Facts You Should Know

I. Derivatives:

A. Power, exponential, natural log

1. $\frac{d}{dx}[x^n] = nx^{n-1}$ 2. $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$
3. $\frac{d}{dx}[e^x] = e^x$ 4. $\frac{d}{dx}[\log_a x] = \frac{d}{dx}\left[\frac{\ln x}{\ln a}\right] = \frac{1}{(\ln a)x}$

B. Trig Functions:

1. $\frac{d}{dx}[\sin x] = \cos x$ 2. $\frac{d}{dx}[\cos x] = -\sin x$

3. $\frac{d}{dx}[\tan x] = \sec^2 x$ 4. $\frac{d}{dx}[\cot x] = -\csc^2 x$

5. $\frac{d}{dx}[\sec x] = \sec x \tan x$ 6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

C. Inverse Trig. Functions:

1. $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$ 2. $\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$

3. $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$ 4. $\frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$

5. $\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}}$ 6. $\frac{d}{dx}[\csc^{-1} x] = -\frac{1}{x\sqrt{x^2-1}}$

D. Hyperbolic Trig. Functions:

1. $\frac{d}{dx}[\sinh x] = \cosh x$ 2. $\frac{d}{dx}[\cosh x] = \sinh x$

3. $\frac{d}{dx}[\tanh x] = \text{sech}^2 x$ 4. $\frac{d}{dx}[\coth\ x] = -\text{csch}\ x$

5. $\frac{d}{dx}[\text{sech}\ x] = -\text{sech}\ x\ \tanh x$ 6. $\frac{d}{dx}[\text{csch}\ x] = -\text{csch}\ x\ \text{coth}\ x$

II. **Antiderivatives:**

A. Power, exponential, natural log

1. $\int x^n \, dx = \frac{1}{n+1} x^{n-1} + c$ 2 $\int \frac{1}{x} \, dx = \ln x + c$ 3. $\int e^x \, dx = e^x + c$

B. Trig Functions:

1. $\int \sin x \, dx = -\cos x + c$
2. $\int \cos x \, dx = \sin x + c$
3. $\int \tan x \, dx = \ln|\sec x| + c$
4. $\int \cot x \, dx = \ln|\sin x| + c$
5. $\int \sec x \, dx = \ln|\sec x + \tan x| + c$
6. $\int \csc x \, dx = \ln|-\csc x + \cot x| + c$
7. $\int \sec^2 x \, dx = \tan x + c$
8. $\int \csc^2 x \, dx = -\cot x + c$
9. $\int \sec x \tan x \, dx = \sec x + c$
10. $\int \csc x \cot x \, dx = -\csc x + c$

C. Inverse Trig. Functions:

1. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$

3. $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

5. $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c$

D. Hyperbolic Trig. Functions:

1. $\int \sinh x \, dx = \cosh x + c$
2. $\int \cosh x \, dx = \sinh x + c$
3. $\int \tanh x \, dx = \ln|\cosh x| + c$
4. $\int \cot x \, dx = \ln|\sinh x| + c$
5. $\int \mathrm{sech}^2 x \, dx = \tanh x + c$
6. $\int \mathrm{csch}^2 x \, dx = -\coth x + c$
7. $\int \mathrm{sech} x \tanh x \, dx = -\mathrm{sech} x + c$
8. $\int \mathrm{csch} x \, \coth x \, dx = -\mathrm{csch} \, x + c$

III. **Identities You should know:**

A. exponential, log

1. $a^x = e^{\ln a^x} = e^{x \ln z}$ 2. $\log_a x = \dfrac{\ln x}{\ln a}$

B. Trig Functions:

1. Right Triangle Identities & Reciprocal Identities

2. Pythagorean identites:
$\sin^2 x + \cos^2 x = 1$, $1 + \cot^2 x = \csc^2 x$, $\tan^2 x + 1 = \sec^2 x$

3. $\frac{1}{2}$−angle identities:
$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$, $cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$

4. Double-angle identities:
$2\sin x \cos x = \sin(2x)$, $\cos^2 x - \sin^2 x = \cos(2x)$,
$2\cos^2 x - 1 = \cos(2x)$, $1 - 2\sin^2 x = \cos(2x)$

5. Sum-of-angles identites:
$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$, $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

6. Product identities:
$\sin x \cos y = \dfrac{1}{2}(\sin(x-y) + \sin(x+y))$, $\sin x \sin y = \dfrac{1}{2}(\cos(x-y) - \cos(x+y))$,
$\cos x \cos y = \dfrac{1}{2}(\cos(x-y) + \cos(x+y))$

C. Hyperobic Trig Functions:

1. Definitions:
$\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{\sinh x}{\cosh x}$
Note that $\sinh(0) = 0$, $\cosh(0) = 1$

2. Reciprocal identities: e.g. $\operatorname{sech} x = \dfrac{1}{\cosh x}$

3. Identities:
$\cosh^2 - \sinh^2 = 1$, $\tanh^2 x + \operatorname{sech}^2 x = 1$, $\coth^2 x - \cosh^2 = 1$,
$\sinh(-x) = \sinh(x)$, $\cosh(-x) = \cosh(x)$, $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$,
$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

IV. **The Chain Rule and its ramifications to integration:**

A. The Chain Rule:

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

1. $\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x \ln a}) = \ln a e^{x \ln a} = \ln a(a^x)$
2. $[\sec(\ln x)]' = \sec(\ln x)\tan(\ln x)\frac{1}{x}$

B. u-substitution:

$\int f'(g(x))g'(x)\,dx = \int f'(u)\,du = f(u) + c = f(g(x)) + c$,
where $u = g(x)$ and $du = g'(x)\,dx$.

1. $\displaystyle\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = \int -\frac{1}{u}\,du = -\ln|u| + c$
$= -\ln|\cos x| + c = \ln|\sec x| + c$, where $u = \cos x$ and $du = -\sin x\,dx$.

2. $\displaystyle\int \sec x\,dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}\,dx = \int \frac{1}{u}\,du = \ln|u| + c$
$= \ln|\sec x + \tan x| + c$, where $u = \sec x + \tan x$ and $du = \sec^2 x + \sec x \tan x$.

3. $\displaystyle\int \frac{4}{2x+3}\,dx = \int \frac{4}{u}\cdot\frac{1}{2}\,du = \int \frac{2}{u}\,du = 2\ln|u| + c = 2\ln|2x+3| + c,$
where $u = 2x + 3$ and $du = 2dx$.

4. $\displaystyle\int \frac{2}{(2x+3)^5}\,dx = \int \frac{4}{u^5}\cdot\frac{1}{2}du = -\frac{2}{4u^4} + c = -\frac{1}{2(2x+3)^4} + c,$
where $u = 2x + 3$ and $du = 2\,dx$.

5. $\displaystyle\int \frac{2}{2x^2+4x+3}\,dx = \int \frac{2}{2(x+1)^2+1}\,dx = \int \frac{2}{(\sqrt{2}(x+1))^2+1}\,dx$
$\displaystyle\int \frac{2}{u^2+1}\cdot\frac{1}{\sqrt{2}}\,du = \sqrt{2}\tan^{-1}(u) + c = \sqrt{2}\tan^{-1}(\sqrt{2}(x+1)) + c$
where $u = \sqrt{2}(x+1)$ and $du = \sqrt{2}\,dx$.

6. $\displaystyle\int \frac{2x+1}{2x^2+4x+3}\,dx = \int \frac{2x+1}{(\sqrt{2}(x+1))^2+1}\,dx,$
with $u = \sqrt{2}(x+1)$, $du = \sqrt{2}\,dx$, and so $x = \frac{1}{\sqrt{2}}u - 1$. Substituting
$\displaystyle\int \frac{2(\frac{1}{\sqrt{2}}u - 1) + 1}{u^2+1}\cdot\frac{1}{\sqrt{2}}\,du = \int \frac{u - \frac{1}{\sqrt{2}}}{u^2+1}\,du = \int \frac{u}{u^2+1}\,du - \frac{1}{\sqrt{2}}\int \frac{1}{u^1+1}\,du$
$\displaystyle\frac{1}{2}\ln u + -\frac{1}{\sqrt{2}}\tan^{-1} u + c = \frac{1}{2}\ln|\sqrt{2}(x+1)| - \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}(x+1)) + c.$