Calculus I Graded Problems

Important: Remember to read the instructions and advice in homework.html!

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§ 2.1 (10 points) Consider the graph of the equation $y = \sin x$, which includes the point $(\pi, 0)$. Suppose we wish to find an equation of the line tangent to the graph at that point.

Choose at least four nearby points on the curve, and compute the approximate slopes of the corresponding secant lines that have $(\pi, 0)$ at one end. From these results, estimate the slope of the tangent line, then use that to write an equation for the tangent line.

(Choose your points carefully. Four is a minimum; you may certainly evaluate more slopes if it will help you. Be sure to clearly state each point you choose and the slope of its corresponding secant line. You **do not** need to show the details of your computation; you should use a calculator or computer for that.)

Hint 1: I very strongly recommend that you check your results by graphing both the given function and your linear equation on the same graph with a calculator or computer. If the result is not a line tangent to the graph at the given point, something has gone wrong!

Hint 2: The correct equation will be relatively simple. If your coefficients are messy decimals, you aren't quite correct! Keep working on it.

§ 2.5 (10 points) Find the values of a and b that make f continuous everywhere. (Be sure to use correct limit notation in your work.)

$$f(x) = \begin{cases} \dfrac{x^2 + x}{x + 1} & \text{if } x < -1 \\ ax^2 + b & \text{if } -1 \le x \le 2 \\ \dfrac{x^2 + x - 6}{x^2 - 3x + 2} & \text{if } x > 2 \end{cases}$$

§ 2.7 (10 points) Use the definition of derivative to find $f'(a)$ if $f(x) = \dfrac{2 - x}{3x + 1}$.

§ 3.1 (10 points) Find the values of m and b that make f differentiable everywhere. (Be sure to use correct limit notation in your work.)

$$f(x) = \begin{cases} x^3 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

§ 3.3 (10 points) Find the values of x for which the curve $y = \dfrac{\sin x}{2 + \cos x}$ has a horizontal tangent line.

§ 3.5 (10 points) Find an equation for the tangent line to the graph of $x^2y^2 = (y+1)^2(2-y)$ at the point $(0, 2)$.

§ 3.9 (10 points) A man is lying on the ground 100 feet from a hot air balloon launch. The balloon is rising at 10 feet per second. Determine the rate of change of the angle of view of the man from the horizontal to the balloon 5 seconds after launch. (Remember to use correct units in your final answer.)

§ 3.10 (10 points) Use a linear approximation (either a linearization or differentials) to estimate the value of $\cos 59°$. Write your answer approximated to at least five places past the decimal point, and compare it to the actual value as evaluated directly with a calculator or computer. (Hint: Remember to convert to radians when necessary!)

§ 4.5 (10 points) Using the techniques of this section, produce a sketch of the graph of the function

$$f(x) = 2 + \frac{1}{x} + \frac{1}{x^4}$$

that shows all important features. Remember to investigate and comment on each of the main items of consideration. (These are the bullet points from class, or steps A–G from the textbook.) Use exact expressions in your analysis, though you might also want to compute decimal approximations as needed to assist your graphing.

§ 4.8 (5 points) Use Newton's Method to estimate the solution of $\cos x = x^3 - 1$ to at least six places past the decimal. Choose an appropriate integer as your initial guess x_0 (I suggest using a graph to guide you), and record each iterant x_i you obtain/use.

§ 5.2 (10 points) For the function $f(x)$ defined below, evaluate $\int_{-4}^{2} f(x)\,dx$ by sketching a graph and interpreting the integral in terms of areas. (Write your answer in exact form.)

$$f(x) = \begin{cases} 3 + \frac{1}{2}x & \text{if } x < -2 \\ 2 & \text{if } -2 \le x < 0 \\ \sqrt{4 - x^2} & \text{if } 0 \le x \le 2 \end{cases}$$