Automatic Control

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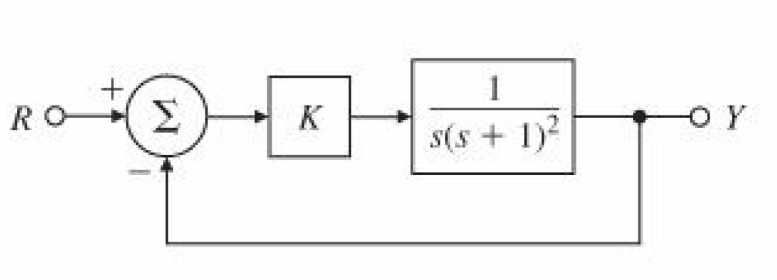
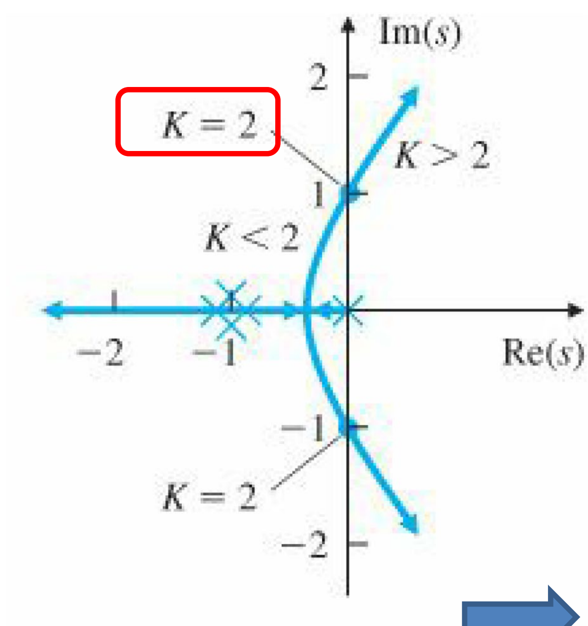
Frequency Response

Bode Plot 3 – Gain Margin and Phase Margin

Applications of the Bode Plot



- $KG(s) = K\frac{1}{s(s+1)^2}$ (Open loop transfer function)



Recall root locus

- $KG(s) = -1 \rightarrow |KG(s)| = 1$
- $\angle G(s) = -180°$

Trajectory of s that satisfies the phase condition

When $K = 2$

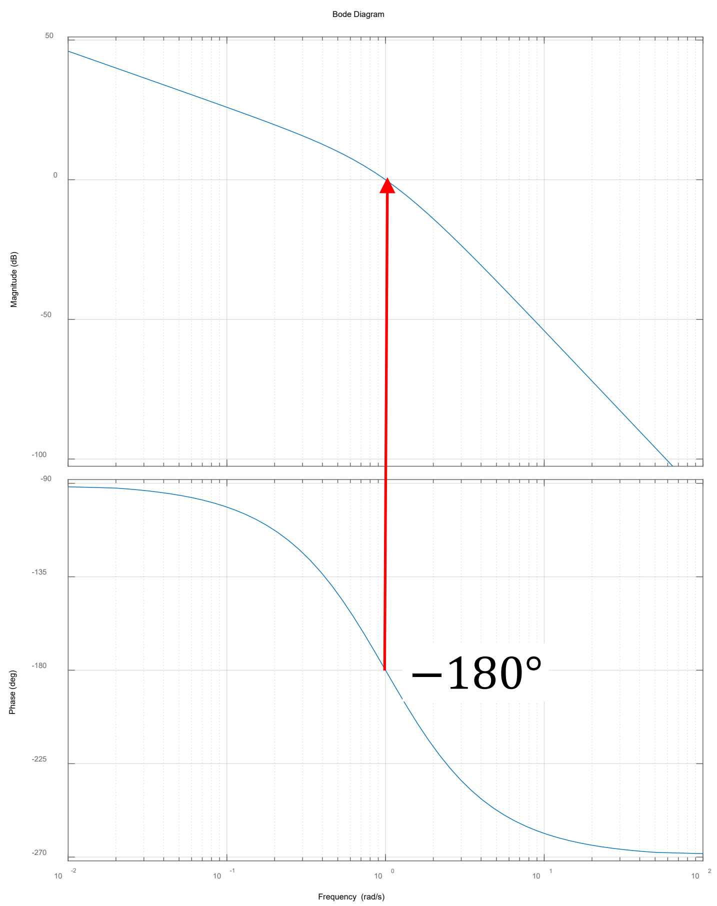
- $s = j1$ (crossover frequency)
- $\angle G(s) = \angle G(j1) = -180°$

Magnitude: 1 when $\omega = 1$
Phase: $-180°$ when $\omega = 1$

Bode Plot Representation

$$KG(s) = \frac{2}{s(s+1)^2}$$

When $\angle KG(j\omega) = -180° \rightarrow |KG(j\omega)| = 1$



If the gain (K) is a value such that the close loop poles are on the imaginary axis

When a bode plot of $KG(s)$ is plotted

Magnitude is 1 when phase is $-180°$

Bode Plot Representation

- Open loop transfer function
  - $KG(s)$

- We are changing K and we know how the closed loop poles move with respect to K from root locus

- We can also draw a Bode plot of $KG(s)$ with different value of Ks
  - Magnitude plot with only move up or down depending on the value of K
  - Phase plot will not change

- For a special value of K that the closed loop poles are on the imaginary axis, $K = K_N$
  - Root locus condition will still be satisfied
    - $|K_N G(s)| = 1$
    - $\angle K_N G(s) = -180°$
  - If a Bode plot is drawn for $K_N G(s)$
    - Magnitude is 1 for the frequency that gives $-180°$

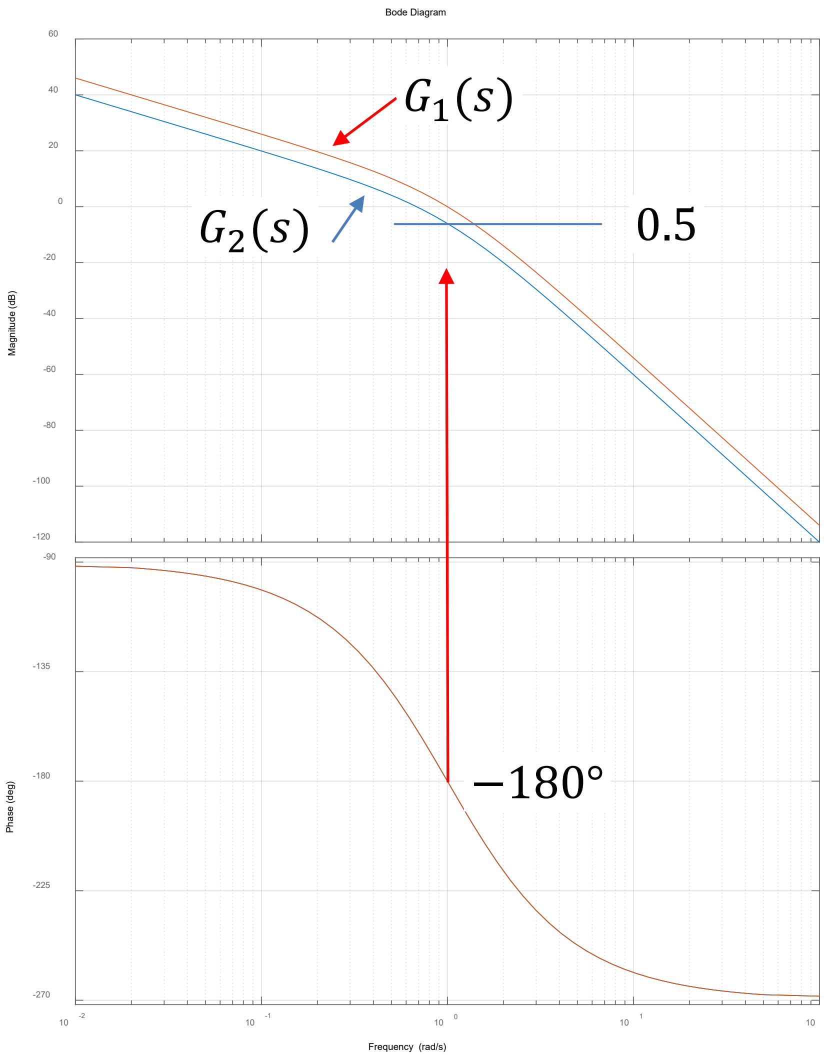
What if $K < K_N$?

- Root locus side
  - Closed loop poles will be stable

- Bode Plot side
  - Magnitude plot will move down
  - Magnitude at $\angle G(\omega_c) = -180°$ is smaller than 1

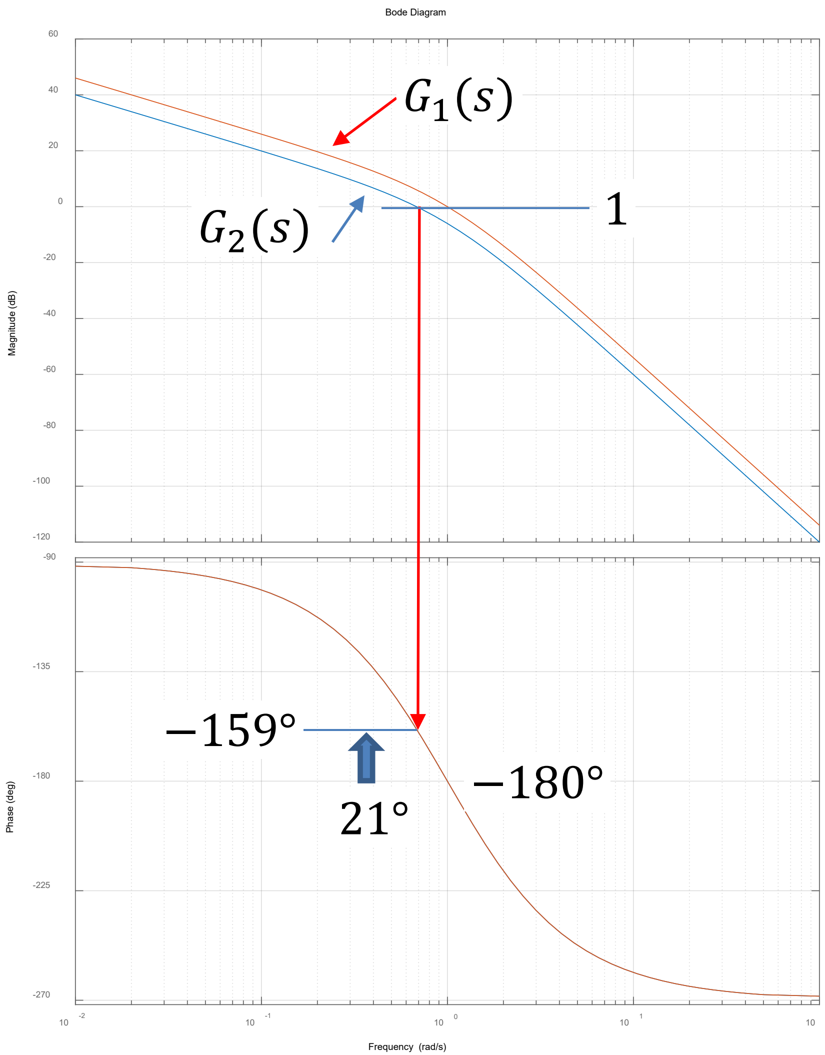
- What is the limit on K until instability occurs?
  - When the magnitude at $\angle G(\omega_c) = -180°$ is 1

Gain Margin



- Compare
  - $G_1(s) = \frac{2}{s(s+1)^2}$
  - $G_2(s) = \frac{1}{s(s+1)^2}$
- At which frequency the phase becomes $-180°$?
  - At $\omega_c = 1$
- At the crossover frequency
  - What is the magnitude of $G_1(j\omega)$?
    - 1
  - What is the magnitude of $G_2(j\omega)$?
    - 0.5
- Is $G_2(s)$ stable?
  - Yes
- How much more can you increase the gain on $G_s(s)$
  - Can multiply up to 2 → Gain margin

Phase Margin



- Compare
  - $G_1(s) = \frac{2}{s(s+1)^2}$
  - $G_2(s) = \frac{1}{s(s+1)^2}$

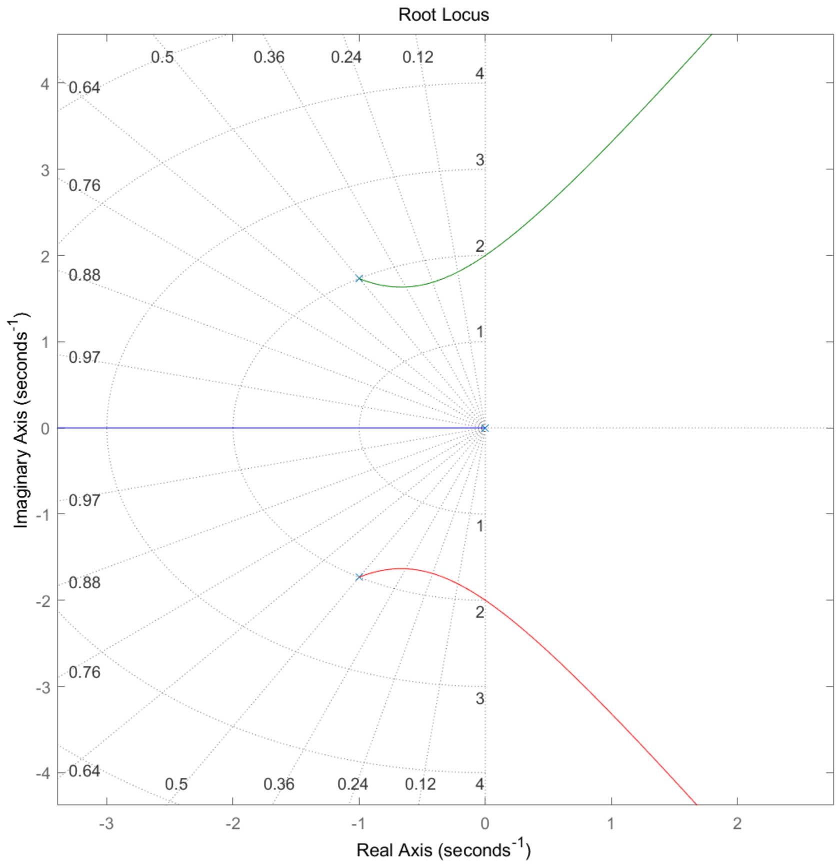
- At which frequency the magnitude becomes 1?
  - For G_1 → at $\omega = 1$ (neutral)
  - For G_2 → at $\omega = 0.68$ (stable)

- When $\omega = 0.68$
  - What is the phase $G_2(j\omega)$?
    - -159

- How much you phase you have left until $-180°$? $G_s(s)$
  - $21°$ → Phase margin

Example

$$G(s) = \frac{1}{s(s^2 + 2s + 4)}$$



Imaginary axis crossing

$$s(s^2 + 2s + 4) + K = 0$$

$$s = j\omega$$

$$j\omega(-\omega^2 + 2j\omega + 4) + K = 0$$

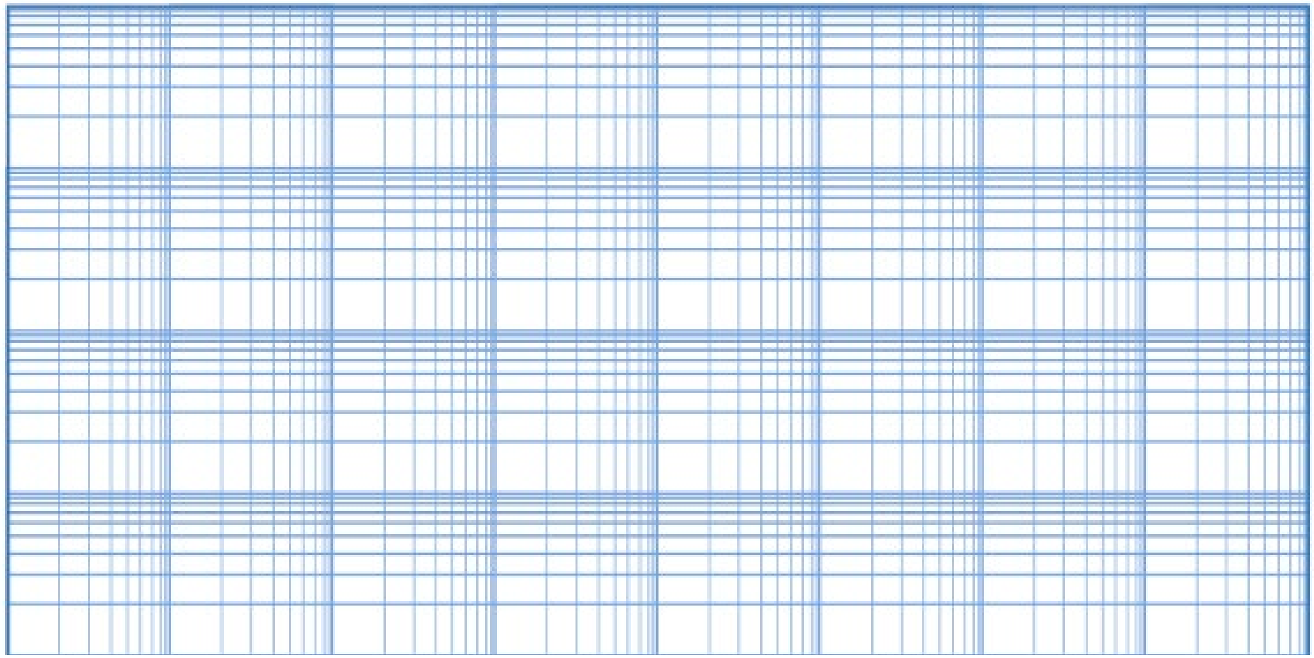
$$-\omega^3 j(-\omega^3 + 4\omega) + K - 2\omega^2 = 0$$

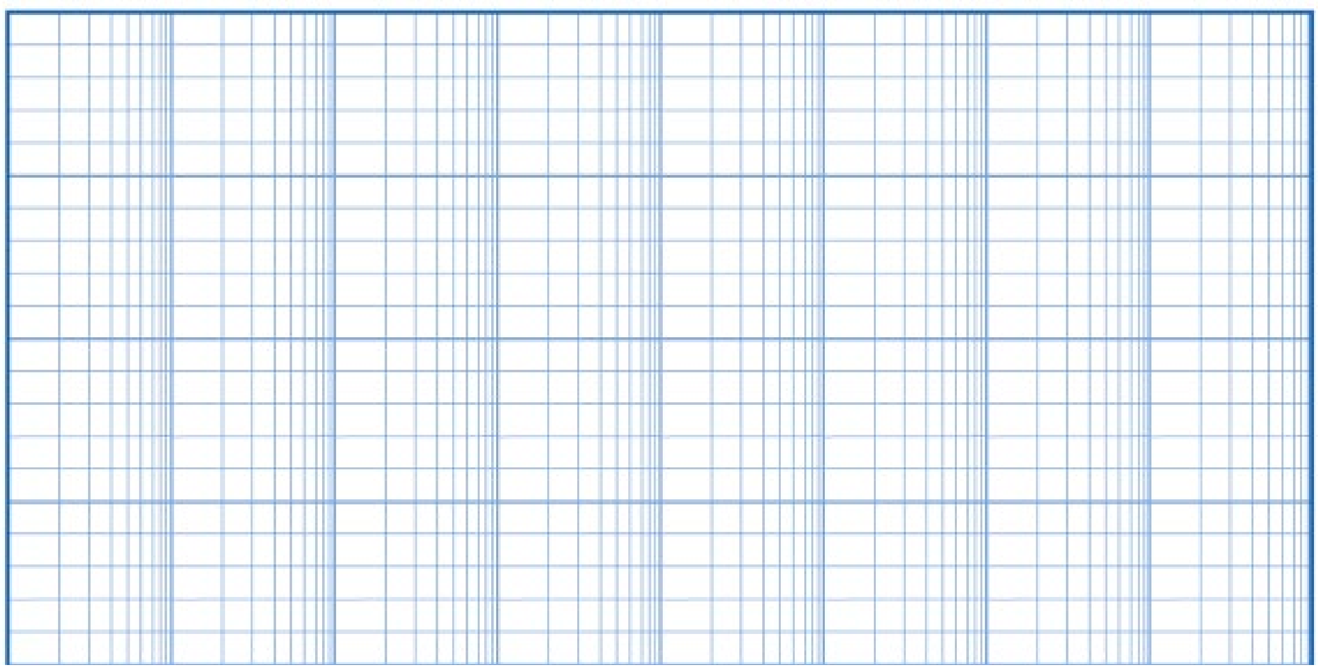
$$\omega = 2, K = 8$$

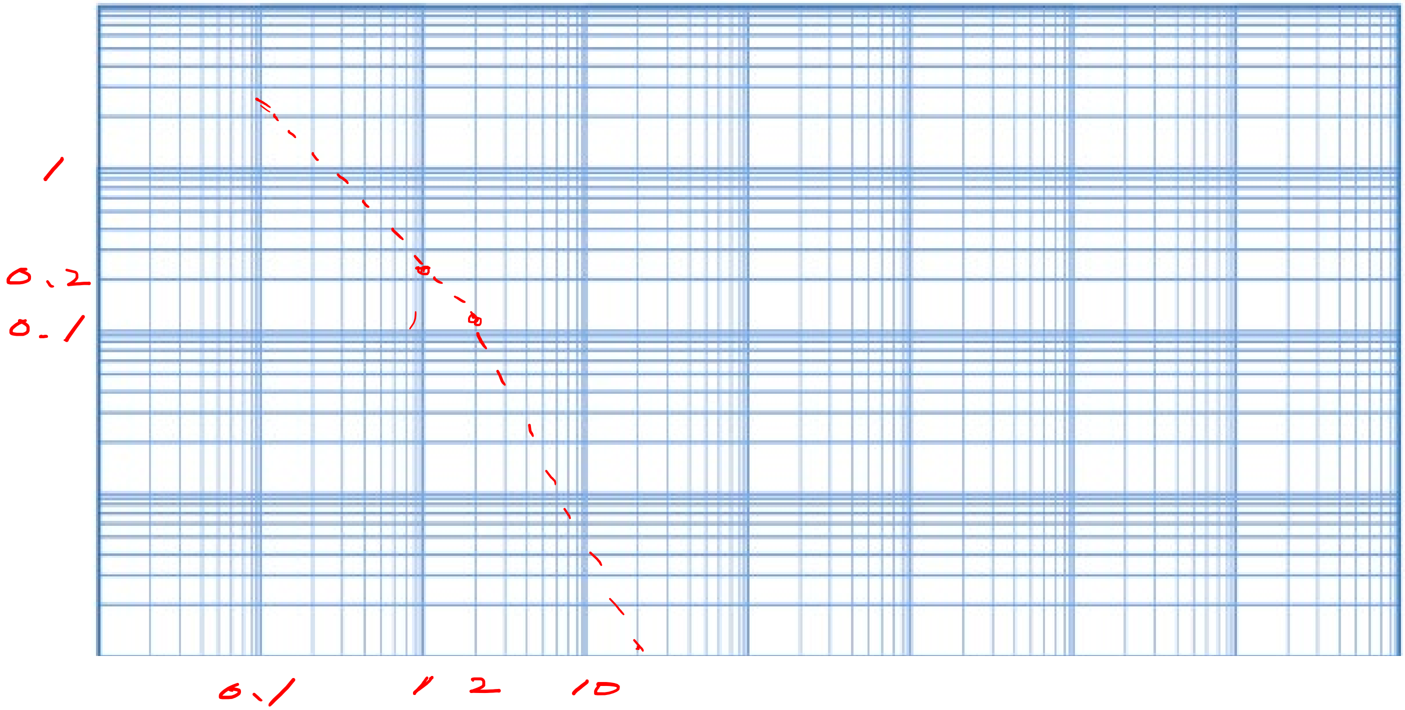
Bode Plot

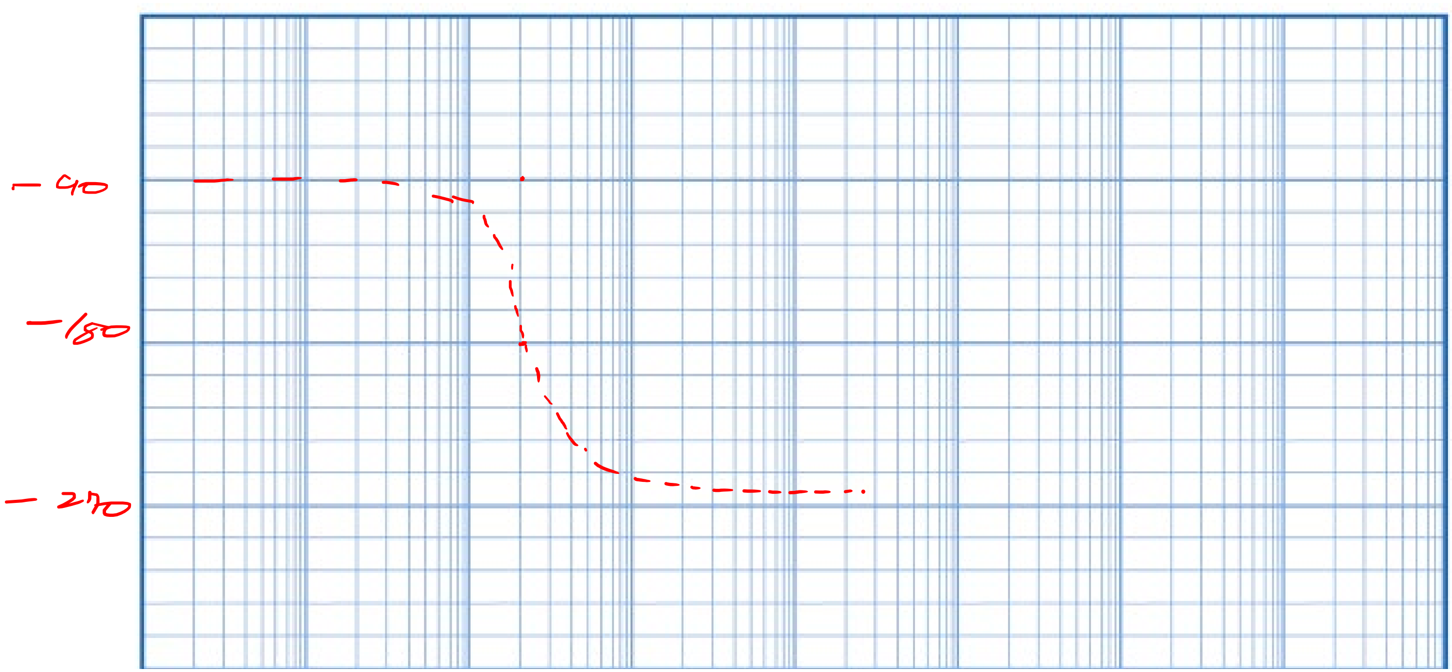
$$G(s) = \frac{1}{s(s^2 + 2s + 4)}$$

- Standard form
  - $G(s) = \frac{1}{s(s^2+2s+4)} = \frac{1}{4}\frac{1}{s\left(\frac{s^2}{4}+2\frac{1}{4}s+1\right)} = \frac{1}{4}\frac{1}{s\left(\left(\frac{s}{2}\right)^2+2\frac{1}{2}\left(\frac{s}{2}\right)+1\right)}$
- Break points (corner frequencies)
  - $s = 1$
    - $|G(j\omega)| = \frac{1}{4}$ $(-12\ \mathrm{dB})$ with slope of -1
    - $\angle G(j\omega) = -90$
  - $s = 2$ → $\zeta = \frac{1}{2}$
    - Slope changes to -3
    - Phase changes from $-90°$ to $-270°$ while

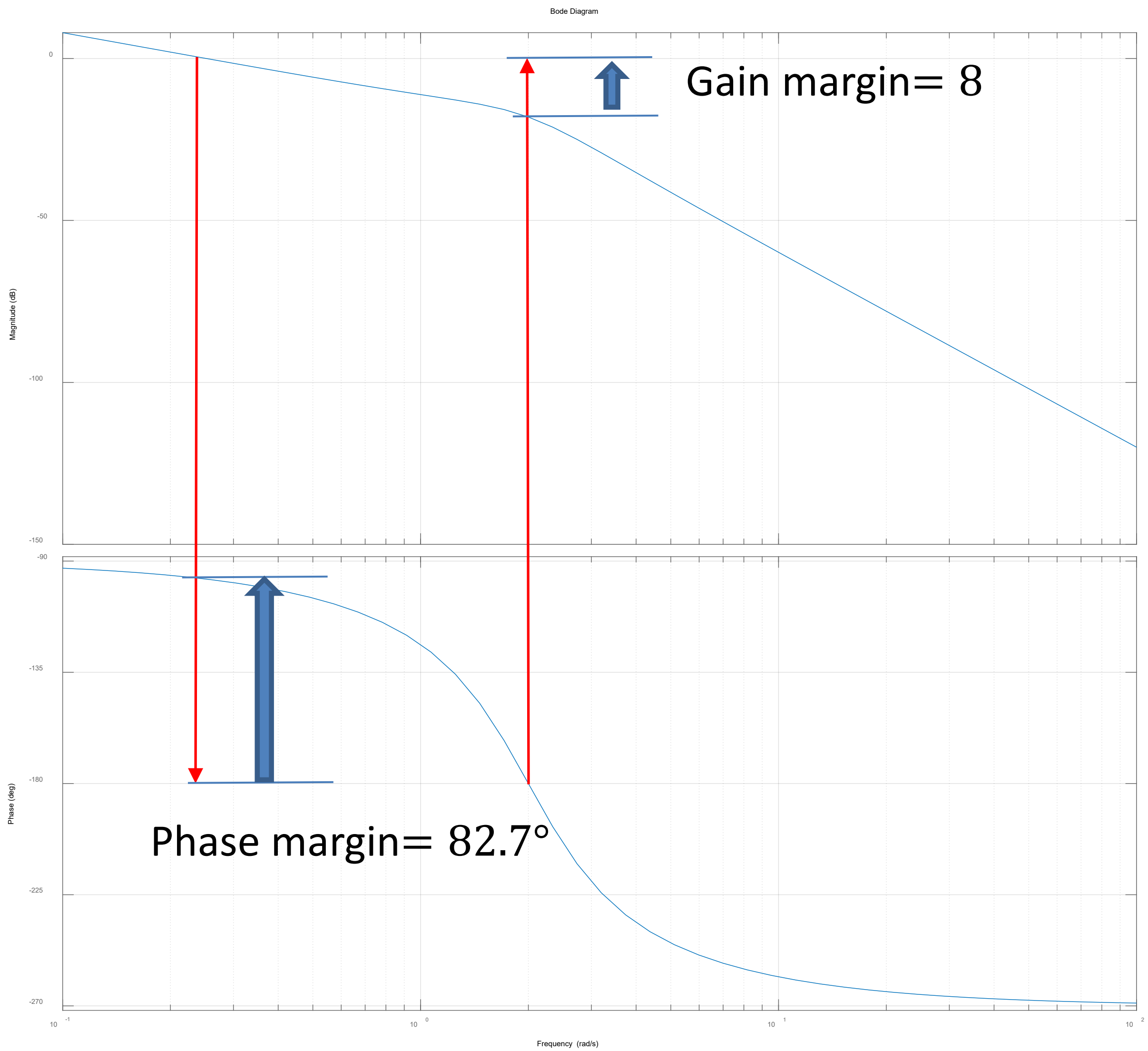





1
0.2
0.1
0.1
1
2
10


-90
-180
-270

Bode Plot


Bode Diagram
Gain margin= 8
Phase margin= 82.7°
Magnitude (dB)
Phase (deg)
0
-50
-100
-150
-90
-135
-180
-225
-270
10 -1
10 0
10 1
10 2
Frequency (rad/s)

Matlabe Commands

```
>> num = 1
num =
    1
>> den = [1 2 4 0]
den =
    1     2     4     0
>> sys = tf(num, den)
sys =
        1
  -----------------
  s^3 + 2 s^2 + 4 s
Continuous-time transfer function.
>> [mag, phase, w] = bode(sys);
>> [Gm , Pm , Wcg , Wcp] = margin(mag, phase, w)
Gm =
   8.0000
Pm =
  82.6871
Wcg =
   2.0000
Wcp =
   0.2520
```

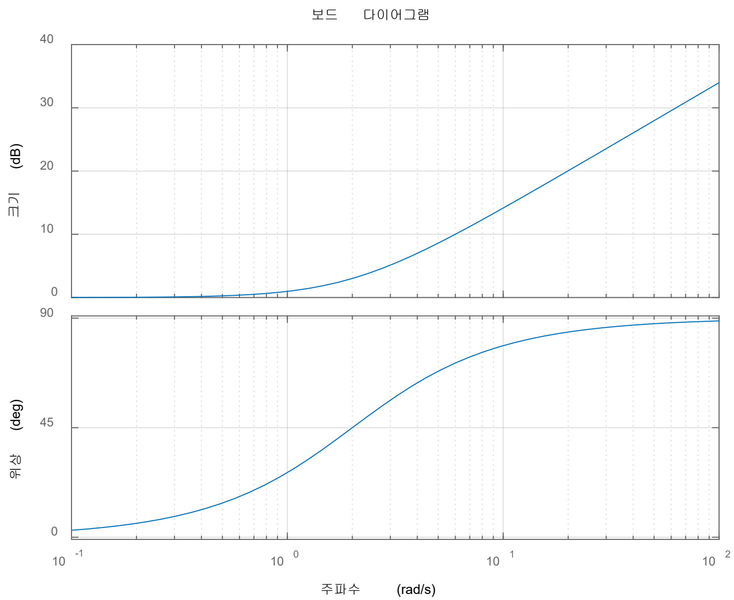
Root Locus vs Bode Plot

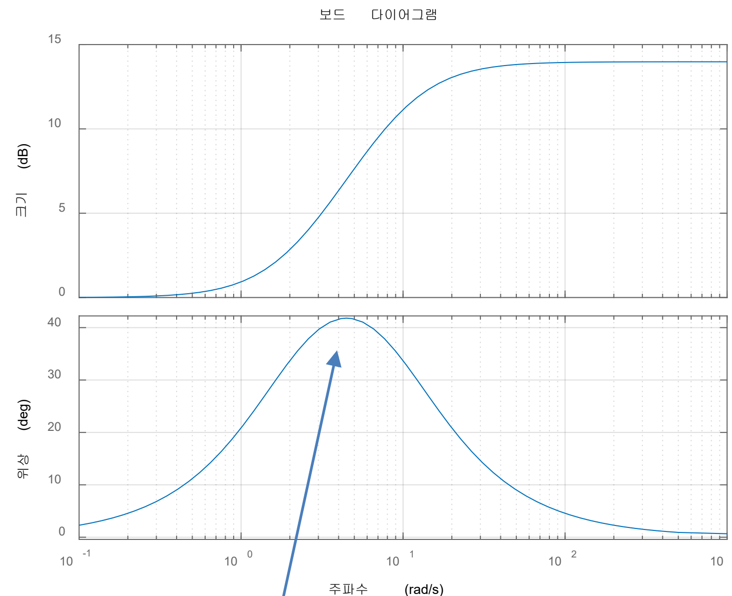
- Root locus
  - Open loop poles and zeros → closed loop poles
  - Find a value of the gain, K, for stability

- Bode plot
  - Can find closed loop stability by evaluating the frequency response of the open loop transfer function, $KG(s)$.
  - Need to know K, and then evaluate stability

→ Can be used for extra small tweaks to improve the characteristics
    - Add phase margin
    - Increase low frequency gain → reduce steady state error

Derivative control vs Lead





$$D(s) = \left(\frac{s}{2} + 1\right)$$

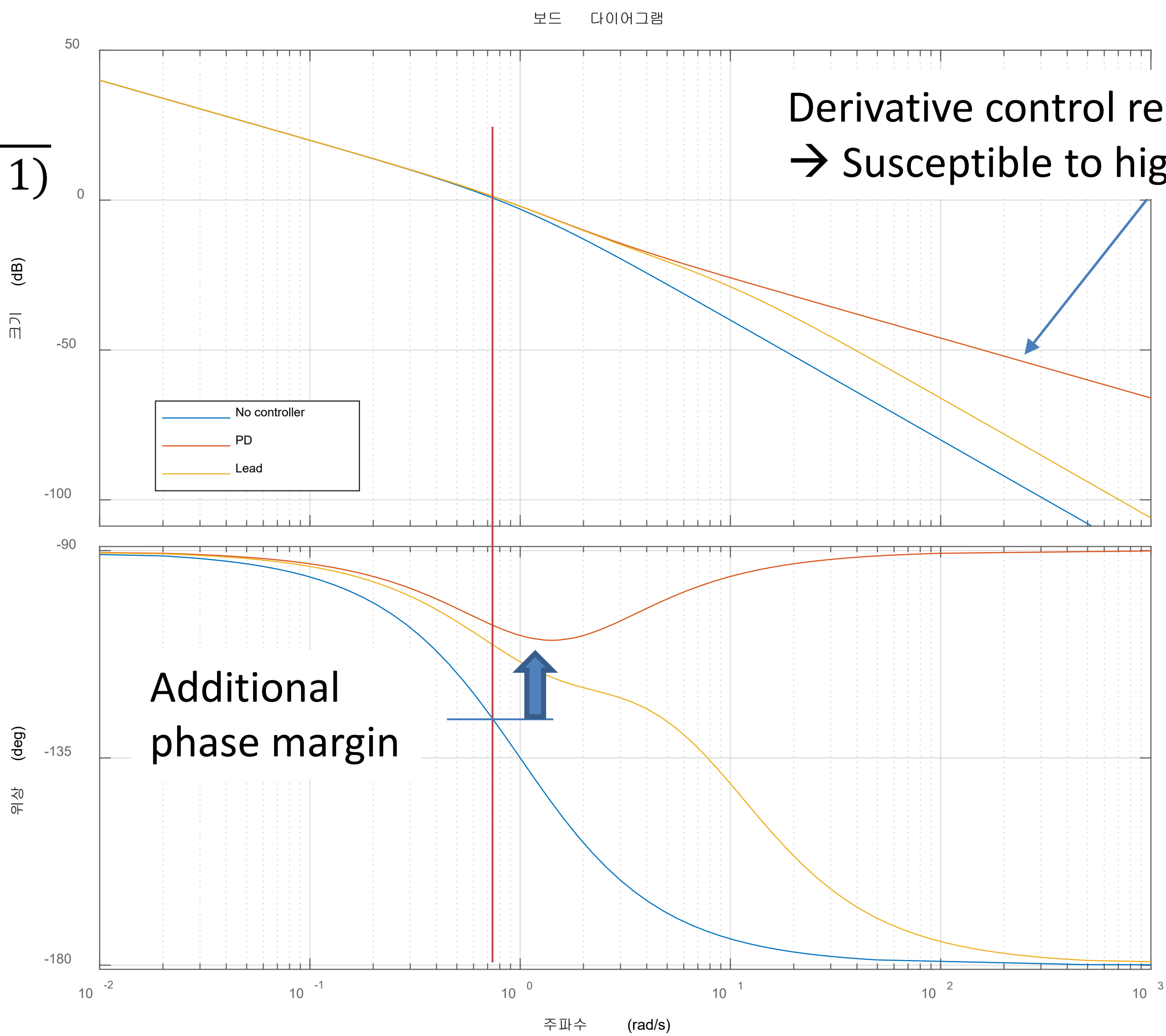
Amount of phase addition can be adjusted

$$D(s) = \frac{\frac{s}{2} + 1}{\frac{s}{10} + 1}$$

PD Control Example

$$G(s) = \frac{1}{s(s+1)}$$

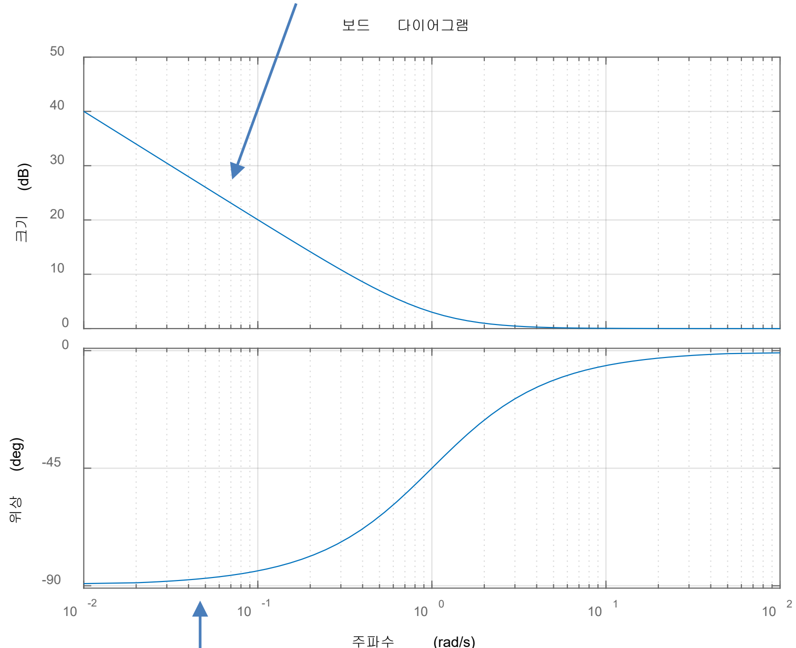
Derivative control reduces the slope
→ Susceptible to high frequency noise



Additional phase margin

Integral Control vs Lag

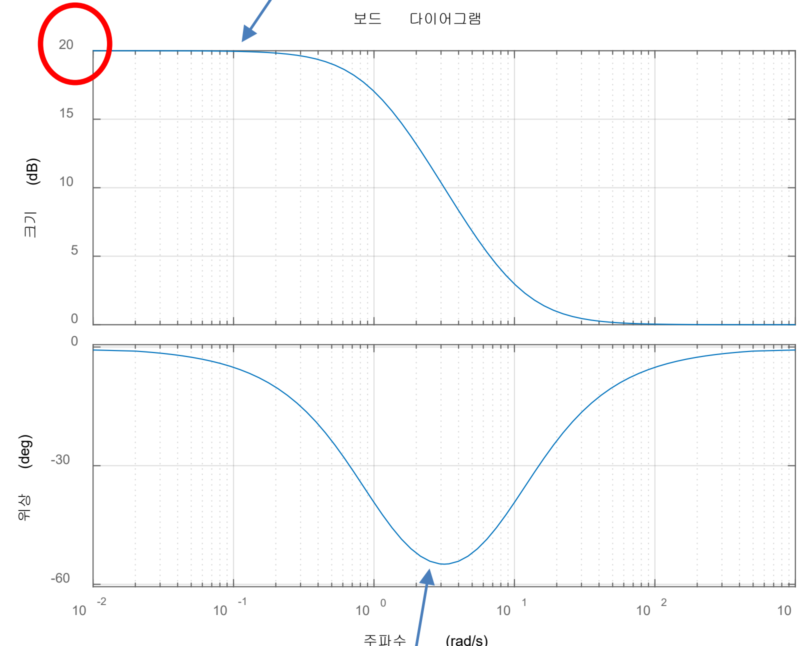
Add low frequency gain



Too much reduction in low frequency phase

$$D(s) = \left(1 + \frac{1}{s}\right) = \frac{s+1}{s}$$

Can add low frequency gain



Amount of phase reduction can be adjusted

$$D(s) = 10\frac{\frac{s}{10}+1}{s+1}$$