Automatic Control

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Dynamics

Rotational Motion

Rotational Motion

- Analogous to

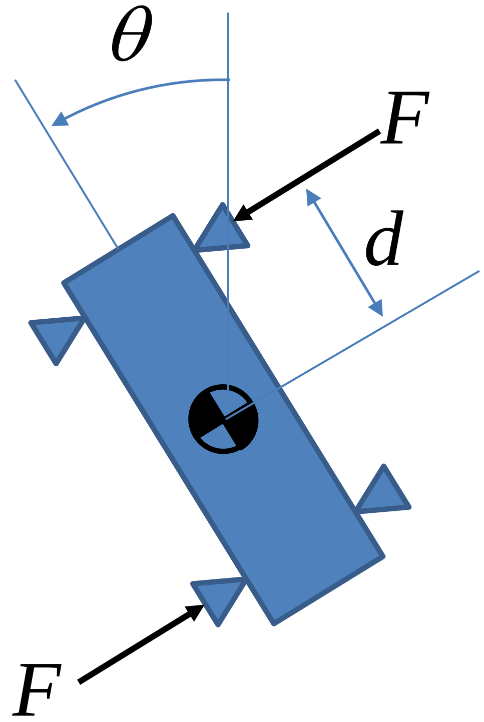
$$\mathbf{F} = m\mathbf{a}$$

- For rotation

$$\mathbf{M} = I\boldsymbol{\alpha}$$

(Comes from the conservation of angular momentum)

Satellite Attitude



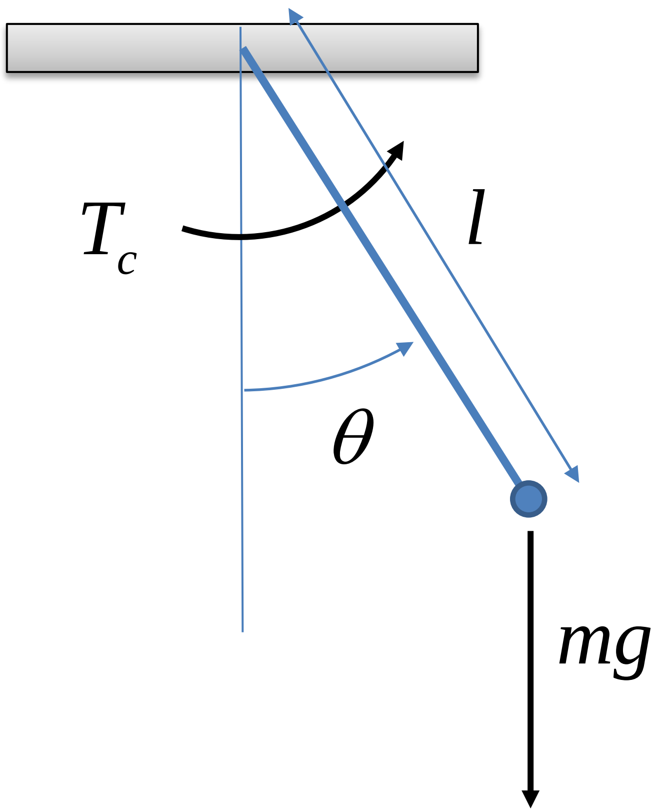
$$I\ddot{\theta} = 2Fd$$

Moment-of-Inertia

- Mass – “resistance to acceleration”
- Moment-of-inertia – “resistance to rotational acceleration”

$$I = \int_A \left(x^2 + y^2\right) dm$$

Pendulum – Point Mass



$$I = \int_A \left(x^2 + y^2\right) dm = ml^2$$

$$\sum \mathbf{M} = T_c - mgl\sin\theta = I\ddot{\theta}$$

$$T_c - mgl\sin\theta = ml^2\ddot{\theta}$$

$$ml^2\ddot{\theta} + mgl\sin\theta = T_c$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

Linearize

- For small angles

$$\sin\theta \approx \theta$$

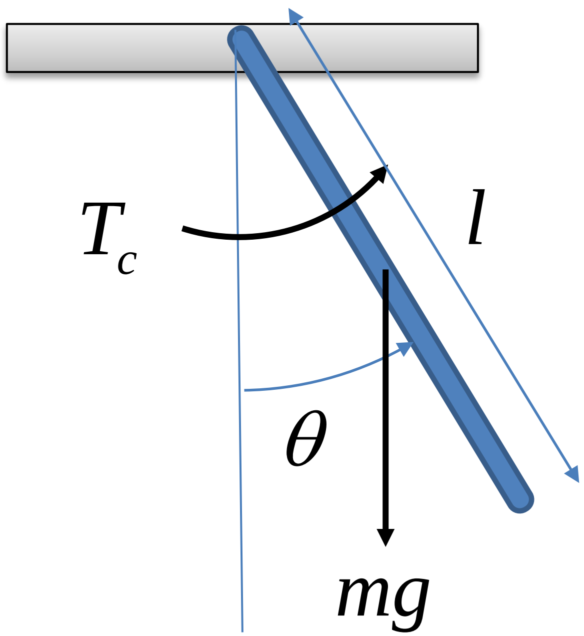
- But how small?

θ	θ (Radians)	$\sin\theta$	Error, %
5	0.0873	0.0872	0.13
10	0.1745	0.1736	0.51
15	0.2618	0.2588	1.15
20	0.3491	0.3420	2.06
25	0.4363	0.4226	3.25
30	0.5236	0.5000	4.72

- Linearized

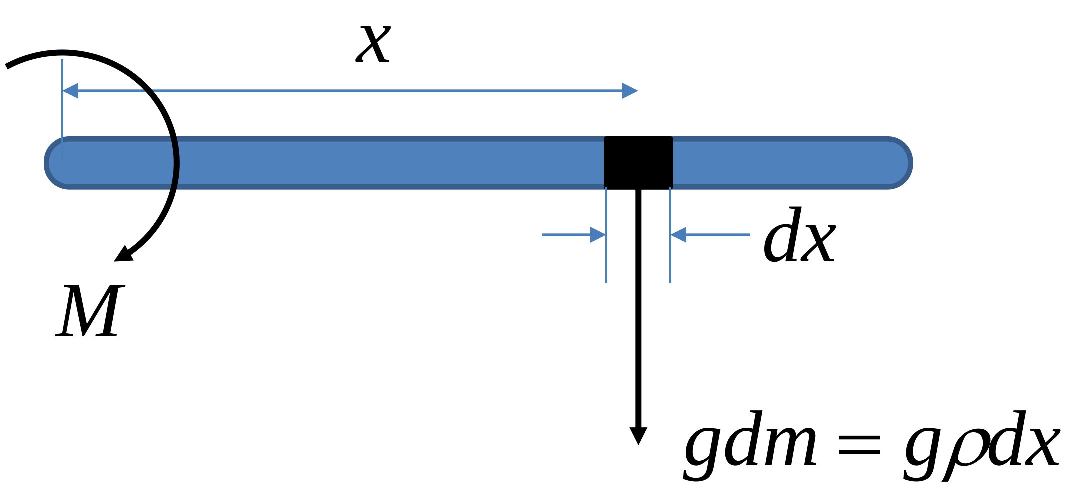
$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

Pendulum - Rod



$$\sum \mathbf{M} = T_c - mg\left(\frac{l}{2}\right)\sin\theta = I\ddot{\theta}$$

Moment-of-Inertia, Moment Arm

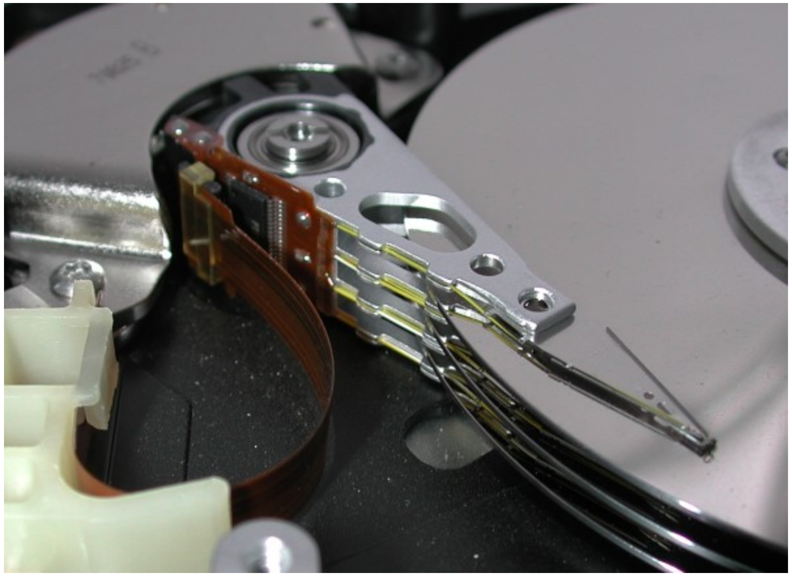


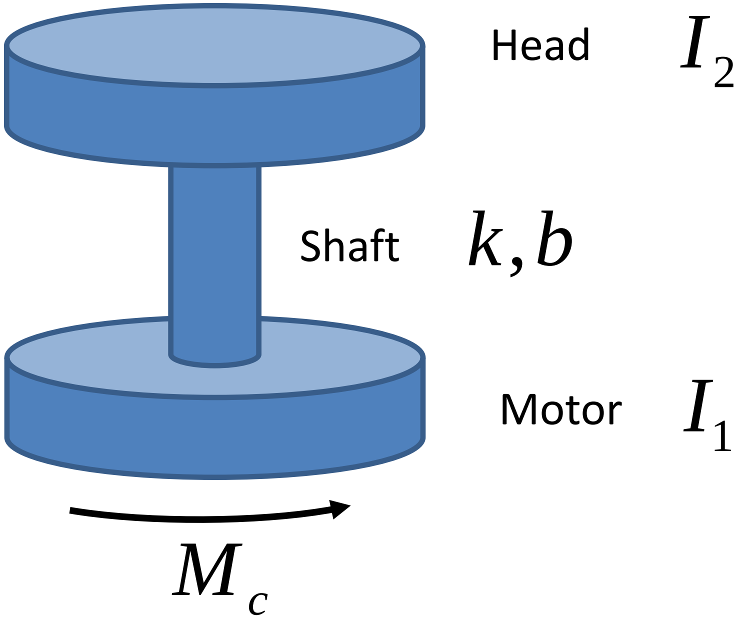
$$\rho l = m$$

$$M = \int_0^l x \bullet g\rho dx = \rho g \int_0^l x dx = \frac{1}{2}\rho g l^2 = \rho g l \times \left(\frac{l}{2}\right) = mg \times \left(\frac{l}{2}\right)$$

$$I = \int_0^l x^2 dm = \int_0^l x^2 \rho dx = \rho \int_0^l x^2 dx = \frac{1}{3}\rho l^3 = \frac{1}{3}(\rho l)l^2 = \frac{1}{3} m l^2$$

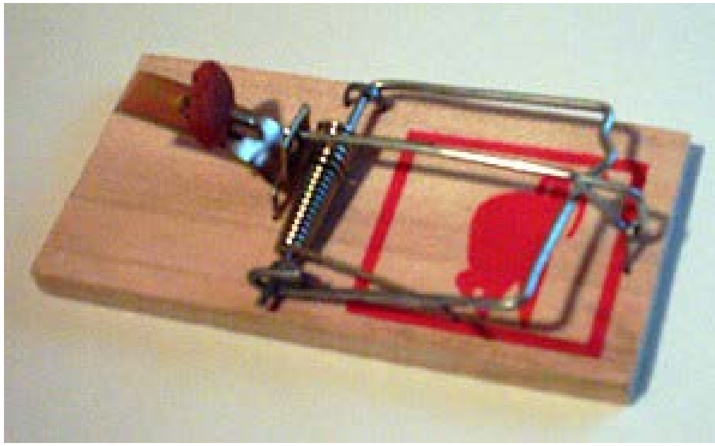
A Two Mass System - Rotation





- Hard disk head control
  - Two rotating mass: motor and head
  - One spring: shaft

Rotational Spring and Damper

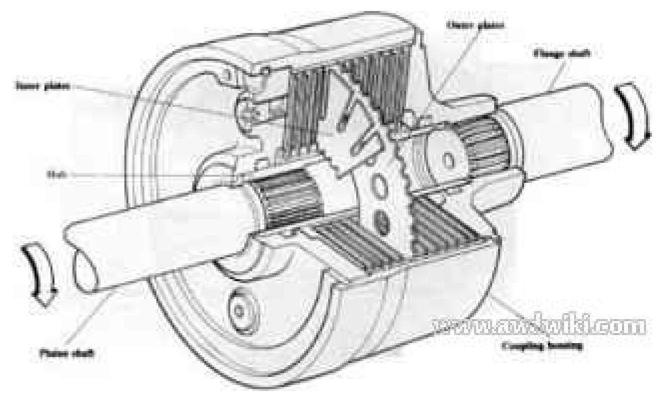


$$M_s = -k\theta$$

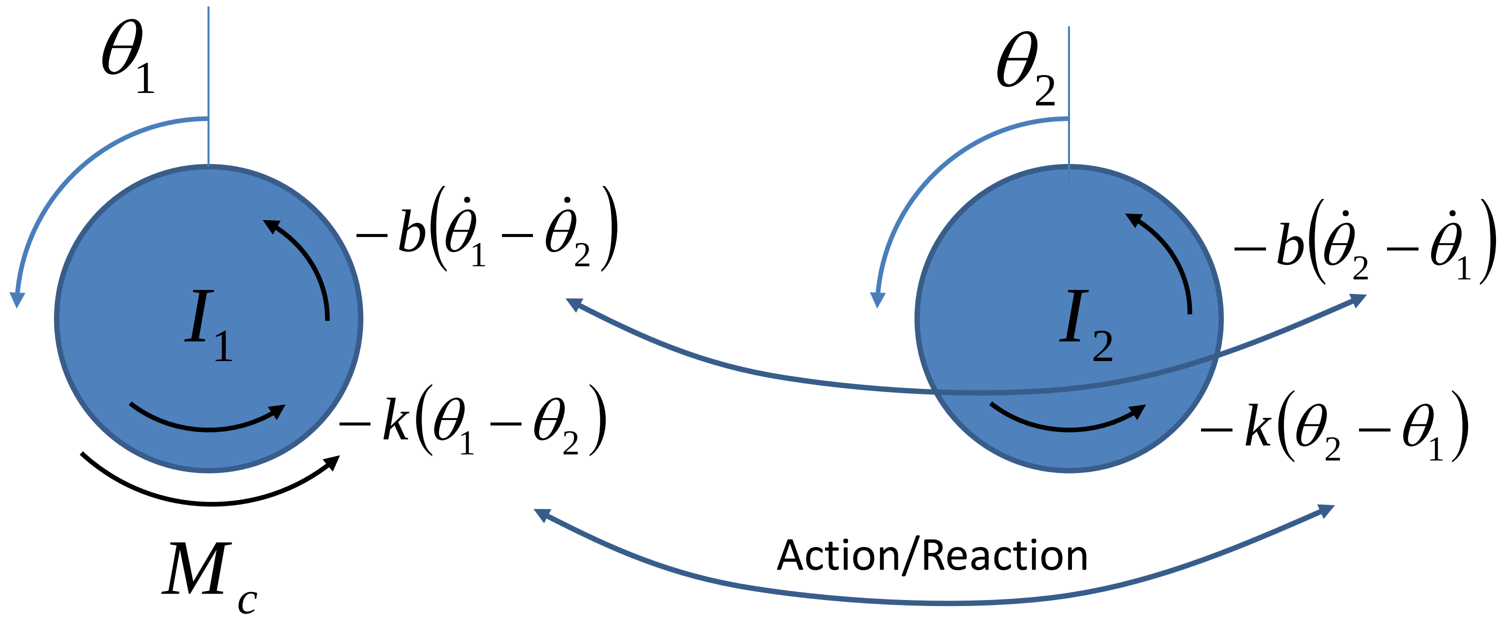
Spring, flexible shaft

$$M_d = -b\dot{\theta}$$

Bearing friction, viscous coupling



Equation of Motion



$$I_1\ddot{\theta}_1 = M_c - b(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2)$$

$$I_2\ddot{\theta} = -b(\dot{\theta}_2 - \dot{\theta}_1) - k(\theta_2 - \theta_1)$$