Calculus I : Spring 2015 **Final Exam**

Instructor: __________________ **Name:** __________________

INSTRUCTIONS : Show all of your work, and give exact answers. No calculators

1. Short answer questions:

   (a) Evaluate $\displaystyle\lim_{t\to 4^-} \frac{|4-t|}{4-t} =$ ________________ (2 pts)

   (b) Suppose that $\sqrt{x} + 2 \le f(x) \le 2x - 13$ for $x > 0$.
   What can you say about $\displaystyle\lim_{x\to 9} f(x)$? ________________ (3 pts)

   (c) Evaluate $\dfrac{d}{dx} \ln 2 =$ ________________ (2 pts)

   (d) Evaluate $\dfrac{d}{dx} \tan^{-1}(2x) =$ ________________ (3 pts)

   (e) Given that $f(-1) = 3$ and $f'(-1) = -2$, use differentials to estimate $f(-0.9)$ ________________ (3 pts)

   (f) Suppose that $f'(x) = -\dfrac{3x(x+2)}{(x-2)^2}$ Is the point $x = -2$ a minimum, maximum, or neither? ________________ (2 pts)

   (g) If $f(2) = 1$, $f(3) = 2$, $f'(x) > 0$ and $f''(x) < 0$, give a sketch of the graph of $f(x)$ for $2 \le x \le 3$ (3 pts)

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(h) Write $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \left(\frac{2i}{n}\right)\right) \cdot \frac{2}{n}$ as an integral ________________ (4 pts)

(i) Evaluate $\int (\cos(x) - \sin(x))\, dx =$ ________________ (2 pts)

(j) Evaluate $\int (e^x + x^e)\, dx =$ ________________ (2 pts)

(k) We try the substitution $u = \frac{x^2}{2}$ to evaluate the integral $\int x \sin\left(\frac{x^2}{2}\right)\, dx$. The differential is $du =$ ________________ (2 pts)

(l) Given the integral $\int \frac{\cos(\ln x)}{x}\, dx$, a suitable substitution is ________________ (2 pts)

12 points

2. **Limits**

(a) (5 pts) Evaluate $\lim_{x\to 1^-} \frac{x^2 - 3x + 2}{x^2 - 2x + 1} =$

(b) (5 pts) Evaluate $\lim_{x\to\infty} (x - \sqrt{x^2 + 3x}) =$

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(c) (10 pts) Use l'Hopital's Rule to evaluate $\lim_{x \to 0} \dfrac{x^2}{1 - \cos(x)} =$

3. **Derivatives**

(a) (5 pts) From the Fundamental Theorem of Calculus, we have $\dfrac{d}{dx} \displaystyle\int_1^x e^{t^2}\, dt = e^{x^2}$.

Find $\dfrac{d}{dx} \displaystyle\int_1^{2x} e^{t^2}\, dt =$

(b) (5 pts) Use *implicit differentiation* to find $\dfrac{dy}{dx}$, given $xy - 2x^2y^3 = 1$.

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(c) (5 pts) If $f(s) = s\sin^{-1}(s)$, find $f'(s)$

(d) (5 pts) Find the derivative of $z = \ln(t^3 + 2)$

(e) (10 pts) Find the *equation* of the tangent line to $y = \dfrac{2x}{x^2 - 1}$ at the point where $x = 2$

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4. **Applications**

(a) (10 pts) Sketch the graph of $y = f(x)$, given the following:

- $f(-1) = 2$; $f(0) = 3$; $\lim_{x\to-\infty} f(x) = 1$; $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to 1^-} f(x) = -\infty$; $\lim_{x\to 1^+} f(x) = +\infty$
- $f'(0) = 0$; $f'(x) > 0$ for $x \in (-\infty, 0)$; $f'(x) < 0$ for $x \in (0, 1) \cup (1, \infty)$
- $f''(-1) = 0$; $f''(x) < 0$ for $x \in (-1, 1)$; $f''(x) > 0$ for $x \in (-\infty, -1) \cup (1, \infty)$

(b) (10 pts) The position of a particle on the x-axis after t minutes is $s(t) = 5 + 3t^2 - t^3$ meters. When is the particle moving to the *left*?

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(c) (10 pts) The volume of a rectangular box with *no lid* and a *square* base is 32 cm^3. Find the minimum possible *surface area* (material needed to make the box).

(d) (10 pts) We are given a rectangle. The height is *increasing* at 0.2 cm/min, the base is *decreasing* at 0.1 cm/min

How is the *area* changing when the height is 2 cm, and the base is 3 cm?

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5. **Integrals**

(a) (5 pts) Evaluate $\displaystyle\int \frac{x}{x^2+4}\,dx =$

(b) (5 pts) Evaluate $\displaystyle\int_0^2 \frac{1}{x^2+4}\,dx =$

(c) (10 pts) Evaluate $\displaystyle\int x\,\sqrt{x-1}\,dx =$

(d) (10 pts) Evaluate $\displaystyle\int_{\ln(\pi/6)}^{\ln(\pi/3)} e^x \sec(e^x)\tan(e^x)\,dx =$