

Here are short notes, first from the authors who codified complexity classes and polynomial time reductions, and second from an earlier CS textbook which addressed polynomial time reductions.

"Computers and Intractability" A Guide to the Theory of NP-Completeness", Michael R. Garey and David S. Johnson, W. H. Freeman and Company, 1979 (twenty-seventh printing). [ISBN-10: 0-7167-1045-5]

Chapter 3 Proving NP-Completeness Results

To prove that $\Pi \in \text{NP-complete}$:

"Given a problem $\Pi \in \text{NP}$, all we need do is show that some already known NP-complete problem Π' can be transformed to Π . Thus, from now on, the process of devising an NP-completeness proof for a decision problem Π will consist of the following four steps:

- (1) Showing that Π is in NP,
- (2) Selecting a known NP-complete problem Π' ,
- (3) Constructing a transformation f from Π' to Π , and
- (4) Proving that f is a (polynomial) transformation." (page 45)

Notation: known NP-complete problem \leq_P new problem

$\Pi' \leq_P \Pi$ (as expressed by Garey and Johnson)

$L' \leq_P L$ (as expressed in CLRS)

"Fundamentals of Computer Algorithms", Ellis Horowitz and Sartaj Sahni, Computer Science Press, 1978 [ISBN: 0-7167-8045-3]

Note: The Instructor has chosen to use the reduction symbol \leq_P where Horowitz and Sahni used \propto because Garey and Johnson defined the symbol \leq_P for all modern usage.

Chapter 11 NP-Hard and NP-Complete Problems

"Definition: Let L_1 and L_2 be problems. L_1 *reduces to* L_2 (also written $L_1 \leq_P L_2$) if and only if there is a way to solve L_1 by a deterministic polynomial time algorithm using a deterministic algorithm that solves L_2 in polynomial time.

This definition implies that if we have a polynomial time algorithm for L_2 then we can solve L_1 in polynomial time. One may readily verify that \leq_P is a transitive relation (i.e., if $L_1 \leq_P L_2$ and $L_2 \leq_P L_3$ then $L_1 \leq_P L_3$)." (page 511)