Electroweak Symmetry Breaking in Gauge-Higgs Unification Models

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Work with M. Peskin

- At the TeV scale, one of the most pressing questions in particle physics is to understand the spontaneous breaking of the electroweak symmetry.

- The Standard Model cannot give answers to:
  - Why is electroweak symmetry broken?
  - Can we calculate the Higgs mass or the Higgs potential?

- Ideally, it would be nice to have a predictive model where we can calculate the Higgs potential in terms of a more fundamental set of parameters, and therefore we can study the conditions of SSB.
- However, it is very hard to achieve with the Higgs as an elementary scalar.
- Consider the Higgs mass term $\mu^2|H|^2$.
  - Divergent radiative corrections.
  - Invariant under all standard symmetries.
- Therefore, we need an unusual (approximate) symmetry to forbid the Higgs mass term.

- SUSY is a possible solution, but highly constrained by LHC.
- What have we seen so far?
  - Light Higgs with mass 125 GeV.
  - a Little Hierarchy, i.e. no new dynamics near the Higgs vev.
- These two observations suggest us to consider the Higgs boson as a composite pseudo-Goldstone boson of a new strong dynamics.
- Is there a calculable, predictive approach to composite Higgs?

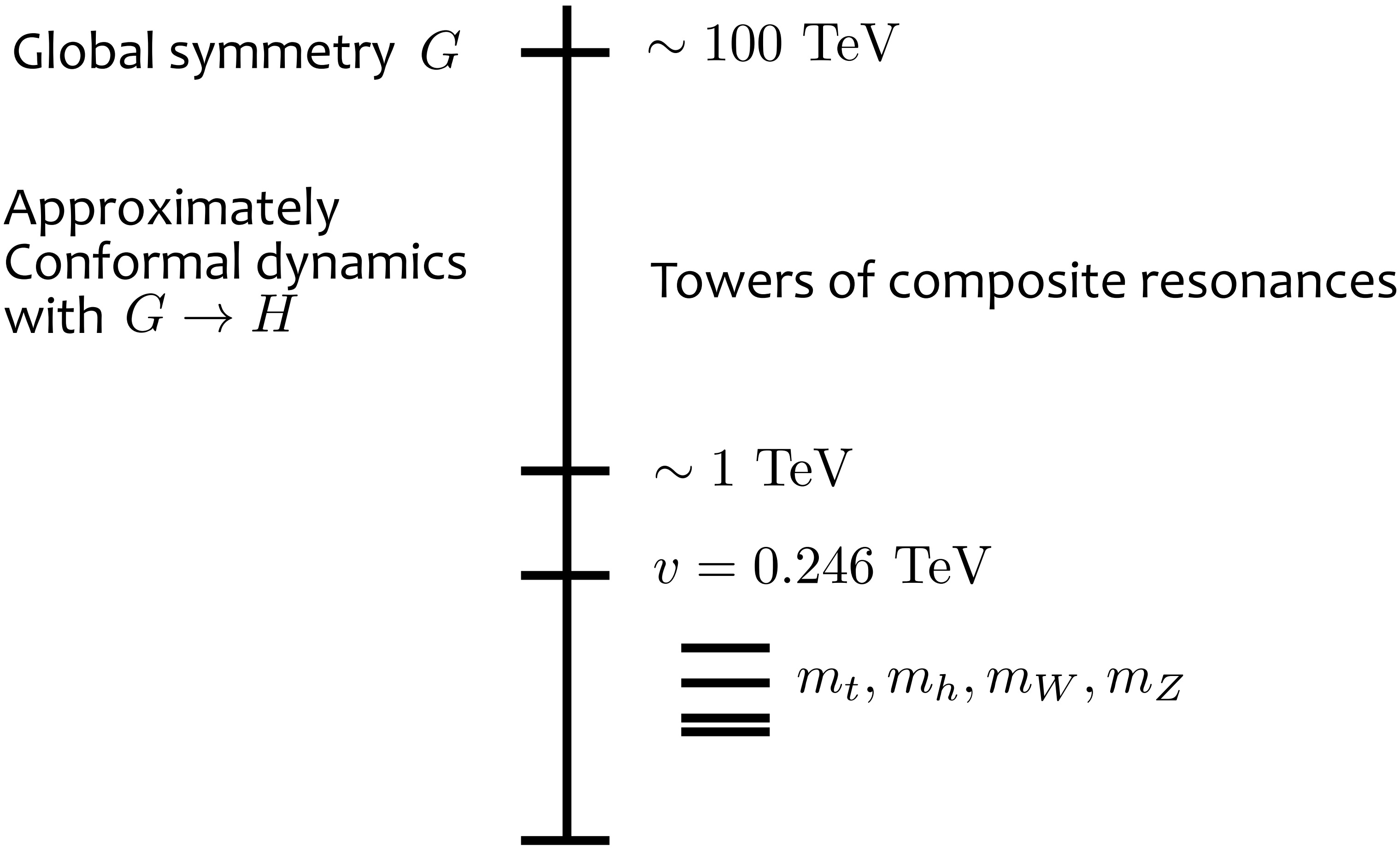
- In this talk, I will discuss an approach to these problems based on following ideas.
  1) Randall-Sundrum geometry.
  2) Higgs as a partner of a vector field.
  3) 2^{nd} order phase transition in fermion condensation.
- These ideas, particularly 1) and 2), were actively pursued in the early 2000's by Hall, Nomura, Agashe, Contino, Pomarol, Hosotani and others.
- With the observed Higgs mass, we would like to advance this theory and make it more predictive.

Randall-Sundrum Geometry

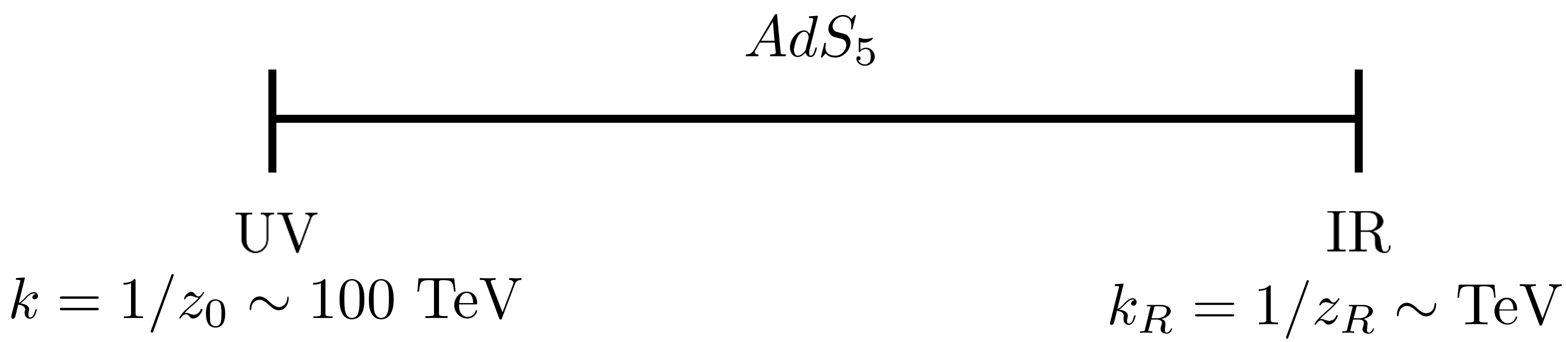
- Realize 4D strong theory with a 5D weakly-coupled theory in AdS5 with two boundaries, as motivated by the AdS/CFT correspondence.

4D		5D
Global symmetries		Gauge symmetries
SM fields	$\leftrightarrow$	Zero-modes
Bound states		Kaluza-Klein states
Higgs boson		A_5^A

Physical Picture in 4D



Realization in 5D



- We consider all fields propagating in the bulk.
- No elementary scalar fields.

Higgs as a Partner of a Vector Field

- Goldstone Higgs in a 4D strong theory
  - From spontaneous breaking of a global symmetry G.
  - (Small) breaking of G will generate the Higgs potential.

- Gauge-Higgs Unification in the 5D theory Hall, Nomura, Tucker-Smith / Hosotani
  - Break the gauge symmetry G by boundary conditions.
  - Then, A_5^A, which transforms as a scalar under 4D Lorentz symmetry, have zero modes.
  - A radiative potential from the top quark can generate vacuum expectation value.

Gauge fields in AdS5

- Under G, we have (A_μ^A, A_5^A).
- We define boundary conditions at z_0, z_R.

  Dirichlet : $(-)$ $A_\mu^A = 0$

  Neumann : $(+)$ $\partial_5 A_\mu^A = 0$

- 5d gauge symmetry enforces that A_5^A have the opposite b.c. to A_μ^A
- $(++)$ b.c. : A_μ^A zero mode → gauge symmetry in 4D theory.
- $(--)$ b.c. : A_5^A zero mode → can get a vev.

Fermions in AdS5 $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

- In 5d, fermions have mass term, $c = m/k$
- The coupled EOM enforces the opposite b.c. between ψ_L, ψ_R
- We define boundary conditions at $z_0, \quad z_R$

  Dirichlet : $(-) \quad \psi_L = 0$

  Neumann : $(+) \quad \psi_R = 0$
- $(++)$ b.c. : ψ_L zero mode
- $(--)$ b.c. : ψ_R zero mode
- (A_μ^A, A_5^A) is equivalent to (ψ_L, ψ_R) with $c = 1/2$
  in terms of the 5D wave-function.

Shape of the Zero Modes

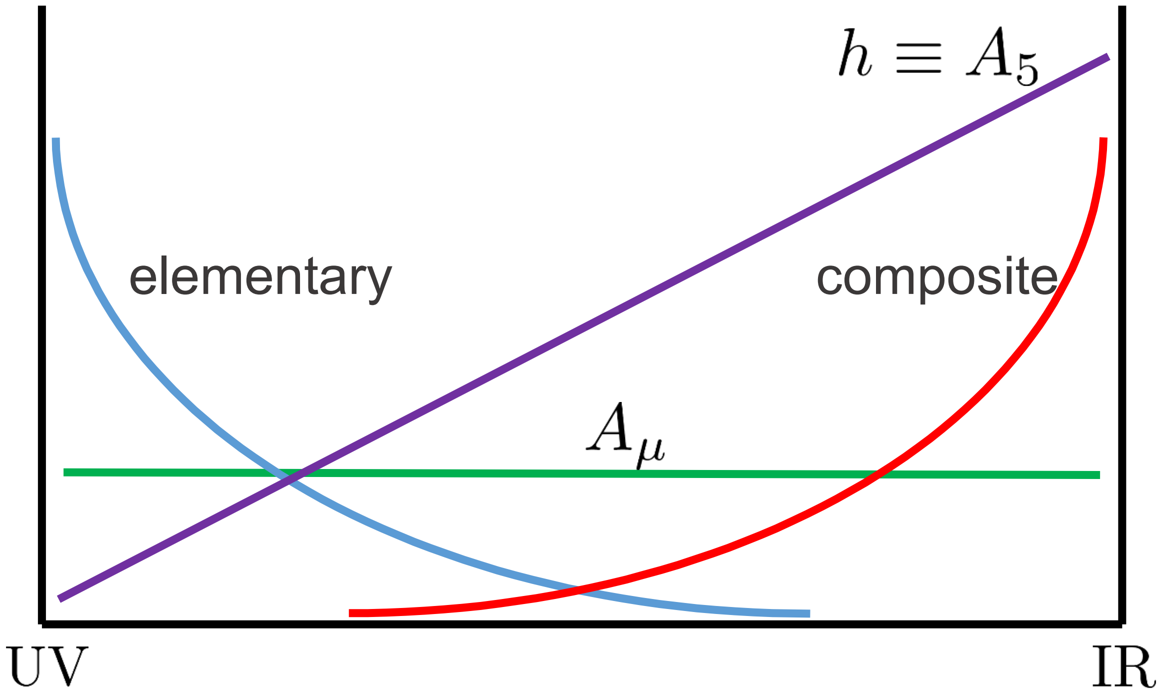
- Massless solutions of the EOM are

$$(++) \;:\; \Psi \sim \begin{pmatrix} u_-(p) z^{2-c} \\ 0 \end{pmatrix} \qquad (--) \;:\; \Psi \sim \begin{pmatrix} 0 \\ u_+(p) z^{2+c} \end{pmatrix}$$

- Then,

$$\int dz \sqrt{g}\,\overline{\Psi}\gamma^0\Psi \sim \int dz \frac{1}{(kz)^4} z^{4\mp 2c} \sim \int \frac{dz}{z} z^{1\mp 2c}$$

- Therefore
  - Left-handed zero mode : UV for c > ½, IR for c < ½
  - Right-handed zero mode : UV for c < -½, IR for c > -½
- Gauge field : c=1/2 fermion
  - A_μ^A flat, A_5^A linear in z



- For strong coupling to the Higgs, a composite zero mode is required.

Higgs potential

- Models of Goldstone bosons and dynamical symmetry breaking have potentials in terms of non-linear sigma field U

$$U = \exp\left(\frac{2i\Pi \cdot T}{f}\right) \qquad h \in \Pi$$

Then, it is natural that $V(\Pi)$ is minimized at $v = 0$ or $v \sim f$,

but we need $0 < v \ll f$ for the Little Hierarchy.

→ Therefore, we must be near a *2nd-order phase transition* in the phase diagram of the model.

- We need competing contributions to the Higgs potential.

Coleman-Weinberg Potential for A5

- Consider a non-zero background 5D gauge field

$$A_M(x^\mu, z) = (0,0,0,0,A_5(z))$$

- By a gauge transformation,
we can remove the background field in the bulk.
But it modifies boundary conditions at either $z = z_0$ or $z = z_R$
by a transformation $$U_W = \exp\left[i g_5 \int_{z_0}^{z_R} dz A_5^a(z) T^a \right]$$

- By integrating out fermions under the twisted b.c.,
we can compute the Coleman-Weinberg potential of Higgs.
It will depend on U_W

SU(2) Example

Contino, Nomura, Pomarol

- Consider a bulk SU(2) gauge theory broken down to U(1) by the b.c.

$$A_\mu^A = \begin{pmatrix} A_\mu^1 \\ A_\mu^2 \\ A_\mu^3 \end{pmatrix} \sim \begin{pmatrix} -- \\ -- \\ ++ \end{pmatrix}$$

- Now turn on $\langle A_5^2 \rangle \neq 0$. This will further break the remaining U(1).

$$U_W = \exp\left[i g_5 \int_{z_0}^{z_R} dz A_5^2(z) \frac{\sigma^2}{2} \right] = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad \theta \sim \frac{v}{f}$$

- Note that U_W is a non-local quantity.
  → Finite CW potential expected.

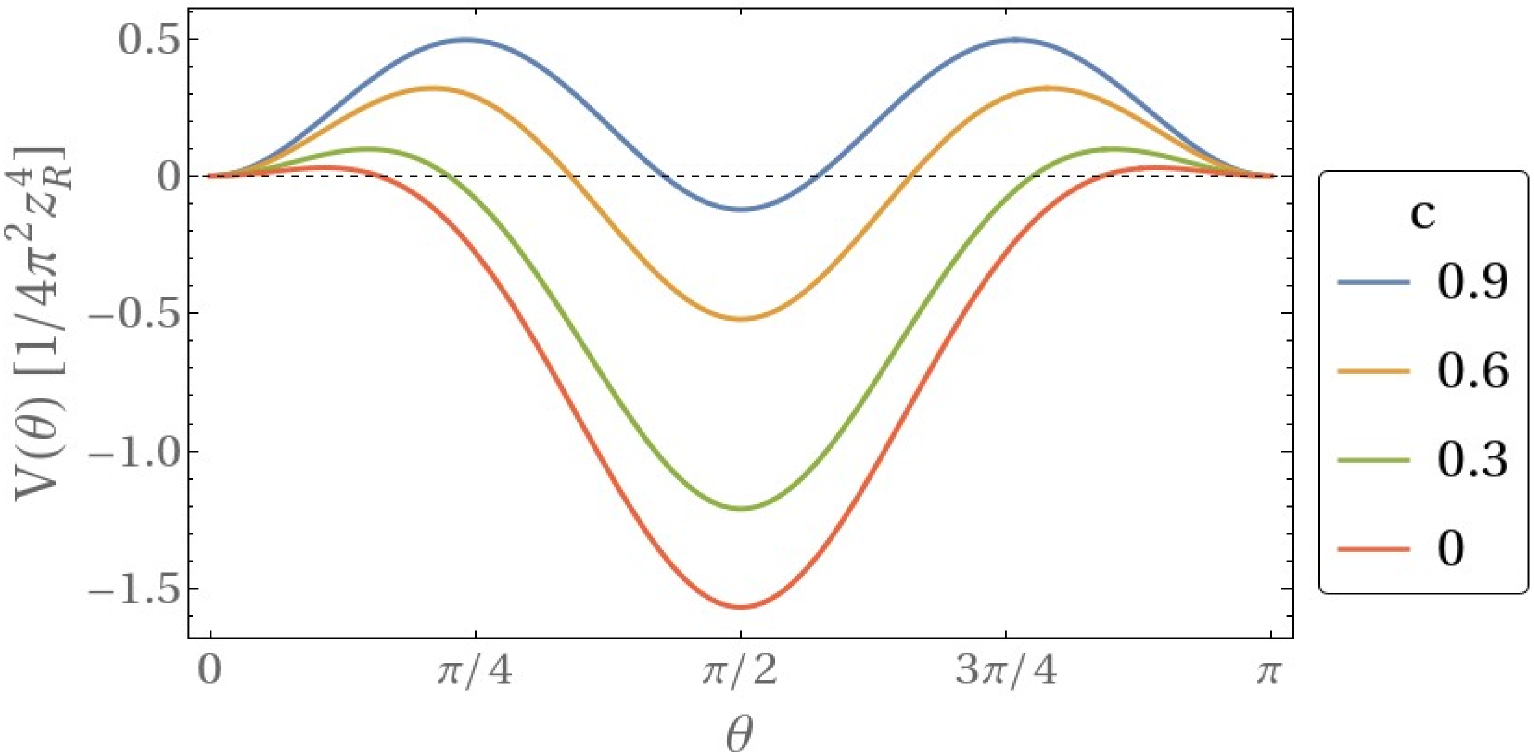
SU(2) Example

- Consider Coleman-Weinberg potential from $\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} ++ \\ -- \end{pmatrix}$
- It includes two massless fermions, and therefore it is energetically favored to pair them up and make massive.
- Indeed, the explicit calculation of CW potential shows

$$V(\theta) = -2 \int \frac{d^4p}{(2\pi)^4} \log\left[1 + \frac{\sin^2\theta}{F(p^2, c)}\right]$$

- $F(p^2, c)$ is a combination of Bessel functions and calculable! It is exponentially increasing for large p → convergent integral.
- $F(p^2, c = 0) = \sinh^2(p(z_R - z_0))$ and the depth of the potential decreases with larger $|c|$
- For the full consideration, we also need to include the potential from gauge fields.

The Complete CW potential of SU(2)



1) Potential minimum always at $\theta = \dfrac{\pi}{2} \rightarrow \; v \sim f$

2) Only first-order phase transition

Competing forces from different fermion multiplets

- Consider two different doublets and their CW potential

$$\Psi_A = \begin{pmatrix} ++ \\ -- \end{pmatrix} \quad \to \quad V_A(\theta) = -2\int \frac{d^4p}{(2\pi)^4} \log\left[1 + \frac{\sin^2\theta}{F_A(p^2,c)}\right]$$

$$\Psi_R = \begin{pmatrix} +- \\ -+ \end{pmatrix} \quad \to \quad V_R(\theta) = -2\int \frac{d^4p}{(2\pi)^4} \log\left[1 - \frac{\sin^2\theta}{F_R(p^2,c)}\right]$$

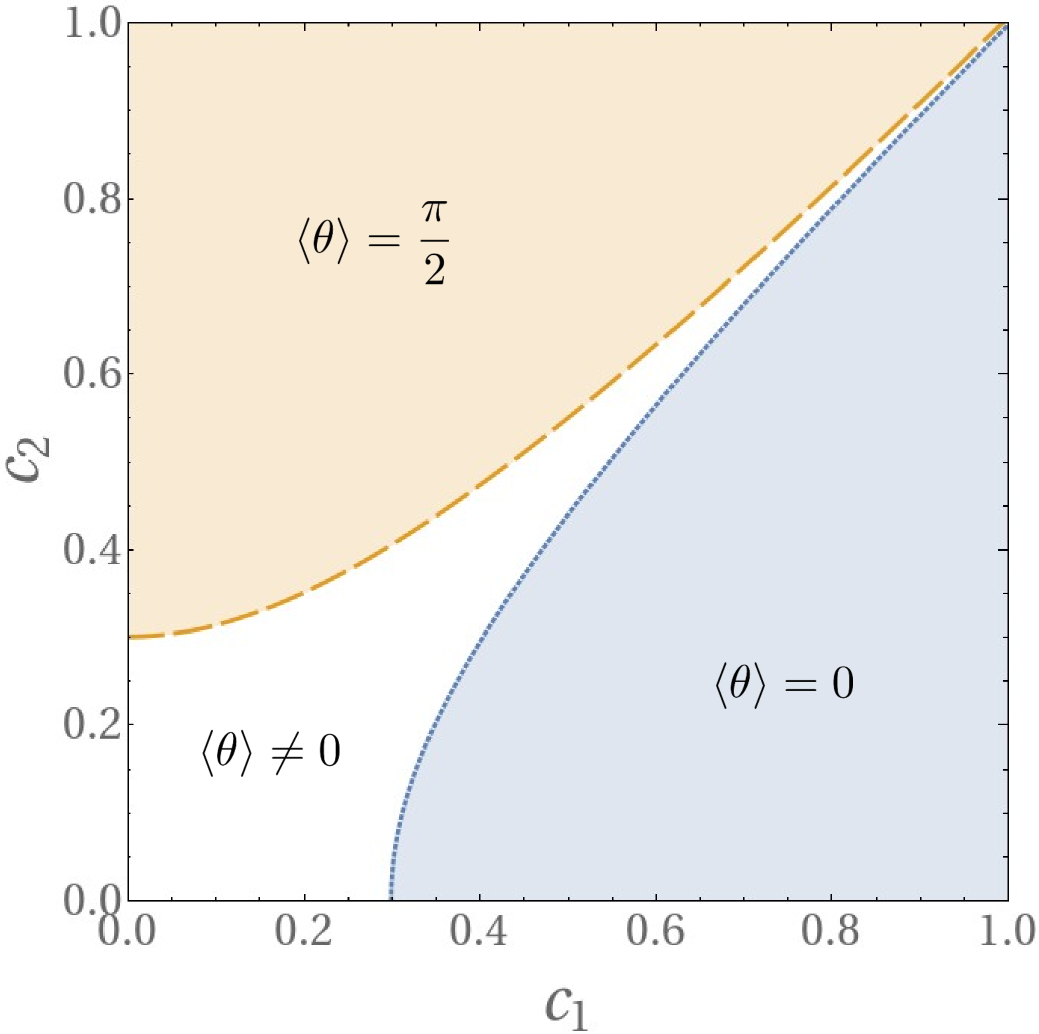
- The new doublet Ψ_R opposes the fermion condensation. It does not have zero modes, but its lowest KK state is typically lighter than the the new physics scale, k_R . In a realistic model, this particle (i.e. vector-like top partner) can be within the reach of collider experiments.

- Small s_W expansion up to s_W^4

$$V_A(s_W, c_1) = -A_A(c_1)s_W^2 + \frac{1}{2}B_A(c_1)s_W^4 + \frac{1}{2}C_A(c_1)s_W^4 \log(1/s_W^2)$$
$$V_R(s_W, c_2) = +A_R(c_2)s_W^2 + \frac{1}{2}B_R(c_2)s_W^4$$

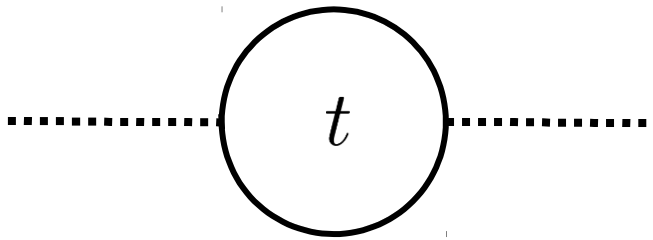
- The line of second-order phase transitions : $A_A(c_1) = A_R(c_2)$
- So, a clever choice of fermion b.c. and c parameters can put the vacuum in the vicinity of the 2nd-order phase transition, and therefore $v \ll f$
- Fine-tuning in 5D might not be a fine-tuning in 4D.

SU(2) Phase Diagram



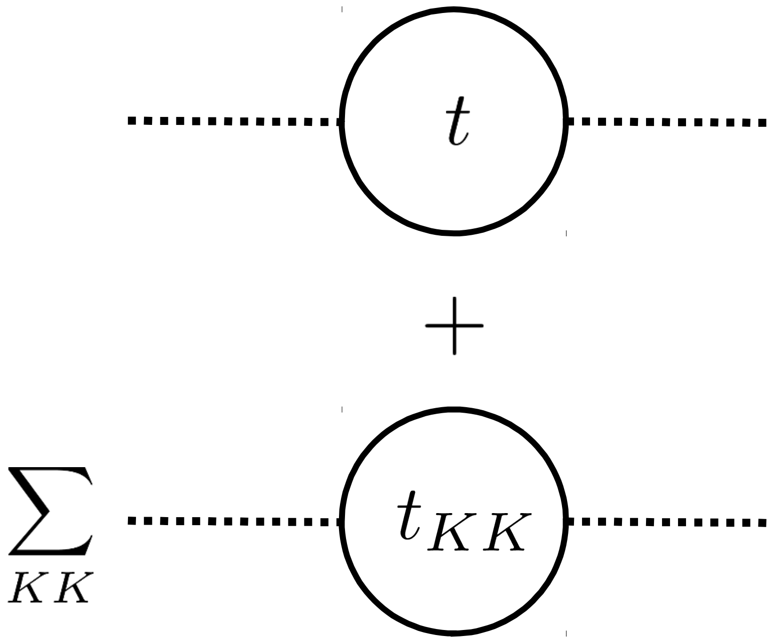
Blue line : Line of 2nd-order phase transition $A_A(c_1) = A_R(c_2)$

$\Psi_A = t, \qquad \Psi_R = T$



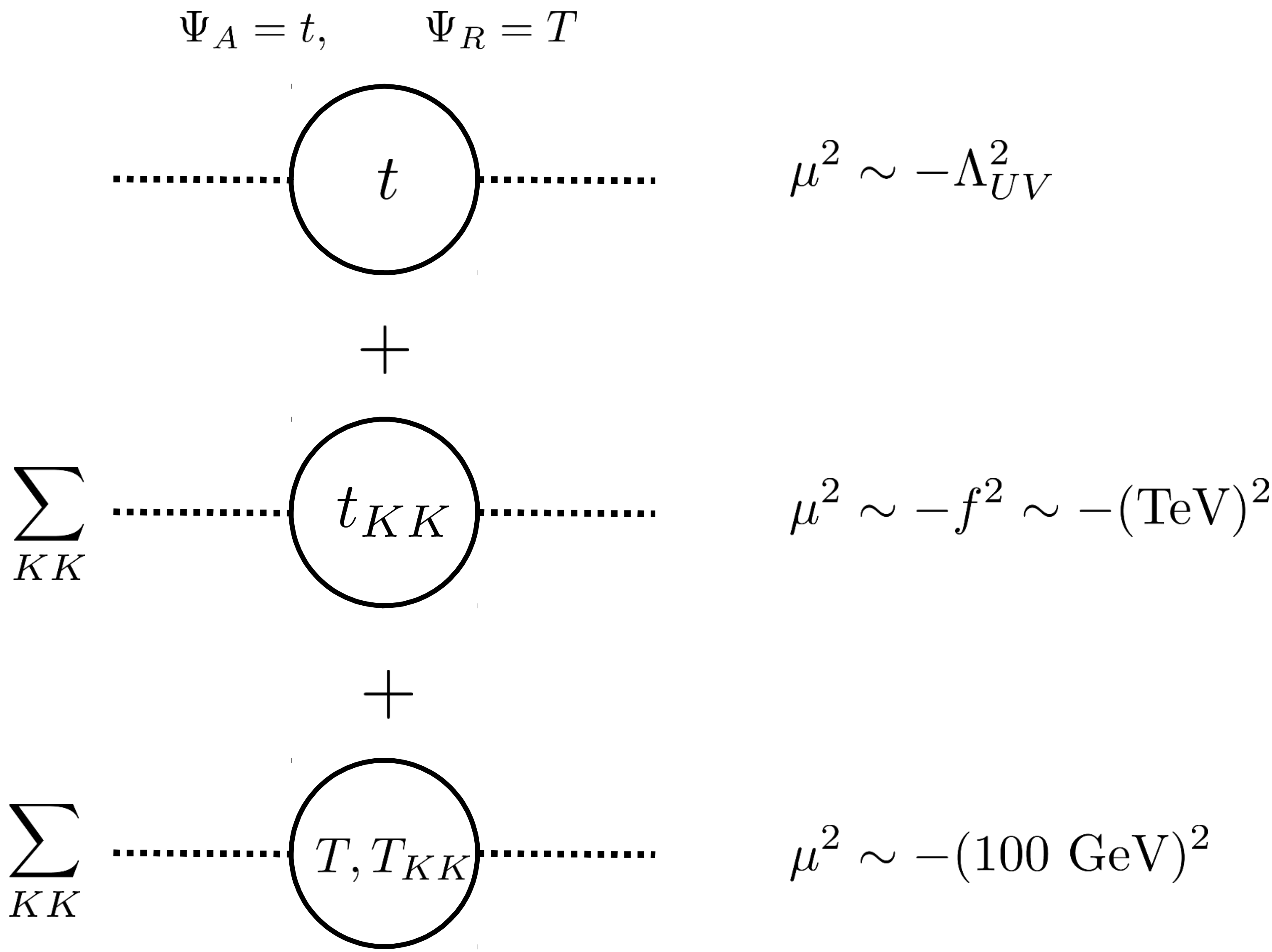
$\mu^2 \sim -\Lambda_{UV}^2$

$\Psi_A = t, \qquad \Psi_R = T$



$\mu^2 \sim -\Lambda_{UV}^2$

$\mu^2 \sim -f^2 \sim -(\mathrm{TeV})^2$


$\Psi_A = t, \qquad \Psi_R = T$
t
$\mu^2 \sim -\Lambda_{UV}^2$
$+$
$\sum_{KK}$
t_{KK}
$\mu^2 \sim -f^2 \sim -(\mathrm{TeV})^2$
$+$
$\sum_{KK}$
T, T_{KK}
$\mu^2 \sim -(100\ \mathrm{GeV})^2$

SO(5) x U(1) Model Agashe, Contino, Pomarol

SM gauge symmetry

$SU(2)_L \times U(1)_Y$

AdS5 bulk

$SO(5) \times U(1)_X$

Custodial symmetry protects T

$SO(4) \times U(1)_X$

UV

$k = 1/z_0 \sim 100 \text{ TeV}$

UV cutoff scale

IR

$k_R = 1/z_R \sim 1 \text{ TeV}$

Scale of the strong interaction

$SO(5)/SO(4)$: Higgs as Goldstone bosons (A_5 zero mode)

$$Y = T_R^3 + X \quad \text{and} \quad Q = T_L^3 + T_R^3 + X$$

Fermions in SO(5)xU(1)

- Embedded in $\mathbf{5_{2/3}} = (\mathbf{2},\mathbf{2})_{\mathbf{2/3}} \oplus (\mathbf{0},\mathbf{0})_{\mathbf{2/3}}$

$$\Psi_t = \begin{bmatrix} \begin{pmatrix} \chi_t(-+) & t_L(++) \\ \chi_b(-+) & b_L(++) \end{pmatrix} \\ t_R(--) \end{bmatrix} \qquad \Psi_T = \begin{bmatrix} \begin{pmatrix} \chi_T(-+) & T(+-) \\ \chi_B(-+) & B(+-) \end{pmatrix} \\ T'(-+) \end{bmatrix}$$

- Custodial symmetry for $Z \to bb$ Agashe, Contino, Da Rold, Pomarol
- Vector-like partners (T, B) of (t_L, b_L) and Q=5/3 particles χ_t, χ_T
- Their contributions to the Higgs potential

$$V_t(h) = -3 \times 2 \int \frac{d^4p}{(2\pi)^4} \log\left[1 + \frac{\frac{1}{2}\sin^2 2\theta}{F_A(p^2, c_t)}\right]$$

$$V_T(h) = -3 \times 2 \int \frac{d^4p}{(2\pi)^4} \log\left[1 - \frac{1-\cos^4\theta}{F_R(p^2, c_T)}\right]$$

Boundary Kinetic Term

- In the setup so far, a single parameter g_5 sets the strength of the new forces as well as $SU(2)_L \times U(1)_Y$ gauge couplings and top Yukawa coupling.
- Then, $m_t \sim m_W \sim m_h$ is expected in this system.
- We can think that some UV dynamics (e.g. GUT) beyond z_0 separate those couplings from that of the strong interaction.
- In order to incorporate this, we introduce boundary kinetic terms on the UV boundary for the $SU(2)_L \times U(1)_Y$ gauge fields and fermions.
- Such UV physics can also introduce mixing on the UV boundary.

Boundary Kinetic Term

- $S_A^{UV} = \int d^4x dz \; a z_0 \delta(z - z_0)\sqrt{-g}\left[-\frac{1}{4} g^{mp} g^{nq} F_{mn} F_{pq}\right]$

  $S_\Psi^{UV} = \int d^4x dz \; a z_0 \delta(z - z_0)\sqrt{-g}\left[\bar{\Psi} i e_A^m \gamma^A \mathcal{D}_m \Psi\right]$

- This gives knobs to control the weak coupling and y_t. For example,

$$g^2 = \frac{g_5^2 k}{\log(z_R/z_0) + a}$$

- That is, the boundary term decreases the coupling of the zero modes to the strong sector. It also decreases the CW potential, particularly the gauge contribution.

- We have boundary terms a_L, a_Y for $SU(2)_L \times U(1)_Y$ and a_t for (t_L, b_L)

Allowed Parameter Space

- There are 9 parameters in this model.

$$z_R, z_0,\ c_t, c_T,\ g_5, g_X,\ a_L, a_Y, a_t$$

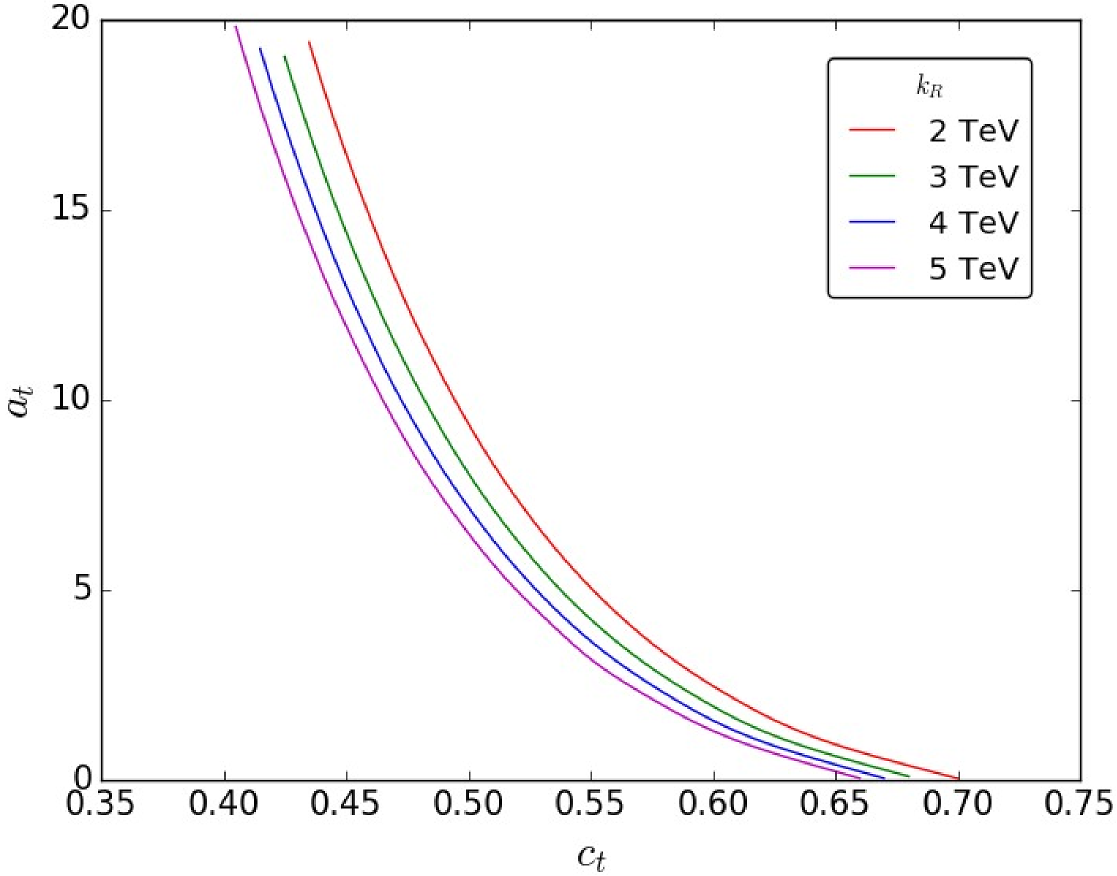
- They are constrained by 5 observables.

$$e, m_W, m_Z, m_t, m_h$$

- It seems to be 4-dimensional parameter space. However,
  - Higgs potential is almost independent of a_Y
  - Observables at TeV scale are only weakly dependent on z_0.

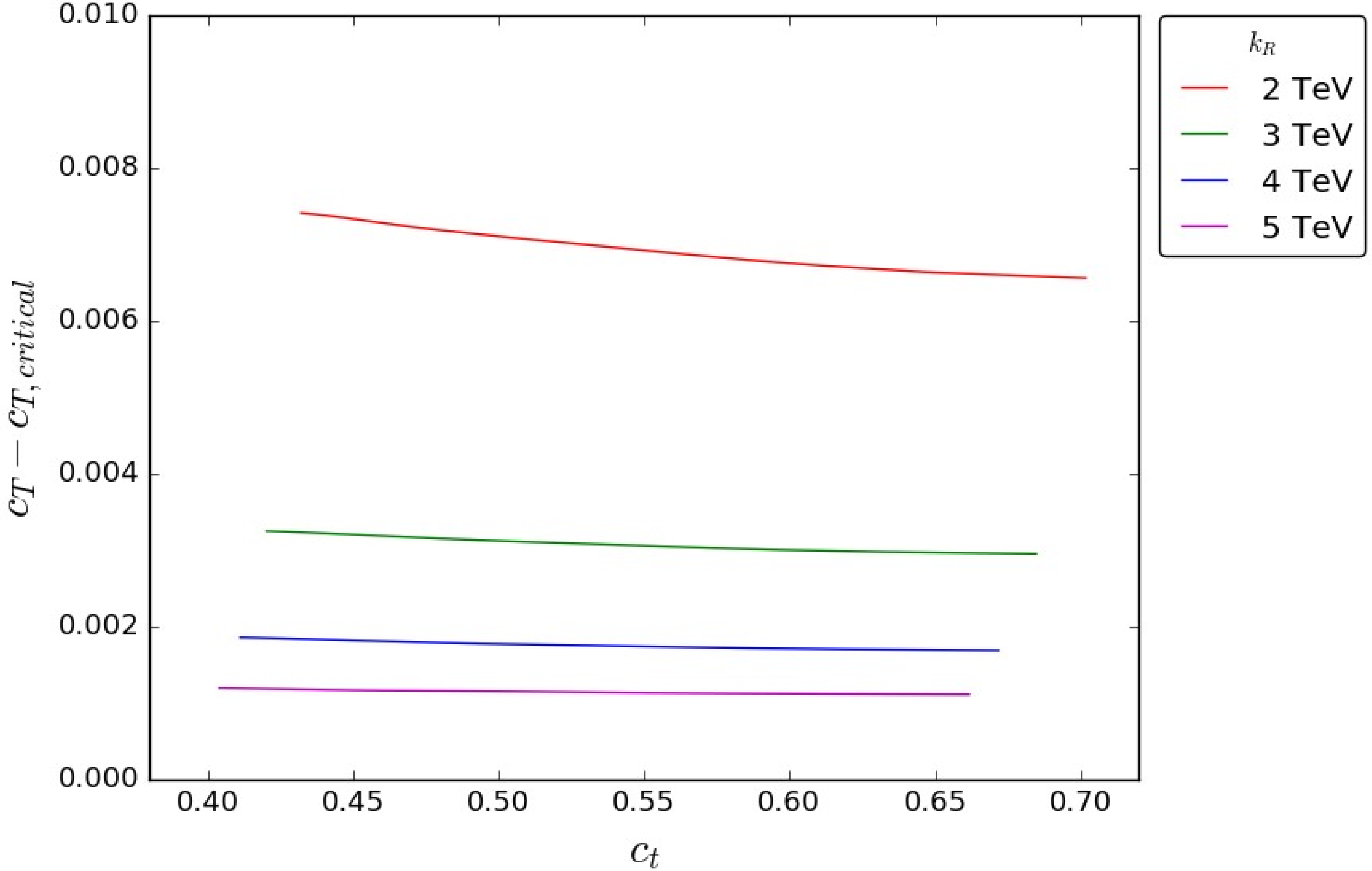
  The exact position of z_0 will be constrained by flavor physics.
  We suggest to use $z_0/z_R = 1/100$.
- Therefore, we have a quasi-2-dimensional parameter space.
- We fix k_R, and fit other parameters.

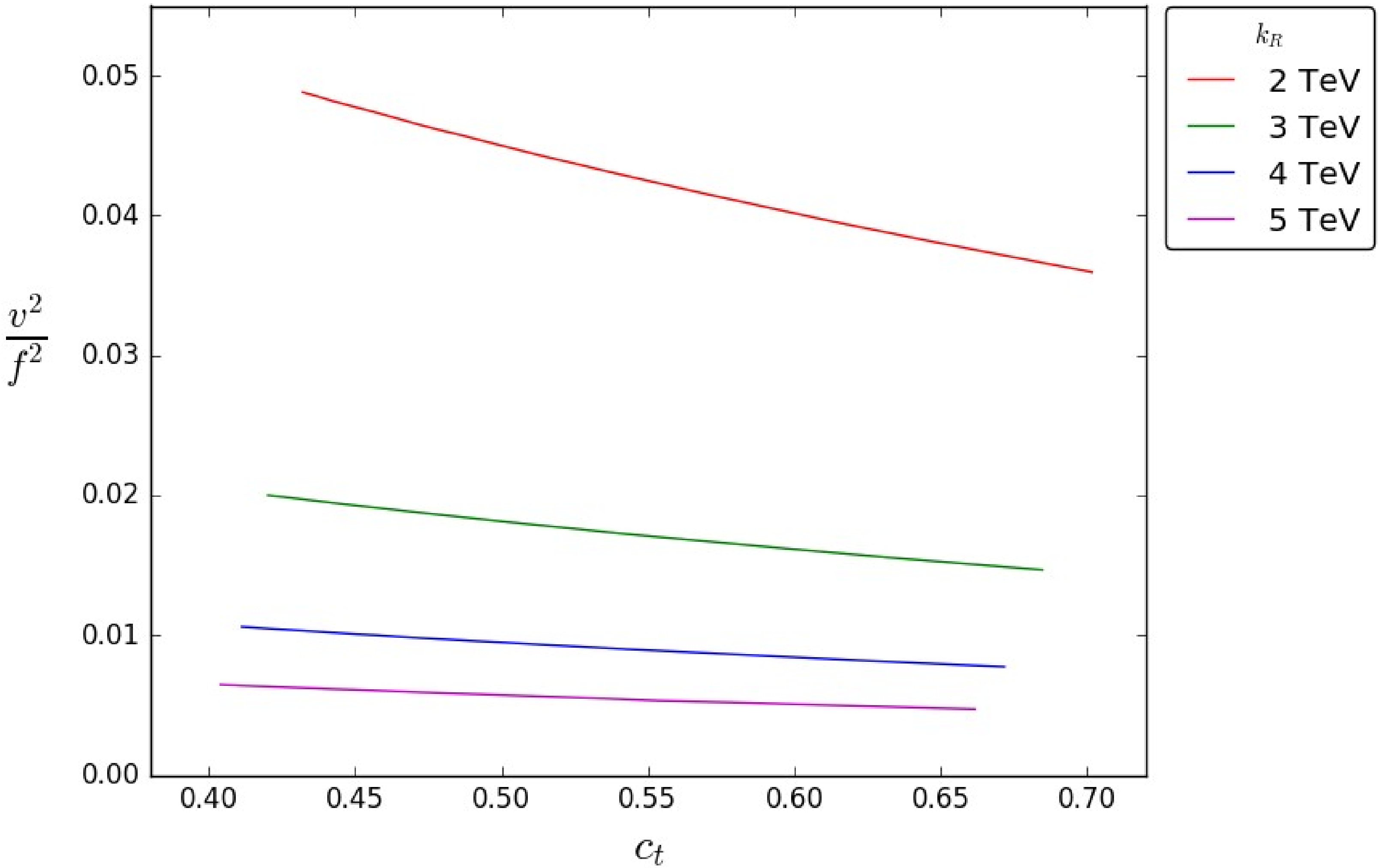
- It turns out that other parameters do not vary much across different k_R.



- That is, the BSM effects scale with k_R.
- Note that the typical Z' mass is $m_{Z'} \sim 2.4 k_R$

Possible measure of tuning, $c_T - c_{T,critical}$





k_R
2 TeV
3 TeV
4 TeV
5 TeV
0.00
0.01
0.02
0.03
0.04
0.05
$\frac{v^2}{f^2}$
0.40
0.45
0.50
0.55
0.60
0.65
0.70
c_t

Precision Electroweak Analysis

- S parameter receives a tree-level correction

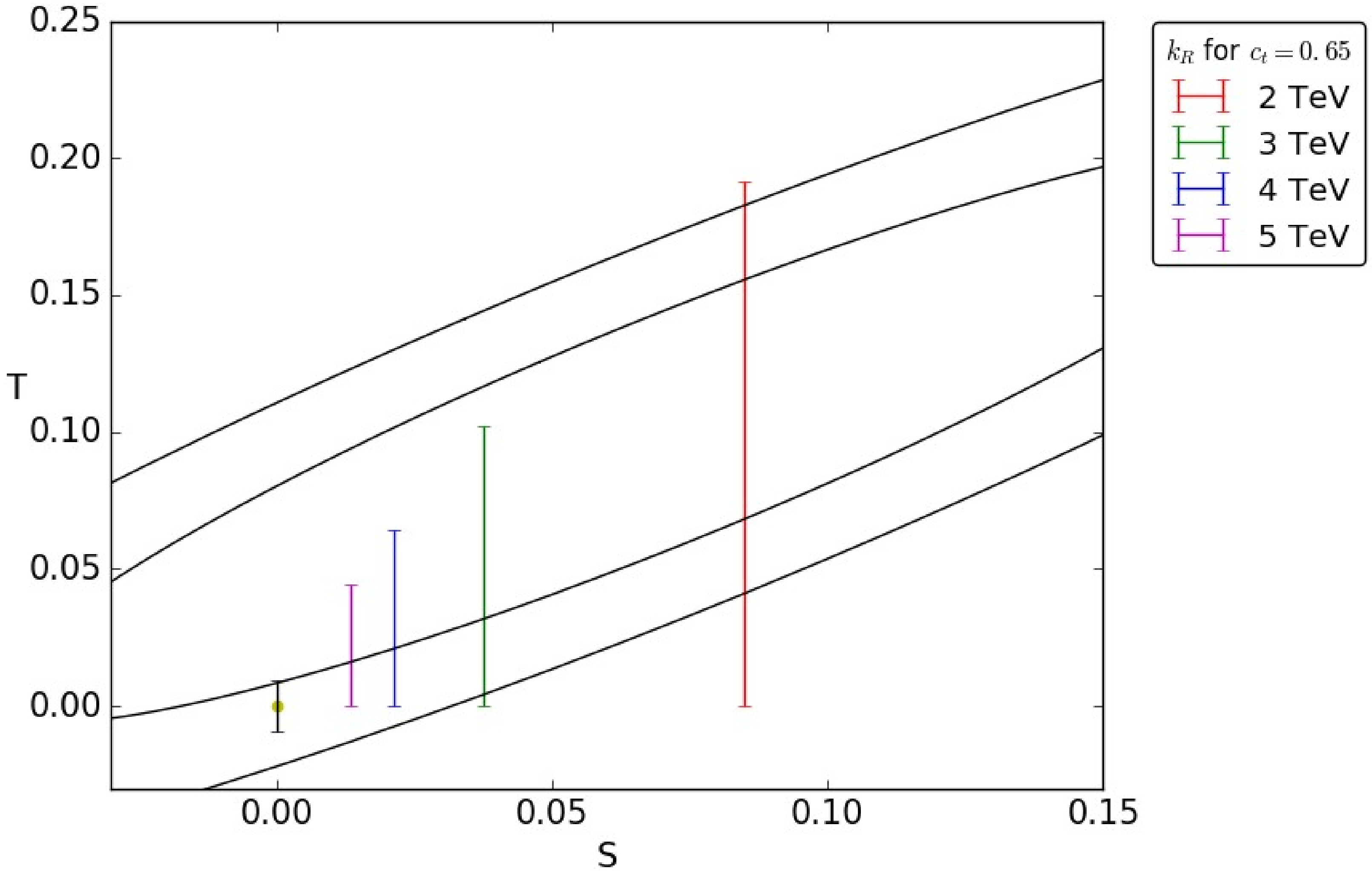
$$S = \pi v^2 z_R^2 \left(\frac{3}{2} - \frac{1}{k\pi R + a_Y} \right)$$

- Fermion loop correction to T parameter
  - It has been argued that the loop correction is convergent so the first KK correction is a good estimate. Carena, Ponton, Santiago, Wagner
  - However, a full 5D calculation shows that it is divergent.
  - The divergent term is subleading in m_t^2.
    Using a cutoff Λ, we can estimate the correction

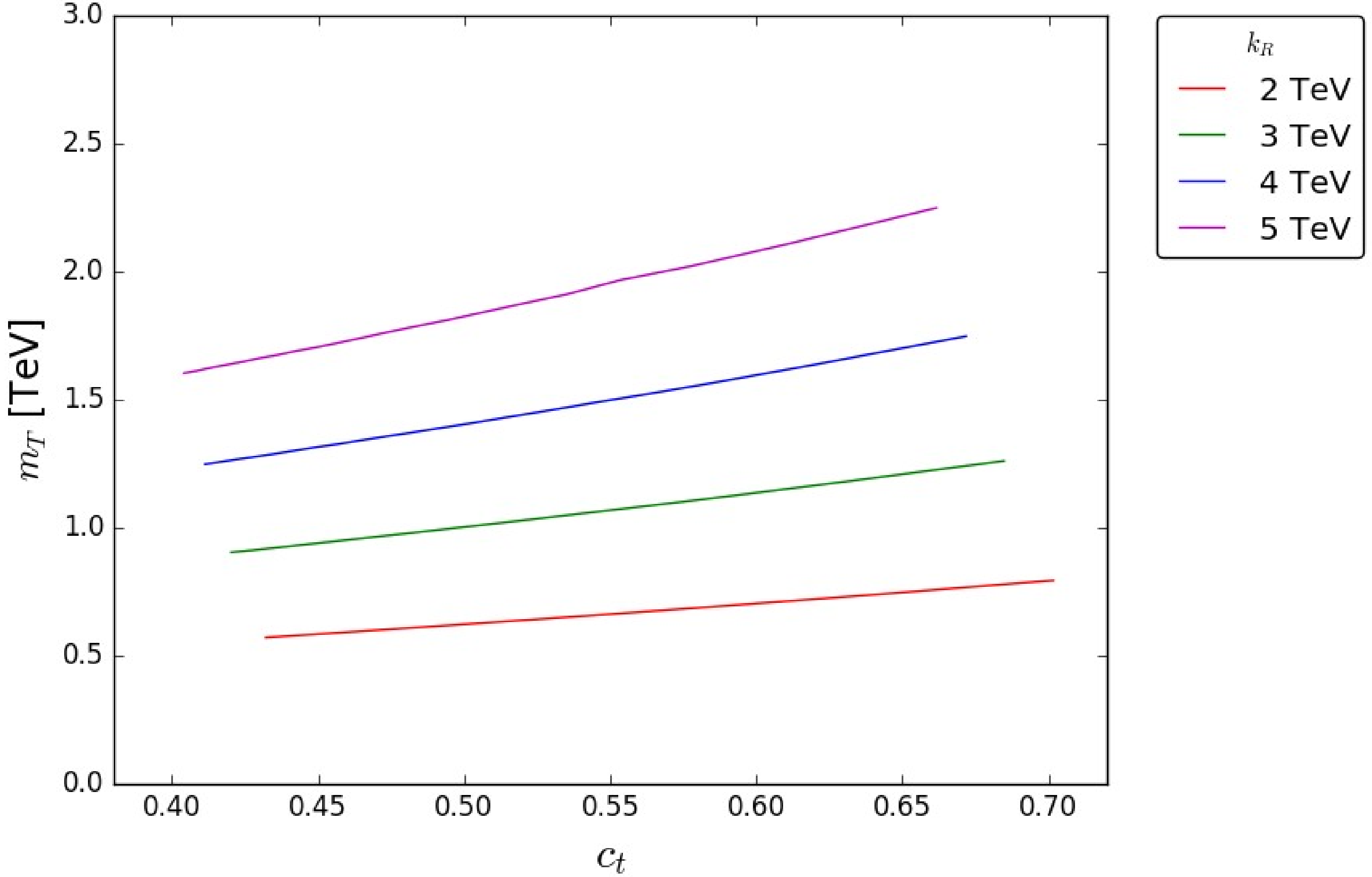
$$T = \frac{3m_t^2}{16\pi s_w^2 c_w^2 m_Z^2} \left[1 + \beta(c_t) m_t^2 z_R^2 \left(\log\left(\frac{\Lambda^2}{m_t^2} \right) - 1 \right) \right]$$

$\beta(c)$ is calculable. We will use $\Lambda \sim k_R$

Precision Electroweak Analysis



Top-Partner Mass



Universal Deviation of Higgs Couplings

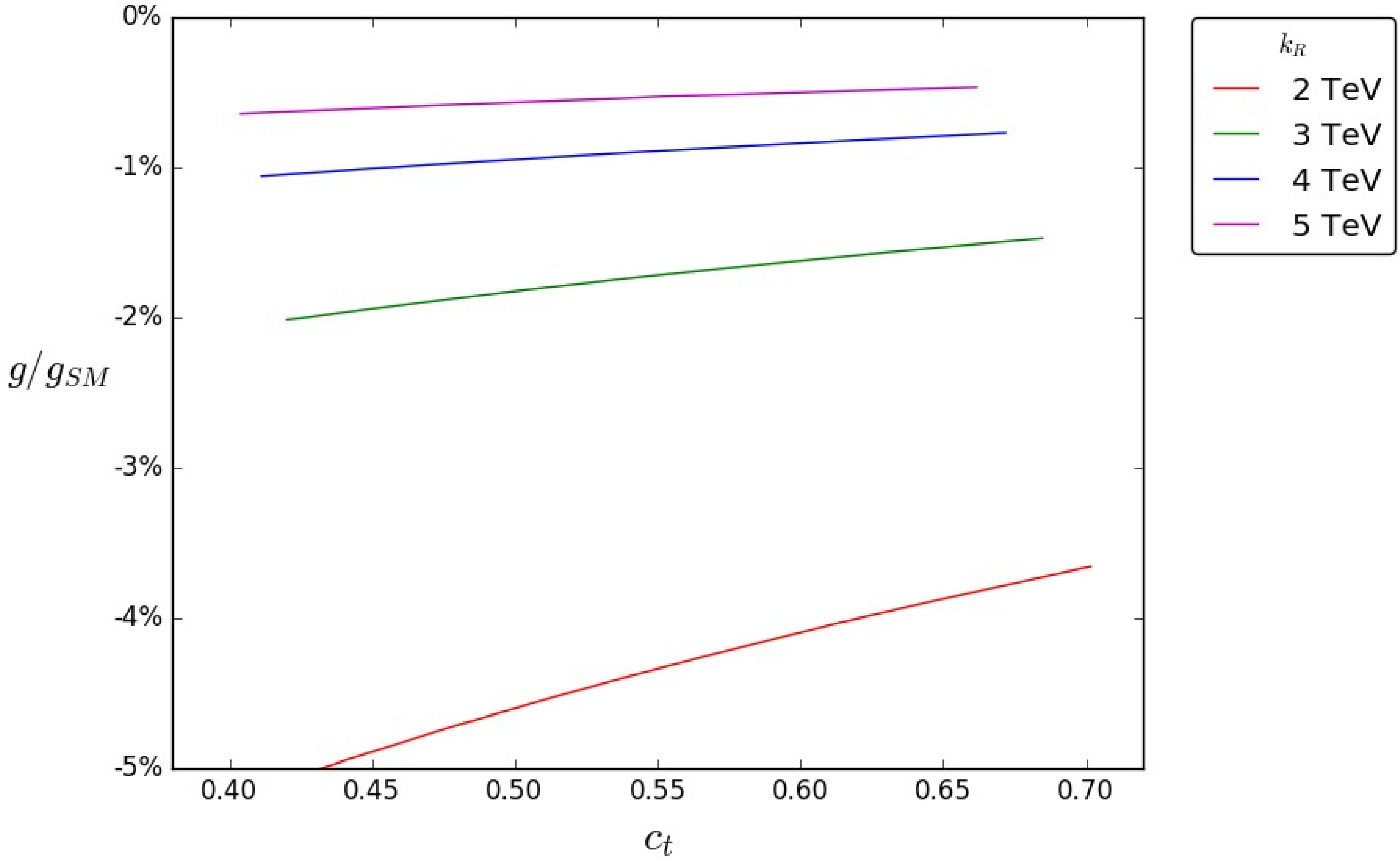
- The bulk gauge group G and fermion embeddings determine the Higgs coupling deviations of massive particles.
- For G=SO(5) and fermions embedded in 5 of SO(5), the masses of gauge bosons and fermions are all of the form

$$\frac{m^2}{f^2} \propto \frac{1}{2} \sin^2 \left(\frac{\sqrt{2}\langle h \rangle}{f} \right)$$

- Therefore, the Higgs coupling deviation in $\bar{t}th, ZZh, WWh$

$$\frac{g}{g_{SM}} = \cos \left(\frac{\sqrt{2}\langle h \rangle}{f} \right) = \sqrt{1 - 2\frac{v^2}{f^2}}$$

Universal Deviation of Higgs Couplings



Natural Mass Hierarchy between top & bottom

- The bottom quark has been massless so far.

$$\Psi_t = \begin{bmatrix} \begin{pmatrix} \chi_t(-+) & t_L(++) \\ \chi_b(-+) & b_L(++) \end{pmatrix} \\ t_R(--) \end{bmatrix}_{2/3}$$

Natural Mass Hierarchy between top & bottom

- The bottom quark has been massless so far.
- Consider an additional multiplet Ψ_b with X = -1/3.

$$\Psi_t = \begin{bmatrix} \begin{pmatrix} \chi_t(-+) & t_L(++) \\ \chi_b(-+) & b_L(++) \end{pmatrix} \\ t_R(--) \end{bmatrix}_{2/3} \qquad \Psi_b = \begin{bmatrix} \begin{pmatrix} t'(-+) & \chi_b(-+) \\ b'(-+) & \psi_b(-+) \end{pmatrix} \\ b_R(--) \end{bmatrix}_{-1/3}$$

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$\sin\beta$

- Mix it with Ψ_t on the UV boundary,
  so that the Higgs pairs up b_L and b_R.
- Consider $c_t < c_b$.
  Then for the left-handed mode, Ψ_t is more composite than Ψ_b.
  Composite operators will be enhanced in the IR and therefore,
  the effect of mixing $\sin\beta$ will be suppressed in the IR.

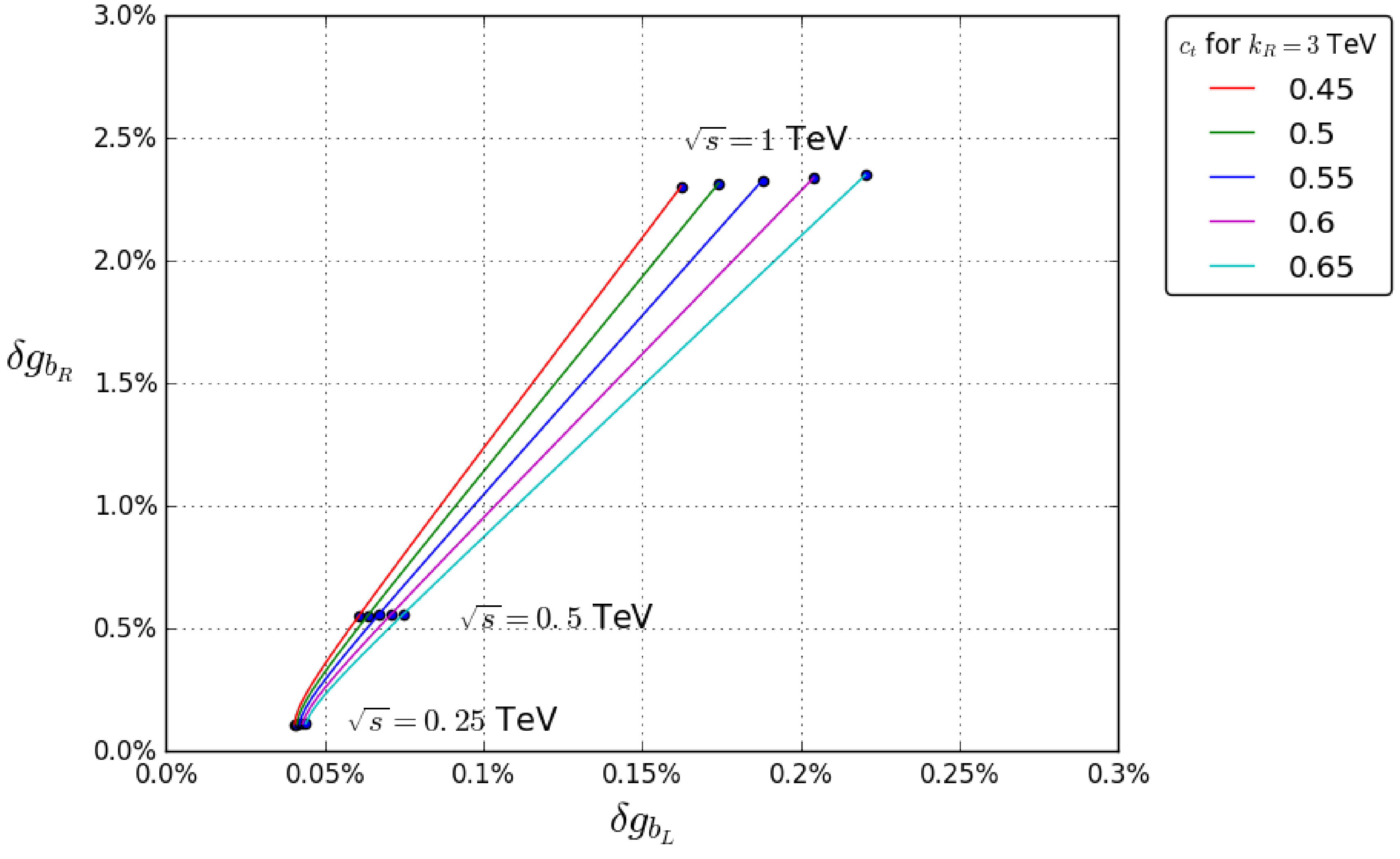
Natural Mass Hierarchy between top & bottom

- Indeed, explicit calculation shows that for $c_t < c_b$

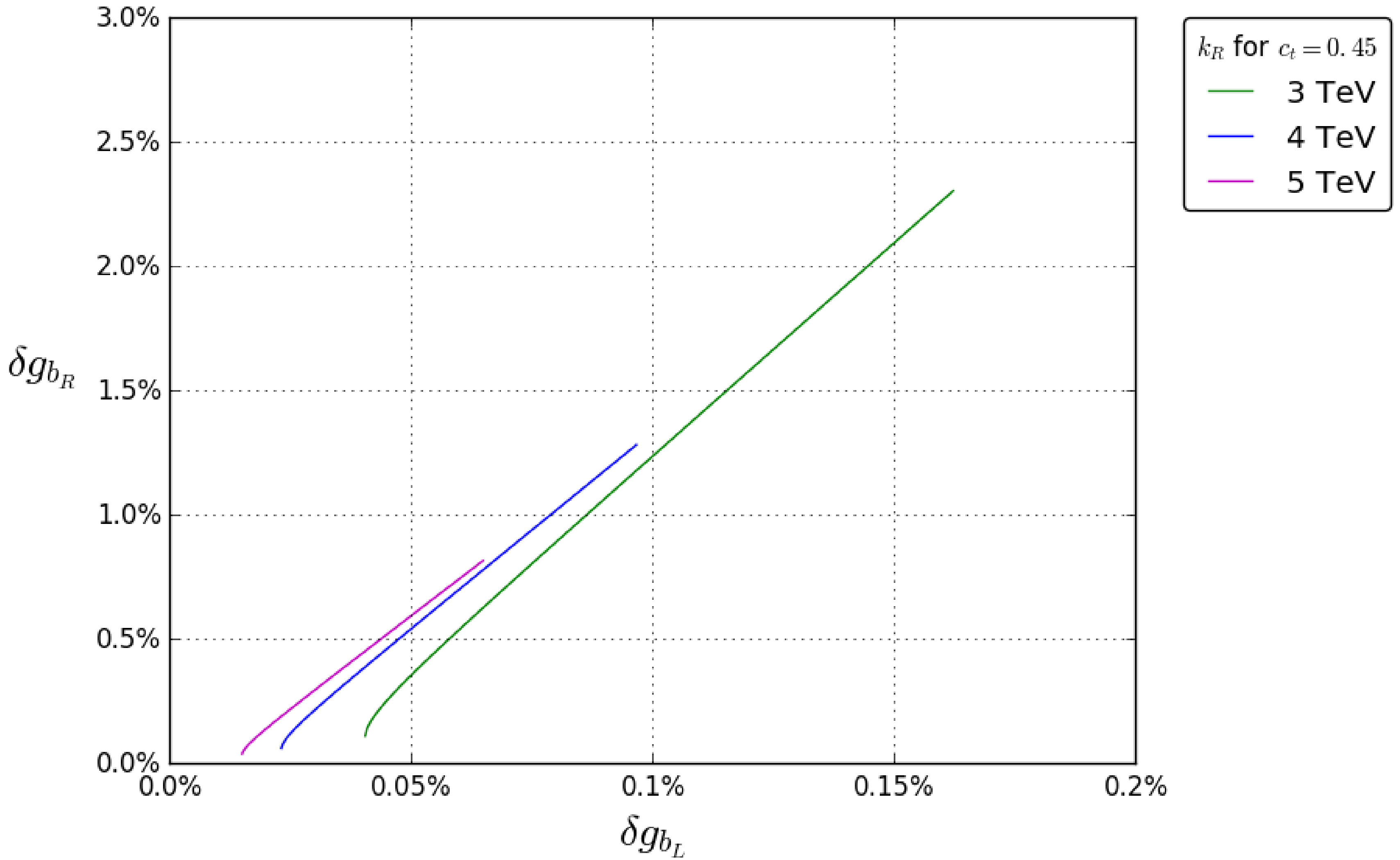
$$\frac{m_b^2}{m_t^2} \sim \tan^2\beta \times \left(\frac{z_0}{z_R}\right)^{2(c_b - c_t)}$$

- β of order-one can generate the large mass hierarchy.
- Note that b_R is a composite right-handed zero mode.
  → possibly, sizable coupling deviation which can be probed at future colliders like ILC.
- For a moment, let's choose $\sin\beta = 0.2$
  and calculate the deviations. cf. Funatsu, Hatanaka, Hosotani, Orikasa

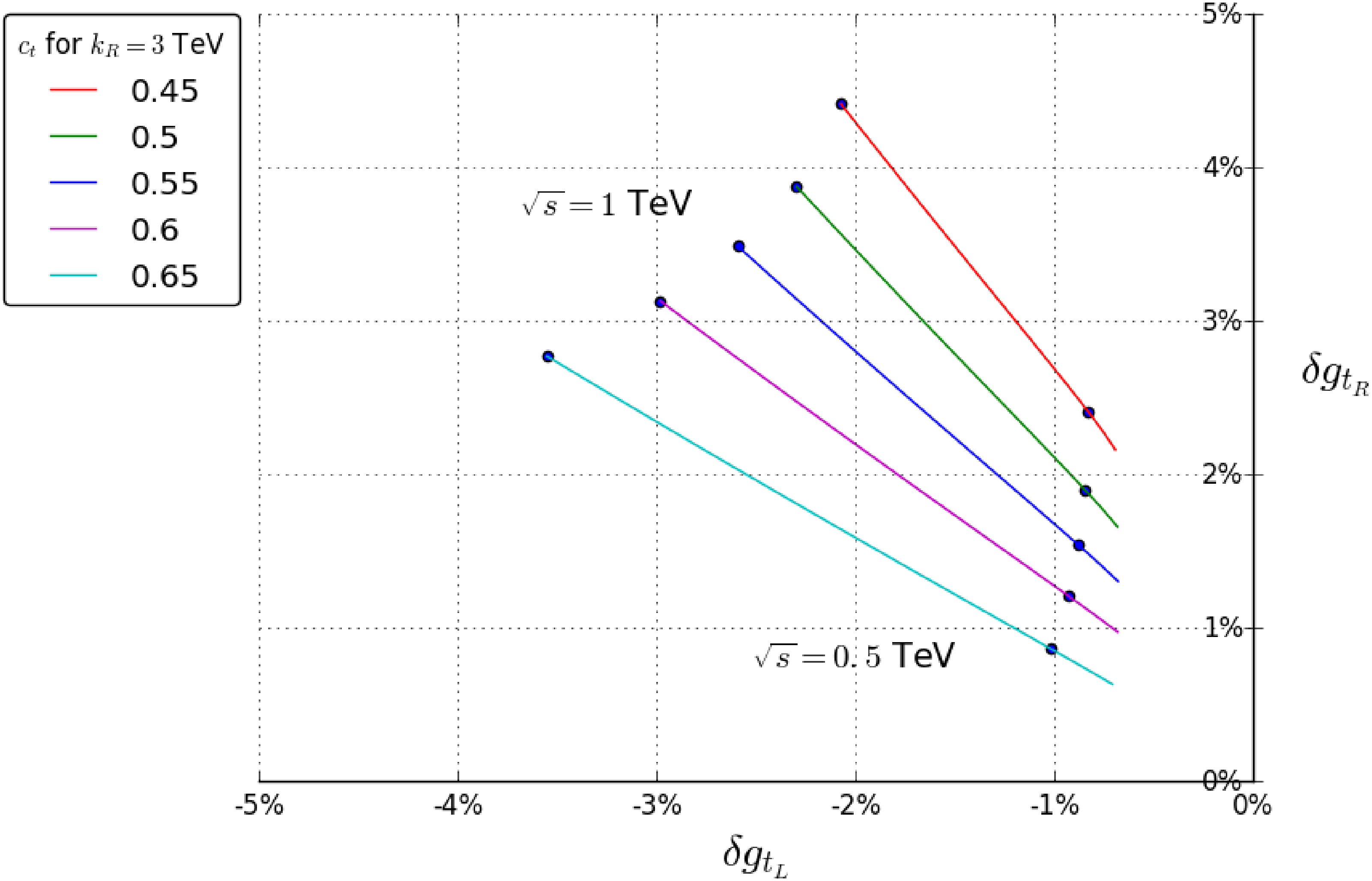
$$\mathbf{e_L^- e_R^+ \rightarrow b_L \bar{b}_R, b_R \bar{b}_L}$$



$$\mathbf{e_L^- e_R^+ \to b_L \bar{b}_R, b_R \bar{b}_L}$$



$$\mathbf{e_L^- e_R^+ \to t_L \bar{t}_R, t_R \bar{t}_L}$$



Full Flavor Structure?

- Similarly, we can explain the weak coupling of the other light flavors to the Higgs sector.
- Connect the light quarks to the composite (t_L, b_L) doublet in the UV, and utilize the enhancement of the composite operator in the IR.
- In this structure, the right-handed singlets are naturally aligned with the Higgs. We expect that this will suppress FCNC. More work is necessary.

Summary

- Gauge-Higgs unification based on $SO(5) \times U(1)_X$ in AdS5 as a dual formulation of a 4D composite Higgs model.
- Finite and calculable Higgs effective potential.
- Competing forces can generate the Little Hierarchy.
- We can make quantitative predictions in this model and they are mostly determined by the scale of new physics.
- The new physics scale is constrained most significantly by the top-partner mass.
- $m_b \ll m_t$ can be naturally realized.
- Coupling deviations can be measured in future colliders.

THANK YOU