CLRS 4th Ed. Notes to accompany Appendix A Summations, A.2 Bounding Summations starting page 1148

A.2-2 on page 1150

Find an asymptotic upper bound on the summation

$\sum_{k=0}^{\lfloor lgn \rfloor} \left\lceil {n}/{2^k} \right\rceil$

When $n = 2^m$ the sum becomes $\frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \cdots + \frac{n}{n} = 2n - 1 = O(n)$

There will always exist a power of 2 which lies between *n* and 2*n* for any choice of *n*.

For example, let *n* = 9, 2*n* =18, then $2^4 = 16$.

Let *n'* denote the smallest power of 2 which is greater than or equal to *n*.

Then $\sum_{k=0}^{\lfloor lgn \rfloor} \left\lceil {n}/{2^k} \right\rceil \le \sum_{k=0}^{\lfloor lgn' \rfloor} \left\lceil {n'}/{2^k} \right\rceil = 2\text{n}' - 1 \le 4n - 1 = O(n)$

A.2-3 on page 1152

Show that the n^{th} Harmonic number is $\Omega(\lg n)$ by splitting the summation.

See derivation A.10 on page 1142, $\ln(n+1) \le H_n \le \ln n + 1$

Split the interval [*n*] into $\lfloor lgn \rfloor - 1$ pieces, starting the i^{th} at $\frac{1}{2^i}$ and going to $\frac{1}{2^{i+1}}$.

$$\sum_{k=1}^{n} \frac{1}{k} \ge \sum_{i=0}^{lgn-1} \sum_{j=0}^{2^i-1} \frac{1}{2^i + j}$$

$$\ge \sum_{i=0}^{lgn-1} \frac{1}{2^i + j}$$

$$\ge \sum_{i=0}^{lgn-1} \frac{1}{2}$$

$$= \frac{1}{2} lgn$$

$$= \Omega(lgn)$$