Homework 7

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Statement of Integrity: I, Joni Vrapi, attempted to answer each question honestly and to the best of my abilities. I cited any, and all, help that I received in completing this assignment.

Problem 1. decision variables are the variables you need to make a decision on - eg how many bowls/mugs do you need to produce to have the most revenue

Problem 2a. If we let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$, then the expected loss for Player 1 can be calculated as $\sum xAy^T$ resulting in:

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \tag{1}$$

Problem 2b. Player 2 playing any strategy other than $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ will allow Player 1 to adjust his strategy to play off of Player 2's strategy, resulting in an expected gain for Player 1. For example, assume Player 2 plays $(1, 0, 0)$. Player 1 would then adjust his strategy to play $(0, 1, 0)$ resulting in an expected gain of 1 (from equation 1). Likewise, it is obvious that regardless of what strategy Player 2 chooses, other than $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, Player 1 will always be able to play off of it towards an expected gain.

Problem 2c. Letting $x = y = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ results in an expected outcome per (1) of 0. This result is indicative of both players winning the same amount of times, on average, resulting in a draw. The sticking point here, however, is that neither player has an incentive to change strategies. If one player were to choose an imbalanced strategy, the other player could alter their strategy to take advantage of it resulting in an expected gain for the other player. Therefore, we have reached a Nash equilibrium.

Problem 2d. No, it is not possible for some mixed strategy where $y' \neq (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ to form a Nash equilibrium because any mixed strategy where rock, paper, or scissors are played in an imbalanced way will allow for the other player to play a strategy where the balance was reversed to accumulate more wins. For example, if Player 2 were to choose slightly more rock than paper or scissors, Player 1 could change their strategy to favor paper by the same amount, resulting in an expected gain.

Problem 3a. From [1] we choose:

Facility-Location()

with $R = U$ and $T = \emptyset$
while R is not empty
    $c = min_{u \in R, s \in T} d_{us}$
    select $s \in R\ T$ and set $U_s \subset R$ that minimizes: $c' = \frac{f_s + \sum_{u \in U_s} d_{us}}{|U_s|}$

    if $c' \leq c$
        select the site s and set U_s used to obtain c' above
        add s to T and delete U_s from R
        set $c_u = c'$ for all $u \in U_s$

    else
        select s and u obtaining the first minimum
        add u to U_s
        set $c_u = c$

Problem 3b. From [1] we consider an optimal solution which contains a subset T^* of sites, and $s \in T^*$ is used to cover a set U_s^* of users. The cost of using s to cover U_s^* is

$$C = f_s + \sum_{u \in U_s^*} d_{us} \tag{2}$$

We will need to compare C with the cost of our greedy algorithm which is $\sum_{u \in U_s^*} c_u$

Lemma:

$$\sum_{u \in U_s^*} c_u \leq \rho(d)(f_s + \sum_{u \in U_s^*} d_{us}) = H(d) \cdot C \tag{3}$$

where $d = |U_s^*|$.

Problem 3c.

References

[1] "Lecture 9." https://cs.gmu.edu/~lifei/teaching/cs684spring17/lec9.pdf. Accessed on 2022-11-25.