CLRS 4th Ed. Notes to accompany Appendix A Summations,

A.2 Bounding Summations starting page 1148

Assumption: *n* is even

$$\sum_{k=1}^{n} = \frac{1}{2}n(n+1) \ (\mathrm{A.1})$$
$$= \Theta(n) \qquad (\mathrm{A.2})$$

Euler is credited with this formula (A.1). Story could be that his elementary school teacher needed a break and assigned the problem “Sum the numbers between 1 and 100” thinking it would take the students a while to complete. Euler realized, though, the following and answered almost immediately.

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1         2         3         4         5         6         7         8         9         10

|___________________________________________________________________________________________|       ∑ = 11
          |_______________________________________________________________________|                 ∑ = 11
                    |___________________________________________________|                           ∑ = 11
                              |_______________________________|                                     ∑ = 11
                                        |___________|                                               ∑ = 11
```

$$Total = \frac{1}{2}10(10+1)$$

Later it is useful to think {(height of the tree) x (work at each level)}, in this case {5x11}.

Now splitting the summation, and seeking a *lower bound*:

$$\sum_{k=1}^{n} k = \sum_{k=1}^{\frac{n}{2}} k + \sum_{k=\frac{n}{2}+1}^{n} k$$
$$\geq \sum_{k=1}^{\frac{n}{2}} 0 + \sum_{k=\frac{n}{2}+1}^{n} n/2 \qquad \text{The first } n/2 \text{ terms are ignored}$$

Using *n* = 10 as an example,

$$\geq 0 + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} + \frac{10}{2} \quad \text{for } k = 6, k = 7, k = 8, k = 9, \text{and } k = 10 \text{ terms in the summation}$$

$$\geq 0 + 25$$
$$= (n/2)^2$$
$$= \Omega(n^2)$$