EE363 Automatic Control: Homework #3

1) *Diagonalization.* We have a linear dynamical system described by

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

with $x \in \mathbb{R}^n$. Also we assume that A has n real eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, and their associated eigenvectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$ are linearly independent.

a) Let an $n \times n$ diagonal matrix Λ be

$$\Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

and $V \in \mathbb{R}^{n \times n}$ be

$$V = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

Check yourself if the following holds.

$$AV = V\Lambda$$

b) Show that there exists a state space realization with a new coordinate $\tilde{x}$

$$\begin{aligned}\dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}u \\ y &= \tilde{C}\tilde{x} + \tilde{D}u\end{aligned}$$

where $\tilde{A}$ is real diagonal. Explicitly state $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ in terms of A, B, C, D, V, and Λ.

2) *This is not a linear algebra class.* In this problem, we will show that the eigenvalues of a block triangular matrix are the eigenvalues of each diagonal blocks. Suppose we have a block triangular matrix A, and without loss of generality we assume that A is block lower triangular.

$$A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$$

We want to show that $\{\lambda_1 \mid A_{11}v_1 = \lambda_1 v_1,\ v_1 \neq 0\} \cup \{\lambda_2 \mid A_{22}v_2 = \lambda_2 v_2,\ v_2 \neq 0\} = \{\lambda \mid Av = \lambda v,\ v \neq 0\}$.

a) Suppose that $A_{22}v_2 = \lambda_2 v_2$ for some $v_2 \neq 0$. Find x_2 such that

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_2 \\ v_2 \end{bmatrix} = \lambda_2 \begin{bmatrix} x_2 \\ v_2 \end{bmatrix}$$

This will prove that λ_2 is an eigenvalue of A. What is the eigenvector of A associated with λ_2?

b) Suppose that $A_{11}v_1 = \lambda_1 v_1$ for some $v_1 \neq 0$ and $A_{22}v_2 \neq \lambda_1 v_2$ for any $v_2 \neq 0$. Find x_1 such that

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ x_1 \end{bmatrix} = \lambda_1 \begin{bmatrix} v_1 \\ x_1 \end{bmatrix}$$

This will prove that λ_1 is an eigenvalue of A. What is the eigenvector of A associated with λ_1?

3) *State space realization.* Find a state space realization of the following transfer function matrices, *i.e.*, find $G(s) : \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. Note: try to reduce the dimension of A as small as possible.

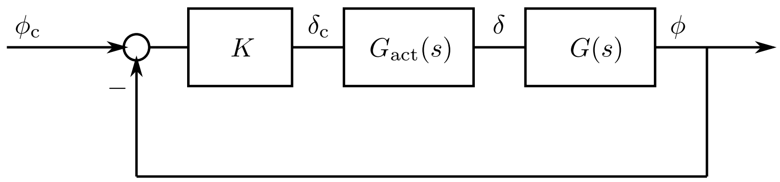
   a) $G(s) = 1/s$
   b) $G(s) = (s-1)/(s+1)$
   c) $G(s) = \begin{bmatrix} 1/s & 1/s \\ 1/s & 1/s \end{bmatrix}$
   d) $G(s) = \begin{bmatrix} (s+1)/s & (2s+1)/s \\ (3s+1)/s & (4s+1)/s \end{bmatrix}$
   e) $G(s) = \begin{bmatrix} (s+1)/s & (2s+1)/s \\ 0 & (4s+1)/s \end{bmatrix}$

4) *Poor control designer × Badly chosen actuator.* We consider a simple roll control systems for a sounding rocket. A simple roll dynamics, which describes *the rotation of the rocket around its axisymmetric axis*, is shown below

$$\dot{p} = L_p p + L_\delta \delta$$
$$\dot{\phi} = p$$

where the state variables, ϕ and p, are the roll angle and the roll rate, respectively, and the input to the system, δ is the fin deflection angle. The coeffcient $L_p < 0$ is called *the aerodynamic damping* which originates from the aerodynamic friction that tends to stop the rotation, and L_δ can be interpreted as *the control effectiveness* of the control fin.

   a) Present the transfer function and a state space description of $G(s)$, the dynamic system from δ to ϕ.
   b) Hereafter, let $L_p = -1$ and $L_\delta = 1$. Briefly explain what happens to the rocket, *i.e.*, p and ϕ, if someone threw a stone and it hit the rocket's control fin.



The above block diagram depicts the control system that we will work on. Your constant gain controller, K, computes the control command δ_c by using the roll tracking error which is the difference between the reference roll command ϕ_c and the actual roll angle ϕ. The computed control command δ_c is sent to the actuator $G_{act}(s)$ which finally generates the control fin angle δ. The ideal actuator will immediately generate the commanded fin angle, *i.e.*, $G_{act}(s) = 1$, however this does not happen in the real world.

   c) Assume the ideal actuator, *i.e.*, $G_{act}(s) = 1$. A poor control designer ignorantly chose a K that resulted in the closed loop damping of $\zeta = 0.5/\sqrt{3}$. What was his/her K?
   d) It turned out that the actuator is far slower than expected, with its dynamics being described by $G_{act}(s) = 2/(s+2)$. Briefly explain the characteristics of the closed loop system (with the K obtained above) under the new actuator model.
   e) Can you come up with a better K?