COMP3005: Computer Vision

Consistent matching

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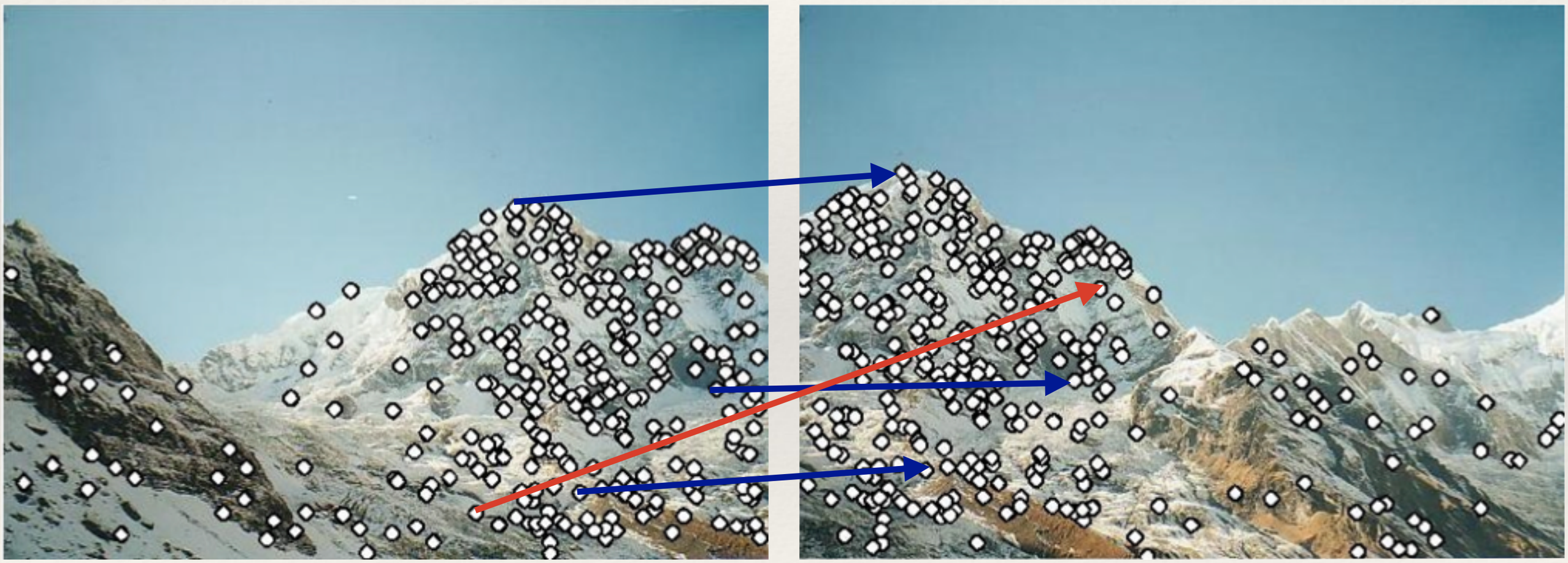
Recap: local interest points, features and matching

- In the previous lectures we've looked at how we can:
  - Locate stable interest points in multiple images
  - Extract highly robust and discriminative local descriptors around each interest point
  - Match interest points across images by comparing their local features

How can we eliminate mismatches?



Feature distinctiveness

- Even the most advanced local feature can be prone to being mismatched.
- There is always a tradeoff in feature distinctiveness.
- If it's too distinctive will not match subtle variations due to noise of imaging conditions.
- If it's not distinctive enough it will match everything.

Constrained matching

- Assume we are given a number of correspondences between the interest points in a pair of images
- Is it possible to estimate which of those correspondences are **inliers** (i.e. correct) or **outliers** (i.e. incorrect/mismatches)?
- What **assumptions** do we have to make?

By assuming a geometric mapping between the two scenes can we recover that mapping and eliminate the mismatches?

Geometric Mappings

What are geometric transforms?

Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

Point Transforms

We're interested in transforms that take the following form:

$$\mathbf{x}' = \mathbf{T}\mathbf{x}$$

Point Transforms

The transform matrix

$$x' = \mathbf{T}x$$

the transformed coordinate

the original coordinate

The Affine Transform

The affine transform is defined as

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{b}$$

or

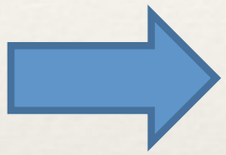
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11}\, a_{12} \\ a_{22}\, a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

The Affine Transform

It's more convenient to write this as a single transform matrix by adding an extra dimension to each vector:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation



$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

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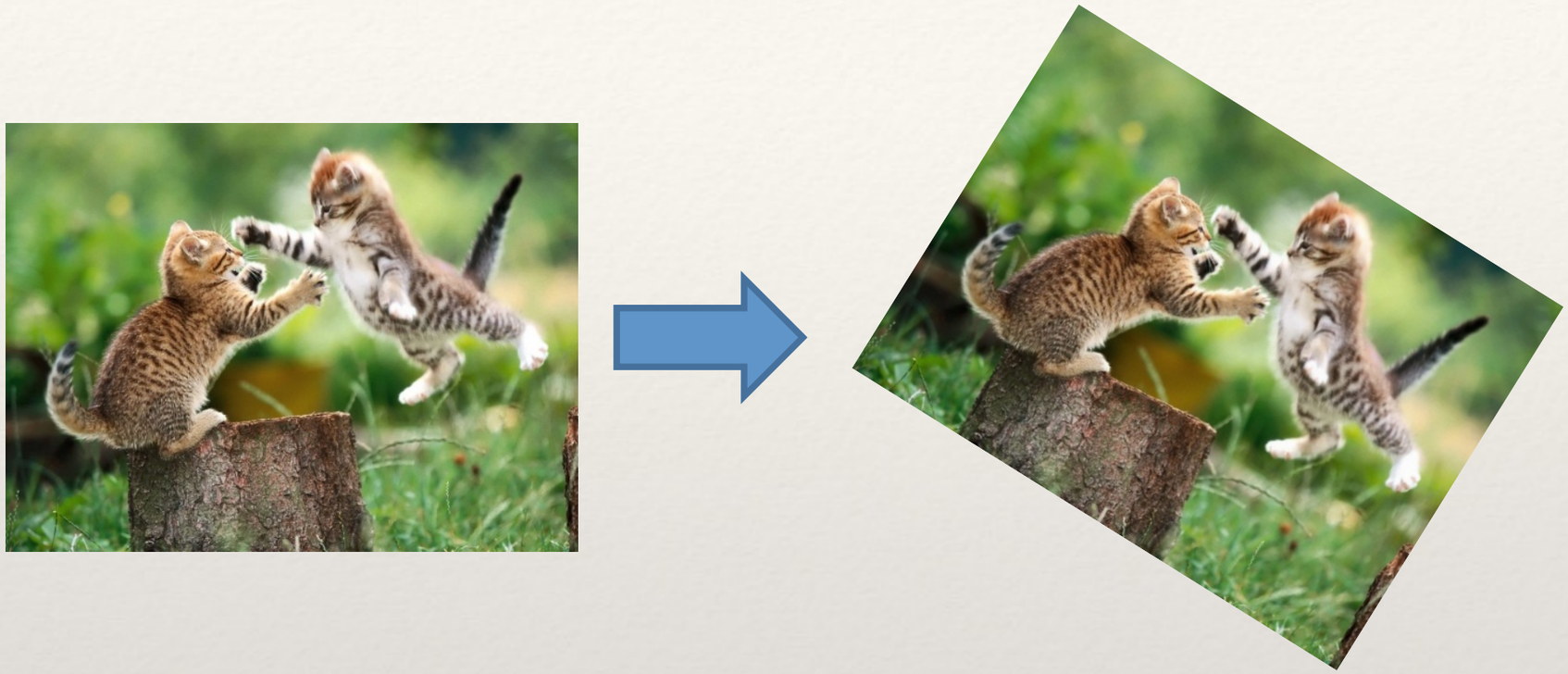
Translation and Rotation



$$\begin{bmatrix} cos(\theta) & -sin(\theta) & t_x \\ sin(\theta) & cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

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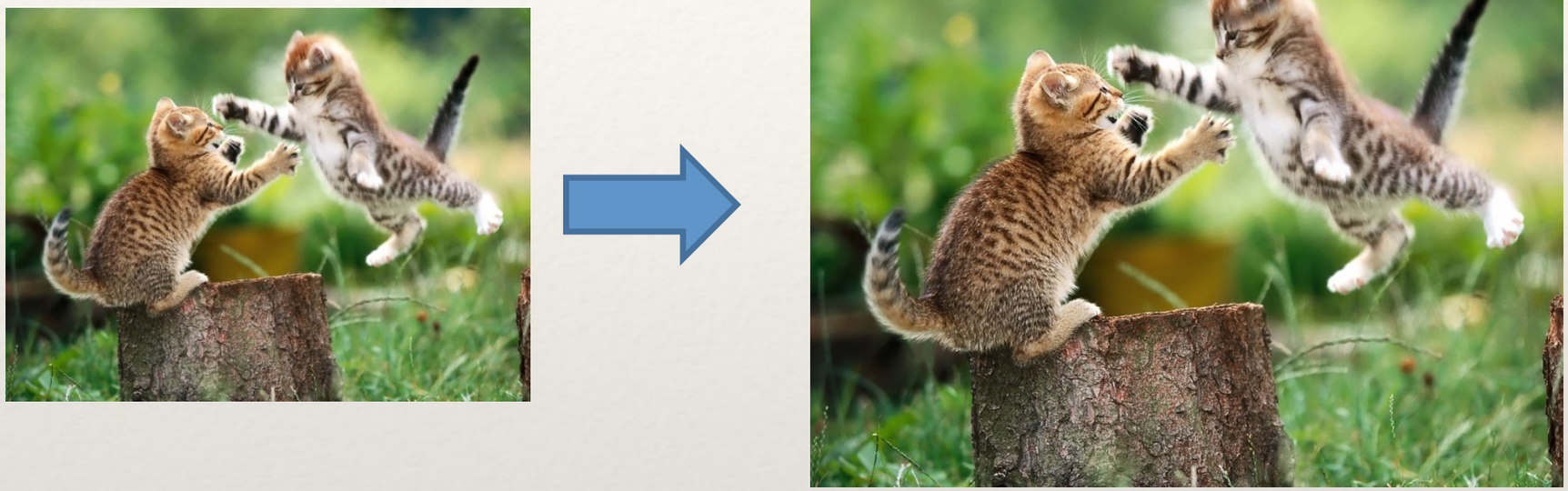
Scaling



$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

Aspect Ratio

$$\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

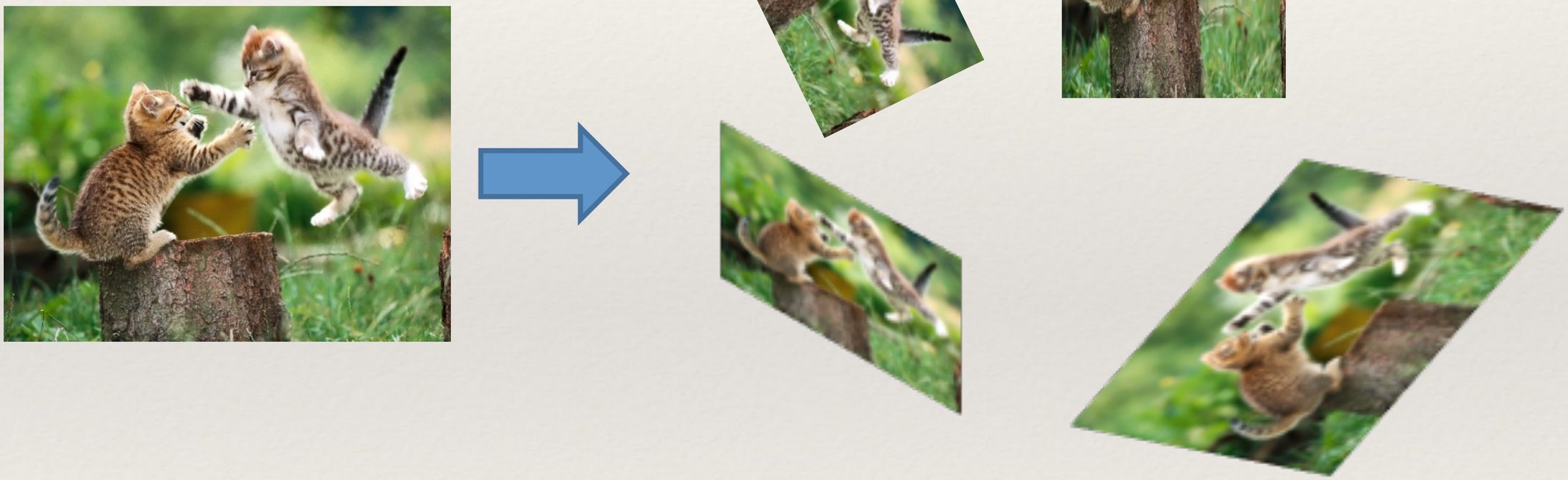
Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

Shear

$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

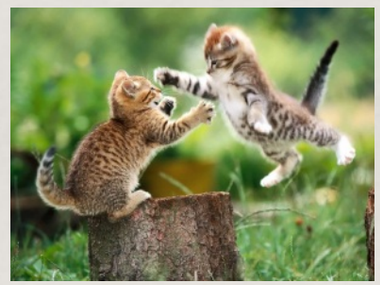
Affine Transform



6 DoF: translation + rotation + scale + aspect ratio + shear

Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

Similarity Transform



4 DoF: translation+rotation+scale

Images from Larry Zitnick's slides: http://courses.cs.washington.edu/courses/csep576/11sp/pdf/Transformations.pdf

What's missing?

More degrees of freedom

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Normalise by w so that the transformed vector is [• , • ,1]

$$x' = u/w$$

$$y' = v/w$$

Homogeneous coordinates

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}$$

The Planar Homography (Projective Transformation)

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$x' = u/w$$

$$y' = v/w$$

Keystone distortions

Recovering a geometric mapping

Simultaneous equations

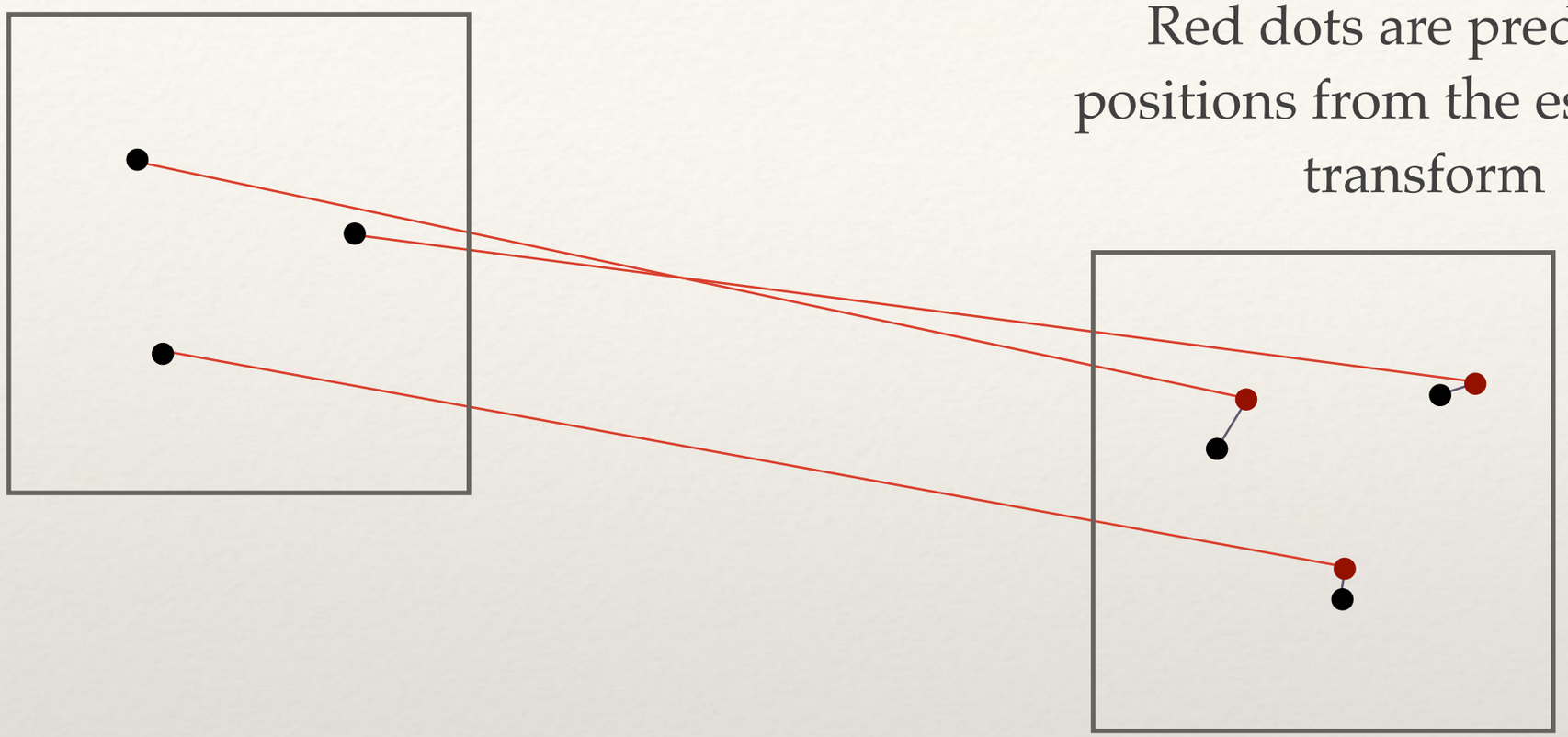
- It is possible to estimate a transform matrix from a set of point matches by solving a set of simultaneous equations
  - Need at least 4 point matches to solve a Homography or 3 to solve an affine transform
- The actual solution technique isn't important…
  - It is important to note that in the presence of noise, and with potentially more matches than required, that we have to solve an *overdetermined system*
  - We need to seek the minimum error or **least-squares** solution

Least-squares

line of best fit (minimises SSE)

residual (difference between observed and predicted)

$$SSE=\Sigma(residual)^2$$



Red dots are predicted positions from the estimated transform

residuals are Euclidean distance between observed and predicted positions

$$SSE=\Sigma(residual)^2$$

Robust Estimation

Problem: Noisy data

- Need a way to deal with estimating a model (i.e. a transform matrix) in the presence of high amounts of noise (i.e. mis-matches)
- Least-squares will be highly suboptimal, and probably find a very bad solution.
- Ideally, we want to identify the correct data (the inliers) and the bad data (the outliers)
  - Then estimate the model using only the good data.

Robust estimation techniques

- The problem of learning a model in the presence of inliers and outliers comes under an area of mathematics called **robust estimation** or **robust model fitting**
- There are a number of different possible techniques
  - …lets look at one of the simplest…

RANSAC: RAndom SAmple Consensus

Assume:

M data items required to estimate model T

N data items in total

Algorithm:

1.) select M data items at random

2.) estimate model T

3.) find how many of the N data items fit T within tolerance *tol*, call this K (i.e. compute how many times the absolute residual is less than *tol*). The points that have an absolute residual less than *tol* are the inliers; the other points are the outliers.

4.) if K is large enough, either accept T, or compute the least-squares estimate using all inliers, and exit with success.

5.) repeat steps 1..4 nIterations times

6.) fail - no good T fit of data

Demo: RANSAC Line Estimation

Further applications of robust local matching

Object recognition & AR

- Object recognition
  - Image of object is matched against scene, and recognised if there is a consistent match
- Augmented reality
  - Data can be added to a scene on the basis of a match

Demo: recognition/tracking/ar

3D reconstruction

- It's possible to estimate depth, and ultimately build a complete 3d scene from sets of point correspondences formed from matching local features

Demo: 3D reconstruction

Matching music/sounds

- Image features can be used to match music!
  - That's how systems like Shazam work
- A recording of a piece of music can be turned into a picture called a spectrogram.
- Local image features can be extracted, described and matched from the spectrogram images

Demo: Spectrogram

Problems with direct local feature matching

Local feature matching is slow!

- Typical image (800x600) might have ~2000 DoG Interest points/SIFT descriptors
  - Each SIFT descriptor is 128 dimensions
- Now assume you want to match a query image against a database of images…
  - The distance between every query feature and every other features needs to be calculated
  - Can this be optimised somehow?

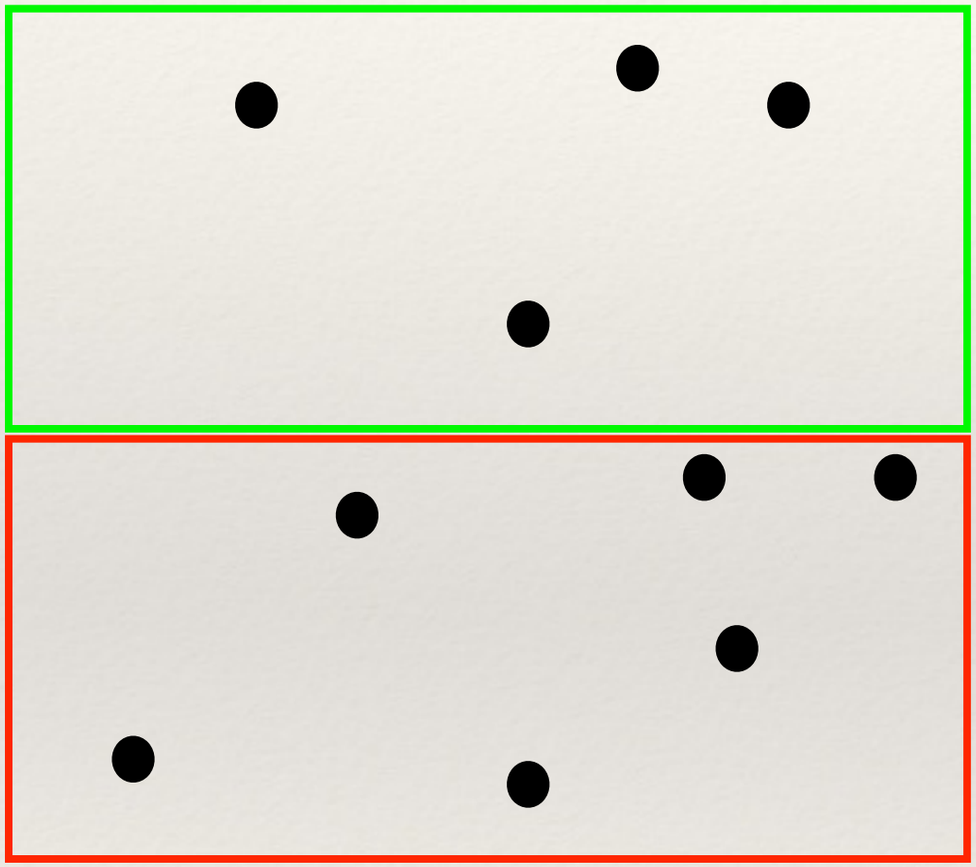
Efficient Nearest Neighbour Search

- How can we **quickly** find the nearest neighbour to a query point in a high dimensional space?
- Index the points in some kind of tree structure?
- Hash the points?
- Quantise the space (*more on this next time*)

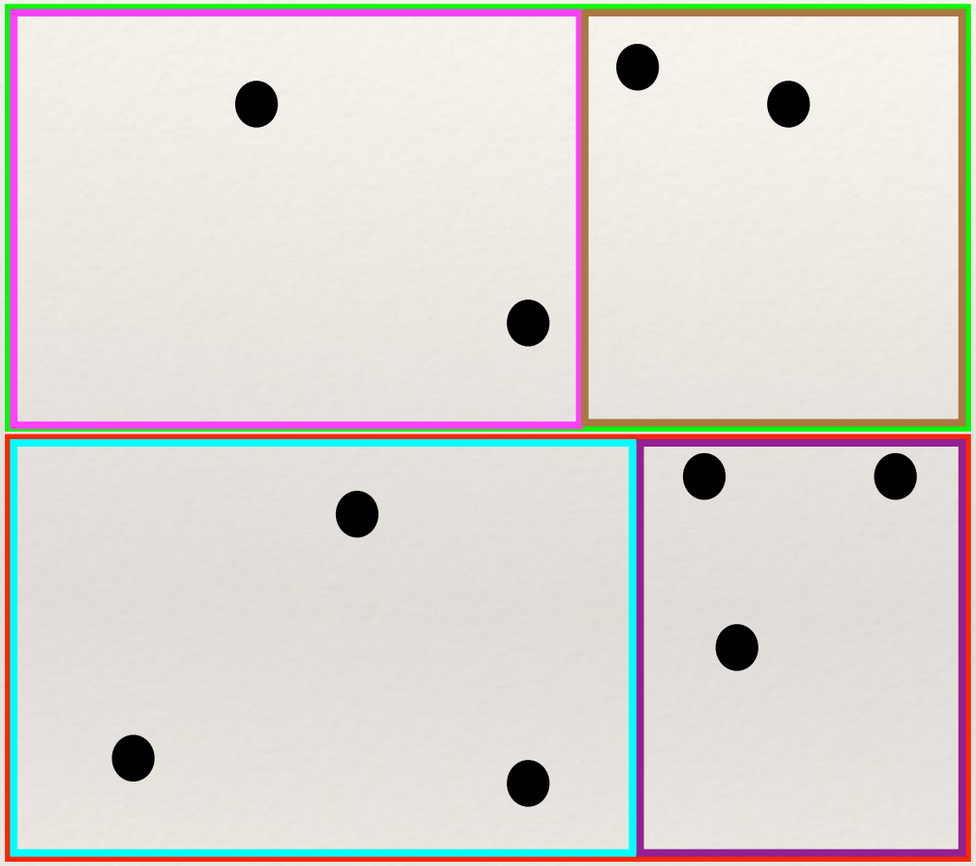
K-D Trees

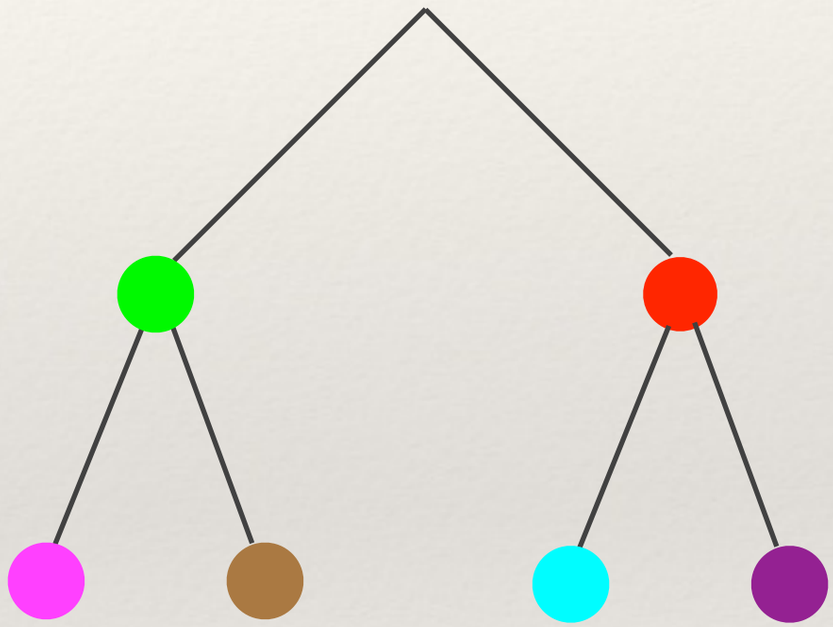
- Binary tree structure that partitions the space along axis-aligned hyperplanes
- Typically take each dimension in turn and splits on the median of the points in the enclosing partition.
- Stop after a certain depth, or when the number of points in a leaf is less than a threshold

K-D Tree



K-D Tree

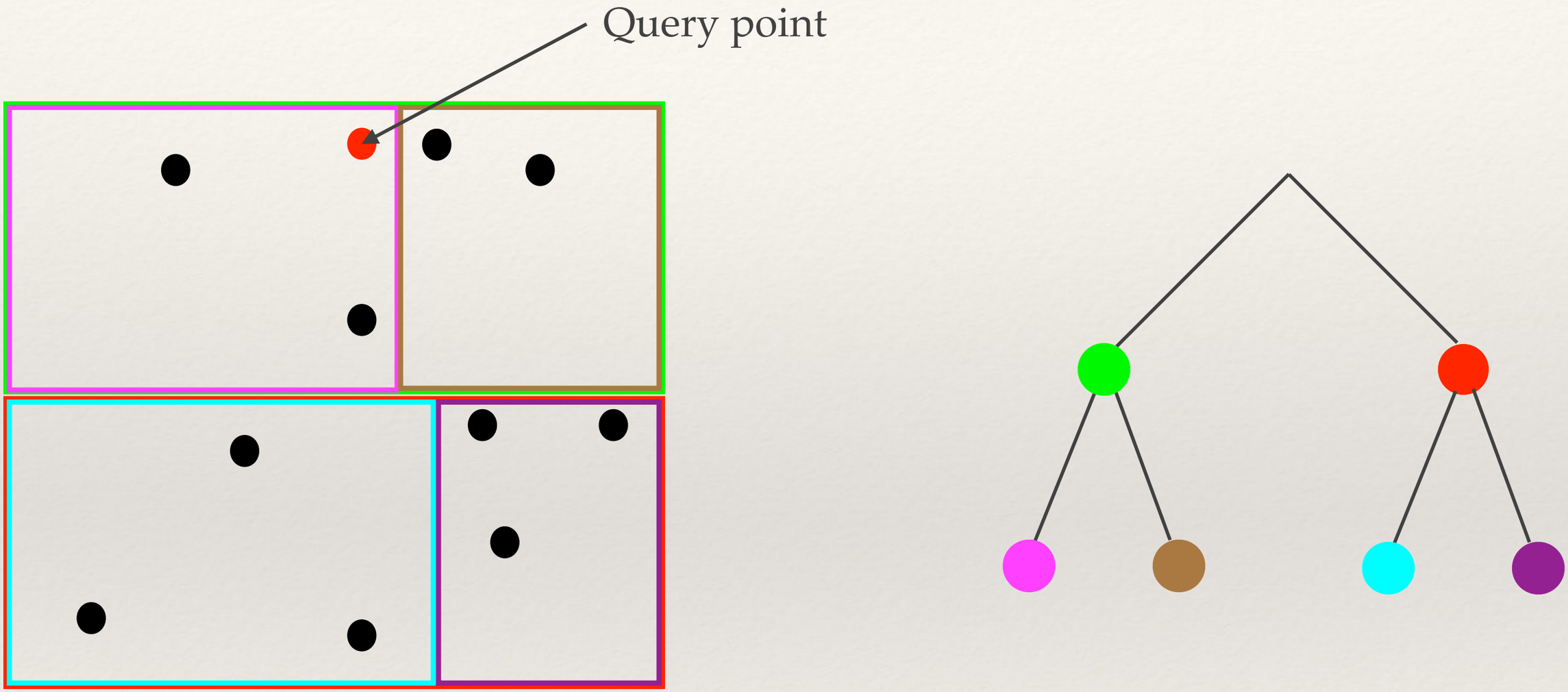




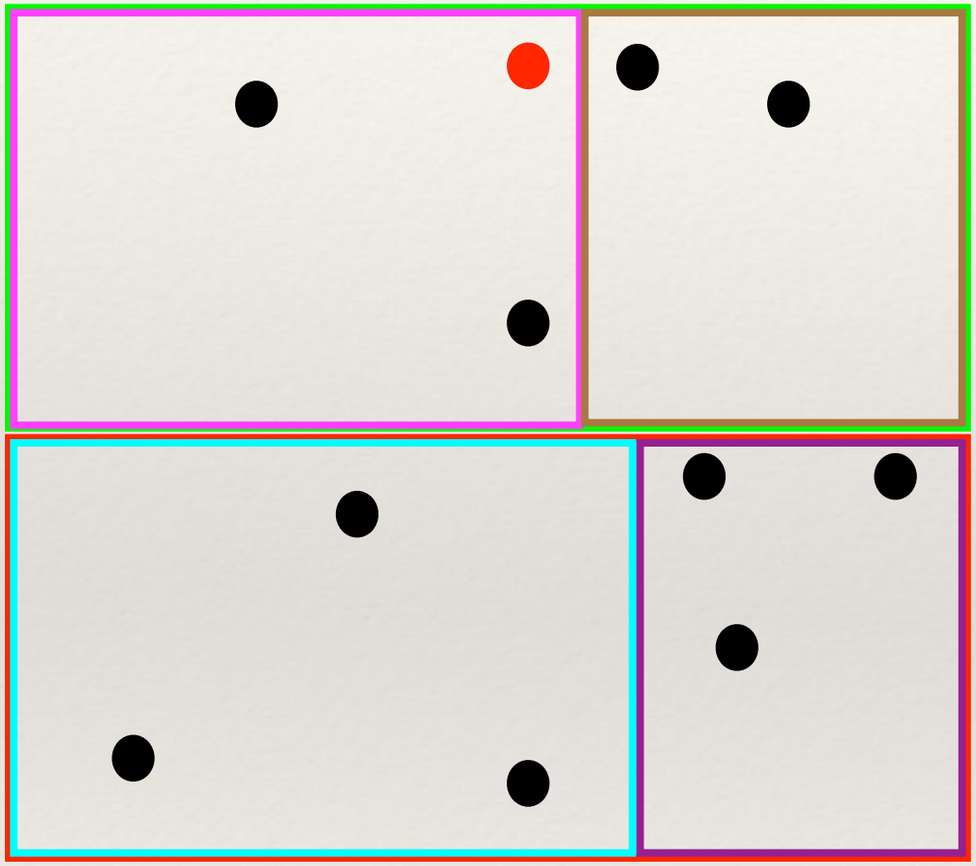
K-D Trees

- Search by walking down the tree until a leaf is hit, and then brute-force search to find the best in the leaf.
  - This is not guaranteed to be the best though…
  - To have to walk back up the tree and see if there are any better matches, and only stop once the root is reached (note you don't have to check a subtree if it's clear that all points in that subtree are further than the current best).

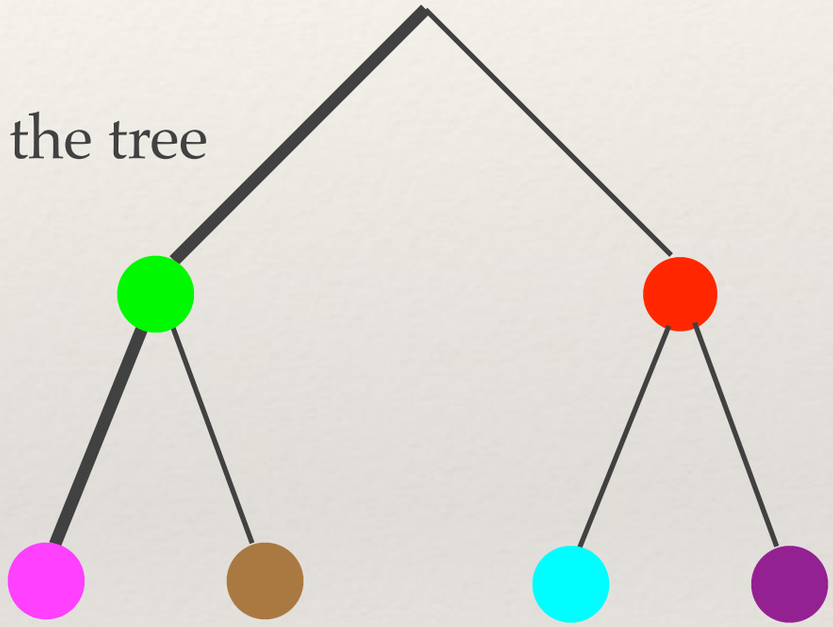
K-D Tree



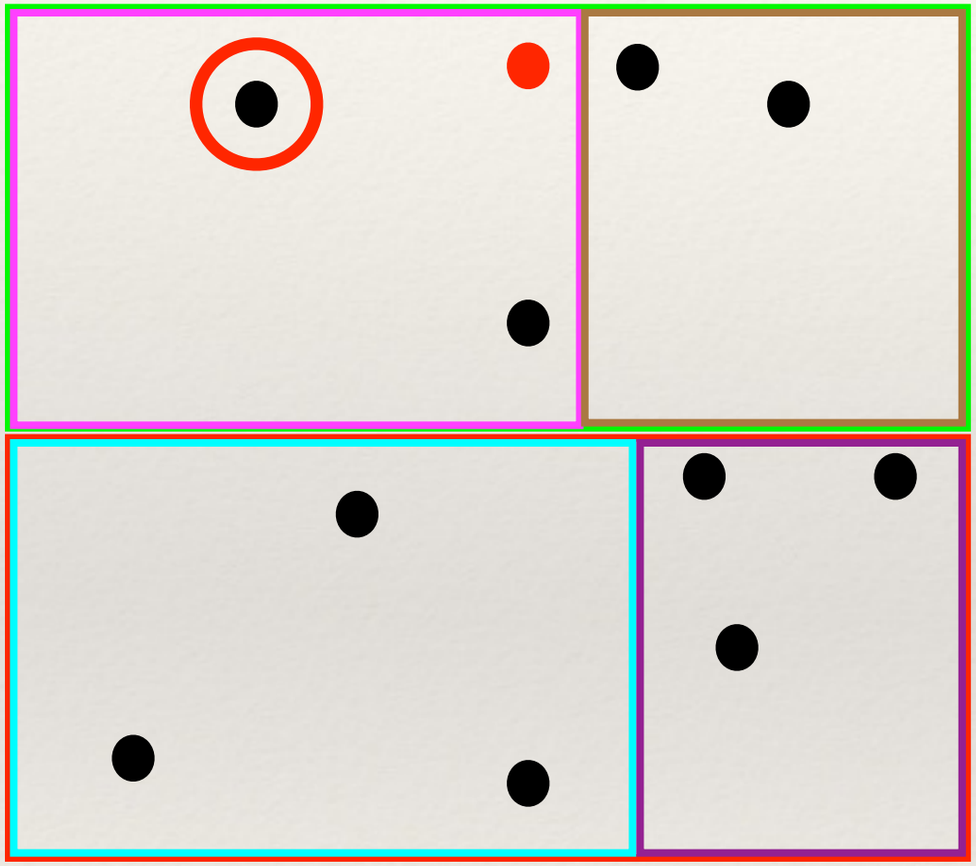
K-D Tree



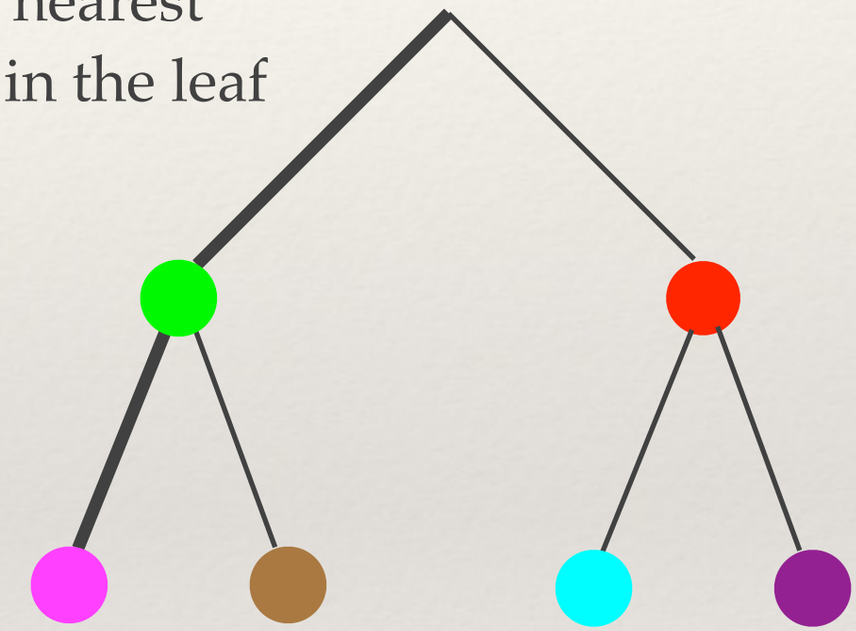
Walk down the tree



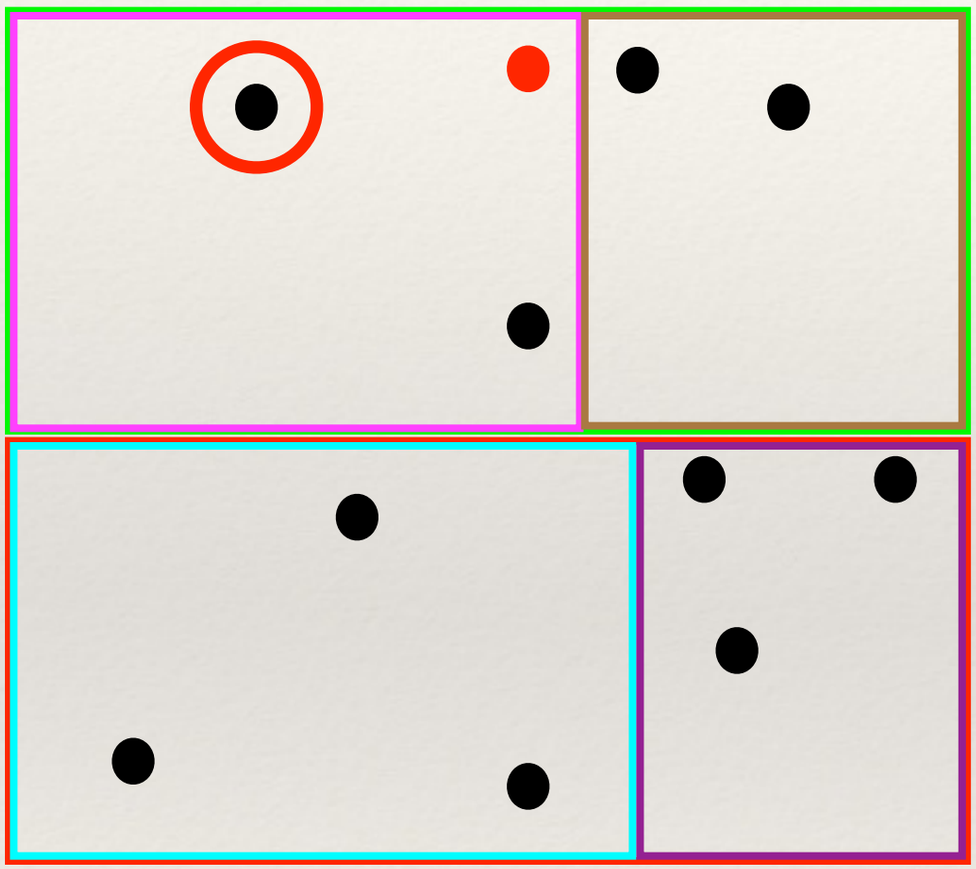
K-D Tree



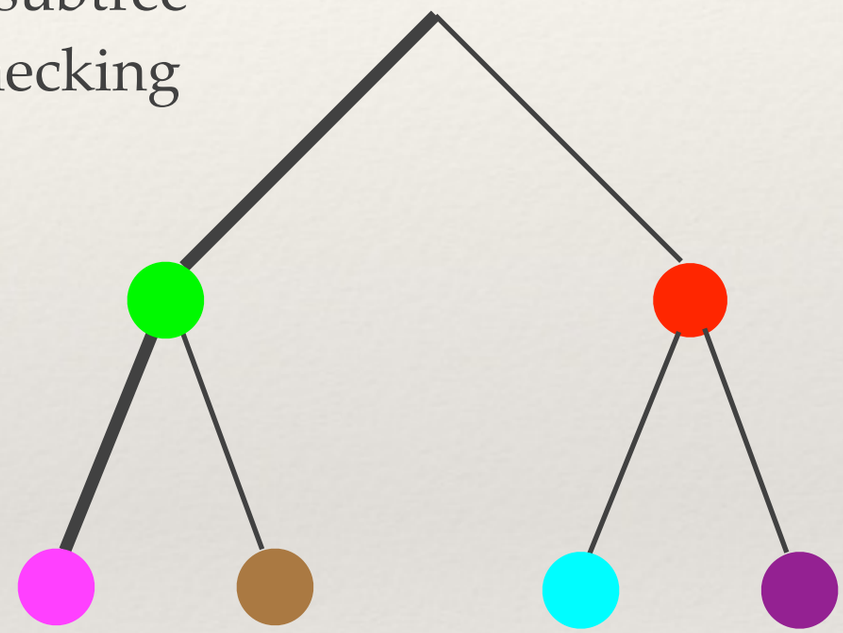
Find the nearest neighbour in the leaf



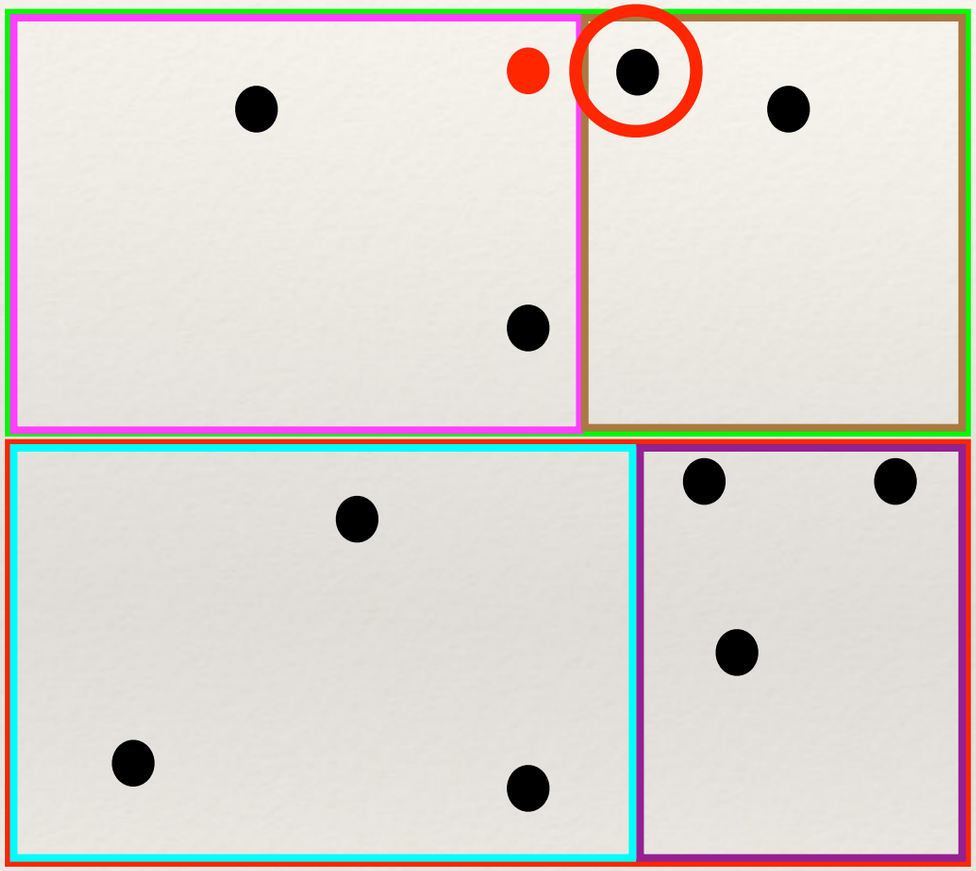
K-D Tree



Backtrack, and see if
the next subtree
needs checking



K-D Tree



Backtrack, and see if
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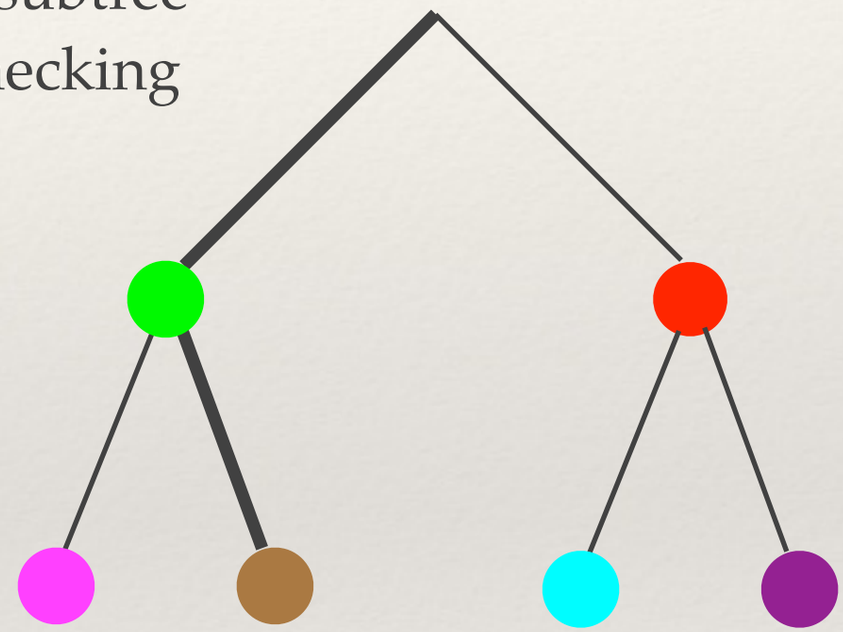
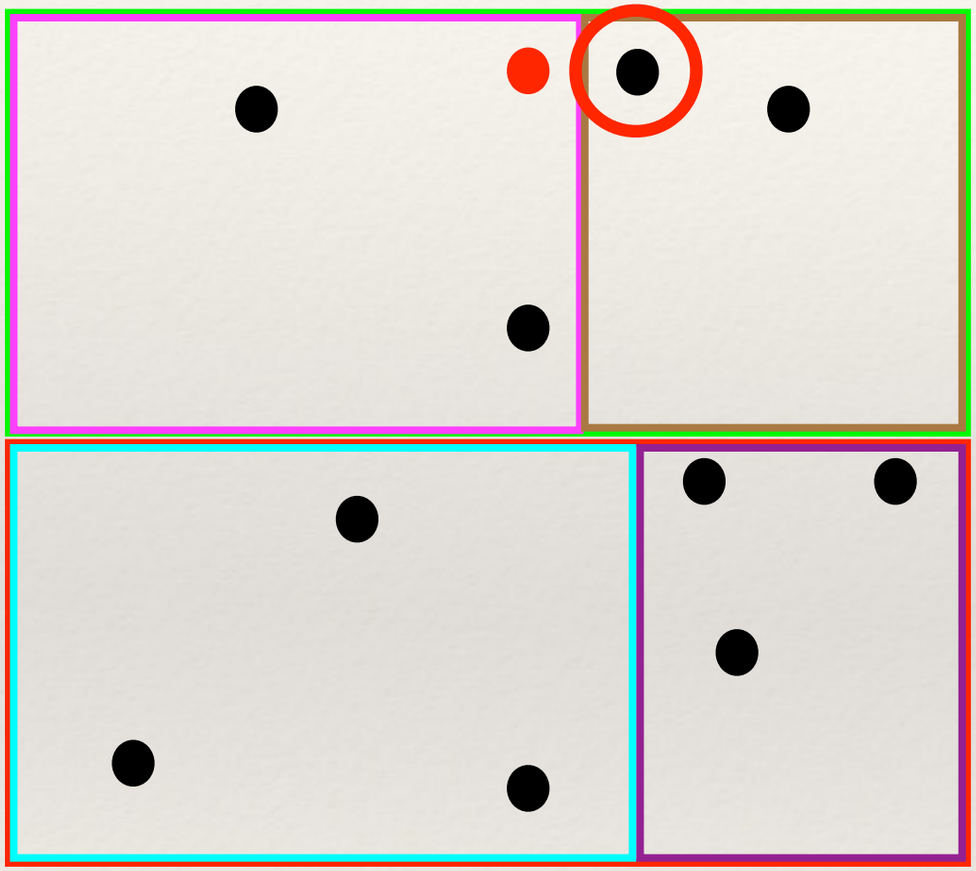
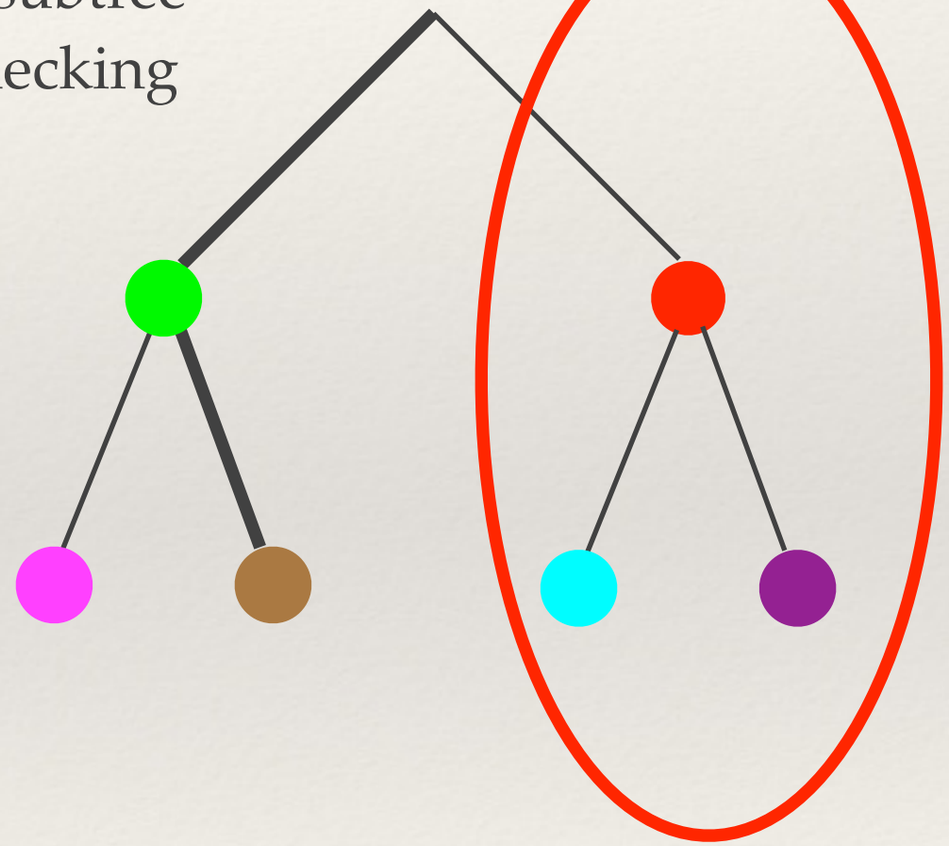


K-D Tree



Backtrack, and see if the next subtree needs checking

No need to check this subtree



K-D Tree problems

- Doesn't scale well to high dimensions
  - You tend to end up needing to search most of the tree
- There are *approximate* versions that won't necessarily return the exact answer that do scale (if you don't mind the potential for mismatch)

Hashing

- Locality Sensitive Hashing (LSH) creates hash codes for vectors such that similar vectors have similar hash codes!

Sketching

- A technique called sketching concatenates binary hashes into a bit string.
  - With the correct LSH function, the Hamming distance between a pair of sketches is proportional to the Euclidean distance between the original vectors
- Can easily compress SIFT features to 128 bits
  - Hamming distance computation is cheap
    - Lookup tables and bitwise operations

Summary

- Inconsistent local feature matches can be removed by assuming some form of constraint holds between the two images
  - This is usually a geometric mapping
    - Affine transform or Homography
      - Can be estimated by finding the least-squares solution of a set of simultaneous equations
  - Robust methods such as RANSAC allow inliers and outliers to be determined whilst learning the mapping
- Interest point matching is slow…
  - K-D Trees and Hashing can help