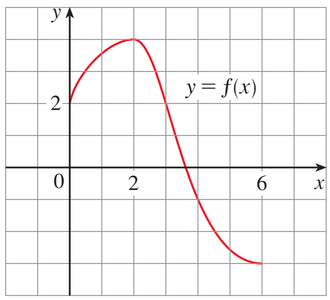
EXERCISES

1. Use the given graph of f to find the Riemann sum with six subintervals. Take the sample points to be (a) left endpoints and (b) midpoints. In each case draw a diagram and explain what the Riemann sum represents.



2. (a) Evaluate the Riemann sum for

$$f(x) = x^2 - x \qquad 0 \leqslant x \leqslant 2$$

with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.

(b) Use the definition of a definite integral (with right endpoints) to calculate the value of the integral

$$\int_0^2 (x^2 - x)\,dx$$

(c) Use the Fundamental Theorem to check your answer to part (b).

(d) Draw a diagram to explain the geometric meaning of the integral in part (b).

3. Evaluate

$$\int_0^1 \left(x + \sqrt{1 - x^2}\right) dx$$

by interpreting it in terms of areas.

4. Express

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin x_i\, \Delta x$$

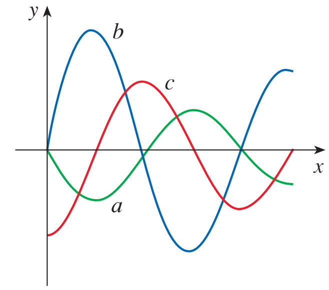
as a definite integral on the interval $[0, \pi]$ and then evaluate the integral.

5. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

T **6.** (a) Write $\int_1^5 (x + 2x^5)\,dx$ as a limit of Riemann sums, taking the sample points to be right endpoints. Use a computer algebra system to evaluate the sum and to compute the limit.

(b) Use the Fundamental Theorem to check your answer to part (a).

7. The figure shows the graphs of f, f', and $\int_0^x f(t)\,dt$. Identify each graph, and explain your choices.

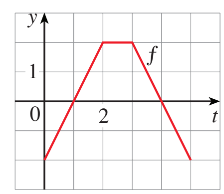


8. Evaluate:

(a) $\displaystyle\int_0^1 \frac{d}{dx}\left(e^{\arctan x}\right) dx$ (b) $\displaystyle\frac{d}{dx}\int_0^1 e^{\arctan x}\,dx$

(c) $\displaystyle\frac{d}{dx}\int_0^x e^{\arctan t}\,dt$

9. The graph of f consists of the three line segments shown. If $g(x) = \int_0^x f(t)\,dt$, find $g(4)$ and $g'(4)$.



10. If f is the function in Exercise 9, find $g''(4)$.

11–42 Evaluate the integral, if it exists.

11. $\displaystyle\int_{-1}^{0} (x^2 + 5x)\,dx$

12. $\displaystyle\int_0^T (x^4 - 8x + 7)\,dx$

13. $\displaystyle\int_0^1 (1 - x^9)\,dx$

14. $\displaystyle\int_0^1 (1 - x)^9\,dx$

15. $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

16. $\displaystyle\int_0^1 \left(\sqrt[4]{u} + 1\right)^2 du$

17. $\displaystyle\int_0^1 y(y^2 + 1)^5\,dy$

18. $\displaystyle\int_0^2 y^2\sqrt{1 + y^3}\,dy$

19. $\displaystyle\int_1^5 \frac{dt}{(t - 4)^2}$

20. $\displaystyle\int_0^1 \sin(3\pi t)\,dt$

21. $\displaystyle\int_0^1 v^2 \cos(v^3)\,dv$

22. $\displaystyle\int_{-1}^{1} \frac{\sin x}{1 + x^2}\,dx$

23. $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t}\,dt$

24. $\displaystyle\int_{-2}^{-1} \frac{z^2 + 1}{z}\,dz$

25. $\displaystyle\int \frac{x}{x^2 + 1}\,dx$

26. $\displaystyle\int \frac{dx}{x^2 + 1}$

27. $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

28. $\displaystyle\int \frac{\csc^2 x}{1+\cot x}\,dx$

29. $\displaystyle\int \sin \pi t \cos \pi t\,dt$

30. $\displaystyle\int \sin x \cos(\cos x)\,dx$

31. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

32. $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

33. $\displaystyle\int \tan x \ln(\cos x)\,dx$

34. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

35. $\displaystyle\int \frac{x^3}{1+x^4}\,dx$

36. $\displaystyle\int \sinh(1+4x)\,dx$

37. $\displaystyle\int \frac{\sec\theta\tan\theta}{1+\sec\theta}\,d\theta$

38. $\displaystyle\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt$

39. $\displaystyle\int x(1-x)^{2/3}\,dx$

40. $\displaystyle\int \frac{x}{x-3}\,dx$

41. $\displaystyle\int_0^3 |x^2-4|\,dx$

42. $\displaystyle\int_0^4 |\sqrt{x}-1|\,dx$

43–44 Evaluate the indefinite integral. Illustrate and check that your answer is reasonable by graphing both the function and its antiderivative (take $C = 0$).

43. $\displaystyle\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$

44. $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

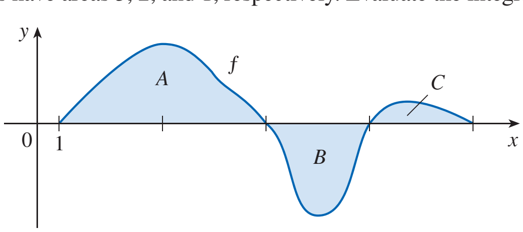
45. Use a graph to give a rough estimate of the area of the region that lies under the curve $y = x\sqrt{x}$, $0 \leqslant x \leqslant 4$. Then find the exact area.

46. Graph the function $f(x) = \cos^2 x \sin x$ and use the graph to guess the value of the integral $\int_0^{2\pi} f(x)\,dx$. Then evaluate the integral to confirm your guess.

47. Find the area under the graph of $y = x^2 + 5$ and above the x-axis, between $x = 0$ and $x = 4$.

48. Find the area under the graph of $y = \sin x$ and above the x-axis, between $x = 0$ and $x = \pi/2$.

49–50 The regions A, B, and C bounded by the graph of f and the x-axis have areas 3, 2, and 1, respectively. Evaluate the integral.



49. (a) $\displaystyle\int_1^5 f(x)\,dx$ (b) $\displaystyle\int_1^5 |f(x)|\,dx$

50. (a) $\displaystyle\int_1^4 f(x)\,dx + \int_3^5 f(x)\,dx$ (b) $\displaystyle\int_1^3 2f(x)\,dx + \int_3^5 6f(x)\,dx$

51–56 Find the derivative of the function.

51. $\displaystyle F(x) = \int_0^x \frac{t^2}{1+t^3}\,dt$

52. $\displaystyle F(x) = \int_x^1 \sqrt{t+\sin t}\,dt$

53. $\displaystyle g(x) = \int_0^{x^4} \cos(t^2)\,dt$

54. $\displaystyle g(x) = \int_1^{\sin x} \frac{1-t^2}{1+t^4}\,dt$

55. $\displaystyle y = \int_{\sqrt{x}}^x \frac{e^t}{t}\,dt$

56. $\displaystyle y = \int_{2x}^{3x+1} \sin(t^4)\,dt$

57–58 Use Property 8 of integrals to estimate the value of the integral.

57. $\displaystyle\int_1^3 \sqrt{x^2+3}\,dx$

58. $\displaystyle\int_2^4 \frac{1}{x^3+2}\,dx$

59–62 Use the properties of integrals to verify the inequality.

59. $\displaystyle\int_0^1 x^2 \cos x\,dx \leqslant \frac{1}{3}$

60. $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\sin x}{x}\,dx \leqslant \frac{\sqrt{2}}{2}$

61. $\displaystyle\int_0^1 e^x \cos x\,dx \leqslant e - 1$

62. $\displaystyle\int_0^1 x \sin^{-1} x\,dx \leqslant \pi/4$

63. Use the Midpoint Rule with $n = 6$ to approximate $\int_0^3 \sin(x^3)\,dx$. Round to four decimal places.

64. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 5]$.

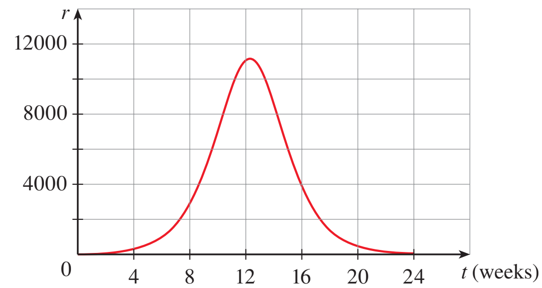
65. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000, and $r(t)$ is measured in barrels per year. What does $\int_{15}^{20} r(t)\,dt$ represent?

66. A radar gun was used to record the speed of a runner at the times given in the table. Use the Midpoint Rule to estimate the distance the runner covered during those 5 seconds.

t (s)	v (m/s)	t (s)	v (m/s)
0	0	3.0	10.51
0.5	4.67	3.5	10.67
1.0	7.34	4.0	10.76
1.5	8.86	4.5	10.81
2.0	9.73	5.0	10.81
2.5	10.22		

67. A population of honeybees increased at a rate of $r(t)$ bees per week, where the graph of r is as shown. Use the

Midpoint Rule with six subintervals to estimate the increase in the bee population during the first 24 weeks.



68. Let

$$f(x) = \begin{cases} -x - 1 & \text{if } -3 \leqslant x \leqslant 0 \\ -\sqrt{1 - x^2} & \text{if } 0 \leqslant x \leqslant 1 \end{cases}$$

Evaluate $\int_{-3}^{1} f(x)\,dx$ by interpreting the integral as a difference of areas.

69. If f is continuous and $\int_0^2 f(x)\,dx = 6$, evaluate

$$\int_0^{\pi/2} f(2\sin\theta)\cos\theta\,d\theta$$

70. The Fresnel function $S(x) = \int_0^x \sin\left(\frac{1}{2}\pi t^2\right) dt$ was introduced in Section 5.3. Fresnel also used the function

$$C(x) = \int_0^x \cos\left(\tfrac{1}{2}\pi t^2\right) dt$$

in his theory of the diffraction of light waves.

(a) On what intervals is C increasing?

(b) On what intervals is C concave upward?

(c) Use a graph to solve the following equation correct to two decimal places:

$$\int_0^x \cos\left(\tfrac{1}{2}\pi t^2\right) dt = 0.7$$

(d) Plot the graphs of C and S on the same screen. How are these graphs related?

71. Estimate the value of the number c such that the area under the curve $y = \sinh cx$ between $x = 0$ and $x = 1$ is equal to 1.

72. Suppose that the temperature in a long, thin rod placed along the x-axis is initially $C/(2a)$ if $|x| \leqslant a$ and 0 if $|x| > a$. It can be shown that if the heat diffusivity of the rod is k, then the temperature of the rod at the point x at time t is

$$T(x, t) = \frac{C}{a\sqrt{4\pi kt}} \int_0^a e^{-(x-u)^2/(4kt)}\,du$$

To find the temperature distribution that results from an initial hot spot concentrated at the origin, we need to compute $\lim_{a \to 0} T(x, t)$. Use l'Hospital's Rule to find this limit.

73. If f is a continuous function such that

$$\int_1^x f(t)\,dt = (x - 1)e^{2x} + \int_1^x e^{-t} f(t)\,dt$$

for all x, find an explicit formula for $f(x)$.

74. Suppose h is a function such that $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, $h''(2) = 13$, and h'' is continuous everywhere. Evaluate $\int_1^2 h''(u)\,du$.

75. If f' is continuous on $[a, b]$, show that

$$2\int_a^b f(x)f'(x)\,dx = [f(b)]^2 - [f(a)]^2$$

76. Find

$$\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + t^3}\,dt$$

77. If f is continuous on $[0, 1]$, prove that

$$\int_0^1 f(x)\,dx = \int_0^1 f(1 - x)\,dx$$

78. Evaluate

$$\lim_{n \to \infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right]$$