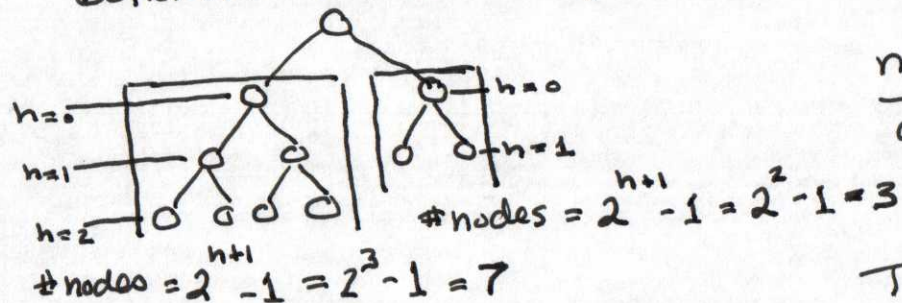


Page 155 "The children's subtrees each have size at most  $2n/3$  - the worst case occurs when the bottom level of the tree is exactly half full ..."

Bottom level half full example:



$$\frac{\text{nodes in LHS tree}}{\text{all nodes in tree}} = \frac{7}{7+3+1} = \frac{7}{11} \leq \frac{2}{3}$$

↑  
root

The number of nodes in a complete binary tree of height  $h$  is  $(2^{h+1} - 1)$

Algebra:

$$\frac{\text{nodes in LHS tree}}{\text{all nodes in tree}} = \frac{2^{h+1} - 1}{(2^{h+1} - 1) + (2^h - 1) + 1} = \frac{2 \cdot 2^h - 1}{2 \cdot 2^h + 2^h - 1} = \frac{2 \cdot 2^h - 1}{3 \cdot 2^h - 1} \leq \frac{2}{3}$$

↑ right subtree is one level less full than the left subtree