

Iteration Method Tutorial

We will use the function in CLRS, Figure 4.1, page 96: $T(n) = 3T(n/4) + cn^2$. The two terms of the function are the **recursive term**, $3T(n/4)$, and the **non-recursive term**, cn^2 .

We would begin our iteration method by rearranging the terms of the asymptotic function to make the iteration analysis easier to manage. Recursion trees always number the depths of the tree starting at 0, the root of the tree.

1. $T(n) = cn^2 + 3T(n/4)$

We have placed the non-recursive term, cn^2 , before the recursive term, $3T(n/4)$.

Next, expand the recursive term $T(n) = cn^2 + 3[T(n/4)]$ of the current iteration. n is now $n/4$. We write the non-recursive term first and then the new recursive expression.

2. $T(n) = cn^2 + 3[c(n/4)^2 + 3T(n/16)]$

If you look at Figure 4.5 (c) you will see the root of the tree is the first non-recursive term in our iteration, the next level of the tree is the next non-recursive term in our iteration, and the next level of the tree, depth=1, the current leaves, are the unexpanded recursive term in our iteration. Notice also that the recursive term in our 1. above was replaced by a non-recursive cost and the recursive cost became children of the nodes now holding a non-recursive cost. n is now $n/16$.

3. $T(n) = cn^2 + 3[c(n/4)^2 + 3\{c(n/16)^2 + 3T(n/64)\}]$

This is our next expansion, not illustrated in Figure 4.5. n is now $n/64$. You need to continue the iteration until you reach a base case, or you discern a function describing cost across all non-recursive nodes at a tree depth. In Figure 4.5, the root is at depth 0. If you sum the work at each depth, by summing the non-recursive costs at that depth, you will arrive at the terms in Figure 4.5 (d) to the right of the recursion tree.

Note that in the iteration method and the recursion tree you need to determine three attributes of the solution: (a) what is the depth of the recursion tree, (b) what is the sum of the non-recursive costs at each depth, and (c) is there a function to described the sums of the non-recursive cost at each depth?