Automatic Control

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Dynamics

Models of Electric Circuit

Models of Electric Circuit

- Electric circuits
  - Interconnections between sources of electric voltage and current, and other electric elements
  - 3 passive elements: resistor (R), inductor (L), capacitor (C)
  - Other elements:
    - Operational amplifiers
    - Transistors

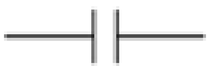
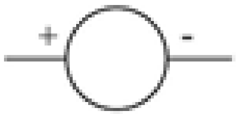
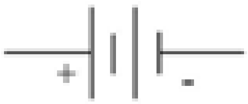
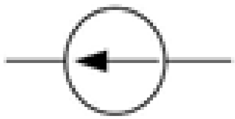
- Why?
  - Actual implementation of control system is in the form of electric circuits

Kirchhoff’s laws

- Kirchhoff
  - Gustav Robert Kirchhoff (1824 – 1887), German physicist
  - Electric circuits, spectroscopy, black-body radiation

- Kirchhoff’s laws
  - KCL (Kirchhoff’s Current Law): The algebraic sum of currents leaving a junction or node equals the algebraic sum of the currents entering that node
  - KVL (Kirchhoff’s Voltage Law): The algebraic sum of all voltages taken around a closed path in a circuit is zero

Elements of Electric Circuits

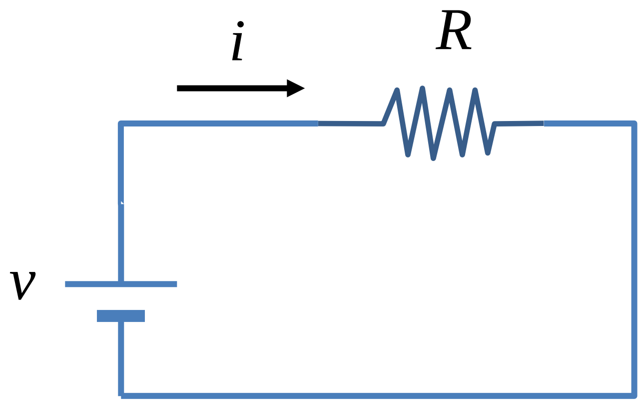
Circuit Element	Symbol	Schematic
Resistor	R	 or 
Inductor	L	
Capacitor	C	
DC Voltage Source	V_s	 or 
DC Current Source	I_s	

$$v = iR$$

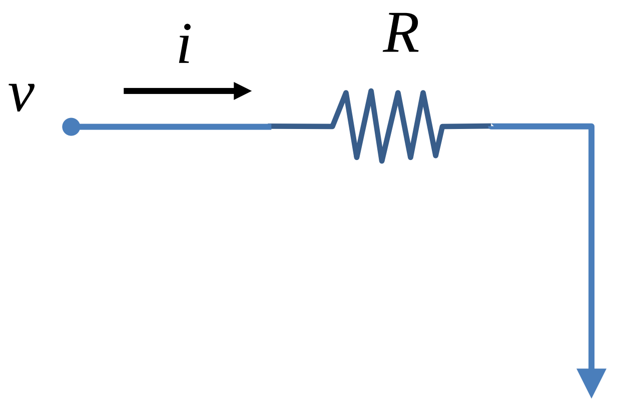
$$v = L\frac{di}{dt}$$

$$i = C\frac{dv}{dt}$$

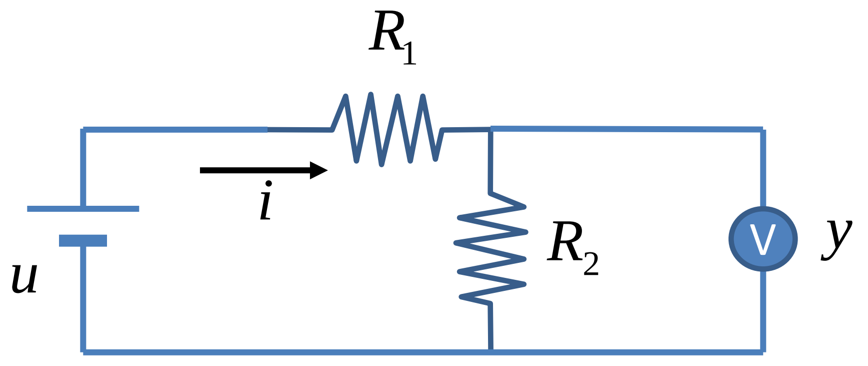
Ohm's Law



$$i = \frac{v}{R}$$

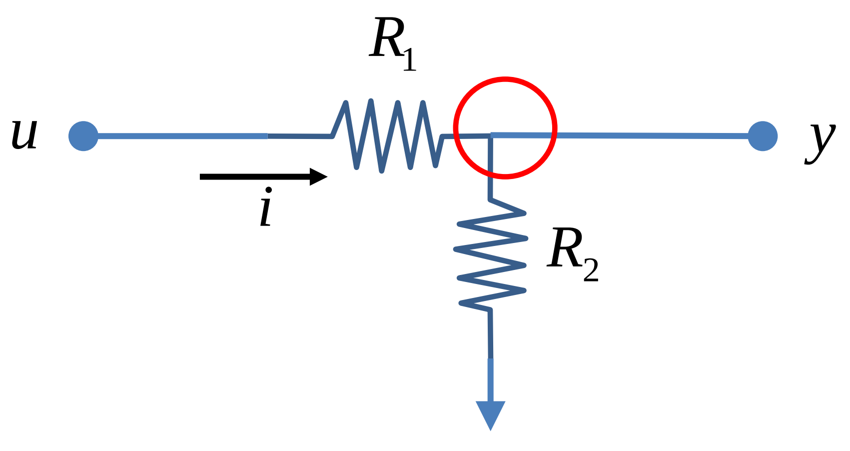


Ohm's Law



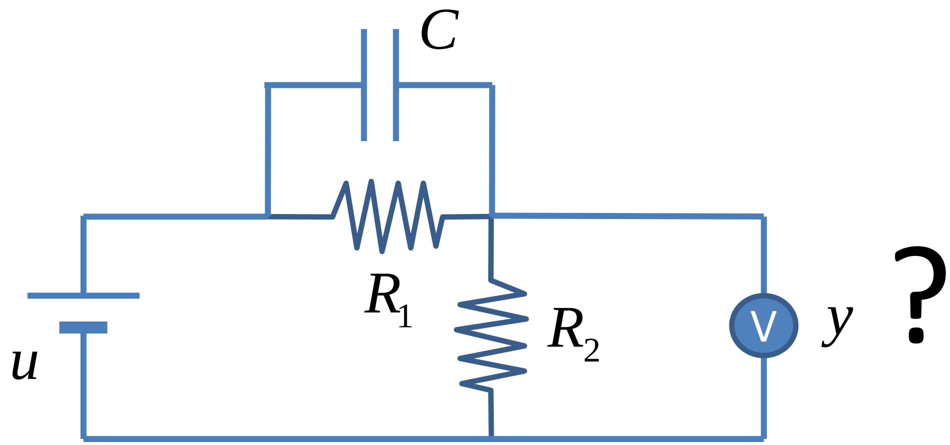
$$i = \frac{u}{R_1 + R_2}$$

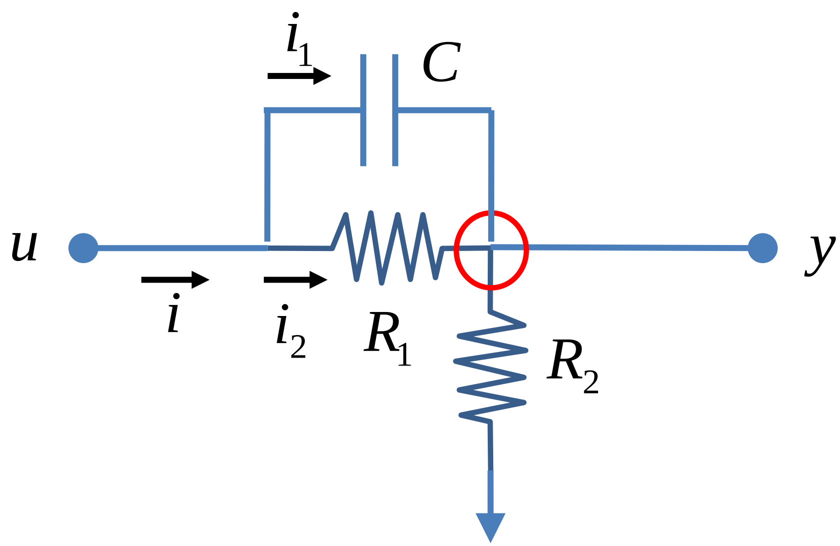
$$y = iR_2$$



$$\frac{u-y}{R_1} = \frac{y-0}{R_2}$$

Circuits with Capacitors





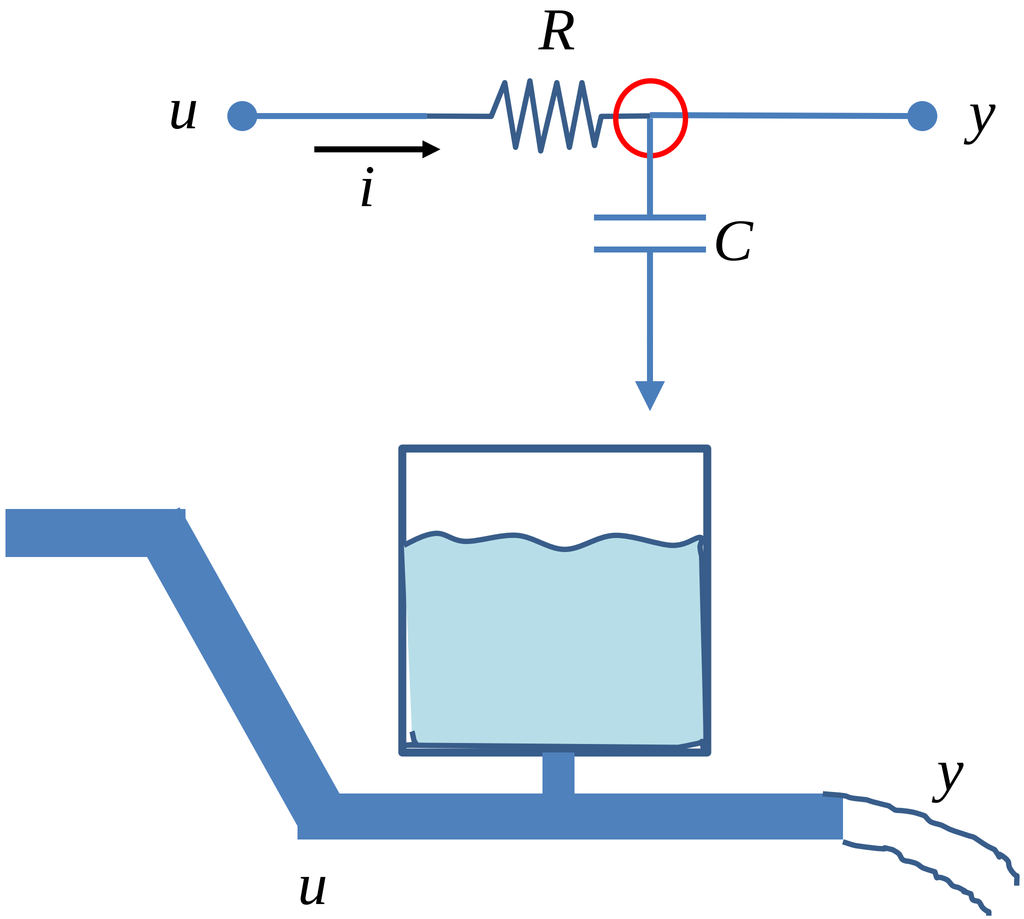
KCL: $i = i_1 + i_2$

$$i_1 = C\frac{d(u-y)}{dt} \qquad i_2 = \frac{(u-y)}{R_1}$$

$$i = \frac{(y-0)}{R_2}$$

$$C\dot{u} + \frac{1}{R_1}u = C\dot{y} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)y$$

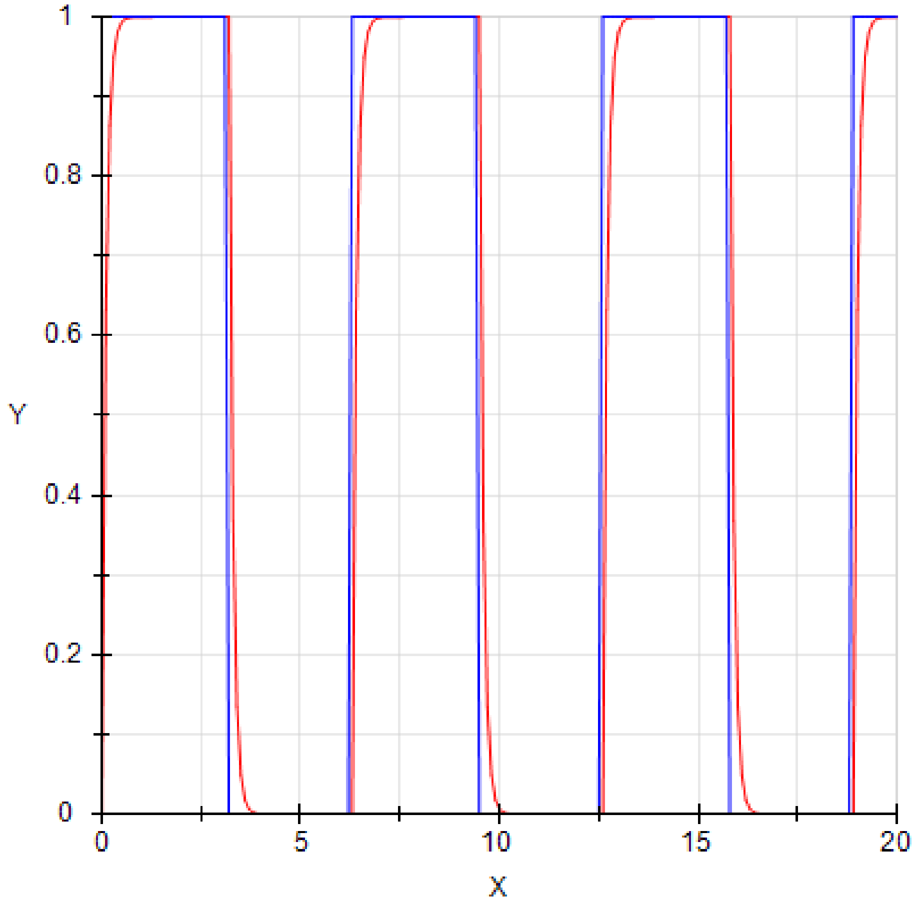
Circuits with Capacitors



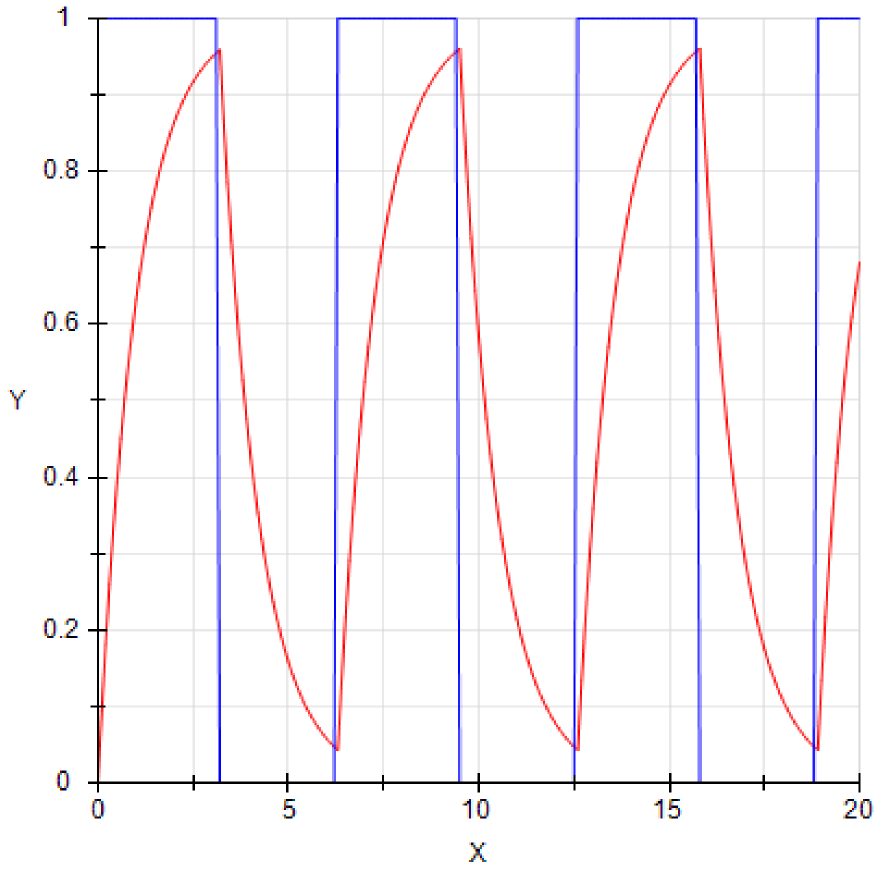
$$\frac{u-y}{R} = C\frac{d(y-0)}{dt}$$

$$y + RC\dot{y} = u$$

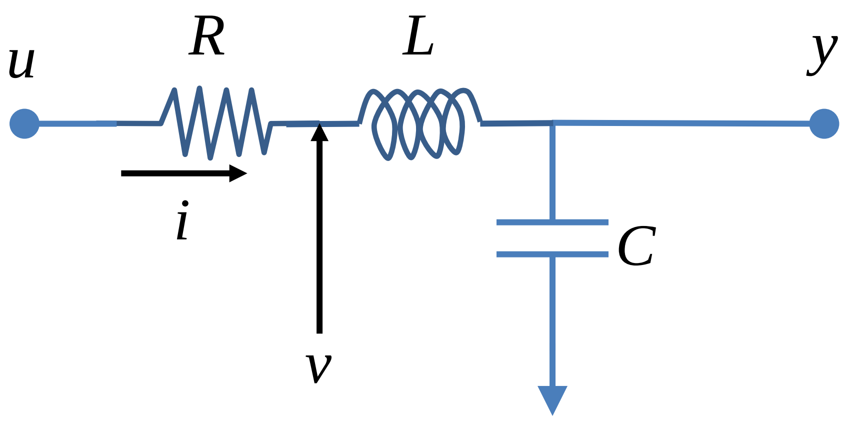
Response – Small Capacitance



Response – Large Capacitance



RLC Circuit



$$i = \frac{u - v}{R}$$

$$L\frac{di}{dt} = (v - y)$$

$$C\frac{dy}{dt} = i$$

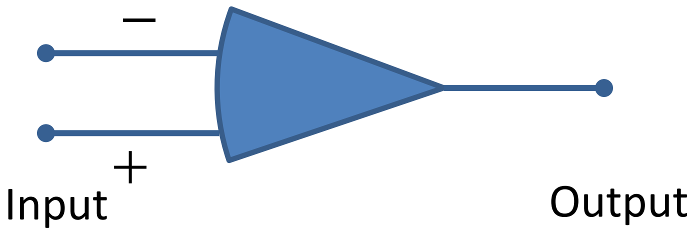
$$LC\ddot{y} + RC\dot{y} + y = u$$

Compare this with

$$m\ddot{y} + b\dot{y} + ky = u$$

Operational Amplifier

- DC-coupled high-gain voltage amplifier with a differential input and a single output
- Building blocks for analogue circuit design



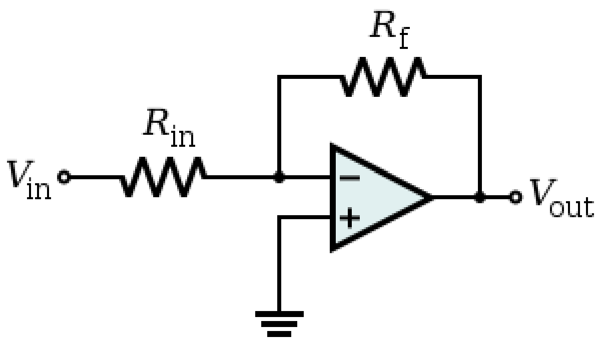
Operational Amplifier

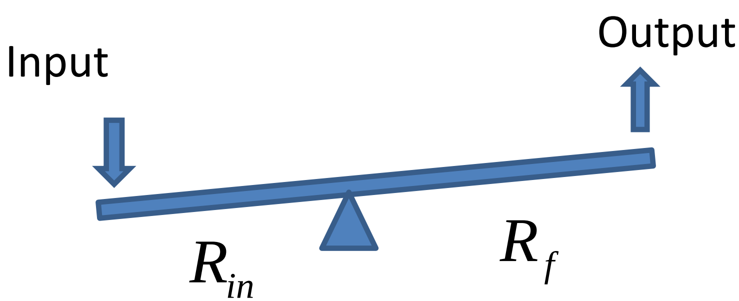
- Text book description
  - Output voltage = (Large gain) x (voltage difference between the two inputs)

- What it actually does
  - OP Amp make sense only in a constructed circuit
  - It adjust the output voltage until the two input voltage becomes the same
  - No current flows in/out through the inputs

OP Amp Circuits

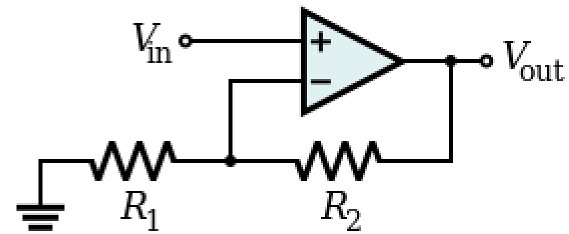
Inverting amplifier

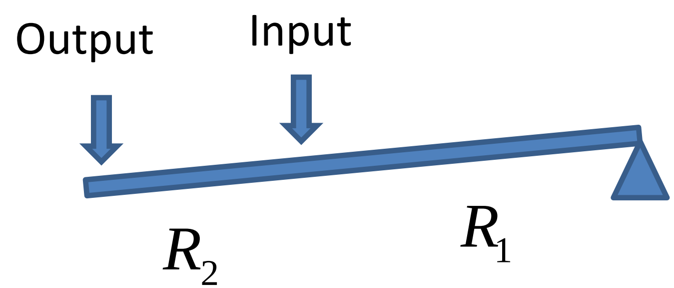




$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

Non-inverting amplifier

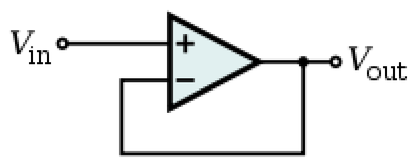




$$V_{out} = \frac{R_1 + R_2}{R_1} V_{in}$$

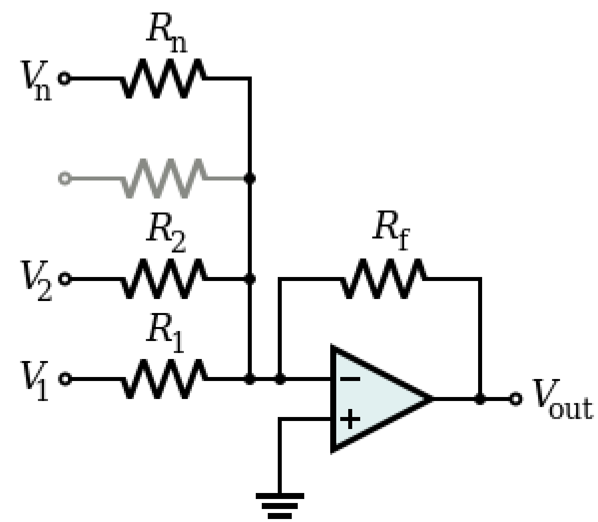
OP Amp Circuits

Voltage follower (unity buffer)



$$V_{out} = V_{in}$$

Summing amplifier

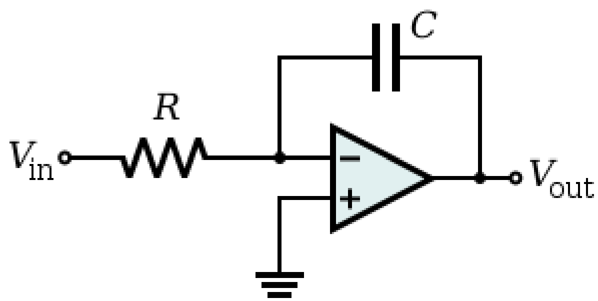


$$V_{out} = -R_f\left(\frac{1}{R_1}V_1 + \frac{1}{R_2}V_2 + \ldots + \frac{1}{R_n}V_n\right)$$


ALESIS
MULTIMIX16USBFX
1
2
3
4
5
6
7
8
9/10
11/12
13/14
15/16
MAIN

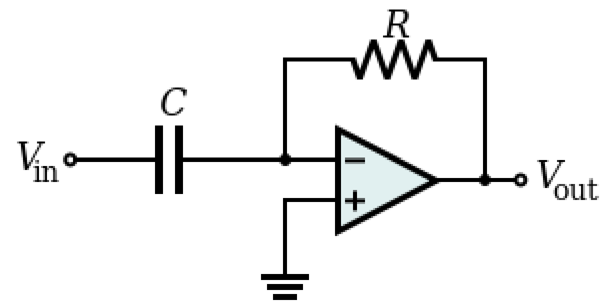
Integrator and Differentiator

Inverting integrator



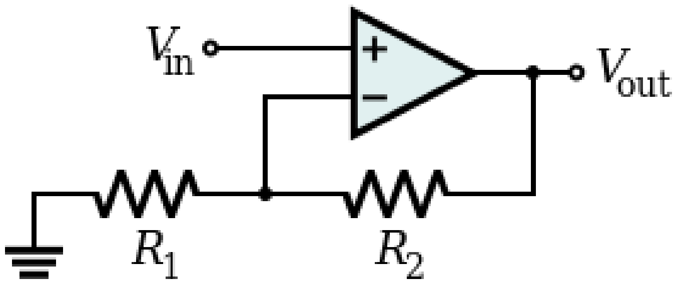
$$V_{out}(t_1) = V_{out}(t_0) - \frac{1}{RC}\int_{t_0}^{t_1} V_{in}(t)\,dt$$

Inverting differentiator



$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Exercise 1



Your pressure sensor’s output voltage ranges from 0 to 0.5 V for the pressures that you want to measure.

Your A/D converter is 5 V. Pick your resistor values so that the non-inverting amplifier scales the sensor output voltage to fit the A/D convertor input voltage.

Exercise 2

To make a PD controller, you need to generate a signal u that is linear combination of input signal e and its derivative.

Construct an op-amp circuit to perform this.

$$u = ae + b\dot{e}$$