Intro to Discrete Math Graded Problems

Important: Remember to read the instructions and advice in homework.html!

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§ 2.1 (10 points) Use Theorem 2.1.1 to fully simplify the logical expression below. Write a clear chain of equivalences, and justify each step by citing the law(s) used by name.

$$\sim[\sim[(p \vee q) \wedge r] \vee \sim q]$$

(Hint: The distributive law *can* be applied here, but it does not help! On the contrary, it will just make things harder for both of us. Look for another approach.)

§ 2.3 (10 points) Write the following argument in symbolic form, then use a truth table to determine whether it is valid; explain your response. If it is invalid, provide a counterexample in plain English.

> If it is chilly this Saturday, then Lisa will wear her wool dress if the hem has been repaired. The forecast for the weekend calls for cool weather, but the hem has not been repaired. Therefore Lisa won't be wearing her wool dress this Saturday.

(Hint: The hardest part of this problem is probably the translation to a symbolic representation, so I strongly recommend checking with me to confirm that you've done that correctly before proceeding!)

§ 2.4 (5 points) Design a circuit for the following I/O table, using only the three basic gate types NOT, AND, and/or OR.

The direct/standard approach, if done correctly, yields a circuit with five gates. For a bonus point, find a correct circuit that uses only *four* gates.

P	Q	R
0	0	1
0	1	0
1	0	0
1	1	1

§ 3.3 (10 points) Consider the predicate $P(x, y) =$ "$y - x = y + x^2$", where the domain for each of the variables x and y is $\mathbb{Z}$, the set of integers. For each of the following statements, determine its truth value (i.e., state whether it is true or false), then state its negation.

1. $\forall y, P(0, y)$
2. $\exists y$ s.t. $P(1, y)$
3. $\forall x, \exists y$ s.t. $P(x, y)$
4. $\exists y$ s.t. $\forall x, P(x, y)$
5. $\forall y, \exists x$ s.t. $P(x, y)$

§ 4.3 (10 points) Using the definition of rational, prove that for any rational numbers r and s, $2r + 3s$ is rational.

(Note: This is the same claim that you'll find in an exercise, but the solution is not the same, since here, you are not permitted to use other theorems or exercises. Also, unlike most other graded problems, this one is comparable to what you may find on a test! I have it here to give you practice and feedback.)

§ 4.5 (10 points) Prove that for all integers a, $a^3 - a$ is divisible by 3. (hint in footnote below[1])

[1] I've thought of two very different proofs for the given statement, both of which are perfectly acceptable. One of them is a relatively direct application of the material of this section. The other approach appears shorter and more elegant, but it hinges on a fact about divisibility that is not proved in the textbook. (Indeed, proving that fact is left as an exercise in this section.) If you wish to take this route, you must include a proof of that key fact.

In other words, make sure that everything you state in a proof is firmly established. If it's already proved or assumed by the textbook, you're good; if it is not, you must support it with a proof of your own.