Math 223, Section 3

Final Exam

12/15/2015

Name: ______________________________

Problem	1	2	3	4	5	6	7	8	Total
Score									
Possible	25	25	25	25	25	25	25	25	200

NOTE: In order to receive full credit, I need to see all your work. Unjustified solutions, or solutions from a calculator, will receive partial or no credit.

1. (25 points) Compute each of the following double integrals.

(a) (15 points)

$$\int_{x=0}^{x=4} \int_{y=\frac{x}{2}}^{2} e^{y^2}\, dy dx.$$

(Hint: you don't know how to integrate e^{y^2}. Nobody does. But there's an easy way to fix this!)

(b) (10 points) The integral of $f(x, y) = (x^2 + y^2)^5$ over the region bounded by $x^2 + y^2 = 4$ with $x \geq 0$.

2. (25 points)

(a) (13 points) Use the Divergence Theorem to compute the flux of $\mathbf{F}$ across S, where

$$\mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$$

and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

(b) (12 points) Use Stokes theorem to evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F}(x, y, z) = (x + y^2)\mathbf{i} + (y + z^2)\mathbf{j} + (z + x^2)\mathbf{k}$$

and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. (As is standard, we are using the counterclockwise orientation when viewed from above.)

3. (25 points) Consider the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.

(a) (10 points) Compute the curvature of $\mathbf{r}(t)$ at the point $(2, 4, 8)$.

(b) (10 points) Compute the unit tangent vector $\mathbf{T}(t)$.

(c) (5 points) Set up, but DO NOT EVALUATE, the integral to compute the length of $\mathbf{r}(t)$ from $(0, 0, 0)$ to $(2, 4, 8)$.

4. (25 points) Let $f(x, y) = x^3y^2 + x^2y^3 + xy$.

(a) (5 points) Compute the gradient of $f(x, y)$ at $(2, 1)$.

(b) (5 points) Compute the directional derivative of $f(x, y)$ at the point $(2, 1)$ in the direction of the point $(5, 5)$.

(c) (5 points) In what direction (your answer should be a unit vector) from $(2, 1)$ does $f(x, y)$ have the largest slope?

(d) (5 points) What is the maximum the directional derivative of $f(x, y)$ at $(2, 1)$ can be?

(e) (5 points) What is the directional derivative of $f(x, y)$ at $(2, 1)$ in a direction perpendicular to the answer from part (c)?

5. (25 points)

(a) (10 points) Find the point of intersection of the two lines

$$\langle 3-2t, -1+2t, 2t\rangle$$

and

$$\langle -s, 3+2s, 3+s\rangle.$$

(b) (10 points) Find the equation of the plane formed by the lines from part (a). (You do NOT need to have the correct answer to part (a) to do this.)

(c) (5 points) Find the angle between the plane from part (a) and the plane $2x - y + 7z = 9$.

6. (25 points) Set up, but DO NOT EVALUATE, the integral to compute each of the following (6 points each and one free point!):

(a) The integral to compute the work of $\mathbf{F}(x, y) = (x^2y)\mathbf{i} + (x^3y^2)\mathbf{j}$ along the path traced by the parabola $y = 3x^2$ from $(0, 0)$ to $(2, 12)$. Your answer should be an integral in terms of t.

(b) The integral of $f(x, y, z) = \sin(x^2+y^2+z^2)$ over the region BELOW the cone $z^2 = x^2+y^2$, above the xy-plane, and within the sphere $x^2+y^2+z^2 = 4$. Your answer should be an integral in spherical coordinates.

(c) The flux of the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ across the surface formed by the part of the plane $x + y + z = 5$ lying within the cylinder $x^2 + y^2 = 1$. Your answer should be in terms of x and y.

(d) The integral(s) to detrmine the x coordinate of the center of mass of the region in the xy-plane in the first quadrant bounded by $3x + 4y = 5$, with density function $\rho(x, y) = xy^2$.

7. (25 points) Consider the vector field

$$\mathbf{F}(x, y, z) = (2xyz + yz^3)\mathbf{i} + (x^2 z + xz^3 + 2y)\mathbf{j} + (x^2 y + 3xyz^2)\mathbf{k}.$$

(a) (10 points) Compute $curl(\mathbf{F})$. What does your answer tell you?

(b) (10 points) Compute the work done by $\mathbf{F}$ along the path

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), t^2 \rangle$$

from $t = 0$ to $t = \pi/2$.

(c) (5 points) Compute the work done by $\mathbf{F}$ along the path from $(0, 0, 0)$, along the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ to $(1, 1, 1)$ and then along the curve $\langle t, t^{1/2}, t^{1/3} \rangle$ from $(1, 1, 1)$ back to $(0, 0, 0)$. (Hint: this is easy! If you take more than twenty seconds to do this problem you're misunderstanding what's going on.)

8. (25 points) Use optimization (correct answers obtained using another method will not receive full credit) to find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

Enjoy your break!