

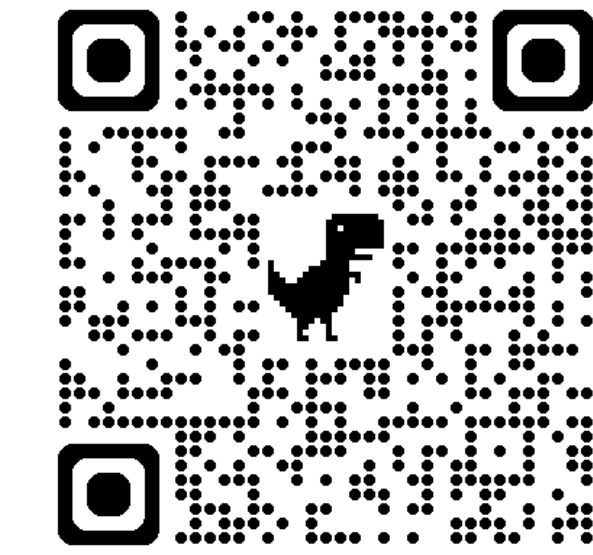
# Periodic Skill Discovery

Jonghae Park<sup>1</sup> Daesol Cho<sup>2</sup> Jusuk Lee<sup>1</sup> Dongseok Shim<sup>1</sup> Inkyu Jang<sup>1</sup> H. Jin Kim<sup>1</sup>

<sup>1</sup>Seoul National University <sup>2</sup>Georgia Institute of Technology



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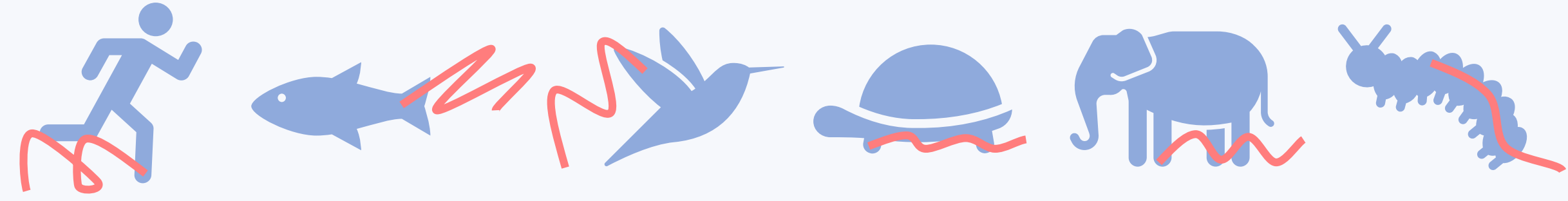


**Project Page**  
(Demos & Code)

## Why Periodic Skills Matter?

A fundamental observation in nature is that nearly all forms of locomotion are inherently periodic.

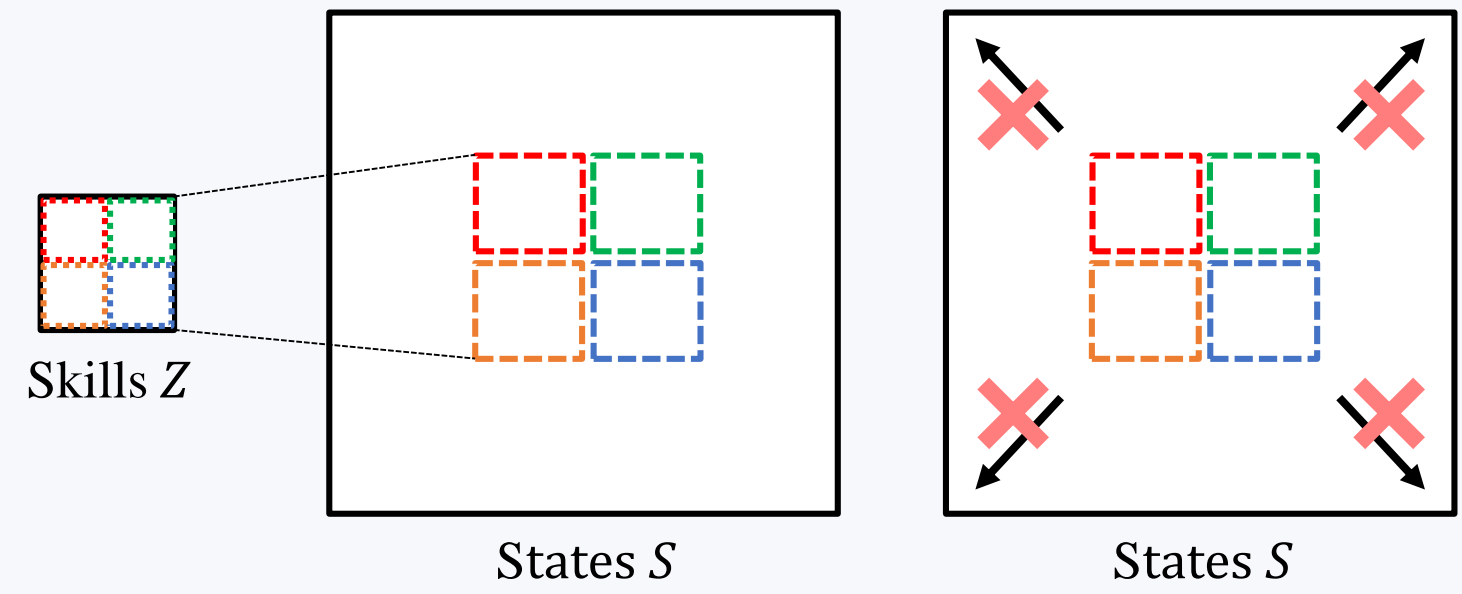
However, existing unsupervised skill discovery methods have rarely addressed **the role of periodicity**.



## Prior works often fail to learn *multi-timescale* behaviors

(1) **Mutual Information (MI)** - based skill discovery (e.g., DIAYN, DADS)

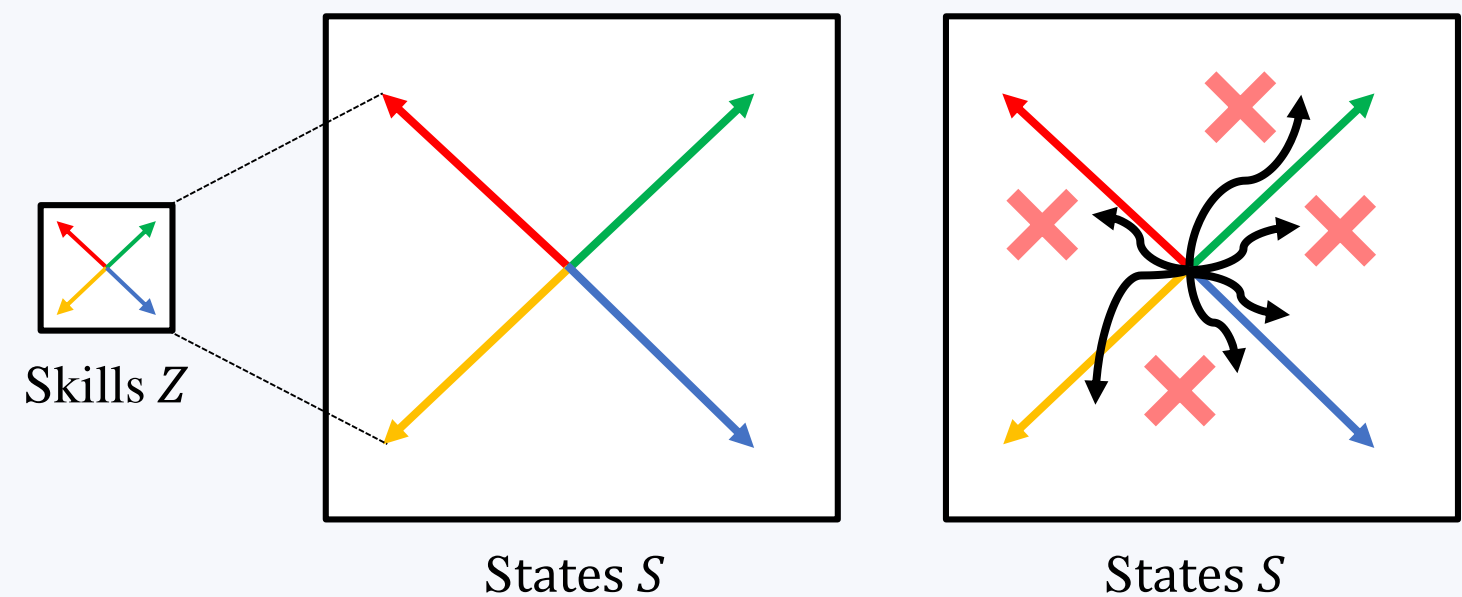
$$I(S; Z) = -H(Z|S) + H(Z) = \mathbb{E}_{z,\tau}[\log p(z|s)] - \mathbb{E}_z[\log p(z)] \\ \geq \mathbb{E}_{z,\tau}[\log q_\theta(z|s)] + (\text{constant}) \simeq \mathbb{E}_{z,\tau} \left[ -\frac{1}{2\sigma^2} \|z - \mu_\theta(s)\|_2^2 \right] + (\text{constant})$$



- (-) No additional motivation for exploration
- (-) Do not consider temporal aspects of skills

(2) **Distance - maximizing** skill discovery (e.g., LSD, CSD, METRA, LGSD)

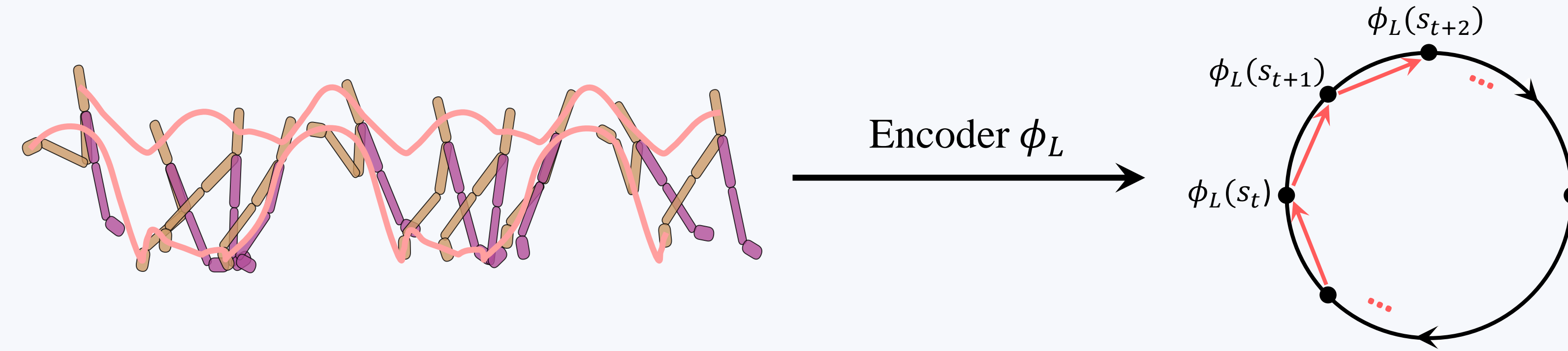
$$\mathcal{J}_{\text{DSD}} := \mathbb{E}_{(z,\tau) \sim \mathcal{D}} \left[ (\phi(s_{t+1}) - \phi(s_t))^T z \right] \quad \text{s.t.} \quad \|\phi(x) - \phi(y)\| \leq d(x, y) \quad \forall x, y \in \mathcal{D}$$



- (-) Prefers *hard-to-achieve* behaviors
- (-) Little incentive to adjust the temporal patterns of skills

## Intuition

- We introduce **Periodic Skill Discovery (PSD)**
- Key intuition: Construct a **circular latent space** for periodic behavior.



## Representations for Periodicity

- Construct a **regular 2L-gon** inscribed in a circle of **diameter L**.

$$\mathcal{J}_{\text{PSD}, \phi} := \mathbb{E}_{p(\tau, L)} \left[ \|\phi_L(s_{t+L}) - \phi_L(s_t)\|_2 - k \|\phi_L(s_{t+L}) + \phi_L(s_t)\|_2 \right] \\ \text{s.t.} \quad \|\phi_L(s_{t+L}) - \phi_L(s_t)\|_2 \leq L, \\ \|\phi_L(s_{t+1}) - \phi_L(s_t)\|_2 \leq L \sin(\pi/2L)$$

## Single-step Intrinsic Reward for the policy $\pi(a | s, L)$

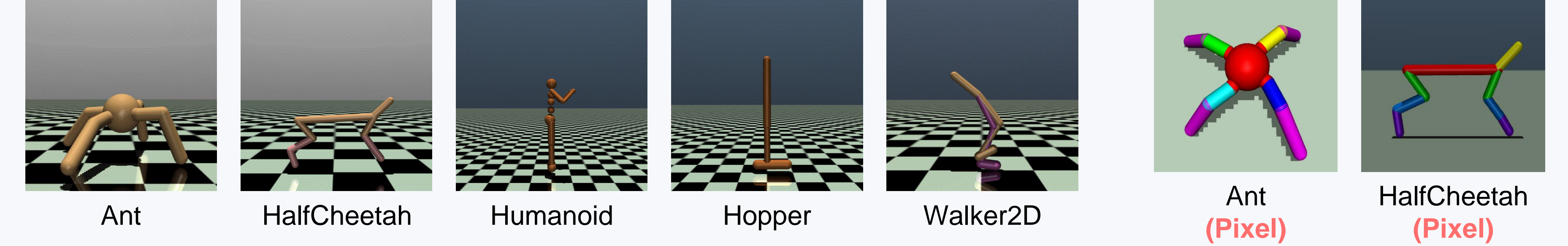
- Define a single-step intrinsic reward encouraging **2L-periodicity**.

$$\Delta := \|\phi_L(s_{t+1}) - \phi_L(s_t)\|_2 - L \sin(\pi/2L) \quad r_{\text{PSD}}(s_t, s_{t+1}, L) := \exp(-\kappa \Delta^2)$$

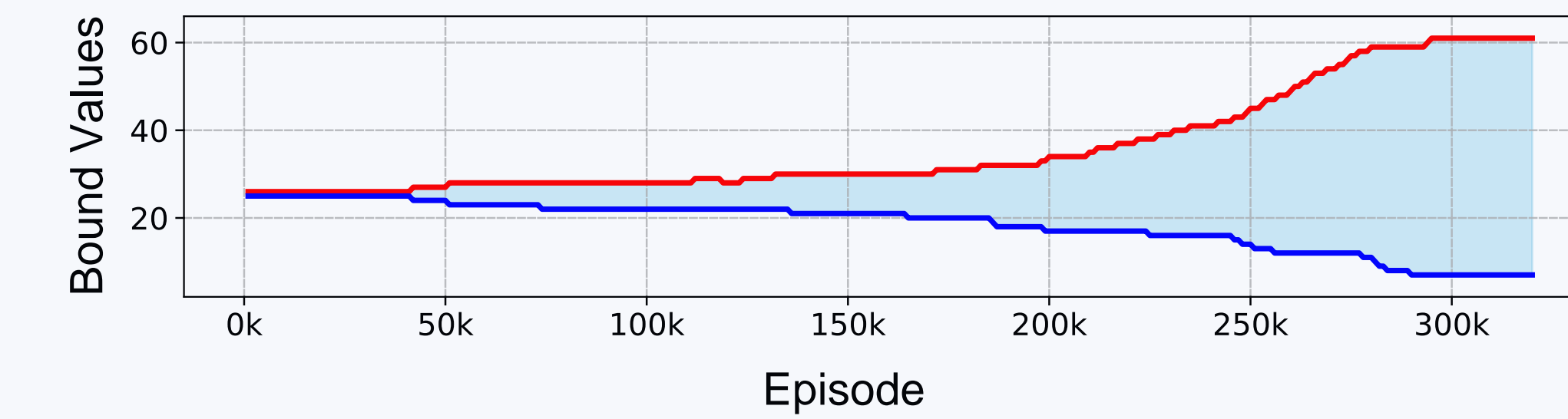
## Adaptive Sampling Method

- To enable the agent to discover a **maximally diverse** range of periods without **any prior knowledge** of its inherent period ranges, we introduce an adaptive sampling method that **dynamically adjusts** the sampling range during training.
- Key idea: Evaluate the policy's **performance on the boundary of the current sampling range**, using  $\sum_{t=0}^{T-1} r_{\text{PSD}}(L_{\text{bound}})$  as the evaluation criterion.

## Benchmark Environments

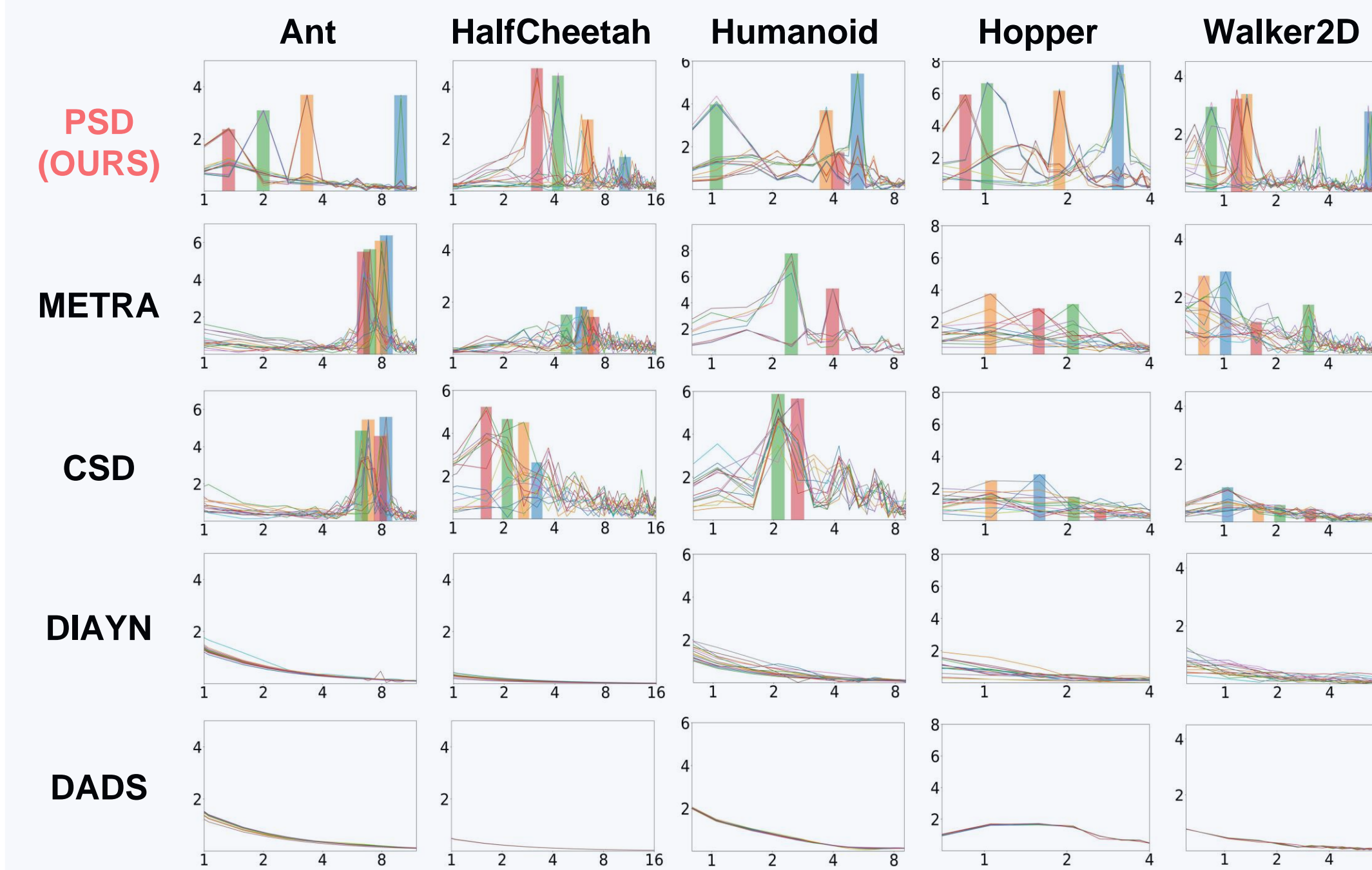


## Learned Sampling Bounds Over Time



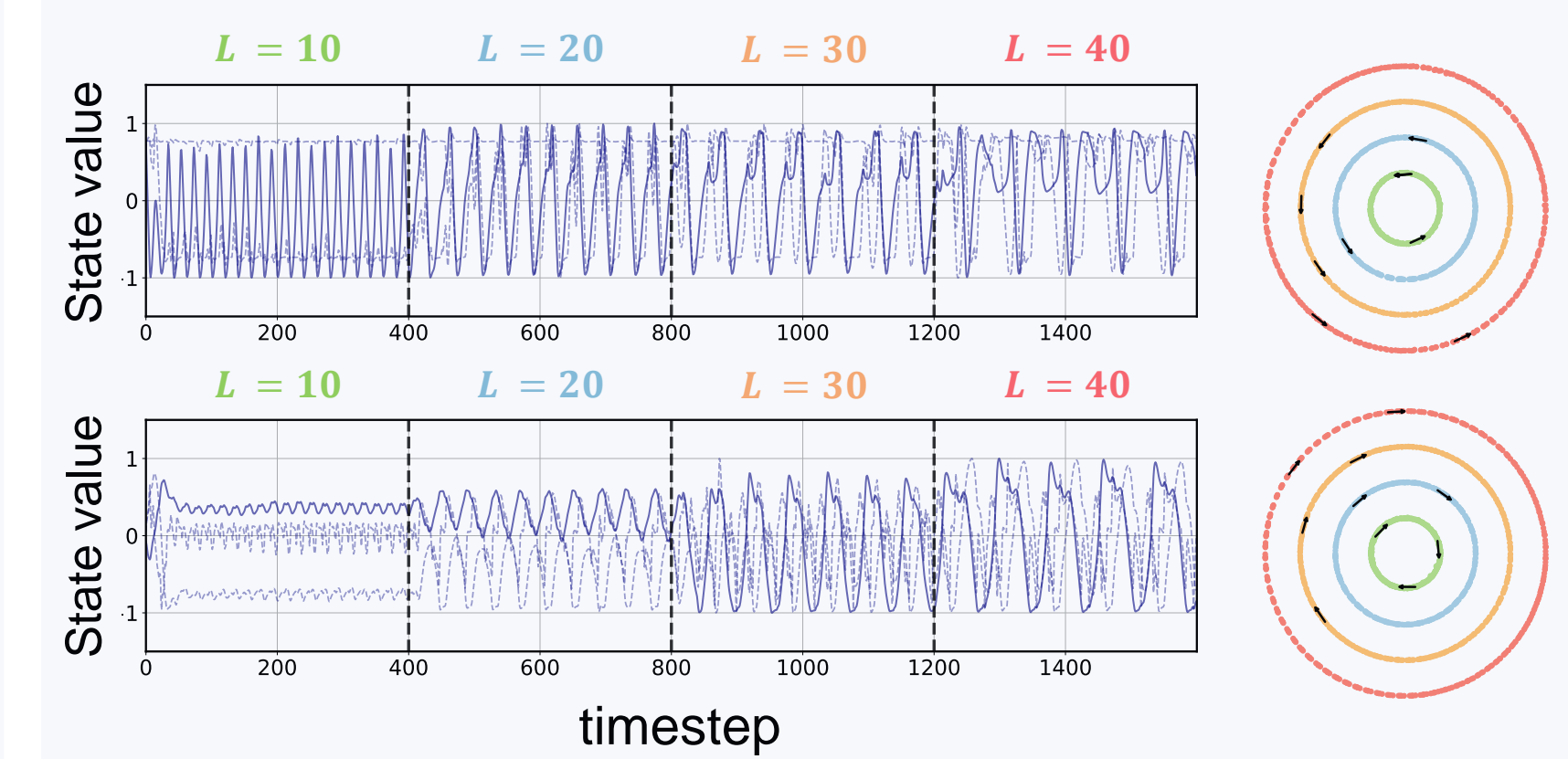
As training progresses, **increasingly challenging periods are proposed** to the agent, enabling the discovery of a wider range of periodic behaviors.

## Frequency-Domain Skill Comparison



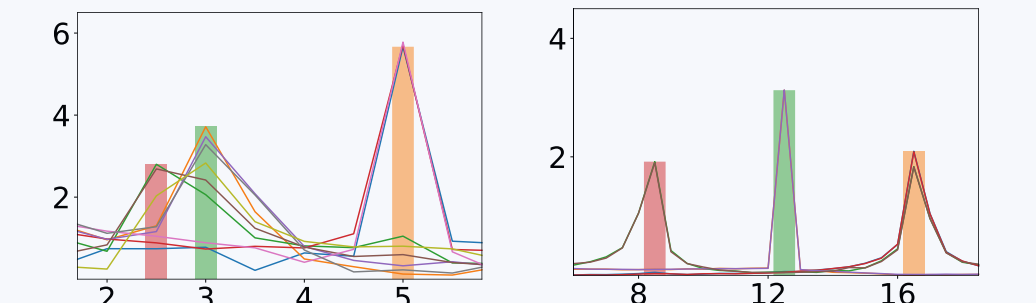
We apply a **Fourier transform** to the skill trajectories (frequency on the x-axis, amplitude on the y-axis). **PSD consistently discovers a wider range of frequencies than the baselines.**

## State Trajectories & Latents (PCA)



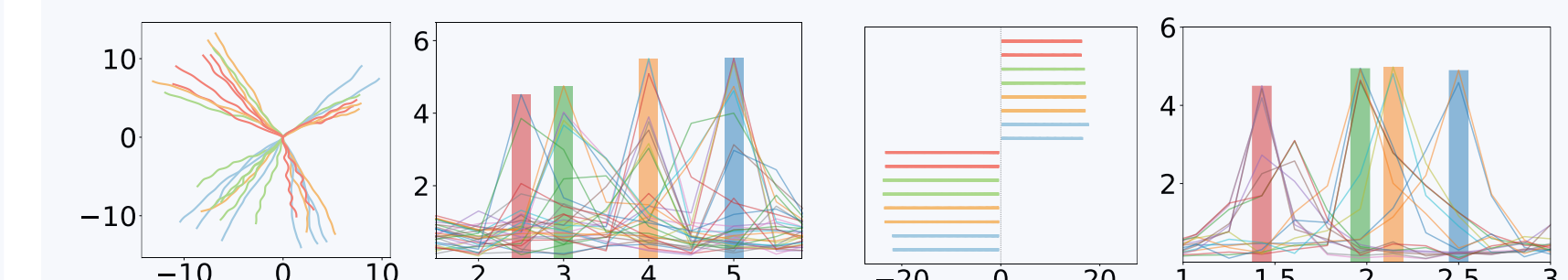
For varying values of L, PSD successfully **constructs a circular latent space** whose diameter is L, and **learns behaviors with the desired period of 2L**.

## PSD with Pixel-based Observations



PSD can discover periodic skills even in **pixel-based observation**. (see our demo)

## PSD combined with METRA (Park et al., 2023)



PSD naturally aligns with METRA, as **both methods capture temporal structure of skills**. (see our paper for details)