## Summary of Asymptotic Notations (CLRS 4th Ed.)

| Asymptotic Bound | $\lim_{n\to\infty} \frac{f(n)}{g(n)} = C$ | Complete Definition | Asymptotic Family Relationships | Is *g*(*n*) an asymptotically tight bound for *f*(*n*)? |
|---|---|---|---|---|
| $\omega$ | $\infty = C$ | $\omega(g(n)) = \left\{ \begin{array}{l} f(n): \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such that} \\ 0 \le cg(n) < f(n) \, \forall \, n \ge n_0 \end{array} \right\}$ | $\omega(g(n)) \subset \Omega(g(n))$ | No; asymptotically larger (see page 61) |
| $\Omega$ | $0 < C \le \infty$ | $\Omega(g(n)) = \left\{ \begin{array}{l} f(n): \exists \text{positive constants } c \text{ and } n_0 \\ \text{such that } 0 \le cg(n) \le f(n) \, \forall \, n \ge n_0 \end{array} \right\}$ | $\Theta(g(n)) \subseteq \Omega(g(n))$ | Maybe |
| $o$ | $0 = C$ | $o(g(n)) = \left\{ \begin{array}{l} f(n): \text{for any positive constant} \\ c > 0, \exists \text{ a constant } n_0 > 0 \text{ such} \\ \text{that } 0 \le f(n) < cg(n) \, \forall \, n \ge n_0 \end{array} \right\}$ | $o(g(n)) \subset O(g(n))$ | No; asymptotically smaller (see page 60) |
| $O$ | $0 \le C < \infty$ | $O(g(n)) = \left\{ \begin{array}{l} f(n): \exists \text{positive constants } c \text{ and } n_0 \\ \text{such that } 0 \le f(n) \le cg(n) \, \forall \, n \ge n_0 \end{array} \right\}$ | $\Theta(g(n)) \subseteq O(g(n))$ | Maybe |
| $\Theta$ | $0 < C < \infty$ | $\Theta(g(n)) = \left\{ \begin{array}{l} f(n): \exists \text{positive constants } c_1, c_2, \text{and } n_0 \\ \text{such that } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \, \forall \, n \ge n_0 \end{array} \right\}$ | Theorem 3.1 For any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ | Yes |

**Notes**:

*f*(n) must be nonnegative whenever *n* is sufficiently large (asymptotically nonnegative)

*g*(n) must be asymptotically nonnegative

if $\forall \, n \ge n_0$ the function *f*(n) is equal to *g*(n) to within a constant factor, then *g*(n) is an asymptotically tight bound for *f*(n)

**Symbol Meanings:**

*f*(*n*) is the function describing the algorithm derived through analysis of the algorithm

*g*(*n*) is the function that is being tested as bounding *f*(*n*) from below, or from above, or from both below and above

$\exists$ - “there exists”

$\forall$ - “for all”

 John E. Boon, Jr.

Iff – “if and only if”