

# **10017** The Never Ending Towers of Hanoi

In 1883, Edward Lucas invented, or perhaps reinvented, one of the most popular puzzles of all times – the Tower of Hanoi, as he called it – which is still used today in many computer science textbooks to demonstrate how to write a recursive algorithm or program. First of all, we will make a list of the rules of the puzzle:

- There are three pegs: A, B and C.
- There are n disks. The number n is constant while working the puzzle.
- All disks are different in size.
- The disks are initially stacked on peg A so that they increase in size from the top to the bottom.
- The goal of the puzzle is to transfer the entire tower from the A peg to the peg C.
- One disk at a time can be moved from the top of a stack either to an empty peg or to a peg with a larger disk than itself on the top of its stack.

Your job will be to write a program which will show a copy of the puzzle on the screen step by step, as you move the disks around. This program has to solve the problem in an efficient way.

**TIP:** It is well known and rather easy to prove that the minimum number of moves needed to complete the puzzle with n disks is  $2^n - 1$ .

#### Input

The input file will consist of a series of lines. Each line will contain two integers n, m. n, lying within the range [1, 250], will denote the number of disks and m, belonging to  $[0, 2^n - 1]$ , will be the number of the last move, you may assume that m will also be less than  $2^{16}$ , and you may also assume that a good algorithm will always have enough time. The file will end at a line formed by two zeros.

## Output

The output will consist again of a series of lines, formatted as show below.

#### NOTES:

- There are 3 spaces between de '=>' and the first number printed. If there isn't any number, there should be no spaces.
- All the disks in a single peg are printed in a single line (not as in the Problem #1 below).
- Print a blank line after every problem.

## Sample Input

64 2

8 45

0 0

# **Sample Output**

```
Problem #1
      64 63 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 45 44 43 42 41
40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15
14 13 12 11 10 9 8 7 6 5 4 3 2 1
B=>
C=>
      64 63 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 45 44 43 42 41
40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15
14 13 12 11 10 9 8 7 6 5 4 3 2
B=>
    1
C=>
      64 63 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 45 44 43 42 41
40 39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15
14 13 12 11 10 9 8 7 6 5 4 3
B=>
     1
C=>
      2
Problem #2
A=>
     8 7 6 5 4 3 2 1
B=>
C=>
      8 7 6 5 4 3 2
A=>
B=>
C=>
      8 7 6 5 4 3
A=>
B=>
      1
C=>
      2
A=>
      8 7 6 5 4 3
B=>
C=>
      2 1
      8 7 6 5 4
A=>
B=>
      3
C=>
      2 1
     8 7 6 5 4 1
A=>
B=>
      3
C=>
      2
      8 7 6 5 4 1
A=>
B=>
      3 2
C=>
```

- A=> 8 7 6 5 4
- B=> 3 2 1
- C=>
- A=> 8 7 6 5
- B=> 3 2 1
- C=> 4
- A=> 8 7 6 5
- B=> 3 2
- C=> 4 1
- A=> 8 7 6 5 2
- B=> 3
- C=> 4 1
- A=> 8 7 6 5 2 1
- B=> 3
- C=> 4
- A=> 8 7 6 5 2 1
- B=>
- C=> 4 3
- A=> 8 7 6 5 2
- B=> 1
- C=> 4 3
- A=> 8 7 6 5
- B=> 1
- C=> 4 3 2
- A=> 8 7 6 5
- B=>
- C=> 4 3 2 1
- A=> 8 7 6
- B=> 5
- C=> 4 3 2 1
- A=> 8 7 6 1
- B=> 5
- C=> 4 3 2
- A=> 8 7 6 1
- B=> 5 2
- C=> 4 3
- A=> 8 7 6

- B=> 5 2 1
- C=> 4 3
- A=> 8 7 6 3
- B=> 5 2 1
- C=> 4
- A=> 8 7 6 3
- B=> 5 2
- C=> 4 1
- A=> 8 7 6 3 2
- B=> 5
- C=> 4 1
- A=> 8 7 6 3 2 1
- B=> 5
- C=> 4
- A=> 8 7 6 3 2 1
- B=> 5 4
- C=>
- A=> 8 7 6 3 2
- B=> 5 4 1
- C=>
- A=> 8 7 6 3
- B=> 5 4 1
- C=> 2
- A=> 8 7 6 3
- B=> 5 4
- C=> 2 1
- A=> 8 7 6
- B=> 5 4 3
- C=> 2 1
- A=> 8 7 6 1
- B=> 5 4 3
- C=> 2
- A=> 8 7 6 1
- B=> 5 4 3 2
- C=>
- A=> 8 7 6
- B=> 5 4 3 2 1
- C=>

- A=> 8 7
- B=> 5 4 3 2 1
- C=> 6
- A=> 8 7
- B=> 5 4 3 2
- C=> 6 1
- A=> 8 7 2
- B=> 5 4 3
- C=> 6 1
- A=> 8 7 2 1
- B=> 5 4 3
- C=> 6
- A=> 8 7 2 1
- B=> 5 4
- C=> 6 3
- A=> 8 7 2
- B=> 5 4 1
- C=> 6 3
- A=> 8 7
- B=> 5 4 1
- C=> 6 3 2
- A=> 8 7
- B=> 5 4
- C=> 6 3 2 1
- A=> 8 7 4
- B=> 5
- C=> 6 3 2 1
- A=> 8 7 4 1
- B=> 5
- C=> 6 3 2
- A=> 8 7 4 1
- B=> 5 2
- C=> 6 3
- A=> 8 7 4
- B=> 5 2 1
- C=> 6 3
- A=> 8 7 4 3

B=> 5 2 1

C=> 6

A=> 8 7 4 3

B=> 5 2

C=> 6 1