Calculus I :	Spring 2015	Final Exam
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Instructor: \_\_\_\_\_ Name: \_\_\_\_\_

#### INSTRUCTIONS: Show all of your work, and give exact answers. No calculators

1. Short answer questions:

(a) Evaluate 
$$\lim_{t\to 4^-} \frac{|4-t|}{4-t} =$$
 \_\_\_\_\_\_ (2 pts)

- (b) Suppose that  $\sqrt{x} + 2 \le f(x) \le 2x 13$  for x > 0. What can you say about  $\lim_{x \to 9} f(x)$ ? \_\_\_\_\_\_ (3 pts)
- (c) Evaluate  $\frac{d}{dx} \ln 2 =$  \_\_\_\_\_ (2 pts)
- (d) Evaluate  $\frac{d}{dx} \tan^{-1}(2x) = \underline{\qquad}$  (3 pts)
- (f) Suppose that  $f'(x) = -\frac{3x(x+2)}{(x-2)^2}$  Is the point x = -2 a minimum, maximum, or neither? \_\_\_\_\_ (2 pts)
- (g) If f(2) = 1, f(3) = 2, f'(x) > 0 and f''(x) < 0, give a sketch of the graph of f(x) for  $2 \le x \le 3$  (3 pts)

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- (h) Write  $\lim_{n\to\infty}\sum_{i=1}^n\left(1+\left(\frac{2i}{n}\right)\right)\cdot\frac{2}{n}$  as an integral \_\_\_\_\_ (4 pts)
- (i) Evaluate  $\int (\cos(x) \sin(x)) dx = \underline{\qquad} (2 \text{ pts})$
- (j) Evaluate  $\int (e^x + x^e) dx =$  \_\_\_\_\_\_ (2 pts)
- (k) We try the substitution  $u = \frac{x^2}{2}$  to evaluate the integral  $\int x \sin\left(\frac{x^2}{2}\right) dx$ . The differential is  $du = \underline{\qquad}$  (2 pts)
- (l) Given the integral  $\int \frac{\cos(\ln x)}{x} dx$ , a suitable substitution is \_\_\_\_\_\_ (2 pts)

12 points

#### 2. Limits

(a) (5 pts) Evaluate 
$$\lim_{x\to 1^-} \frac{x^2 - 3x + 2}{x^2 - 2x + 1} =$$

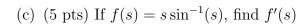
(b) (5 pts) Evaluate 
$$\lim_{x\to\infty} (x - \sqrt{x^2 + 3x}) =$$

(c) (10 pts) Use l'Hopital's Rule to evaluate  $\lim_{x\to 0} \frac{x^2}{1-\cos(x)} =$ 

### 3. Derivatives

(a) (5 pts) From the Fundamental Theorem of Calculus, we have  $\frac{d}{dx} \int_1^x e^{t^2} dt = e^{x^2}$ . Find  $\frac{d}{dx} \int_1^{2x} e^{t^2} dt =$ 

(b) (5 pts) Use implicit differentiation to find  $\frac{dy}{dx}$ , given  $xy - 2x^2y^3 = 1$ .



(d) (5 pts) Find the derivative of  $z = \ln(t^3 + 2)$ 

(e) (10 pts) Find the equation of the tangent line to  $y = \frac{2x}{x^2 - 1}$  at the point where x = 2

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## 4. Applications

- (a) (10 pts) Sketch the graph of y = f(x), given the following:
  - f(-1) = 2; f(0) = 3;  $\lim_{x \to -\infty} f(x) = 1$ ;  $\lim_{x \to \infty} f(x) = 0$ ;  $\lim_{x \to 1^{-}} f(x) = -\infty$ ;  $\lim_{x \to 1^{+}} f(x) = +\infty$
  - f'(0) = 0; f'(x) > 0 for  $x \in (-\infty, 0)$ ; f'(x) < 0 for  $x \in (0, 1) \cup (1, \infty)$
  - f''(-1) = 0; f''(x) < 0 for  $x \in (-1, 1)$ ; f''(x) > 0 for  $x \in (-\infty, -1) \cup (1, \infty)$

(b) (10 pts) The position of a particle on the x-axis after t minutes is  $s(t) = 5 + 3t^2 - t^3$  meters. When is the particle moving to the *left*?

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(c)	(10 pts) The volume of a rectangular box with $no\ lid$ and a $square\ lid$	base is $32 cm^3$ . Find
	the minimum possible $surface$ $area$ (material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the minimum possible $surface$ $area$ (material needed to make the bound of the material needed to make the material needed to material needed to make the material nee	
(d)	(10 pts) We are given a rectangle. The height is $increasing$ at 0.2 $decreasing$ at 0.1 cm/min	
	How is the <i>area</i> changing when the height is 2 cm, and the base is 3	3 cm?
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# 5. Integrals

(a) (5 pts) Evaluate 
$$\int \frac{x}{x^2+4} dx =$$

(b) (5 pts) Evaluate 
$$\int_0^2 \frac{1}{x^2 + 4} dx =$$

(c) (10 pts) Evaluate 
$$\int x \sqrt{x-1} dx =$$

(d) (10 pts) Evaluate 
$$\int_{\ln(\pi/6)}^{\ln(\pi/3)} e^x \sec(e^x) \tan(e^x) dx =$$