Precalculus Final Exam Review: Part 2

1. Solve the inequality by using a number line and the behavior of the graph at each zero. Write the solution in interval notation. $\dfrac{1}{x+5} < \dfrac{x-2}{x-7}$

2. Algebraically find $f^{-1}(x)$, where $f(x) = \dfrac{x}{x+1}$.

3. Solve for x: $\left(\dfrac{1}{2}\right)^{3x} = 8^{x-2}$

4. Determine the domain of the function: $y = \ln\left(\dfrac{x-2}{x+3}\right)$

5. Use the properties of logarithms to write the given expression as a single term:
   $\ln\left(x^2 - 4\right) - \ln(x+2)$

6. Solve for x. Clearly identify any extraneous solutions. If there are no solutions, so state.
   $\ln(x+7) + \ln 9 = 2$

7. Solve for x: $7^x = 4^{2x-1}$

8. Use the formula for arc length to find the value of the unknown quantity.
   $\theta = 320°$; $s = 52.5$ km

9. Use the formula for area of a circular sector to find the value of the unknown quantity.
   $A = 437.5$ cm^2; $r = 12.5$ cm

10. At carnivals and fairs, the *Gravity Drum* is a popular ride. People stand along the wall of a circular drum with radius 12 ft, which begins spinning very fast, pinning them against the wall. The drum is then turned on its side by an armature, with the riders screaming and squealing with delight. As the drum is raised to a near-vertical position, it is spinning at a rate of 35 rpm. (a) What is the angular velocity in radians? (b) What is the linear velocity (in miles per hour) of a person on this ride?

11. Without the use of a calculator, state the exact value of the trigonometric functions for the given angle.

    (a) $\cos\left(\dfrac{\pi}{6}\right)$

    (b) $\cos\left(\dfrac{5\pi}{6}\right)$

    (c) $\cos\left(\dfrac{7\pi}{6}\right)$

    (d) $\cos\left(\dfrac{11\pi}{6}\right)$

    (e) $\cos\left(\dfrac{13\pi}{6}\right)$

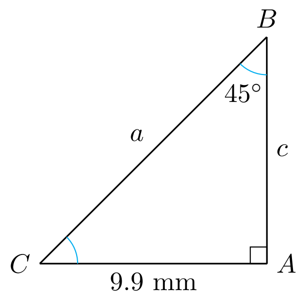
    (f) $\cos\left(-\dfrac{\pi}{6}\right)$

    (g) $\cos\left(-\dfrac{5\pi}{6}\right)$

    (h) $\cos\left(-\dfrac{23\pi}{6}\right)$

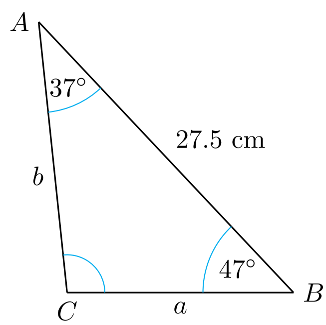
12. Given $\left(\frac{\sqrt{2}}{3}, -\frac{\sqrt{7}}{3}\right)$ is a point on the unit circle corresponding to t, find the value of all six circular functions of t.

13. Solve the triangle using trigonometric functions of the acute angle θ. Give a complete answer (in table form) using exact values.



14. Given $\theta = -150°$, find the exact value of $\sin\theta$, $\cos\theta$, and $\tan\theta$ using reference angles.

15. Solve the triangle. Give your answer in table form.



16. Given the following information about a triangle, completely solve the triangle. Give your answer in table form: side $a = 75$ cm, angle $C = 38°$, side $b = 32$ cm

17. Sketch a graph of $y = \sec t$, for $t \in [-2\pi, 2\pi]$.

18. Sketch a graph of $y = \tan t$, for $t \in [-2\pi, 2\pi]$.

19. Sketch a graph of $y = \tan(2t)$, for $t \in [-\pi, \pi]$.

20. Sketch a graph of $y = \tan\left(\frac{t}{2}\right)$, for $t \in [-\pi, \pi]$.

21. Sketch the graph of $y = 3\sin(3t)$ for $t \in [0, 3\pi]$.

22. Identify the amplitude, period, vertical shift, horizontal shift, and primary interval of $y = 4\sin\left(\frac{\pi t}{8} + \frac{\pi}{4}\right) - 2$.

23. Sketch $y = 2\cos(\frac{\pi t}{4} + \frac{3\pi}{4})$, for $t \in [0, 8]$.

24. In Oslo, Norway, the number of hours of daylight reaches a low of 6 hr in January and a high of nearly 18.8 hr in July. (a) Find a sinusoidal equation model for the number of daylight hours each month. Assume $t = 0$ corresponds to January 1. (b) Sketch the graph of the model.

25. Verify the identity: $\sin^2 x \cot^2 x + \sin^2 x = 1$

26. Verify the identity: $\dfrac{\sin x}{1+\sin x} - \dfrac{\sin x}{1-\sin x} = -2\tan^2 x$

27. Find the exact value of the expression using a sum or difference identity. Some simplifications may involve using symmetry and the formulas for negatives.

$$\cos\left(-\frac{5\pi}{12}\right)$$

28. Find the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ using the information given.

$$\sin(2\theta) = \frac{24}{25};\ 2\theta \text{ in QII}$$

(Assume that 2θ is an angle in the interval $[0, 2\pi)$.)

29. Rewrite in terms of an expression containing only cosines to the power 1: $\sin^4 x \cos^2 x$

30. Find the exact values of $\sin\left(\dfrac{\theta}{2}\right)$, $\cos\left(\dfrac{\theta}{2}\right)$, and $\tan\left(\dfrac{\theta}{2}\right)$ using the information given:

$$\sin\theta = \frac{12}{13};\ \theta \text{ is obtuse}$$

31. Evaluate without a calculator and answer in radians: $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

32. Evaluate without a calculator and answer in radians: $\sin^{-1}\left(\cos\left(\dfrac{2\pi}{3}\right)\right)$

33. Evaluate the expression. Draw a right triangle and label the sides to assist.

$$\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{12+x^2}}\right)\right)$$

34. Find all solutions in $[0, 2\pi)$: $4\cos^2 x = 3$

35. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $\sqrt{3}\sin x \tan(2x) - \sin x = 0$

36. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $2\cos\left(\dfrac{1}{2}x\right)\cos x - 2\sin\left(\dfrac{1}{2}x\right)\sin x = 1$

37. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $2\cos^2\left(\frac{x}{3}\right) + 3\sin\left(\frac{x}{3}\right) - 3 = 0$

38. When it was first constructed in 1889, the Eiffel Tower in Paris, France was the tallest structure in the world. In 1975, the CN Tower in Toronto, Canada became the world's tallest structure. The CN tower is 153 ft less than twice the height of the Eiffel Tower, and the sum of their heights is 2799 ft. How tall is each tower?

39. Write the system associated with the given RREF matrix and then use the system to find the solution.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

40. Write the system associated with the given RREF matrix and then use the system to find the solution.

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

41. Compute the determinant of the matrix:

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & -1 \\ 2 & 1 & -4 \end{bmatrix}$$

42. Solve the system of equations using Cramer's rule, if possible: $\begin{cases} 4x + y = -11 \\ 3x - 5y = -60 \end{cases}$

43. Write out the first four terms in the sequence: $a_n = \dfrac{(n+3)!}{(2n)!}$

44. Find the number of terms in the arithmetic sequence using the given information: $a_1 = 2$, $a_n = -22$, $d = -3$

45. At 5 PM in Coldwater, the temperature was a chilly 36°F. If the temperature decreased by 3°F every half-hour for the next 7 hours, at what time did the temperature hit 0°F?

46. Find S_{12} for the geometric sequence with $a_1 = 8$ and $r = -2$.

47. A new photocopier under heavy use will depreciate about 25% per year (meaning it holds 75% of its value each year). If the copier is purchased for \$7000, how much is it worth 4 years later? How many years until its value is less than \$1246?

48. Use the binomial theorem to fully expand: $(2x - 3)^4$