mixtures

Questions

- Mouth tone versus body tone
- Boundary condition of pipe body resonance solution: air column starts at lower lip so both ends are at ambient pressure ($\Delta P = 0$)
- End correction? Pipe appears (sounds) a bit longer than it is
- Derivation and utility of sum and difference tones
- Temperament - scheme to adjust interval sizes to accommodate pure octaves

How pipes make sound

- Air enters pipe toe and pass through flue
- Air stream impinges on the pipe's upper lip, creating vortex-like structures (eddies)
- These structures contain a broad spectrum of frequencies
- Only the pipe's natural frequencies are excited/amplified.

Diagram of pipe

Deriving the resonance frequencies of a pipe

The acoustic pressure, that is the deviation from the ambient ($p \equiv P - p_0$), obeys the acoustic wave equation:

$$\frac{d^2P}{dt^2} = \frac{1}{c^2}\frac{d^2P}{dt^2}$$

Since the boundary conditions of the organ pipe are that the pressure deviation goes to zero (i.e. $P(x = 0, L) = 0$), solutions are sine waves. In medium with sound speed c, the dispersion relation is: $f = \frac{c}{\lambda}$

The resonant frequencies of this pipe is then given by: $f_n = \frac{nc}{2L}$

Comparison of theoretical and experimental spectra

	Number of half steps
Unison/octave	1, 8, 15, 22, ...
3^{rd}	3, 10, 17, 24, ...
5^{rd}	5, 12, 19, 26, ...

Get pitch class via modulo 7 (ex. $mod_7(24) = 3$ so the 24th is a version of the third.)

Related terminologies

Pipe length

Half-step distance

Diatonic distance

Partials/harmonics

Mixture basics

Contemporary American Organ (p. 67)

Mutation: ranks sounding pitches other than a unison/octave Mixture: combination of unisons/octaves and mutations controlled via one drawknob

Mixture size range from two ranks (12th and 15th) to 8 ranks.

Smallest pipe has speaking length of less than three-eighths of an inch. When a mutation voice is desired to be smallest than this length, instead made an octave lower. This is deemed a break. Happens one or multiple times across the span of the mixture.

Other factors in mixture design: * Pipe types: diapason/principal, flute, etc (different harmonic contents produce varying mixture characters) * Scaling (also affects harmonic content) * Voicing (ex. soft, loud, aggressive, etc.)

Mixture examples

Mixtures with no breaks

- Raushquinte or Grave Mixture
  - Only 12th and 15th
  - *Raush* in German, means intoxication, ecstasy, drunkeness
- Cornet
  - 3 to 5 ranks with a versino of a third on top
  - Common pitch constituents are (12-15-17) and (1-8-12-15-17)
- Sesquialtera
  - 12th and 17th for which series they support (ex. to support an 8' stop, pitches $2\frac{2}{3}$ and $1\frac{3}{5}$ are played)

Mixtures with breaks

- Mixture/Fourniture/Plein Jeu
- Scharf/Acuta
- Carillon/Glockenspiel
- Harmonics

Resultants Provides some of the overtone series to give the illusion of a non-existent, lower fundamental tone.

Compton Harmonics

(Arthur) Harrison Harmonics Page 303 of Barnes:
32' Grand Cornet on Pedal CCC plays: * Bourdon 16': 16 $10\frac{2}{3}$ * Gemshorn: $10\frac{2}{3}$ $5\frac{1}{3}$ 4 $3\frac{1}{5}$

(All octaves and fifths.)

Table 2: Monkies live in the zoo.

Description	Diatonic number	Chromatic distance	Pipe length (ft)	Common Names
Root, unison	1	0	8	Unison
Octave	8	12	4	Super octave
Oct + 5th	12	17	$2\frac{2}{3}$	Nazard, quint
2 octaves	17	24	2	Fifteenth
2 oct + major 3rd	17	28	$1\frac{3}{5}$	Tierce
2 oct + 5th	19	31	$1\frac{1}{3}$	Larigot
2 oct + min 7th	Flatted 21st	34	$1\frac{1}{7}$	Septime
3 oct	22	38	1	None

Labial (lip) pipe: Synonym for flue pipe

Temperament

A temperament is a system of tuning that adjusts intermediate interval sizes while retaining pure octaves.

Just Intonation

Pythagorean

Meantone

Well-temperament

Equal temperament

Uses a single ratio between adjacent notes. Let this ratio be R. Let the first note in a scale be f_0, the second be f_1, and so on.

$$R = \frac{f_1}{f_0} = \frac{f_2}{f_1} = ... = \frac{f_{12}}{f_{11}}$$
$$R^{12} = \frac{f_{12}}{f_0}$$

Since we want octaves to be equally spaced, adjacent octaves will have a frequency ratio of two. Therefore:

$$R = \sqrt[12]{2} = 1.0594630943592953$$

We may equivalently write the twelfth root of two as $2^{\frac{1}{12}}$. This notation will be a bit more compact in the following section.

Now that the ratio R has been solved for, we can write out the first few frequencies.

$f_1 = 2^{\frac{1}{12}} f_0$

$f_2 = 2^{\frac{1}{12}} f_1 = \left(2^{\frac{1}{12}}\right)^2 f_0 = 2^{\frac{1}{6}} f_0$

$f_3 = 2^{\frac{1}{12}} f_2 = 2^{\frac{1}{12}} 2^{\frac{1}{6}} f_0 = 2^{\frac{1}{4}} f_0$

And so on.

A typical value for reference frequency is 440 Hertz at 70 degrees Fahrenheit.

Scale degree	Fraction	Decimal approx.
0	1	1.000
1	$2^{\frac{1}{12}}$	1.059
2	$2^{\frac{1}{6}}$	1.112
3	$2^{\frac{1}{4}}$	1.189
4	$2^{\frac{1}{3}}$	1.260
5	$2^{\frac{5}{12}}$	1.334
6	$2^{\frac{1}{2}}$	1.414
7	$2^{\frac{7}{12}}$	1.498
8	$2^{\frac{2}{3}}$	1.587
9	$2^{\frac{3}{4}}$	1.682
10	$2^{\frac{5}{6}}$	1.782
11	$2^{\frac{11}{12}}$	1.888
12	2	2.000