ASE2010 Applied linear algebra: Homework #2

1) *Orthogonal projection.* Let x be an n-vector and $u_1, \ldots, u_k$ with $k < n$ be orthonormal n-vectors. The projection of x onto the span of $u_1, \ldots, u_k$ is

$$\hat{x} = (u_1^T x)u_1 + \cdots + (u_k^T x)u_k \in \mathbb{R}^n$$

and the projection of x onto the span of $u_1, \ldots, u_{k-1}$ is

$$\tilde{x} = (u_1^T x)u_1 + \cdots + (u_{k-1}^T x)u_{k-1} \in \mathbb{R}^n.$$

Show that $\hat{x}$ is closer to x than $\tilde{x}$ in that

$$\|\hat{x} - x\| \leq \|\tilde{x} - x\|.$$

2) *VMLS Exercises.*

a) **5.1** *Linear independence of stacked vectors.*

b) **5.4** *Norm of linear combination of orthonormal vectors.*

c) **5.5** *Orthogonalizing vectors.*

d) **5.6** *Gram–Schmidt algorithm.*

e) **5.8** *Early termination of Gram–Schmidt algorithm.*

f) **6.3** *Block matrix.*

g) **6.8** *Cash flow to back account balance.*

h) **6.10** *Resource requirements.*

i) **6.12** *Skew-symmetric matrices.*

j) **6.14** *Norm of matrix-vector product.*

k) **6.17** *Stacked matrix.*

l) **6.18** *Vandermonde matrices.*