Homework 2

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Statement of Integrity: I, Joni Vrapi, attempted to answer each question honestly and to the best of my abilities. I cited any, and all, help that I received in completing this assignment.

Problem 1. A language is *decidable* if a Turing machine accepts strings that are in the language, and rejects strings that are not in the language. In other words, the Turing machine will halt on all inputs [1]. From this, we can see that an *undecidable* language is one in which a Turing machine will not halt. Suppose we have a Turing machine M which can decide a language L. We can therefore create another Turing machine M', by running M and switching its accepts to rejects and vice versa, that decides L^C. M will always halt if it decides L, therefore we do not need to worry L^C will cause M to never halt. If decidable languages are closed under complementation, then undecidable languages must be as well.

Problem 2. A transitive relation R on a set X occurs when $\forall a, b, c \in X \mid (aRb) \wedge (bRc) \implies (aRc)$ [2]. If we let $f(x)$ be the polynomial time reduction function $\mid x \in L_1 \iff f(x) \in L_2$. We can then define $g(x)$ to be the polynomial time reduction function $\mid x \in L_2 \iff g(x) \in L_3$. We can then compute, in polynomial time, $g \circ f \mid x \in L_1 \iff g(f(x)) \in L_3$. $\therefore L_1 \leq_P L_3$, so $\leq_P$ is transitive.

References

[1] "Turing machines." https://brilliant.org/wiki/turing-machines/. Accessed on 2022-09-27.

[2] "Transitive relation." https://en.wikipedia.org/wiki/Transitive_relation. Accessed on 2022-09-27.