§ 6.2: Properties of Sets

Theorem 6.2.1: For all sets A, B, and C:
Inclusion of Intersection: $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
Inclusion in Union: $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
Transitive Property of Subsets: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Procedural Versions of Set Definitions: Let $X, Y \subseteq U$ and $x, y \in U$.

$$\begin{aligned}
x \in X \cup Y &\iff x \in X \text{ or } x \in Y \\
x \in X \cap Y &\iff x \in X \text{ and } x \in Y \\
x \in X - Y &\iff x \in X \text{ and } x \notin Y \\
x \in X^c &\iff x \notin X \\
(x, y) \in X \times Y &\iff x \in X \text{ and } y \in Y
\end{aligned}$$

Example A: Prove that for all sets A and B, $A \subseteq A \cup B$.

Theorem 6.2.2: Set Identities. For all sets A, B, and C, subsets of a universal set U:

1.	Commutative laws:	$A \cup B = B \cup A$	$A \cap B = B \cap A$
2.	Associative laws:	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
3.	Distributive laws:	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4.	Identity laws:	$A \cup \emptyset = A$	$A \cap U = A$
5.	Complement laws:	$A \cup A^c = U$	$A \cap A^c = \emptyset$
6.	Double complement law:	$(A^c)^c = A$	
7.	Idempotent laws:	$A \cup A = A$	$A \cap A = A$
8.	Universal bound laws:	$A \cup U = U$	$A \cap \emptyset = \emptyset$
9.	De Morgan's laws:	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
10.	Absorption laws:	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
11.	Complements of U and $\emptyset$:	$U^c = \emptyset$	$\emptyset^c = U$
12.	Set difference law:	$A - B = A \cap B^c$	

Theorem 6.2.4: If E is a set with no elements and A is any set, then $E \subseteq A$.

Corollary 6.2.5: There is only one set with no elements.

Example B: (#32) For all sets A, B, and C, if $A \subseteq B$ and $B \cap C = \emptyset$, then $A \cap C = \emptyset$.