ASE3093 Automatic Control: Homework #5

1) *Complementary Filter in the Frequency Domain.* Suppose two sensors, $x_1(t)$ and $x_2(t)$, are both measuring the same underlying physical signal (e.g., an orientation angle), but with different error characteristics across the frequency spectrum:

- $x_1(t)$ is obtained from an accelerometer — accurate at low frequencies but dominated by high-frequency noise,
- $x_2(t)$ is obtained from a gyroscope — reliable at high frequencies but subject to low-frequency drift.

To fuse these signals in a frequency-sensitive manner, a *complementary filter* is used. The estimated signal $\hat{x}(s)$ is formed as:

$$\hat{x}(s) = G_{\mathrm{LP}}(s)x_1(s) + G_{\mathrm{HP}}(s)x_2(s)$$

where:

$$G_{\mathrm{LP}}(s) = \frac{1}{\tau s + 1}, \qquad G_{\mathrm{HP}}(s) = \frac{\tau s}{\tau s + 1}$$

These filters are complementary, satisfying:

$$G_{\mathrm{LP}}(s) + G_{\mathrm{HP}}(s) = 1$$

a) Show that $G_{\mathrm{LP}}(s) + G_{\mathrm{HP}}(s) = 1$ for all s.

b) Sketch or plot the Bode magnitude plots of $G_{\mathrm{LP}}(s)$ and $G_{\mathrm{HP}}(s)$ for $\tau = 0.1$.

c) Describe the behavior of both filters in the following frequency regimes:
   - Low frequencies ($\omega \ll \frac{1}{\tau}$),
   - High frequencies ($\omega \gg \frac{1}{\tau}$),
   - Crossover frequency ($\omega = \frac{1}{\tau}$).

d) Explain how the complementary filter leverages the strengths of both x_1 and x_2 to provide a more robust and accurate estimate $\hat{x}(t)$ across the entire frequency range.

2) *Bode plots.* For each of the following systems, sketch the Bode magnitude and phase plots by hand. Then verify your results using a computer-based tool (e.g., python or MATLAB).

a) $$G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}$$

b) $$G(s) = \frac{1000(s+1)}{s(s+2)(s^2+8s+64)}$$

c) $$G(s) = \frac{4s(s+10)}{(s+50)(4s^2+5s+4)}$$

d) $$G(s) = \frac{s+2}{s^2(s+20)}$$

e) $$G(s) = \frac{(s+0.5)(s+1.5)}{s(s^2+2s+2)(s+5)(s+15)}$$

f) $$G(s) = \frac{s+1}{(s+2)(s+10)}$$

g) $$G(s) = \frac{s-1}{(s-2)(s+10)}$$