Features

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Records and embedding

Raw data

- *raw data* pairs are (u, v), with $u \in \mathcal{U}$, $v \in \mathcal{V}$
- $\mathcal{U}$ is set of all possible input values
- $\mathcal{V}$ is set of all possible output values
- each u is called a *record*
- typically a record is a tuple, or list, $u = (u_1, u_2, \ldots, u_r)$
- each u_i is a *field* or *component*, which has a *type*, *e.g.*, real number, Boolean, categorical, ordinal, word, text, audio, image, parse tree (more on this later)
- *e.g.*, a record for a house for sale might consist of

  (address, photo, description, house/apartment?, lot size, . . . , # bedrooms)

Feature map

- learning algorithms are applied to (x, y) pairs,

$$x = \phi(u), \qquad y = \psi(v)$$

- $\phi : \mathcal{U} \to \mathbf{R}^d$ is the *feature map* for u
- $\psi : \mathcal{V} \to \mathbf{R}$ is the *feature map* for v
- feature maps transform *records* into *vectors*
- feature maps usually work on each field separately,

$$\phi(u_1, \ldots, u_r) = (\phi_1(u_1), \ldots, \phi_r(u_r))$$

- ϕ_i is an *embedding* of the type of field i into a vector

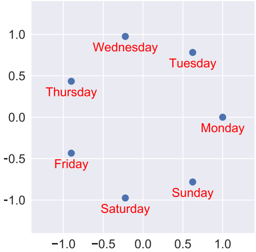
Embeddings

- embedding puts the different field types on an equal footing, *i.e.*, vectors
- some embeddings are simple, *e.g.*,
  - for a number field ($\mathcal{U} = \mathbf{R}$), $\phi_i(u_i) = u_i$
  - for a Boolean field, $\phi_i(u_i) = \begin{cases} 1 & u_i = \text{TRUE} \\ -1 & u_i = \text{FALSE} \end{cases}$
- others are more sophisticated
  - text to TFID histogram
  - `word2vec` (maps words into vectors)
  - pre-trained ImageNet NN (maps images into vectors)

(more on these later)

More embeddings

- color to (R, G, B)
- geolocation data: $\phi(u) =$(Lat,Long) in $\mathbf{R}^2$ or embed in $\mathbf{R}^3$
- day of week:



Faithful embeddings

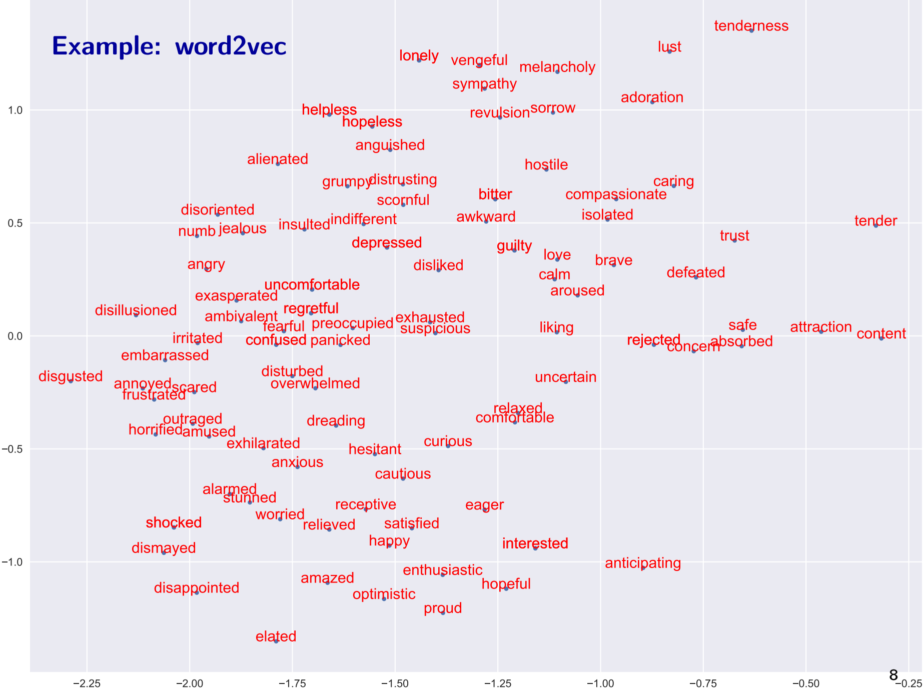
a *faithful* embedding satisfies

- $\phi(u)$ is near $\phi(\tilde{u})$ when u and $\tilde{u}$ are 'similar'
- $\phi(u)$ is not near $\phi(\tilde{u})$ when u and $\tilde{u}$ are 'dissimilar'

- lefthand concept is *vector distance*
- righthand concept depends on field type, application

- interesting examples: names, professions, companies, countries, languages, ZIP codes, cities, songs, movies
- we will see later how such embeddings can be constructed

Example: word2vec



Standardized embeddings

usually assume that an embedding is *standardized*

- entries of $\phi(u)$ are centered around 0
- entries of $\phi(u)$ have RMS value around 1
- roughly speaking, entries of $\phi(u)$ ranges over ± 1

- with standarized embeddings, entries of feature map

$$\phi(u_1, \ldots, u_r) = (\phi_1(u_1), \ldots, \phi_r(u_r))$$

  are all comparable, *i.e.*, centered around zero, standard deviation around one
- $\mathrm{rms}(\phi(u) - \phi(\tilde{u}))$ is reasonable measure of how close records u and $\tilde{u}$ are

Standardization or z-scoring

- suppose $\mathcal{U} = \mathbf{R}$ (field type is real numbers)
- for data set $u^1, \ldots, u^n \in \mathbf{R}$

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u^i \qquad \mathrm{std}(u) = \left(\frac{1}{n}\sum_{i=1}^{n}(u^i - \bar{u})^2\right)^{\frac{1}{2}}$$

- the *z-score* or *standardization* of u is the embedding

$$x = \mathrm{zscore}(u) = \frac{1}{\mathrm{std}(u)}(u - \bar{u})$$

- ensures that embedding values are centered at zero, with standard deviation one
- z-scored features are very easy to interpret: $x = \phi(u) = +1.3$ means that u is 1.3 standard deviations above the mean value

Standardized data matrix

- suppose all d (real) features have been standardized
- columns of $n \times d$ feature matrix X have zero mean, RMS value one
- $(1/n)X^T X = \Sigma$ is the *feature correlation matrix*
- $\Sigma_{ii} = 1$ (since each column of X has RMS value 1, and so norm $\sqrt{n}$)
- Σ_{ij} is *correlation coefficient* of ith and jth raw features

Log transform

- old school rule-of-thumb: if field u is positive and ranges over wide scale, embed as $\phi(u) = \log u$ (or $\log(1+u)$) (and then standarize)
- examples: web site visits, ad views, company capitalization
- interpretation as faithful embedding:
  - 20 and 22 are similar, as are 1000 and 1100
  - but 20 and 120 are not similar
  - *i.e.*, you care about fractional or relative differences between raw values

  (here, log embedding is faithful, affine embedding is not)
- can also apply to output or label field, *i.e.*, $y = \psi(v) = \log v$ if you care about percentage or fractional errors; recover $\hat{v} = \exp(\hat{y})$

Example: House price prediction

- we want to predict house selling price v from record $u = (u_1, u_2)$
  - $u_1 =$ area (sq. ft.)
  - $u_2 = \#$ bedrooms
- we care about relative error in price, so we embed v as $\psi(v) = \log v$ (and then standardize)
- we standardize fields u_1 and u_2

$$x_1 = \frac{u_1 - \mu_1}{\sigma_1}, \qquad x_2 = \frac{u_2 - \mu_2}{\sigma_2}$$

  - $\mu_1 = \bar{u}_1$ is mean area
  - $\mu_2 = \bar{u}_2$ is mean number of bedrooms
  - $\sigma_1 = \mathrm{std}(u_1)$ is std. dev. of area
  - $\sigma_2 = \mathrm{std}(u_2)$ is std. dec. of $\#$ bedrooms

(means and std. dev. are over our data set)

Example: House price regression model

- regression model: $\hat{y} = \theta_1 + \theta_2 x_1 + \theta_3 x_2$
- in terms of original raw data:

$$\hat{v} = \exp\left(\theta_1 + \theta_2 \frac{u_1 - \mu_1}{\sigma_1} + \theta_3 \frac{u_2 - \mu_2}{\sigma_2}\right)$$

- $\mathtt{exp}$ undoes log embedding of house price

Vector embeddings

Vector embeddings for real field

- we can embed a field u into a vector $x = \phi(u) \in \mathbf{R}^k$
- useful even when $\mathcal{U} = \mathbf{R}$ (real field)
- polynomial embedding:

$$\phi(u) = (1, u, u^2, \ldots, u^d)$$

- piecewise linear embedding:

$$\phi(u) = (1, (u)_-, (u)_+)$$

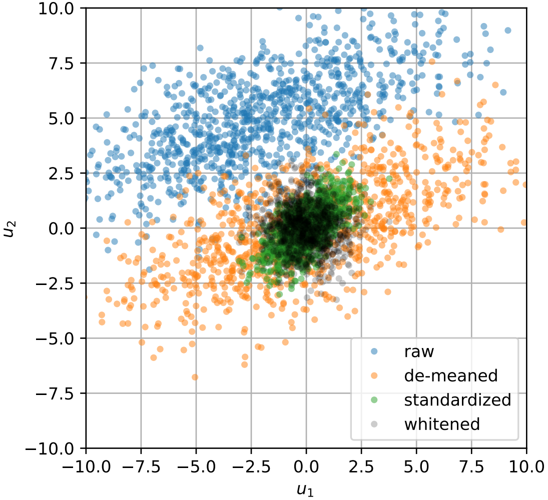
  where $(u)_- = \min(u, 0)$, $(u)_+ = \max(u, 0)$

- regression with these features yield polynomial and piecewise linear predictors

Whitening

- analog of standardization for raw data $\mathcal{U} = \mathbf{R}^d$
- start with raw data, $n \times d$ matrix U
- $\bar{u} = U^T \mathbf{1}/n$ is vector of column means
- $\tilde{U} = U - \mathbf{1}\bar{u}^T$ is de-meaned data matrix
- $\tilde{U} = QR$ is its QR factorization
- $X = \sqrt{n}Q = \sqrt{n}\tilde{U}R^{-1}$ defines embedding $x^i = \phi(u^i)$
  - columns of X have zero mean and RMS value one
  - columns of X are orthogonal
  - features are uncorrelated
  - feature correlation matrix is $\Sigma = I$

Whitening example



Categorical data

- data field is *categorical* if it only takes a finite number of values
- *i.e.*, $\mathcal{U}$ is a finite set $\{\alpha_1, \ldots, \alpha_k\}$
- examples:
  - $\textsc{true}/\textsc{false}$ (two values, also called Boolean)
  - $\textsc{apple}$, $\textsc{orange}$, $\textsc{banana}$ (three values)
  - $\textsc{monday}, \ldots, \textsc{sunday}$ (seven values)
  - ZIP code (40000 values)
- *one-hot embedding for categoricals*: $\phi(\alpha_i) = e_i \in \mathbf{R}^k$

$$\phi(\textsc{apple}) = (1, 0, 0), \quad \phi(\textsc{orange}) = (0, 1, 0), \quad \phi(\textsc{banana}) = (0, 0, 1)$$

- standardizing these features handles *unbalanced* data

Ordinal data

- ordinal data is categorical, with an order
- example: *Likert scale*, with values

  STRONGLY DISAGREE, DISAGREE, NEUTRAL, AGREE, STRONGLY AGREE

- can embed into **R** with values $-2, -1, 0, 1, 2$
- or treat as categorical, with one-hot embedding into $\mathbf{R}^5$
- example: number of bedrooms in house
  - can be treated as a real number
  - or as an ordinal with (say) values $1, \ldots, 6$

Feature engineering

How feature maps are constructed

- start by embedding each field

$$\phi(u_1, \ldots, u_r) = (\phi_1(u_1), \ldots, \phi_r(u_r))$$

- can then standardize, if needed
- use *feature engineering* to create new features from existing ones

Creating new features

- product features: $x_{\mathrm{new}} = x_i x_j$ (models *interactions* between features)
- max features: $x_{\mathrm{new}} = \max(x_i, x_j)$ (can also use min)
- positive/negative parts:

$$x_{\mathrm{new}+} = (x_i)_+ = \max(x_i, 0), \qquad x_{\mathrm{new}-} = (x_i)_- = \min(x_i, 0)$$

- random features:
  - choose random matrix R
  - new features are $(Rx)_+$ or $(Rx)_-$

Un-embedding

Un-embedding

- we embed v as $y = \psi(v)$, $\psi : \mathcal{V} \to \mathbf{R}$
- we need to 'invert' this operation, and go from $\hat{y}$ to $\hat{v}$
- when the inverse function exists, we use $\psi^{-1} : \mathbf{R} \to \mathcal{V}$
- example: log embedding $y = \log v$ has inverse $v = \exp y$
- prediction stack:
  1. *embed*: given record u, feature vector is $x = \phi(u)$
  2. *predict*: $\hat{y} = g(x)$
  3. *un-embed:* $\hat{v} = \psi^{-1}(\hat{y})$
- final predictor is $\hat{v} = \psi^{-1}(g(\phi(u)))$

Un-embedding

- in many cases, the inverse of ψ function doesn't exist
- for example, embedding a Boolean or ordinal into **R**
- for the purposes of un-embedding, we define

$$\psi^{-1}(y) = \underset{v \in \mathcal{V}}{\mathrm{argmin}} \|y - \psi(v)\|$$

  i.e., we choose the value of v for which $\psi(v)$ is closest to y
- example: embed $\textsc{true} \mapsto 1$ and $\textsc{false} \mapsto -1$
- un-embed via

$$\psi^{-1}(y) = \begin{cases} \textsc{true} & \text{if } y > 0 \\ \textsc{false} & \text{otherwise} \end{cases}$$

Example: Un-embedding one-hot

- *one-hot embedding*: $\phi(u) = e_u$ for $\mathcal{U} = \{1, \ldots, d\}$
- un-embed

$$\phi^{-1}(x) = \operatorname*{argmin}_u ||x - e_u||_2 = \operatorname*{argmax}_u x_u$$