1 Many-Valued Logic

Digital circuits are designed and implemented using the binary logics, which is shown as a bivalent logic which must be either **“absolute true”** or **“absolute false”**, because of its simplicity and convenience. For example, for the statement “the earth is round” the binary logic will give a value of “true”. Moreover, the voltage mode binary logic allows for optimal noise margins that come from using only the two voltage references for the logic values, thus making it simpler to analyze between a logic “1” and logic “0”. However, there are some circumstances where the binary logic breaks down. For instance, traffic lights have 3 states (green, yellow, red), which is reasonably problematic to implement using bivalent logic. Hereby, the multi-valued logics comes.

In science, a non-classic many-valued logic is represented as a propositional calculus in which no restrictions about the number of truth values are present: it allows larger sets of truth degrees. In modern era, the main MVL systems are represented by Łukasiewicz logics, Gödel logics, t-Norm based systems, three-valued systems, Dunn/Belnap’s 4-valued system, and product systems. In electronic devices’ structure, an interconnection cost and delay of circuits is a big challenge. Thus, high-radix systems could be beneficial towards solving the problem by providing a better relationship between circuits. The same phenomenon is present in other relevant areas such as signal processing, data structures and discrete system algorithms.

For example, considering decimal number “322” and its representations in other radixes, it is seen that the number of digits is inversely proportional to the base of the number itself: as radix increases amount of digits decreases.

$$322_{10} = 11002_4 = 101000010_2$$

One of the other factors that could help to achieve the increase of data density is the number of functions that can be represented by a single logic gate. The total number of functions F is given by formula $F = r^{r^n}$.

Input number	Binary (radix = 2)	Quaternary (radix = 4)
1	$2^{2^1} = 4$	$4^{4^1} = 256$
2	$2^{2^2} = 16$	$4^{4^2} = 4294967296$

Table 1. Total number of different logic functions

As it can be seen from the table, the amount of functions that can be represented by quaternary is much larger than binary number. Accordingly, MVL not only reduces the total number of connections, but also causes reduction of total number of gates compared to its binary implementation, as several binary gates potentially can be replaced by a single MVL gate.

Though the potential of Multi-Valued Logic gates looks very promising because of its several major advantages, the feasibility of the systems depends on these factors:

- Availability of reliable gateway implementations.
- Adequate synthesis tools and techniques.

2 Multi-Valued Gates

To the date, researchers and developers has proposed MVL for devices such as memories, combinational circuits (adders, multipliers) and programmable devices. If the first discoverer of MVL application on memory was Intel, releasing their products which could store two bits of information per cell with four voltage levels, Samsung launched their NAND-based flash based memory in face of 840 Evo Series SSDs that could store 3 bits of information per cell. Such technologies turned out to be beneficial because of its possibility to pack larger amount of information into small areas. The technology was referred as Multi-Level Cell, which allowed to store two or more bits of information per cell instead of one by using intermediate voltage levels. Consequently, it had positive impact on cost-capacity relationship.

While talking about standard gates, despite the multiple researches and application attempts, only few of them could be classified as real alternatives to a regular basic CMOS (Complementary Metal-Oxide-Semiconductor) gates. Such implementations of Multi-Valued Logic circuits are categorized as voltage-mode and current-mode operations.

The work principle of voltage-mode implementation is based on having several stable intermediate voltage levels between fully charged and fully discharged, which sometimes requires nonstandard technologies. The main advantage of such technique is little amount of power consumption, which happens only during logic level switching.

On the other side, the work principle of current-mode implementation is based on Kirchhoff’s current law, where logic levels are represented by different currents that can be involved on addition and subtraction operations in order to obtain the desired output. The major advantage of such technique is a simplicity of arithmetic operations, while a higher power consumption is tend to be a disadvantage, as the current constantly flows through the circuit.

Reference List

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