# Equivariant Graph Neural Networks

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# Outline

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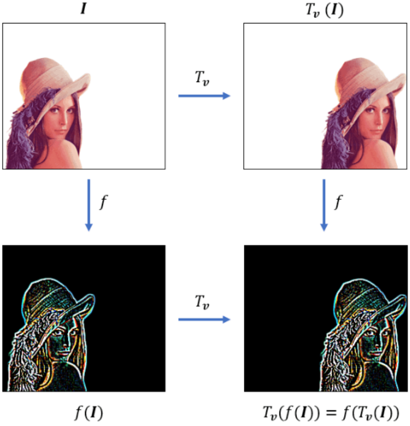

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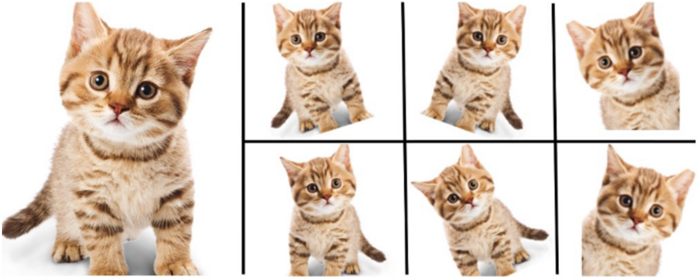
- A cat is a cat no matter how you look at it.



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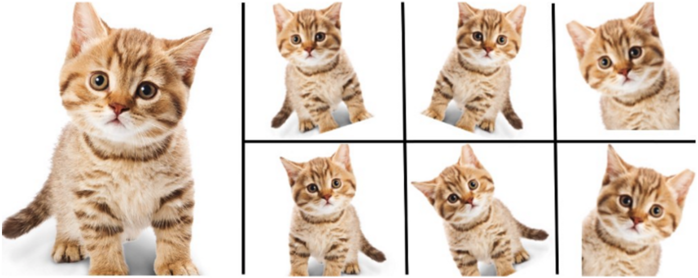

- It is acceptable to assume that being invariant to the rotation of the cat is a good property for a classification network.

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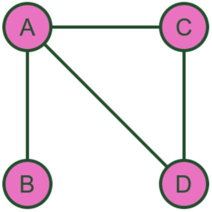
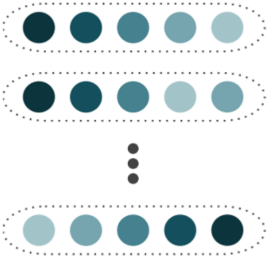
- Our focus today is on sets and graph data.




Simple graph

| | A | B | C | D |
|---|---|---|---|---|
| **A** | 0 | 1 | 1 | 1 |
| **B** | 1 | 0 | 0 | 0 |
| **C** | 1 | 0 | 0 | 1 |
| **D** | 1 | 0 | 1 | 0 |



# The Permutation Group $S_n$

- The permutation group $S_n$ is the group of all permutations of $n$ elements.
- It has $n!$ elements, representing the $n!$ ways to order $n$ elements.
- Given a set $X = \{x_1, x_2, \ldots, x_n\}$, a permutation $\pi \in S_n$ is a bijection $\pi : X \to X$
- e.g. $x = (x_1, x_2, x_3)$, and $\pi = (1, 2, 3) \in S_3$ is the permutation that maps $1 \to 2$, $2 \to 3$ and $3 \to 1$.
- We denote the **action** of $\pi$ on $x$ as $\pi x = (x_3, x_1, x_2)$.

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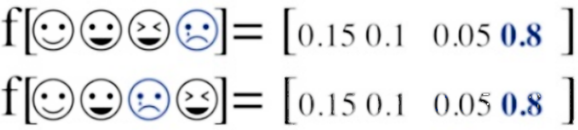

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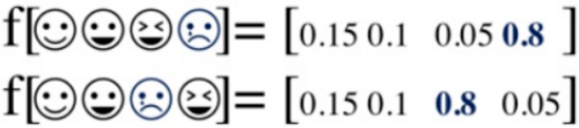

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- An equivariant neural network is a function $f : \mathbb{R}^{n \times d} \to \mathbb{R}^{n \times d'}$ such that $\boldsymbol{P}f(X) = f(\boldsymbol{P}X)$.

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- The action of $\boldsymbol{P}$ on $(A, X)$ is $(\boldsymbol{P}^{\boldsymbol{T}} A \boldsymbol{P}, \boldsymbol{P} X)$.

# Conclusion

- end

Thank You!