Precalculus Final Exam Review: Part 1

1. Solve the inequality by using a number line and the behavior of the graph at each zero. Write the solution in interval notation. $(x+1)^2(x-4) \geq 0$

2. Algebraically find $f^{-1}(x)$, where $f(x) = \dfrac{x+2}{1-x}$.

3. Algebraically find $f^{-1}(x)$. Then, prove that you found the correct inverse by finding $f(f^{-1}(x))$ and $f^{-1}(f(x))$. $f(x) = \dfrac{(x-1)^3}{8}$

4. Solve for x: $(e^{2x-4})^3 = \dfrac{e^{x+5}}{e^2}$

5. Determine the domain of the function: $y = \log\big(9 - x^2\big)$

6. Use the properties of logarithms to write the following expression as sum or difference of simple logarithmic terms: $\ln\left(\dfrac{7x\sqrt{3-4x}}{2(x-1)^3}\right)$

7. Solve for x. Clearly identify any extraneous solutions. If there are no solutions, so state. $\ln 21 = 1 + \ln(x-2)$

8. Solve for x: $\left(\dfrac{1}{5}\right)^{x-1} = \left(\dfrac{1}{2}\right)^{3-x}$

9. Use the formula for arc length to find the value of the unknown quantity. $\theta = 3.5$; $r = 280$ m

10. A water sprinkler is set to shoot a stream of water a distance of 12 m and rotate through an angle of $40°$. (a) What is the area of the lawn it waters? (b) For $r = 12$ m, what angle is required to water twice as much area? (c) For $\theta = 40°$, what range for the water stream is required to water twice as much area?

11. At the park two blocks from our home, the kids' round-a-bout has a radius of 56 in. About the time the kids stop screaming, "Faster, Daddy, faster!" I estimate the round-a-bout is turning at $\frac{3}{4}$ revolutions per second. (a) What is the related angular velocity? (b) What is the linear velocity (in miles per hour) of Eli and Reno, who are "hanging on for dear life" at the rim of the round-a-bout?

12. Without the use of a calculator, state the exact value of the trigonometric functions for the given angle.

(a) $\tan\left(\frac{\pi}{3}\right)$

(b) $\tan\left(\frac{2\pi}{3}\right)$

(c) $\tan\left(\frac{4\pi}{3}\right)$

(d) $\tan\left(\frac{5\pi}{3}\right)$

(e) $\tan\left(\frac{7\pi}{3}\right)$

(f) $\tan\left(-\frac{\pi}{3}\right)$

(g) $\tan\left(-\frac{4\pi}{3}\right)$

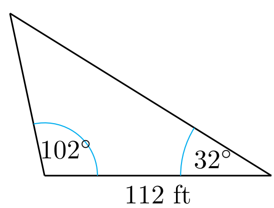
(h) $\tan\left(-\frac{10\pi}{3}\right)$

13. Given $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is a point on the unit circle corresponding to t, find the value of all six circular functions of t.

14. Given that $\cos\theta = \frac{2}{3}$, determine the value of the other five trigonometric functions of the acute angle θ. Answer in exact form (a diagram will help).

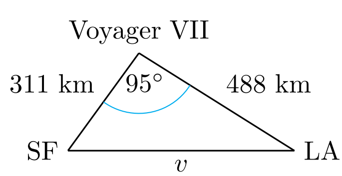
15. Given that $\tan\theta = -\frac{12}{5}$ and $\cos\theta > 0$, find the values of x, y, and r. Clearly indicate the quadrant of the terminal side of θ, then state the values of the six trigonometric functions of θ.

16. An ancient fortress is built on a steep hillside, with the base of the fortress walls making a 102° angle with the hill. At the moment the fortress casts a 112-ft shadow, the angle of elevation from the tip of the shadow to the top of the wall is 32°. What is the distance from the base of the fortress to the top of the wall?



17. Assume that we know the given information about a triangle: side $a = 320$ cm, side $b = 290$ cm, and angle $A = 50°$. Also assume that angle $B_1 = 44°$. Will one or two triangles satisfy the given information? Be sure to explain how you found your answer, showing all calculations and work.

18. Voyager VII measures its distance from Los Angeles and from San Francisco using radio waves as shown. Using an on-board sighting device, the satellite determines angle V is 95°. How many kilometers separate Los Angeles and San Francisco?



19. Sketch a graph of $y = \cos t$, for $t \in [-2\pi, 2\pi]$.

20. Sketch a graph of $y = \csc t$, for $t \in [-2\pi, 2\pi]$.

21. Sketch a graph of $y = \cos(2t)$, for $t \in [0, 2\pi]$.

22. Sketch a graph of $y = 2\cos(2t)$, for $t \in [0, 2\pi]$.

23. Sketch a graph of $y = 2 + 2\cos(2t)$, for $t \in [0, 2\pi]$.

24. Sketch the graph of $y = -2\cos\left(\frac{t}{2}\right)$ for $t \in [0, 4\pi]$.

25. Identify the amplitude, period, vertical shift, horizontal shift, and primary interval of $y = \pi\cos(2\pi t - \pi) + 1$.

26. Sketch $y = -3\sin(\pi t)$ for $t \in [0, 4]$.

27. In Geneva, Ohio, the daily temperature in March ranges from an average high of 39°F to an average low of 29°F. (a) Find a sinusoidal equation model for the daily temperature. Assume $t = 0$ corresponds to noon. (b) Sketch the graph of the model.

28. Verify the identity: $\dfrac{1 + \cos x}{\sin x + \cos x \sin x} = \csc x$

29. Verify the identity: $\dfrac{\cot x - \tan x}{\cot^2 x - \tan^2 x} = \sin x \cos x$

30. Given α and β are obtuse angles with $\sin\alpha = \dfrac{28}{53}$ and $\cos\beta = -\dfrac{13}{85}$, find: (a) $\sin(\alpha - \beta)$, (b) $\cos(\alpha + \beta)$, and (c) $\tan(\alpha - \beta)$

31. Find the exact value of $1 - 2\sin^2\left(\dfrac{\pi}{8}\right)$.

32. Rewrite in terms of an expression containing only cosines to the power 1: $\sin^2 x \cos^2 x$

33. Use a half-angle identity to find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for the given value of θ: $\theta = \dfrac{11\pi}{12}$

34. Evaluate without a calculator and answer in radians: $\tan^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$

35. Evaluate without a calculator and answer in radians: $\cos^{-1}\left(\cos\left(\dfrac{5\pi}{4}\right)\right)$

36. Evaluate the expression. Draw a right triangle and label the sides to assist.
$\sin\left(\cos^{-1}\left(-\dfrac{7}{25}\right)\right)$

37. Find all solutions in $[0, 2\pi)$: $8\tan x + 7\sqrt{3} = -\sqrt{3}$

38. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $2\cos x \sin x - \cos x = 0$

39. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $\sqrt{3}\tan(2x) = -\sqrt{3}$

40. Solve the equation for the principal root, all solutions in $[0, 2\pi)$, and all real solutions: $5\csc^2 x - 5\cot x - 5 = 0$

41. Solve the system: $\begin{cases} 5x - 6y = 2 \\ x + 2y = 6 \end{cases}$

42. Write the system associated with the given RREF matrix and then use the system to find the solution.

$$\left[\begin{array}{cccc} 1 & -\frac{5}{2} & -2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

43. Write the system associated with the given RREF matrix and then use the system to find the solution.

$$\left[\begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

44. In triangle ABC, the sum of angles A and C is equal to three times angle B. Angle C is 10 degrees more than twice angle B. Set up a system of equations (or a matrix) to solve for the measure of each angle.

45. Compute the determinant of the matrix:

$$\begin{bmatrix} 4 & -7 \\ 3 & -5 \end{bmatrix}$$

46. Use Cramer's rule to solve the system of equations, if possible: $\begin{cases} y + 2z = 1 \\ 2x - y + z = -2 \\ 3x - 2z = 3 \end{cases}$

47. Find the first four terms of the recursive sequence:

$$\begin{cases} a_1 = -1 \\ a_n = (a_{n-1})^2 + 3 \end{cases}$$

48. Find the common difference d and the value of a_1 using the given information: $a_3 = 7$, $a_7 = 19$

49. Evaluate the sum: $\sum_{n=1}^{30} (3n - 4)$

50. Assuming $r > 0$, find the common ratio r and the value of a_1 using the given information: $a_3 = 324$, $a_7 = 64$

51. Determine whether the infinite geometric series has a finite sum. If so, find the sum: $\sum_{k=1}^{\infty} \frac{3}{4}\left(\frac{2}{3}\right)^k$

52. Evaluate: $\binom{22}{3}$

53. Use the binomial theorem to write the first three terms: $\left(v^2 - \frac{1}{2}w\right)^{12}$