Math 223, 9:45 section

Final Exam

8/5/16

Name: ____

Problem	1	2	3	4	5	6	Total
Score							
Possible	40	40	30	30	30	30	100

NOTE: I need to see all of your work for each problem. Unjustified work will receive little or no credit.

1. (40 points) Let $f(x, y) = x^2y - x^3y^2$.

(a) (10 points) Compute the tangent plane to $f(x, y)$ at the point $(1, 1)$.

(b) (10 points) Use linear approximation to estimate $f(1.1, 0.97)$.

(c) (10 points) Compute the directional derivative of f at $(1, 1)$ in the direction of the vector $\mathbf{u} = \frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$.

(d) (10 points) What is the maximum rate of change of f at the point $(1, 1)$?

2. (40 points) Let $\mathbf{F}(x, y) = (x + 2y)\mathbf{i} + (x - 2y)\mathbf{j}$.

(a) (10 points) Show that $\mathbf{F}$ is NOT conservative.

(b) (20 points) Compute the work done by $\mathbf{F}$ on an object moving from $(0, 0)$ to $(2, 4)$ along a straight line.

(c) (10 points) Compute the work done by $\mathbf{F}$ on an object moving along the circular path $x = 3\cos(t)$, $y = 3\sin(t)$, where $0 \leq t \leq 2\pi$.

3. (30 points) Find each of the following (10 points each)

(a) The plane that contains the points $(1, 0, 1)$, $(1, 1, 0)$, and $(0, 1, 1)$.

(b) The point of intersection of the line $x = 2 + t$, $y = -3 + 2t$, $z = 1 - t$ with the plane $x + 2y + 3z = 4$,

(c) The angle between the planes $x + 2y + 3z = 4$ and $x - 2y - 3z = 6$.

4. (30 points) Let $\mathbf{r}(t) = t^2\mathbf{i} + \cos(t)\mathbf{j} + \sin(t)\mathbf{k}$.

(a) Set up, but DO NOT EVALUATE, the integral to compute the length of $\mathbf{r}(t)$ from $t = 0$ to $t = 2\pi$.

(b) Compute $\mathbf{T}(t)$.

(c) Compute the curvature $\kappa(t)$. Don't bother to simplify your answer.

5. (30 points) Set up, but DO NOT EVALUATE, each of the following integrals.

(a) (7 points) The integral(s) to find the x coordinate of the center of mass of the top half of the circle $x^2 + y^2 = 4$ with density function $\rho(x, y) = \cos(x^2 + y^2)$. Your answer should be in polar coordinates.

(b) (7 points) The integral of $f(x, y, z) = (x^2 + y^2 + z^2)^{3/2}$ over the region E, where E is the region within the top half of the sphere $x^2 + y^2 + z^2 = 9$ and inside the cone $z^2 = x^2 + y^2$. Your answer should be in spherical coordinates.

(c) (8 points) Switch

$$\int_{x=0}^{x=25} \int_{y=-\sqrt{x}}^{\sqrt{x}} (x + y^2) dy dx$$

to $dxdy$.

(d) (8 points) The integral of $f(x, y, z) = z(x^2 + y^2)$ over the region E, where E is within the cylinder $x^2 + y^2 = 4$ and the top half of the sphere $x^2 + y^2 + z^2 = 9$, in cyclindrical coordinates.

6. (30 points)

(a) (15 points) Compute $\int_S y dS$, where S is the part of the paraboloid $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 4$.

(b) (15 points) Use the Divergence Theorem to compute the surface integral $\mathbf{F} \cdot d\mathbf{S}$ (i.e. the flux of $\mathbf{F}$ across the surface S) if

$$\mathbf{F}(x, y, z) = (\cos(z) + xy^2)\mathbf{i} + xe^{-z}\mathbf{j} + (\sin(y) + x^2 z)\mathbf{k}$$

and S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

Enjoy the rest of your summer!