Calculus III Graded Problems

Important: Remember to read the instructions and advice in homework.html!

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§ 12.2 (10 points) Alice and Bob are carrying a 30 kg mass via a pair of ropes tied to a handle atop the object. Alice is standing a bit to the side, holding her rope at a 40° angle away from vertical; Bob is standing directly across from Alice, but slightly closer to the object, with his rope only 35° away from vertical. Find the approximate magnitude of the tension in each rope, rounded to the nearest whole newton. (Use the vector techniques of this section, **not** the Laws of Sines or Cosines.)

Notes: The weight of an object is the force it experiences from gravity; its magnitude can be calculated as $W = mg$, where m is the mass of the object, and $g \approx 9.8\,\mathrm{m/s^2}$ is gravitational acceleration. The SI unit of force is the newton: $1\,\mathrm{N} = 1\,\mathrm{kg \cdot m/s^2}$. Since you are asked for an approximate value, you should of course use a calculator to obtain decimal approximations and perform any necessary arithmetic.

§ 12.3 (10 points) Given the triangle with vertices $A(-1, 3, 0)$, $B(1, 2, 2)$, and $C(3, 4, 6)$, find the approximate interior angle at each vertex, rounded to the nearest degree. (Use the vector techniques of this section, **not** a direct application of the Law of Cosines.)

§ 12.5 (12 points) Complete the following (4 points each).

1. Find parametric equations for the line passing through the points $(1, 2, 1)$ and $(-2, 11, 7)$.

2. Find the point at which the above line intersects the plane $x + 2y + 5z = 40$.

3. At approximately what angle do the above line and plane intersect? (Round to the nearest degree.)

§ 12.6 (10 points) A surface consists of the set of points $P(x, y, z)$ which are equidistant from the point $(2, 0, 0)$ and the yz-plane. Find a simplified equation to describe this surface. Identify the type of surface and the location of its center or vertex, as appropriate.

§ 13.4 (10 points) A projectile is launched from starting position $\mathbf{r}_0 = \langle 3, 5, 8 \rangle$ m with initial velocity $\mathbf{v}_0 = \langle 2, -1, 4 \rangle$ m/s. Approximately how long does it take for the projectile to land, and what are the approximate coördinates of its landing position? (Assume that the z-axis points upward, so that gravity induces an acceleration in the negative z direction with magnitude $g = 9.8\,\mathrm{m/s^2}$. Ground level is at $z = 0$.)

Note: Be sure to use the techniques of vector calculus: start with an acceleration function $\mathbf{a}(t)$, integrate to obtain a position function $\mathbf{r}(t)$, etc. This is not only to test your understanding of these specific techniques, but also because in the past, when students tried to apply other formulae to this problem, they usually did so incorrectly! I want to save all of us the headache of dealing with that, so I'm making this method an explicit requirement.

§ 14.1 (10 points) Find the domain and range of the function

$$f(x, y) = \sqrt{x^2 + y^2 - 4}.$$

(Remember that domains and ranges are *sets*, and should be written as such.)

Sketch a graph of the domain and two level curves in the xy-plane. Label the level curves with their corresponding function values. (Choose curves in the interior of the domain—its boundary does not count.)

§ 14.3 (7 points) Find $\dfrac{\partial^2 f}{\partial x \partial y}$ for the function

$$f(x, y) = \int_{y-x}^{xy} \cosh t^2 \, dt.$$

(I recommend trying this on your own first, but if you encounter difficulties, see the hint I tucked into the footnote[1] below so you can avoid "spoilers" until you're ready for them.)

§ 14.4 (7 points) The interior of a cylindrical steel cargo drum is measured to be 57 cm in diameter and 85 cm in height. The estimated maximum errors in these measurements are ± 0.3 cm for the diameter and ± 0.1 cm for the height. Use differentials to estimate the *approximate* maximum error in the calculated volume of the drum. State both the absolute and the percent (relative) errors.

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[1] Your usual approach to evaluating integrals probably encounters some difficulty here, since $\cosh t^2$ has no elementary antiderivative. Recall that the Fundamental Theorem of Calculus comes in *two* parts! The one you use most *can* be applied here, but it takes some care, and you'll have to be a little bit indirect with it. Alternatively, you can try applying the other part. If you need a refresher on that, take a look at the content of § 5.3. Basic exercises include #9–20; once you get the idea there, take a look at #67–71. (All references are for 9th Edition Early Transendentals; other editions will differ slightly.)

§ 14.7 (15 points) Find the absolute maximum and minimum values of the function

$$f(x, y) = 12x + 6y - x^3 - y^2$$

on the rectangular domain $R = [-3, 3] \times [0, 8]$.

Be sure to show your work—at least, the important parts. Once you find all the points of interest, I don't need to see the full details of how you evaluate f at each of them. Feel free to use technology to accelerate those arithmetic computations however you choose; for that portion of the problem, it's sufficient to write a series of statements of the form "$f(a, b) = c$".

If you're uncertain how to interpret the notation used above, you may be rusty on Cartesian products; see the footnote[2] for a brief refresher.

§ 15.1 (10 points) Evaluate the integral

$$\iint_R 20x^7y^4 \cos(\pi x^4y^5)\, dA$$

where $R = [0, 1/2] \times [0, 1]$. (Hint: Choose your order of integration carefully!)

[2] Recall (from § 12.1 lecture) that the Cartesian product of two sets is the set of all ordered pairs in which the first entry is an element of the first set and the second entry is an element of the second set. In symbols,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

We have been using two special Cartesian products a great deal in this course already:

$$\begin{aligned} \mathbb{R}^2 &= \mathbb{R} \times \mathbb{R} &&= \{(x, y) \mid x, y \in \mathbb{R}\} \\ \mathbb{R}^3 &= \mathbb{R} \times \mathbb{R} \times \mathbb{R} &&= \{(x, y, z) \mid x, y, z \in \mathbb{R}\} \end{aligned}$$

If you'd like to see some examples and graphs, this notation is also used in § 15.1, where Figures 2 and 6 illustrate rectangles R defined in a similar manner in the nearby text.