```python
import time

import numpy
import matplotlib.pyplot as plt
import numpy as np
import scipy.special as sp_spec
import scipy.stats as sp_stats
```

Assignment 1A. Problem 1.4.19 SVI.

Generate data

The cell below generates data for the LDA model. Note, for simplicity, we are using N_d = N for all d.

```python
def generate_data(D, N, K, W, eta, alpha):
    # sample K topics
    beta = sp_stats.dirichlet(eta).rvs(size=K)  # size K x W

    theta = np.zeros((D, K))   # size D x K

    w = np.zeros((D, N, W))
    z = np.zeros((D, N), dtype=int)
    for d in range(D):
        # sample document topic distribution
        theta_d = sp_stats.dirichlet(alpha).rvs(size=1)
        theta[d] = theta_d
        for n in range(N):
            # sample word to topic assignment
            z_nd = sp_stats.multinomial(n=1, p=theta[d, :]).rvs(size=1).argmax(axis=1)[0]

            # sample word
            w_nd = sp_stats.multinomial(n=1, p=beta[z_nd, :]).rvs(1)

            z[d, n] = z_nd
            w[d, n] = w_nd

    return w, z, theta, beta

D_sim = 500
N_sim = 50
K_sim = 2
W_sim = 5

eta_sim = np.ones(W_sim)
eta_sim[3] = 0.0001  # Expect word 3 to not appear in data
eta_sim[1] = 3.  # Expect word 1 to be most common in data
alpha_sim = np.ones(K_sim) * 1.0
w0, z0, theta0, beta0 = generate_data(D_sim, N_sim, K_sim, W_sim, eta_sim, alpha_sim)
w_cat = w0.argmax(axis=-1)  # remove one hot encoding
unique_z, counts_z = numpy.unique(z0[0, :], return_counts=True)
unique_w, counts_w = numpy.unique(w_cat[0, :], return_counts=True)

# Sanity checks for data generation
print(f"Average z of each document should be close to theta of document. \n Theta of doc 0: {theta0[0]} \n Mean z of doc 0: {counts_z/N_sim}")
print(f"Beta of topic 0: {beta0[0]}")
print(f"Beta of topic 1: {beta0[1]}")
print(f"Word to topic assignment, z, of document 0: {z0[0, 0:10]}")
print(f"Observed words, w, of document 0: {w_cat[0, 0:10]}")
print(f"Unique words and count of document 0: {[f'{u}: {c}' for u, c in zip(unique_w, counts_w)]}")
```

```
Average z of each document should be close to theta of document.
 Theta of doc 0: [0.54132269 0.45867731]
 Mean z of doc 0: [0.5 0.5]
Beta of topic 0: [0.11430351 0.69431184 0.08117848 0.         0.11020617]
Beta of topic 1: [0.10644659 0.37798951 0.47896521 0.         0.03659869]
Word to topic assignment, z, of document 0: [1 1 0 1 0 1 0 1 0 1]
Observed words, w, of document 0: [1 2 4 1 1 1 1 1 1 2]
Unique words and count of document 0: ['0: 9', '1: 27', '2: 11', '4: 3']
```

```python
import torch
import torch.distributions as t_dist

def generate_data_torch(D, N, K, W, eta, alpha):
    """
    Torch implementation for generating data using the LDA model. Needed for sampling larger datasets.
    """
    # sample K topics
    beta_dist = t_dist.Dirichlet(torch.from_numpy(eta))
    beta = beta_dist.sample([K])  # size K x W

    # sample document topic distribution
    theta_dist = t_dist.Dirichlet(torch.from_numpy(alpha))
    theta = theta_dist.sample([D])

    # sample word to topic assignment
    z_dist = t_dist.OneHotCategorical(probs=theta)
    z = z_dist.sample([N]).reshape(D, N, K)

    # sample word from selected topics
    beta_select = torch.einsum("kw, dnk -> dnw", beta, z)
    w_dist = t_dist.OneHotCategorical(probs=beta_select)
    w = w_dist.sample([1])

    w = w.reshape(D, N, W)

    return w.numpy(), z.numpy(), theta.numpy(), beta.numpy()
```

Helper functions

```python
def log_multivariate_beta_function(a, axis=None):
    return np.sum(sp_spec.gammaln(a)) - sp_spec.gammaln(np.sum(a, axis=axis))
```

CAVI Implementation, ELBO and initialization

```python
def initialize_q(w, D, N, K, W):
    """
    Random initialization.
    """
    phi_init = np.random.random(size=(D, N, K))
    phi_init = phi_init / np.sum(phi_init, axis=-1, keepdims=True)
    gamma_init = np.random.randint(1, 10, size=(D, K))
    lmbda_init = np.random.randint(1, 10, size=(K, W))
    return phi_init, gamma_init, lmbda_init

def update_q_Z(w, gamma, lmbda):
    D, N, W = w.shape
    K, W = lmbda.shape
    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma, axis=1, keepdims=True))  # size D x K
    E_log_beta = sp_spec.digamma(lmbda) - sp_spec.digamma(np.sum(lmbda, axis=1, keepdims=True))   # size K x W
    log_rho = np.zeros((D, N, K))
    w_label = w.argmax(axis=-1)
    for d in range(D):
        for n in range(N):
            E_log_beta_wdn = E_log_beta[:, int(w_label[d, n])]
            E_log_theta_d = E_log_theta[d]
            log_rho_n = E_log_theta_d + E_log_beta_wdn
            log_rho[d, n, :] = log_rho_n

    phi = np.exp(log_rho - sp_spec.logsumexp(log_rho, axis=-1, keepdims=True))
    return phi

def update_q_theta(phi, alpha):
    E_Z = phi
    D, N, K = phi.shape
    gamma = np.zeros((D, K))
    for d in range(D):
        E_Z_d = E_Z[d]
        gamma[d] = alpha + np.sum(E_Z_d, axis=0)  # sum over N
    return gamma

def update_q_beta(w, phi, eta):
    E_Z = phi
    D, N, W = w.shape
    K = phi.shape[-1]
    lmbda = np.zeros((K, W))
    for k in range(K):
        lmbda[k, :] = eta
        for d in range(D):
            for n in range(N):
                lmbda[k, :] += E_Z[d,n,k] * w[d,n]  # Sum over d and n
    return lmbda

def calculate_elbo(w, phi, gamma, lmbda, eta, alpha):
    D, N, K = phi.shape
    W = eta.shape[0]
    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma, axis=1, keepdims=True))  # size D x K
    E_log_beta = sp_spec.digamma(lmbda) - sp_spec.digamma(np.sum(lmbda, axis=1, keepdims=True))  # size K x W
    E_Z = phi  # size D, N, K
    log_Beta_alpha = log_multivariate_beta_function(alpha)
    log_Beta_eta = log_multivariate_beta_function(eta)
    log_Beta_gamma = np.array([log_multivariate_beta_function(gamma[d, :]) for d in range(D)])
    dg_gamma = sp_spec.digamma(gamma)
    log_Beta_lmbda = np.array([log_multivariate_beta_function(lmbda[k, :]) for k in range(K)])
    dg_lmbda = sp_spec.digamma(lmbda)

    neg_CE_likelihood = np.einsum("dnk, kw, dnw", E_Z, E_log_beta, w)
    neg_CE_Z = np.einsum("dnk, dk -> ", E_Z, E_log_theta)
    neg_CE_theta = -D * log_Beta_alpha + np.einsum("k, dk ->", alpha - 1, E_log_theta)
    neg_CE_beta = -K * log_Beta_eta + np.einsum("w, kw ->", eta - 1, E_log_beta)
    H_Z = -np.einsum("dnk, dnk ->", E_Z, np.log(E_Z))
    gamma_0 = np.sum(gamma, axis=1)
    dg_gamma0 = sp_spec.digamma(gamma_0)
    H_theta = np.sum(log_Beta_gamma + (gamma_0 - K) * dg_gamma0 - np.einsum("dk, dk -> d", gamma - 1, dg_gamma))
    lmbda_0 = np.sum(lmbda, axis=1)
    dg_lmbda0 = sp_spec.digamma(lmbda_0)
    H_beta = np.sum(log_Beta_lmbda + (lmbda_0 - W) * dg_lmbda0 - np.einsum("kw, kw -> k", lmbda - 1, dg_lmbda))
    return neg_CE_likelihood + neg_CE_Z + neg_CE_theta + neg_CE_beta + H_Z + H_theta + H_beta

def CAVI_algorithm(w, K, n_iter, eta, alpha):
  D, N, W = w.shape
  phi, gamma, lmbda = initialize_q(w, D, N, K, W)

  # Store output per iteration
  elbo = np.zeros(n_iter)
  phi_out = np.zeros((n_iter, D, N, K))
  gamma_out = np.zeros((n_iter, D, K))
  lmbda_out = np.zeros((n_iter, K, W))

  for i in range(0, n_iter):

    ###### CAVI updates #######

    # q(Z) update
    phi = update_q_Z(w, gamma, lmbda)

    # q(theta) update
    gamma = update_q_theta(phi, alpha)

    # q(beta) update
    lmbda = update_q_beta(w, phi, eta)

    # ELBO
    elbo[i] = calculate_elbo(w, phi, gamma, lmbda, eta, alpha)

    # outputs
    phi_out[i] = phi
    gamma_out[i] = gamma
    lmbda_out[i] = lmbda

  return phi_out, gamma_out, lmbda_out, elbo

n_iter0 = 100
K0 = K_sim
W0 = W_sim
eta_prior0 = np.ones(W0)
alpha_prior0 = np.ones(K0)
phi_out0, gamma_out0, lmbda_out0, elbo0 = CAVI_algorithm(w0, K0, n_iter0, eta_prior0, alpha_prior0)
final_phi0 = phi_out0[-1]
final_gamma0 = gamma_out0[-1]
final_lmbda0 = lmbda_out0[-1]
```

```python
precision = 3
print(f"----- Recall label switching - compare E[theta] and true theta and check for label switching -----")
print(f"Final E[theta] of doc 0 CAVI:  {np.round(final_gamma0[0] / np.sum(final_gamma0[0], axis=0, keepdims=True), precision)}")
print(f"True theta of doc 0:          {np.round(theta0[0], precision)}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----")
print(f"Final E[beta] k=0: {np.round(final_lmbda0[0, :] / np.sum(final_lmbda0[0, :], axis=-1, keepdims=True), precision)}")
print(f"Final E[beta] k=1: {np.round(final_lmbda0[1, :] / np.sum(final_lmbda0[1, :], axis=-1, keepdims=True), precision)}")
print(f"True beta k=0: {np.round(beta0[0, :], precision)}")
print(f"True beta k=1: {np.round(beta0[1, :], precision)}")
```

```
----- Recall label switching - compare E[theta] and true theta and check for label switching -----
Final E[theta] of doc 0 CAVI:  [0.394 0.606]
True theta of doc 0:          [0.541 0.459]
----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----
Final E[beta] k=0: [0.117 0.249 0.623 0.    0.011]
Final E[beta] k=1: [0.107 0.764 0.004 0.    0.126]
True beta k=0: [0.114 0.694 0.081 0.    0.11 ]
True beta k=1: [0.106 0.378 0.479 0.    0.037]
```

SVI Implementation

Using the CAVI updates as a template, finish the code below.

```python
def update_q_Z_svi(batch, w, gamma, lmbda):
    """
    SVI update for q(Z).
    """
    D_batch, N_batch, _ = batch.shape
    K = gamma.shape[-1]

    E_log_theta = sp_spec.digamma(gamma) - sp_spec.digamma(np.sum(gamma, axis=1, keepdims=True))
    E_log_beta = sp_spec.digamma(lmbda) - sp_spec.digamma(np.sum(lmbda, axis=1, keepdims=True))

    log_rho = np.zeros((D_batch, N_batch, K))
    w_label = batch.argmax(axis=-1)

    for d in range(D_batch):
        for n in range(N_batch):
            E_log_beta_wdn = E_log_beta[:, int(w_label[d, n])]
            E_log_theta_d = E_log_theta[d]
            log_rho_n = E_log_theta_d + E_log_beta_wdn
            log_rho[d, n, :] = log_rho_n

    phi_batch = np.exp(log_rho - sp_spec.logsumexp(log_rho, axis=-1, keepdims=True))
    return phi_batch

def update_q_theta_svi(batch, phi, alpha):
    """
    SVI update for q(theta).
    """
    D_batch, _, K = phi.shape

    E_Z_batch = phi
    gamma_batch = np.zeros((D_batch, K))

    for d in range(D_batch):
        E_Z_d = E_Z_batch[d]
        gamma_batch[d] = alpha + np.sum(E_Z_d, axis=0)  # sum over N
    return gamma_batch

def update_q_beta_svi(batch, w, phi, eta):
    """
    SVI update for q(beta).
    """
    D_batch, N_batch, W = batch.shape
    K = phi.shape[-1]

    lmbda_batch = np.zeros((K, W))

    for k in range(K):
        lmbda_batch[k, :] = eta
        for d in range(D_batch):
            for n in range(N_batch):
                lmbda_batch[k, :] += phi[d, n, k] * batch[d, n]  # Sum over d and n
    return lmbda_batch

def SVI_algorithm(w, K, S, n_iter, eta, alpha):
    """
    Stochastic Variational Inference (SVI) algorithm for LDA.
    """
    D, N, W = w.shape
    phi, gamma, lmbda = initialize_q(w, D, N, K, W)

    # Store output per iteration
    elbo = np.zeros(n_iter)
    phi_out = np.zeros((n_iter, D, N, K))
    gamma_out = np.zeros((n_iter, D, K))
    lmbda_out = np.zeros((n_iter, K, W))

    for i in range(n_iter):
        # Sample batch (subsample documents)
        batch_indices = np.random.choice(D, size=S, replace=False)
        batch_w = w[batch_indices]

        # SVI updates
        phi_batch = update_q_Z_svi(batch_w, w, gamma, lmbda)
        gamma_batch = update_q_theta_svi(batch_w, phi_batch, alpha)
        lmbda_batch = update_q_beta_svi(batch_w, w, phi_batch, eta)

        # Update global variables
        phi[batch_indices] = phi_batch
        gamma[batch_indices] = gamma_batch
        lmbda = lmbda_batch

        # ELBO
        elbo[i] = calculate_elbo(w, phi, gamma, lmbda, eta, alpha)

        # Store outputs
        phi_out[i] = phi
        gamma_out[i] = gamma
        lmbda_out[i] = lmbda

    return phi_out, gamma_out, lmbda_out, elbo
```

CASE 1

Tiny dataset

```python
np.random.seed(0)

# Data simulation parameters
D1 = 50
N1 = 50
K1 = 2
W1 = 5
eta_sim1 = np.ones(W1)
alpha_sim1 = np.ones(K1)

w1, z1, theta1, beta1 = generate_data(D1, N1, K1, W1, eta_sim1, alpha_sim1)

# Inference parameters
n_iter_cavi1 = 100
n_iter_svi1 = 100
eta_prior1 = np.ones(W1) * 1.
alpha_prior1 = np.ones(K1) * 1.
S1 = 5 # batch size

start_cavi1 = time.time()
phi_out1_cavi, gamma_out1_cavi, lmbda_out1_cavi, elbo1_cavi = CAVI_algorithm(w1, K1, n_iter_cavi1, eta_prior1, alpha_prior1)
end_cavi1 = time.time()

start_svi1 = time.time()
phi_out1_svi, gamma_out1_svi, lmbda_out1_svi, elbo1_svi = SVI_algorithm(w1, K1, S1, n_iter_svi1, eta_prior1, alpha_prior1)
end_svi1 = time.time()

final_phi1_cavi = phi_out1_cavi[-1]
final_gamma1_cavi = gamma_out1_cavi[-1]
final_lmbda1_cavi = lmbda_out1_cavi[-1]
final_phi1_svi = phi_out1_svi[-1]
final_gamma1_svi = gamma_out1_svi[-1]
final_lmbda1_svi = lmbda_out1_svi[-1]
```

Evaluation

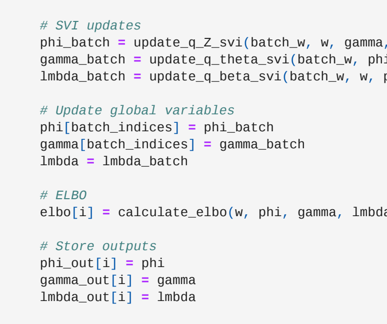
Do not expect perfect results in terms expectations being identical to the "true" theta and beta. Do not expect the ELBO plot of your SVI alg to be the same as the CAVI alg. However, it should increase and be in the same ball park as that of the CAVI alg.

```python
np.set_printoptions(formatter={'float': lambda x: "{0:0.3f}".format(x)})
print(f"----- Recall label switching - compare E[theta] and true theta and check for label switching -----")
print(f"E[theta] of doc 0 SVI:  {final_gamma1_svi[0] / np.sum(final_gamma1_svi[0], axis=0, keepdims=True)}")
print(f"E[theta] of doc 0 CAVI: {final_gamma1_cavi[0] / np.sum(final_gamma1_cavi[0], axis=0, keepdims=True)}")
print(f"True theta of doc 0:    {theta1[0]}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----")
print(f"E[beta] SVI k=0:    {final_lmbda1_svi[0, :] / np.sum(final_lmbda1_svi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] SVI k=1:    {final_lmbda1_svi[1, :] / np.sum(final_lmbda1_svi[1, :], axis=-1, keepdims=True)}")
print(f"E[beta] CAVI k=0:   {final_lmbda1_cavi[0, :] / np.sum(final_lmbda1_cavi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] CAVI k=1:   {final_lmbda1_cavi[1, :] / np.sum(final_lmbda1_cavi[1, :], axis=-1, keepdims=True)}")
print(f"True beta k=0:      {beta1[0, :]}")
print(f"True beta k=1:      {beta1[1, :]}")
```

```
----- Recall label switching - compare E[theta] and true theta and check for label switching -----
E[theta] of doc 0 SVI:  [0.686 0.314]
E[theta] of doc 0 CAVI: [0.475 0.525]
True theta of doc 0:    [0.676 0.324]
----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----
E[beta] SVI k=0:    [0.119 0.152 0.284 0.364 0.081]
E[beta] SVI k=1:    [0.146 0.170 0.273 0.293 0.118]
E[beta] CAVI k=0:   [0.276 0.347 0.129 0.095 0.154]
E[beta] CAVI k=1:   [0.075 0.011 0.351 0.503 0.059]
True beta k=0:      [0.185 0.291 0.214 0.183 0.128]
True beta k=1:      [0.136 0.075 0.291 0.434 0.063]
```

```python
plt.plot(list(range(1, n_iter_cavi1 + 1)), elbo1_svi[np.arange(0, n_iter_svi1, int(n_iter_svi1 / n_iter_cavi1))])
plt.plot(list(range(1, n_iter_cavi1 + 1)), elbo1_cavi)
plt.title("ELBO plot")
plt.xlabel("iterations")
plt.ylabel("ELBO")
plt.show()
```



CASE 2

Small dataset

```python
np.random.seed(0)

# Data simulation parameters
D2 = 1000
N2 = 50
K2 = 3
W2 = 10
eta_sim2 = np.ones(W2)
alpha_sim2 = np.ones(K2)

w2, z2, theta2, beta2 = generate_data(D2, N2, K2, W2, eta_sim2, alpha_sim2)

# Inference parameters
n_iter_cavi2 = 100
n_iter_svi2 = 100
eta_prior2 = np.ones(W2) * 1.
alpha_prior2 = np.ones(K2) * 1.
S2 = 100 # batch size

start_cavi2 = time.time()
phi_out2_cavi, gamma_out2_cavi, lmbda_out2_cavi, elbo2_cavi = CAVI_algorithm(w2, K2, n_iter_cavi2, eta_prior2, alpha_prior2)
end_cavi2 = time.time()

start_svi2 = time.time()
phi_out2_svi, gamma_out2_svi, lmbda_out2_svi, elbo2_svi = SVI_algorithm(w2, K2, S2, n_iter_svi2, eta_prior2, alpha_prior2)
end_svi2 = time.time()

final_phi2_cavi = phi_out2_cavi[-1]
final_gamma2_cavi = gamma_out2_cavi[-1]
final_lmbda2_cavi = lmbda_out2_cavi[-1]
final_phi2_svi = phi_out2_svi[-1]
final_gamma2_svi = gamma_out2_svi[-1]
final_lmbda2_svi = lmbda_out2_svi[-1]
```

Evaluation

Do not expect perfect results in terms expectations being identical to the "true" theta and beta. Do not expect the ELBO plot of your SVI alg to be the same as the CAVI alg. However, it should increase and be in the same ball park as that of the CAVI alg.

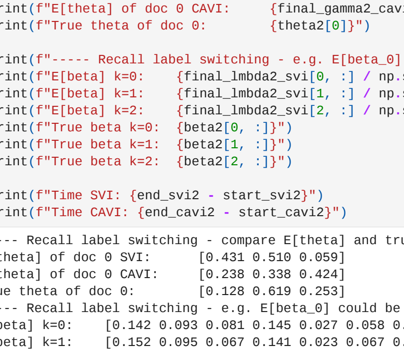
```python
np.set_printoptions(formatter={'float': lambda x: "{0:0.3f}".format(x)})
print(f"----- Recall label switching - compare E[theta] and true theta and check for label switching -----")
print(f"E[theta] of doc 0 SVI:      {final_gamma2_svi[0] / np.sum(final_gamma2_svi[0], axis=0, keepdims=True)}")
print(f"E[theta] of doc 0 CAVI:     {final_gamma2_cavi[0] / np.sum(final_gamma2_cavi[0], axis=0, keepdims=True)}")
print(f"True theta of doc 0:        {theta2[0]}")

print(f"----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----")
print(f"E[beta] k=0:    {final_lmbda2_svi[0, :] / np.sum(final_lmbda2_svi[0, :], axis=-1, keepdims=True)}")
print(f"E[beta] k=1:    {final_lmbda2_svi[1, :] / np.sum(final_lmbda2_svi[1, :], axis=-1, keepdims=True)}")
print(f"E[beta] k=2:    {final_lmbda2_svi[2, :] / np.sum(final_lmbda2_svi[2, :], axis=-1, keepdims=True)}")
print(f"True beta k=0:  {beta2[0, :]}")
print(f"True beta k=1:  {beta2[1, :]}")
print(f"True beta k=2:  {beta2[2, :]}")

print(f"Time SVI: {end_svi2 - start_svi2}")
print(f"Time CAVI: {end_cavi2 - start_cavi2}")
```

```
----- Recall label switching - compare E[theta] and true theta and check for label switching -----
E[theta] of doc 0 SVI:      [0.431 0.510 0.059]
E[theta] of doc 0 CAVI:     [0.238 0.338 0.424]
True theta of doc 0:        [0.128 0.619 0.253]
----- Recall label switching - e.g. E[beta_0] could be fit to true theta_1. -----
E[beta] k=0:    [0.142 0.093 0.081 0.145 0.027 0.058 0.018 0.183 0.161 0.093]
E[beta] k=1:    [0.152 0.095 0.067 0.141 0.023 0.067 0.021 0.181 0.163 0.090]
E[beta] k=2:    [0.164 0.086 0.063 0.144 0.030 0.060 0.021 0.184 0.177 0.071]
True beta k=0:  [0.067 0.105 0.077 0.066 0.046 0.087 0.048 0.186 0.277 0.040]
True beta k=1:  [0.139 0.067 0.074 0.230 0.007 0.008 0.002 0.158 0.134 0.181]
True beta k=2:  [0.295 0.123 0.047 0.116 0.010 0.078 0.012 0.222 0.057 0.041]
Time SVI: 5.615252256393433
Time CAVI: 44.14750862121582
```

```python
plt.plot(list(range(1, n_iter_cavi2 + 1)), elbo2_svi[np.arange(0, n_iter_svi2, int(n_iter_svi2 / n_iter_cavi2))])
plt.plot(list(range(1, n_iter_cavi2 + 1)), elbo2_cavi)
plt.title("ELBO plot")
plt.xlabel("iterations")
plt.ylabel("ELBO")
plt.show()
```



CASE 3

Medium small dataset, one iteration for time analysis.

```python
np.random.seed(0)

# Data simulation parameters
D3 = 10**4
N3 = 500
K3 = 5
W3 = 10
eta_sim3 = np.ones(W3)
alpha_sim3 = np.ones(K3)

w3, z3, theta3, beta3 = generate_data_torch(D3, N3, K3, W3, eta_sim3, alpha_sim3)

# Inference parameters
n_iter3 = 1
eta_prior3 = np.ones(W3) * 1.
alpha_prior3 = np.ones(K3) * 1.
S3 = 100 # batch size

start_cavi3 = time.time()
phi_out3_cavi, gamma_out3_cavi, lmbda_out3_cavi, elbo3_cavi = CAVI_algorithm(w3, K3, n_iter3, eta_prior3, alpha_prior3)
end_cavi3 = time.time()

start_svi3 = time.time()
phi_out3_svi, gamma_out3_svi, lmbda_out3_svi, elbo3_svi = SVI_algorithm(w3, K3, S3, n_iter3, eta_prior3, alpha_prior3)
end_svi3 = time.time()

final_phi3_cavi = phi_out3_cavi[-1]
final_gamma3_cavi = gamma_out3_cavi[-1]
final_lmbda3_cavi = lmbda_out3_cavi[-1]
final_phi3_svi = phi_out3_svi[-1]
final_gamma3_svi = gamma_out3_svi[-1]
final_lmbda3_svi = lmbda_out3_svi[-1]
```

```python
print(f"Examine per iteration run time.")
print(f"Time SVI: {end_svi3 - start_svi3}")
print(f"Time CAVI: {end_cavi3 - start_cavi3}")
```

```
Examine per iteration run time.
Time SVI: 2.003345251083374
```

```
Time CAVI: 67.85830616950989
```