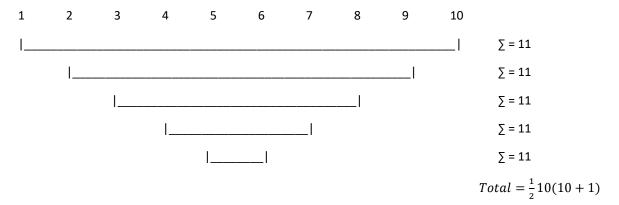
CLRS 4th Ed. Notes to accompany Appendix A Summations,

A.2 Bounding Summations starting page 1148

Assumption: n is even

$$\sum_{k=1}^{n} = \frac{1}{2}n(n+1) \text{ (A. 1)}$$
$$= \Theta(n) \qquad \text{(A. 2)}$$

Euler is credited with this formula (A.1). Story could be that his elementary school teacher needed a break and assigned the problem "Sum the numbers between 1 and 100" thinking it would take the students a while to complete. Euler realized, though, the following and answered almost immediately.



Later it is useful to think {(height of the tree) x (work at each level)}, in this case {5x11}.

Now splitting the summation, and seeking a *lower bound*:

$$\sum_{k=1}^{n} k = \sum_{k=1}^{\frac{n}{2}} k + \sum_{k=\frac{n}{2}+1}^{n} k$$

$$\geq \sum_{k=1}^{\frac{n}{2}} 0 + \sum_{k=\frac{n}{2}+1}^{n} n/2 \qquad \text{The first } n/2 \text{ terms are ignored}$$

Using n = 10 as an example,

$$\geq 0 + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} + \frac{10}{2}$$
 for $k = 6, k = 7, k = 8, k = 9$, and $k = 10$ terms in the summation $\geq 0 + 25$ $= (n/2)^2$ $= \Omega(n^2)$