Math 223, Section 4

Final Exam

5/9/2017

Name: ______________________________

Problem	1	2	3	4	5	6	7	8	Total
Score									
Possible	25	25	25	25	25	25	25	25	200

NOTE: In order to receive full credit, I need to see all your work. Unjustified solutions, or solutions from a calculator, will receive partial or no credit.

1. (25 points) Let S be the surface of the tetrahedron in the first octant formed by the three coordinate planes (i.e. the xy-plane, the xz-plane, and the yz-plane) and the plane $x + y + z = 1$ (note S is closed). Let

$$\mathbf{F}(x, y, z) = \langle 3x^2 + 2xy + z^3, -4xy - 2y^2, 2yz - xz \rangle.$$

Compute the total flux of $\mathbf{F}$ across S.

2. (25 points) Let's play with lines and planes and surfaces.

(8 ponits) Find the equation of the plane that contains the points $(2, 4, -1)$, $(5, 2, 2)$, and $(8, 0, 0)$.

(8 points) Find the line of intersection of the planes $x + y - 2z = 0$ and $x - y + 3z = 1$.

(9 points) Identify by name each of the following surfaces:

(i) $x = z^2 + y^2$.

(ii) $x^2 = z^2 + y^2$.

(iii) $x^2 - y^2 + z^2 = 1$.

3. (25 points)

(a) (8 points) Set up in spherical coordinates, but DO NOT EVALUATE, the integral of the function $f(x, y, z) = \sin(x^2 + y^2 + z^2)$ over the inside of the part of the sphere $x^2 + y^2 + z^2 = 16$ inside the first octant.

(b) (8 points) Set up, but DO NOT EVALUATE, the integrals to find the center of mass of the region bounded by $y = x^2$ and $x = y^2$, if the density function is given by $\rho(x, y) = e^{x+y}$.

(c) Compute the work done by the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + y\mathbf{j} + yz\mathbf{k}$ along the straight line from $(1, 0, -1)$ to $(3, 4, 2)$. Note: I DO want you to evaluate this integral.

4. (25 points) Let $f(x, y) = \sin(xy) + x^3y^2$.

(a) (5 points) Compute f_x and f_y.

(b) (5 points) Compute the gradient of f at $(1, \pi)$.

(c) (5 points) Compute the directional derivative of f in the direction of the vector $\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$ at the point $(1, \pi)$.

(d) (5 points) In what direction is the slope of $f(x, y)$ the steepest at $(1, \pi)$? Your answer should be a unit vector.

(e) (5 points) Let $x = e^t$ and $y = 2t$. Compute $\frac{df}{dt}$.

5. (25 points)

(a) (13 points) Let $\mathbf{F}(x,y) = (\frac{-y^3}{3} + x^5\sin(x))\mathbf{i} + (\frac{x^3}{3} + e^{y^2})\mathbf{j}$. Let C be the unit circle centered at the origin, traversed counterclockwise. Use Green's theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

(b) (12 points) Let S be the surface of the hemisphere $x^2+y^2+z^2=1$, with $z \geq 0$. Use Stokes' theorem to compute the flux of $curl(\mathbf{F})$ across S (with the upward orientation) if

$$\mathbf{F}(x,y,z) = \langle -y, 2x, x+z \rangle.$$

6. (25 points) Let $\mathbf{r}(t) = (t^2+1)\mathbf{i} + 2t\mathbf{j} + e^t\mathbf{k}$.

(a) (7 points) Compute the curvature $\kappa(t)$ for this vector curve.

(b) (6 points) Set up, but DO NOT EVALUATE, the integral to compute the length of this curve from $t = 0$ to $t = 3$.

(c) (6 points) Compute $\mathbf{T}(t)$ for this curve.

(d) (6 points) Give the formulas for $\mathbf{N}(t)$ and $\mathbf{B}(t)$. You do NOT need to compute them for this curve, I just want to see that you know the formulas.

7. (25 points) Let

$$\mathbf{F}(x, y, z) = (2xyze^{x^2yz})\mathbf{i} + (x^2ze^{x^2yz} + z^3)\mathbf{j} + (x^2ye^{x^2yz} + 3yz^2)\mathbf{k}.$$

(a) (6 points) Show that $\mathbf{F}$ is conservative.

(b) (7 points) Find the potential function f for $\mathbf{F}$.

(c) (6 points) Compute the work done by $\mathbf{F}$ along the path $\langle t, t^2, t^3\rangle$ from $(0, 0, 0)$ to $(1, 1, 1)$.

(d) (6 points) Compute the work done by $\mathbf{F}$ along the path from $(1, 0, 0)$, along the helix $\langle \cos(t), \sin(t), t\rangle$ to $(1, 0, 2\pi)$, and then back to $(1, 0, 0)$ along a straight line.

8. (25 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

Here's the punchline: He's so excited, he jumps up, and his butt falls off!

Enjoy your summer!