# Chapter 2: The Logic of Compound Statements

## Section 2.1: Logical Form and Logical Equivalence

**Example A:** Logical Form
**Argument 1:** If it is raining, then I am carrying an umbrella. It is raining. Therefore, I am carrying an umbrella.
**Argument 2:** If $x$ is an integer, then $x$ is rational. $x$ is an integer. Therefore, $x$ is rational.
**Form:** If $p$, then $q$. $p$. Therefore, $q$.

**Definition:** A **statement** (or **proposition**) is a sentence that is true or false, but not both.

**Example B:** $p =$ "It is raining." $q =$ "I am carrying an umbrella."

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**Definition:** A **tautology** is a statement form that is always true. A **contradiction** is a statement form that is always false.

**Theorem 2.1.1: Logical Equivalences.** Given any statement variables $p$, $q$, and $r$, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

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| 1. | Commutative laws: | $p \wedge q \equiv q \wedge p$ | $p \vee q \equiv q \vee p$ |
| 2. | Associative laws: | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ |
| 3. | Distributive laws: | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |
| 4. | Identity laws: | $p \wedge \mathbf{t} \equiv p$ | $p \vee \mathbf{c} \equiv p$ |
| 5. | Negation laws: | $p \vee \sim p \equiv \mathbf{t}$ | $p \wedge \sim p \equiv \mathbf{c}$ |
| 6. | Double negative law: | $\sim(\sim p) \equiv p$ | |
| 7. | Idempotent laws: | $p \wedge p \equiv p$ | $p \vee p \equiv p$ |
| 8. | Universal bound laws: | $p \vee \mathbf{t} \equiv \mathbf{t}$ | $p \wedge \mathbf{c} \equiv \mathbf{c}$ |
| 9. | De Morgan's laws: | $\sim(p \wedge q) \equiv \sim p \vee \sim q$ | $\sim(p \vee q) \equiv \sim p \wedge \sim q$ |
| 10. | Absorption laws: | $p \vee (p \wedge q) \equiv p$ | $p \wedge (p \vee q) \equiv p$ |
| 11. | Negations of $\mathbf{t}$ and $\mathbf{c}$: | $\sim \mathbf{t} \equiv \mathbf{c}$ | $\sim \mathbf{c} \equiv \mathbf{t}$ |