EE363 Automatic Control: Homework #2

1) *Linearity of Laplace transform.* Suppose we are given a signal $f(t)$ and its corresponding Laplace transform $\mathcal{L}\{f(t)\} = F(s)$.

   a) Prove that $\mathcal{L}\{kf(t)\} = kF(s)$ for any $k \in \mathbb{R}$.

   a) Prove that $\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$.

2) *Drawing exercise.* You will sketch the step response of the following plant.

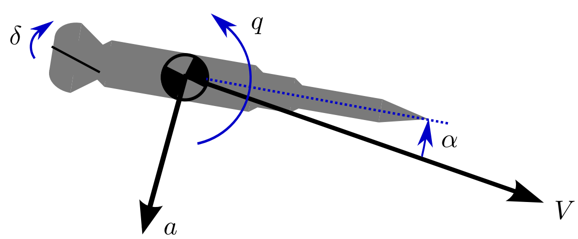
$$G(s) = \frac{s^2 - 26}{(s+10)(s^2+3s+4)}$$

   a) Find the partial fraction expansion, *i.e.*, express $G(s)$ as a sum of two simple fractional expressions.
   b) Sketch the step responses of the two components, and then sum them together to get the step response of $G(s)$.

3) *Longitudinal dynamics of a rocket.* The short period longitudinal dynamics of a rocket can be described as follows:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta$$

   where α is the *angle of attack*, roughly the misalign angle between the rocket's body axis and the velocity axis, and q is the *pitch rate*, simply the angular rate of the rocket. The input δ is the rocket's *fin deflection*, which is used as the control input to this system, and the coefficients $Z_\alpha, Z_\delta, M_\alpha, M_q, M_\delta$ are assumed to be constant. (Actually you don't necessarily have to understand these; all you have to know is that this is some linear dynamical system with the state $x = \begin{bmatrix} \alpha & q \end{bmatrix}^T$ and the input $u = \delta$.)



   a) Find the transfer function describing the dynamics from δ to q.
   b) The rocket's lateral acceleration a is given by $a = V(\dot{\alpha} - q)$ where V is the rocket's speed. Find a state-space description of the system describing the dynamics from δ to a. (Hint. a is in fact a linear combination of the state variable x and the control input u.)
   c) Let $Z_\alpha = -1$, $M_\alpha = 12$, and $M_q = -2$. Is the system stable?

4) *An important thing that your professor forgot to mention in class.* Suppose you have a system $G(s)$ as follows and let its response to the unit step input be $y(t)$.

$$G(s) = \frac{1}{s-2}$$

   a) Try to find $\lim_{t\to\infty} y(t)$ by using the Final Value Theorem.
   b) Directly find $y(t)$ from the inverse Laplace transform, then evaluate $\lim_{t\to\infty} y(t)$.
   c) Did you get the same results? If not, refer to your textbook and explain why.