ASE7030 Convex optimization: Homework #3

1) *Convexity.* Suppose an $n \times n$ symmetric matrix A and let the quadratic form associated with it be:

$$f(x) = x^T A x$$

Then it is crystal clear that $f(x)$ is convex if $A \geq 0$, and we can also show that in this case its epigraph and the sublevel sets are all convex.

Now suppose A is not positive semidefinite and it has exactly one strictly negative eigenvalue. Say, $\lambda_1 < 0$, and $\lambda_2, \ldots, \lambda_n \geq 0$.

a) Explain that the epigraph of $f(x)$ is not convex.

$$\textbf{epi } f = \left\{ (x,t) \in \mathbb{R}^{n+1} \mid x^T A x \leq t \right\}$$

b) Show that t-sublevel set of $f(x)$ with fixed t is convex. *Hint: This is true.*

$$L_t^-(f) = \left\{ x \in \mathbb{R}^n \mid x^T A x \leq t \right\}$$