Homework 2

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Statement of Integrity: I, Joni Vrapi, attempted to answer each question honestly and to the best of my abilities. I cited any, and all, help that I received in completing this assignment.

Problem 1. A language is *decidable* if a Turing machine accepts strings that are in the language, and rejects strings that are not in the language. In other words, the Turing machine will halt on all inputs [1]. From this, we can see that an *undecidable* language is one in which a Turing machine will not halt. Suppose we have a Turing machine M which can decide a language L. We can therefore create another Turing machine M', by running M and switching its accepts to rejects and vice versa, that decides L^C. M will always halt if it decides L, therefore we do not need to worry L^C will cause M to never halt. If decidable languages are closed under complementation, then undecidable languages must be as well.

Problem 2. A transitive relation R on a set X occurs when $\forall a, b, c \in X \mid (aRb) \wedge (bRc) \implies (aRc)$ [2]. If we let $f(x)$ be the polynomial time reduction function $\mid x \in L_1 \iff f(x) \in L_2$. We can then define $g(x)$ to be the polynomial time reduction function $\mid x \in L_2 \iff g(x) \in L_3$. We can then compute, in polynomial time, $g \circ f \mid x \in L_1 \iff g(f(x)) \in L_3$. $\therefore L_1 \leq_P L_3$, so $\leq_P$ is transitive.

Problem 3a. $L_1 = \{a^n b^n \mid 0 \leq n \leq 1000\}$
Since $n \leq 1000$, L_1 is finite. All finite languages are regular. All regular languages belong to Type 3.

Problem 3b. $L_2 = \{a^n b^n \mid n \geq 0\}$
Since $n \geq 0$, L_2 is infinite. This is not a regular language as it does not satisfy the pumping lemma. There is, however, a context free grammar. Context free grammars are of Type 2.

Problem 3c. $L_3 = \{a^n b^m \mid n, m \geq 0\}$
L_3 can be written as a regular expression in the form $a^* b^*$, therefore this is regular and regular grammars are of Type 3.

Problem 4. By contradiction, assume that there exists some maximal clique K of graph G such that $\chi(G) < |K|$. Let's also assume a proper coloring [3] C of G which takes $\chi(G)$ colors. Using the pigeonhole principle [4], let the vertices of K be the pigeons, while the colors assigned to them are the holes. At least two vertices in K should be the same color, and we can label them V_1 and V_2. $\therefore C(V_1) = C(V_2)$. From the definition of a clique, we know that every pair of vertices in K are adjacent, therefore V_1 and V_2 are adjacent in K as well as G. But, as stated before, we also have $C(V_1) = C(V_2)$, which is a contradiction to the fact that C is a proper coloring. Therefore, we can conclude that $\chi(G)$ is no less than the size of any maximal clique of G.

Problem 5.

References

[1] "Turing machines." https://brilliant.org/wiki/turing-machines/. Accessed on 2022-09-27.

[2] "Transitive relation." https://en.wikipedia.org/wiki/Transitive_relation. Accessed on 2022-09-27.

[3] "Definition:proper coloring." https://proofwiki.org/wiki/Definition:Proper_Coloring. Accessed on 2022-09-28.

[4] "Pigeonhole principle." https://en.wikipedia.org/wiki/Pigeonhole_principle#Uses_and_applications. Accessed on 2022-09-28.