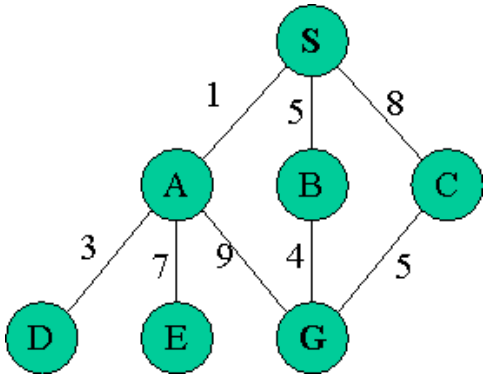
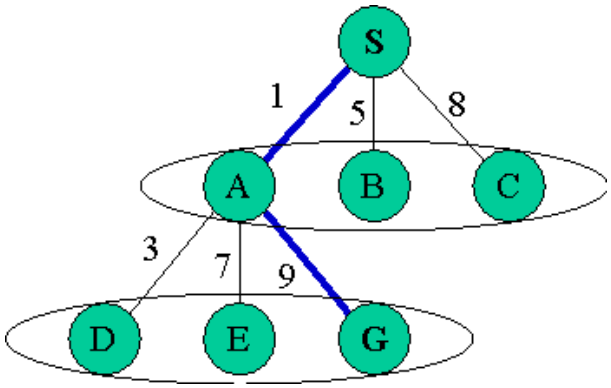
Optimal Graph Search

As suggested above, suppose we wish to find a path from the source vertex s to a particular vertex, identified by some characteristic or set of properties? In addition, suppose we wish to find the path that minimizes the cost along the edges? For example, suppose we are presented with the following graph:



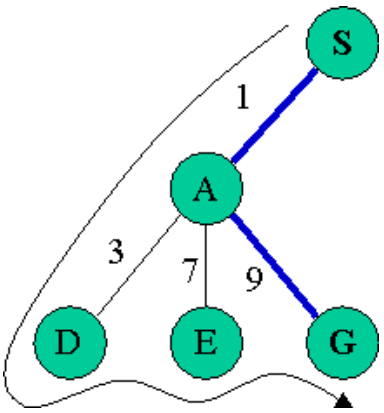
In this example, we are searching from the vertex designated by "S" and want to find the optimal path to the vertex designated by "G." Simple inspection reveals that the path S–B–G yields the optimal cost path with a total cost of $5 + 4 = 9$.

Suppose we apply breadth-first search to this graph. If we do that, we obtaining a graph with the following edges explored:



In this figure, we point out that breadth-first search explores the graph in levels, and the expansion of a vertex's adjacent vertices occur in left-to-right fashion. This means that goal vertex "G" will be found along the path through "A" before either of the two remaining paths because of the blind way the edges are ordered. Consequently, path S–A–G is found with a cost of 10. This is a suboptimal path.

What about depth-first search? Will performance be better if we use this alternative algorithm? Since we explained above that at least breadth-first search is optimal with uniform cost but depth-first search is not even optimal then, it seems unlikely depth-first search will improve our situation. In fact, the following is the resulting graph identifying the explored edges:



As we see, for this graph, depth-first search returns exactly the same suboptimal path as breadth-first search. What is interesting is that fewer vertices needed to be explored to get this path. Note, however, that such a result is not guaranteed; however, depth-first search is often found empirically to find a path more quickly than breadth-first search unless the paths explored in the tree prior to reaching the goal go significantly deeper than the position of the goal vertex.

We will revisit this problem of finding an optimal path in a graph later in this discussion.