Intro to Discrete Math Graded Problems

Important: Remember to read the instructions and advice in homework.html!

---

§ 2.1 (10 points) Use Theorem 2.1.1 to fully simplify the logical expression below. Write a clear chain of equivalences, and justify each step by citing the law(s) used by name.

$$\sim[\sim[(p \vee q) \wedge r] \vee \sim q]$$

(Hint: The distribute law *can* be applied here, but it does not help! On the contrary, it will just make things harder for both of us. Look for another approach.)

§ 2.3 (10 points) Write the following argument in symbolic form, then use a truth table to determine whether it is valid; explain your response. If it is invalid, provide a counterexample in plain English.

> If it is cool this Friday, then Craig will wear his leather jacket if the pockets are mended. The forecast for Friday calls for cool weather, but the pockets have not been mended. Therefore Craig won't be wearing his leather jacket this Friday.

(Hint: The hardest part of this problem is probably the translation to a symbolic representation, so I strongly recommend checking with me to confirm that you've done that correctly before proceeding!)

§ 2.4 (10 points) Design a circuit for the following I/O table, using only the three basic gate types NOT, AND, and/or OR.

The direct/standard approach, if done correctly, yields a circuit with five gates; this is worth 9 points. For full credit, find a circuit that uses no more than four gates.

P	Q	R
0	0	1
0	1	0
1	0	0
1	1	1

§ 3.3 (10 points) Consider the predicate

$$P(x, y) = \text{“}y - x = y + x^2\text{”}$$

where the universe (domain) for each of the variables x and y is $\mathbb{Z}$, the set of integers. For each of the following statements, determine its truth value (i.e., state whether it is true or false), then state its negation.

1. $\forall y, P(0, y)$
2. $\exists y$ s.t. $P(1, y)$
3. $\forall x, \exists y$ s.t. $P(x, y)$
4. $\exists y$ s.t. $\forall x, P(x, y)$
5. $\forall y, \exists x$ s.t. $P(x, y)$