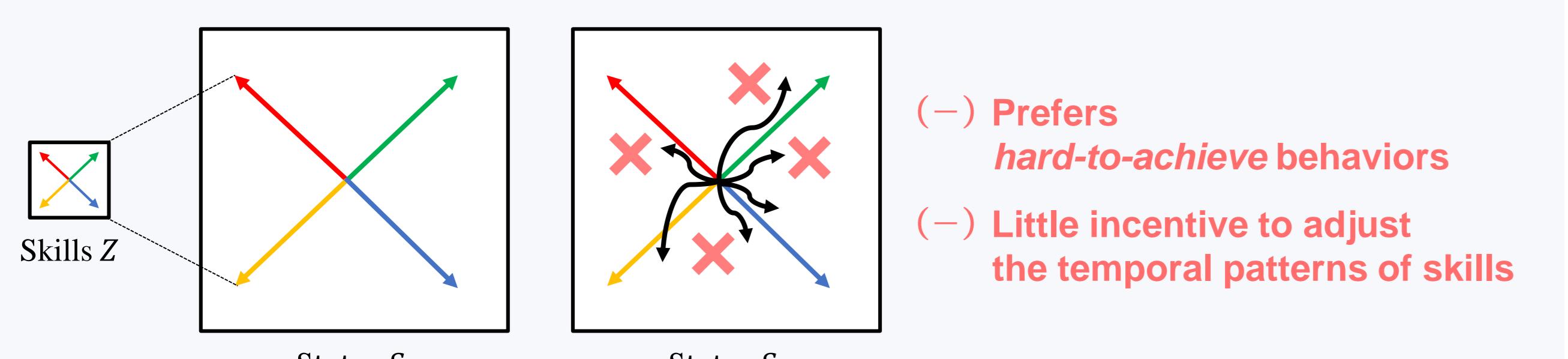
(1) Mutual Information (MI) - based skill discovery (e.g., DIAYN, DADS)

$$I(S; Z) = -H(Z|S) + H(Z) = \mathbb{E}_{z,\tau}[\log p(z|s)] - \mathbb{E}_z[\log p(z)] \\ \geq \mathbb{E}_{z,\tau}[\log q_\theta(z|s)] + (\text{constant}) \simeq \mathbb{E}_{z,\tau}\left[-\frac{1}{2\sigma^2}\|z - \mu_\theta(s)\|_2^2\right] + (\text{constant})$$



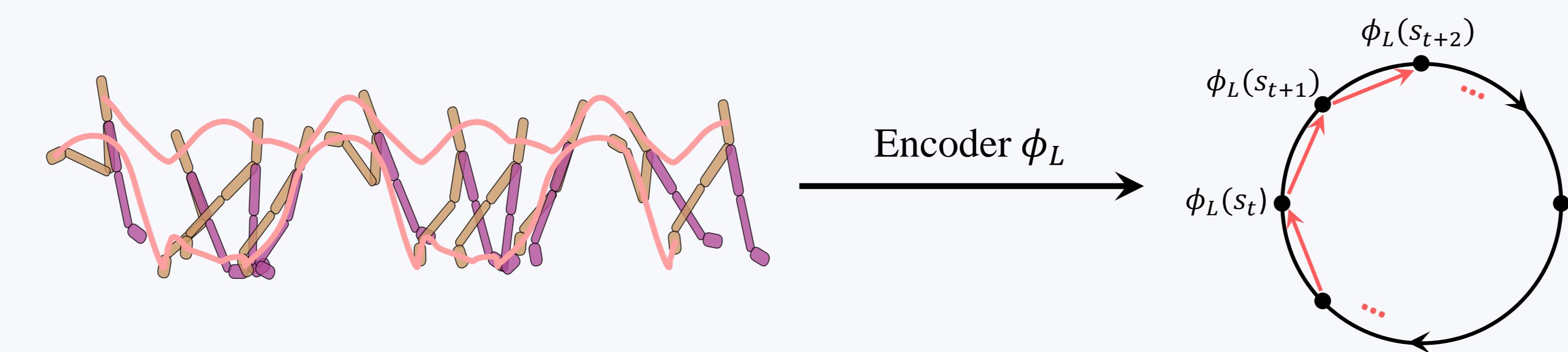
(2) Distance - maximizing skill discovery (e.g., LSD, CSD, METRA, LGSD)

$$\mathcal{J}_{DSD} := \mathbb{E}_{(z,\tau) \sim \mathcal{D}}[(\phi(s_{t+1}) - \phi(s_t))^\top z] \quad \text{s.t.} \quad \|\phi(x) - \phi(y)\| \leq d(x, y) \quad \forall x, y \in \mathcal{D}$$



Intuition

- We introduce Periodic Skill Discovery (PSD)
- Key intuition: Construct a **circular latent space** for periodic behavior.



Representations for Periodicity

- Construct a **regular $2L$ -gon** inscribed in a circle of diameter L .

$$\mathcal{J}_{PSD, \phi} := \mathbb{E}_{p(\tau, L)} \left[\|\phi_L(s_{t+L}) - \phi_L(s_t)\|_2 - k \|\phi_L(s_{t+L}) + \phi_L(s_t)\|_2 \right] \\ \text{s.t.} \quad \|\phi_L(s_{t+L}) - \phi_L(s_t)\|_2 \leq L, \\ \|\phi_L(s_{t+1}) - \phi_L(s_t)\|_2 \leq L \sin(\pi/2L)$$

Single-step Intrinsic Reward for the policy $\pi(a | s, L)$

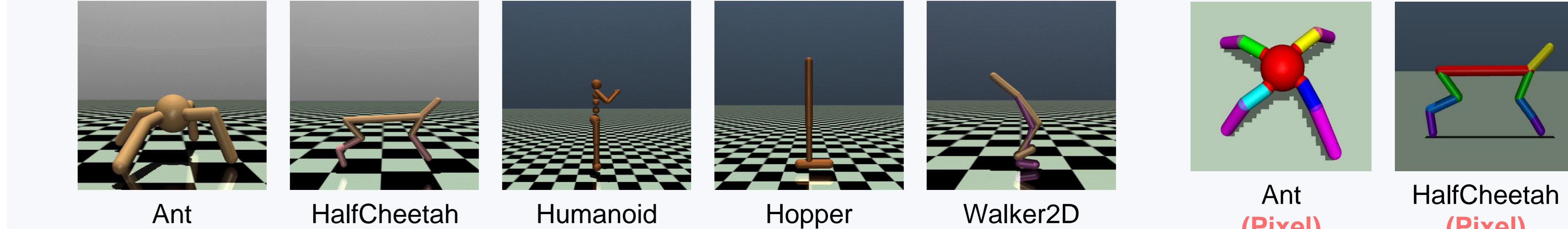
- Define a single-step intrinsic reward encouraging $2L$ -periodicity.

$$\Delta := \|\phi_L(s_{t+1}) - \phi_L(s_t)\|_2 - L \sin(\pi/2L) \quad r_{PSD}(s_t, s_{t+1}, L) := \exp(-\kappa \Delta^2)$$

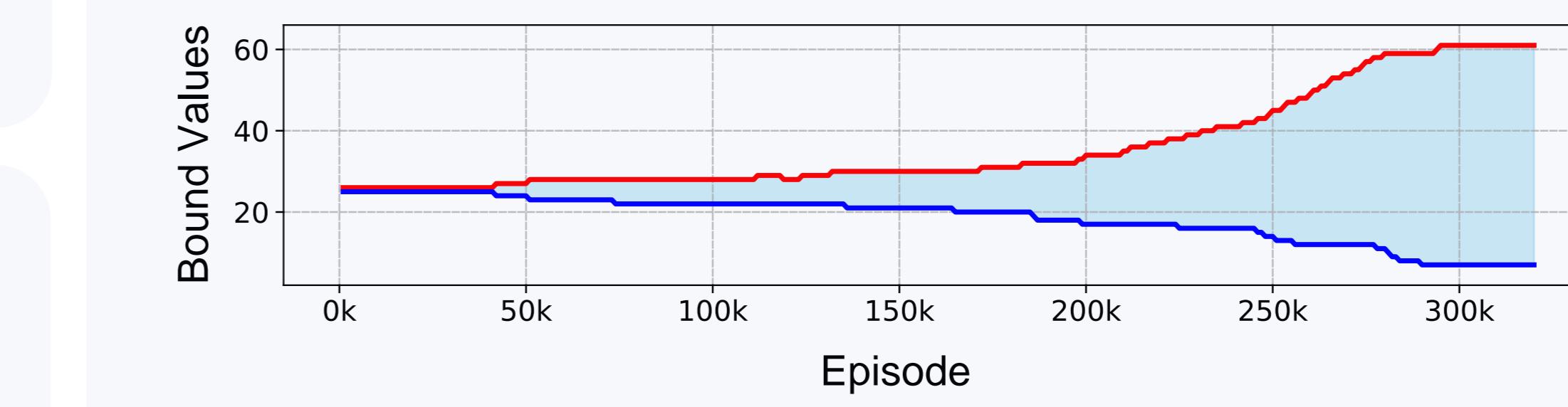
Adaptive Sampling Method

- To enable the agent to discover a **maximally diverse** range of periods without **any prior knowledge** of its inherent period ranges, we introduce an adaptive sampling method that **dynamically adjusts** the sampling range during training.
- Key idea: Evaluate the policy's performance on the **boundary** of the current sampling range, using $\sum_{t=0}^{T-1} r_{PSD}(L_{bound})$ as the evaluation criterion.

Benchmark Environments

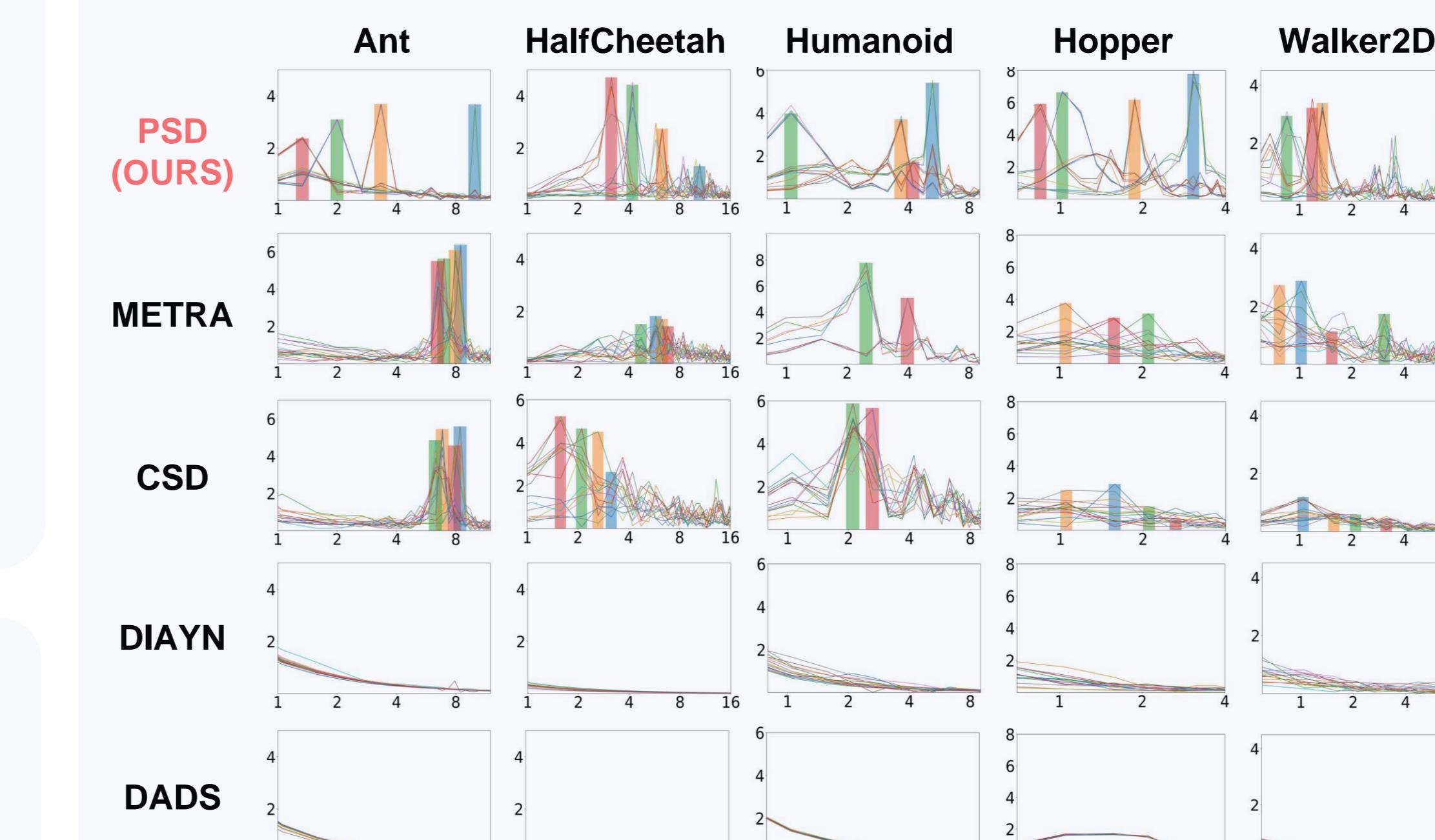


Learned Sampling Bounds Over Time



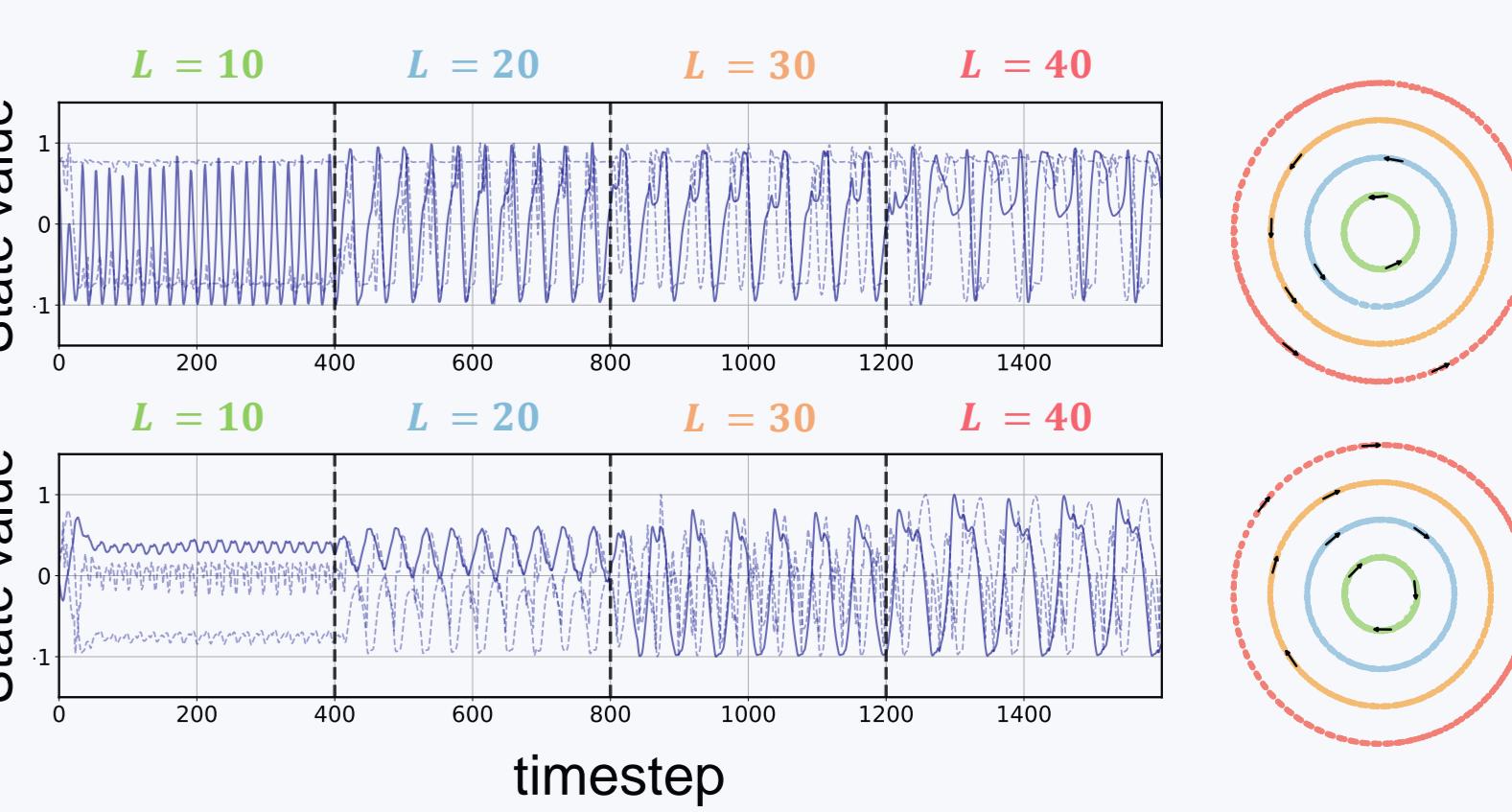
As training progresses, **increasingly challenging periods are proposed** to the agent, enabling the discovery of a wider range of periodic behaviors.

Frequency-Domain Skill Comparison



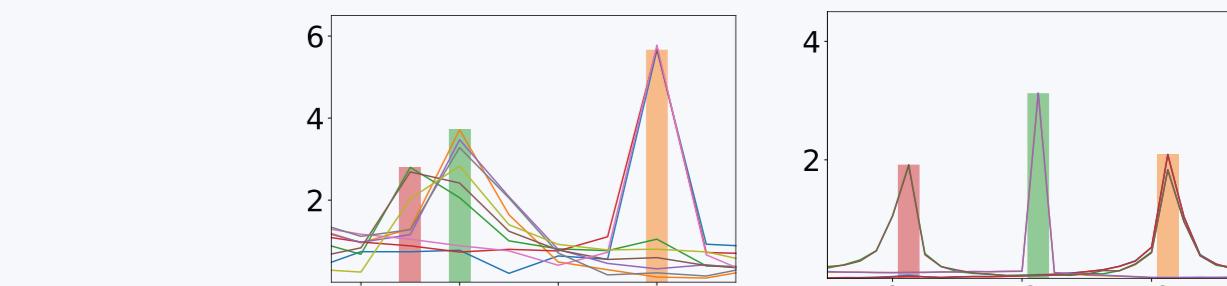
We apply a **Fourier transform** to the skill trajectories (frequency on the x-axis, amplitude on the y-axis). PSD consistently discovers a wider range of frequencies than the baselines.

State Trajectories & Latents (PCA)



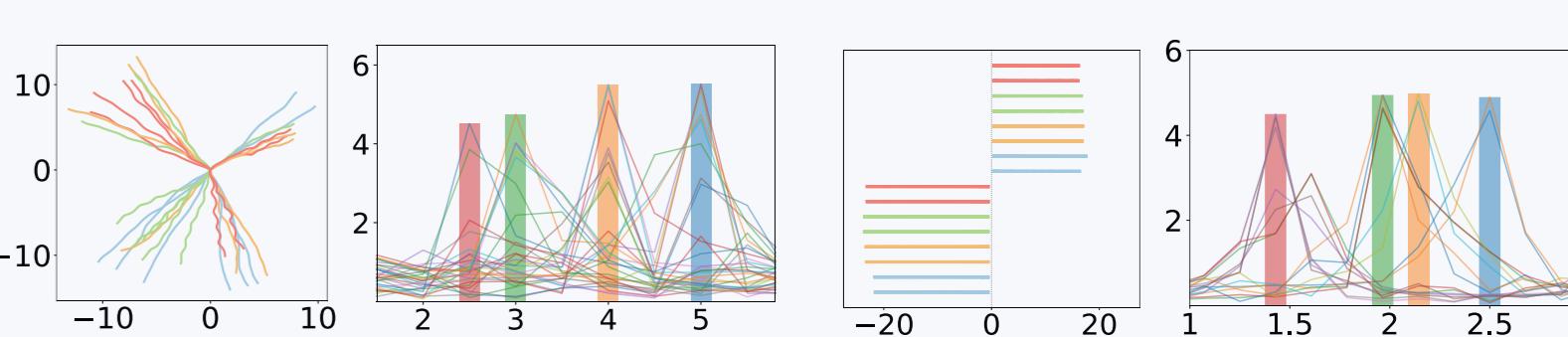
For varying values of L , PSD successfully constructs a **circular latent space** whose diameter is L , and learns behaviors with the desired period of $2L$.

PSD with Pixel-based Observations



PSD can discover periodic skills even in pixel-based observation. (see our demo)

PSD combined with METRA (Park et al., 2023)



PSD naturally aligns with METRA, as both methods capture temporal structure of skills. (see our paper for details)