Calculus I : Spring 2016 **Final Exam**

Instructor: _____ **Name:** _____

INSTRUCTIONS : Show all of your work, and give exact answers. No calculators

1. Short answer questions:

   (a) Evaluate $\lim_{t \to -\infty} \dfrac{|5+t|}{5+t} =$ _____ (2 pts)

   (b) Suppose that $\dfrac{\sin(x)}{x} \leq f(x) \leq 1$ for $0 < x < 1$.
   What can you say about $\lim_{x \to 0^+} f(x)$? _____ (3 pts)

   (c) Evaluate $\dfrac{d}{dx}\left(e^3\right) =$ _____ (2 pts)

   (d) Evaluate $\dfrac{d}{dx}\left(\sin^{-1}(3x)\right) =$ _____ (3 pts)

   (e) Given that $f(3) = 5$ and $f'(3) = 4$, use differentials to estimate $f(2.8)$ _____
   (3 pts)

   (f) Suppose that $f'(x) = -\dfrac{x+3}{(x+1)(x-3)}$ Identify the point $x = -3$ as a minimum, maximum, or neither. _____ (2 pts)

Page 1 Total (15)

(g) Sketch the graph of $f(x) = \dfrac{1-2x}{(x-3)^2}$ close to $x = 3$ (3 pts)

(h) If $s(t)$ is the position on the x-axis of a particle at time t, then the quantity $\dfrac{s(t+h)-s(t)}{h}$ represents _____ _____ _____ (2 pts)

(i) Evaluate $\displaystyle\int_6^6 (t^3+t^2)\,dt =$ _____ (3 pts)

(j) Evaluate $\displaystyle\int \left(\frac{1}{x}+\frac{1}{x^2}\right)\,dx =$ _____ (3 pts)

(k) We try the substitution $u = \ln(x)$ to evaluate the integral $\displaystyle\int \frac{(\ln(x))^2}{x}\,dx$. The differential is $du =$ _____ (2 pts)

(l) Given the integral $\displaystyle\int e^x \cos(e^x)\,dx$, a suitable substitution is _____ (2 pts)

Page 2 Total (15)

2. **Limits**

(a) (5 pts) Evaluate $\lim_{x\to 2^-} \dfrac{x^2-5x+6}{2x^2-x-6} =$

(b) (5 pts) Evaluate $\lim_{x\to\infty} \dfrac{3x^2-2x+5}{5x^2-\sqrt{x^4+3x^2}} =$

(c) (10 pts) Use l'Hopital's Rule to evaluate $\lim_{x\to 0} \dfrac{x^2}{1+2x-e^{2x}} =$

3. **Derivatives**

(a) (5 pts) From the Fundamental Theorem of Calculus, we have $\frac{d}{dx}\int_1^x \sin(t^2)\ dt = \sin(x^2)$.
Find $\frac{d}{dx}\int_1^{4x} \sin(t^2)dt =$

(b) (5 pts) Use *implicit differentiation* to find $\frac{dy}{dx}$, given $x^2y + xy^6 = 2$.

(c) (5 pts) If $f(s) = s\tan^{-1}(s)$, find $f'(s)$

(d) (5 pts) Find the derivative of $z = \ln(\sec(t) + \tan(t))$

(e) (5 pts) Find the *equation* of the tangent line to $y = x^3 + x^2 + 2x + 1$ at $x = 1$

(f) (5 pts) Use *logarithmic differentiation* to find $f'(x)$, where $f(x) = \dfrac{e^{x^2}\sqrt{1-3x}}{(x+1)^8}$

Page 5 Total (15)

4. **Applications**

(a) (10 pts) Sketch the graph of $y = f(x)$, given the following:

- $f(1) = 0$; $f(3) = 3$; $f(4) = 2$; $f(5) = 1$; $\lim_{x\to-\infty} f(x) = 0$; $\lim_{x\to\infty} f(x) = +\infty$; $\lim_{x\to0^-} f(x) = +\infty$; $\lim_{x\to0^+} f(x) = -\infty$
- $f'(3) = 0$; $f'(5) = 0$; $f'(x) > 0$ for $x \in (-\infty, 0) \cup (0, 3) \cup (5, +\infty)$; $f'(x) < 0$ for $x \in (3, 5)$
- $f''(4) = 0$; $f''(x) > 0$ for $x \in (-\infty, 0) \cup (4, +\infty)$; $f''(x) < 0$ for $x \in (0, 4)$

(b) (10 pts) Find the absolute maximum and minimum of $x(x^2 - 12)$ on the interval [0,3].

(c) (10 pts) A farmer wishes to fence a rectangular pasture of area 2800 m^2. One side of the fence is along a road, and that side costs \$5 per meter. The other 3 sides cost \$2 per meter.

Find the minimum possible cost.

(d) (10 pts) We are given a rectangle. The height is *increasing* at 0.1 cm/min, the base is *decreasing* at 0.2 cm/min

At what rate is the *diagonal* changing when the height is 3 cm, and the base is 4 cm?

5. **Integrals**

(a) (5 pts) Evaluate $\int \frac{x}{x^2+3}\, dx$

(b) (5 pts) Evaluate $\int_0^1 \frac{1}{x^2+3}\, dx$

(c) (10 pts) Evaluate $\int \frac{x}{\sqrt{x-2}}\, dx$

(d) (5 pts) Evaluate $\int \cos(x)\ \sin(\sin(x))\, dx$

(e) (5 pts) Evaluate $\int_1^2 (3t^2+1)\, dt$