

자동제어(Automatic Control)

4장 블록선도와 신호흐름선도

교재: Automatic Control Systems

블록선도

WHY?

1. 단순성 & 융통성
2. 시스템의 구성이나 연결관계 표현 또는 전달함수와 함께 시스템 전체의 인과관계 표시
3. 선형 및 비선형시스템의 모델링

Block Diagrams

4-1-1 Modeling of Typical Elements of Block Diagrams in Control Systems

The common elements in block diagrams of most control systems include

- Comparators
- Blocks representing individual component transfer functions, including
 - Reference sensor (or input sensor)
 - Output sensor
 - Actuator
 - Controller
 - Plant (the component whose variables are to be controlled)
- Input or reference signals¹
- Output signals
- Disturbance signal
- Feedback loops

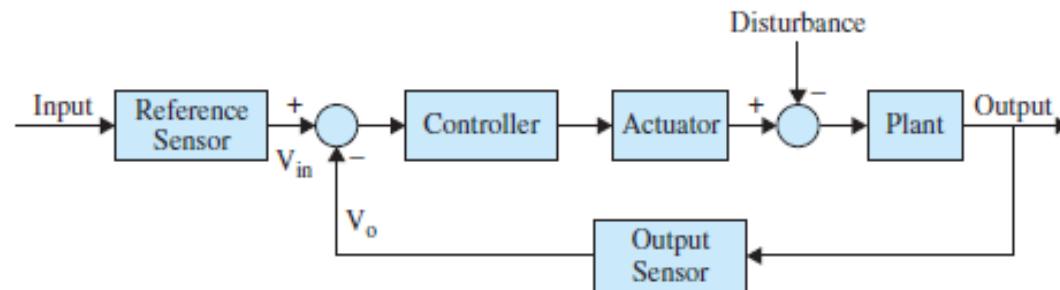
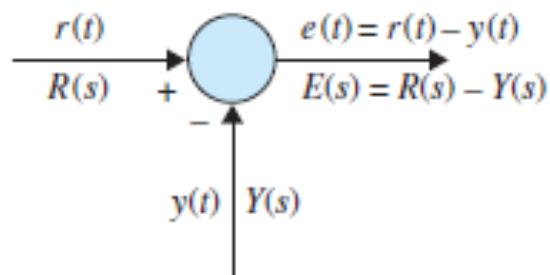
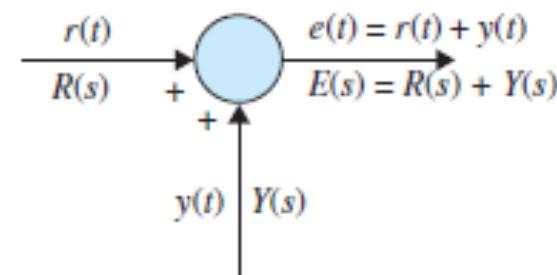


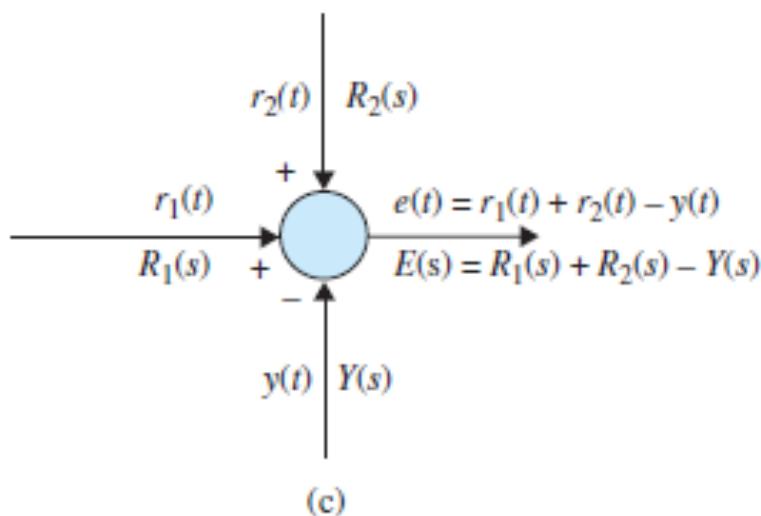
Figure 4-2 Block diagram representation of a general control system.



(a)



(b)



A comparator
performs addition
and subtraction

Figure 4-3 Block diagram elements of typical sensing devices of control systems. (a) Subtraction. (b) Addition. (c) Addition and subtraction.

In Laplace domain, the following input-output relationship can be written for the system in Fig. 4-4:

$$X(s) = G(s)U(s) \quad (4-4)$$

If signal $X(s)$ is the output and signal $U(s)$ denotes the input, the transfer function of the block in Fig. 4-4 is

$$G(s) = \frac{X(s)}{U(s)} \quad (4-5)$$

Typical block elements that appear in the block diagram representation of most control systems include **plant**, **controller**, **actuator**, and **sensor**.

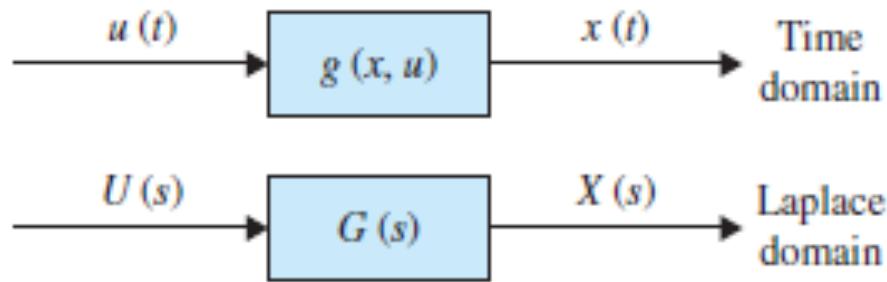


Figure 4-4 Time and Laplace domain block diagrams.

EXAMPLE 4-1-1

$$X(s) = A(s)G_2(s)$$

$$A(s) = U(s)G_1(s)$$

$$X(s) = G_1(s)G_2(s)$$

$$G(s) = \frac{X(s)}{U(s)}$$

$$G(s) = G_1(s)G_2(s) \quad (4-6)$$

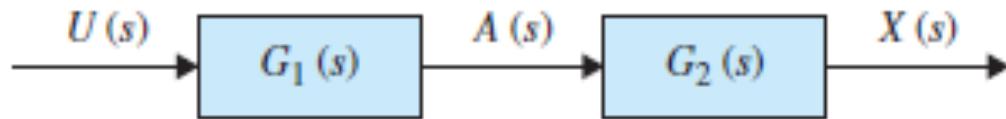


Figure 4-5 Block diagrams $G_1(s)$ and $G_2(s)$ connected in series—a cascade system.

EXAMPLE 4-1-2

$$A_1(s) = U(s)$$

$$A_2(s) = A_1(s)G_1(s)$$

$$A_3(s) = A_1(s)G_2(s)$$

$$X(s) = A_2(s) + A_3(s)$$

$$X(s) = U(s)(G_1(s) + G_2(s))$$

$$G(s) = \frac{X(s)}{U(s)}$$

(4-7)

$$G(s) = G_1(s) + G_2(s)$$

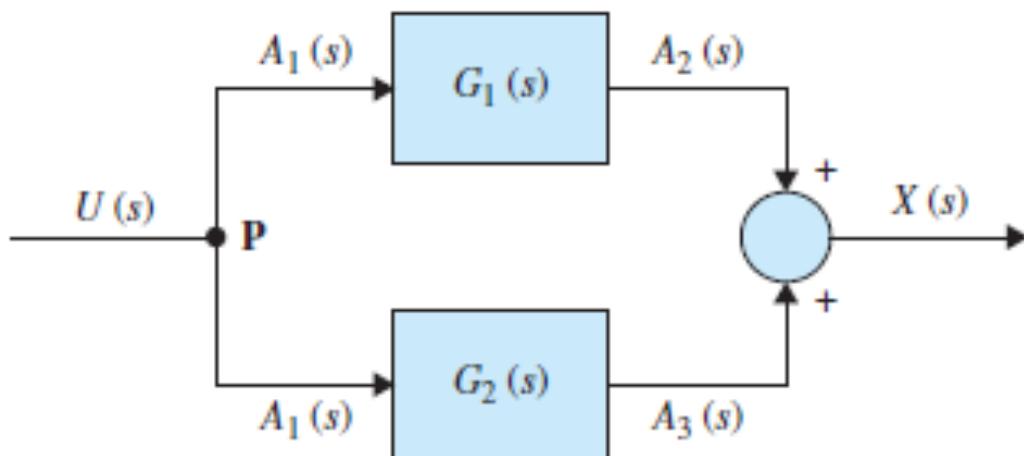
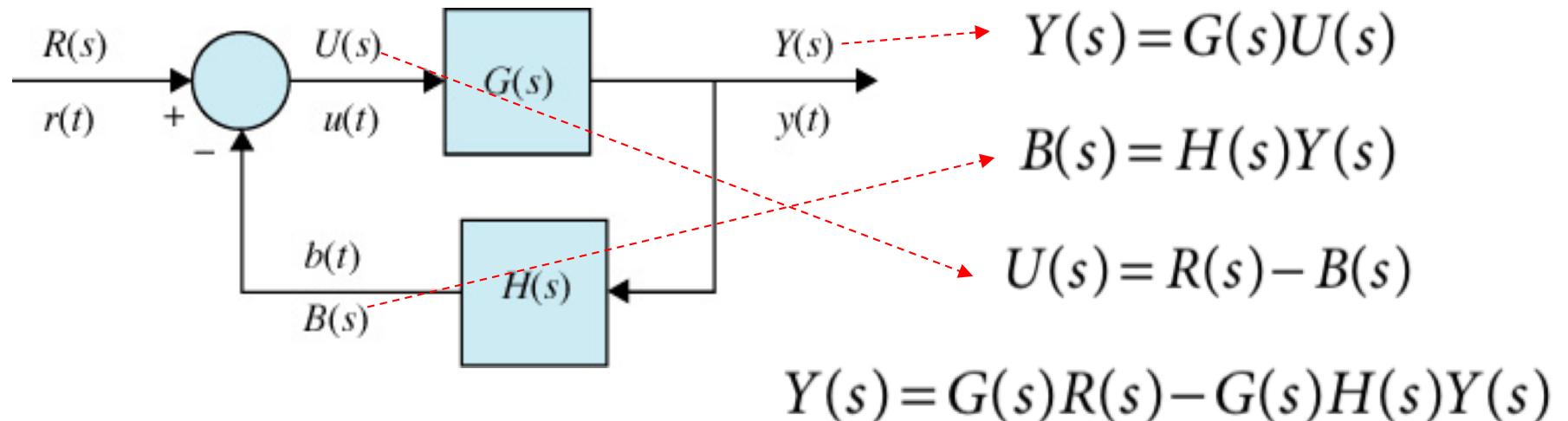


Figure 4-6 Block diagrams $G_1(s)$ and $G_2(s)$ connected in parallel.

피드백 제어시스템의 기본 블록선도



$r(t), R(s)$ =기준입력(지시)

$y(t), Y(s)$ =출력(제어변수)

$b(t), B(s)$ =피드백신호

$u(t), U(s)$ =실행신호 = 오차신호 $e(t), E(s)$ [$H(s)=1$ 일 때]

$H(s)$ =피드백전달함수

$G(s)H(s)=L(s)$ =루프전달함수

$G(s)$ =전방경로전달함수

$M(s)=Y(s)/R(s)$ =폐루프전달함수 또는 시스템전달함수

$$M(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

3-1-2 Relation between Mathematical Equations and Block Diagrams

$$\omega_n^2 U(s) - 2\zeta\omega_n X(s)s - \omega_n^2 X(s) = X(s)s^2 \quad (4-16)$$

Figure 4-8 Graphical representation of Eq. (4-16) using a comparator.

$$\frac{V(s)}{U(s)} = \frac{\omega_n^2 s}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4-20)$$

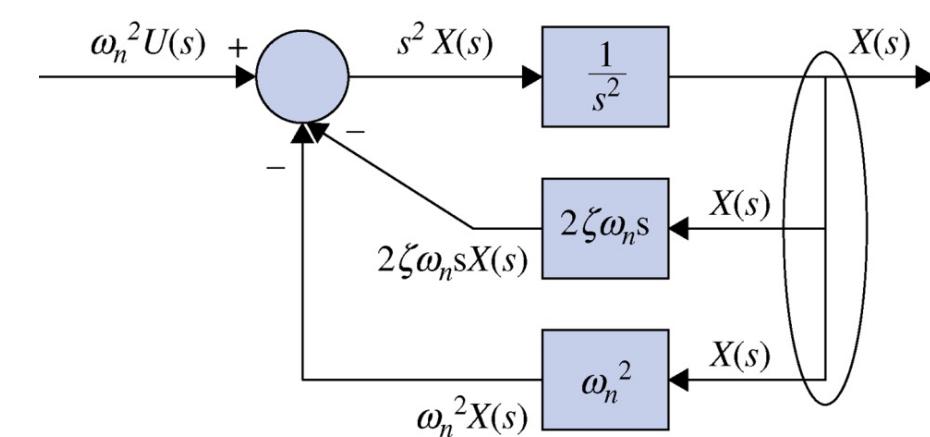


Figure 4-9 Addition of blocks $1/s^2$, $2\zeta\omega_n s$, and ω_n^2 to the graphical representation of Eq. (4-16).

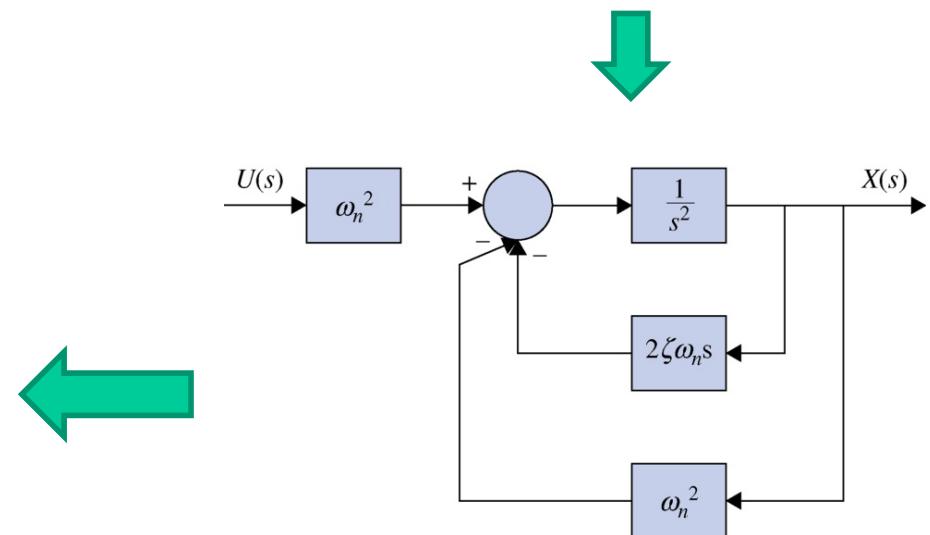


Figure 4-10 Block diagram representation of Eq. (4-16) in Laplace domain.

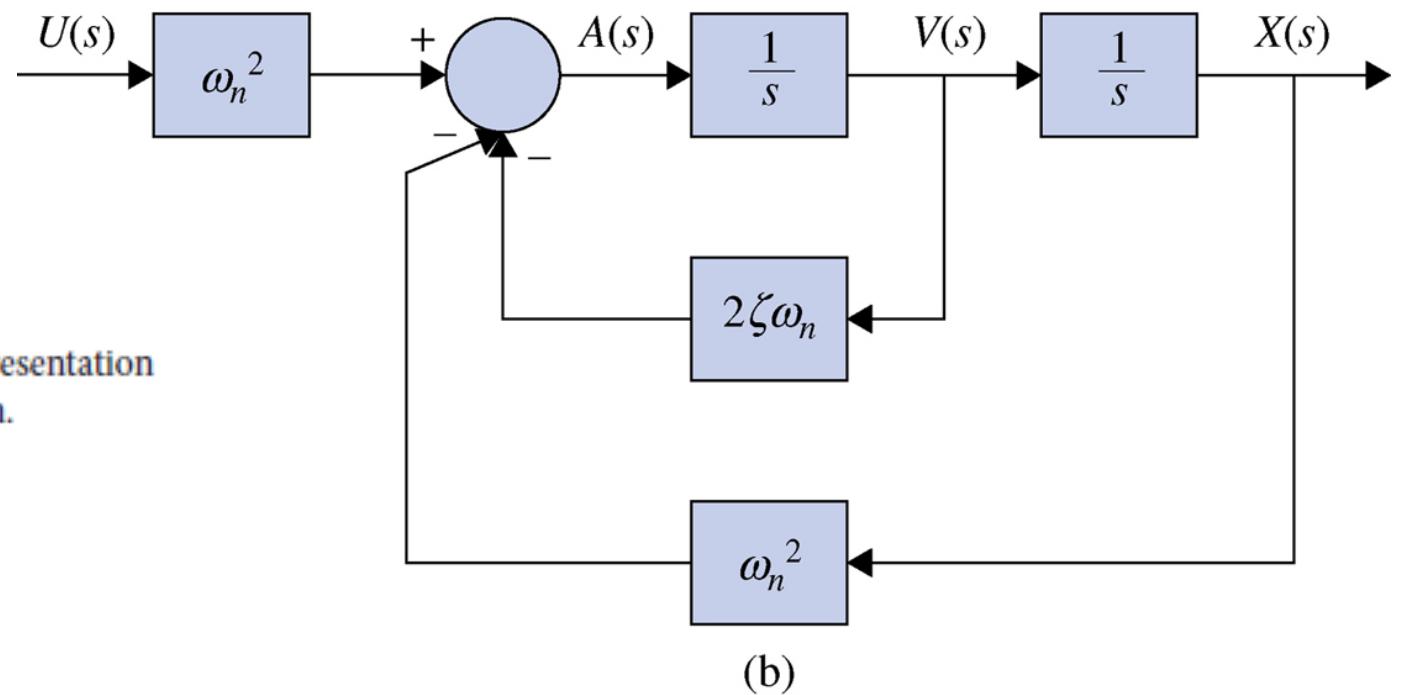
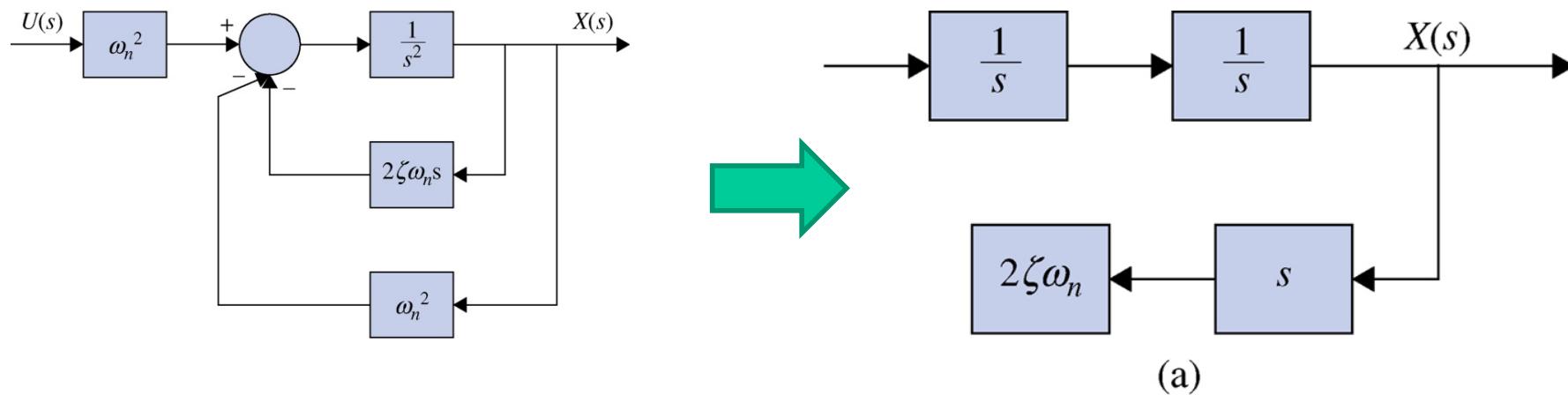


Figure 4-10 Block diagram representation of Eq. (4-16) in Laplace domain.

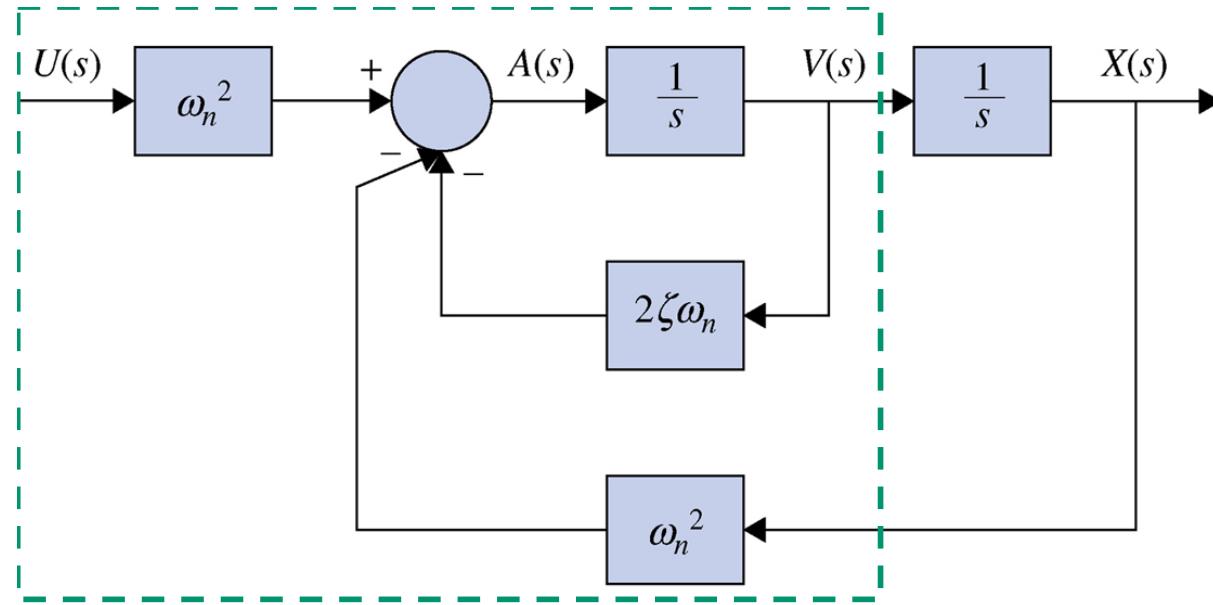


Figure 4-11 (a) Factorization of $1/s$ term in the internal feedback loop of Fig. 4-10. (b) Final block diagram representation of Eq. (4-16) in Laplace domain.

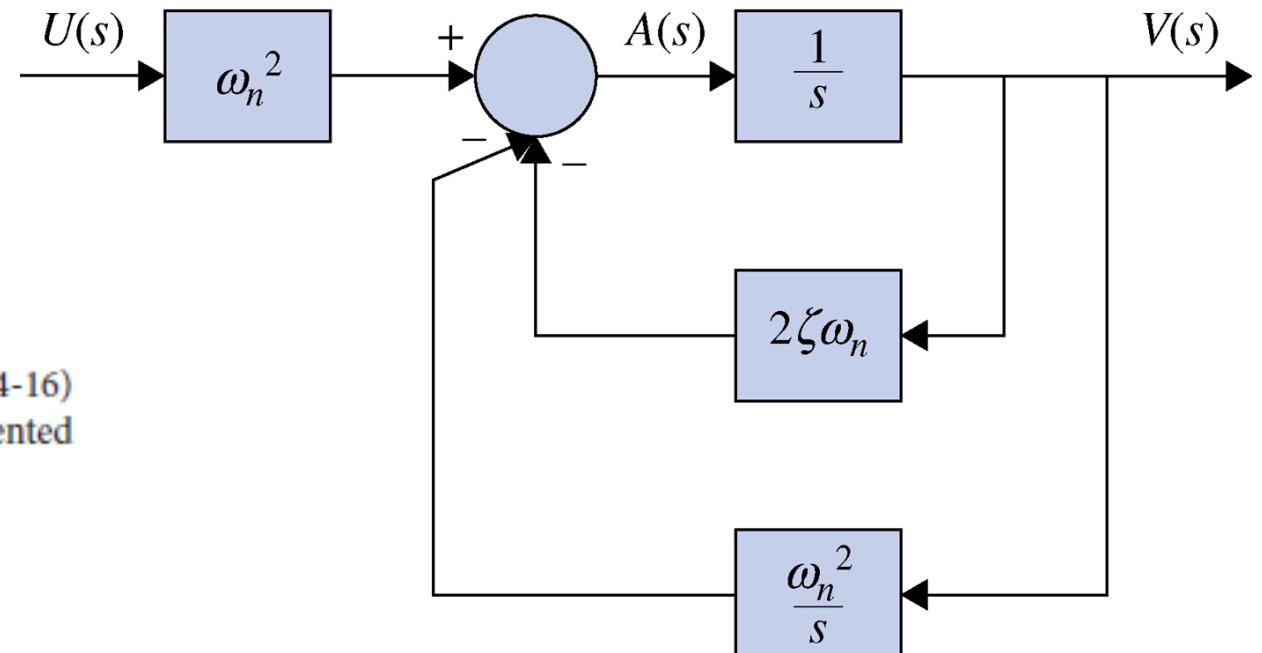
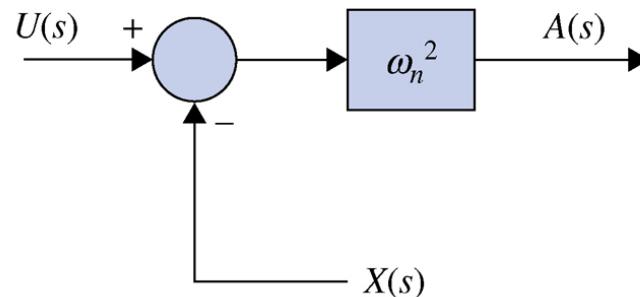
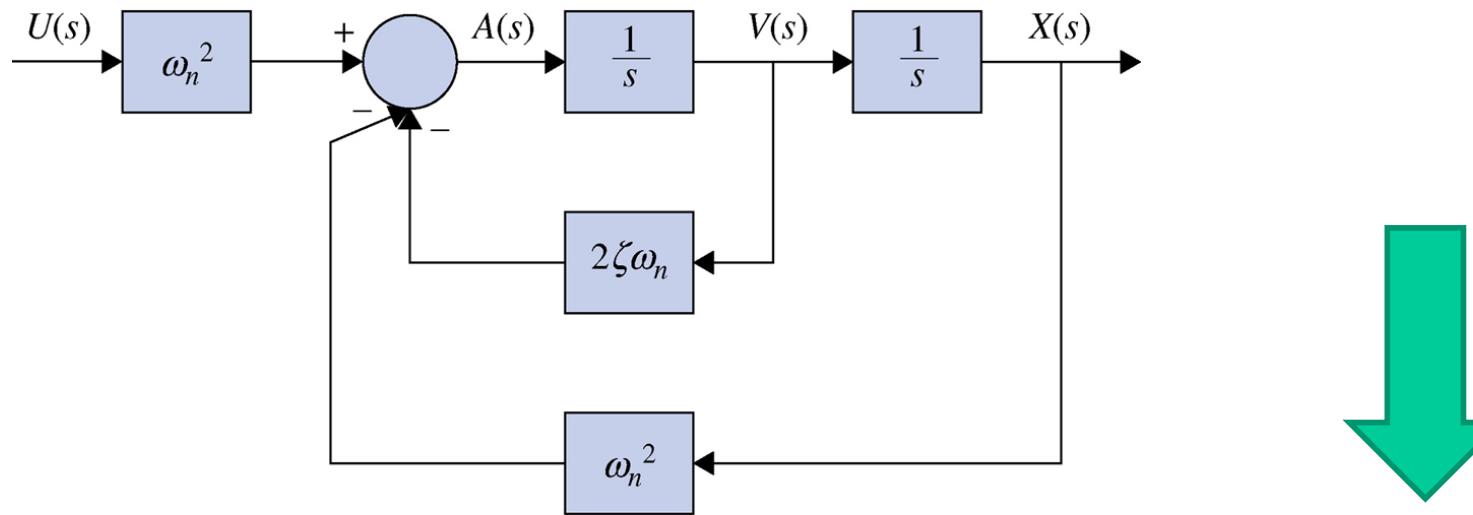
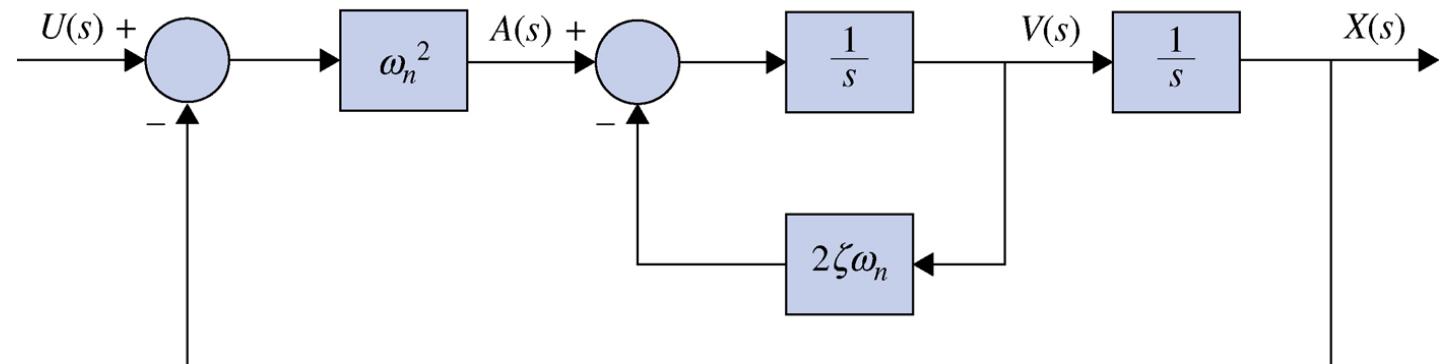


Figure 4-12 Block diagram of Eq. (4-16) in Laplace domain with $V(s)$ represented as the output.



(a)



(b)

Figure 4-13 (a) Factorization of ω_n^2 . (b) Alternative block diagram representation of Eq. (4-16) in Laplace domain.

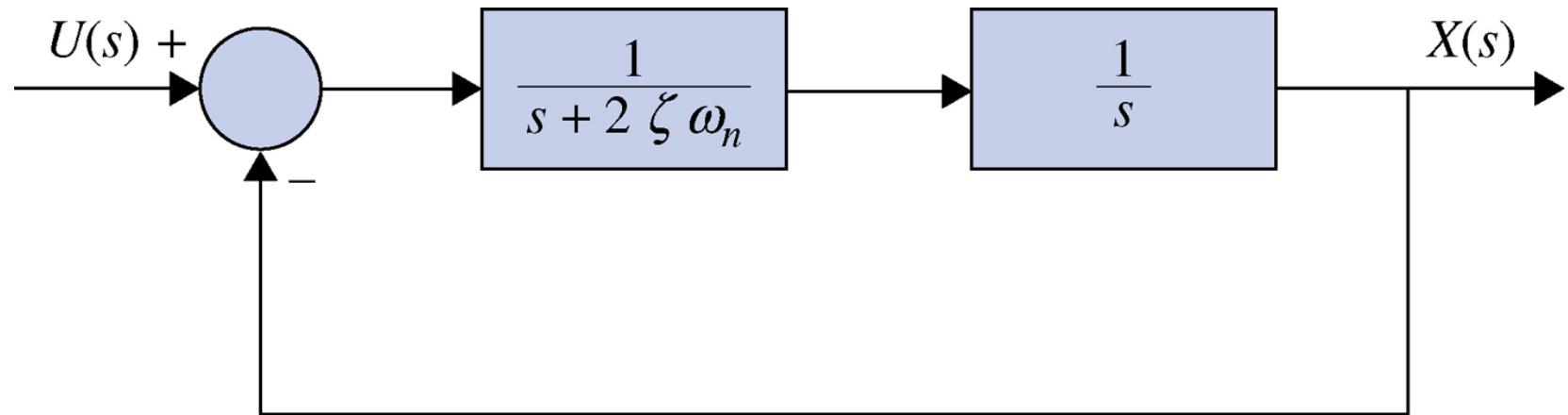
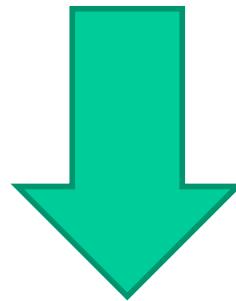
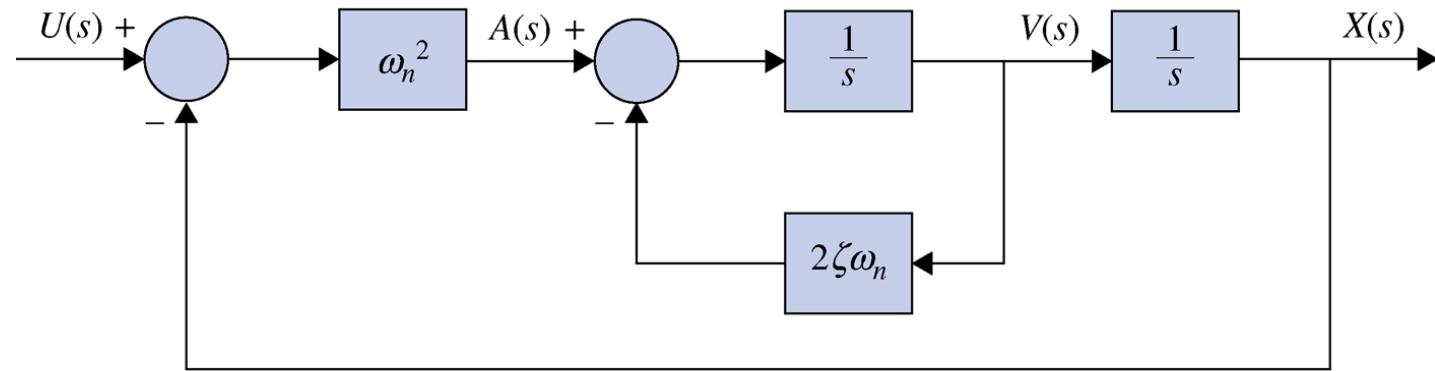
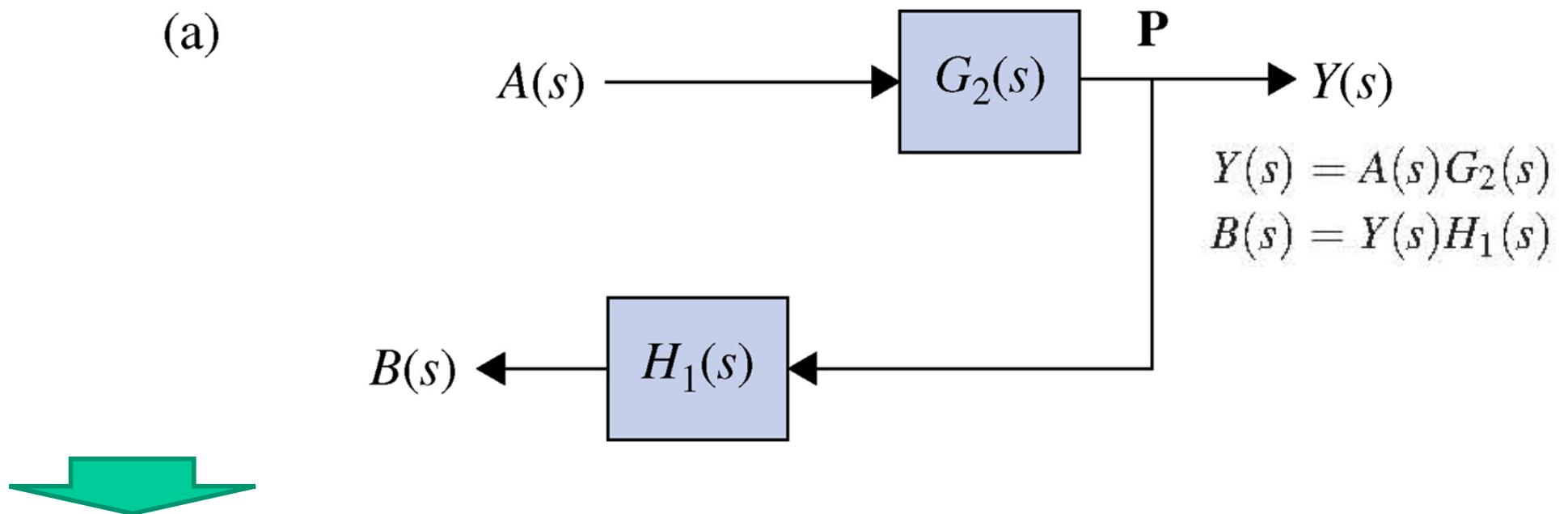


Figure 4-14 A block diagram representation of $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.

4-1-3 Block Diagram Reduction: Branch point relocation

(a)



(b)

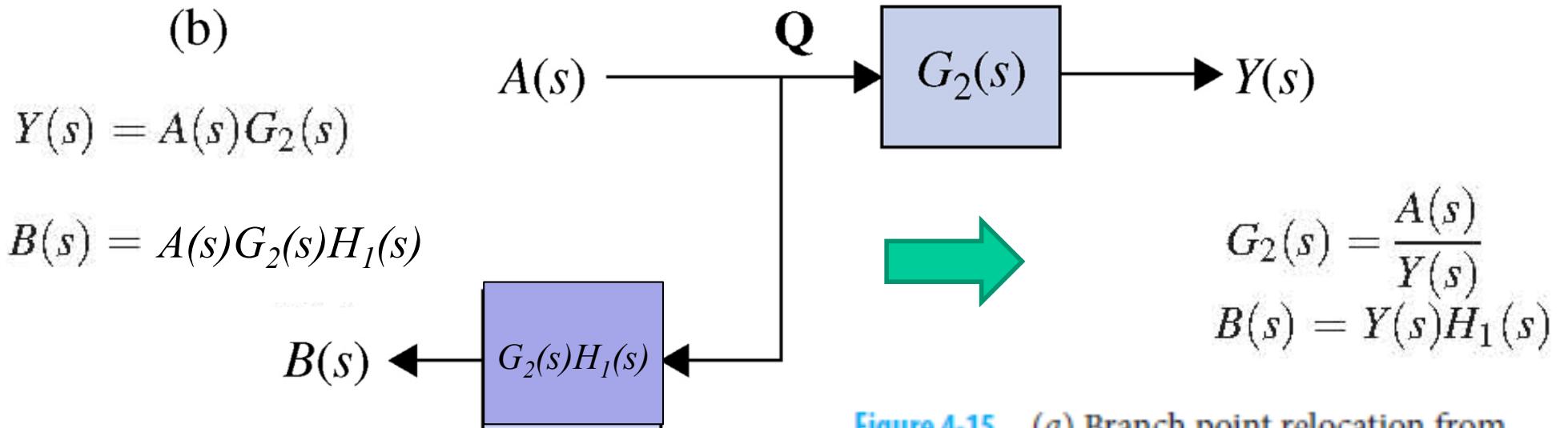
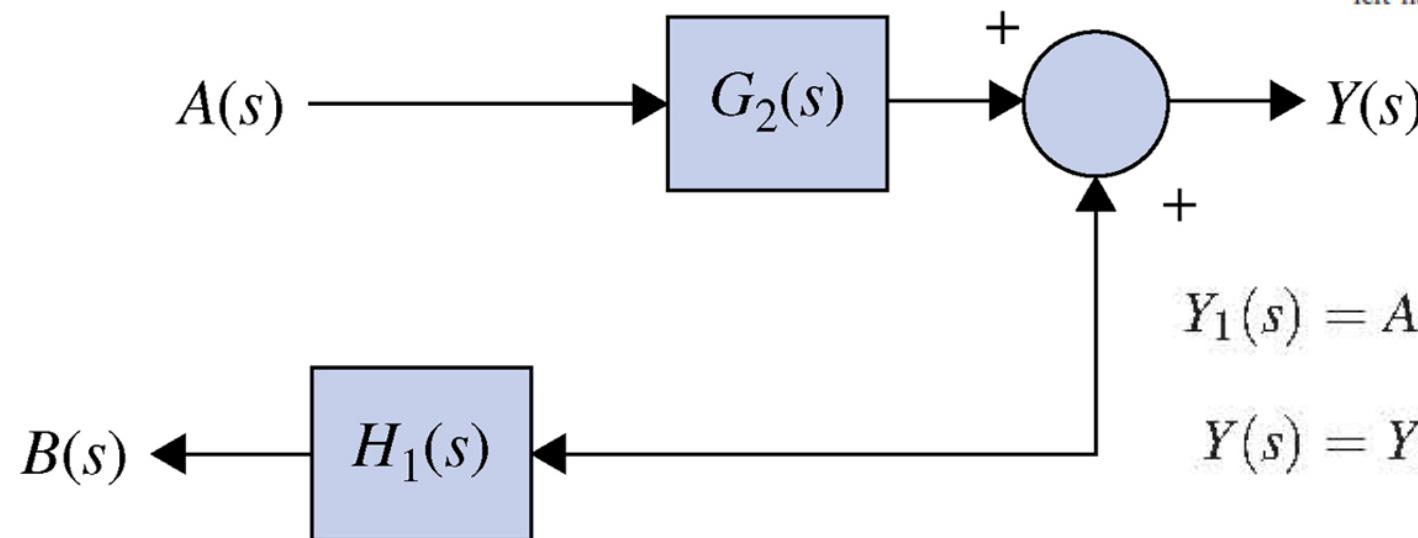


Figure 4-15 (a) Branch point relocation from point P to (b) point Q.

3-1-3 Block Diagram Reduction: Comparator relocation

Figure 4-16 (a) Comparator relocation from the right-hand side of block $G_2(s)$ to (b) the left-hand side of block $G_2(s)$.

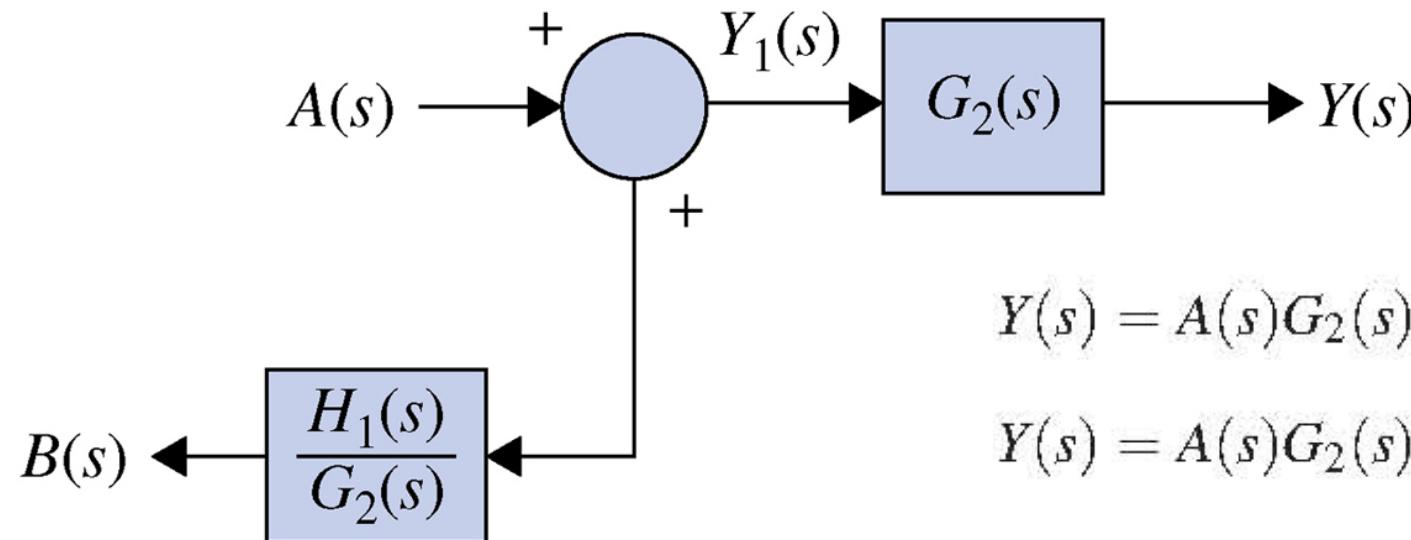
(a)



$$Y_1(s) = A(s) + B(s) \frac{H_1(s)}{G_2(s)}$$

$$Y(s) = Y_1(s)G_2(s)$$

(b)



$$Y(s) = A(s)G_2(s) + B(s) \frac{H_1(s)}{G_2(s)} G_2(s)$$

$$Y(s) = A(s)G_2(s) + B(s)H_1(s)$$

EXAMPLE 3-1-5 Find the input–output transfer function of the system

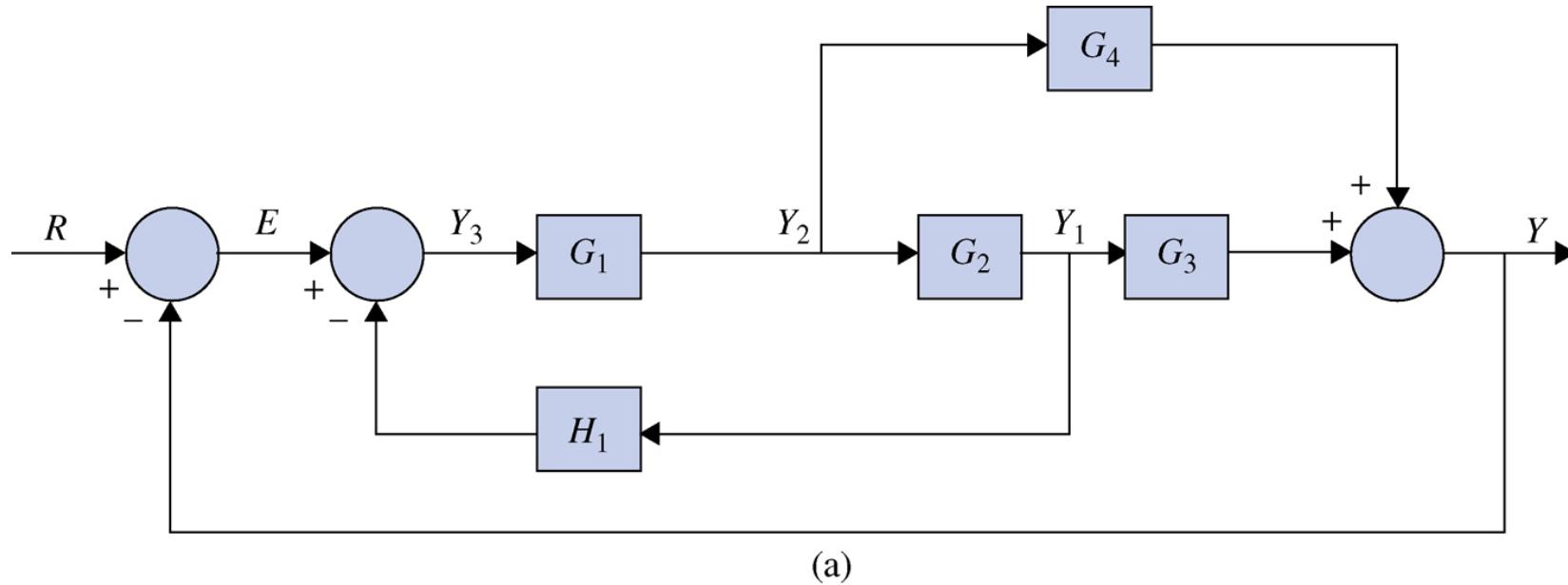
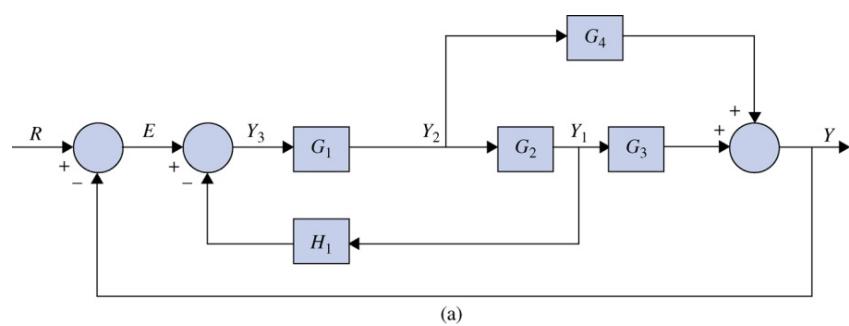
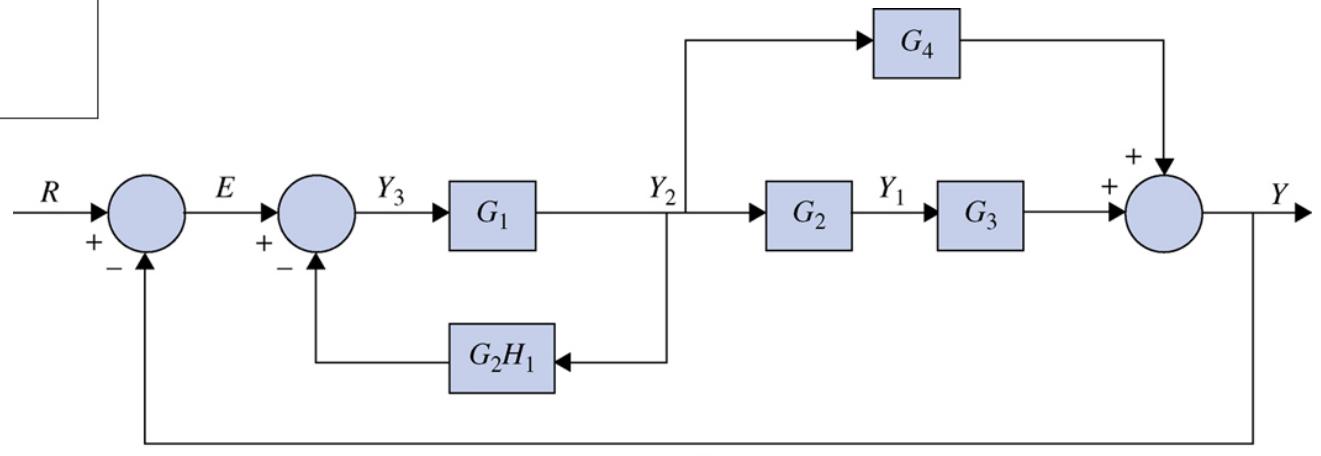


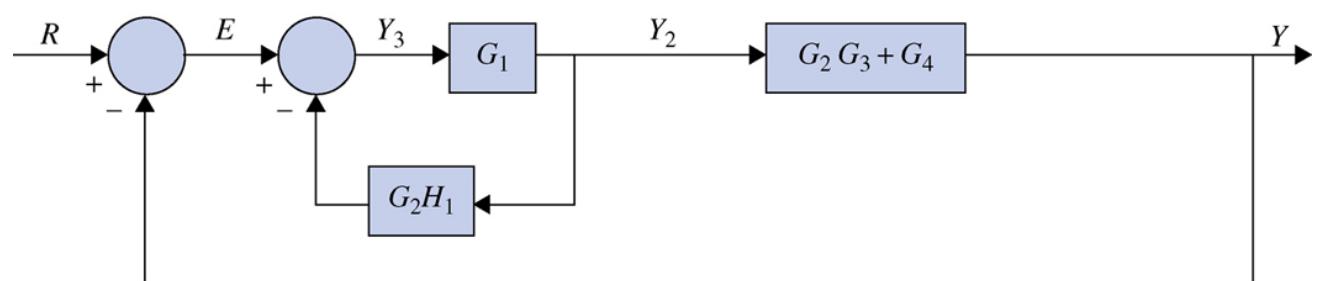
Figure 4-17 (a) Original block diagram. (b) Moving the branch point at Y_1 to the left of block G_2 .
(c) Combining the blocks G_1 , G_2 , and G_3 . (d) Eliminating the inner feedback loop.



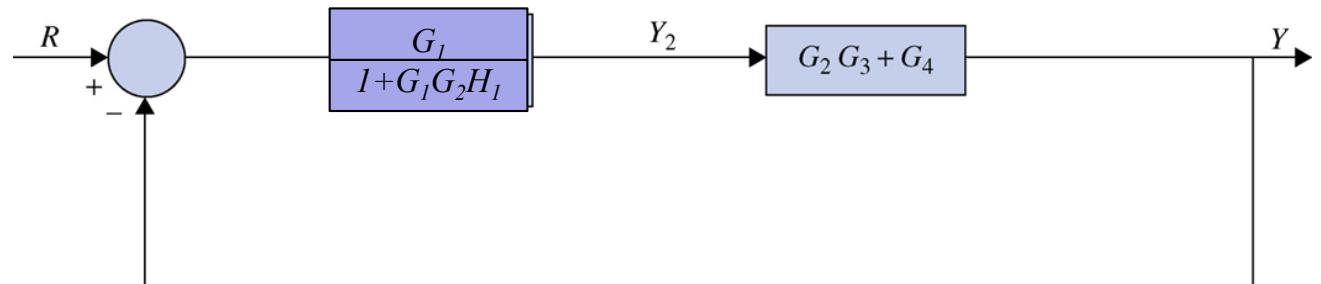
(a)



(b)



(c)



(d)

$$\frac{Y(s)}{E(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_1 G_4}$$

Figure 4-17 (a) Original block diagram. (b) Moving the branch point at Y_1 to the left of block G_2 . (c) Combining the blocks G_1 , G_2 , and G_3 . (d) Eliminating the inner feedback loop.

4-1-4 Block Diagram of Multi-Input Systems—Special Case: Systems with a Disturbance

Super Position: For linear systems, the overall response of the system under multi-inputs is the summation of the responses due to the individual inputs, i.e., in this case,

$$Y_{total} = Y_R|_{D=0} + Y_D|_{R=0} \quad (4-28)$$

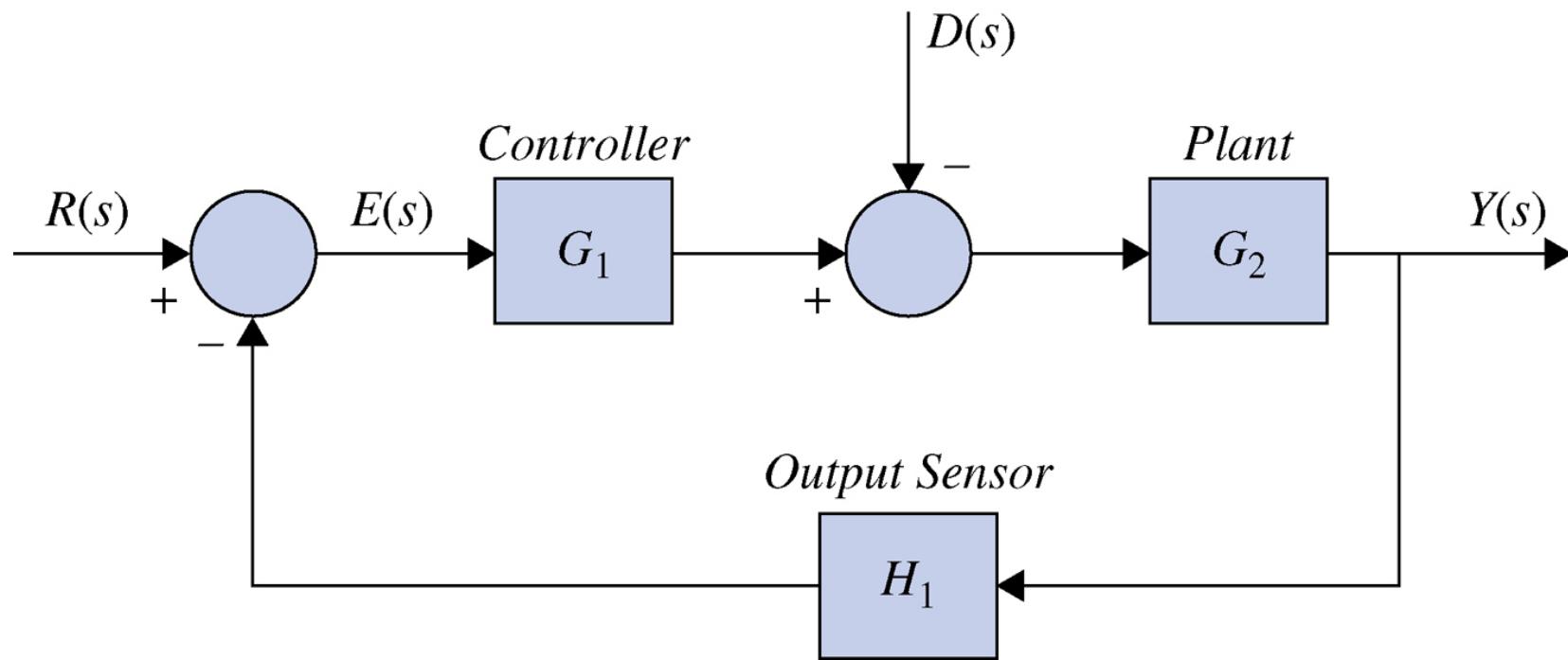
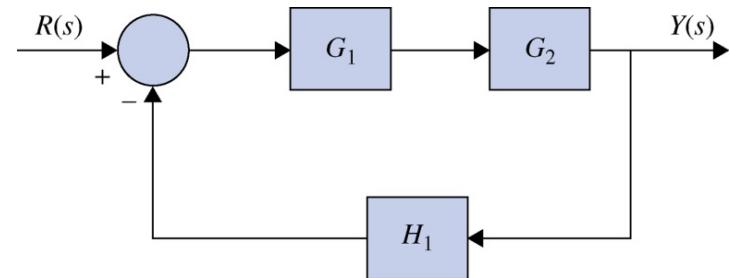


Figure 4-19 Block diagram of the system
in Fig. 4-18, when $D(s)=0$.

When $D(s) = 0$, the block diagram is simplified (Fig. 3-20) to give the transfer function

$$\frac{Y(s)}{R(s)} = \frac{G_1(s) G_2(s)}{1 + G_1(s) G_2 H_1(s)} \quad (3-29)$$



When $R(s) = 0$, the block diagram is rearranged to give (Fig. 3-21):

$$\frac{Y(s)}{D(s)} = \frac{-G_2(s)}{1 + G_1(s) G_2(s) H_1(s)} \quad (3-30)$$



As a result, from Eq. (3-28) to Eq. (3-32), we ultimately get

$$Y_{total} = \frac{Y(s)}{R(s)} \Big|_{D=0} R(s) + \frac{Y(s)}{D(s)} \Big|_{R=0} D(s) \quad (4-31)$$

$$Y(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s) + \frac{-G_2}{1 + G_1 G_2 H_1} D(s)$$

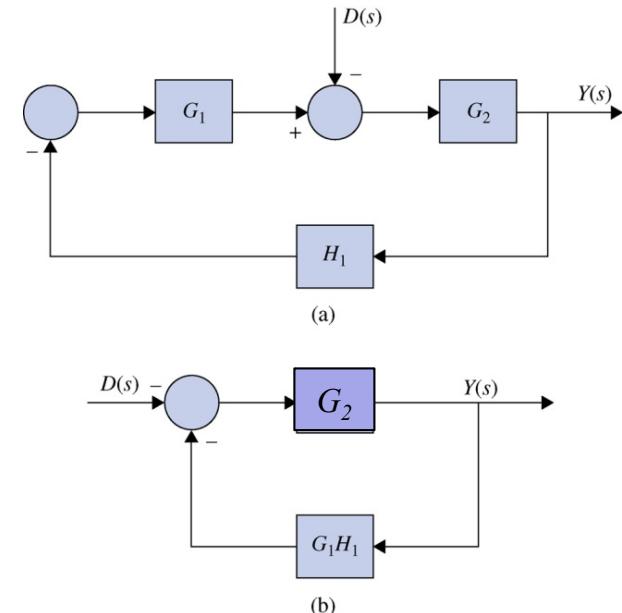
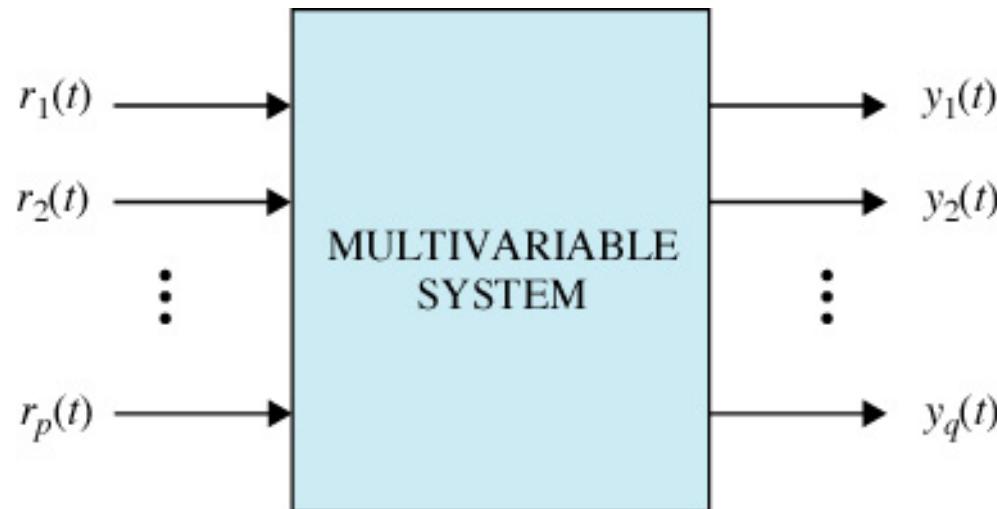
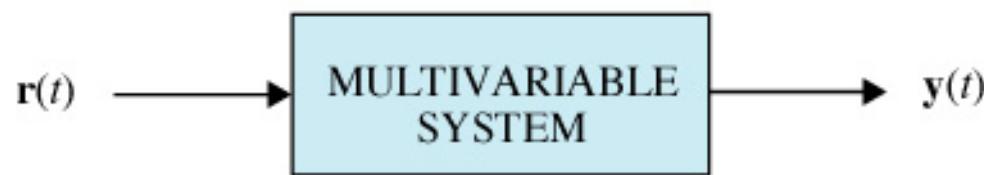


Figure 4-20 Block diagram of the system in Fig. 4-18, when $R(s) = 0$.

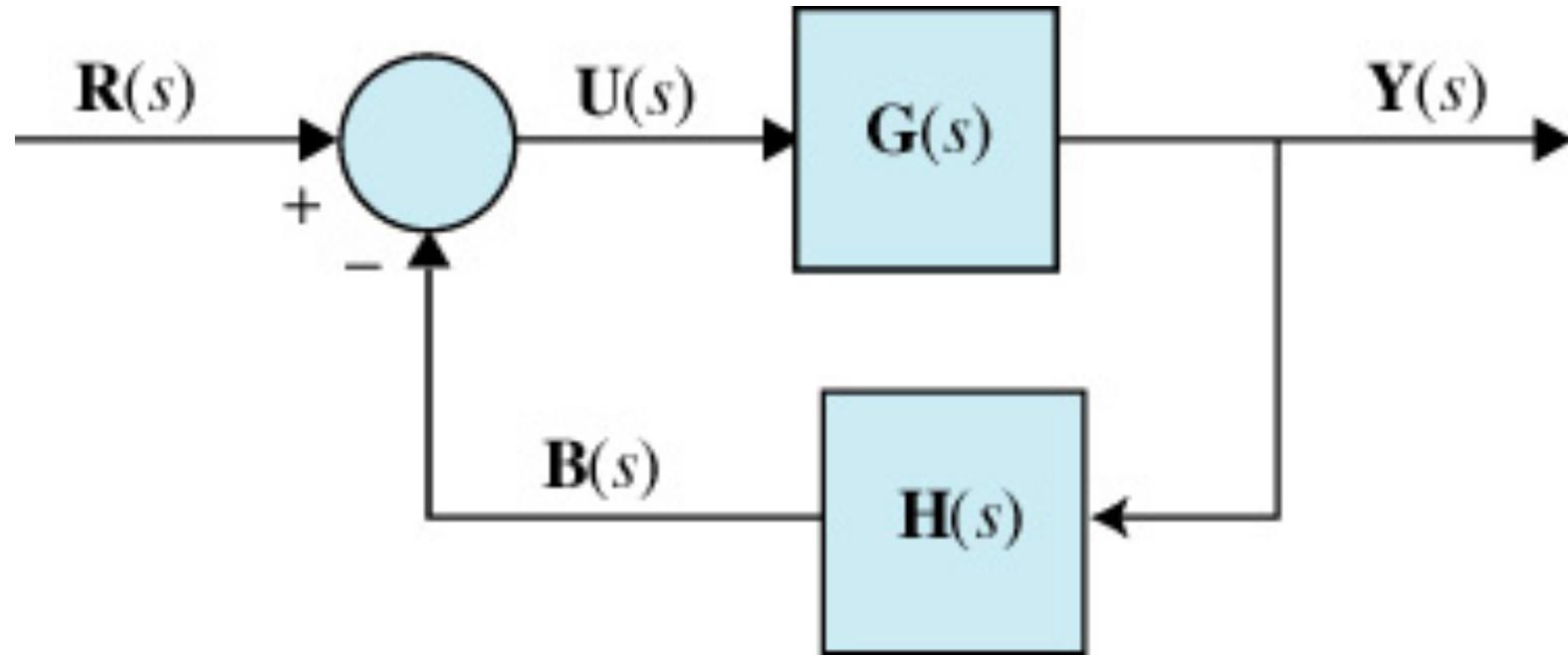
다변수시스템의 블록선도와 전달함수



(a)



(b)



$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s)$$

$$\mathbf{U}(s) = \mathbf{R}(s) - \mathbf{B}(s)$$

$$\mathbf{B}(s) = \mathbf{H}(s)\mathbf{Y}(s)$$

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{R}(s) - \mathbf{G}(s)\mathbf{H}(s)\mathbf{Y}(s)$$

$$\mathbf{Y}(s) = [\mathbf{I} + \mathbf{G}(s)\mathbf{H}(s)]^{-1} \mathbf{G}(s)\mathbf{R}(s)$$

$$\mathbf{M}(s) = [\mathbf{I} + \mathbf{G}(s)\mathbf{H}(s)]^{-1} \mathbf{G}(s)$$

$$\mathbf{Y}(s) = \mathbf{M}(s)\mathbf{R}(s)$$

- 예제 3-1

$$\mathbf{G}(s) = \begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s} \\ 2 & \frac{1}{s+2} \end{bmatrix} \quad \mathbf{H}(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{I} + \mathbf{G}(s)\mathbf{H}(s) = \begin{bmatrix} 1 + \frac{1}{s+1} & -\frac{1}{s} \\ 2 & 1 + \frac{1}{s+2} \end{bmatrix} = \begin{bmatrix} \frac{s+2}{s+1} & -\frac{1}{s} \\ 2 & \frac{s+3}{s+2} \end{bmatrix}$$

$$\mathbf{M}(s) = [\mathbf{I} + \mathbf{G}(s)\mathbf{H}(s)]^{-1} \mathbf{G}(s) = \frac{1}{\Delta} \begin{bmatrix} \frac{s+3}{s+2} & \frac{1}{s} \\ -2 & \frac{s+2}{s+1} \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s} \\ 2 & \frac{1}{s+2} \end{bmatrix}$$

$$\Delta = \frac{s+2}{s+1} \frac{s+3}{s+2} + \frac{2}{s} = \frac{s^2 + 5s + 2}{s(s+1)}$$

$$\mathbf{M}(s) = \frac{s(s+1)}{s^2 + 5s + 2} \begin{bmatrix} \frac{3s^2 + 9s + 4}{s(s+1)(s+2)} & -\frac{1}{s} \\ 2 & \frac{3s+2}{s(s+1)} \end{bmatrix}$$

Signal-Flow Graphs (신호흐름선도)

- 블록선도의 단순화된 표현
- 선형대수방정식의 변수 사이의 입출력관계를 도식적으로 나타내는 방법
- N개의 대수방정식 선형시스템

$$y_j = \sum_{k=1}^N a_{kj} y_k \quad j = 1, 2, \dots, N$$

j 번째 결과 = $\sum_{k=1}^N (k$ 에서 j 까지의 이득) \times (k 번째 원인)

출력 = \sum 이득 \times 입력

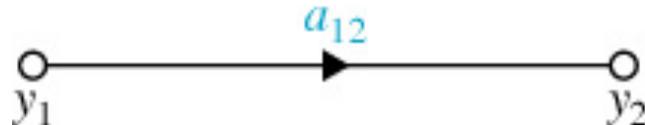
$$Y_j(s) = \sum_{k=1}^N G_{kj}(s) Y_k(s) \quad j = 1, 2, \dots, N$$

Signal-Flow Graphs (신호흐름선도)

- 기본요소

- 마디(node): 변수
 - 가지(branch): 이득과 방향

$$y_2 = a_{12} y_1$$



- 예제 3-2

$$y_2 = a_{12}y_1 + a_{32}y_3$$

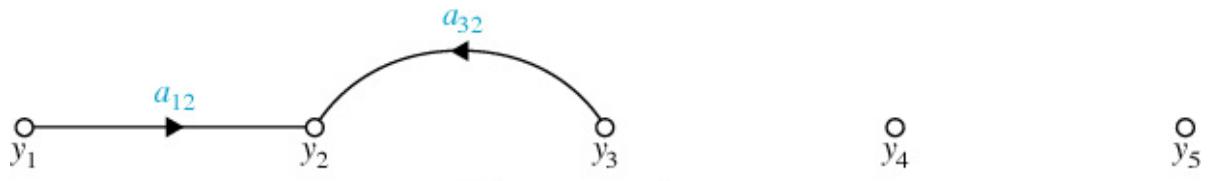
$$y_3 = a_{23}y_2 + a_{43}y_4$$

$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$

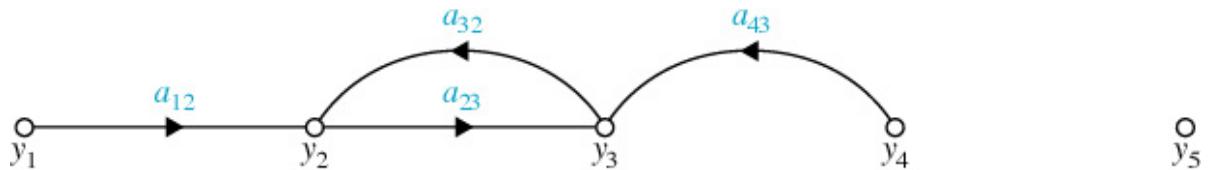
$$y_5 = a_{25}y_2 + a_{45}y_4$$

Figure 3-7 (p. 50)

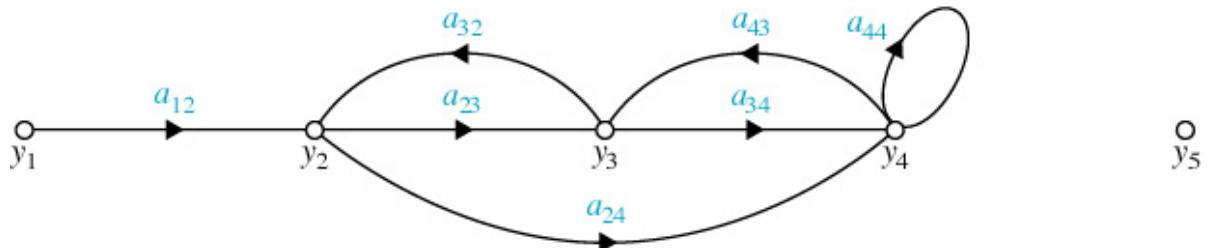
Step-by-step construction of the signal-flow graph in Eq. (3-27).



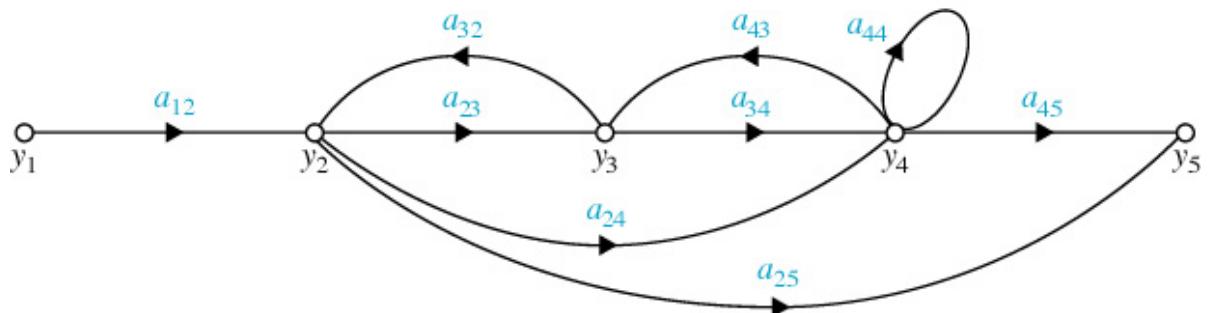
$$(a) y_2 = a_{12}y_1 + a_{32}y_3$$



$$(b) y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4$$



$$(c) y_2 = a_{12}y_1 + a_{32}y_3 \quad y_3 = a_{23}y_2 + a_{43}y_4 \quad y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4$$



(d) Complete signal-flow graph

4-2 SIGNAL-FLOW GRAPHS (SFGs)

Transfer Function	Block Diagram	Signal Flow Diagram
One block System $\frac{Y(s)}{R(s)} = G(s)$	(a) 	(b)
Cascade $\frac{Y(s)}{R(s)} = G_1(s) G_2(s)$	(c) 	(d)
Parallel $\frac{Y(s)}{R(s)} = G_1(s) + G_2(s)$	(e) 	(f)
Feedback $\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$	(g) 	(h)

Figure 4-23 Block diagrams and their SFG equivalent representations. (a) Input-output representation in block diagram form. (b) Equivalent input-output representation in SFG form. (c) A cascade block diagram representation. (d) Equivalent cascade SFG representation. (e) A parallel block diagram representation. (f) Equivalent parallel SFG representation. (g) A negative feedback block diagram representation. (h) Equivalent negative feedback SFG representation.

기본성질

1. 선형시스템에만 적용
2. 방정식은 원인과 결과 꼴의 대수방정식
3. 마디는 변수, 원인과 결과의 순서로 왼쪽에서 오른쪽,
입력에서 출력으로
4. 신호는 가지의 화살표 방향으로
5. 마디 y_k 에서 y_j 로 향하는 가지는 변수 y_j 의 변수 y_k 에
대한 의존관계를 나타냄
6. 마디 y_k 와 y_j 사이의 가치를 따라 이동하는 신호 y_k 는
가지이득 a_{kj} 가 곱해져서 마디 y_j 에서 신호 $a_{kj}y_k$ 가 전달

정의

- 입력마디(or source): 밖으로 나가는 가지들만을 갖는 마디
- 출력마디(or sink): 들어오는 가지들만을 갖는 마디
- 경로(path): 같은 방향으로의 연속적인 가지들의 집합
- 전방경로(forward path): 입력마디에서 출발하여 출력마디에서 끝나는 경로로서 어느 마디도 한 번 이상 거쳐서는 안됨.

정의

- 루프(loop): 같은 마디에서 출발하고 끝나는 경로로서 다른 마디는 한 번 이상 거쳐서는 안됨.
- 경로이득(path gain): 경로를 이루는 가지들의 이득의 곱
- 비접촉루프(nontouching loop): 신호흐름선도의 두 부분이 공통의 마디를 공유하지 않을 경우.

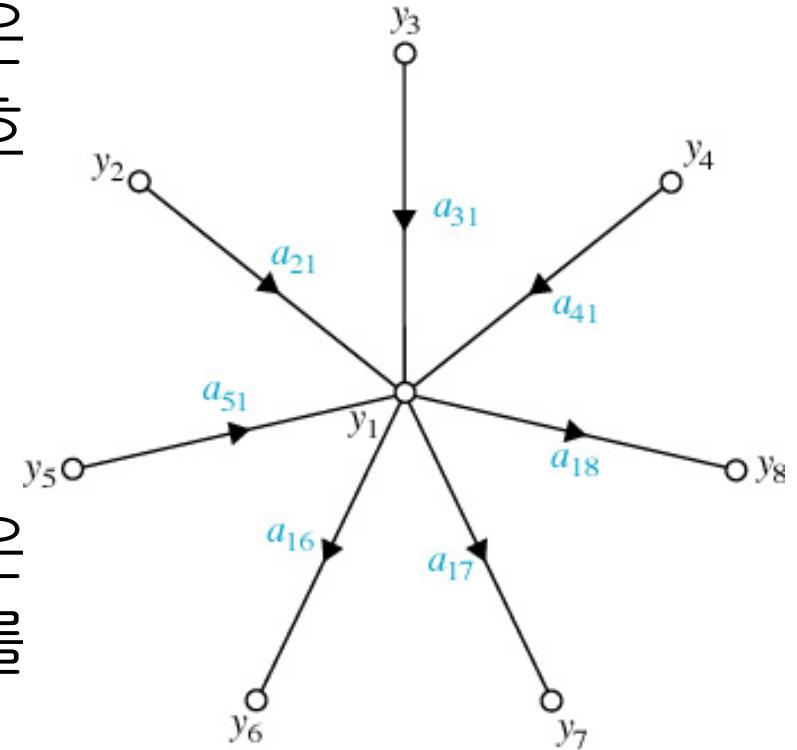
신호흐름선도의 연산법

1. 마디로 표시된 변수의 값은
마디로 들어오는 모든 신호
합과 같다.

- $y_1 = a_{21}y_2 + a_{31}y_3 + a_{41}y_4 + a_{51}y_5$

2. 마디로 표시된 변수의 값은
마디를 나가는 모든 가지를
따라 전달된다.

- $y_6 = a_{16}y_1, y_7 = a_{17}y_1, y_8 = a_{18}y_1$



-
-
3. 두 마디를 연결하는 같은 방향으로의 병렬가지는
병렬가지들이 이득의 합과 같은 이득을 갖는 하나의
가지로 대치 가능
4. 같은 방향으로 직렬로 연결된 가지들은 가지이득들의
곱과 같은 이득을 하나의 가지로 대치 가능

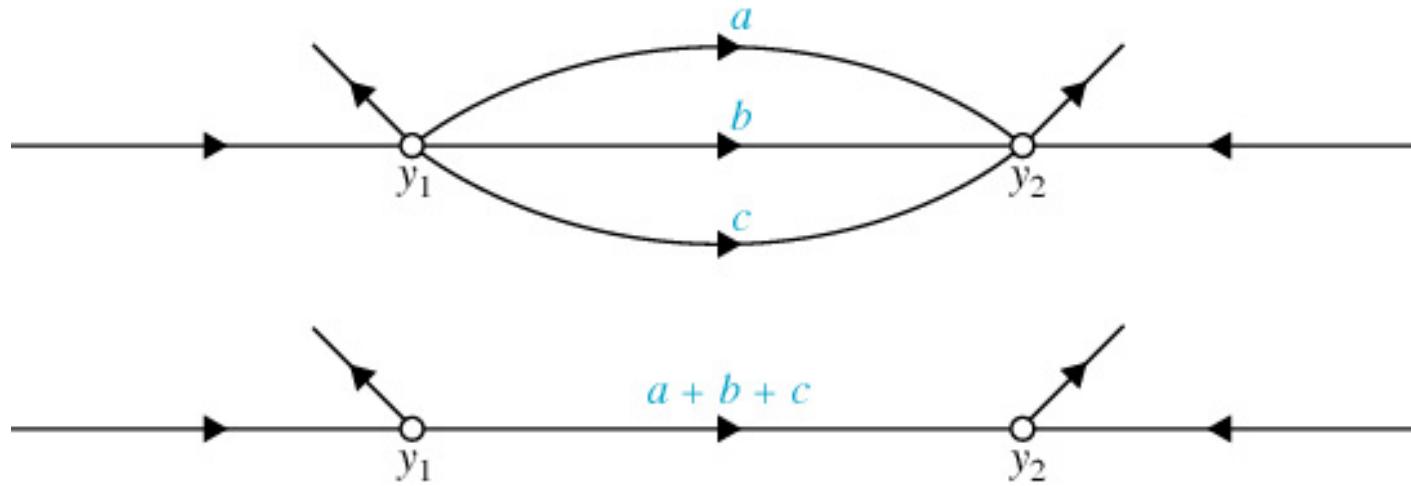


Figure 4-25 Signal-flow graph with parallel paths replaced by one with a single branch.

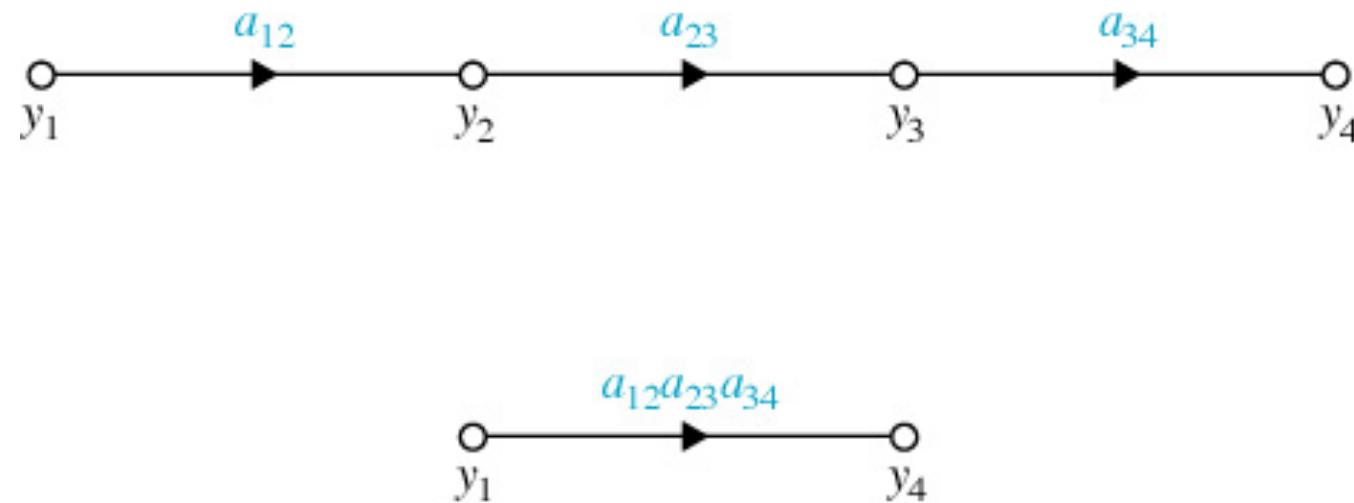
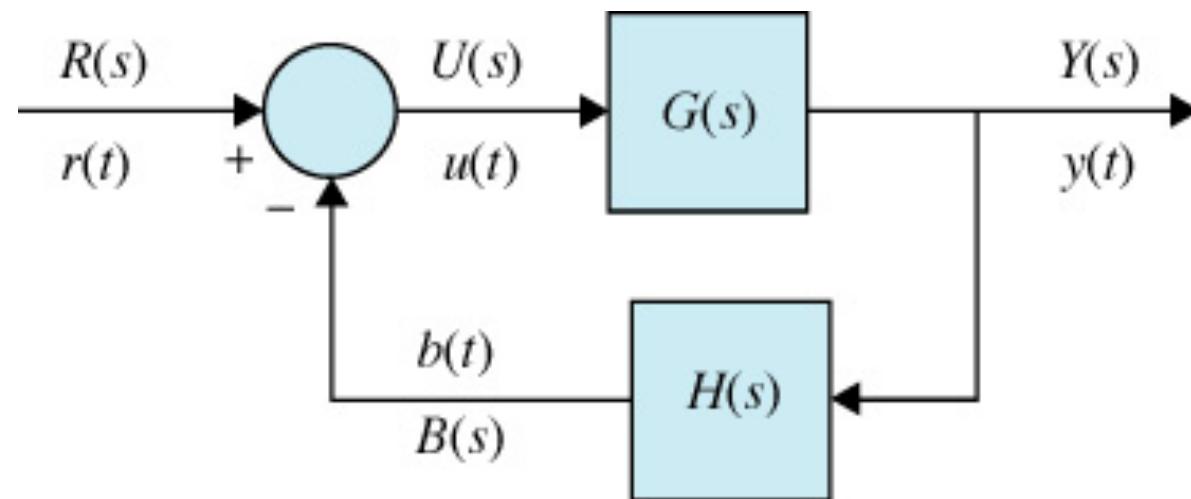
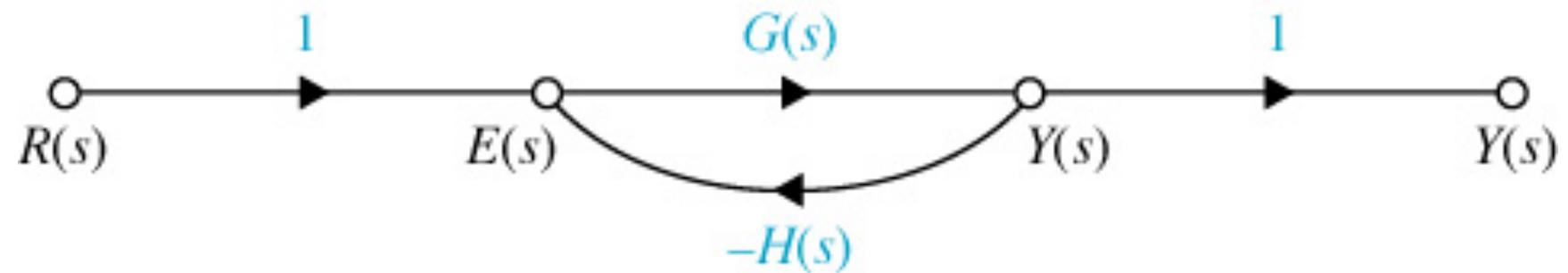
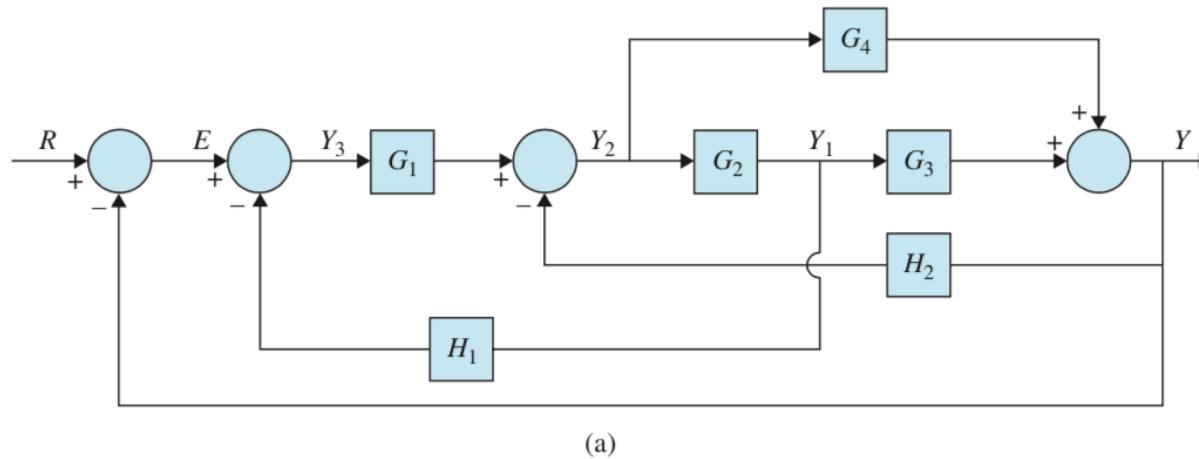


Figure 4-26 Signal-flow graph with cascade unidirectional branches replaced by a single branch.

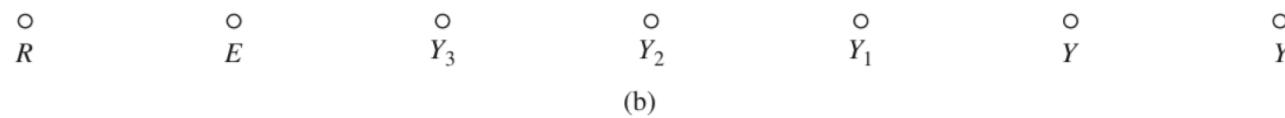
피드백 제어시스템의 신호흐름선도



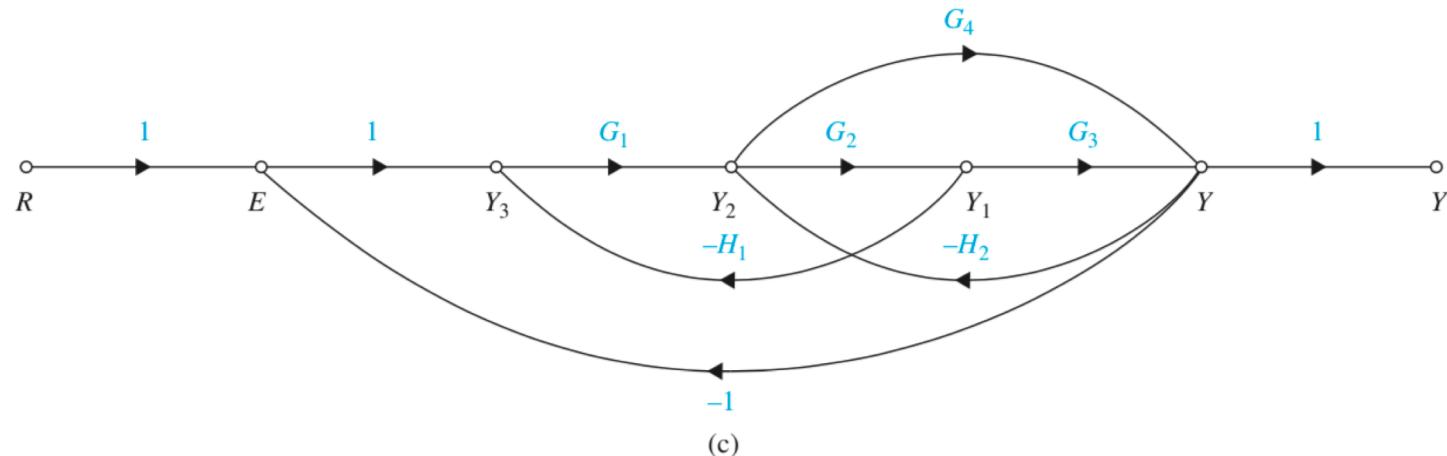
Example 4-2-1



(a)



(b)



(c)

신호흐름선도의 일반이득공식

- 일반이득공식
- 여기서

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^N \frac{M_k \Delta_k}{\Delta}$$

y_{in} =입력마디 변수

y_{out} =출력마디 변수

$M=y_{in}$ 과 y_{out} 사이의 이득

$N=y_{in}$ 과 y_{out} 사이의 전방경로의 총수

$M_k=y_{in}$ 과 y_{out} 사이의 k 번째 전방경로의 이득

$$\Delta = 1 - \sum_i L_{i1} + \sum_j L_{j2} - \sum_k L_{k3} + \dots \quad (3-32)$$



정 $L_{mr}=r$ 개의 비접촉루프의 가능한 m 번째($m=i, j, k, \dots$) 조합의 이득곱.

$$\Delta = 1 - (\text{각각의 모든 루프이득의 합}) + (\text{두 개의 비접촉루프의 가능한 모든 조합의 이득곱의 합}) - (\text{세 개의 비접촉루프의 가능한 모든 조합의 이득곱의 합}) + \dots \quad (3-33)$$

Δ_k = k 번째 전방경로와 접촉하지 않는 신호흐름선도에 대한 Δ

- 예제 4-2-3

1. $R(s)$ 와 $Y(s)$ 사이에는 오직 하나의 전방경로만 있으며 그 전방경로이득은

$$M_1 = G(s) \quad (3-34)$$

2. 오직 하나의 루프만 있으며 루프이득은

$$L_{11} = -G(s)H(s)$$

3. 오직 하나의 루프뿐이므로 비접촉루프들은 없다. 더구나 전방경로가 그 루프와 접촉되어 있다. 따라서 $\Delta_1=1$ 이고

$$\Delta = 1 - L_{11} = 1 + G(s)H(s) \quad (3-36)$$

$$\frac{Y(s)}{R(s)} = \frac{M_1 \Delta_1}{\Delta} = \frac{G(s)}{1 + G(s)H(s)}$$

- 예제 4-2-4 y_1 과 y_5 사이의 두 개의 전방경로와 전방경로이득은

$$M_1 = a_{12}a_{23}a_{34}a_{34} \quad \text{전방경로: } y_1 - y_2 - y_3 - y_4 - y_5$$

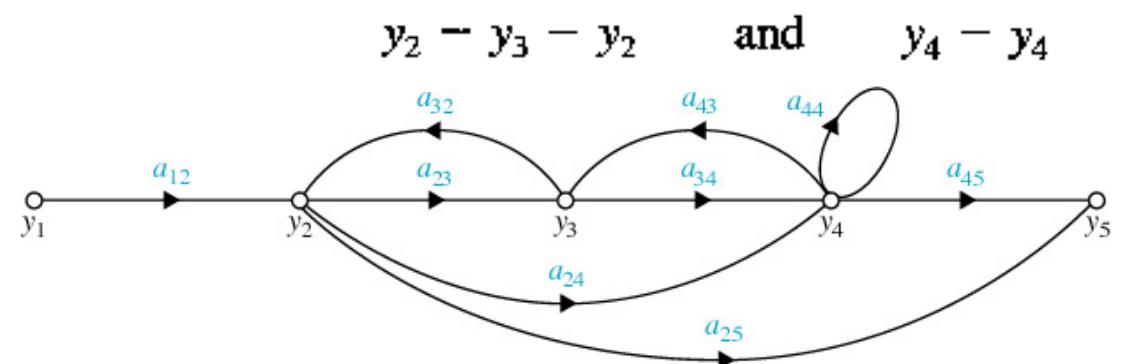
$$M_2 = a_{12}a_{25} \quad \text{전방경로: } y_1 - y_2 - y_5$$

$$M_3 = a_{12}a_{24}a_{45} \quad \text{전방경로: } y_1 - y_2 - y_4 - y_5$$

신호흐름선도의 네 개의 루프가 그림 3-10에 나와 있다. 루프이득은

$$L_{11} = a_{23}a_{32} \quad L_{21} = a_{34}a_{43} \quad L_{31} = a_{24}a_{43}a_{32} \quad L_{41} = a_{44}$$

비접촉루프가 한 쌍, 즉



(d) Complete signal-flow graph

그래서 두 개의 비접촉루프이들의 곱은

$$L_{12} = a_{23}a_{32}a_{44}$$

모든 루프들은 전방경로 M_1 및 M_3 와 접촉하고 있다. 따라서 $\Delta_1 = \Delta_3 = 1$.
 $y_3 - y_4 - y_3$ 와 $y_4 - y_4$ 루프는 전방경로 M_2 와 접촉하고 있지 않다. 따라서

$$\Delta_2 = 1 - a_{34}a_{43} - a_{44} \quad (3-39)$$

식 (3-31)에 이들을 대입하면

$$\frac{y_5}{y_1} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta} = \frac{(a_{12}a_{23}a_{34}a_{45}) + (a_{12}a_{25})(1 - a_{34}a_{43} - a_{44}) + a_{12}a_{24}a_{45}}{1 - (a_{23}a_{32} + a_{34}a_{43} + a_{24}a_{32}a_{43} + a_{44}) + a_{23}a_{32}a_{44}} \quad (3-40)$$

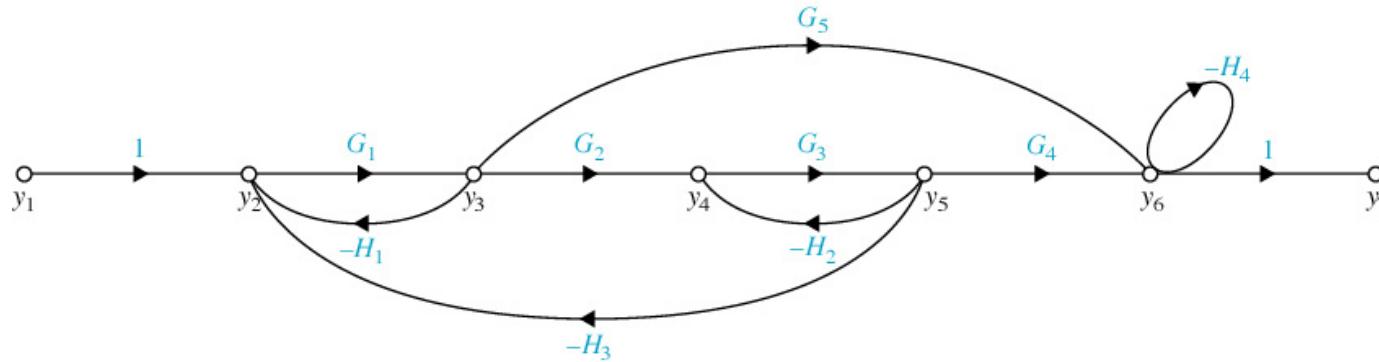
$$\begin{aligned} \Delta &= 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + L_{12} \\ &= 1 - (a_{23}a_{32} + a_{34}a_{43} + a_{24}a_{32}a_{43} + a_{44}) + a_{23}a_{32}a_{44} \end{aligned} \quad (3-41)$$

y_2 를 출력으로 하면

$$\frac{y_2}{y_1} = \frac{a_{12}(1 - a_{34}a_{43} - a_{44})}{\Delta} \quad (3-42)$$



비입력마디와 출력마디 사이에 일반이득공식의 적용



Let y_{in} be an input and y_{out} be an output node of an SFG. The gain, y_{out}/y_2 , where y_2 is not an input, may be written as

$$\frac{y_{out}}{y_2} = \frac{\frac{y_{out}}{y_{in}}}{\frac{y_2}{y_{in}}} = \frac{\frac{\sum M_k \Delta_k|_{\text{from } y_{in} \text{ to } y_{out}}}{\Delta}}{\frac{\sum M_k \Delta_k|_{\text{from } y_{in} \text{ to } y_2}}{\Delta}}$$

Because Δ is independent of the inputs and the outputs, the last equation is written as

$$\frac{y_{out}}{y_2} = \frac{\sum M_k \Delta_k|_{\text{from } y_{in} \text{ to } y_{out}}}{\sum M_k \Delta_k|_{\text{from } y_{in} \text{ to } y_2}}$$

Simplified Gain Formula

Condition: *when all loops and forward paths are touching,*

EXAMPLE 4-2-8 For Example 4-2-5, where there are non-touching loops, as seen in Fig. 4-33, the simplified gain formula can be used by eliminating the non-touching loops after some block diagram manipulations.

The two forward paths between y_1 and y_7 , and the forward-path gains are now

Forward path	Gain
$y_1 - y_2 - y_3 - y_4 - y_5 - y_6 - y_7$	$M_1 = G_1 G_2 G_3 G_4 G_5$
$y_1 - y_2 - y_3 - y_4 - y_7$	$M_2 = G_1 G_2 G_3$

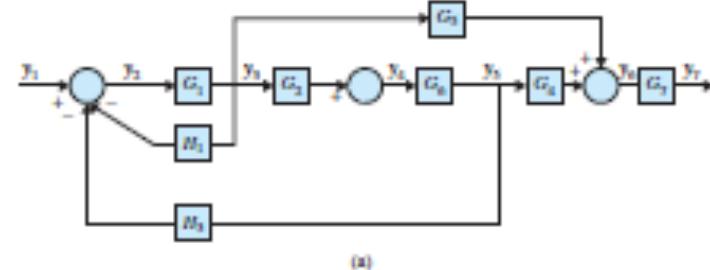
The two touching loops of the SPG are shown in Fig. 4-33. The loop gains are

Loop	Gain
$y_2 - y_3 - y_2$	$L_{11} = -G_1 H_1$
$y_2 - y_3 - y_4 - y_5 - y_2$	$L_{21} = -G_1 G_2 G_3 H_1$

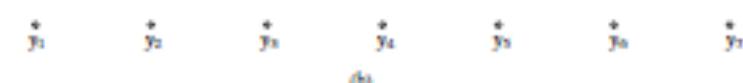
Note in this case

$$G_k = \frac{G_k}{1+G_k H_1} \quad \text{and} \quad G_7 = \frac{1}{1+H_1}$$

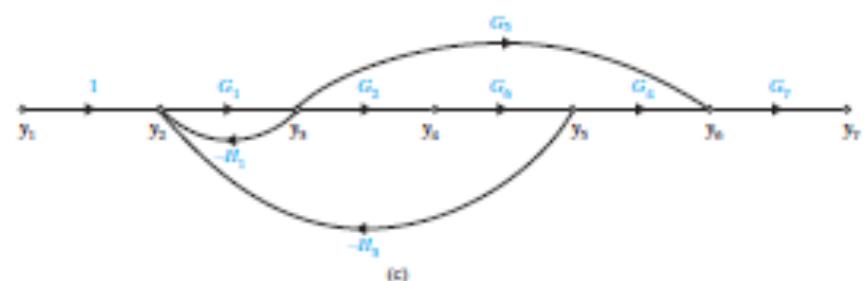
$$M = \frac{y_{\text{out}}}{y_{\text{in}}} = \sum \frac{\text{Forward Path Gains}}{1 - \text{Loop Gains}}$$



(a)



(b)



(c)

Figure 4-33 (a) Modified block diagram of the control system in Fig. 4-32 to eliminate the non-touching loops. (b) Signal nodes representing the variables. (c) Equivalent signal-flow graph.

State Diagram (상태선도)

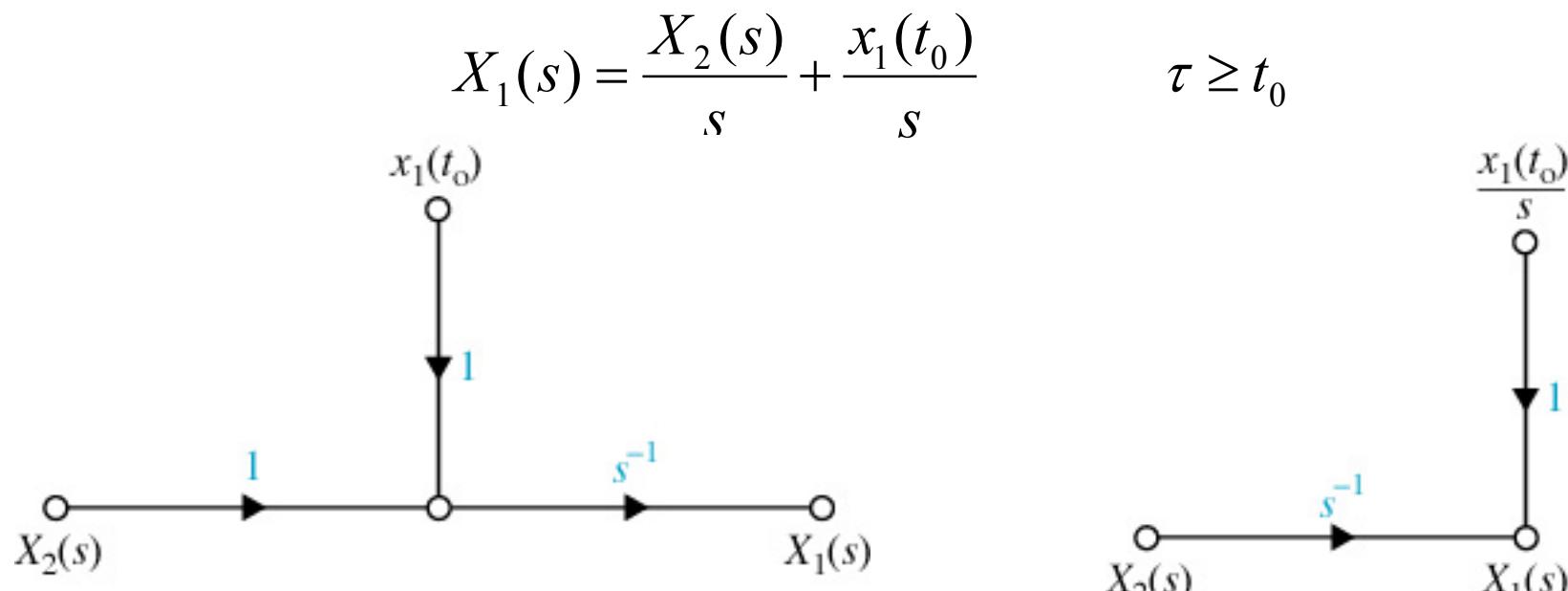
- 신호흐름선도의 확장: 상태방정식, 미분방정식을 나타냄
- 라플라스변환된 상태방정식을 이용하여 신호흐름선도의 법칙에 따라 구성
- $x_1(t)$ 와 $x_2(t)$ 가 1차 미분의 관계에 있다면,

$$\frac{dx_1(t)}{dt} = x_2(t)$$

$$\begin{aligned}x_1(t) &= \int_{t_0}^t x_2(\tau) d\tau + x_1(t_0) & X_1(s) &= \mathcal{L}\left[\int_{t_0}^t x_2(\tau) d\tau\right] + \frac{x_1(t_0)}{s} \\&&&= \mathcal{L}\left[\int_0^t x_2(\tau) d\tau - \int_{t_0}^{t_0} x_2(\tau) d\tau\right] + \frac{x_1(t_0)}{s} \\&&&= \frac{X_2(s)}{s} - \mathcal{L}\left[\int_0^{t_0} x_2(\tau) d\tau\right] + \frac{x_1(t_0)}{s}\end{aligned}$$

State Diagram (상태선도)

- 적분기의 과거는 $x_1(t_0)$ 로 나타내고, 상태천이는 $\tau=t_0$ 에서 시작된다고 가정하면, $0 < \tau < t_0$ 에 대해 $x_2(\tau)=0$ 이다.



- 적분기 출력이 s^{-1} 배의 입력과 초기조건 $x_1(t_0)/s$ 의 합

State Diagram (상태선도)의 용도

1. 시스템의 미분방정식으로부터 쉽게 구성된다. 따라서 상태변수와 상태방정식이 금방 구해진다.
2. 시스템의 전달함수로부터 그려질 수 있다. (전달함수의 분해)
3. 컴퓨터에 의해 시스템의 프로그래밍에 이용된다.
4. 라플라스변환 영역에서의 상태천이방정식은 상태선도로부터 신호흐름선도 이득 공식에 의해서 구할 수 있다.
5. 시스템의 전달함수는 상태선도로부터 구해질 수 있다.
6. 상태방정식과 출력방정식은 상태선도로부터 구해질 수 있다.

미분방정식으로부터 상태선도 구하기

$$\frac{d^n y(t)}{dt^n} + a_n \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_2 \frac{dy(t)}{dt} + a_1 y(t) = r(t)$$

- 상태선도를 구하려면 식을 정리해서,

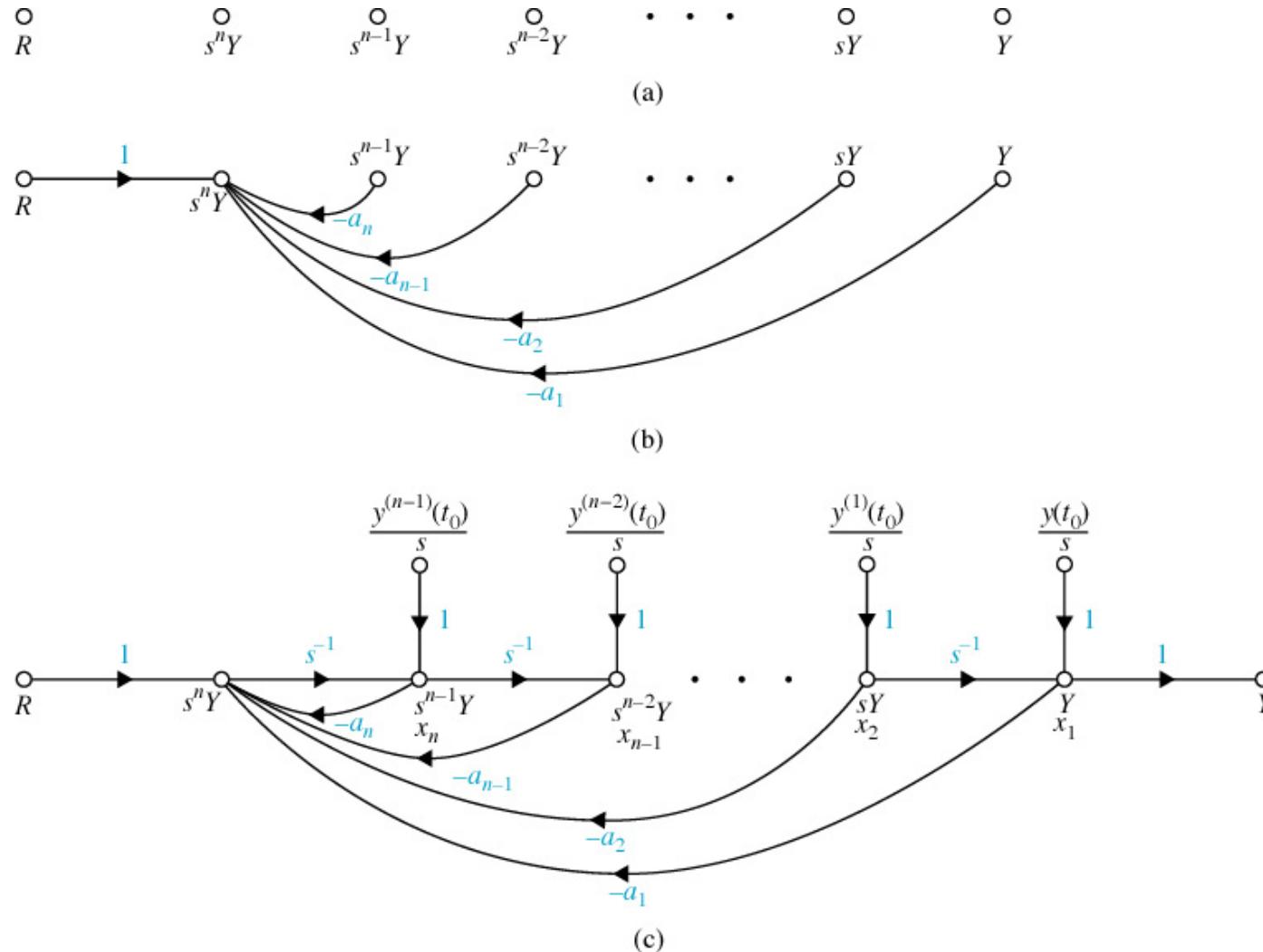
$$\frac{d^n y(t)}{dt^n} = -a_n \frac{d^{n-1} y(t)}{dt^{n-1}} - \cdots - a_2 \frac{dy(t)}{dt} - a_1 y(t) + r(t)$$

$R(s)$, $s^n Y(s)$, $s^{n-1} Y(s)$, ..., $s Y(s)$ 및 $Y(s)$ 를 나타내는 마디들을
왼쪽에서 오른쪽으로 그림 3-19(a)와 같이 배열한다.

- 적분기의 출력은 상태변수 x_1, x_2, \dots, x_n 으로 정의된다.

Figure 3-19 (p. 60)

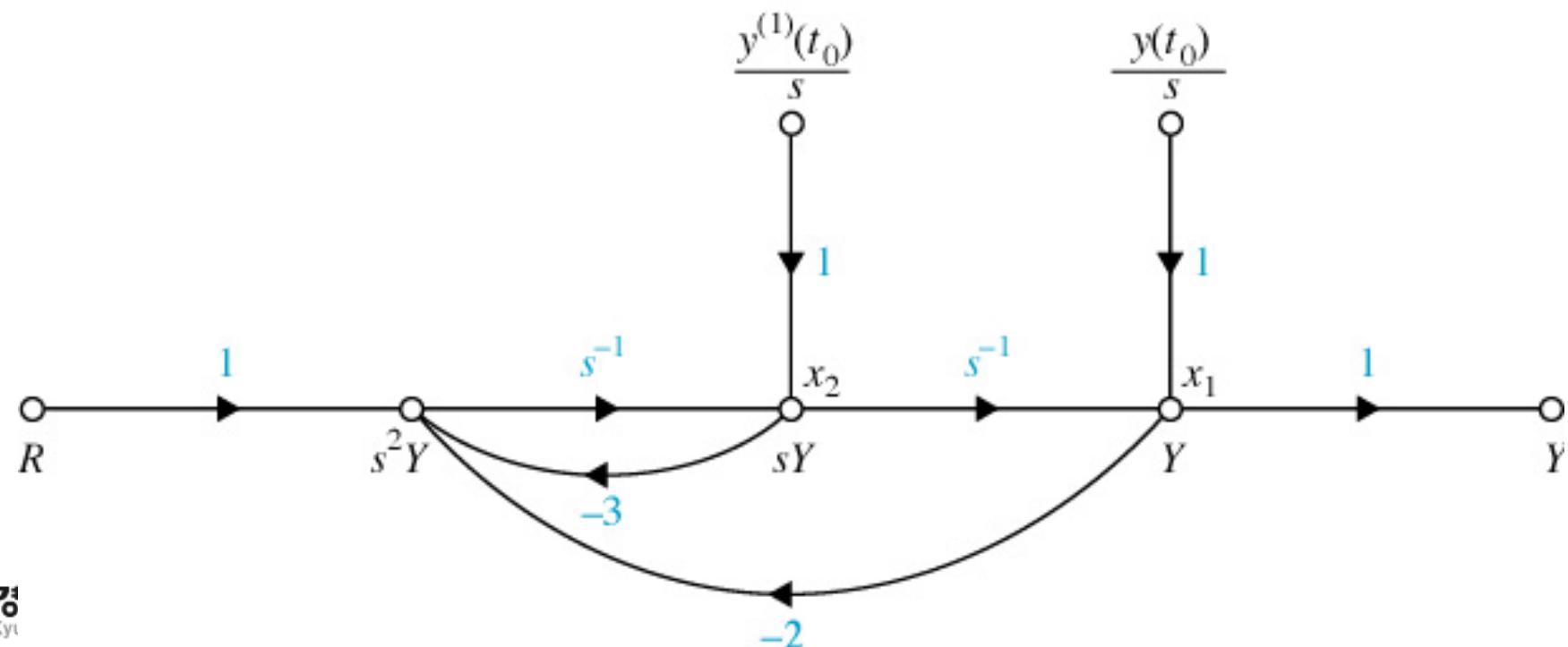
State-diagram representation of the differential equation of Eq. (3-58).



• 예제 3-8

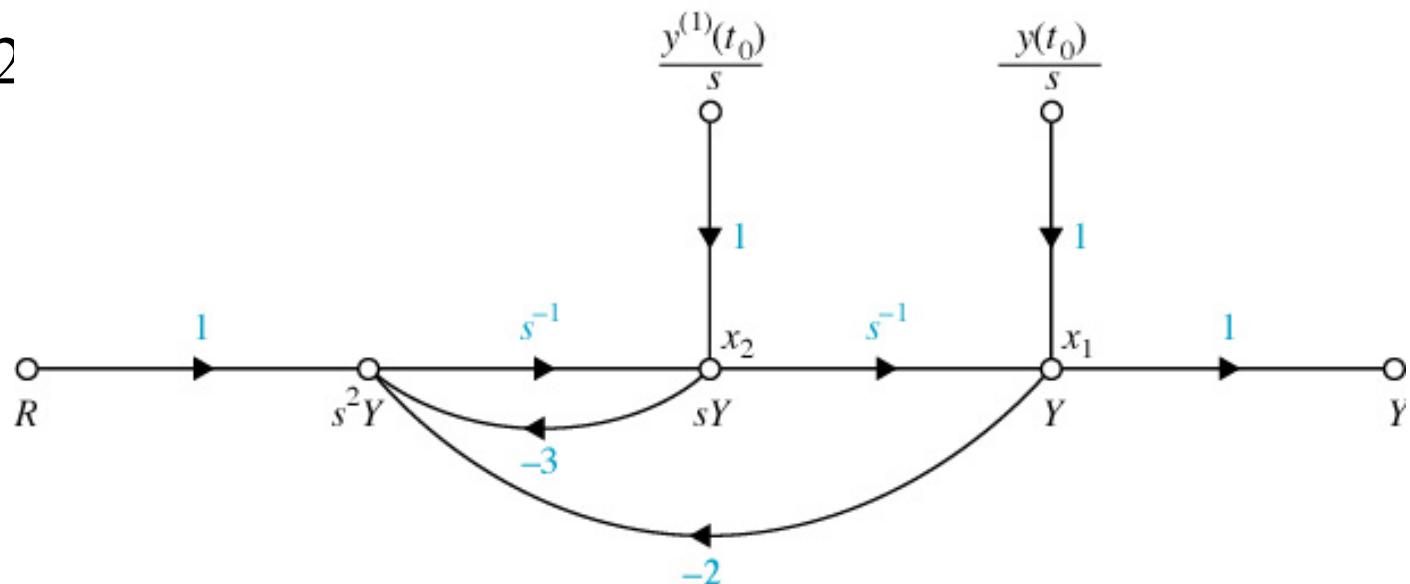
$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = r(t)$$

$$\frac{d^2y(t)}{dt^2} = -3\frac{dy(t)}{dt} - 2y(t) + r(t)$$



상태선도에서 전달함수 구하기

- 입력과 출력 사이의 전달함수는 다른 모든 입력과 초기상태를 0으로 놓고 이득공식에 의하여 상태선도로부터 얻어진다.
- 예제 4-3-2



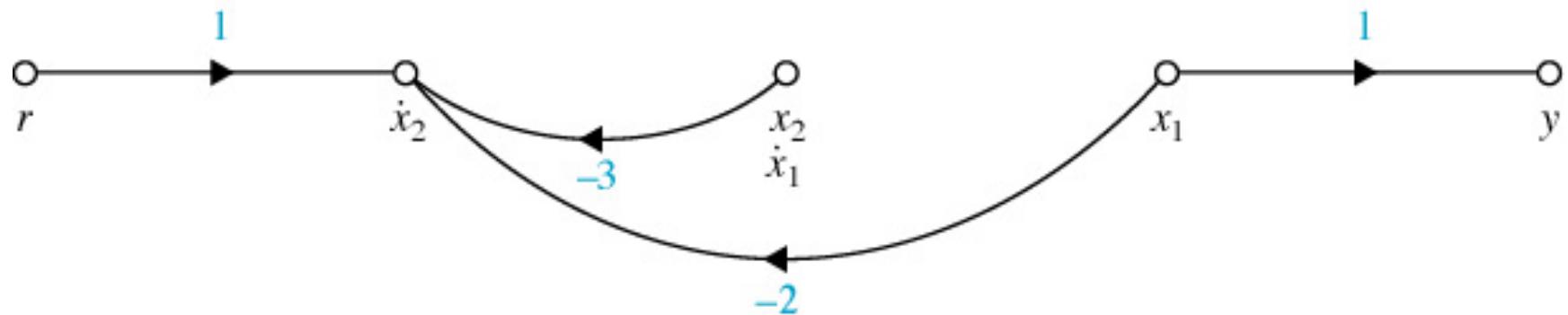
$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + 3s + 2}$$

상태선도에서 상태와 출력방정식 구하기

- 상태방정식과 출력방정식은 이득공식에 의하여 상태선도로부터 구할 수 있다.
 - 상태방정식: $\frac{dx(t)}{dt} = ax(t) + br(t)$
 - 출력방정식: $y(t) = cx(t) + dr(t)$
1. 상태선도에서 상태와 출력방정식 구하기

1. 상태와 출력방정식은 라플라스 연산자 s 와 초기상태를 포함하지 않으므로 상태선도에서 초기상태와 이득 s^{-1} 의 적분기 가지를 제거한다.
2. 상태방정식에 대해 상태변수의 도함수는 상태방정식의 좌변에 나타나므로 이들 변수를 출력마디로 한다. 출력방정식에 있는 출력 $y(t)$ 는 당연히 출력마디 변수임.
3. 상태변수와 입력은 상태와 출력방정식의 우변에 나타나므로 이들 변수를 입력마디로 간주한다.
4. 이득공식을 상태선도에 적용한다.

- 예제 3-10

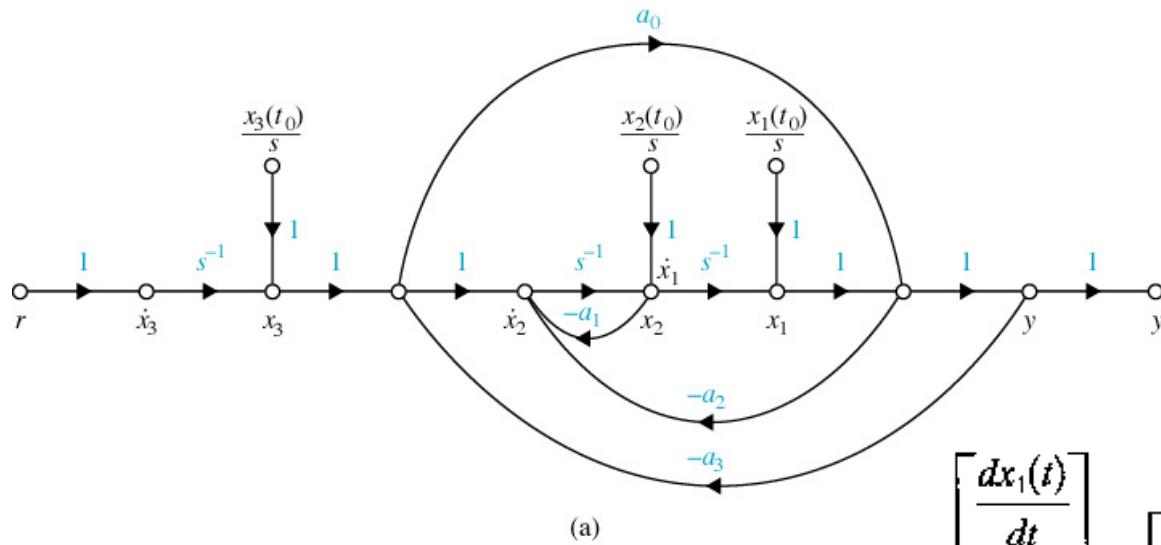


$$\frac{dx_1(t)}{dt} = x_2(t)$$

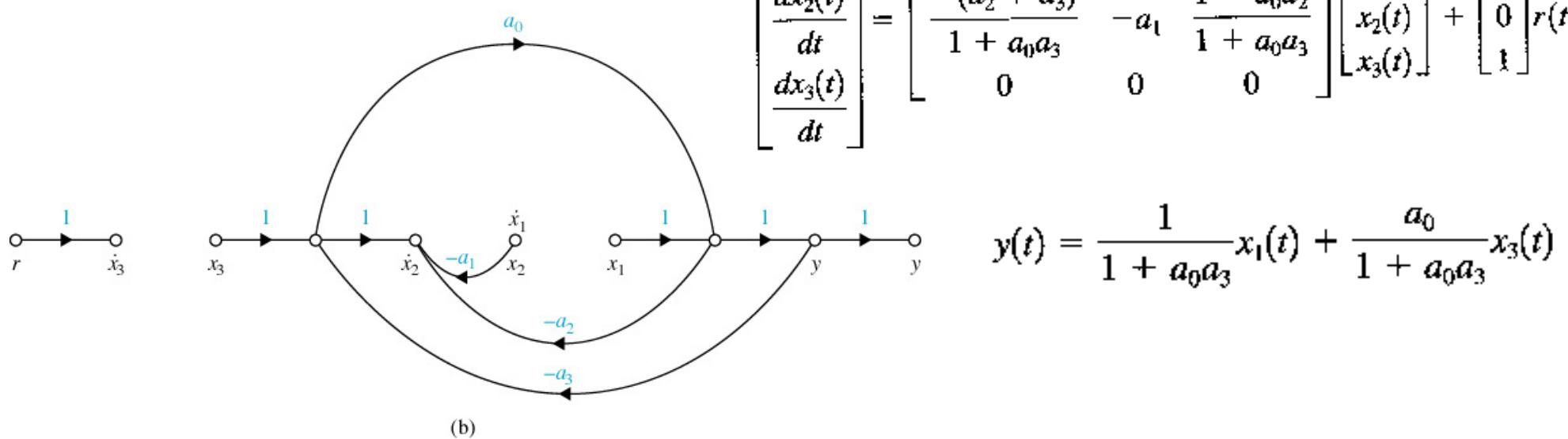
$$\frac{dx_2(t)}{dt} = -2x_1(t) - 3x_2(t) + r(t)$$

$$y(t) = x_1(t)$$

• 예제 4-3-4



$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{(a_2 + a_3)}{1 + a_0 a_3} & -a_1 & \frac{1 - a_0 a_2}{1 + a_0 a_3} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$



$$y(t) = \frac{1}{1 + a_0 a_3} x_1(t) + \frac{a_0}{1 + a_0 a_3} x_3(t)$$

Figure 4-39

(a) State diagram.

(b) State diagram in part (a) with all initial states and integrators left out.

Example 4-4-1

$$\ddot{y}(t) + \frac{B}{M}y(t) + \frac{K}{M}\dot{y}(t) = \frac{1}{M}f(t) \quad (4-105)$$

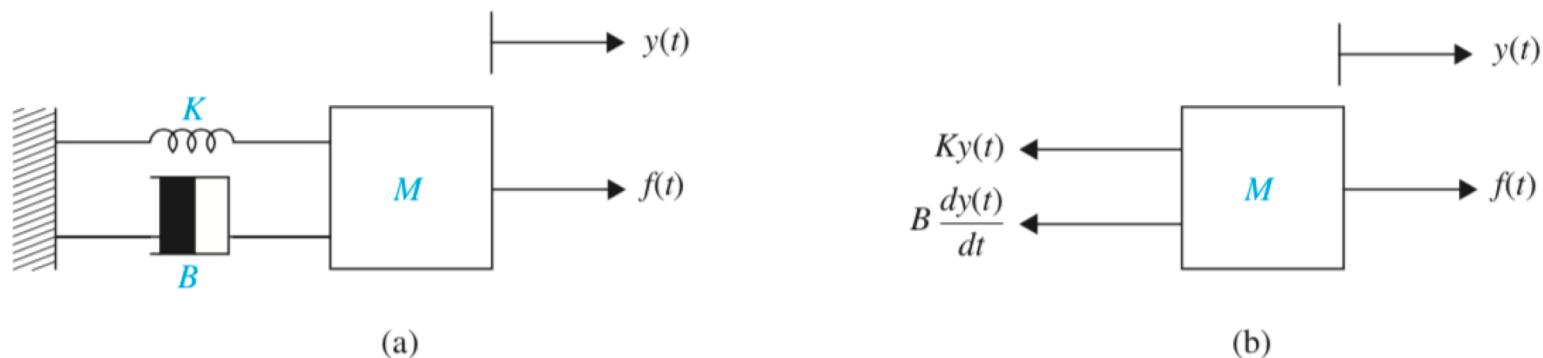


Figure 4-40 (a) Mass-spring-friction system. (b) Free-body diagram.

where $y(t)$ is the output, $\frac{f(t)}{M}$ is considered the input, and $\dot{y}(t)=\left(\frac{dy(t)}{dt}\right)$ and $\ddot{y}(t)=\left(\frac{d^2y(t)}{dt^2}\right)$ represent velocity and acceleration, respectively.

- For zero initial conditions,

$$Y(s)\left(s^2 + \frac{B}{M}s + \frac{K}{M}\right) = \frac{F(s)}{M}$$

$$\frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Example 4-4-1

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

where

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

and

$$\mathbf{u}(t) = \frac{f(t)}{M}$$

The output equation is

$$y(t) = x_1(t)$$

So Eq. (4-107) is rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{f(t)}{M}$$

Example 4-4-1

$$\begin{aligned}\frac{dx_1(t)}{dt} &= x_2(t) \\ \frac{dx_2(t)}{dt} &= -\frac{K}{M}x_1(t) - \frac{B}{M}x_2(t) + \frac{1}{M}f(t) \\ y(t) &= x_1(t)\end{aligned}\tag{4-112}$$

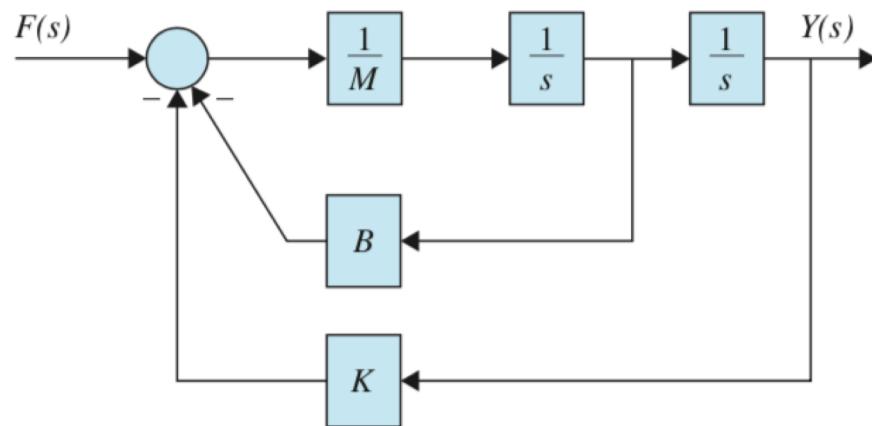


Figure 4-41 Block diagram representation of mass-spring-damper system of Eq. (4-106).

Example 4-4-1

For nonzero initial conditions, Eq. (4-112) has a different Laplace transform representation that may be written as

$$\begin{aligned}sX_1(s) - x_1(0) &= X_2(s) \\ sX_2(s) - x_2(0) &= -\frac{B}{M}X_2(s) - \frac{K}{M}X_1(s) + \frac{1}{M}F(s) \\ Y(s) &= X_1(s)\end{aligned}\tag{4-115}$$

The corresponding SFG representation for Eq. (4-115) is shown in Fig. 4-43.

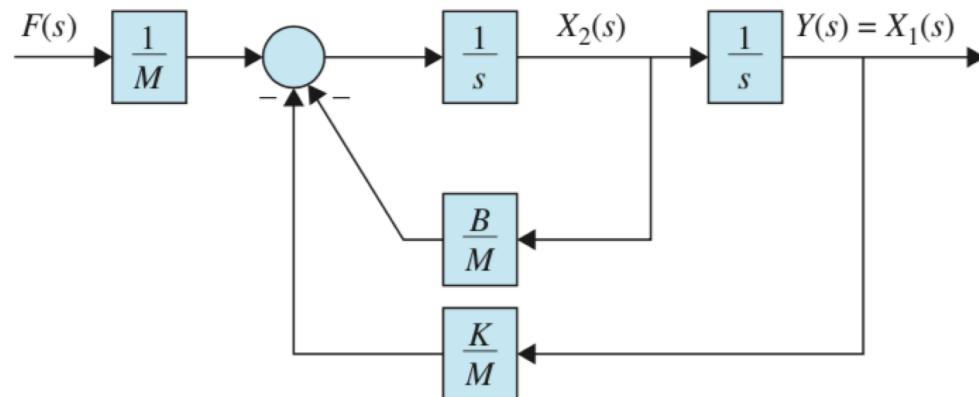


Figure 4-42 Block diagram representation of mass-spring-damper system of shown in Fig. 4-41.

Example 4-4-1

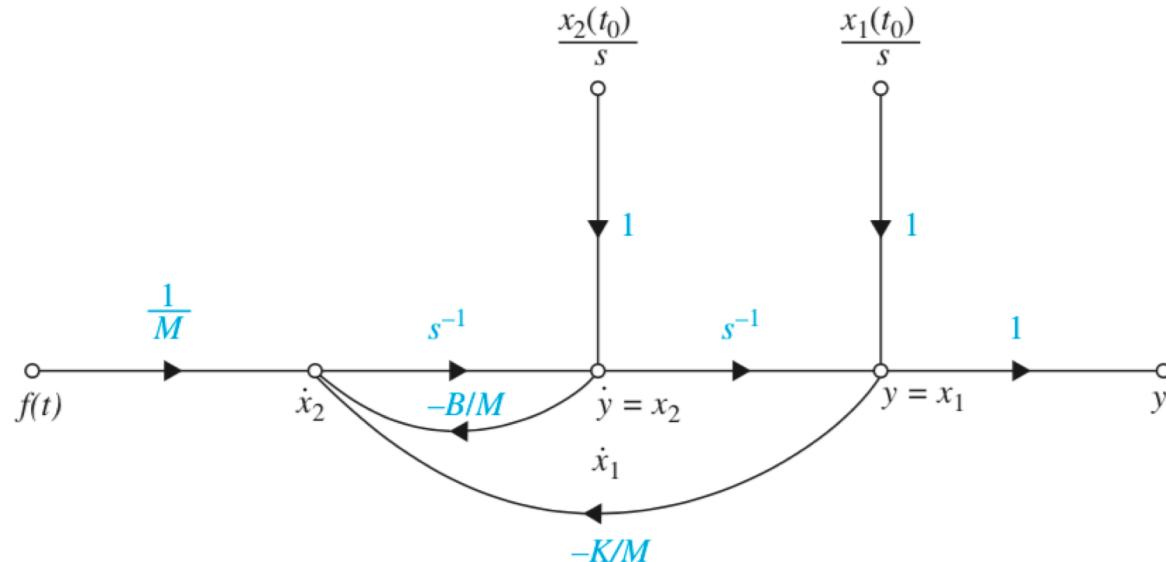


Figure 4-43 SFG representation of mass-spring-damper system of Eq. (4-115) with nonzero initial conditions $x_1(t_0)$ and $x_2(t_0)$.

Upon simplifying Eq. (4-115) or by applying the gain formula to the SFG representations of the system, the output becomes

$$Y(s) = \frac{1}{Ms^2 + Bs + K} F(s) + \frac{Ms}{Ms^2 + Bs + K} x_1(t_0) + \frac{M}{Ms^2 + Bs + K} x_2(t_0) \quad (4-116)$$