Summary of Master Method (CLRS 4th Ed.)

Case	Ratio check	Inequality that must be checked	Theorem 4.1 text page 102	Notes (see pages 103-104)
1	$\frac{f(n)}{n^{log_b a}} = n^{-\varepsilon}, \varepsilon > 0$	$f(n) \le cn^{log_b a}$	If $f(n) \in O\left(n^{log_b a-\varepsilon}\right)$ for some constant $\varepsilon > 0$, then $T(n) \in \Theta\left(n^{log_b a}\right)$ **[Total cost dominated by cost of the recursion tree leaves.]**	Not only must $f(n)$ be smaller than $n^{log_b a}$, it must be polynomially smaller
2	$\frac{f(n)}{n^{log_b a}} = lg^k n, k \ge 0$	$c_1 n^{log_b a} \le f(n) \le c_2 n^{log_b a}$	If $f(n) \in \Theta\left(n^{log_b a}\, lg^k\, n\right)$ where $k \ge 0$, then $T(n) \in \Theta\left(n^{log_b a}\, lg^{k+1}\, n\right)$ **[Total cost evenly distributed over recursion tree levels.]**	Most common situation occurs when $k =$ 0 then $T(n) \in \Theta\left(n^{log_b a} \lg n\right)$
3	$\frac{f(n)}{n^{log_b a}} = n^{\varepsilon}, \varepsilon > 0$	$cn^{log_b a} \le f(n)$	If $f(n) \in \Omega\left(n^{log_b a+\varepsilon}\right)$ for some constant $\varepsilon > 0$, and $af\left(\frac{n}{b}\right) \le cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) \in \Theta\left(f(n)\right)$ **[Total cost dominated by cost of the recursion tree root.]**	Not only must $f(n)$ be larger than $n^{log_b a}$, it must be polynomially larger and satisfy the "regularity" condition $af\left(\frac{n}{b}\right) \le cf(n)$ If $f(n)$ has the form n^i $c = \left(\frac{a}{b^i}\right)$ which is < 1

Notes:

Applies only to recurrences $T(n)$ on $n \in \mathbb{N}$ by $T(n) = aT(n/b) + f(n)$, where $a > 0$ and $b > 1$ are constants, and $f(n)$ is a driving function that is defined and nonnegative on all sufficiently large reals.

See page 66 for definition $lg^k\, n \equiv (lg\, n)^k$

The Critical Exponent, $E = \frac{log\, a}{log\, b} = log_b\, a$, and the Watershed function $n^E = n^{log_b a}$.

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If $f(n)$ is smaller than $n^{log_b a}$ but not polynomially smaller, or

if $f(n)$ is larger than $n^{log_b a}$ but not polynomially larger, or

if the "regularity" condition $af\left(\frac{n}{b}\right) \le cf(n)$ is not satisfied,

the Master method cannot be used to solve the recurrence.