Math 223, Section 4

Final Exam

12/13/2016

Name: ____

Problem	1	2	3	4	5	6	7	8	First half total
Score									
Possible	12	13	12	13	12	13	12	13	100
Problem	9	10	11	12	13	14	15	16	Total
Score									
Possible	12	13	12	13	12	13	12	13	200

NOTE: In order to receive full credit, I need to see all your work. Unjustified solutions, or solutions from a calculator, will receive partial or no credit.

1. (12 points) Let $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Compute

(a) $|\mathbf{a}| =$ ____________________________

(b) $\mathbf{a} \cdot \mathbf{b} =$ ____________________________

(c) $\text{proj}_{\mathbf{a}}\mathbf{b} =$ ____________________________

Now suppose $\mathbf{u} \cdot \mathbf{v} = 7$ and $\mathbf{u} \times \mathbf{v} =< 1, 2, 3 >$. Use the properties of the dot product and cross product to find

(d) $3\mathbf{v} \times \mathbf{u} =$ ____________________________

(e) $3\mathbf{v} \cdot \mathbf{u} =$ ____________________________

(f) $\mathbf{u} \times (19\mathbf{u} + \mathbf{v}) =$ ____________________________

2. (13 points) Let $f(x, y, z) = \dfrac{x - yz}{xy}$. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial^2 f}{\partial z \partial x}$. Simplify your results.

3. (12 points) Let $\mathbf{r}(t) =< \cos t, t + 1, \sin t >$. Compute the unit tangent vector $\hat{T}$ and the curvature κ and evaluate them at the point where $t = \pi$.

4. (13 points) Find an equation for the plane that goes through the points $(1, 1, 1)$, $(3, 1, 2)$ and $(4, -2, 2)$.

5. (12 points) Let $f(x, y) = 3(x-1)^2 + 2(y+3)^2 + 7$. Find an equation for the tangent plane of f at the point $(x, y) = (2, -2)$.

6. (13 points) Let $f(x, y, z) = x^2y + xyz + y^3z^2$.

(a) Find the gradient of f.

(b) Evaluate the gradient of f at the point $P = (0, 1, 2)$.

(c) Find the rate of change of f at the point $P = (0, 1, 2)$ in the direction given by the vector $< 3, 1, -4 >$.

7. (12 points) Find and classify the critical points of $f(x,y) = xy - 2x - 2y - x^2 - y^2$.

8. (13 points) Find the maximum and minimum values of $f(x,y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$.

9. (12 points) First switch the order of integration and then evaluate $I = \int_0^9 \int_{\sqrt{y}}^3 \cos(x^3)\, dx dy$.

10. (13 points) Set up (BUT DO NOT EVALUATE) the triple integral that represents the volume of the solid region V that lies under the plane $z = 1 + x + y$ and above the base region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 4$. DO NOT EVALUATE.

11. (12 points) Evaluate $I = \int_C (xy)\, ds$, where C is the straight line segment in the xy-plane that goes from $(1, 0)$ to $(3, -1)$.

12. (13 points) The vector field $\mathbf{F}(x, y) = (ye^x + 2x + y)\mathbf{i} + (e^x + x + \cos y)\mathbf{j}$ is conservative.

(a) Find a function $V(x, y)$ such that $\mathbf{F} = \nabla V$.

(b) Evaluate $I = \int_C \mathbf{F} \cdot d\mathbf{r}$, where C is parameterized by $x = 2t$, $y = t + 2$ for $0 \leq t \leq 1$.

13. (12 points) Use the Divergence Theorem to set up and evaluate the integral for the net flux of $\mathbf{F} = \left\langle \frac{x^3}{3}, \frac{y^3}{3}, \frac{z^3}{3} \right\rangle$ leaving the sphere of radius 2.

14. (13 points) Use CYLINDRICAL COORDINATES to set up and evaluate the triple integral for the volume of the solid that is bounded above by $z = 10 - x^2 - y^2$ and below by $z = 1$. Evaluate to get a number, but you don't need to simplify that number.

15. (12 points) Use Green's Theorem to set up and evaluate the vector line integral of $\mathbf{F} =< y + e^{\sqrt{x}}, 2x + \cos(y^2) >$ over the closed curve C which is the circle of radius 1 in the xy-plane.

16. (13 points) Set up (BUT DO NOT EVALUATE) the integral for the surface area of the portion of the surface $z = 2x^2 - 3y^2 + 12$ that lies above the domain $D = [1, 3] \times [4, 7]$. DO NOT EVALUATE.

Here's the punchline: He was okay. It turns out he was used to hardships. Enjoy your break!