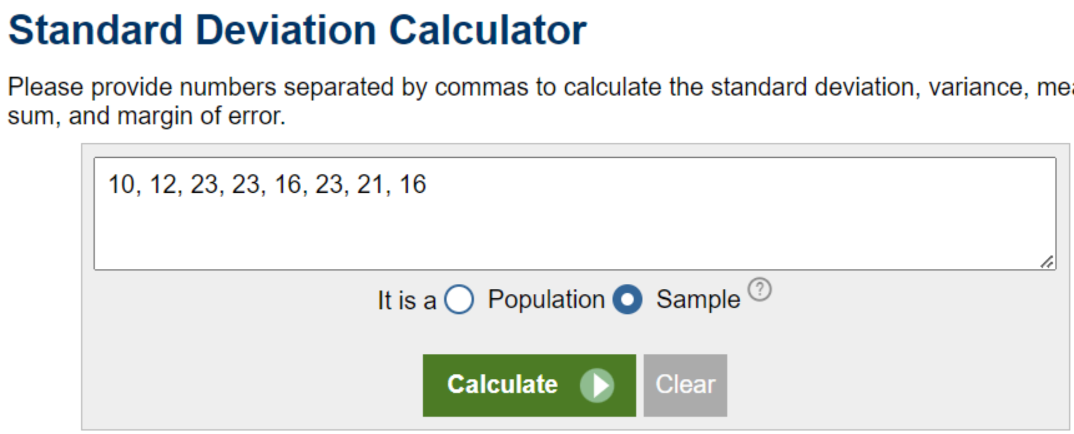
Confidence Interval of Population Mean (Unknown σ)

The confidence interval of the population mean is given as $[\bar{X} - E, \bar{X} + E]$, where E is called the margin of error. When the population standard deviation, $\boldsymbol{\sigma}$**, is unknown**, the margin of error can be computed using the sample standard deviation, $\boldsymbol{s}$, the critical value, t_c, and the sample size n.

The sample standard deviation, $\boldsymbol{s}$, can be obtained from data using the following applets. You may use the direct formula to calculate, but it is demanding. I recommend using the following link. The sample standard deviation could be provided in the problem statement.

- Applets for the sample standard deviation
  https://www.calculator.net/standard-deviation-calculator.html



Ex1. Regional researchers want to know the average incubation period for COVID-19 (coronavirus disease) in a small county. Note that the incubation period is defined by the CDC as "the time from exposure to the causative agent until the first symptoms develop." Suppose the researchers obtained a sample of 11 patients with an identifiable exposure time in this county. The population standard deviation is assumed to be unknown.

7, 11, 12, 3, 10, 5, 7, 9, 4, 8, 1

Q1. What is the sample size? 11

Q2. What is the target parameter?

The average incubation period for all COVID-19 people in a small county.

Q3. What is the sample mean of the data?

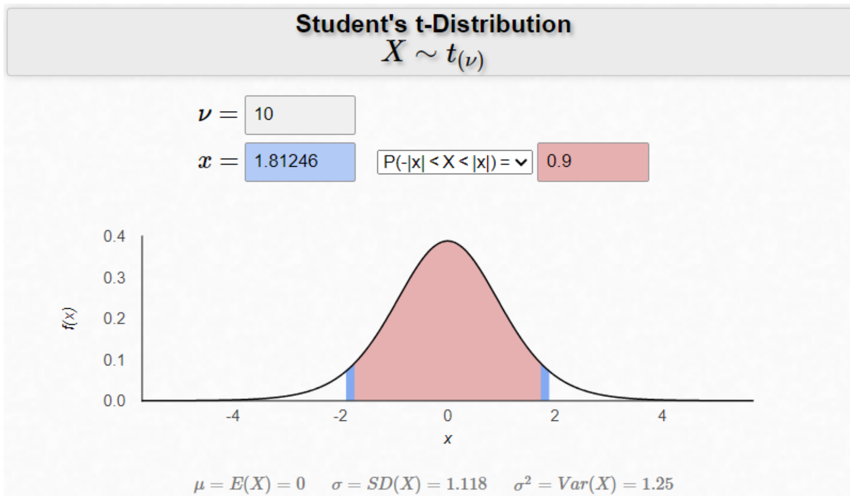
7

Q4. What is the sample standard deviation of the data?

3.4641

The critical value t_c can be obtained from the t-distribution with degrees of freedom $n-1$. The critical value t_c satisfies $P(-t_c < t < t_c) = c$ when t follows t_{n-1}.

- Applets for the student t-distribution
  https://homepage.divms.uiowa.edu/~mbognar/applets/t.html



The margin of error can be computed by the following formula.

$$E = t_c \frac{s}{\sqrt{n}}$$

Then, the confidence interval is $[\bar{X} - t_c \frac{s}{\sqrt{n}}, \bar{X} + t_c \frac{s}{\sqrt{n}}]$

(Continue on Ex1) The researchers want to construct a 90% confidence interval of the population mean using this dataset.

Q1. What is the degrees of freedom of t-distribution? 10

Q2. What is the critical value? 1.81246

Q3. What is the 90% confidence interval?

$$\left[7 - 1.81246\frac{3.4641}{\sqrt{11}}, 7 + 1.81246\frac{3.4641}{\sqrt{11}}\right] = [5.1069, 8.8931]$$

Q4. When the researchers wanted to build a 95% confidence interval, what is the confidence interval? Is it wider or narrower than the results in Q3?

$$\left[7 - 2.228139\frac{3.4641}{\sqrt{11}}, 7 + 2.228139\frac{3.4641}{\sqrt{11}}\right] = [4.6728, 9.3272]$$

Q5. The researchers collected more data sample in this region and found that the sample mean and the sample standard deviation does not change. Will they have a narrower or wider confidence interval? The more sample size, the narrower confidence interval is.