Precalculus Final Exam Review: Part 3

1. Algebraically find $f^{-1}(x)$. Then, prove that you found the correct inverse by finding $f(f^{-1}(x))$ and $f^{-1}(f(x))$. $f(x) = \sqrt[3]{2x+1}$

2. Solve for x: $e^x(e^x + e) = \dfrac{e^x + e^{3x}}{e^{-x}}$

3. Determine the domain of the function: $y = \ln\left(\sqrt{5-3x}\right)$

4. Solve for x: $\log_4(x+2) - \log_4(3) = \log_4(x-1)$

5. Solve for x. Clearly identify any extraneous solutions. If there are no solutions, so state. $\log(x+8) + \log x = \log(x+18)$

6. Solve for x: $5^{2x+1} = 9^{x+1}$

7. The city of Pittsburgh, Pennsylvania is directly north of West Palm Beach, Florida. Pittsburgh is at 40.3° north latitude, while West Palm Beach is at 26.4° north latitude. Assuming the Earth has a radius of 3960 mi, how far apart are these cities?

8. Use the formula for area of a circular sector to find the value of the unknown quantity. $\theta = 5$; $r = 6.8$ km

9. Without the use of a calculator, state the exact value of the trigonometric functions for the given angle.

   (a) $\sin\left(\dfrac{\pi}{4}\right)$

   (b) $\sin\left(\dfrac{3\pi}{4}\right)$

   (c) $\sin\left(\dfrac{5\pi}{4}\right)$

   (d) $\sin\left(\dfrac{7\pi}{4}\right)$

   (e) $\sin\left(\dfrac{9\pi}{4}\right)$

   (f) $\sin\left(-\dfrac{\pi}{4}\right)$

   (g) $\sin\left(-\dfrac{5\pi}{4}\right)$

   (h) $\sin\left(-\dfrac{11\pi}{4}\right)$

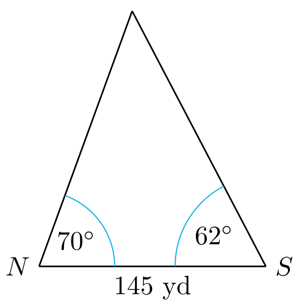
10. Given $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ is a point on the unit circle corresponding to t, find the value of all six circular functions of t.

11. In the wiring of an apartment complex, electrical wire is being pulled from a spool with radius 1 decimeter (1 dm = 10 cm). (a) What length (in decimeters) is removed as the spool turns through 5 rad? (b) How many decimeters are removed in one complete turn ($t = 2\pi$) of the spool?

12. A person standing near the top of the Eiffel Tower notices a car wreck some distance from the tower. If the angle of depression from the person's eyes to the wreck is 32°, how far away is the accident from the base of the tower? (The height of the Eiffel Tower is approximately 300.6 m.)

13. Given that $\cos\theta = \dfrac{4}{5}$ and $\sin\theta < 0$, find the values of x, y, and r. Clearly indicate the quadrant of the terminal side of θ, then state the values of the six trigonometric functions of θ.

14. When the Goodyear Blimp is viewed from the field-level bleachers near the southern end-zone of a football stadium, the angle of elevation is 62°. From the field-level bleachers near the northern end-zone, the angle of elevation is 70°. Find the height of the blimp if the distance from the southern bleachers to the northern bleachers is 145 yd.



15. Given the following information about a triangle, completely solve the triangle. Give your answer in table form: side $a = 15\sqrt{3}$ in., side $b = 6\sqrt{3}$ in., side $c = 10\sqrt{3}$ in.

16. Sketch a graph of $y = \sin t$, for $t \in [-2\pi, 2\pi]$.

17. Sketch a graph of $y = \cot t$, for $t \in [-2\pi, 2\pi]$.

18. Sketch the graph of $y = \sin(2t)$ for $t \in [0, 2\pi]$.

19. Sketch a graph of $y = -\sin(2t)$ for $t \in [0, 2\pi]$.

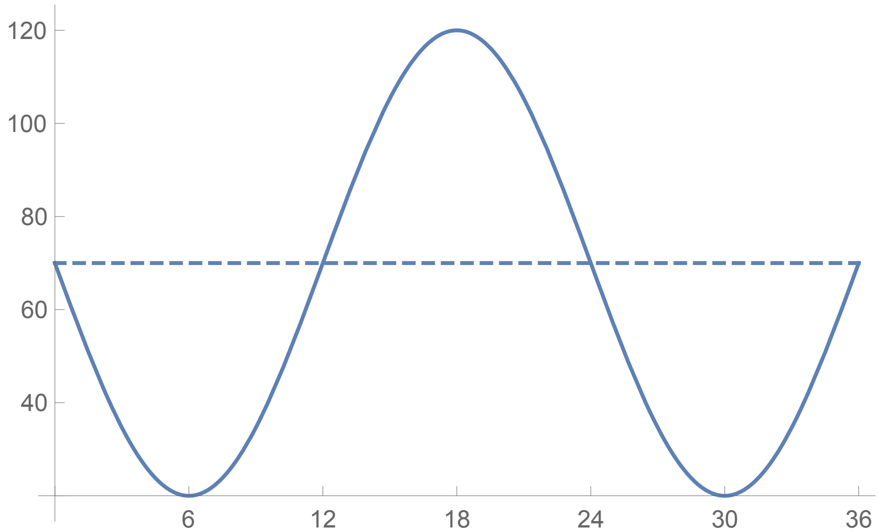
20. Sketch a graph of $y = 1 - \sin(2t)$ for $t \in [0, 2\pi]$.

21. Sketch the graph of $y = -2\sin(\pi t)$ for $t \in [0, 4]$.

22. Identify the amplitude, period, vertical shift, horizontal shift, and primary interval of $y = -3\cos\left(\frac{\pi}{2}t + \frac{3\pi}{8}\right)$.

23. Sketch $y = \frac{2}{3}\cos(4t) + 1$ for $t \in [0, 2\pi]$

24. Find the equation of the graph below. Please write your answer in the form $y = A\sin(Bt + C) + D$.



25. Verify the identity: $\dfrac{\sin x \cos x + \cos x}{\sin x + \sin^2 x} = \cot x$

26. Verify the identity: $\dfrac{\cos\theta}{1 - \sin\theta} = \sec\theta + \tan\theta$

27. For $\cos\alpha = -\dfrac{7}{25}$ with terminal side in QII and $\cot\beta = \dfrac{15}{8}$ with terminal side in QIII, find: (a) $\sin(\alpha + \beta)$ and (b) $\tan(\alpha + \beta)$

28. Find the exact values of $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ using the information given.

$$\sin\theta = \frac{5}{13};\ \theta \text{ in QII}$$

29. Rewrite in terms of an expression containing only cosines to the power 1: $3\cos^4 x$

30. Use a half-angle identity to find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the given value of θ: $\theta = 22.5°$

31. Evaluate without a calculator and answer in radians: $\arcsin\left(\dfrac{\sqrt{3}}{2}\right)$

32. Evaluate without a calculator: $\cot\left(\cos^{-1}\left(-\dfrac{1}{2}\right)\right)$

33. Evaluate the expression. Draw a right triangle and label the sides to assist.

$$\sin\left(\tan^{-1}\left(\frac{\sqrt{5}}{2}\right)\right)$$

34. Find all solutions in $[0, 2\pi)$: $4\sin^2 x = 1$

35. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $\sec^2 x - 6\sec x = 16$

36. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $4\sin^2 x - 4\cos^2 x = 2\sqrt{3}$

37. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $\cos(2x) + 2\sin^2 x - 3\sin x = 0$

38. Solve the system and identify the system as consistent, inconsistent, or dependent:
$$\begin{cases} 1.2x + 0.4y = 5 \\ 0.5y = -1.5x + 2 \end{cases}$$

39. Write the system associated with the given RREF matrix and then use the system to find the solution.
$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 8 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

40. Write the system associated with the given RREF matrix and then use the system to find the solution.
$$\begin{bmatrix} 3 & -4 & 2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

41. Compute the determinant of the matrix:
$$\begin{bmatrix} -2 & 3 & 4 \\ 0 & 6 & 2 \\ 1 & -1.5 & -2 \end{bmatrix}$$

42. Solve the system of equations using Cramer's rule, if possible: $\begin{cases} \frac{x}{8} + \frac{y}{4} = 1 \\ \frac{y}{5} = \frac{x}{2} + 6 \end{cases}$

43. Expand and evaluate the series: $\sum_{j=3}^{7} 2j$

44. Identify the first term a_1 and the common difference d. Then, write the expression for the general term a_n and use it to find the 6th, 10th, and 12th terms of the sequence: $7, 4, 1, -2, -5, \ldots$

45. Find the number of terms in the geometric sequence: $2, -6, 18, -54, \ldots, -486$

46. Determine whether the infinite geometric series has a finite sum. If so, find the sum: $3 + 6 + 12 + 24 + \cdots$

47. Evaluate: $\binom{20}{17}$

48. Use the binomial theorem to fully expand: $(1 - 2i)^3$ (note that $i = \sqrt{-1}$)