Mood’s Median Test

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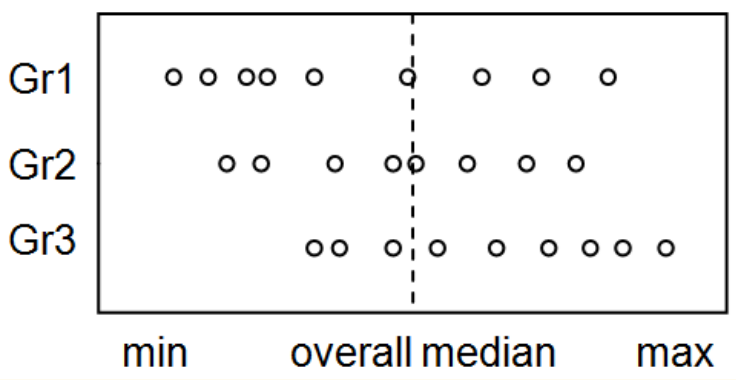
Mood's Median Test- What is it?

The Mood's median test compares whether K independent samples are drawn from populations with equal medians.

Therefore, Mood's median nonparametric hypothesis test is a special case of Pearson's Chi-squared test. It can also be considered as an alternative to the one-way ANOVA.



Assumptions for test

(Science Direct)

1. Data should include only one categorical factor
2. Response variable should be continuous
3. The population distributions that the samples were drawn from have similar shapes

Notes:

- More useful when there are < 20 observations
- Sample sizes can be unequal
- The populations do not have to be normally distributed

Hypotheses for Mood's Median Test

H_0: All population medians are equal

H_1: At least one median is not equal to the other medians

```
H₀ : M₁ = M₂ = M₃ = ..... Mₖ  ; M= Median
H₁ : At least two of them show significant difference.
```

Test Statistic

TEST STATISTIC

$$X_0^2 = \sum_{i=1}^{k} \frac{(O_{AMi} - E_{AMi})^2}{E_{AMi}} + \sum_{i=1}^{k} \frac{(O_{BMi} - E_{BMi})^2}{E_{BMi}}$$

O_{iAM} = Observed Frequencies of ith sample above Median
O_{iBM} = Observed Frequencies of ith sample below Median

E_{iAM} = Expected Frequencies of ith sample above Median
E_{iBM} = Expected Frequencies of ith sample below Median

REJECTION CRITERIA $X_0^2 > X_{\sigma,k-1}^2$

We reject the Null Hypothesis if Test Statistic Chi-square is greater than the critical value at a given level of significance (alpha) and K-1 degrees of freedom.

When is this test good to use?

ADVANTAGES:

1. This test can be applied for more than two samples
2. It is more useful when the data contains few outliers because this test only focuses on the median value instead of ranks
3. It can be used to test for symmetrical data
4. It doesn't require equal variances across the independent samples

Note: Usually researchers prefer Wilcoxon Rank Sum test or Mann-Whitney U test as they provide more robust results when compared to Mood's Median Test. (Six Sigma)

Steps for the test

1. Set Hypothesis
2. Set alpha level, find degrees of freedom, and compute χ^2 critical value
3. Calculate overall median
4. Construct a 2xK contingency table for observed values > and <= the overall median
5. Find the expected values
6. Calculate χ^2 Test statistic
7. Make a conclusion based on alpha level and test statistic

Example

(Geeks for Geeks)

35 Students were asked to provide ratings for a restaurant chain in different areas of the city.

There were:
11 Students for Restaurant A,
12 Students for Restaurant B, and
12 Students for Restaurant C.

The ratings are given on 4 conditions: Cleanliness, Taste, Service, and Price.

Each condition can get a maximum of 5 points, hence a restaurant can get a Maximum rating of 20 points.

Test whether the Median ratings are the same for the 3 Restaurant chains.

Area A	Area B	Area C
17	19	12
16	15	16
13	15	18
10	17	13
19	16	13
18	12	15
16	10	19
14	19	20
15	12	11
17	13	13
18	14	17
	15	18

Hypothesis, Critical Value, and Overall Median

1. Set Hypothesis

$H_0 : M_A = M_B = M_C$, where M_i = Median of i^{th} restaurant.

H_1 : At least one median is not equal to the other medians.

1. Set alpha level, find degrees of freedom, and compute Chi-Squared critical value

Set alpha level = 0.05; Degrees of freedom = K-1 = 3-1 = 2

Critical Chi-Square value = $\chi^2_{0.05,\,2} = 5.991$

Rejection area: Reject H_0 if observed χ^2 Test Statistic is greater than 5.991

1. Calculate overall median

There are 35 data points so the median will be the 18^{th} element after arranging in ascending order:

~~10~~, ~~10~~, ~~11~~, ~~12~~, ~~12~~, ~~12~~, ~~13~~, ~~13~~, ~~13~~, ~~13~~, ~~13~~, ~~14~~, ~~14~~, ~~15~~, ~~15~~, ~~15~~, ~~15~~, ***15***, ~~16~~, ~~16~~, ~~16~~, ~~16~~, ~~17~~, ~~17~~, ~~17~~, ~~17~~, ~~18~~, ~~18~~, ~~18~~, ~~18~~, ~~19~~, ~~19~~, ~~19~~, ~~19~~, ~~20~~

4. Construct a Contingency Table

Construct a 2xK table using the observed values with two columns – one showing the number of ratings above the overall median for each restaurant and the other showing the number of ratings below the overall median for each restaurant.

Observed				
	Area A	Area B	Area C	Totals
> overall median	7	4	6	17
<= overall median	4	8	6	18
Totals	11	12	12	**35**

5. Find Expected Values

Expected value of each cell = (Column total * Row total) / N

Expected	Area A	Area B	Area C	Totals
> overall median	5.34	5.83	5.83	17
<= overall median	5.66	6.17	6.17	18
Totals	11	12	12	35

6. Compute test statistic

$$\chi_0^2 = \frac{(7 - 5.34)^2}{5.34} + \frac{(4 - 5.83)^2}{5.83} + \frac{(6 - 5.83)^2}{5.83} + \frac{(4 - 5.66)^2}{5.66} + \frac{(8 - 6.17)^2}{6.17} + \frac{(6 - 6.17)^2}{6.17}$$

$\chi^2_0 = 2.125$

7. Draw a conclusion

$2.125 < 5.991$, therefore we fail to reject H_0 and conclude that the true median scores of each restaurant are equal.

Package: RVAideMemoire

R Code

(RCompanion)

```r
Type <- c('A','A','A','A','A','A','A','A','A','A','A',
          'B','B','B','B','B','B','B','B','B','B','B','B',
          'C','C','C','C','C','C','C','C','C','C','C','C')

Value <- c(17,16,13,10,19,18,16,14,15,17,18,
           19,15,15,17,16,12,10,19,12,13,14,15,
           12,16,18,13,13,15,19,20,11,13,17,18)

dat <- data.frame(Group,Value)

library(RVAideMemoire)
mood.medtest(Value ~ Type, data = dat, exact = FALSE)
```

```
> mood.medtest(Value ~ Type, data=data, exact = FALSE)

        Mood's median test

data:  Value by Type
X-squared = 2.1247, df = 2, p-value = 0.3456
```

Works Cited

https://www.sciencedirect.com/science/article/abs/pii/S0167715214002855

https://ritsokiguess.site/pasias/mood-median-test.html

https://sixsigmastudyguide.com/moods-median-non-parametric-hypothesis-test/

https://www.geeksforgeeks.org/moods-median-test/

https://rcompanion.org/handbook/F_05.html

Questions?