



AHEAD OF WHAT'S POSSIBLE™

Virtual  
**GRCoN**20

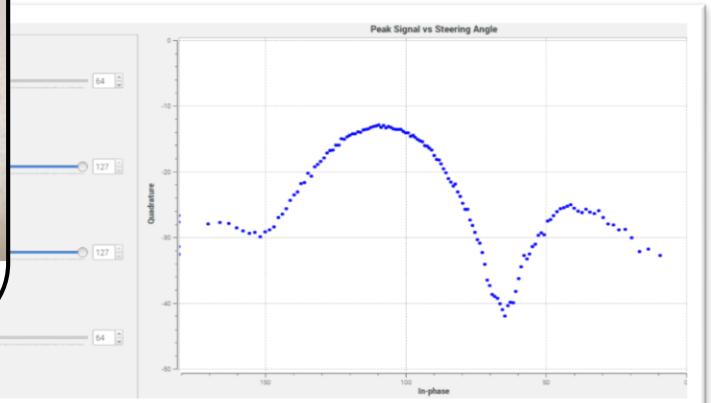
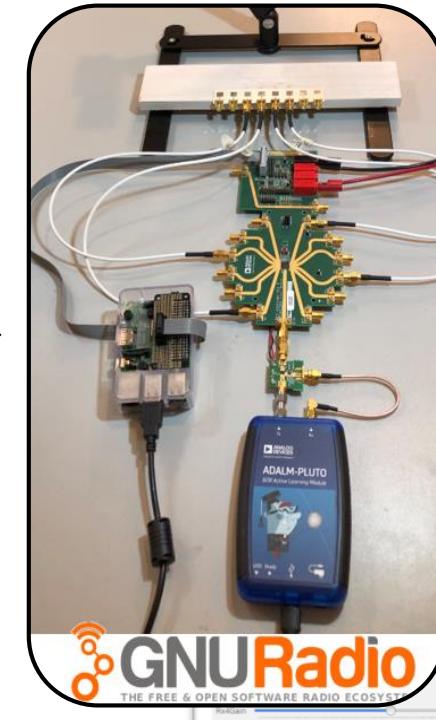
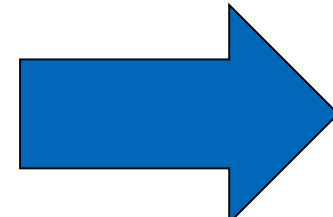
Phased Array  
Beamforming Workshop  
*Understanding and Prototyping*

Jon Kraft, Analog Devices

Sept 17, 2020

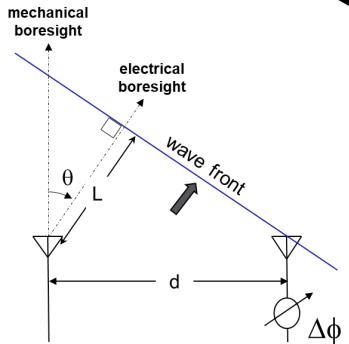
# Goals for this Workshop

1. Gain an **intuitive** understanding of beamforming concepts
2. **Hands on** experimenting with these concepts
3. Quickly **prototype** your own phased array system

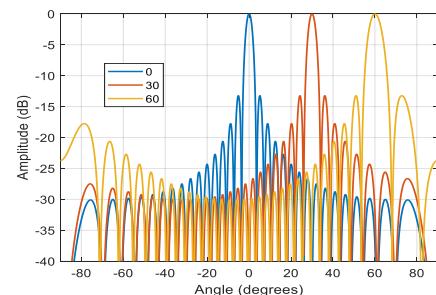


# Overview of the Phased Array Workshop

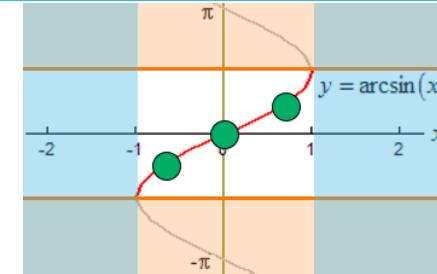
## Steering Angle



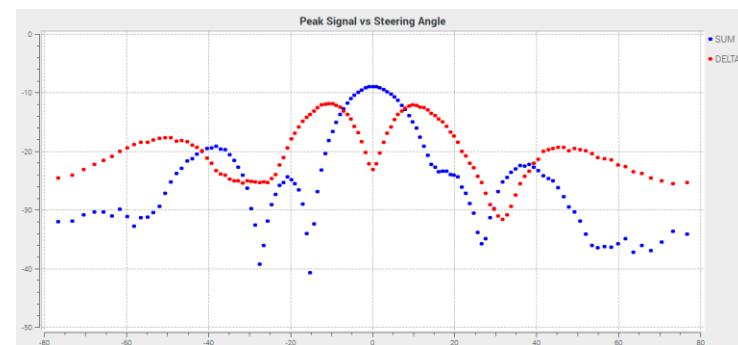
## Antenna Patterns



## Antenna Impairments

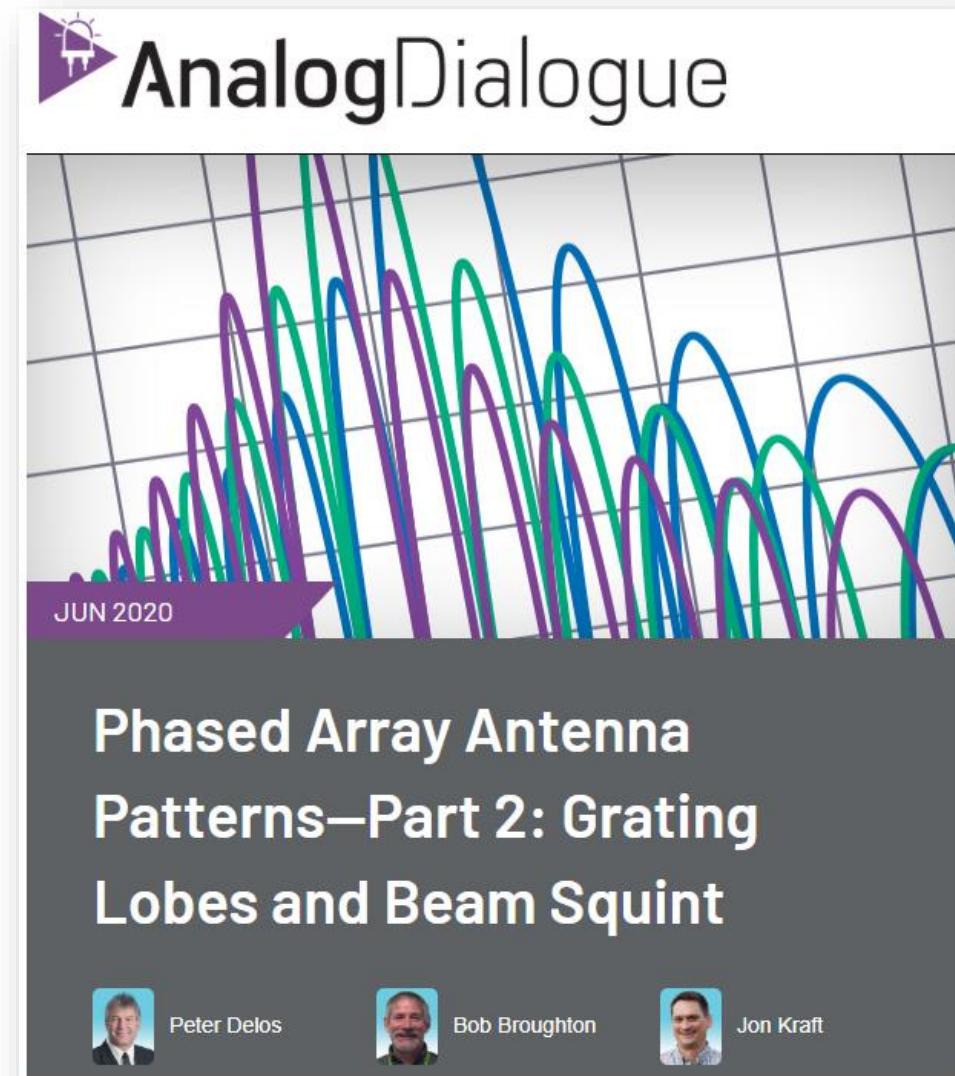


## Monopulse Tracking



# Acknowledgements

- ▶ **Bob Broughton** – A very patient teacher!
- ▶ **Pete Delos** – lead author of our Analog Dialogue Articles
- ▶ **Analog Device's System Development Group (SDG)**
  - This team creates our hardware+software ecosystem to get from prototypes to production
  - **Robin Getz, Travis Collins, Michael Hennerich, Mark Thoren**, and many more
- ▶ **Ozan Gurbuz**: designer of the X band patch antenna for this workshop. This antenna is available to all, link later.
- ▶ **Paul Clark**: Author of the Field Expedient SDR Books. These books are a great step by step guide through GNU Radio operation and SDR concepts. [www.fieldxp.com](http://www.fieldxp.com)



<https://www.analog.com/en/analog-dialogue.html>

# What is beamforming?

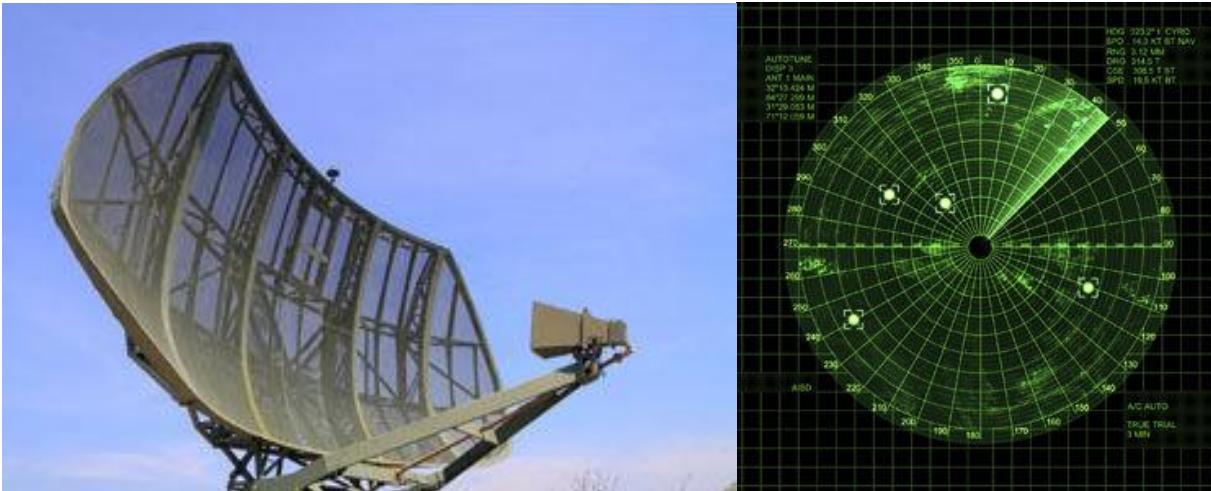
## Who uses it?

## Why does it matter?

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# What is Phased Array Beamforming?

- Rotating Antennas (mechanical gimbals)



- Phased Array antennas accomplish the same, but without mechanical movement

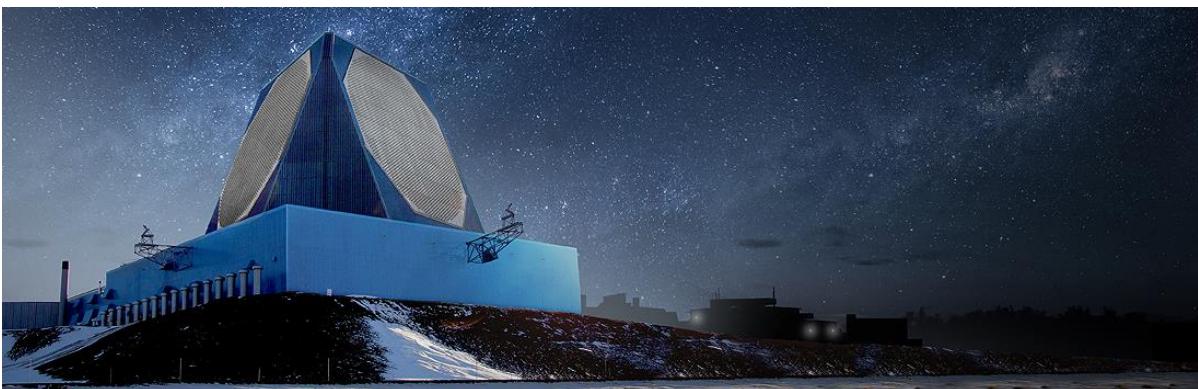
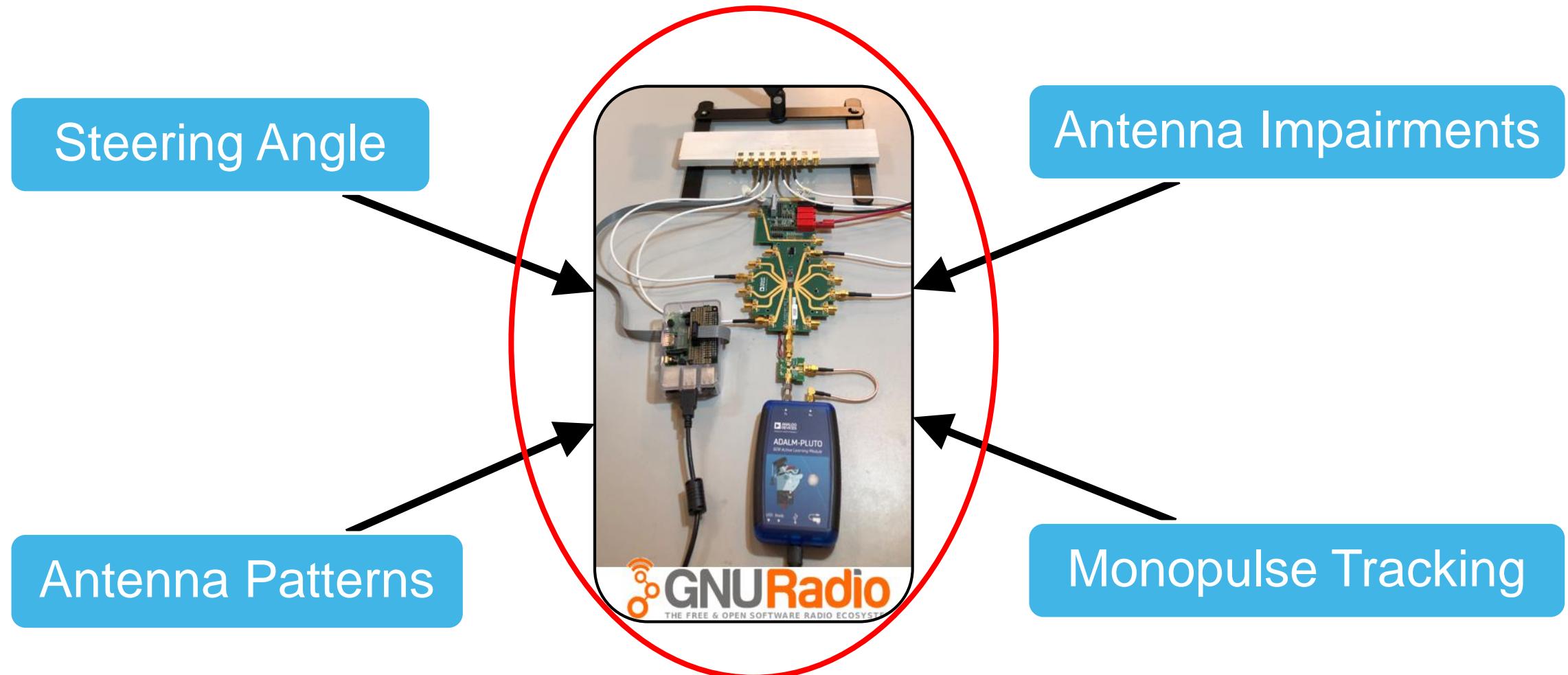


figure from <https://www.analog.com/en/technical-articles/an-interview-with-analog-devices-discussing-rf-electronics-for-phased-array-applications.html>

# Where is Phased Array Beamforming Used?

- Mobile Communications
- RADAR
- Satellite Communications





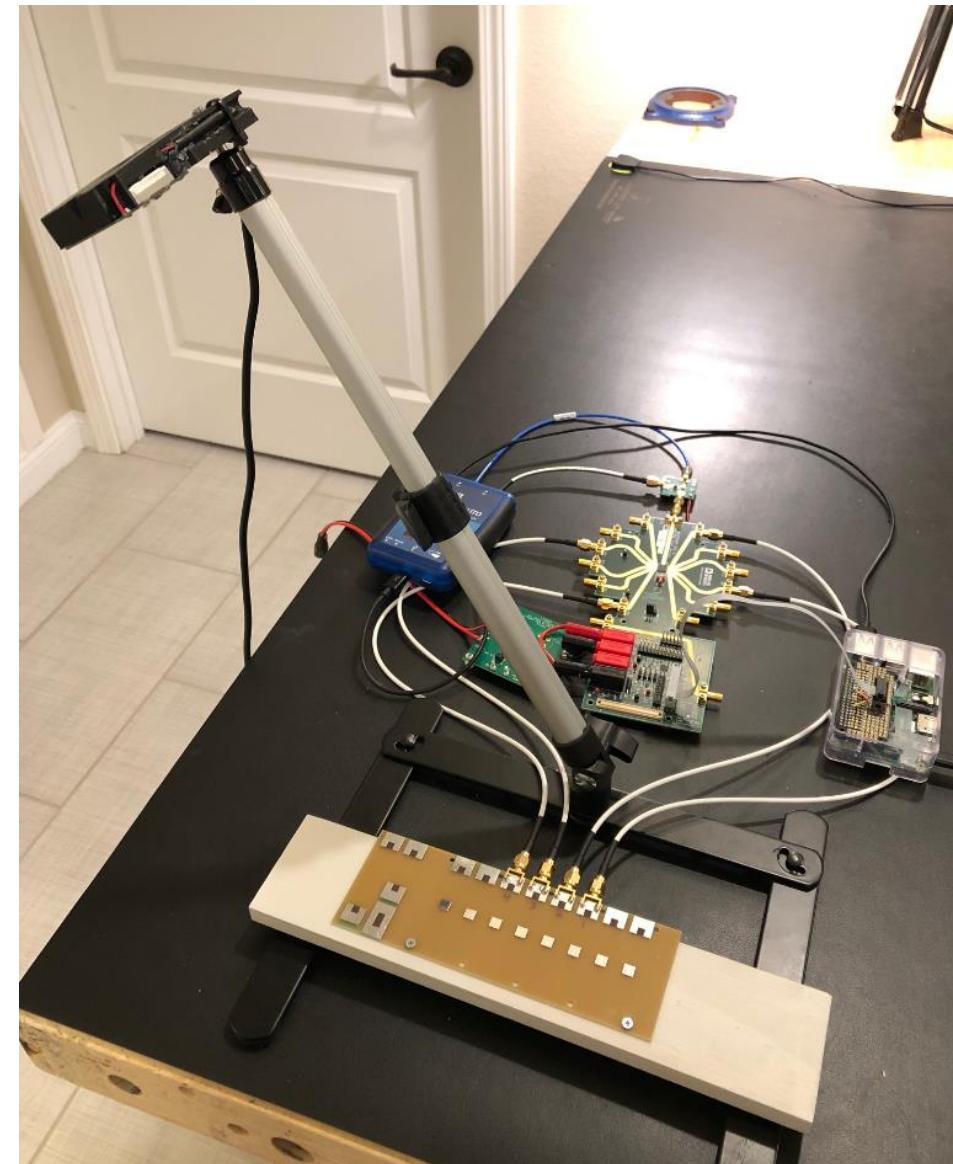
# Overview of Building Instructions:

- Bill of Materials (BOM)
- Assemble the Eval Boards
- Raspberry Pi Setup
- RF Source Assembly
- Antenna Assembly

YOU CAN BUILD THIS!

Instructions at:

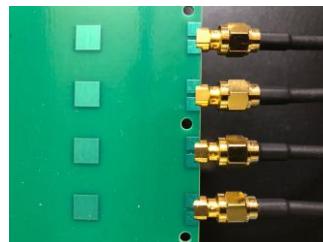
<http://www.github.com/JonKraft/PhasedArray>



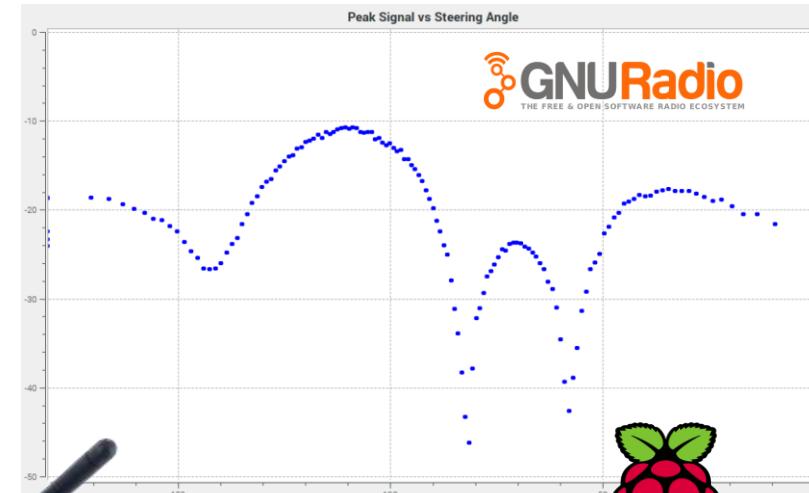
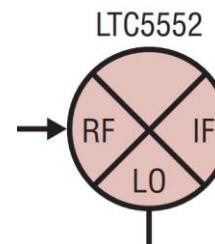
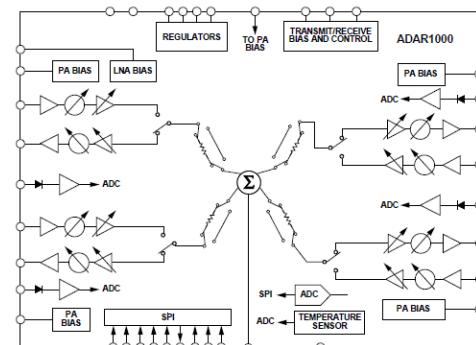
# Let's Experience the Math!

- ▶ Can we build our own beamformer and see this math “Hands On”?
- ▶ How would we do that?
  - Antenna
  - Beamformer
  - Mixer (for higher frequencies)
  - Transceiver (or data converter)
  - And don't forget SOFTWARE!!!!

4 Patch Antenna

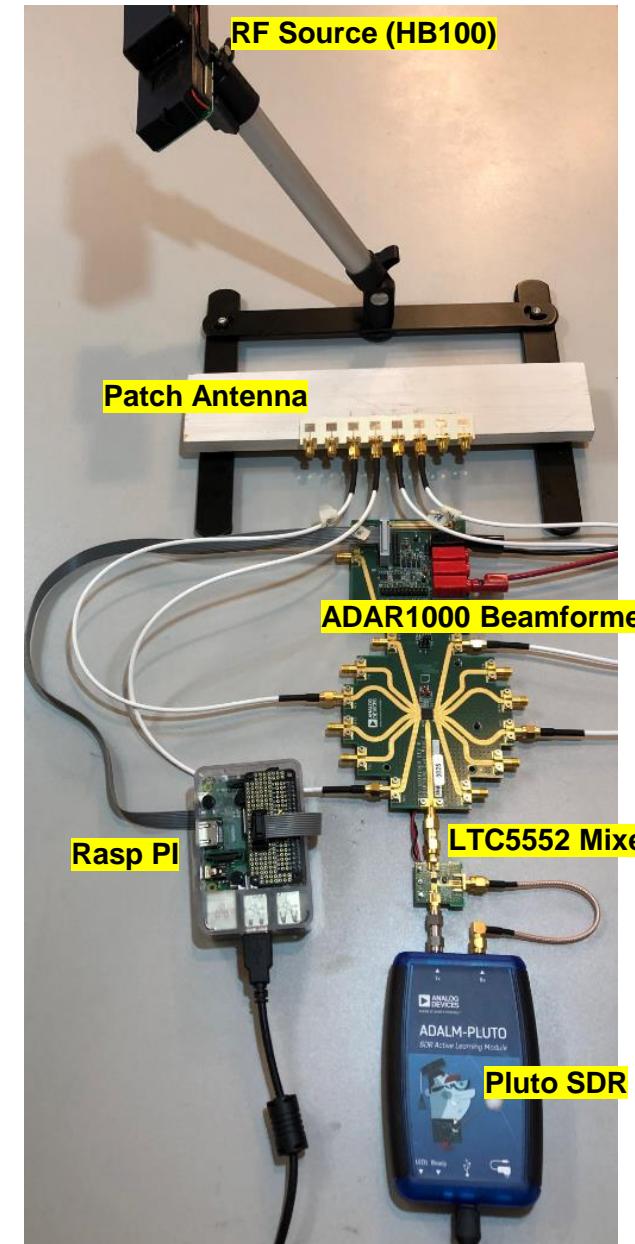
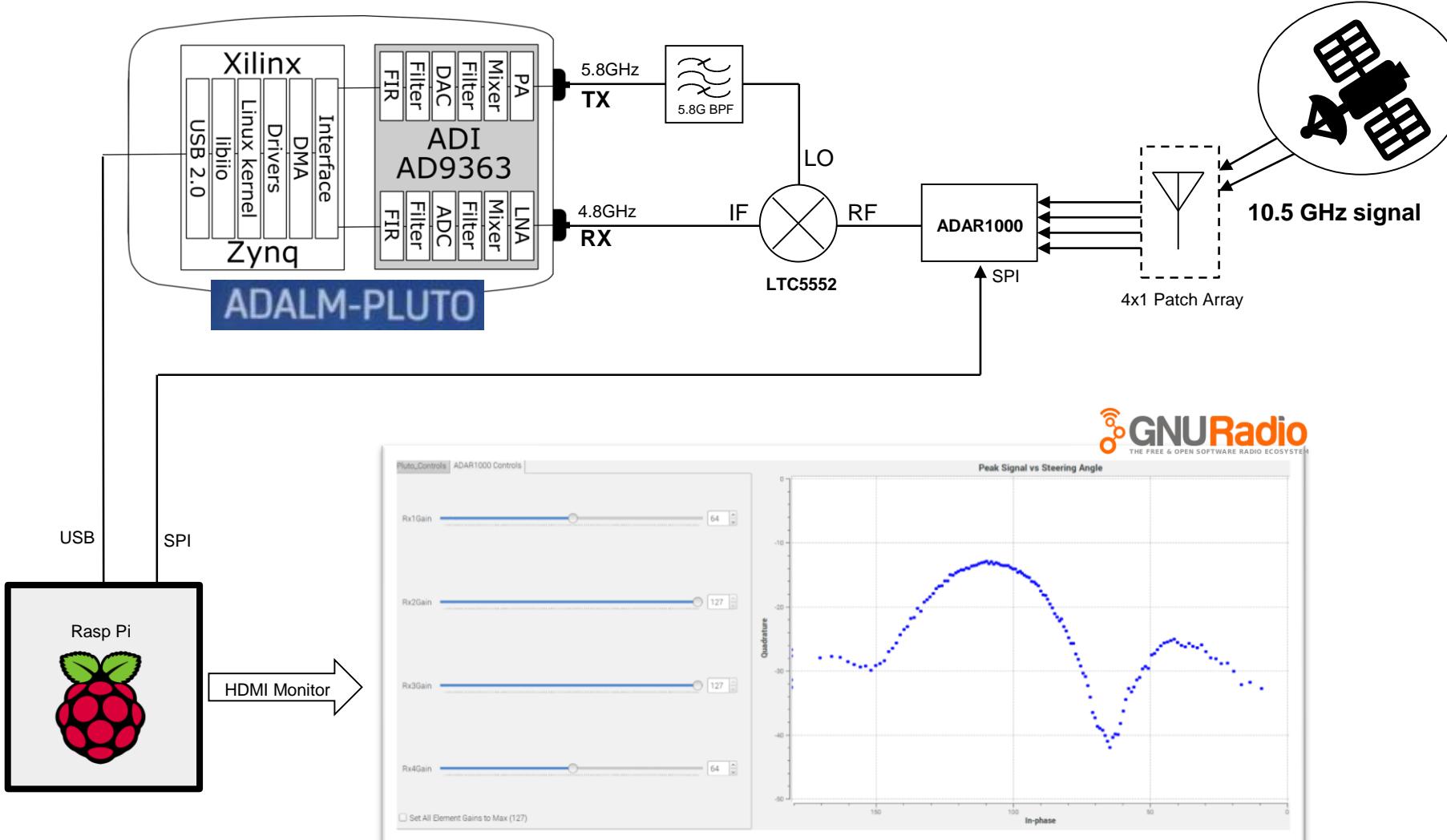


ADAR1000



**YOU CAN BUILD THIS!** Complete step by step instructions, with software, at [www.github.com/jonkraft/phasedarray](https://www.github.com/jonkraft/phasedarray)

# Simple Phased Array Setup



# Bill of Materials:

## Beamformer, Mixer, and SDR

Qty	Description	Other info	Part Number	Link
1	ADAR1000 Eval Board		EVAL-ADAR1000	<a href="https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/EVAL-ADAR1000.html#eb-overview">https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/EVAL-ADAR1000.html#eb-overview</a>
1	Pluto SDR		ADALM-PLUTO	<a href="https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/ADALM-PLUTO.html">https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/ADALM-PLUTO.html</a>
1	LTC5552 Mixer		DC2668A	<a href="https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/dc2668a.html">https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/dc2668a.html</a>
1	LT3045 3.3V LDO Board		DC2491A	<a href="https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/dc2491a.html">https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/dc2491a.html</a>
6	18" SMA Cable		415-0033-018	<a href="https://www.digikey.com/product-detail/en/TE Connectivity/ACX1240-ND/114-ND/457274">https://www.digikey.com/product-detail/en/TE Connectivity/ACX1240-ND/114-ND/457274</a>
1	SMA Connector		ACX1240-ND	
1	AC to DC wall wart			<a href="https://www.amazon.com/gp/product/B01N7RS0NG/ref=ppx_yo_dt_b_asin_title_o09_s00?ie=UTF8&amp;psc=1">https://www.amazon.com/gp/product/B01N7RS0NG/ref=ppx_yo_dt_b_asin_title_o09_s00?ie=UTF8&amp;psc=1</a>
1	2.1x5.5mm barrel jack			
1	Banana Jack			
1	Banana cable			
10	Bumper			
1	Raspberry Pi			<a href="https://www.amazon.com/gp/product/B01M27459S/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1">https://www.amazon.com/gp/product/B01M27459S/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1</a>
1	Raspberry Pi Polarity keeper			
1	rectangular male connector			
2	Ribbon cable connector			
1	Ribbon cable connector			

YOU CAN BUILD THIS!

Instructions at:

<http://www.github.com/JonKraft/PhasedArray>

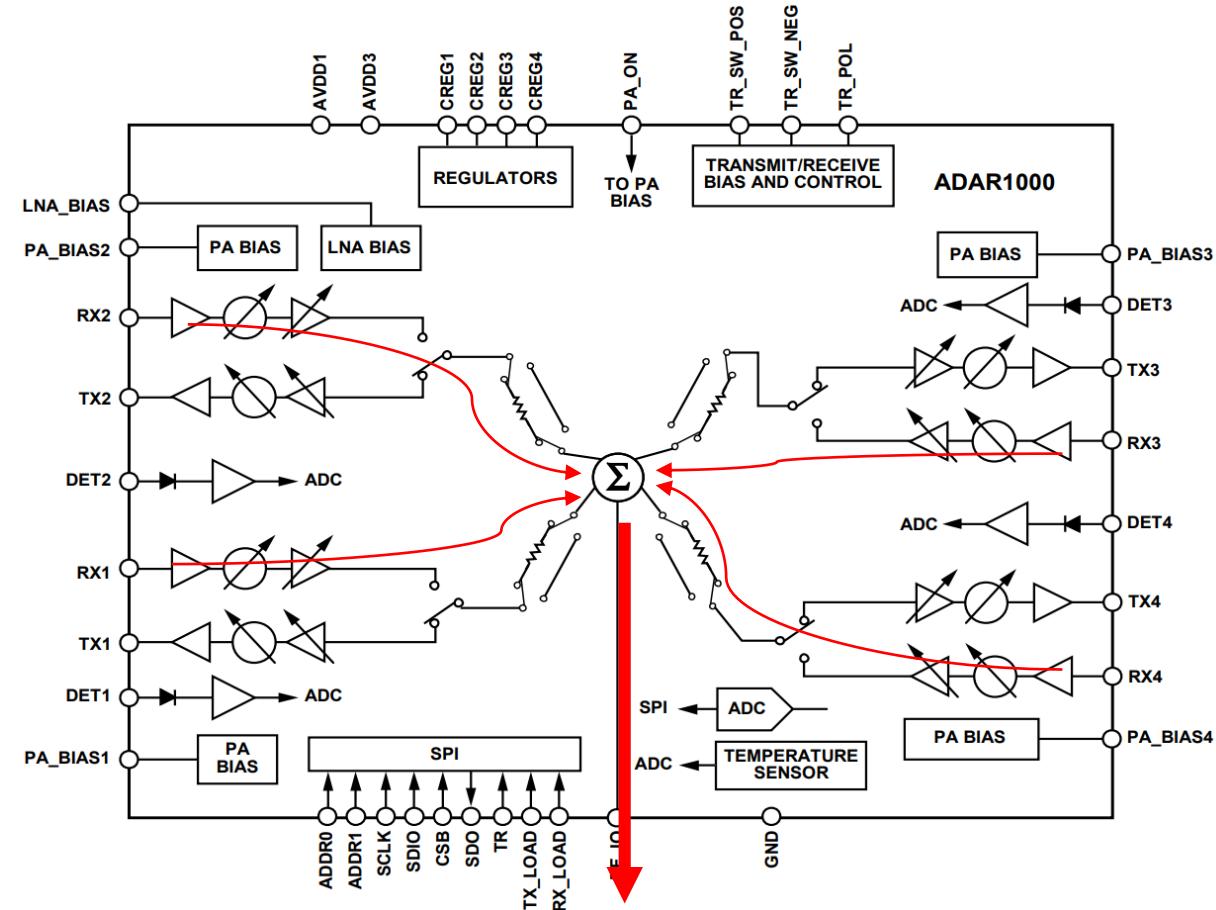
## Antenna Board, RF Source, and Stand

Qty	Description	Other info	Part Number	Link
4	CONN SMA JACK STR 50OHM EDGE MNT		314-1703-ND	<a href="https://www.digikey.com/product-detail/en/BU-1420701851/314-1703-ND/9950117/?itemSeq=310517966">https://www.digikey.com/product-detail/en/BU-1420701851/314-1703-ND/9950117/?itemSeq=310517966</a>
1	8 element 10.525GHz Patch Antenna			
1	10.525GHz RF Source			<a href="https://www.amazon.com/gp/product/B00FFW4AZ4/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1">https://www.amazon.com/gp/product/B00FFW4AZ4/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1</a>
1	Power cable for RF Source		AE10621-ND	<a href="https://www.digikey.com/product-detail/en/assmann-wsw-components/AK670-OE-BLACK/AE10621-ND/2391700">https://www.digikey.com/product-detail/en/assmann-wsw-components/AK670-OE-BLACK/AE10621-ND/2391700</a>
1	Stand for RF Source			<a href="https://www.amazon.com/gp/product/B07JR2Q1G1/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1">https://www.amazon.com/gp/product/B07JR2Q1G1/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&amp;psc=1</a>
1	Adapter to hold RF Source to Stand			<a href="https://www.amazon.com/gp/product/B07RJW34WB/ref=ppx_yo_dt_b_asin_title_o00_s02?ie=UTF8&amp;psc=1">https://www.amazon.com/gp/product/B07RJW34WB/ref=ppx_yo_dt_b_asin_title_o00_s02?ie=UTF8&amp;psc=1</a>
4	Magnet to hold antenna to stand		469-1063-ND	<a href="https://www.digikey.com/product-detail/en/radial-magnet-inc/8221/469-1063-ND/5400502">https://www.digikey.com/product-detail/en/radial-magnet-inc/8221/469-1063-ND/5400502</a>

# ADAR1000: 4 Channel Analog Beamformer

## ADAR1000 Features

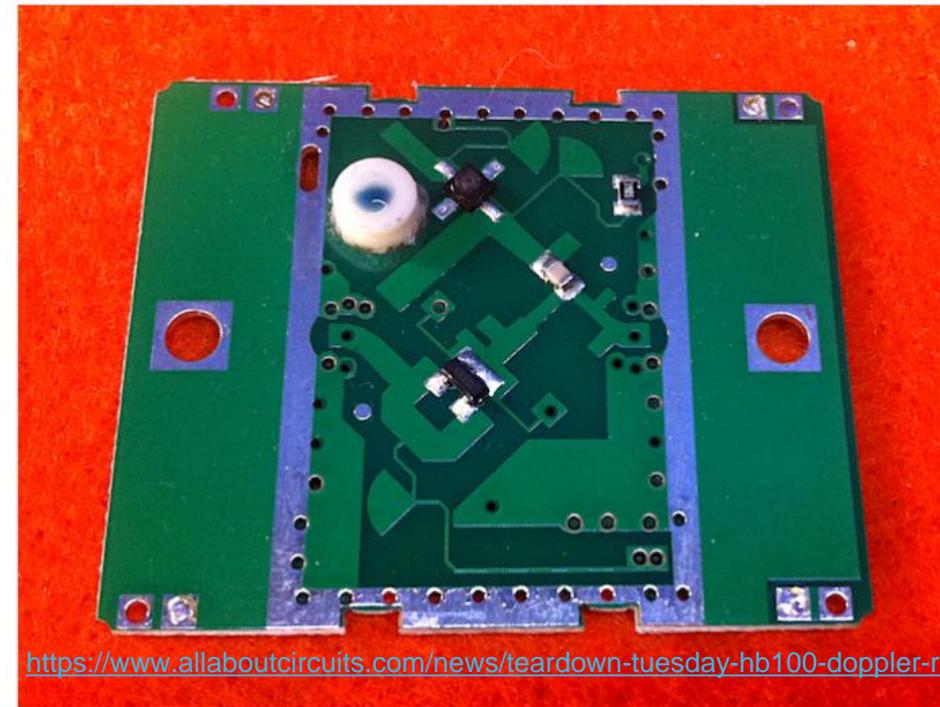
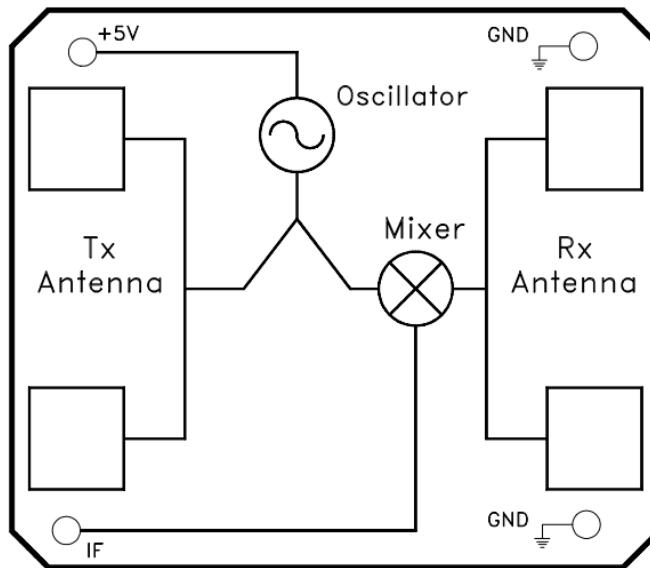
- ▶ 8 GHz to 16 GHz frequency range
- ▶ 360° phase adjustment range
- ▶ 2.8° phase resolution
- ▶  $\geq 31$  dB gain adjustment range



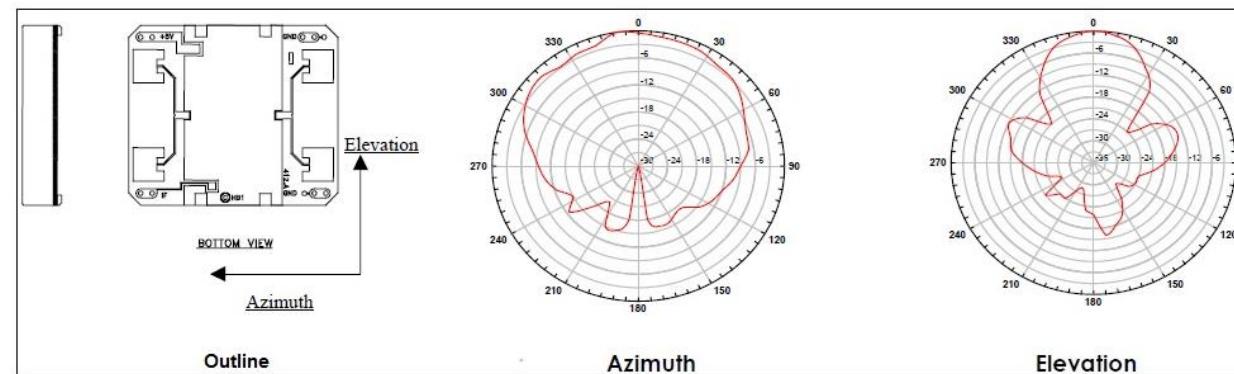
# 10.5GHz RF Source

## ► Use the ultra fun HB100!

- \$3 (includes shipping!) on Ebay
- Draws 40mA from 5V
- Generates a poor quality 10.5GHz tone
- It's good enough for us though!

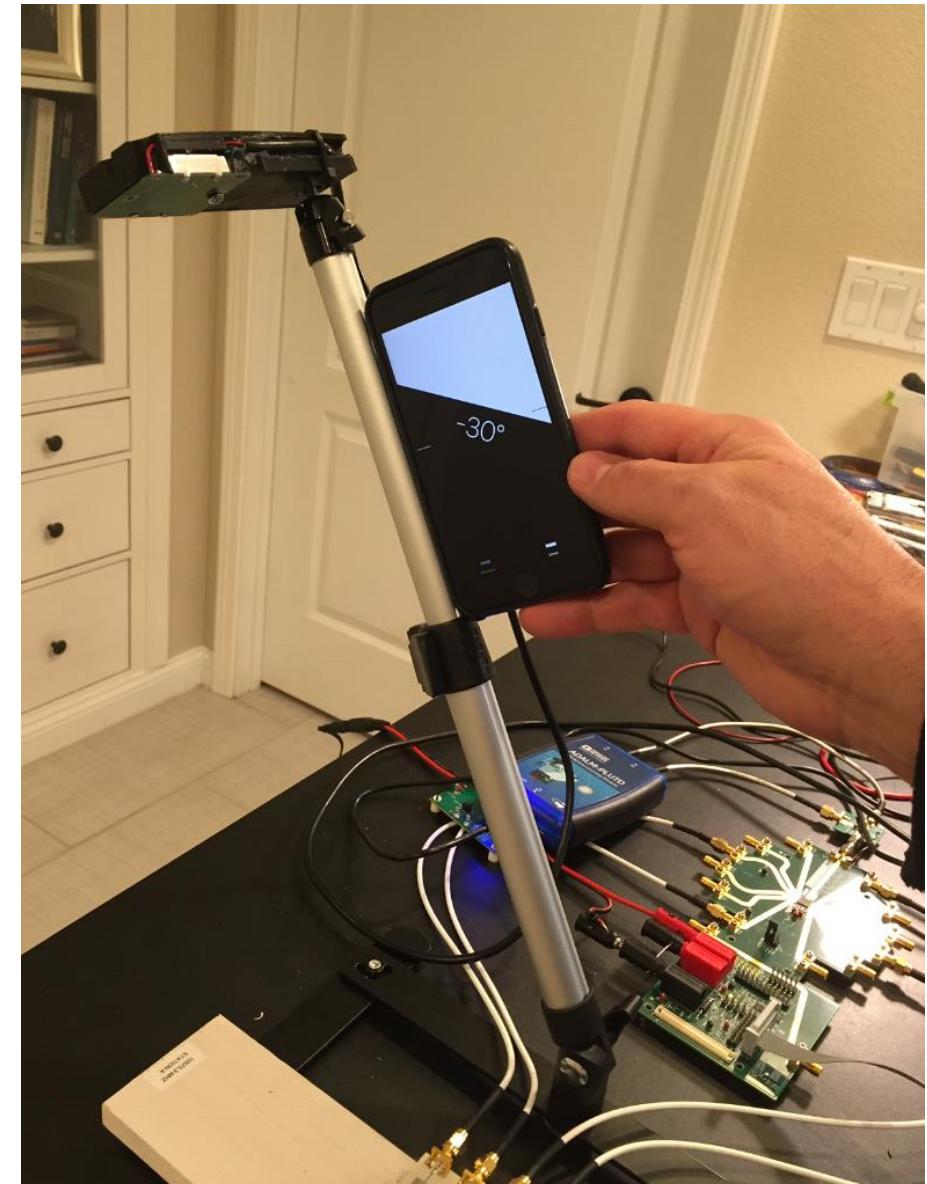
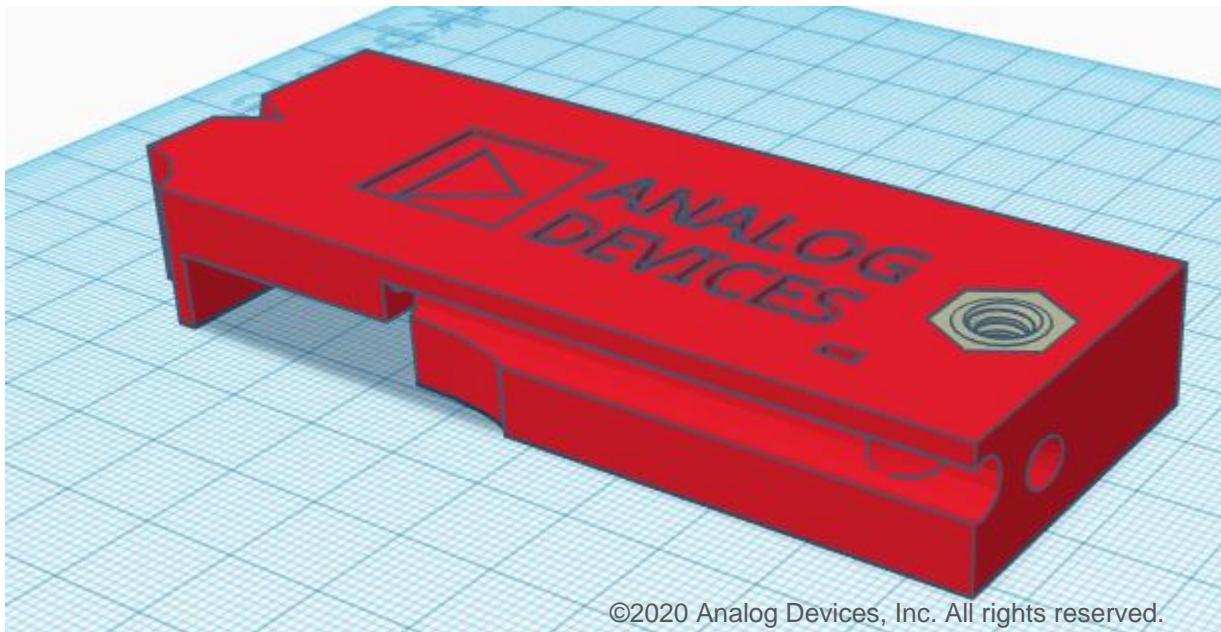


What sorcery is this?



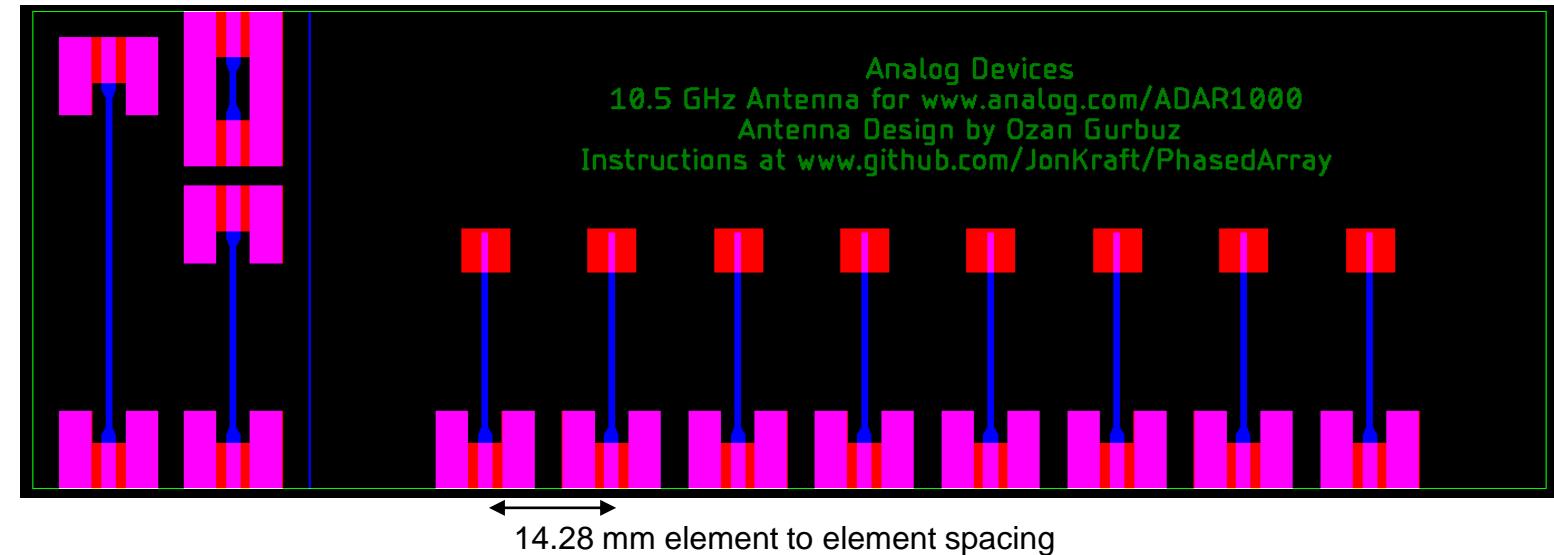
# HB100 RF Source Setup

- ▶ Add wires to 5V and GND (see next slide)
- ▶ Mount it to a stand:
  - This one works well:
  - [https://www.amazon.com/gp/product/B07JR2Q1G1/ref=ppx\\_yo\\_dt\\_b\\_asin\\_title\\_o00\\_s01?ie=UTF8&psc=1](https://www.amazon.com/gp/product/B07JR2Q1G1/ref=ppx_yo_dt_b_asin_title_o00_s01?ie=UTF8&psc=1)
- ▶ Then attach it with a 3D printed holder
  - .stl file available at [www.github.com/jonkraft/phasedarray](https://www.github.com/jonkraft/phasedarray)



# Patch Antenna

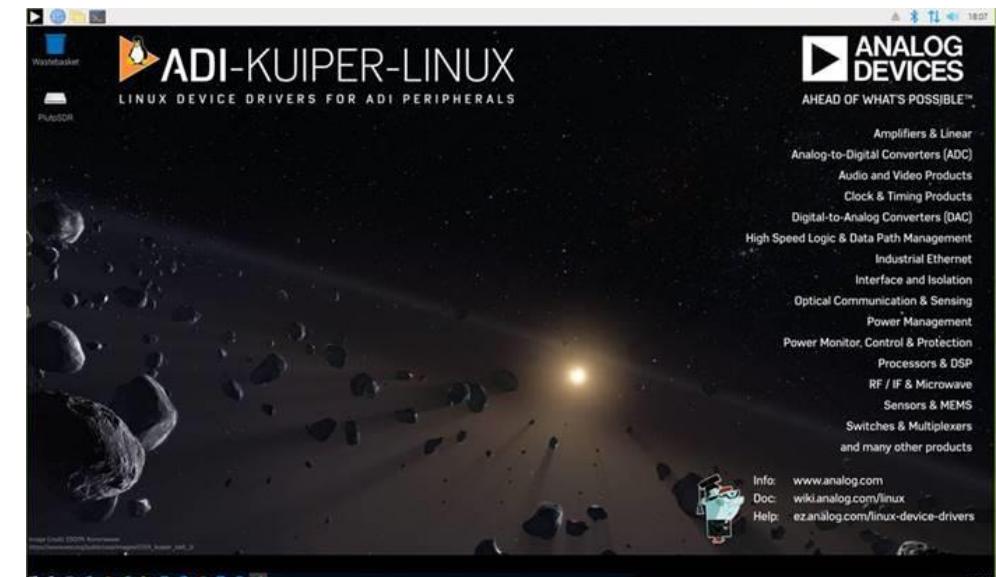
- A 10.5GHz patch antenna, designed for this lab by Ozan Gurbuz
- Contact your local Analog Devices sales person! They can get you one.
- The gerber files, to make your own, are available at:
  - [www.github.com/jonkraft/phasedarray](https://www.github.com/jonkraft/phasedarray)
- This antenna can also be ordered directly from PCBWAY:
  - Use this link: [https://www.pcbway.com/project/shareproject/10\\_5GHz\\_X\\_Band\\_Patch\\_Antenna.html](https://www.pcbway.com/project/shareproject/10_5GHz_X_Band_Patch_Antenna.html)



- Also thanks to Kent Britain [www.wa5vjb.com](http://www.wa5vjb.com) who made a fantastic 10GHz antenna for this
  - He does amazing custom antenna designs and is very reasonably priced and approachable.

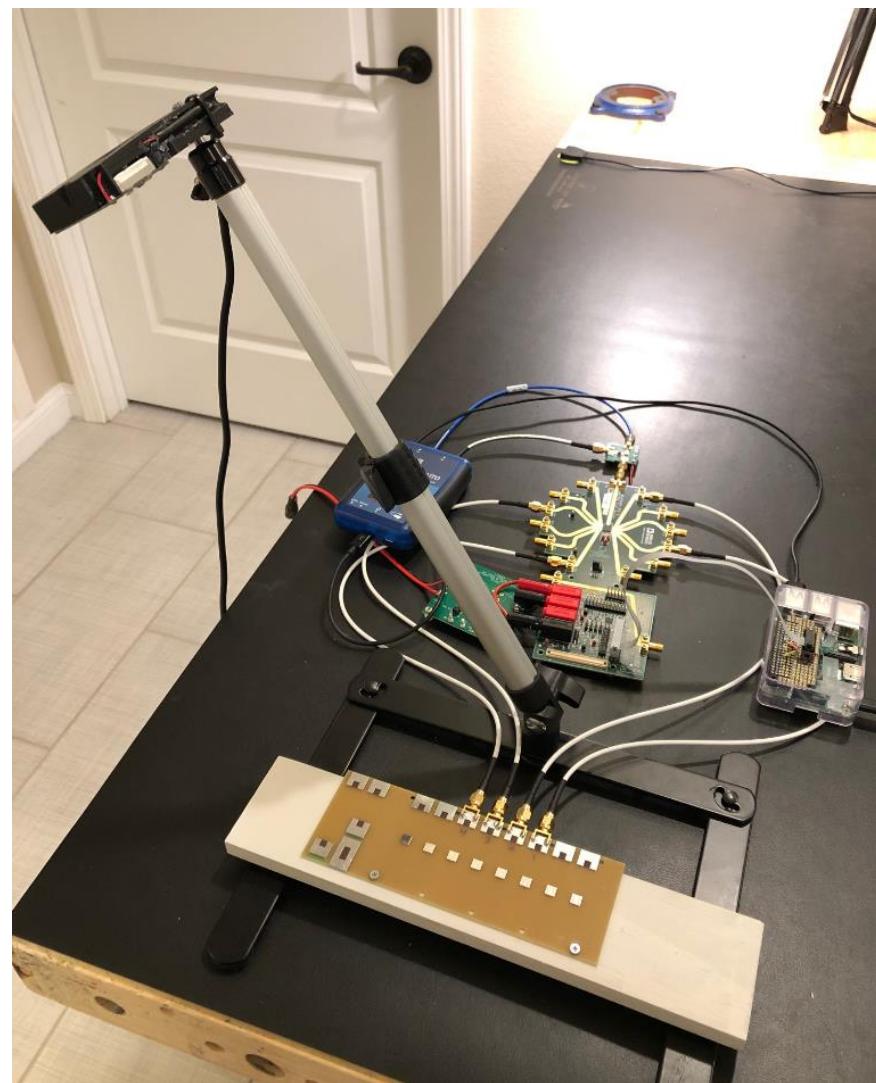
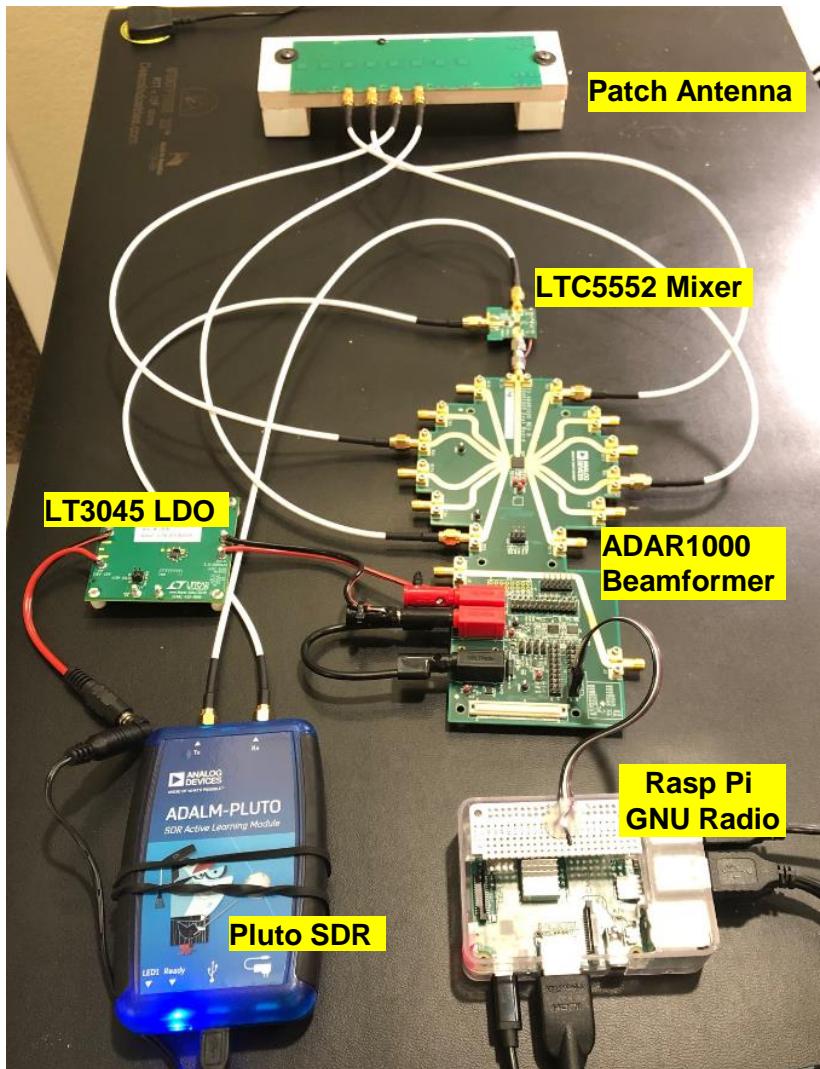
# Raspberry Pi Setup

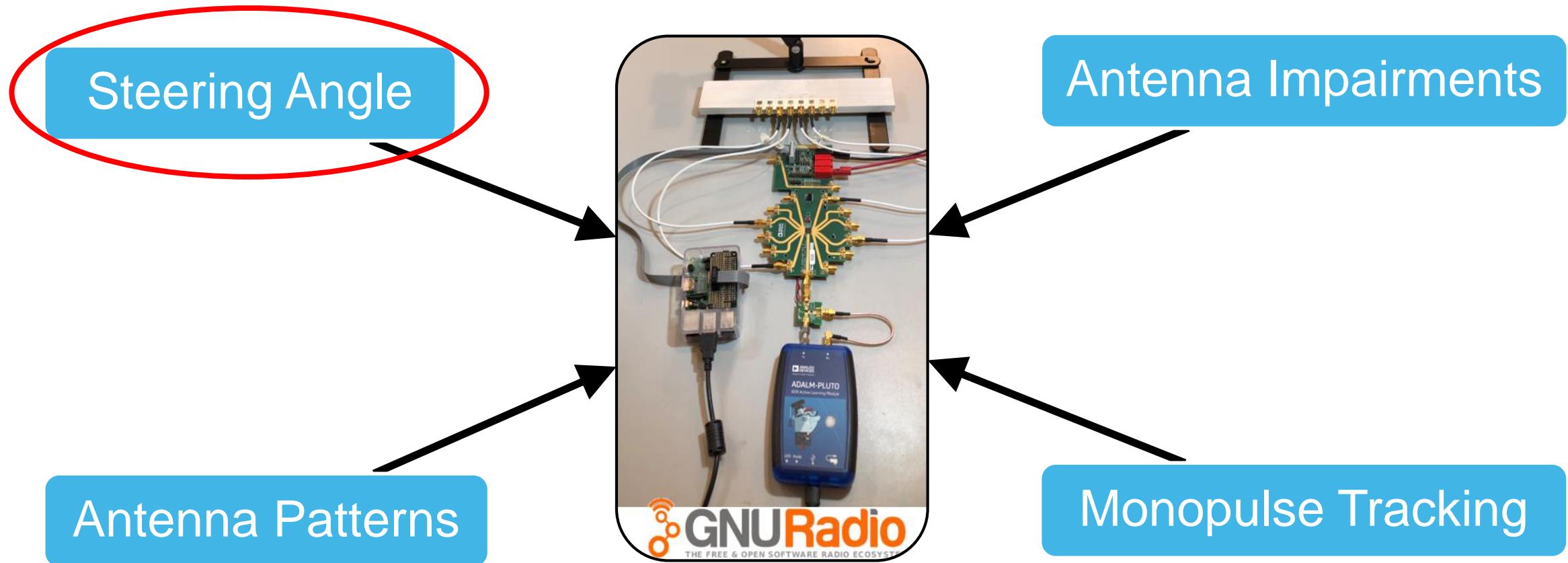
- › Raspberry Pi 3 or 4 will work.
- › 3 Options for Install
  - Install Manually:
    - <https://github.com/jonkraft/Pluto-Install-for-Raspberry-Pi>
  - Use ADI-Kuiper-Linux
    - Everything is preinstalled: GNURadio 3.8, IIO scope, LIBIIO, LIBM2K, PYADI-IIO, etc.
    - [https://wiki.analog.com/resources/tools-software/linux-software/adi-kuiper\\_images](https://wiki.analog.com/resources/tools-software/linux-software/adi-kuiper_images)
  - Complete out of the box, ready to go:
    - <https://download.analog.com/phased-array-lab/raspi.7z>
    - This has everything you need, including all GNU Radio 3.8 files, already loaded in there. No other installs are required, it'll work out of the box!



**For more info on Kuiper, please watch Mark Thoren's GRCon 2020 "Python for the Rest of Us"**

# Fully Assembled Lab Station:





# What is Phased Array Beamforming?

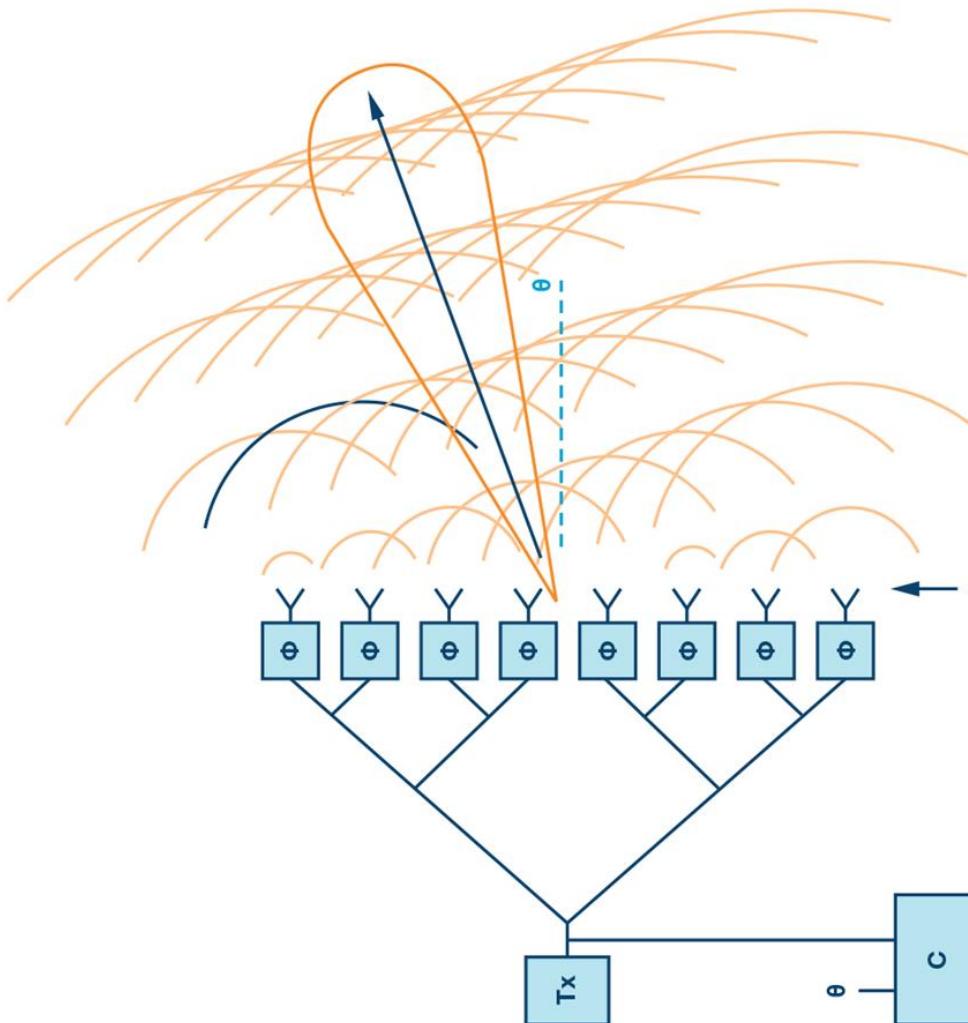
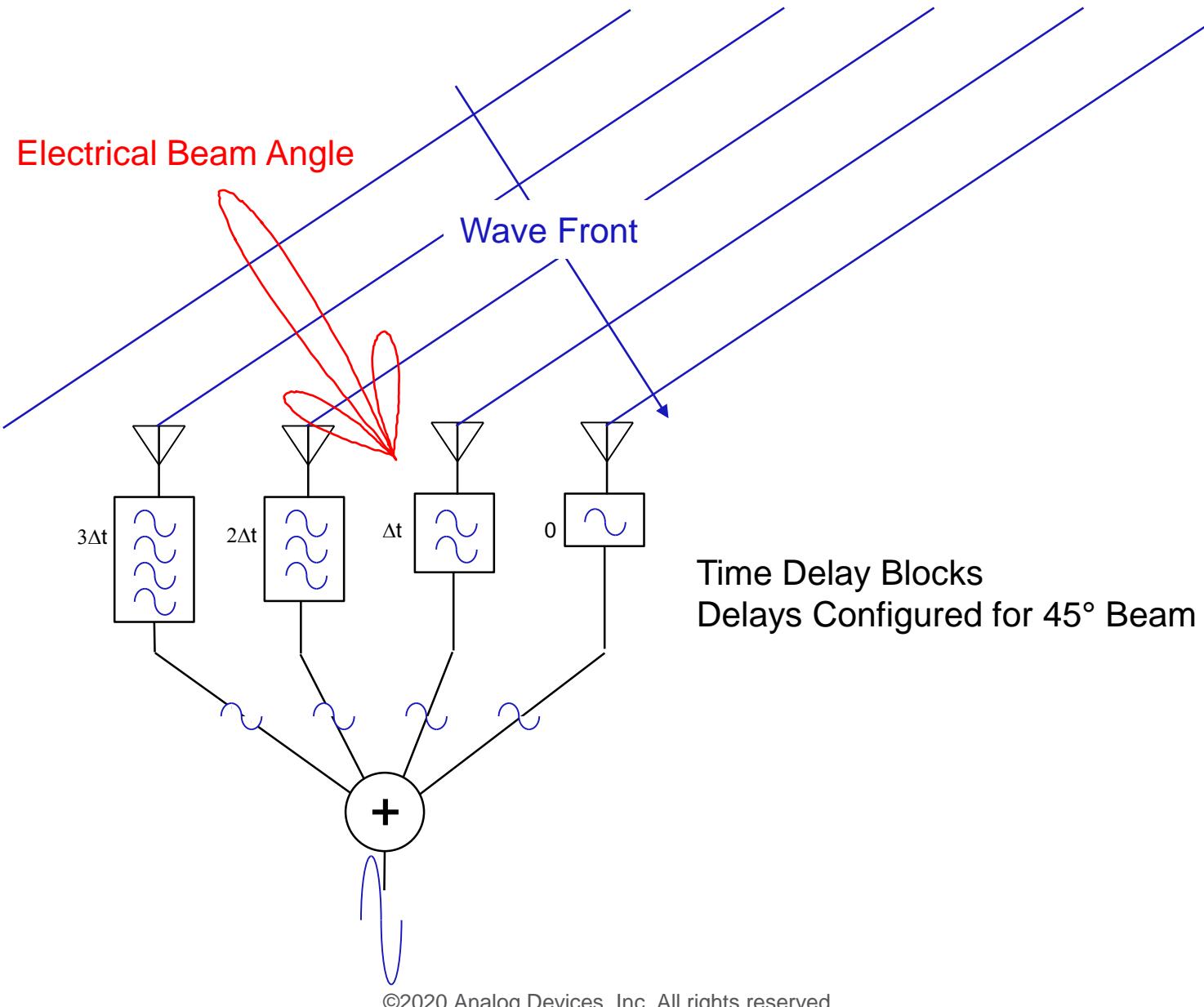
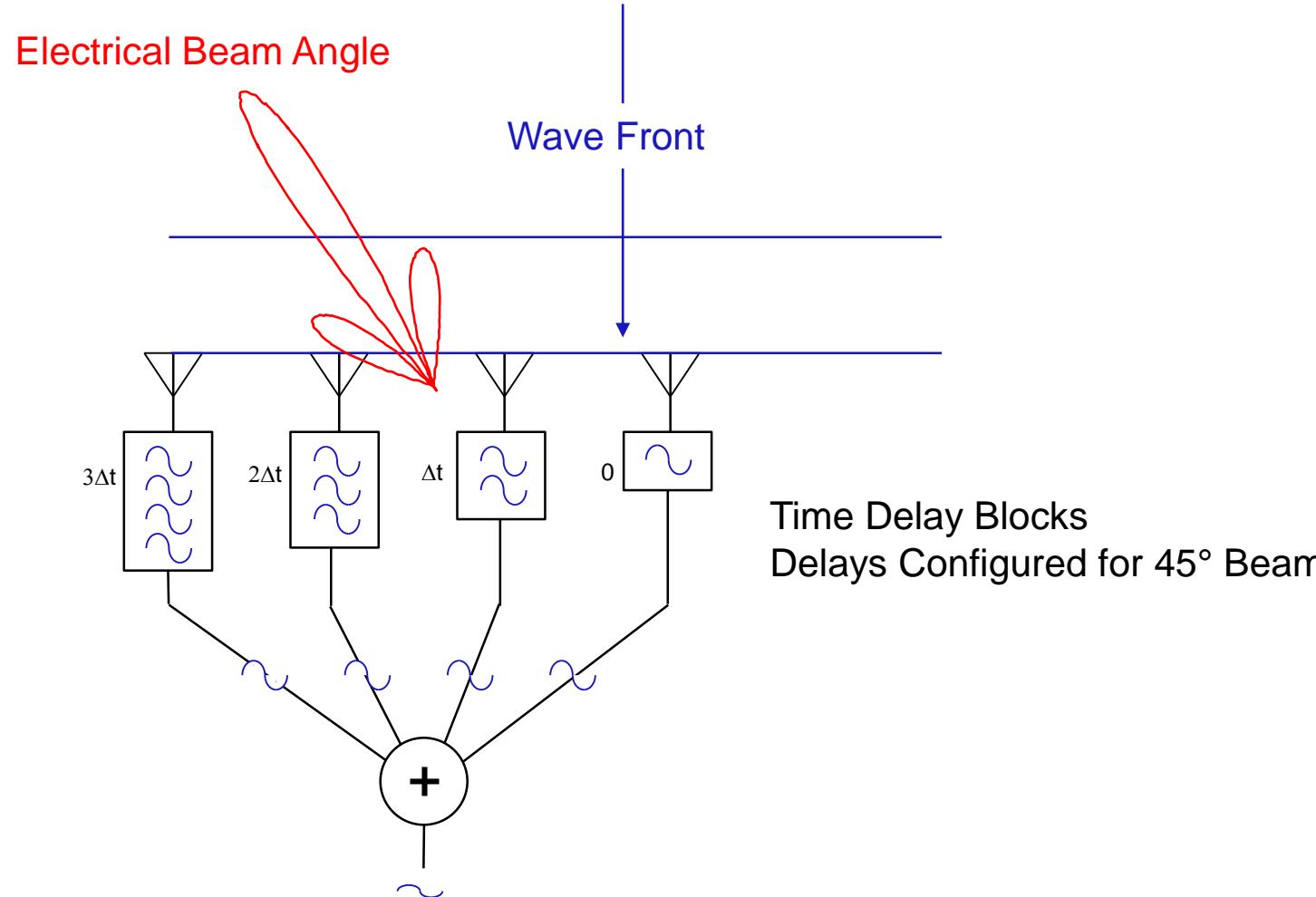


figure from <https://www.analog.com/en/analog-dialogue/articles/phased-array-beamforming-ics-simplify-antenna-design.html>

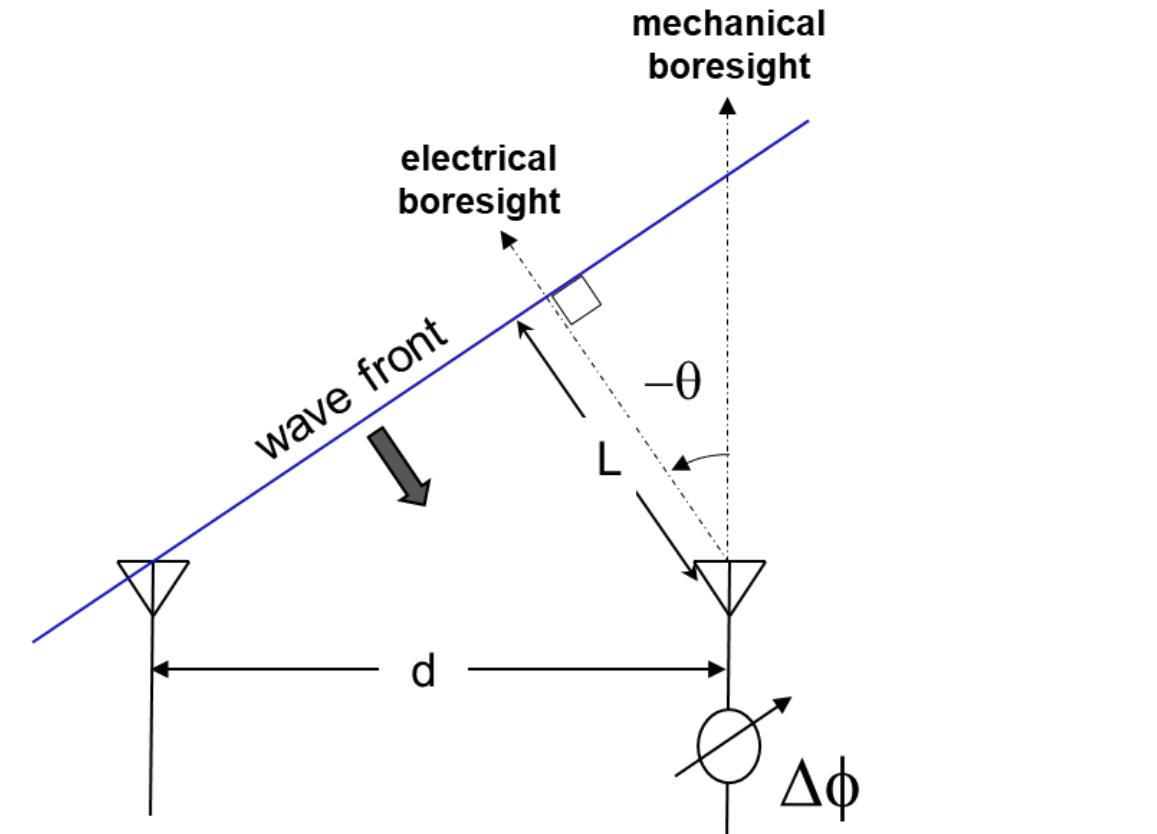
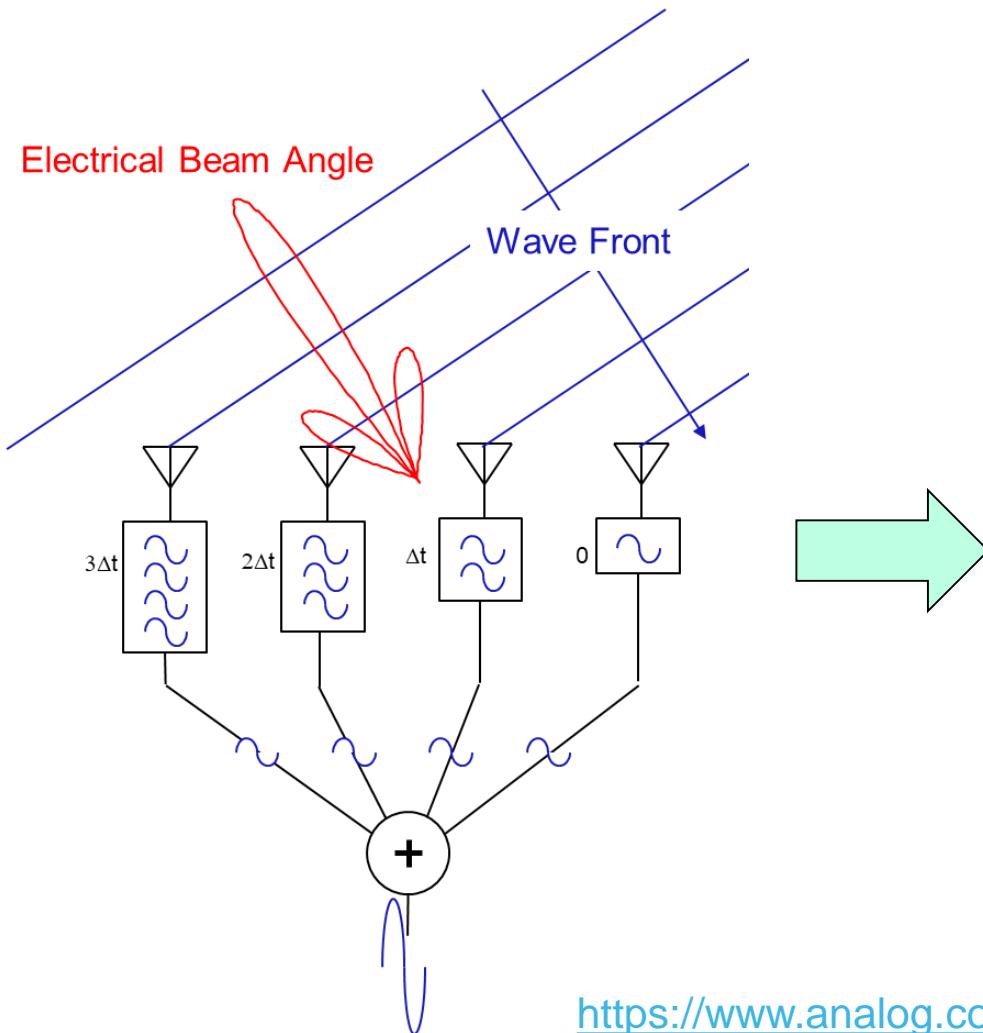
# Understanding Steering Angle: Math and Theory



# Understanding Steering Angle: Math and Theory



# Understanding Steering Angle: Math and Theory



$\theta$  - beam electrical angle

$L$  - incremental propagation distance between elements  
 $d$  - distance between elements

<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part1.html>

# Understanding Steering Angle: Math and Theory

There are 3 ways to describe this delay:

1. An incremental distance to travel:

$$L = d \sin \theta$$

2. A time delay between elements:

$$\Delta t = L / c = d \sin \theta / c$$

$$\rightarrow \theta = \sin^{-1}(\Delta t c / d)$$

3. A phase shift between elements:

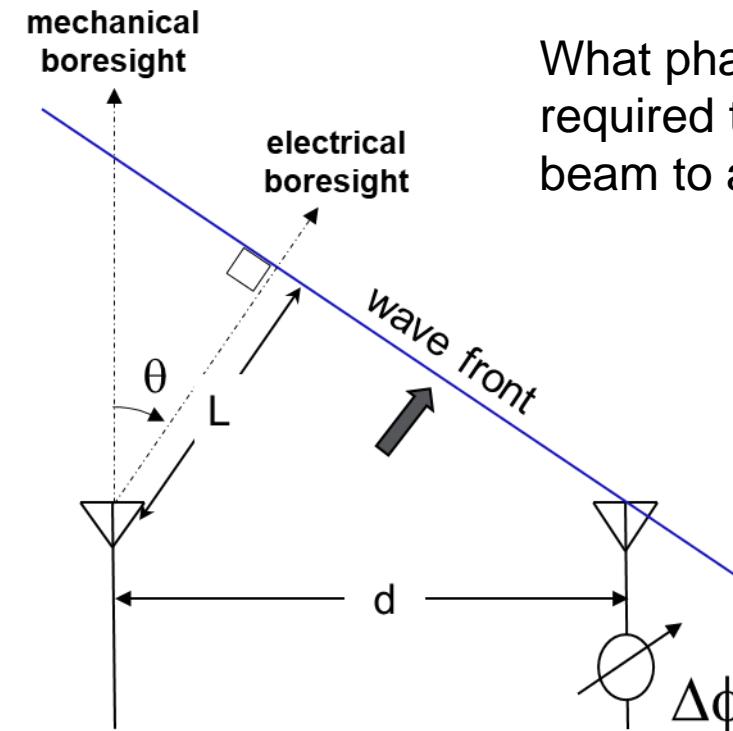
$$\Delta\phi = 2\pi L / \lambda = 2\pi f L / c$$

$$\rightarrow \Delta\phi = 2\pi f d \sin \theta / c$$

$$\rightarrow \theta = \sin^{-1}(\Delta\phi c / (2\pi f d))$$

Rewrite  $\Delta\phi$  relative to wavelength

$$\Delta\phi = \frac{2\pi d \sin \theta}{\lambda}$$



What phase shift,  $\Delta\phi$ , is required to steer the beam to an angle  $\theta$ ?

$\theta$  - beam electrical angle

$\Delta t$  - incremental time delay between elements

$\Delta\phi$  - incremental phase shift between elements

L - incremental propagation distance between elements

d - distance between elements

C - speed of light  $3 \times 10^8$  m/s

# Understanding Steering Angle: Math and Theory

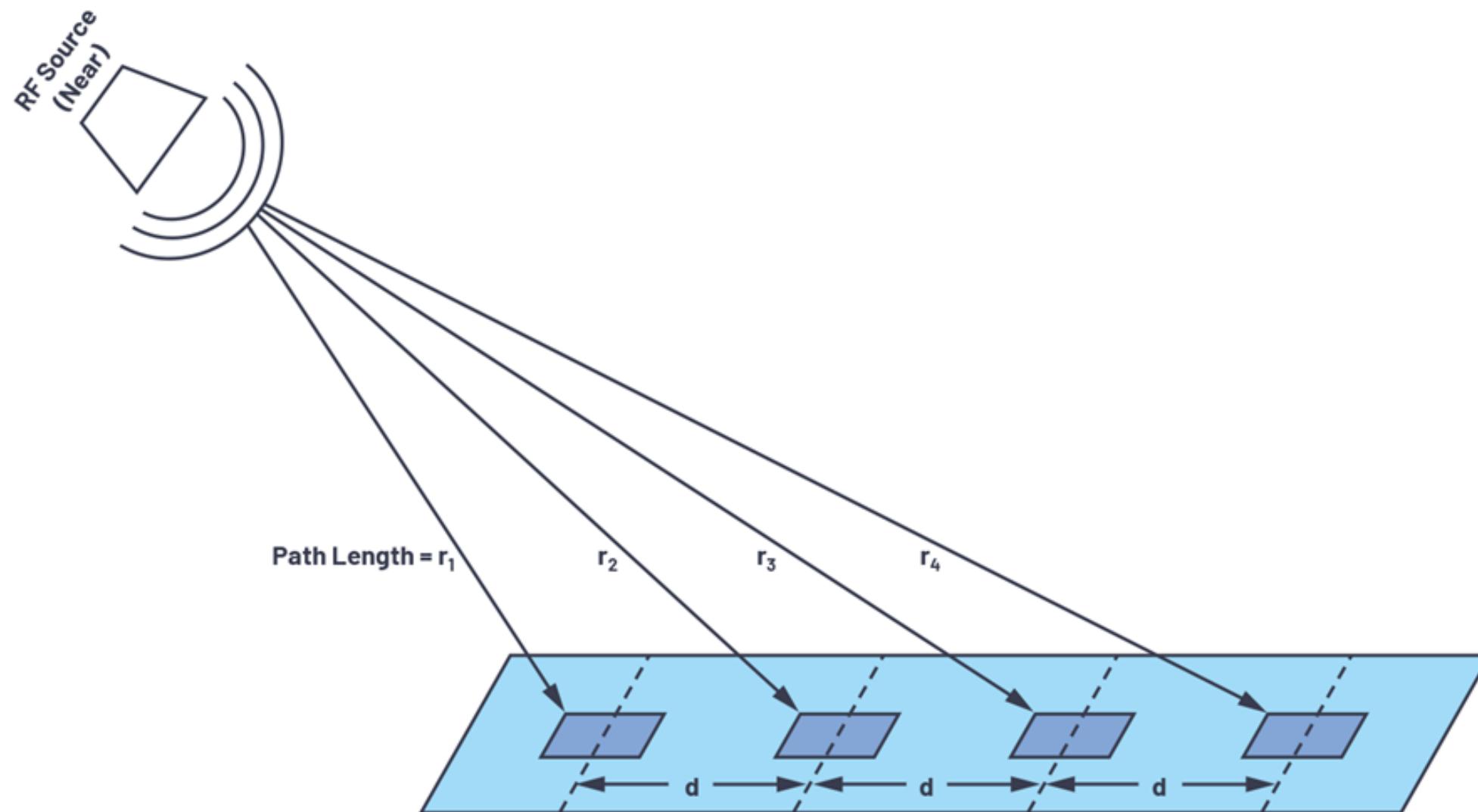
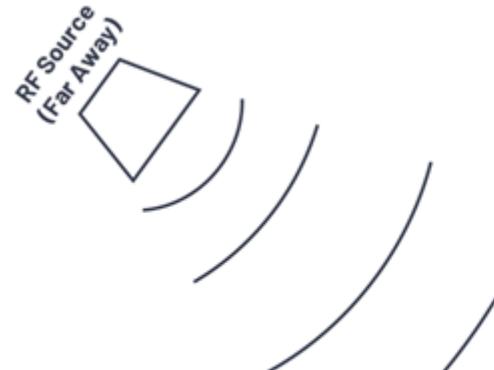


Figure from "Phased Array Antenna Patterns—  
Part 1: Linear Array Beam Characteristics and Array Factor"

# Understanding Steering Angle: Math and Theory

- ▶ In the “far field”
  - All of our lines are parallel
  - Therefore all thetas are equal.



- ▶ How far is far?
  - 4 elements, 15mm spacing at 10.5GHz means far field is at >142 mm

<https://www.everythingrf.com/rf-calculators/antenna-near-field-distance-calculator>

$$\text{Far Field} \geq \frac{2D^2}{\lambda}$$

D = Antenna dimensions (Can be the length or diameter of the antenna)

Remember L is the incremental propagation distance between elements :  $L = d \sin(\theta)$

Therefore:

$$r_2 = r_1 + L$$

$$r_3 = r_2 + L = r_1 + 2L$$

$$r_4 = r_3 + L = r_1 + 3L$$

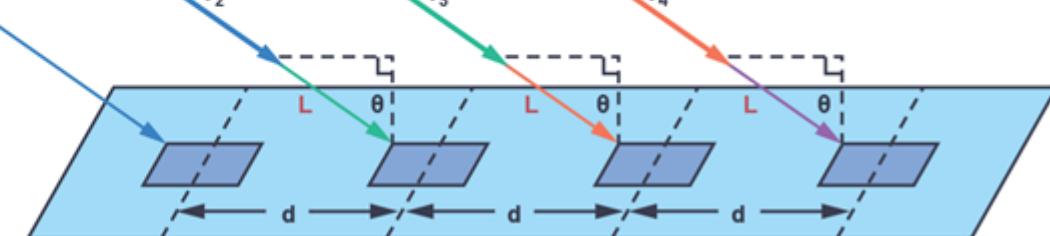
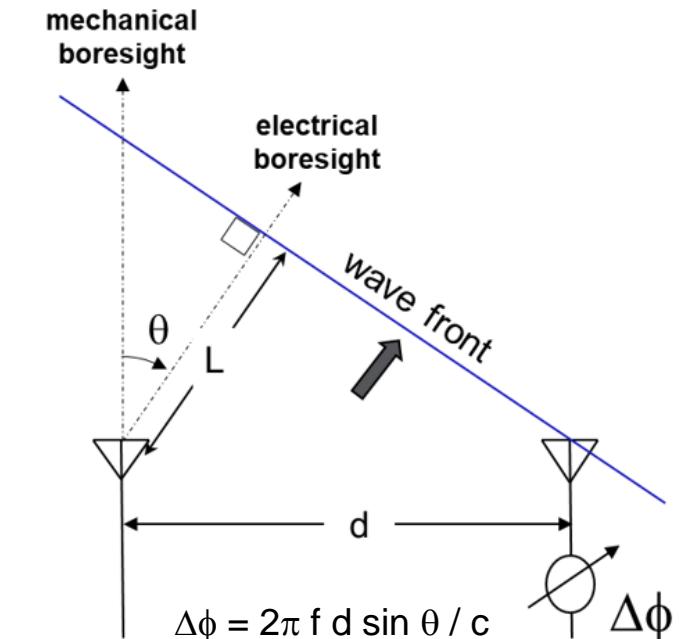
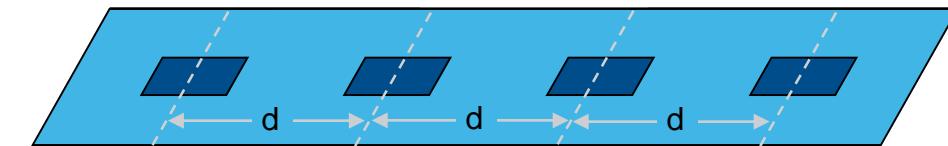
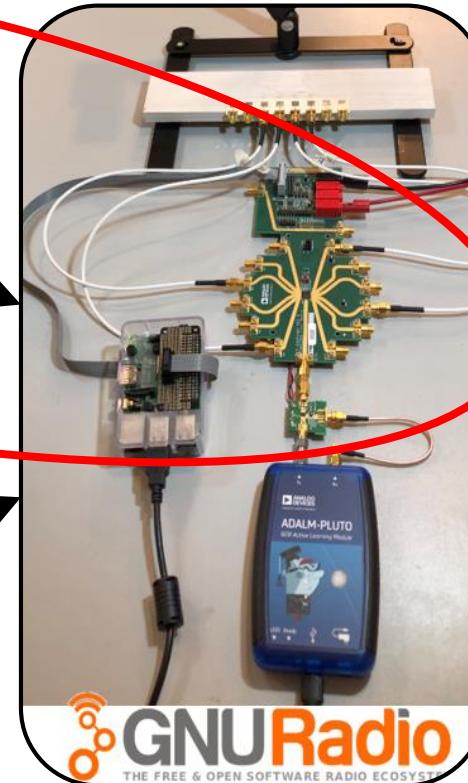


Figure from “Phased Array Antenna Patterns—  
Part 1: Linear Array Beam Characteristics and Array Factor”

# Understanding Steering Angle: Math and Theory

- Let's figure out what we expect for a phase delta:
  - Remember:  $\Delta\phi = 2\pi f d \sin \theta / c$
  - If we position our RF source at  $\theta = 30^\circ$ , then what is  $\Delta\phi$ ?
    - $\theta = 30^\circ = 0.52 \text{ rad}$
  - **d = 0.015 m** (the antenna was designed for  $d = \lambda/2$  at 10GHz)
    - $\lambda = c/f = (3 \times 10^8 \text{ m/s}) / (10 \text{ GHz}) = 0.03 \text{ m}$
  - **f = 10.5 GHz**
- Therefore:
  - $\Delta\phi = 2\pi f d \sin \theta / c = 2\pi * 10.5 \times 10^9 * 0.015 * \sin(0.52) / 3 \times 10^8$   
 $\rightarrow \Delta\phi = 1.64 \text{ rad} = 94^\circ$
- What does this mean?
  - If our RF source is at  $\theta = 30^\circ$ , then our maximum signal will be realized if we sum all our elements such that each element is shifted by  $94^\circ$  from its neighbor.
  - But don't take Math's word for it! Let's try it for ourselves!





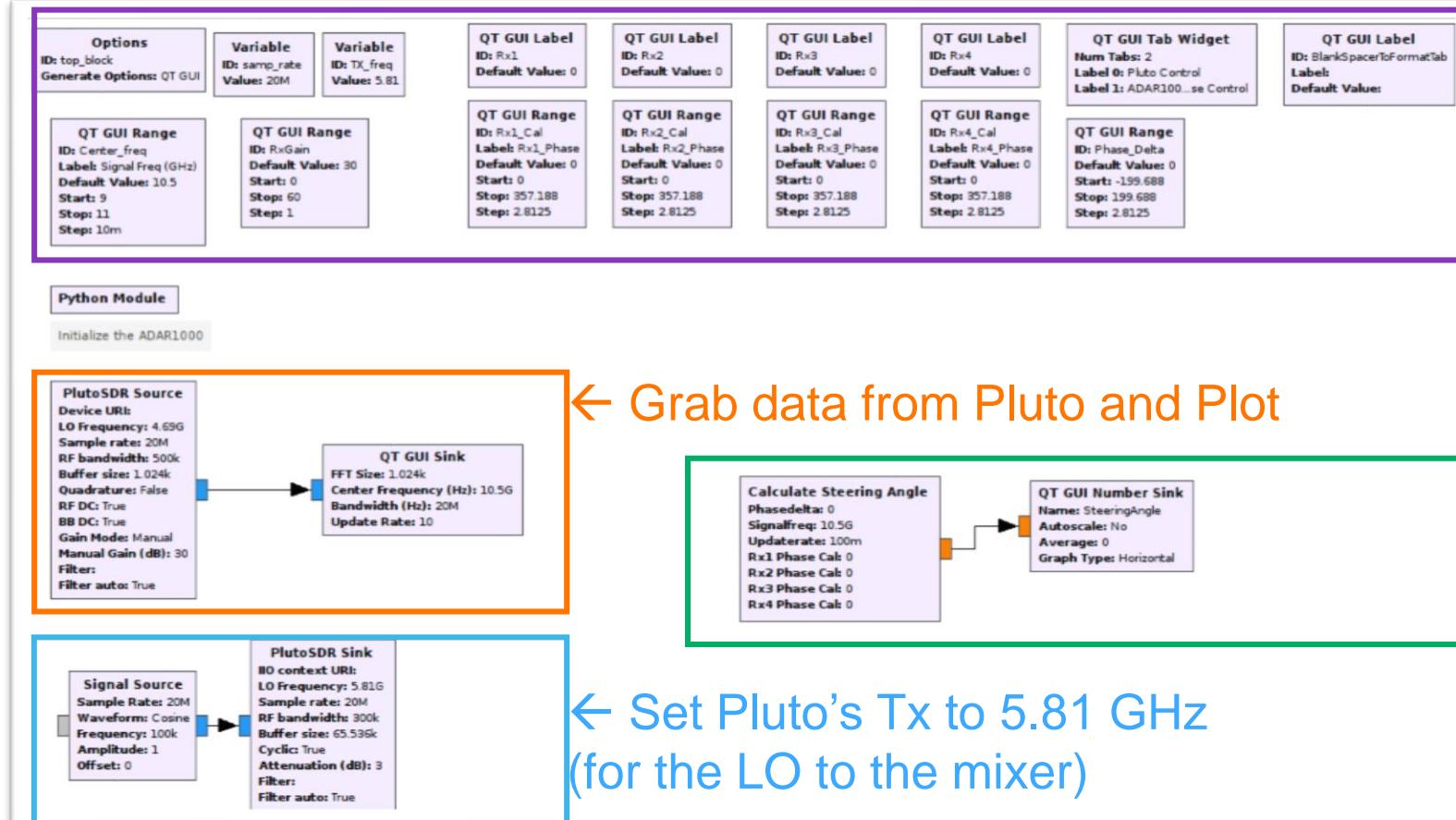
Steering Angle

Antenna Impairments

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# Steering Angle: GNU Radio Experiments

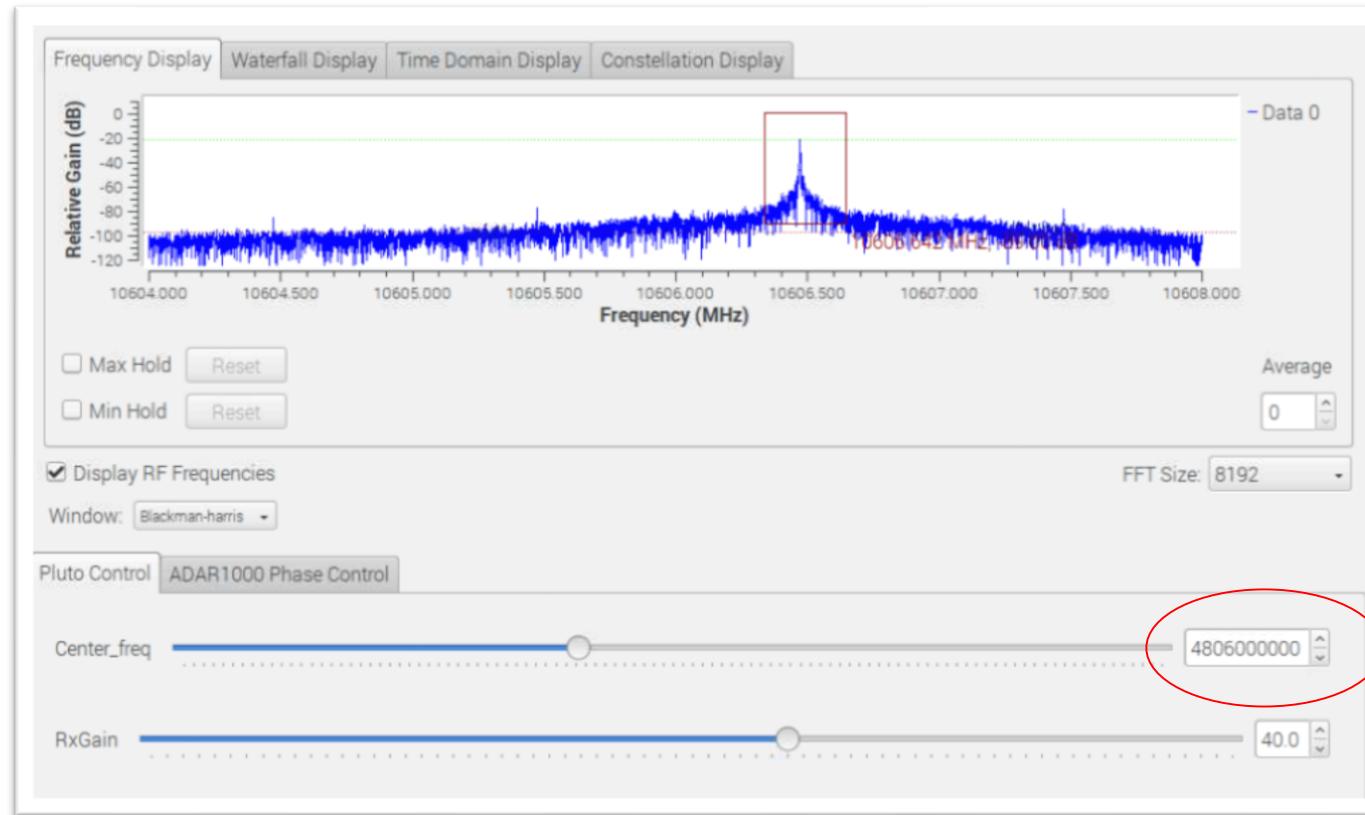


## Variables and GUI Objects

- This is a Python block which:
1. Programs the ADAR1000 for the phases we set from the controls above.
  2. Calculates steering angle from those phases.

# Understanding Steering Angle: Lab Experiments

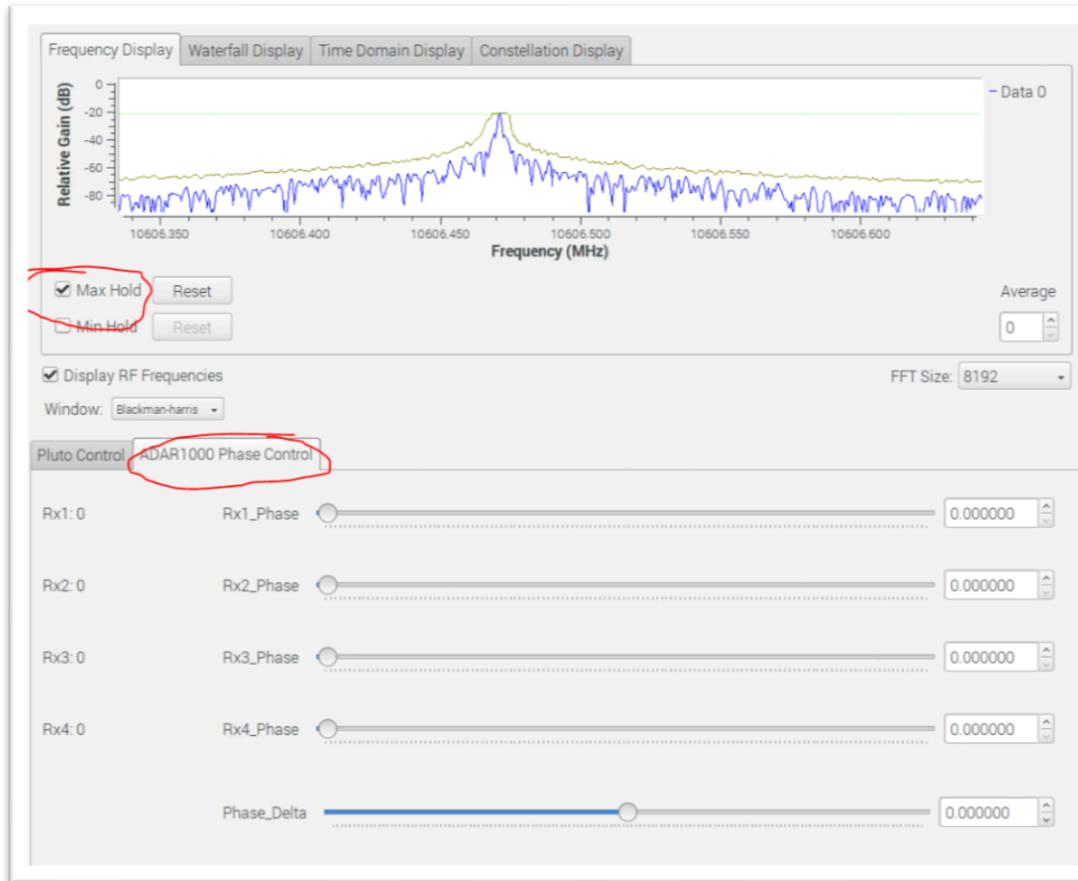
- ▶ Adjust the frequency and Gain of Pluto, if needed.
- ▶ Zoom in on the peak:



Adjust to find the RF Source frequency

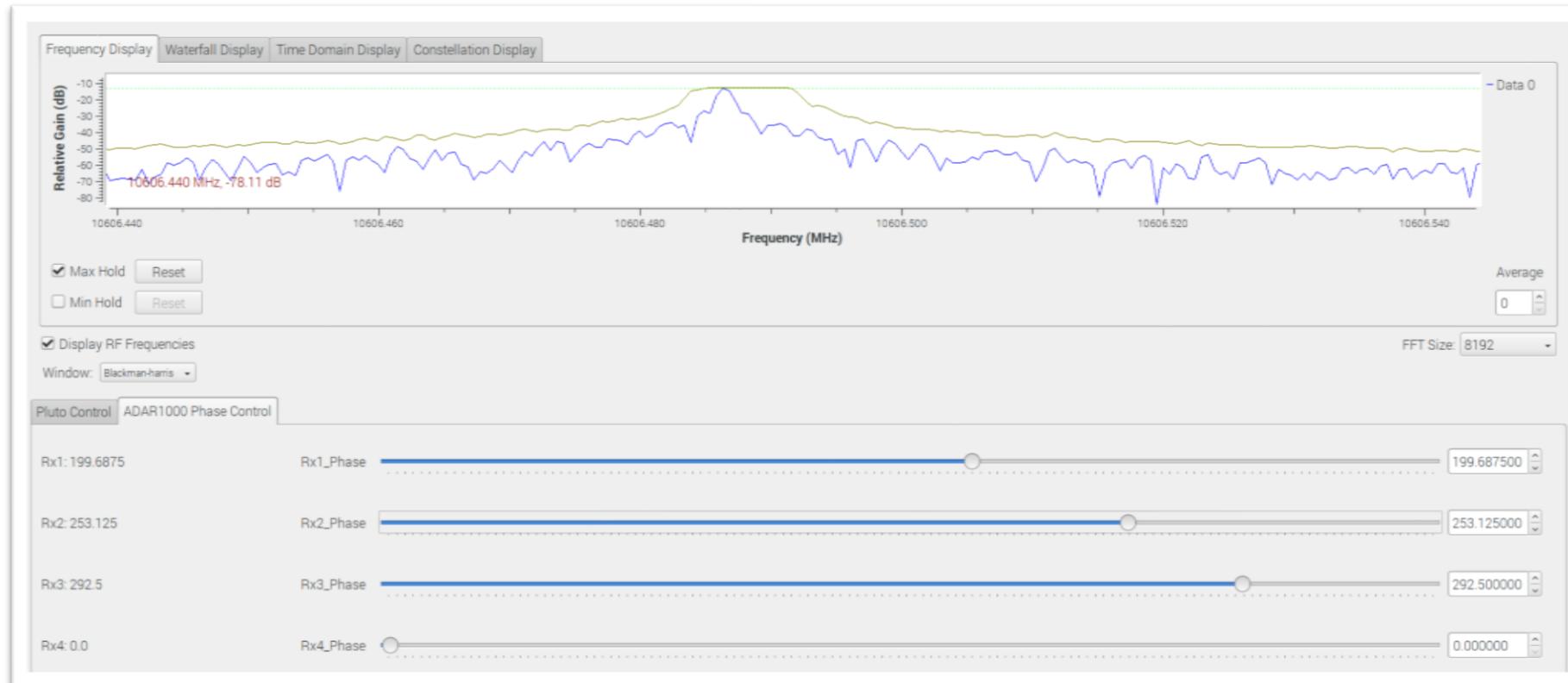
# Understanding Steering Angle: Lab Experiments

- ▶ Select “Max Hold” and the “ADAR1000 Phase Control” Tab



# Understanding Steering Angle: Lab Experiments

- ▶ Grab the “Rx\_Phase” sliders and move them around.
  - You are changing the phase delay of each individual ADAR1000 antenna element
  - You should see the peak FFT response rising and falling
- ▶ Why does the peak rise and fall in response to each element’s phase?

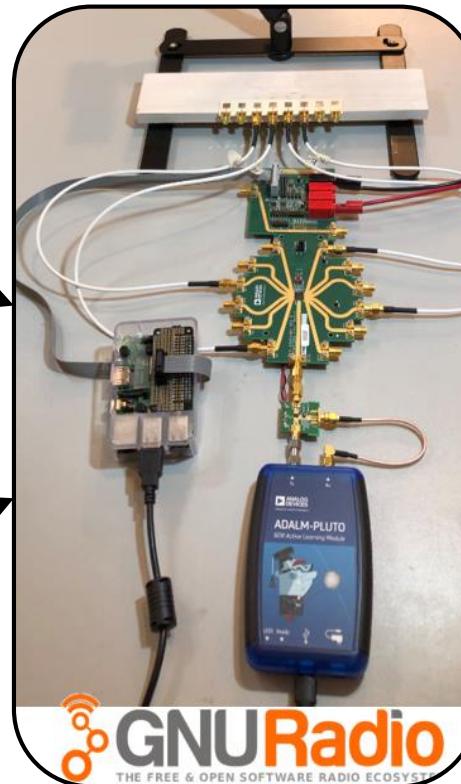


Steering Angle

Antenna Impairments

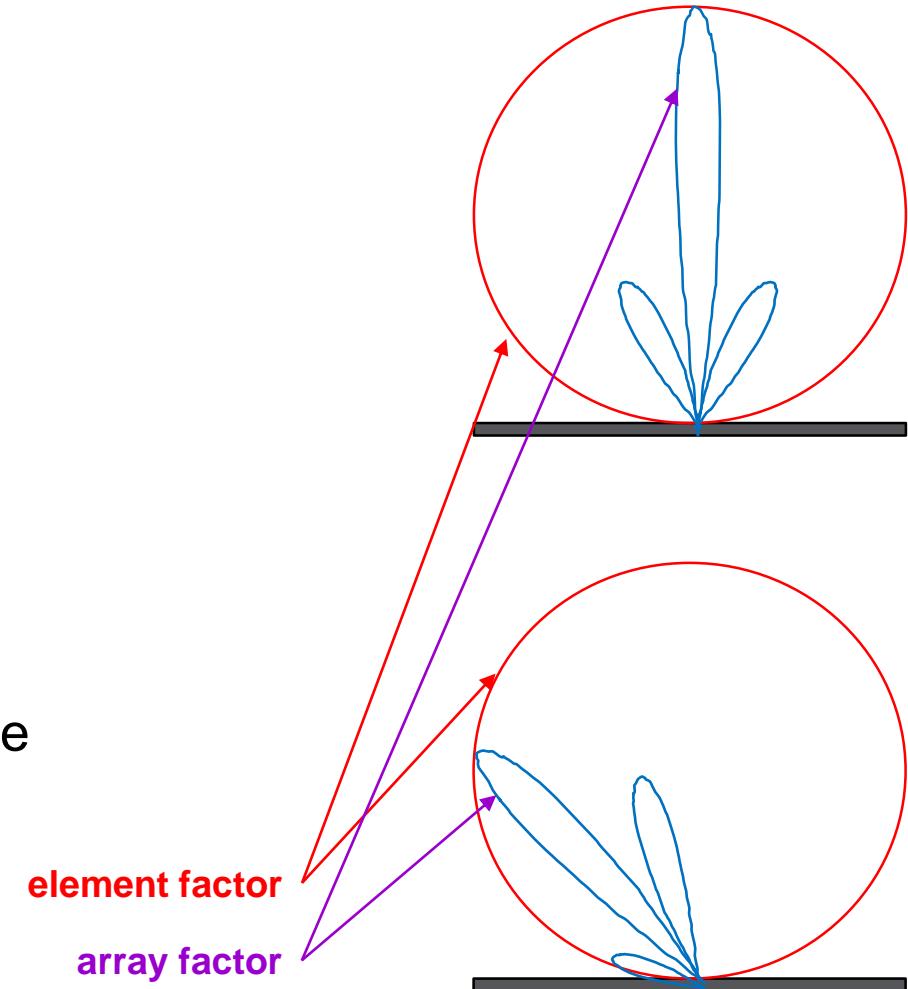
Antenna Patterns

Monopulse Tracking



# Understanding Beam Width: Math and Theory

- Let's work out the math to see if that graph is what we expect from our antenna array
- "Element Factor" -  $G_E(\theta)$ 
  - the radiating pattern of a single element in the array
- "Array Factor" -  $G_A(\theta)$ 
  - Determined by array geometry and beam weights (amplitude and phase)
- The Phased Array Gain (dB)  $\rightarrow G(\theta) = G_E(\theta) + G_A(\theta)$
- Assuming all our elements are the same, let's focus on the Array Factor,  $G_A(\theta)$

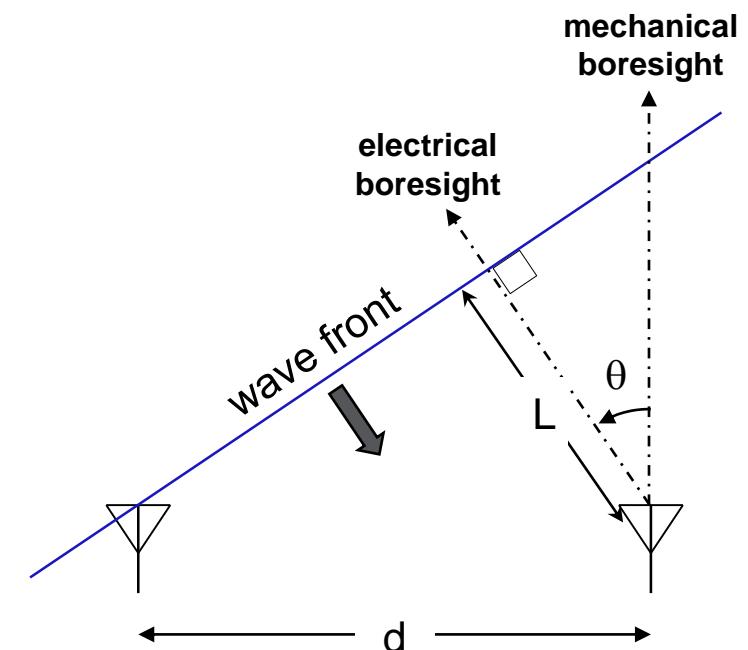


<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part1.html>

# Understanding Beam Width: Math and Theory

- Recall that each element receives a signal that may be delayed relative to the element next to it.
- And the array factor is the summation of all those signals. If each signal has the same amplitude, with shifted phases, then, for our 4 element array, the Array Factor is:
  - $G_A = e^{j0 \cdot \Delta\phi} + e^{j1 \cdot \Delta\phi} + e^{j2 \cdot \Delta\phi} + e^{j3 \cdot \Delta\phi}$
  - Where the first phase delta is zero, so this simplifies to:
  - $G_A = 1 + e^{j\Delta\phi} + e^{j2 \cdot \Delta\phi} + e^{j3 \cdot \Delta\phi}$
- $G_A = e^{j(N-1) \cdot \Delta\phi / 2} \cdot \frac{\sin(N \cdot \Delta\phi / 2)}{\sin(\Delta\phi / 2)}$
- Array gain is at a maximum when  $\Delta\phi = 0$ , so for small values of  $\Delta\phi$ , the magnitude of  $G_A$  is:
  - $|G_{A(\Delta\phi = 0)}| = \frac{\sin(N \cdot 0)}{\sin(0)} = N$
- Then normalizing that for other steering angles near  $\Delta\phi = 0$ 
  - $|G_{A(NORM)}| = \frac{\sin(N \cdot \Delta\phi / 2)}{N \cdot \sin(\Delta\phi / 2)}$

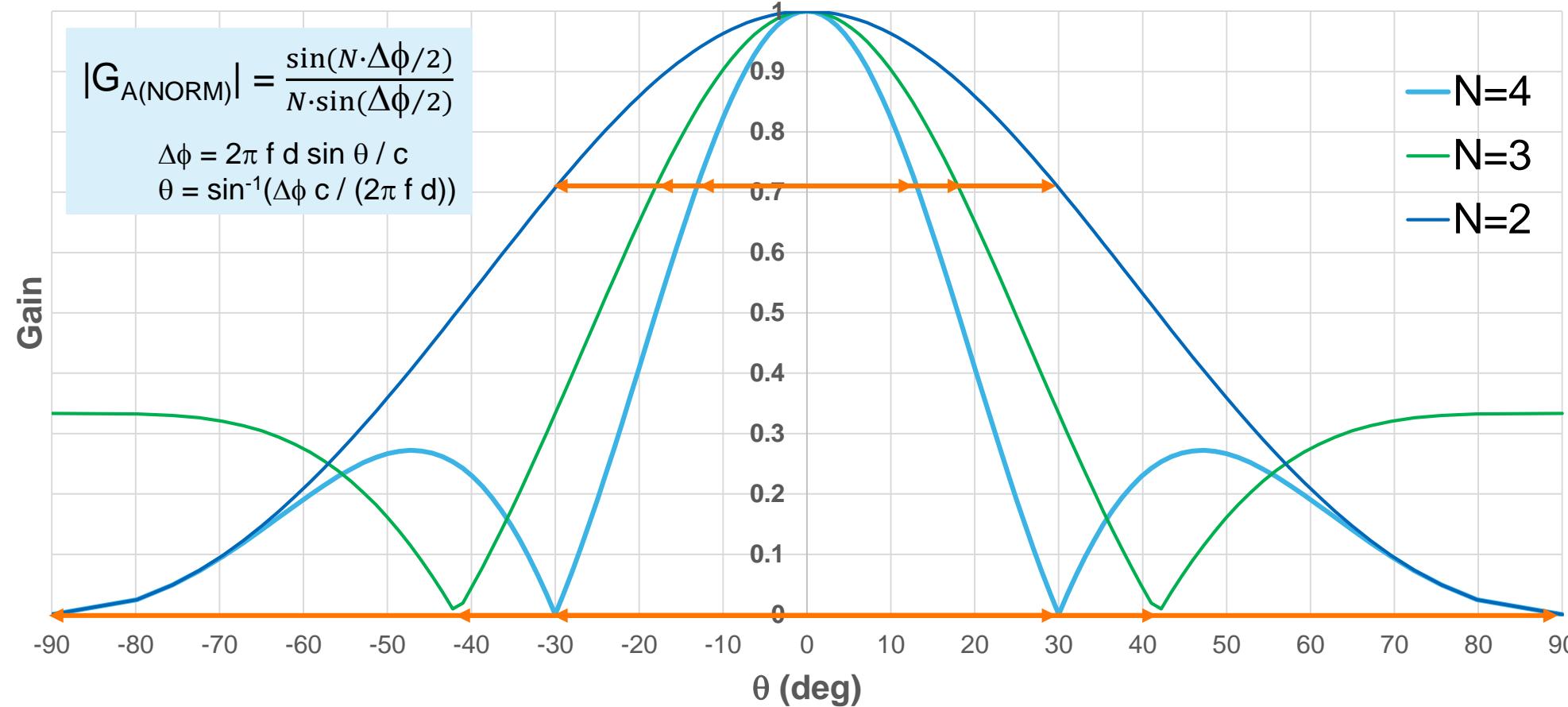
see <http://www.waves.utoronto.ca/prof/svhum/ece422/notes/15-arrays2.pdf>



Remember:  
 $\theta = \sin^{-1}(\Delta\phi c / (2\pi f d))$   
 $\Delta\phi = 2\pi f d \sin \theta / c$

# Understanding Beam Widths: Math and Theory

## Normalized Array Gain for d=15mm



# Understanding Beam Width: Math and Theory

- ▶ Halfpower Beam Width (HPBW)
  - Main lobe beamwidth, measured 3dB down from its peak

$$\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 1/\sqrt{2}$$

- ▶ Note Effect of Changing N

	<b>f=10.5GHz</b>
<b>N=4</b>	HPBW=25°
<b>N=3</b>	HPBW=34°
<b>N=2</b>	HPBW=56°

$$d = 15 \text{ mm}$$

# Understanding Beam Width: Math and Theory

## ► First Null Beam Width (FNBW)

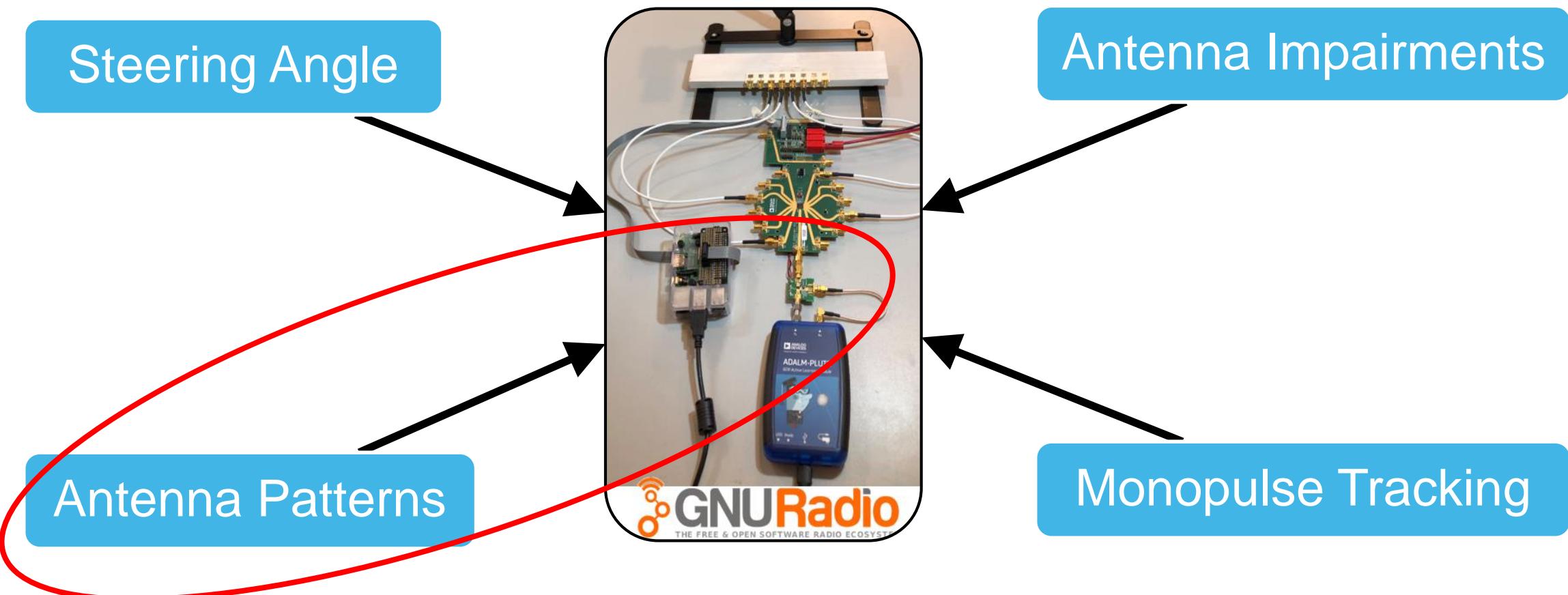
- Spacing between main lobe nulls

- $\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 0$

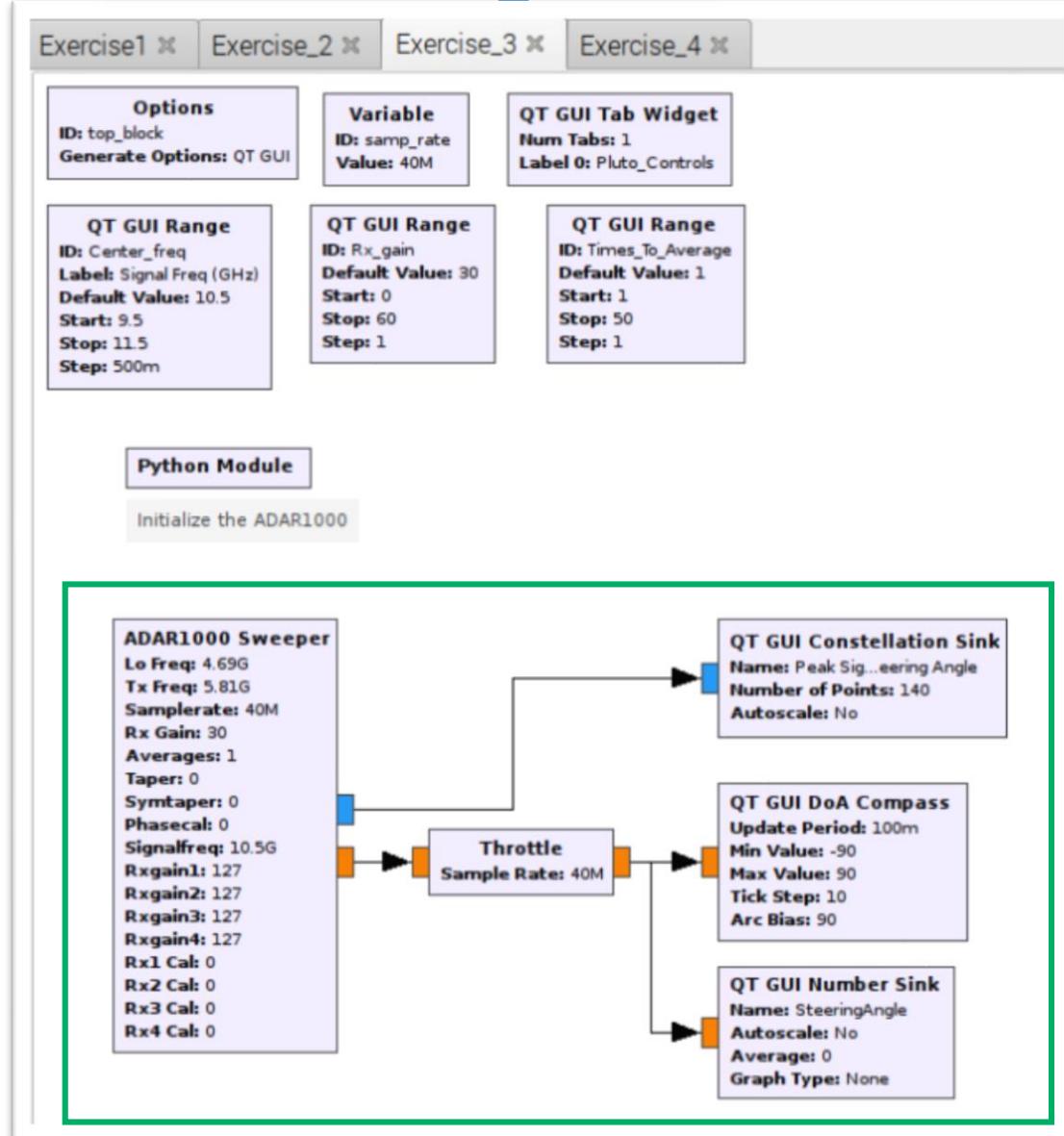
## ► Note Effect of Changing N

	<b>f=10.5GHz</b>
<b>N=4</b>	HPBW=56°
<b>N=3</b>	HPBW=78°
<b>N=2</b>	HPBW=137°

$$d = 15 \text{ mm}$$



# Understanding Beam Width: Lab Experiments

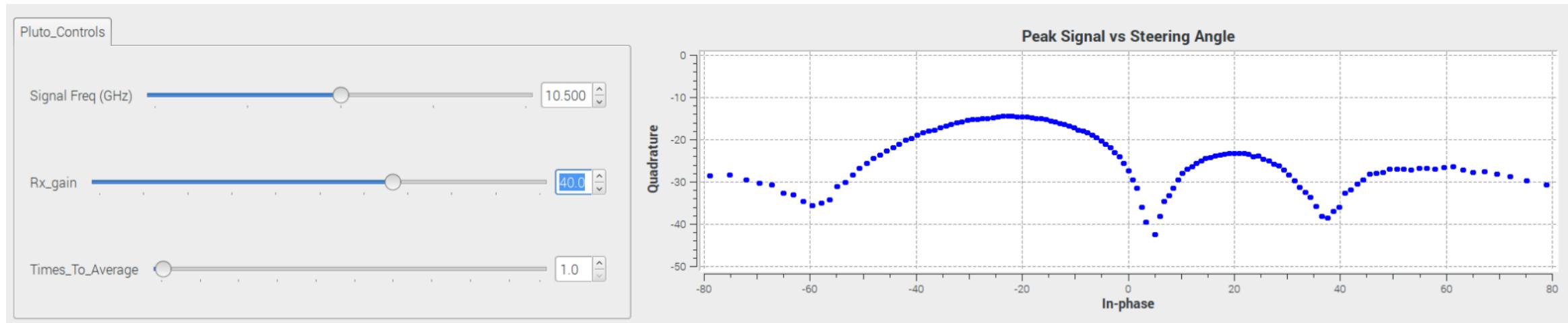


Nearly everything has been replaced by this "ADAR1000 Sweeper" block

- This is a Python file that controls Pluto and the ADAR1000
- It is much faster to change the ADAR1000 beams and grab Pluto data with this script, than the plug ins we used before
- This script also calculates and plots our beam

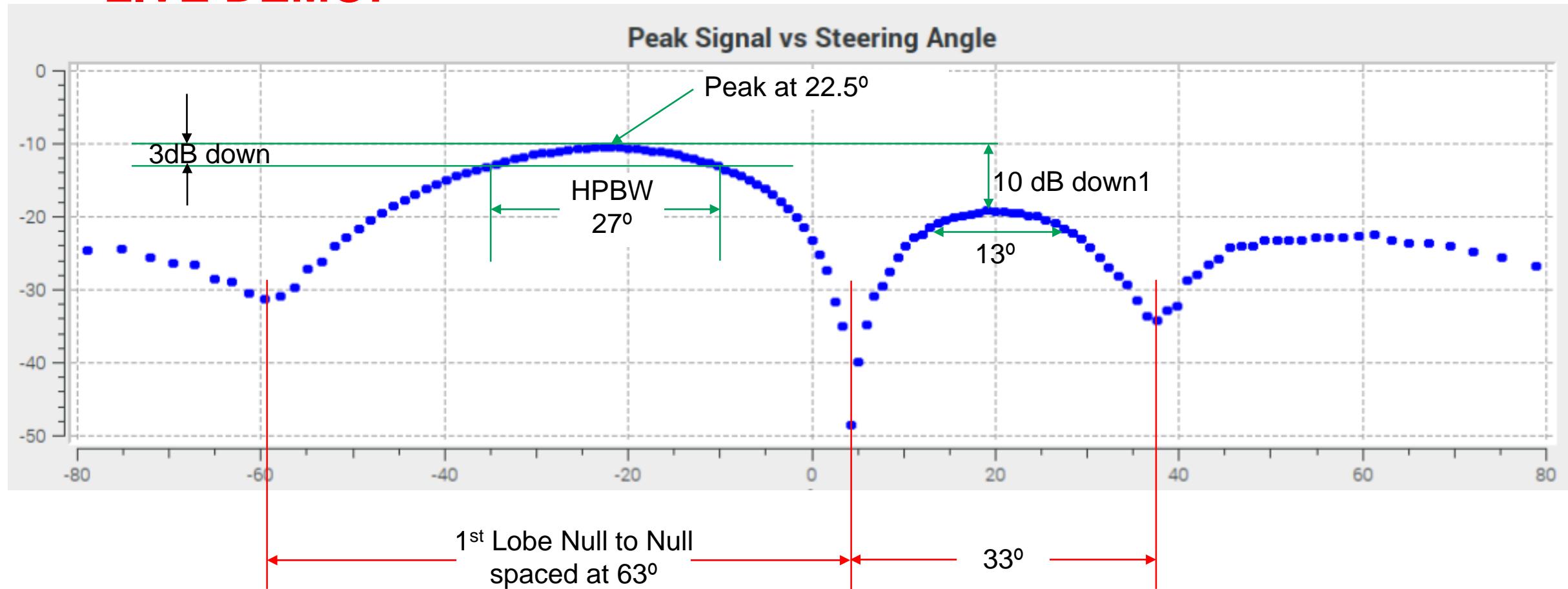
# Understanding Beam Width: Lab Experiments

- ▶ Make observations on:
  - The width of the “main” lobe
  - Number of peaks and nulls
  - Distance between Null to Null
- ▶ Ignore the “In-Phase” and “Quadrature” axis labels. These are really “Steering Angle (theta)” and “Peak Signal (dB)”
  - Just an unfortunate side effect of using a constellation plot to view x-y data.... Does anyone have a solution this?????



# Understanding Beam Width: Lab Experiments

LIVE DEMO!



# Understanding Beam Width: Math and Theory

- ▶ Halfpower Beam Width (HPBW)
  - Main lobe beamwidth, measured 3dB down from its peak

$$\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 1/\sqrt{2}$$

- ▶ Note Effect of Changing N
  - Beamwidth
  - Number of Peaks/Nulls
  - Peak Gain

	<b>f=10.5GHz</b>
<b>N=4</b>	HPBW=25°
<b>N=3</b>	HPBW=34°
<b>N=2</b>	HPBW=56°

*d = 15 mm*

# Understanding Beam Width: Math and Theory

## ► First Null Beam Width (FNBW)

- Spacing between main lobe nulls

- $\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 0$

## ► Note Effect of Changing N

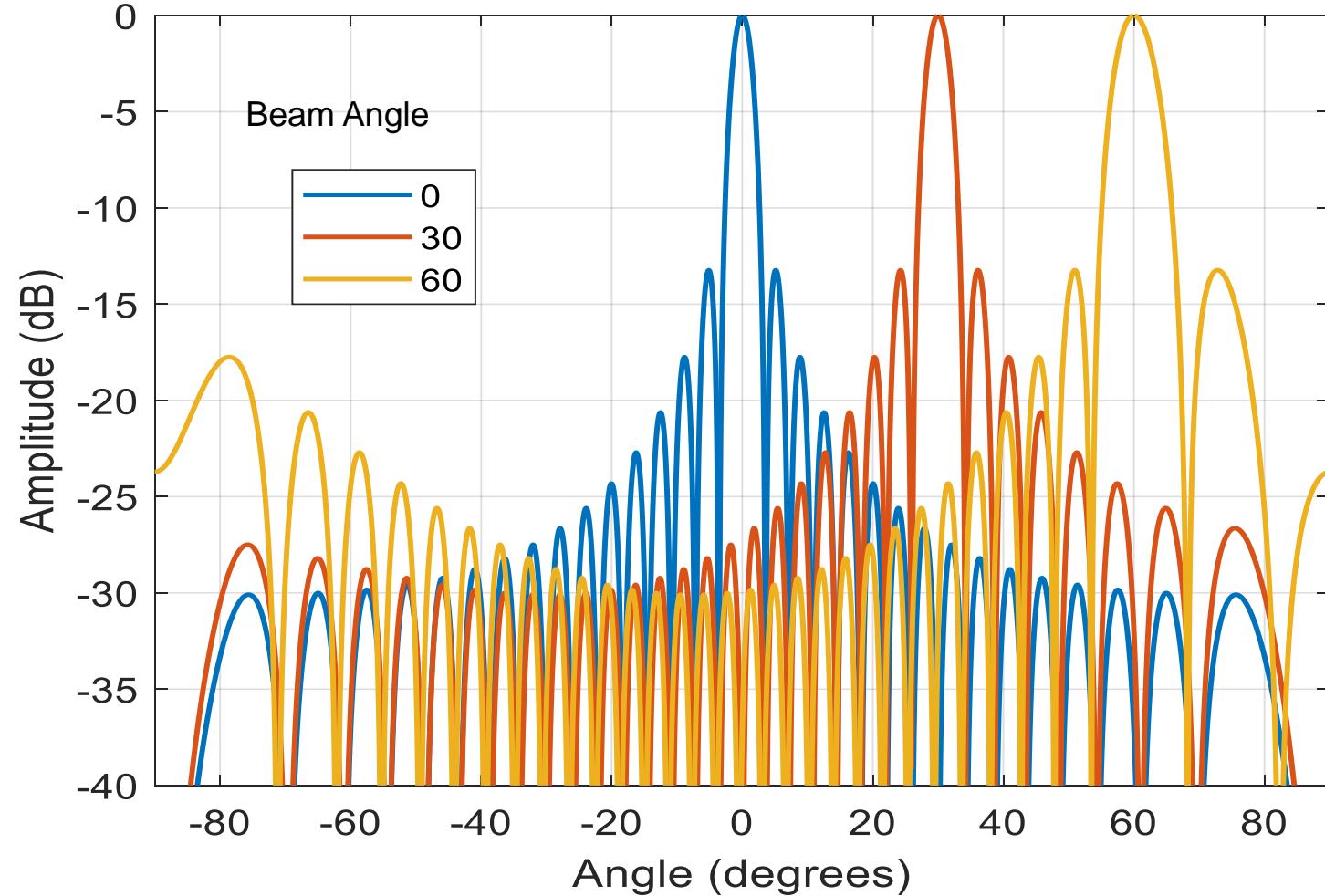
	<b>f=10.5GHz</b>
<b>N=4</b>	HPBW=56°
<b>N=3</b>	HPBW=78°
<b>N=2</b>	HPBW=137°

$$d = 15 \text{ mm}$$

# Beam Width vs Angle

$$AF[\theta, \Delta\phi] = \frac{\sin\left(N \left[ \frac{\pi d}{\lambda} \sin(\theta) - \frac{\Delta\phi}{2} \right] \right)}{N \sin\left( \frac{\pi d}{\lambda} \sin(\theta) - \frac{\Delta\phi}{2} \right)}$$

Normalized array factor of a 32 element linear array at several beam angles,  $d=\lambda/2$



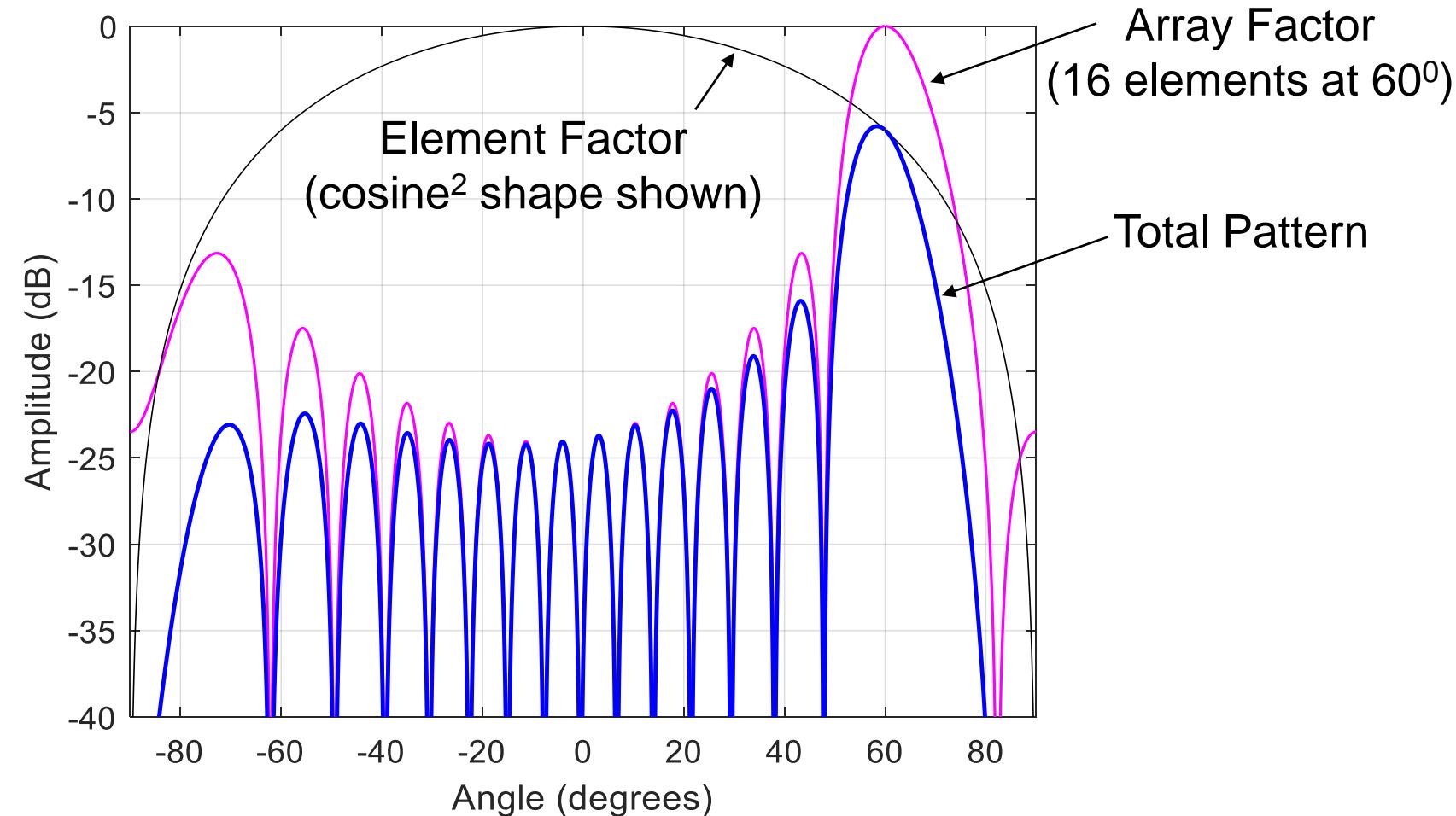
# Array Factor + Element Factor

The Phased Array Gain (dB)

$$G(\theta) = G_E(\theta) + G_A(\theta)$$

## Observations

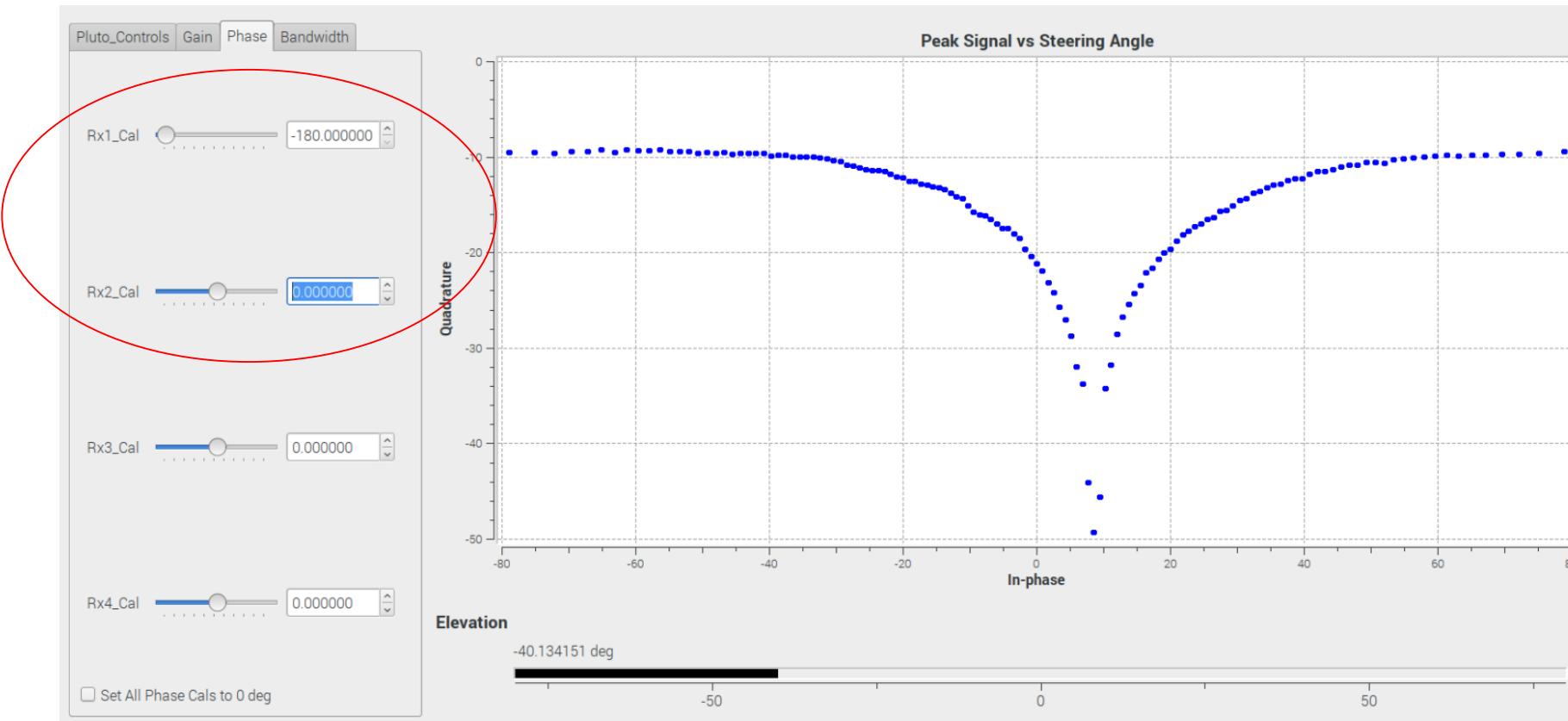
- 1) The main beam loses amplitude at the rate of the element factor
- 2) The sidelobes on boresight have no amplitude loss.
- 3) The result is the sidelobe performance of the overall array degraded off boresight



<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part1.html>

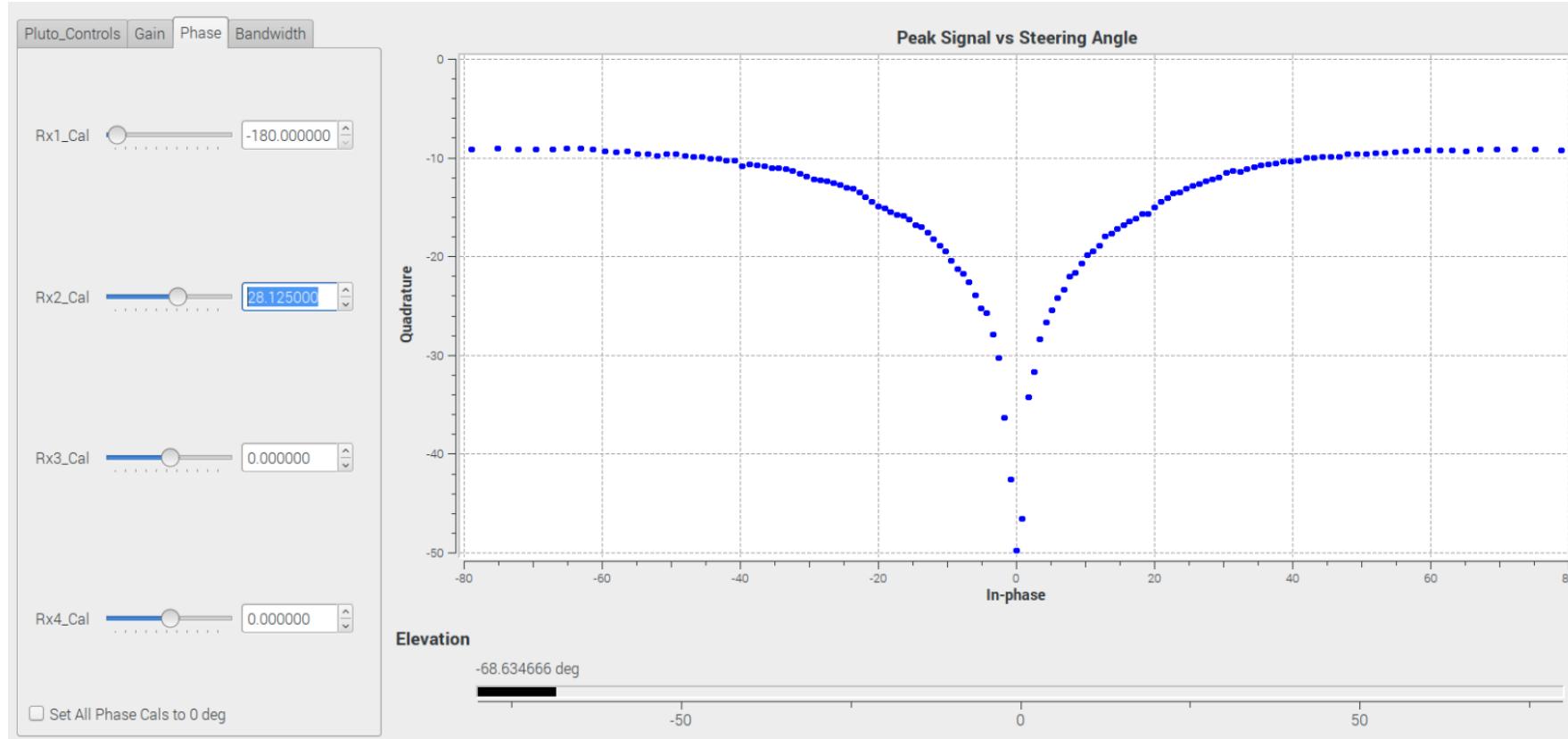
# We Need to Have a Talk About Calibration

# Simple Calibration

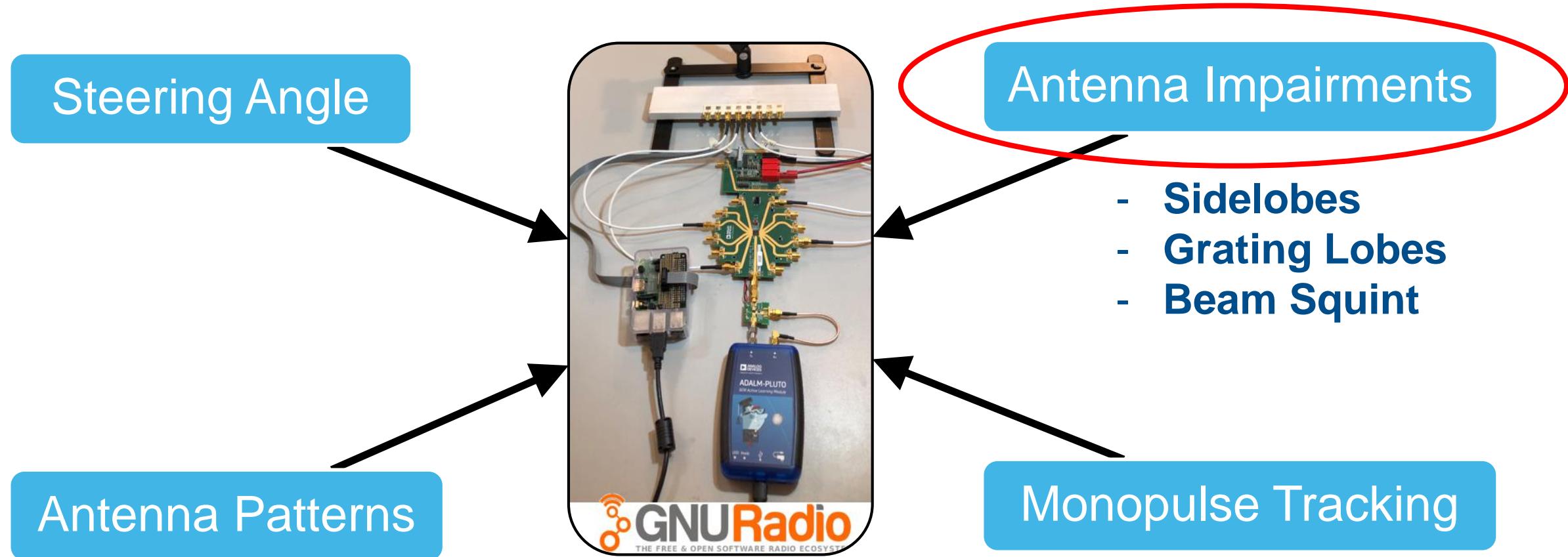


- Phase mismatch (almost entirely from the SMA cables) will cause a distorted antenna plot
- Enable just two channels, then phase match them, then proceed to the next channel
- Matching Rx1 and 2 is much easier if we look for the null (which is sharp), vs. the peak (which is broad and flat)
- The null is found by subtracting Rx1 from Rx2
  - Set gain of Rx3 and Rx4 to 0
  - Then give Rx1 a -180 deg phase shift

# Simple Calibration



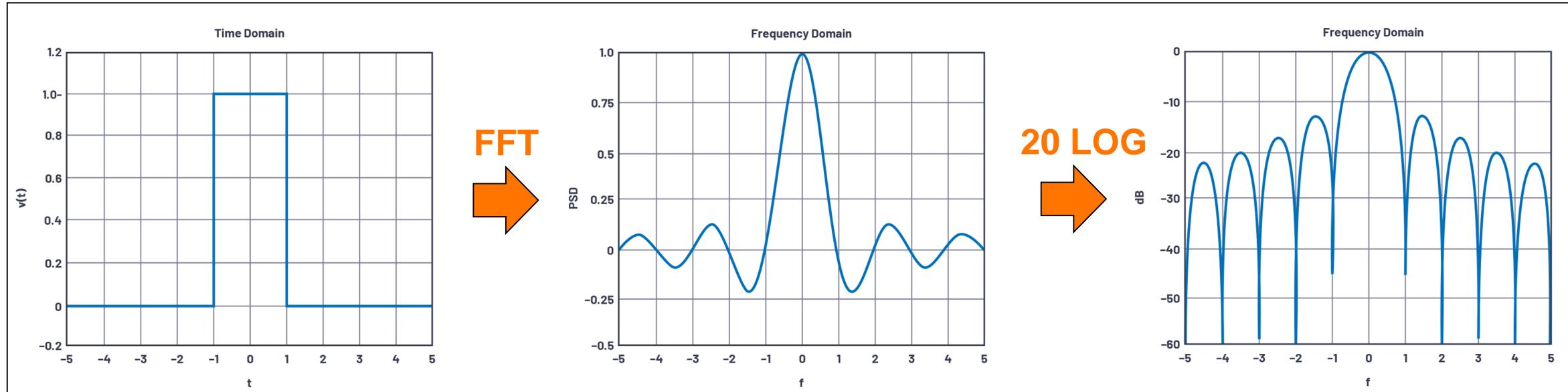
- ▶ Now dial in Rx2 until the peak aligns with broadside.
- ▶ This is the phase offset that needs to be set for Rx2



# Side Lobes → Combat with Tapering

# Side Lobes

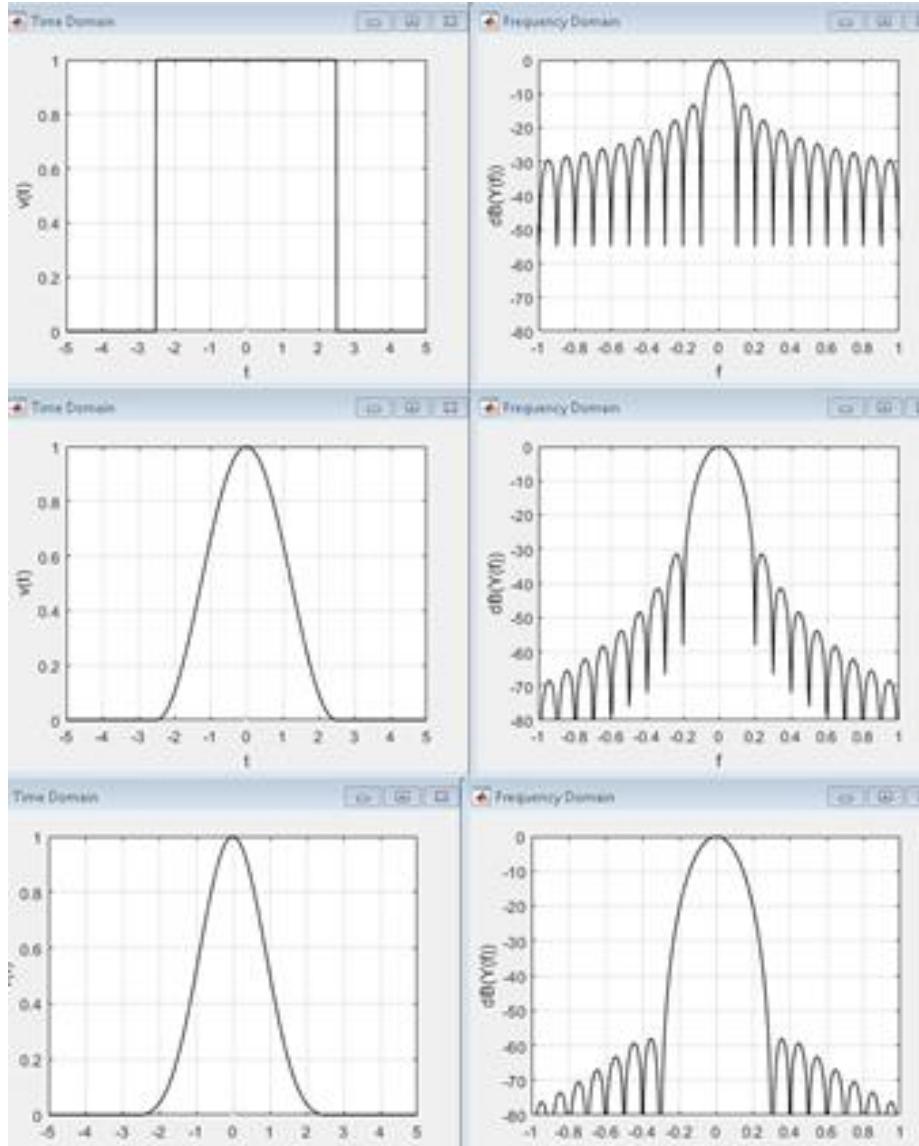
- With all elements at the same gain, we have effectively have a box car window.
  - This is analogous to rectangular window FFT



- Time domain pulse → frequency domain  $\sin(x)/x$  → first sidelobe -13dBc, etc.
- As pulse becomes wider...
  - Main lobe narrows
  - Sidelobes move in
  - Sidelobe levels remain unchanged

<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part3.html>

# Understanding Beam Tapering: Window Functions



Boxcar – 1<sup>st</sup> sidelobe @ -13dBc  
Narrowest main lobe

Hanning – 1<sup>st</sup> sidelobe < -30dBc  
Main lobe broadens

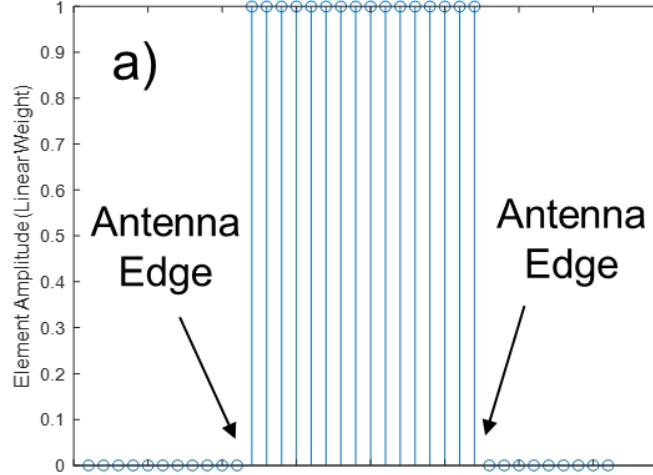
Blackman – Lowest sidelobes  
Broadest main lobe

Note: windowing losses not shown in these examples

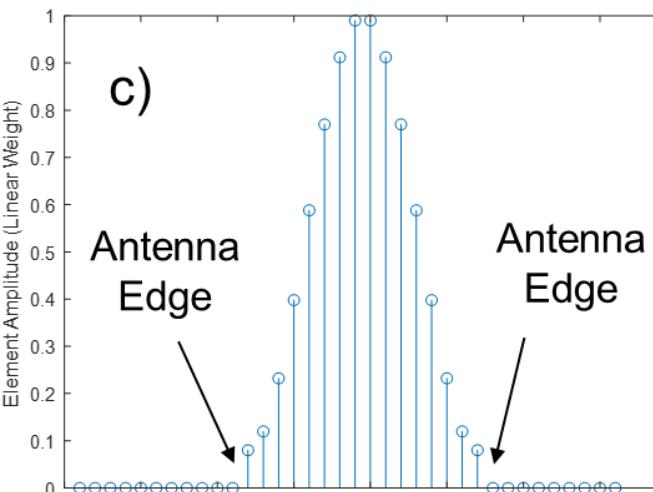
# Side Lobe Control: Beam Tapering

Uniform  
Weighting

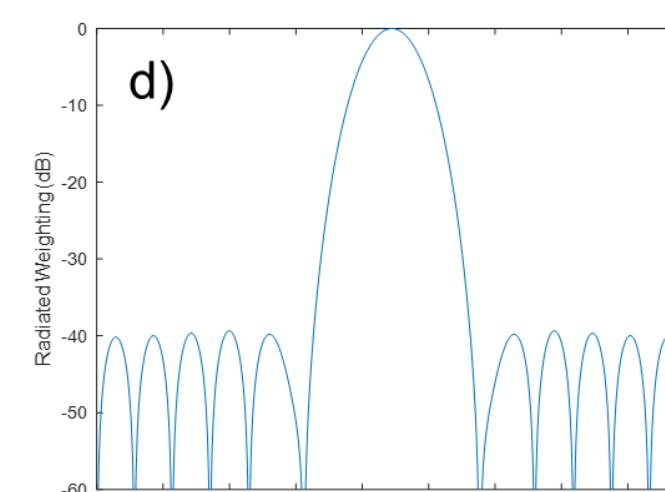
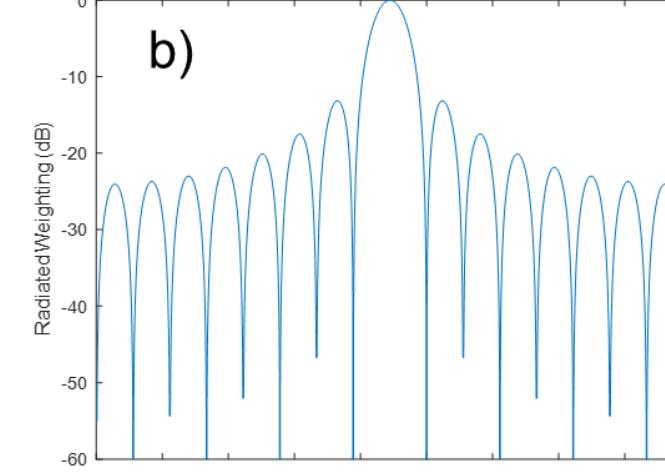
Field Domain  
(Element Amplitude)



Hamming  
Weighting

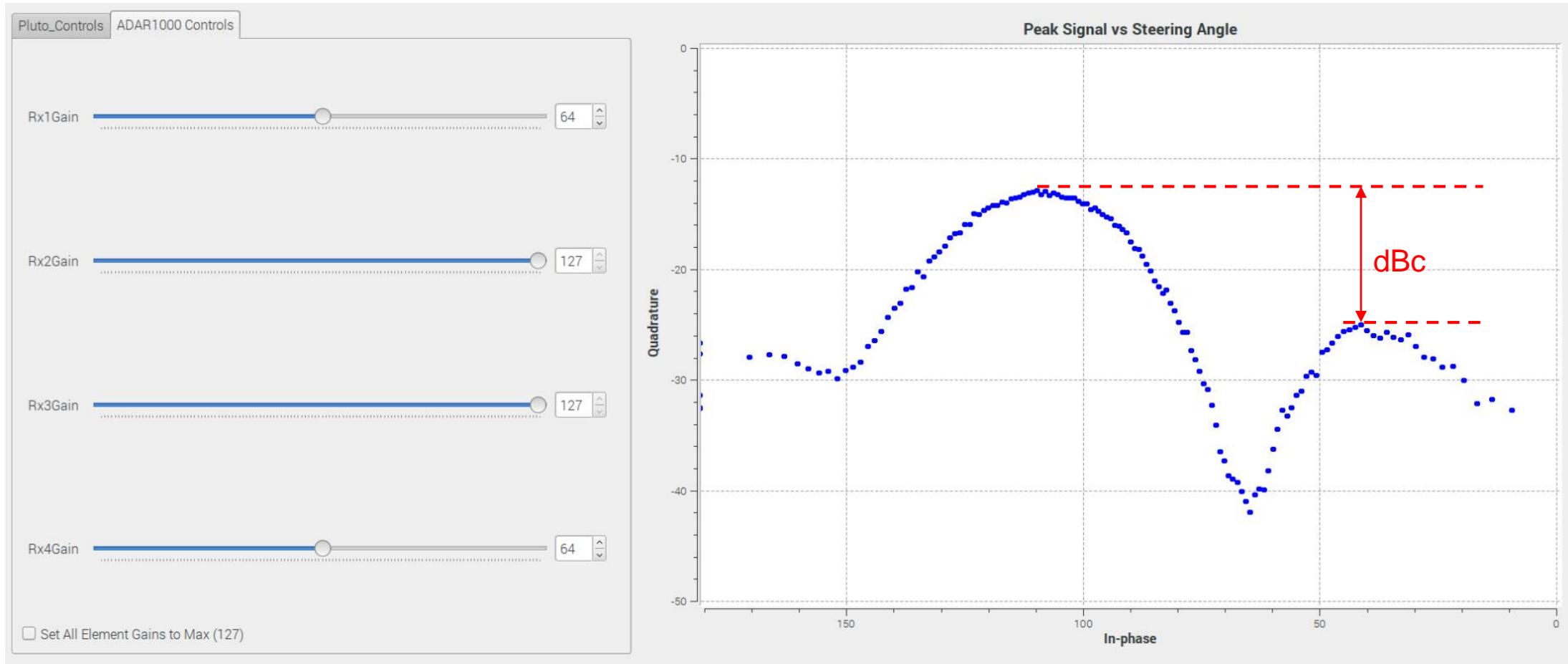


Spatial Domain  
(Radiated Pattern Weighting)



# Understanding Beam Tapering: Lab

- ▶ Click on the ADAR1000 Controls Tab
- ▶ We now have amplitude control of each element
- ▶ Now, let's observe the effect if Rx1 and Rx4 are  $\frac{1}{2}$  the gain of Rx2 and Rx3:



# Understanding Beam Tapering: Lab

- ▶ Notice the following cases:
  - All Gains = 127
  - Rx1,4 = 30, and Rx2,3 = 127 (Blackman Taper)
  - Rx1,4 = 55, and Rx2,3 = 127 (Hamming Taper)

There's only so much tapering we can do with 4 elements!

# Grating Lobes → Combat with Spacing

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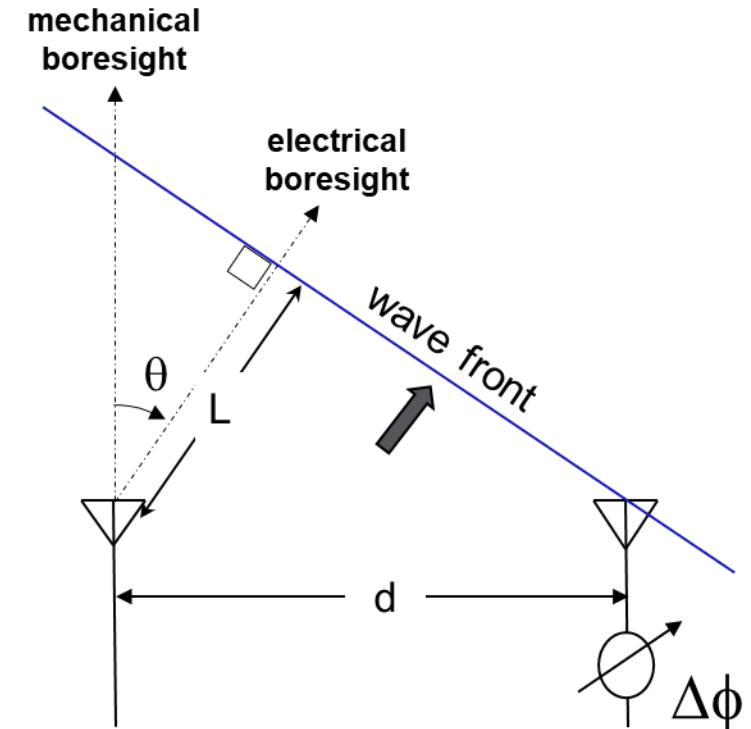
# Understanding Grating Lobes: Math and Theory

- ▶ Grating Lobes are analogous to aliasing on an ADC
  - From sampling theory, we need minimum sample rate of 2x frequency of interest
- ▶ Grating Lobes are “Spatial Aliasing”
  - Each antenna element is a spatial “sample”
  - Need two “samples” per wavelength to avoid spatial aliasing (aka Grating Lobes)
    - For  $<\lambda/2$  element spacing, no grating lobes occur
    - For  $>\lambda/2$  element spacing, grating lobes will appear at the opposite horizon
- ▶ To calculate where the grating lobes will appear, we need to return to our old friend, Math.



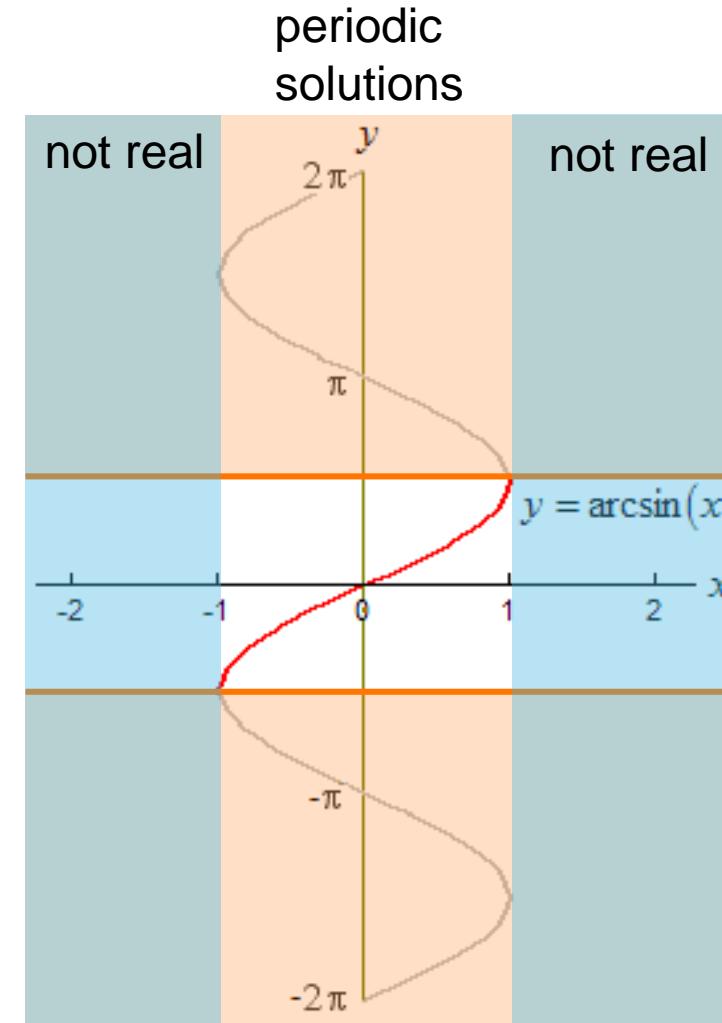
# Understanding Grating Lobes: Math and Theory

- ▶ Let's start with our familiar equations:
  - $\Delta\phi = (2\pi d / \lambda) \cdot \sin(\theta)$
  - Then our steering angle is
    - $\theta = \sin^{-1}(\Delta\phi / 2\pi \cdot \lambda/d)$
  - But  $\Delta\phi$  wraps around every  $2\pi$ , so we can really say:
    - $\theta = \sin^{-1}(m \cdot \Delta\phi / 2\pi \cdot \lambda/d)$ , for  $m=0, \pm 1, \pm 2$ , etc.
- ▶ But remember that the arcsin is a weird function.....



# Understanding Grating Lobes: Math and Theory

- ▶ Arcsin is SIN rotated 90 deg
- ▶ Valid values are along the red portion
- ▶ Arcsin result limited to  $\pm \pi/2$ 
  - We ignore the periodic solutions
- ▶ Arcsin argument limited to  $\pm 1$ 
  - Beyond  $\pm 1$  is complex (non-real)
  - Ignore because we can't have a “non-real” steering angle!



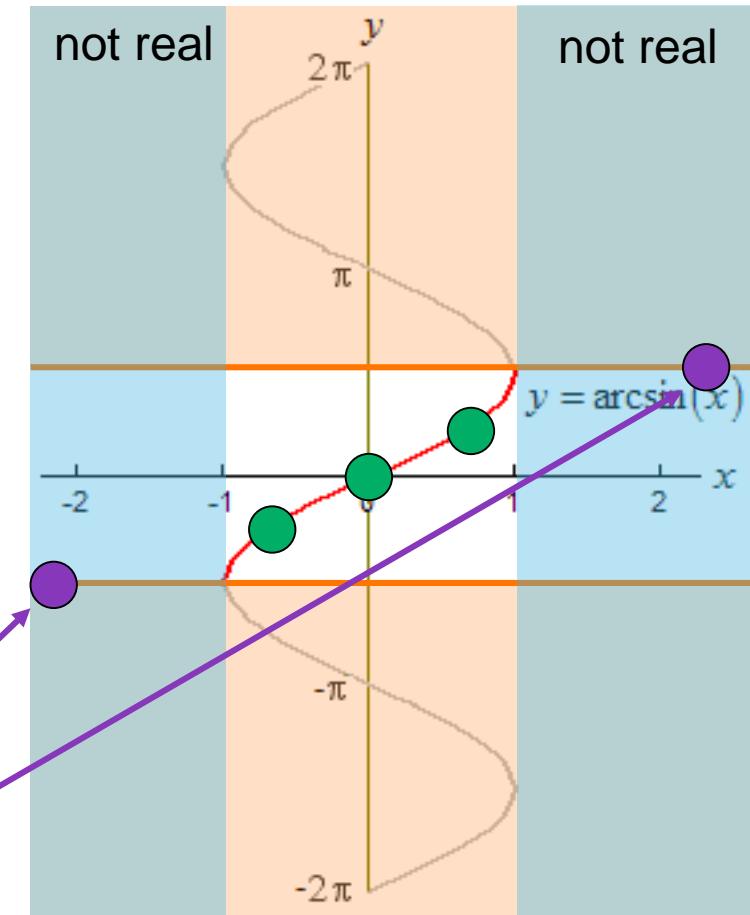
<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part2.html>

# Understanding Grating Lobes: Math and Theory

For Broadside (i.e. steering angle = 0°)

$$\theta_{\text{MAIN}} = \sin^{-1}(m \lambda/d), \text{ for } m=0, \pm 1, \pm 2, \text{ etc.}$$

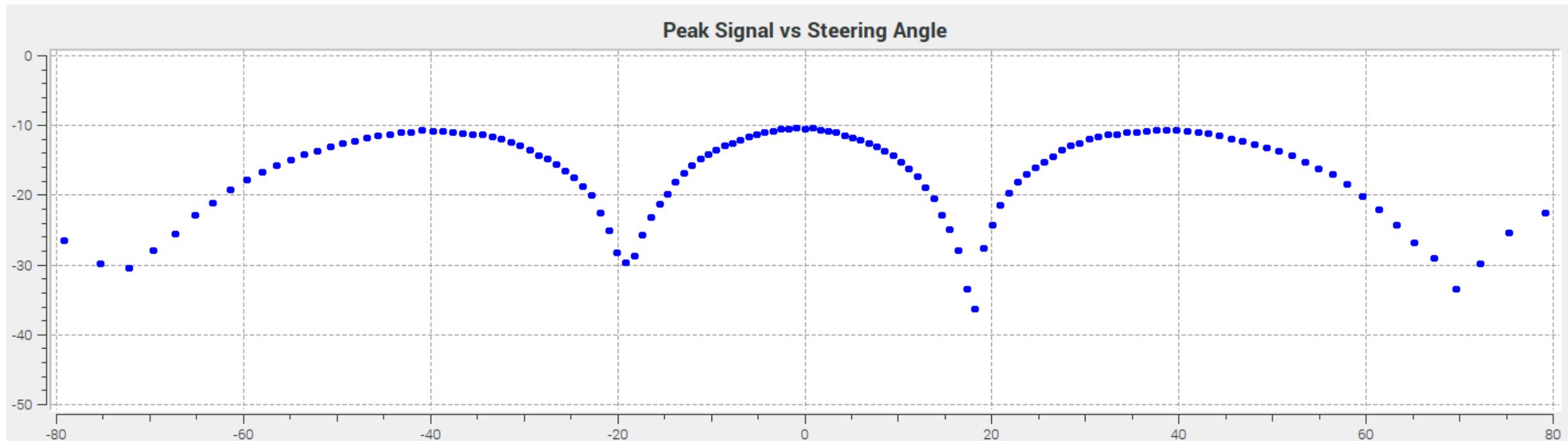
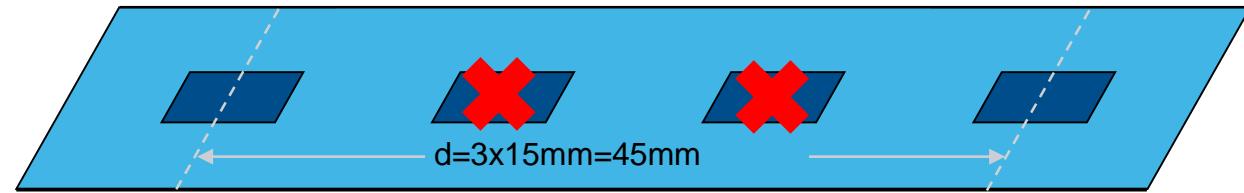
- ▶ Case 1:  $\lambda = 30\text{mm}$ , and  $d=15\text{mm}$  ( $\lambda/d = 2$ )
  - This yields only one real solution (only  $m=0$  gives valid solution)
- ▶ Case 2:  $\lambda = 30\text{mm}$ , and  $d=45\text{mm}$  ( $\lambda/d = 0.66$ )
  - $\theta(\text{main lobe}) = \sin^{-1}(m * 0.66)$ , is valid for  $m=0$  **AND**  $\pm 1$
  - We will see 3 main lobes!
    - Located at  $\theta = 0, \theta = \pm 0.72 \text{ rad} (\pm 41^\circ)$
  - FYI,  $m= \pm 2$  gives  $\pi/2 - i*0.78$ , which we ignore



<https://www.analog.com/en/analog-dialogue/articles/phased-array-antenna-patterns-part2.html>

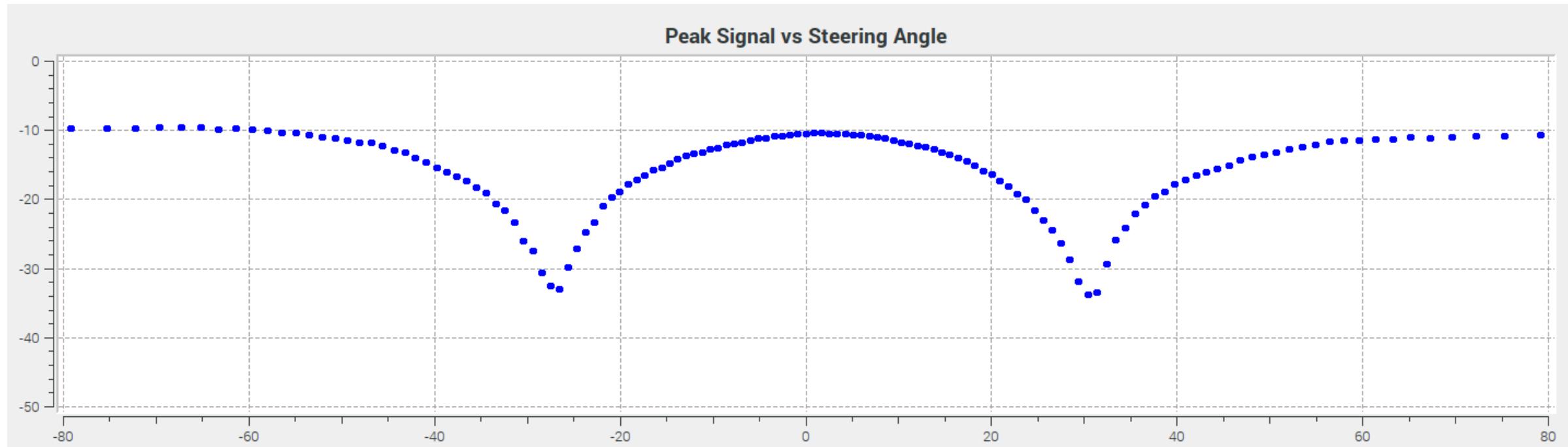
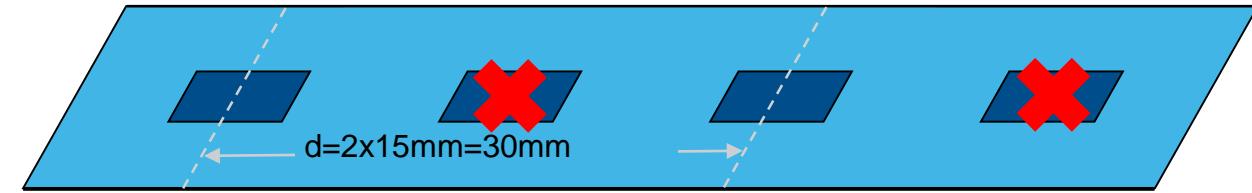
# Understanding Grating Lobes: Lab

- Set the radiating element to  $0^\circ$  (full broadside)
- Now set Rx1 and Rx4 to 127, and Rx 2 and 3 to 0
  - Now our  $d = 3 * 15 \text{ mm} = 45 \text{ mm}$ . So  $\lambda/d$  is about 0.66
- We expect to see main lobes at  $\sin^{-1}(0)$  **and**  $\sin^{-1}(\pm 1 \cdot 0.0285/0.045)$  which is  $0^\circ$  and  $\pm 40^\circ$



# Understanding Grating Lobes: Lab

- Set the radiating element to  $0^\circ$  (full broadside)
- ▶ Set every other element to 0: Rx1 = 127, Rx2 = 0, Rx3 = 127, and Rx4 = 0
  - Now our  $d = 2 * 15\text{mm} = 30 \text{ mm}$ . So  $\lambda/d$  is about 1
- ▶ We expect to see main lobes at  $\sin^{-1}(0.1)$  and  $\sin^{-1}(\pm 1.1)$  which is  $0^\circ$  and  $\pm 90^\circ$



# Understanding Grating Lobes: Math and Theory

- ▶ Can tolerate wider element spacing if the max beam angle is constrained

$$d_{max} = \frac{\lambda}{1 + |\sin\theta_{max}|}$$

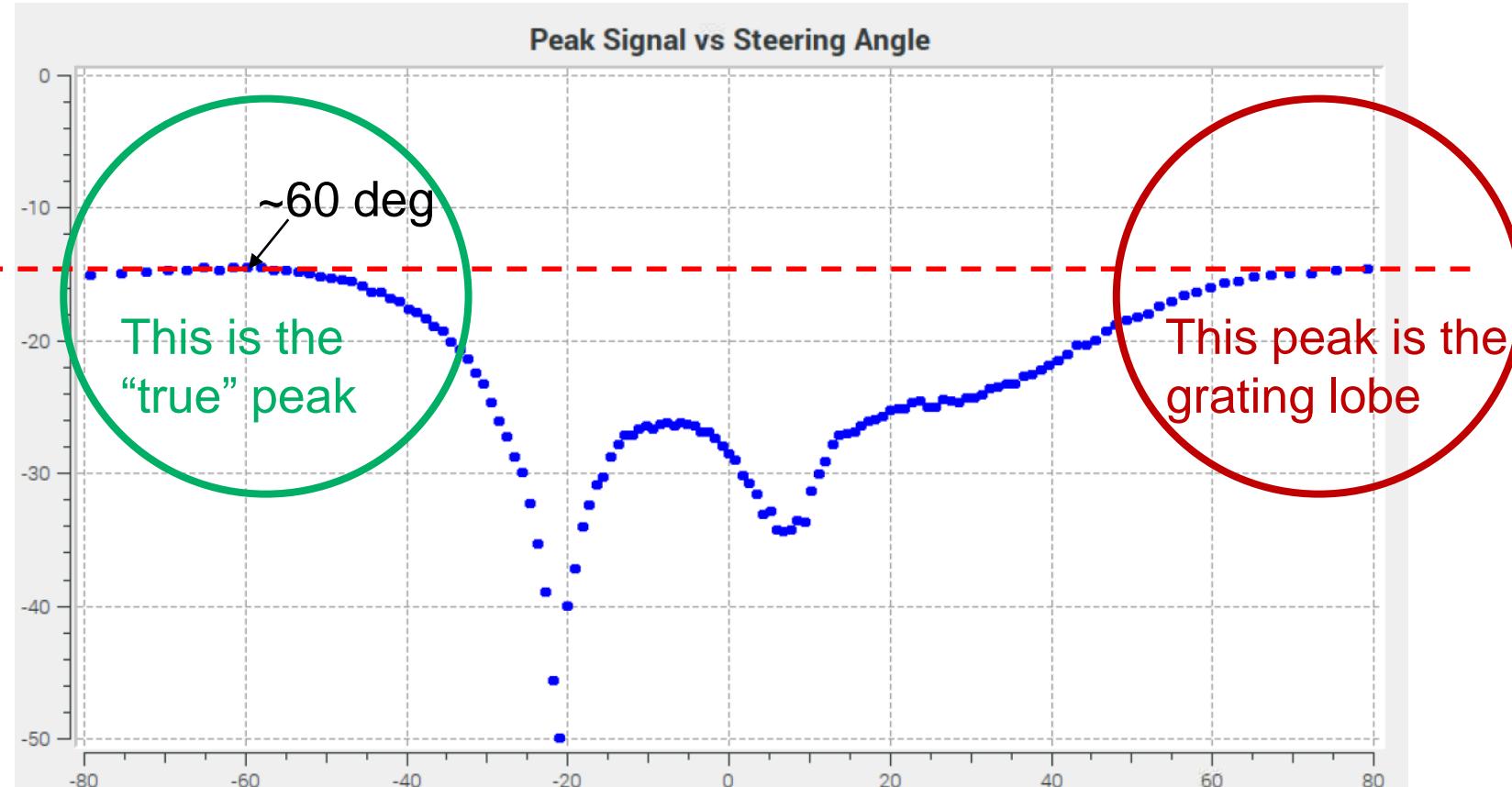
Derivation in “Phased Array Antenna Patterns—  
Part 2: Grating Lobes and Beam Squint”

- ▶ 10 GHz wavelength ( $\lambda$ ) is 30mm  $\rightarrow d$  for 0.5  $\lambda$ =15mm
- ▶ 10.5 GHz wavelength ( $\lambda$ ) is 28.5mm  $\rightarrow$  spacing is **0.53  $\lambda$**
- ▶ So if  $f=10.5\text{GHz}$ , and  $d=15\text{mm}$ , then:
  - $\theta_{max} = \arcsin(3E8/(10.5E9*0.015)-1) = 65 \text{ deg}$
  - We expect to see a grating lobe, on the opposite horizon, as we steer the beam to **65 deg**

# Understanding Grating Lobes: Math and Theory

- ▶ Let's try it now!
- ▶ Bring the radiator closer to the horizon and note when you have two equal peaks
- ▶ Use iPhone angle measure app!

$f=10.492\text{GHz}$



# Beam Squint → Combat with Time Delays

---

# Understanding Beam Squint: Math and Theory

- The same delay can be solved for in two ways:

- As a time delay:

$$\theta = \sin^{-1}(\Delta t c / d)$$

This is not a function of frequency

- As a phase delay:

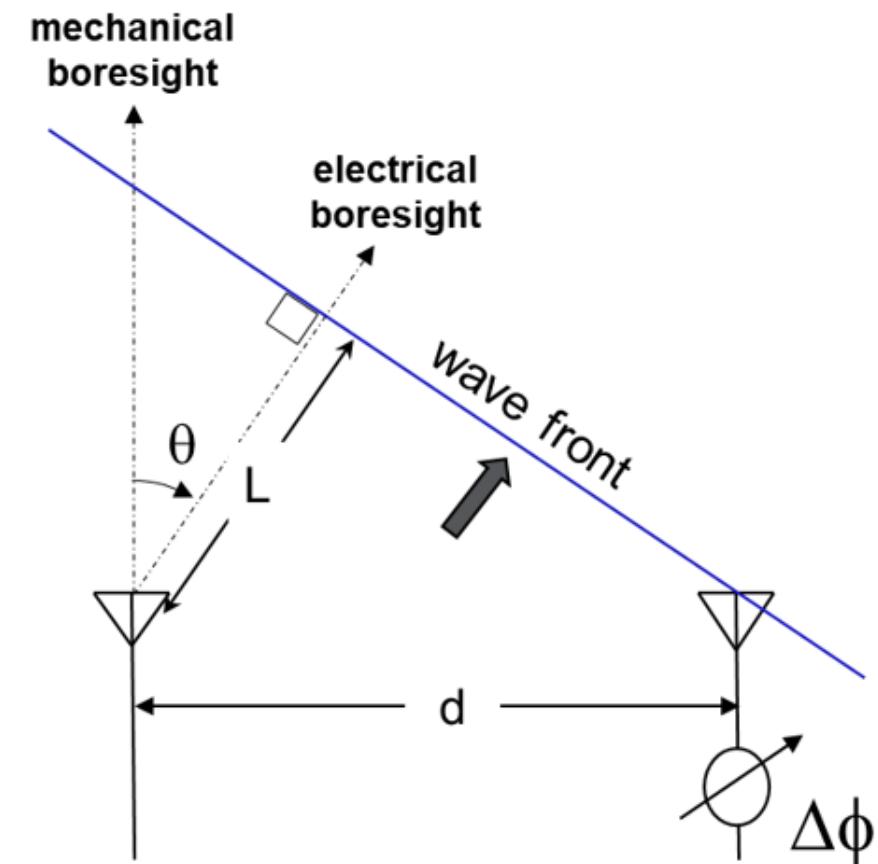
$$\theta = \sin^{-1}(\Delta\phi c / (2\pi f d))$$

This is a function of frequency

If we change frequency, the steering angle “changes”

- That is beam squint:

- The beam angle changing as a function of frequency.
- A true time delay beamformer doesn't have that problem because it doesn't have that frequency dependence.
- Mechanical boresight also does not have beam squint – as the phase shift is 0, and so there is no frequency component.



Remember:

$$\theta = \sin^{-1}(\Delta\phi c / (2\pi f d))$$

$$\Delta\phi = 2\pi f d \sin \theta / c$$

# Understanding Beam Squint: Math and Theory

- ▶ Beam deviation (beam squint) vs frequency can be calculated as:

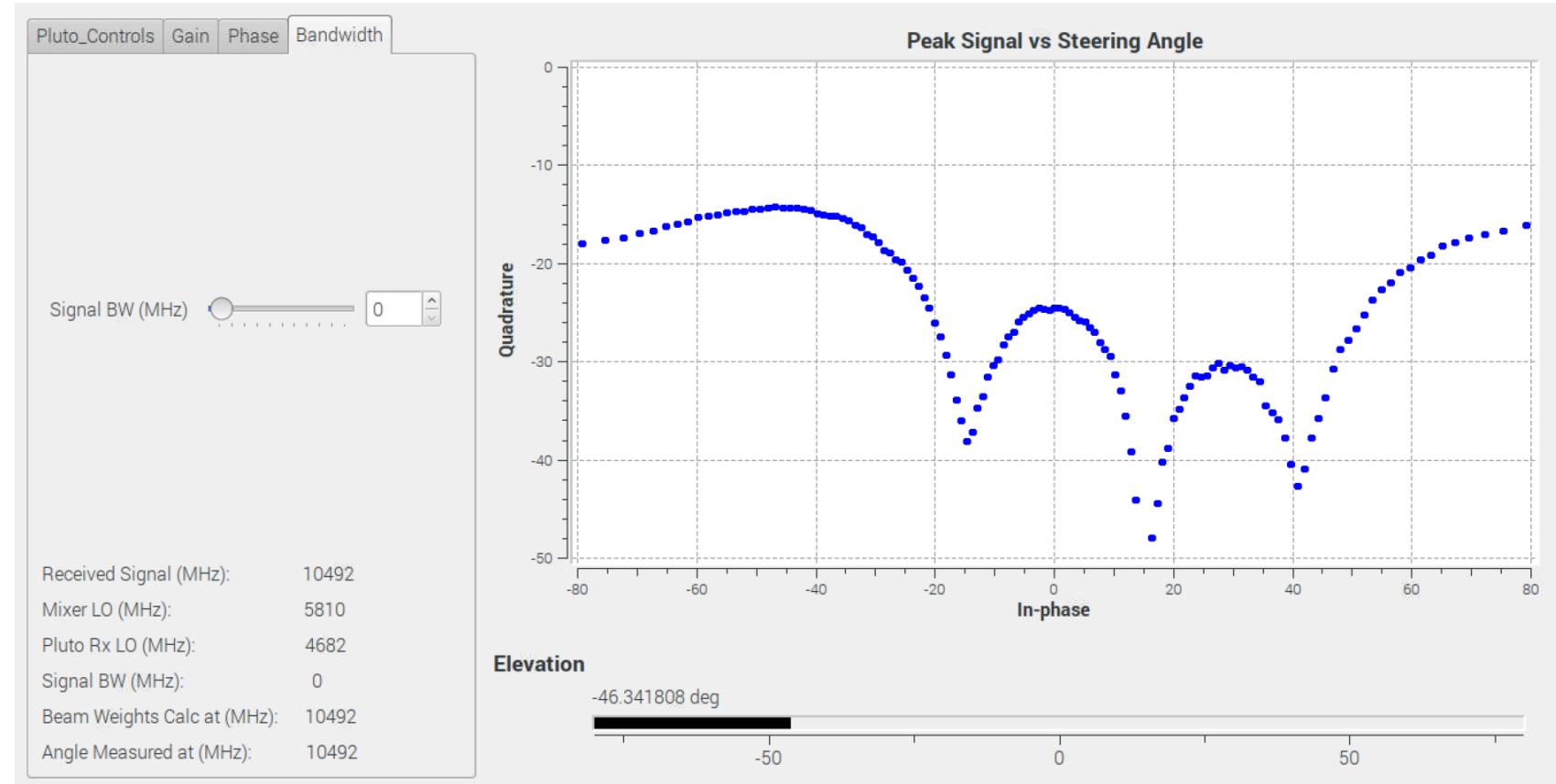
$$\Delta\theta = \arcsin\left(\frac{f_0}{f} \sin \theta_0\right) - \theta_0$$

- ▶ For example
  - Let's set our carrier frequency to be 10.5 GHz, and  $f_0 = 10$  GHz (500 MHz of BW)
  - We want to steer the beam to +/- 45° from mechanical boresight
  - $\Delta\theta = \arcsin(10.5/10 * \sin(45^\circ)) - 45^\circ = 3^\circ$
  - The beam will shift 3° at 10.5 GHz vs 10 GHz

<https://www.analog.com/en/analog-digital/articles/phased-array-antenna-patterns-part2.html>

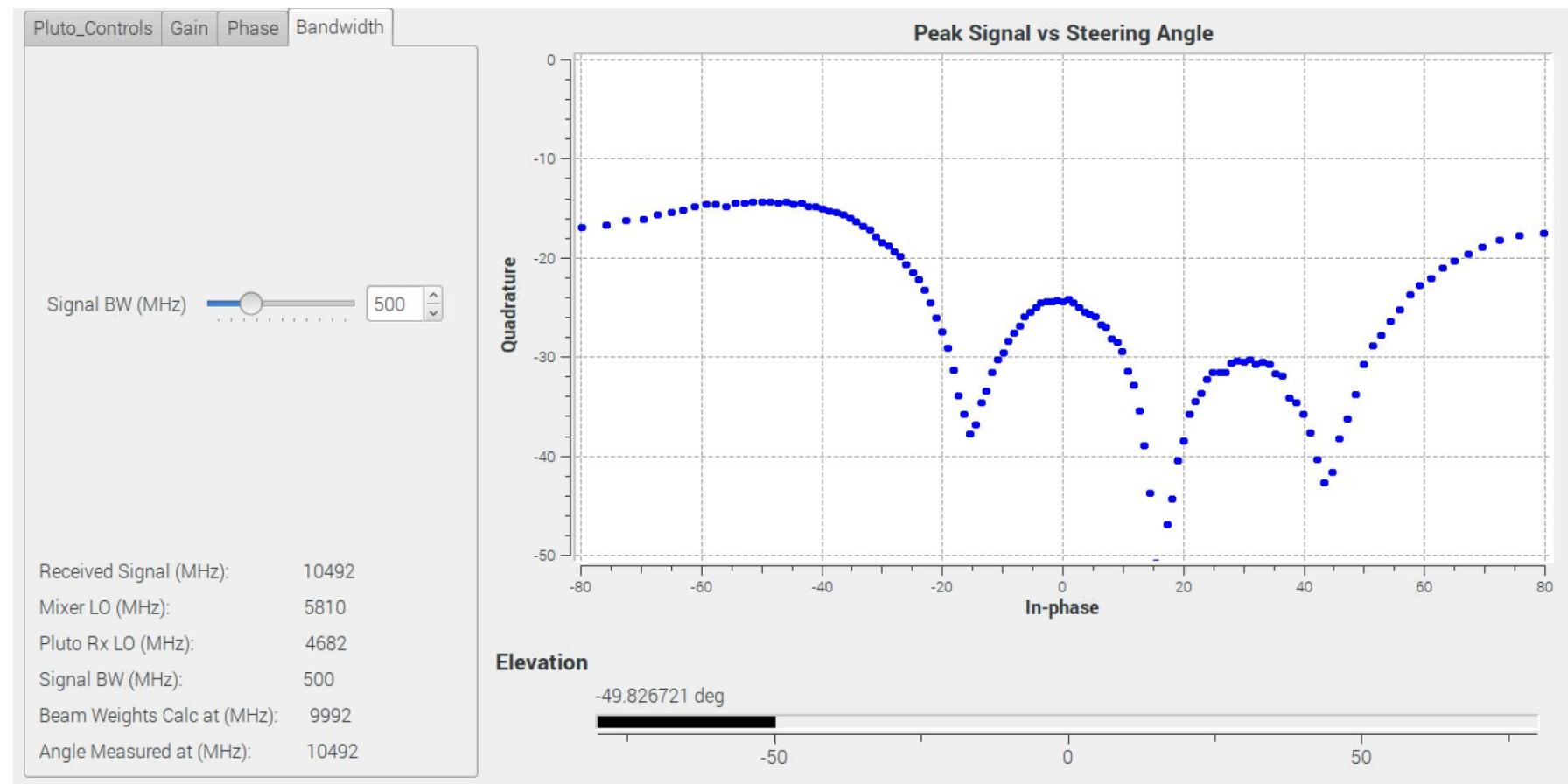
# Understanding Beam Squint: Lab

- ▶ Set theta to about 45 deg
- ▶ Click on the “Bandwidth” tab
- ▶ With BW = 0 MHz:
  - We are calculating the angle for just a narrow band sine wave



# Understanding Beam Squint: Lab

- Now change the signal BW to 500MHz
- This recalculates the angles for:
  - Signal – 500MHz
  - We are now looking at the furthest corner of our bandwidth
- The new angle has changed!
  - The change is: $-46.3 - (-49.8) = 3.5^\circ$
  - Just about what we calculated!

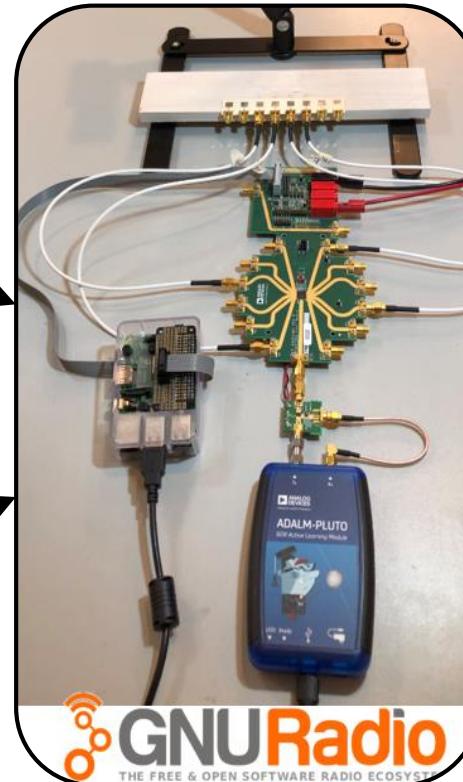


Steering Angle

Antenna Patterns

Antenna Impairments

Monopulse Tracking



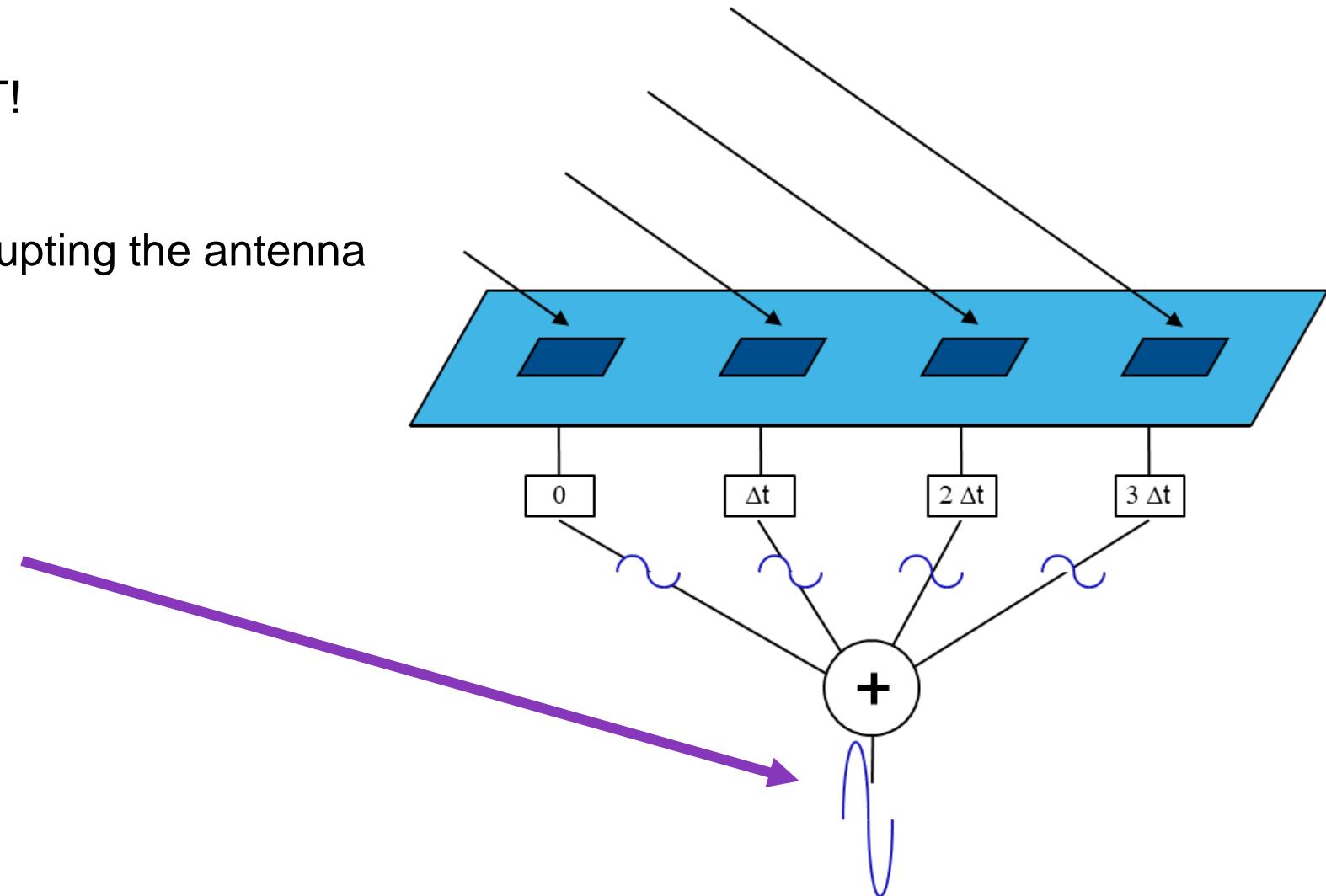
- Monopulse Tracking Intro
- Types of Beamformers
- X Microwave
- Monopulse Tracker in Action!

# Understanding Monopulse Tracking

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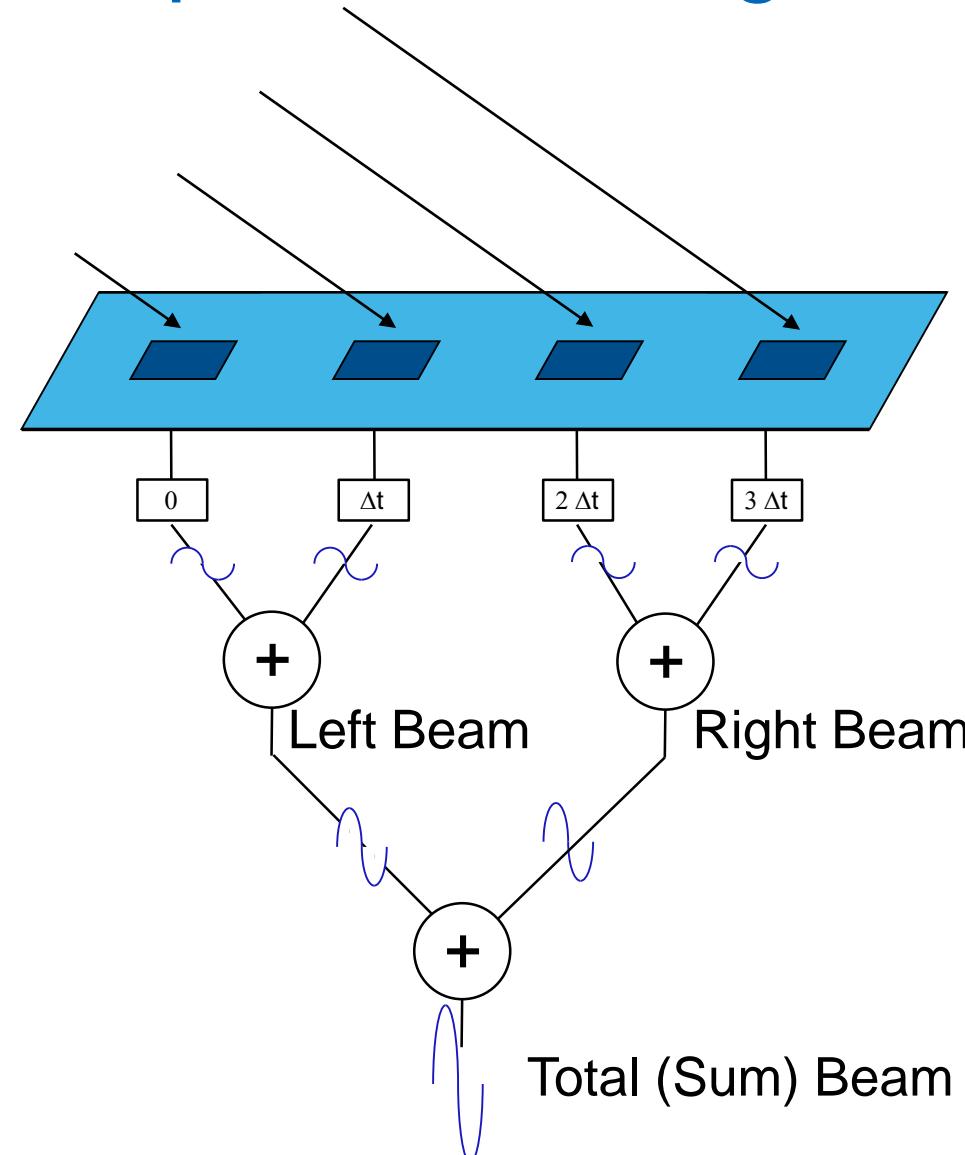
# Understanding Monopulse Tracking: Theory

- We've sweeping the beam a LOT!
  - This disrupts the antenna
  - Ideally want to track without disrupting the antenna
  - → Monopulse Tracking!
- 
- Right now, we have one beam:



# Understanding Monopulse Tracking: Theory

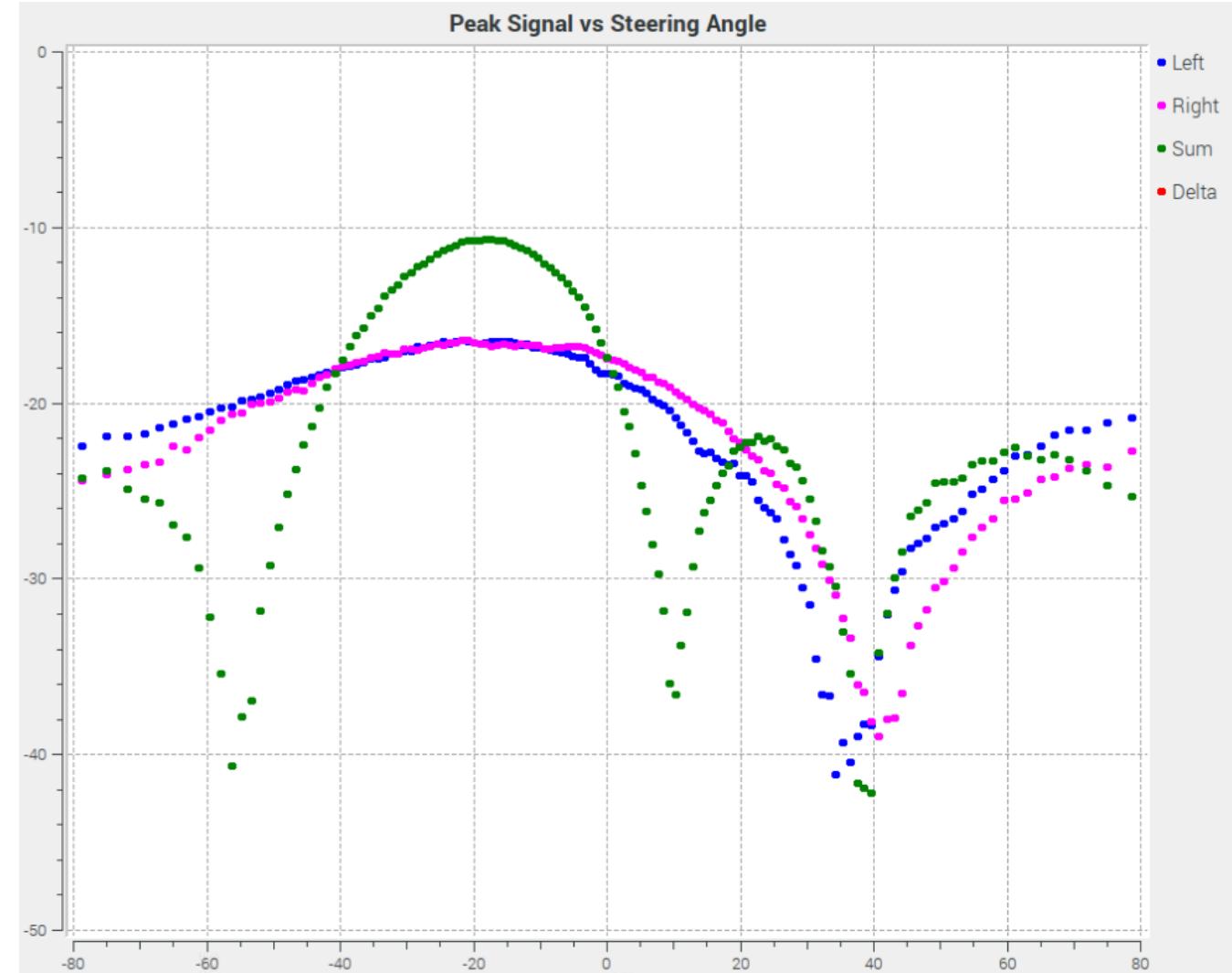
- ▶ But we could make 2 beams instead:
  - “Left” Beam
  - “Right” Beam



- ▶ Summing would give us the same total

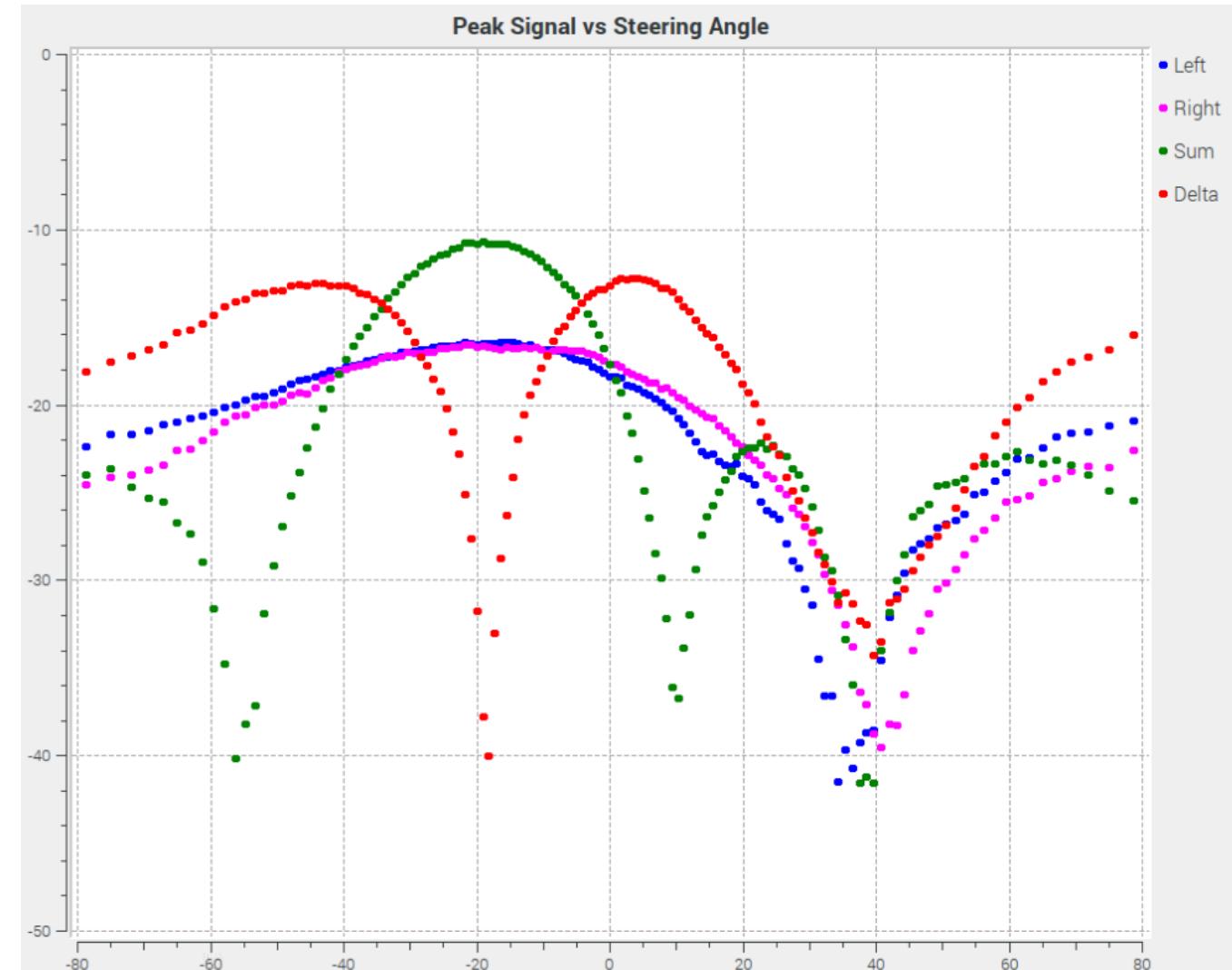
# Understanding Monopulse Tracking: Lab

- ▶ Each beam, separately, just looks like a two element array
  - **Blue** and **Pink** Curves
- ▶ Adding gives one of two results:
  - Phase aligned = Beamwidth improves to look like **N=4 array**
    - This **IS** what we want!!
  - Not phased aligned = slightly higher gain version of N=2 array
    - This is **NOT** what we want!!
- ▶ So we want to keep beams **phase synchronous!**



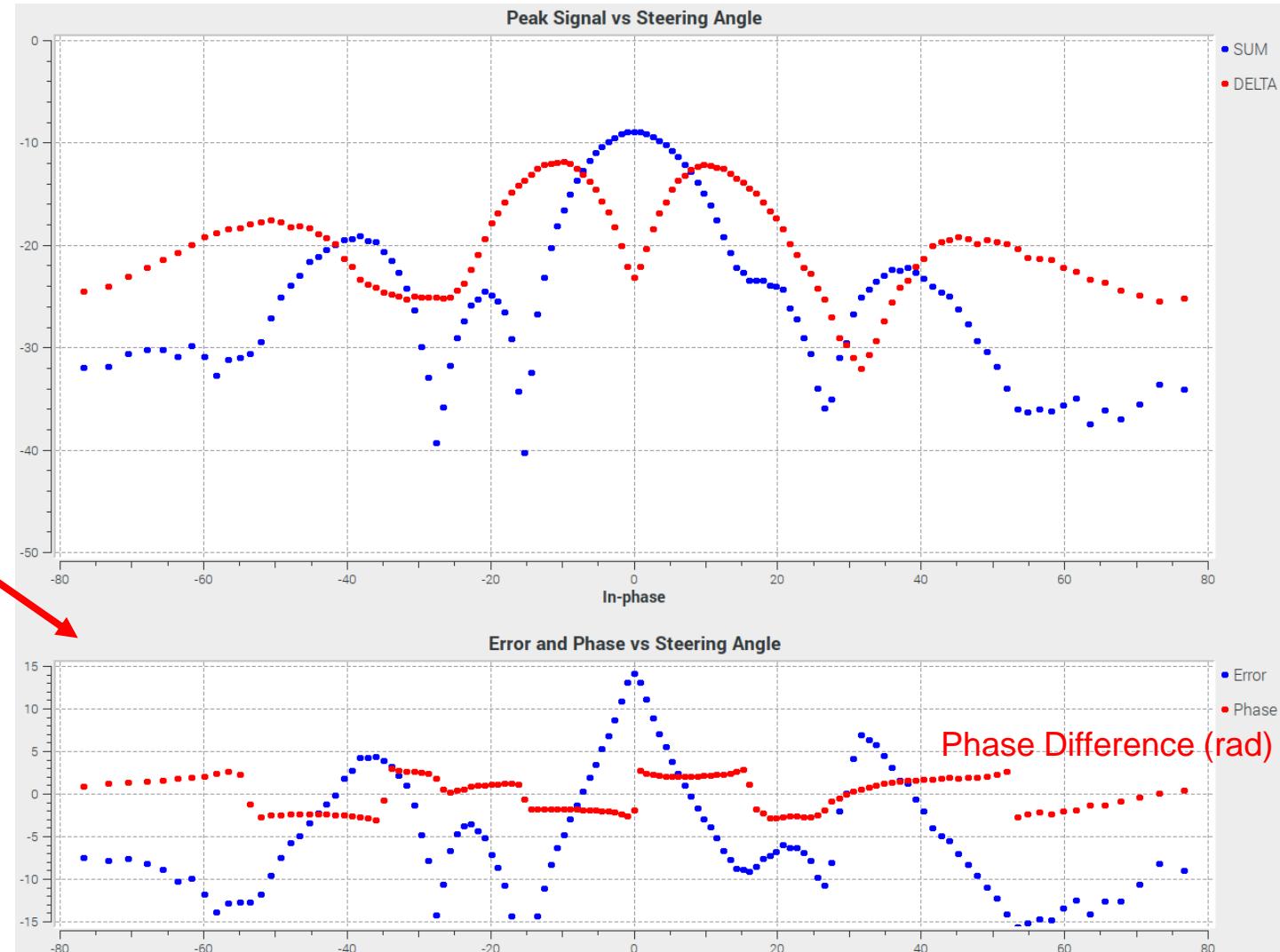
# Understanding Monopulse Tracking: Lab

- ▶ But what if we took the difference instead?
- ▶ This gives the red “Delta” plot
- ▶ The sharp null corresponds to the direction of arrival (DOA) peak.
- ▶ We can use this to developing our “Monopulse” tracking algorithm.
- ▶ But that means we need to upgrade our setup to “Hybrid”



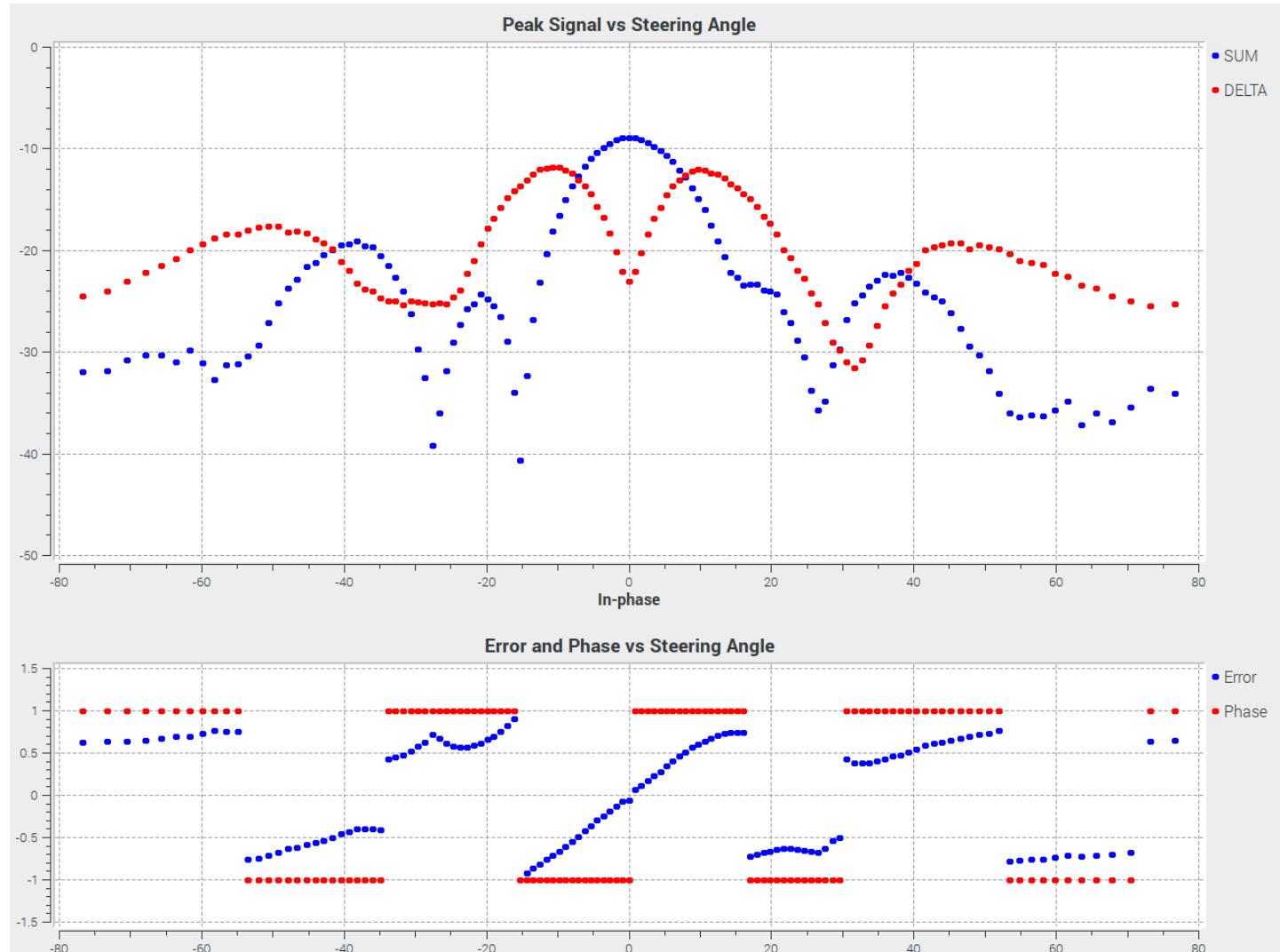
# Understanding Monopulse Tracking: Lab

- ▶ But let's look at the difference in magnitude and phase:
  - “Error” = “Sum” – “Delta”
  - “Phase” =  $(\text{phase}(\text{Sum}) - \text{phase}(\text{Delta}))$
- ▶ “Error” is a V shaped curve
- ▶ We just need the sign of the phase shift
- ▶ Now we can make a true error function!



# Understanding Monopulse Tracking: Lab

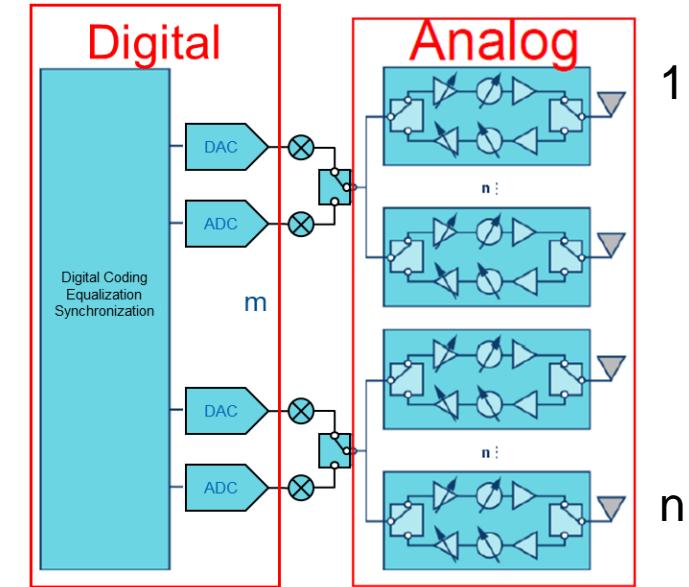
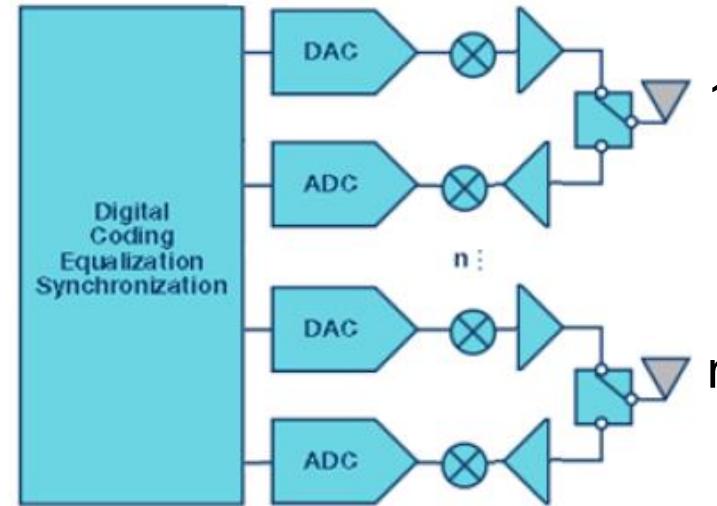
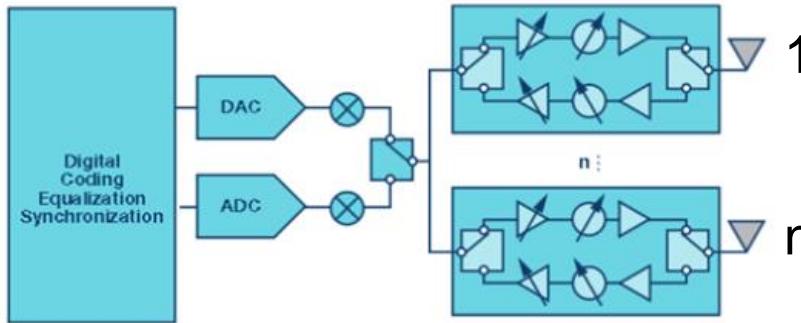
- ▶ Phase is now plotted as just the sign of the phase difference
- ▶ The Error function takes into account the sign of phase and is normalized.
- ▶ Now, we can finally use this error equation to determine when, and by how much, to steer our beam to recenter it.



# Analog, Digital, and Hybrid Beamformers

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# Beamforming architectures



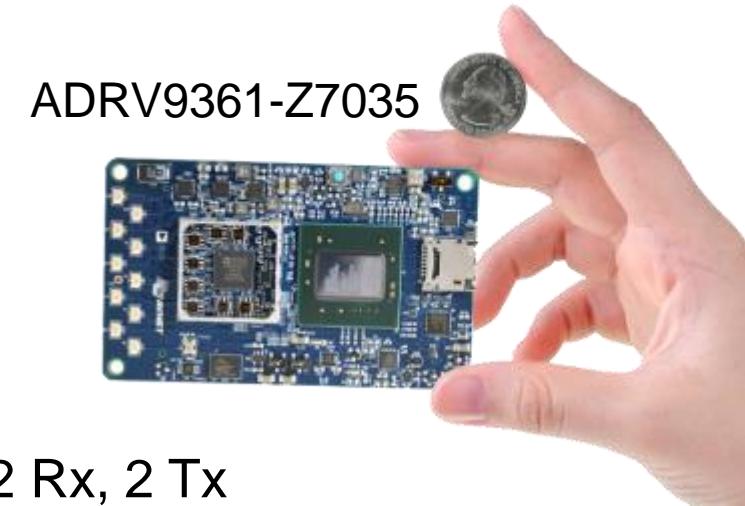
Analog Beamforming	Digital Beamforming	Hybrid Beamforming
Beam formed by weighting RF paths	Beam formed by weighting digital paths	Digital combining of multiple analog beams
Single set of data converters	Separate data converters for each element	$1 < m < n$ sets of data converters
Low power/complexity	Highest power / complexity	Moderate power/complexity
Good for coverage	Highest capacity / flexibility	Compromise between analog and digital
Single narrow beam	Wide analog beamwidth, narrow digital beams	Best choice with existing technology

# Understanding Monopulse Tracking: Lab

- ▶ To implement monopulse on our two beam linear array, we need:
  - Two ADAR1000s (one for “Right” beam, one for “Left” beam)
  - Two ADC channels (one for each beam, so we can digitally calculate sum and delta).
  - Putting these together will result in a “Hybrid” Beamformer!
- ▶ Fortunately, the big brother to Pluto, ADRV9361-Z7035 will work great here:
  - <https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/adrv9361-z7035.html>

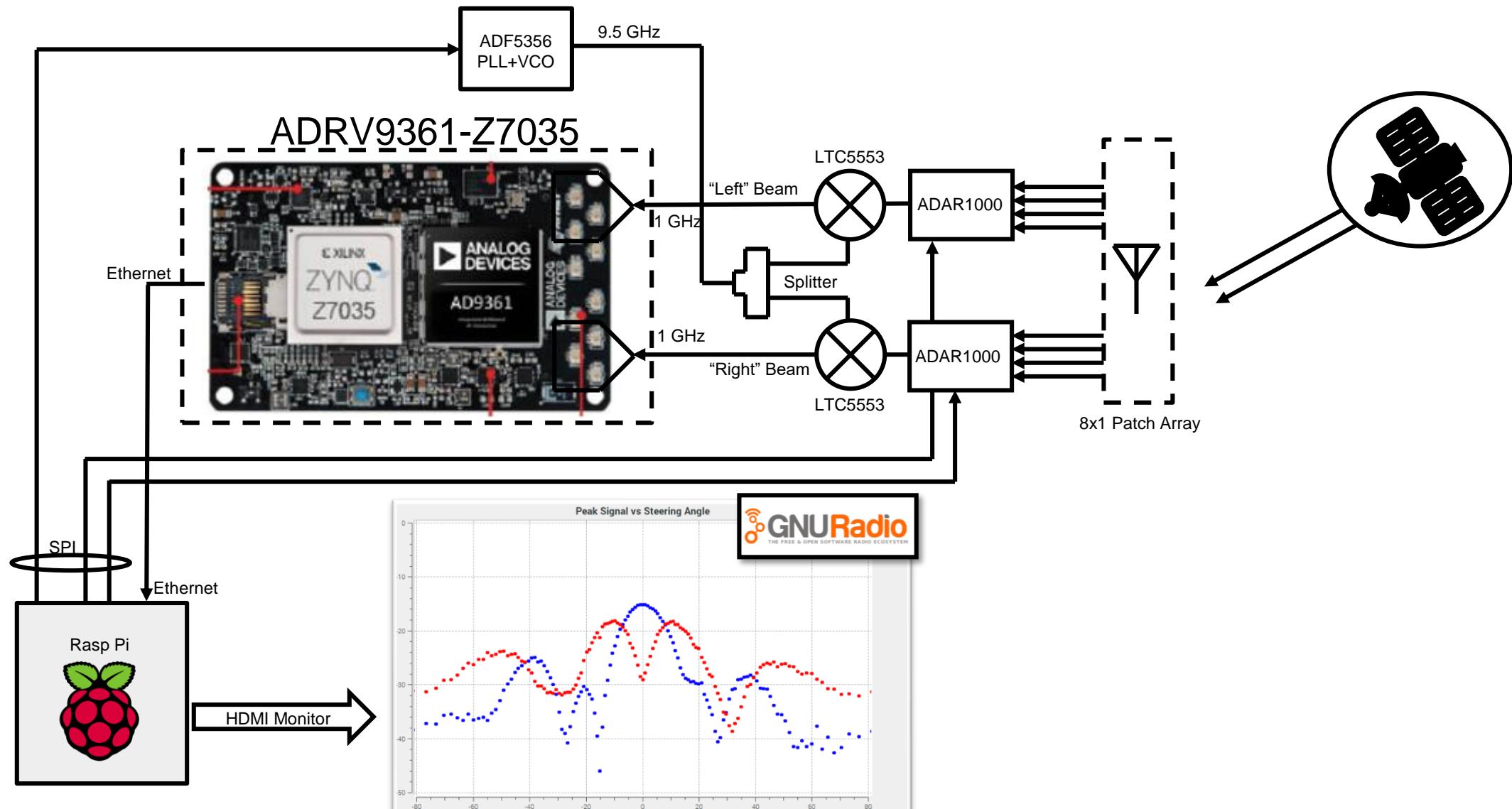


1 Rx, 1 Tx



2 Rx, 2 Tx

# Monopulse Tracker Setup:



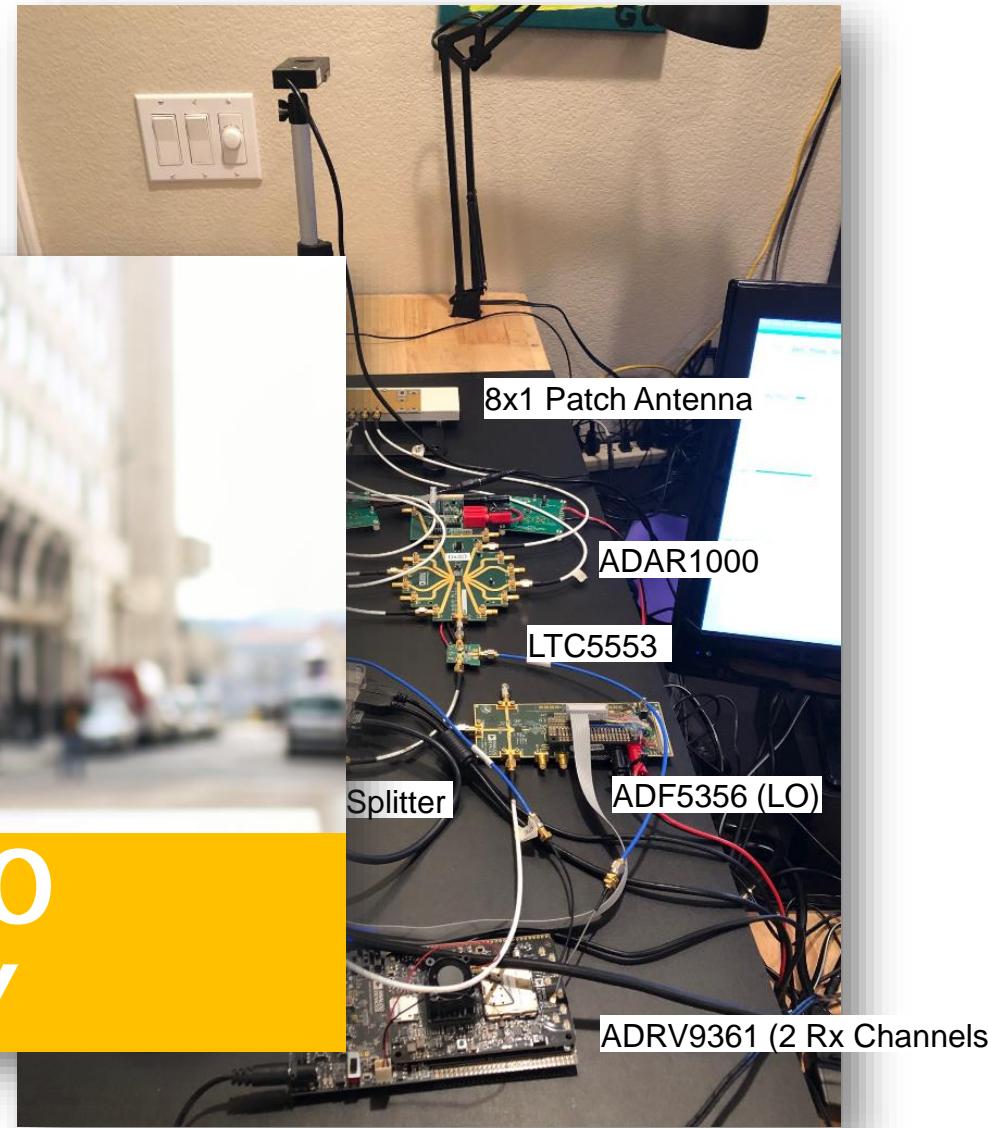
# Understanding Monopulse Tracking: Lab

Our setup becomes a bit more complicated....



THERE HAS GOT TO  
BE A BETTER WAY

Photo free to use, courtesy of Andrea Piacquadio, [www.pexels.com](http://www.pexels.com)

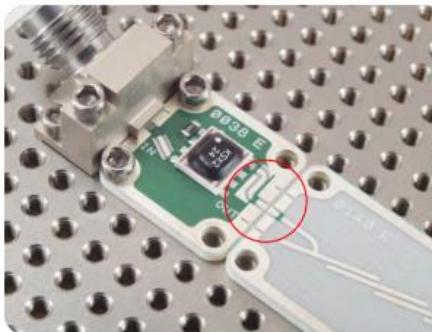


# *Rapid Prototyping with X Microwave:*

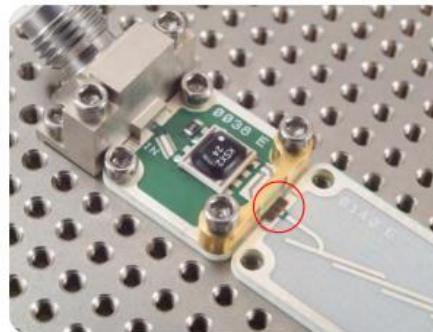
How to **instantly** generate RF Layouts and a custom prototypes

- ▶ X Microwave is doing great stuff.
  - <https://www.xmicrowave.com/>
  - It is RF breadboarding at up to 67GHz!

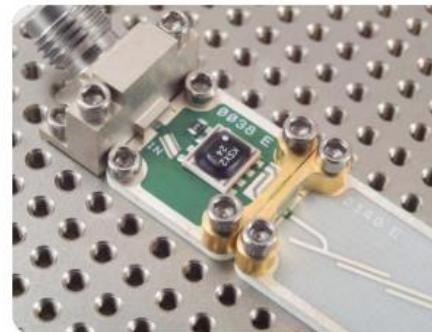
Launch-to-Launch Solderless Interconnect



1. Line up the Launch



2. Place the G-S-G Jumper



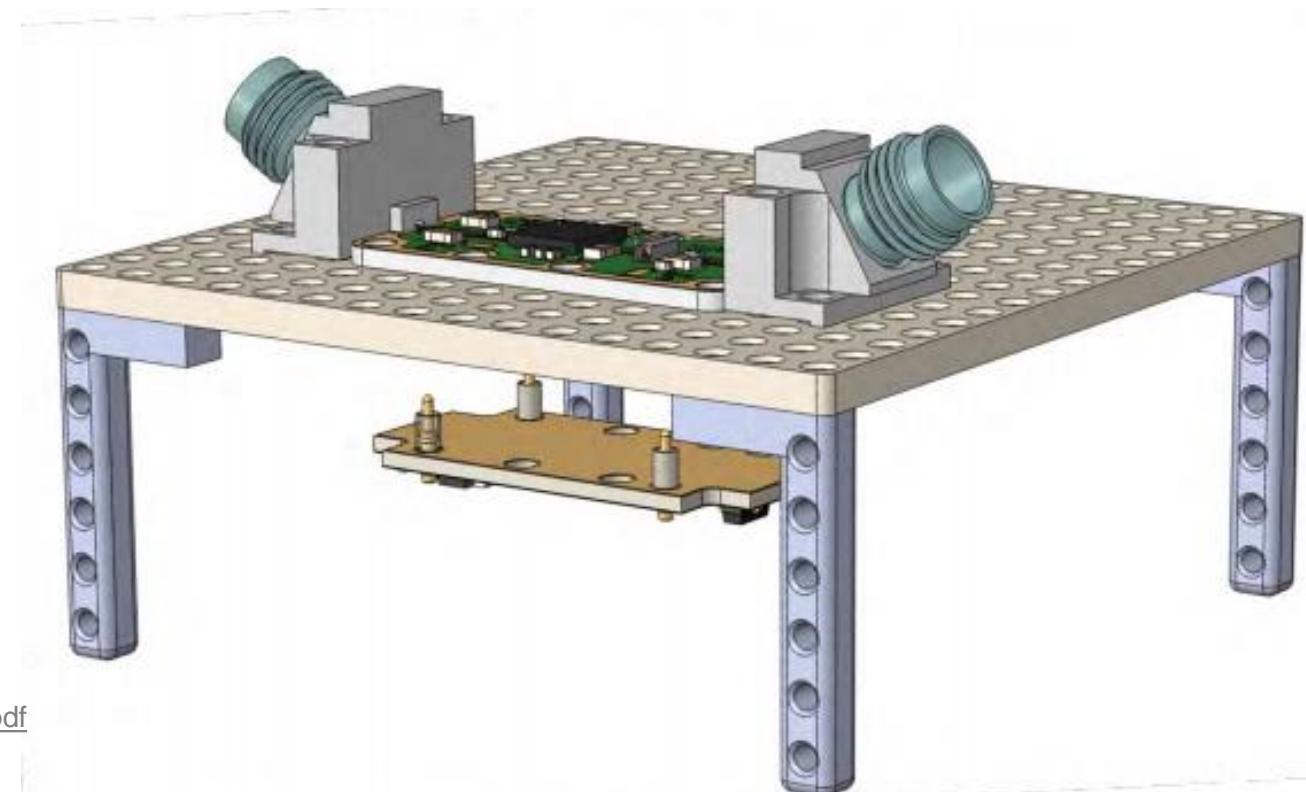
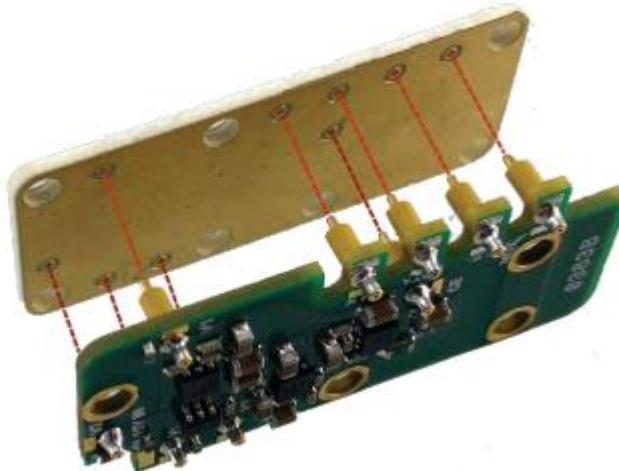
3. Attach the Anchors

[https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium\\_Luther2019.pdf](https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium_Luther2019.pdf)

- ▶ Many manufacturers are included, but the largest portfolio is from Analog Devices:
  - Over 400 Unique ADI Parts in their library: [ADI/HMC/LTC RF parts](#)
  - Amplifiers, synthesizers, couplers, mixers, filters, digital step attenuators, switches, transmission lines, etc.

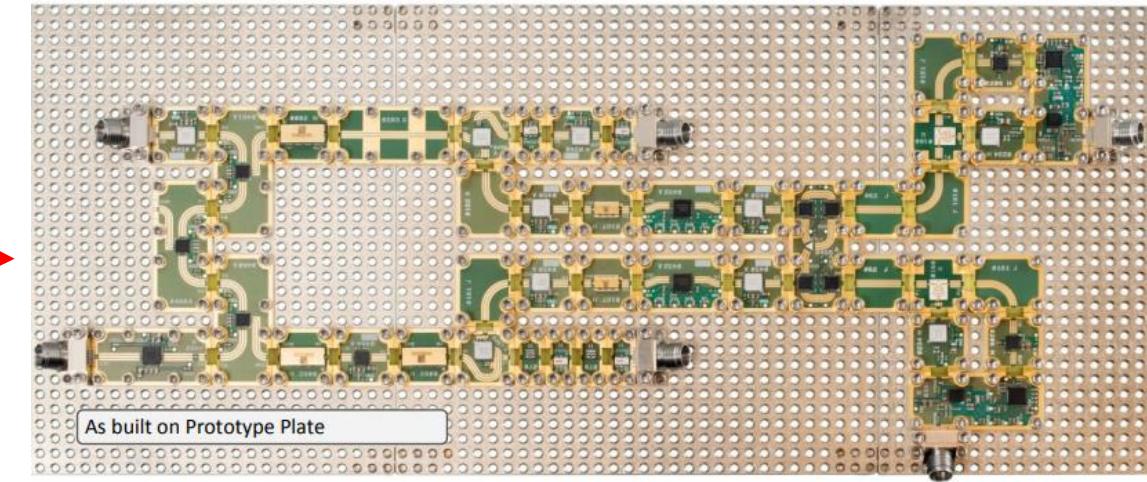
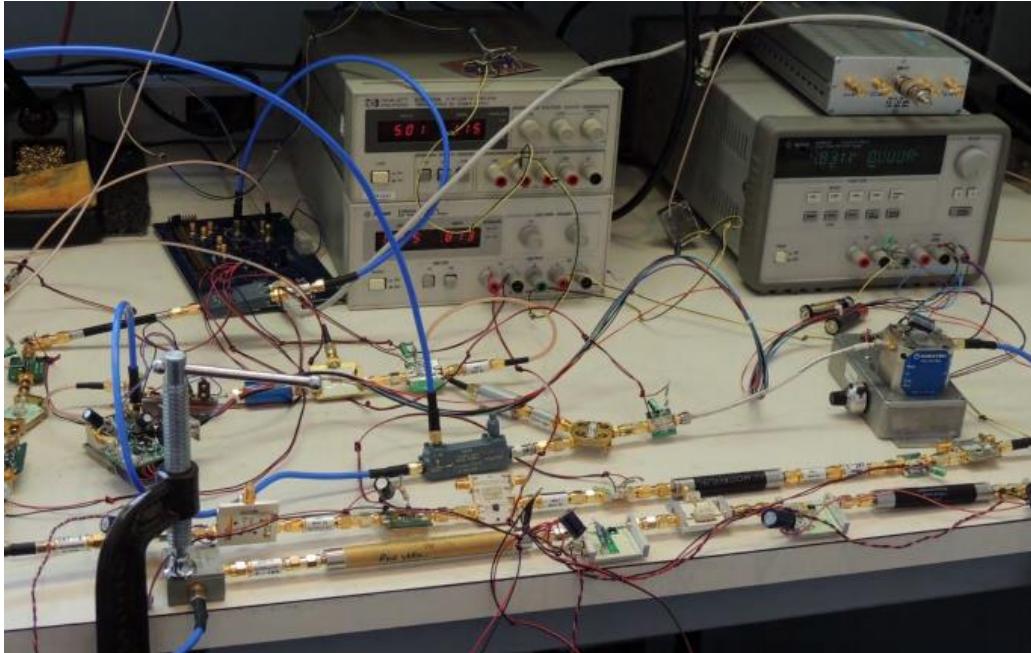
# X Microwave: Power and Data Routing

- ▶ Power and SPI comes from an accessory board on the bottom.
- ▶ It connects via pogo pins to spots on the main RF board.
- ▶ So the RF path is kept pristine



[https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium\\_Luther2019.pdf](https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium_Luther2019.pdf)

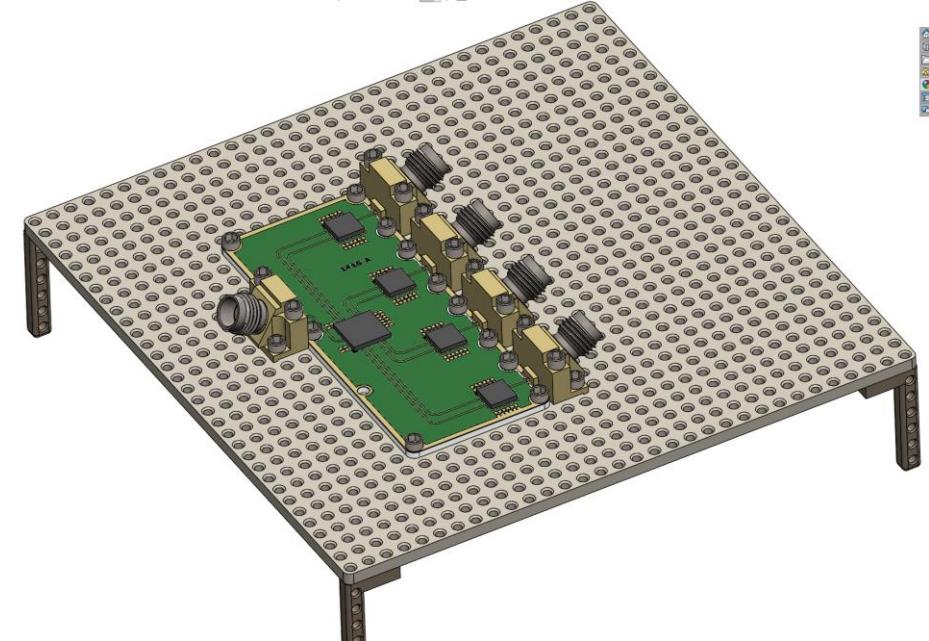
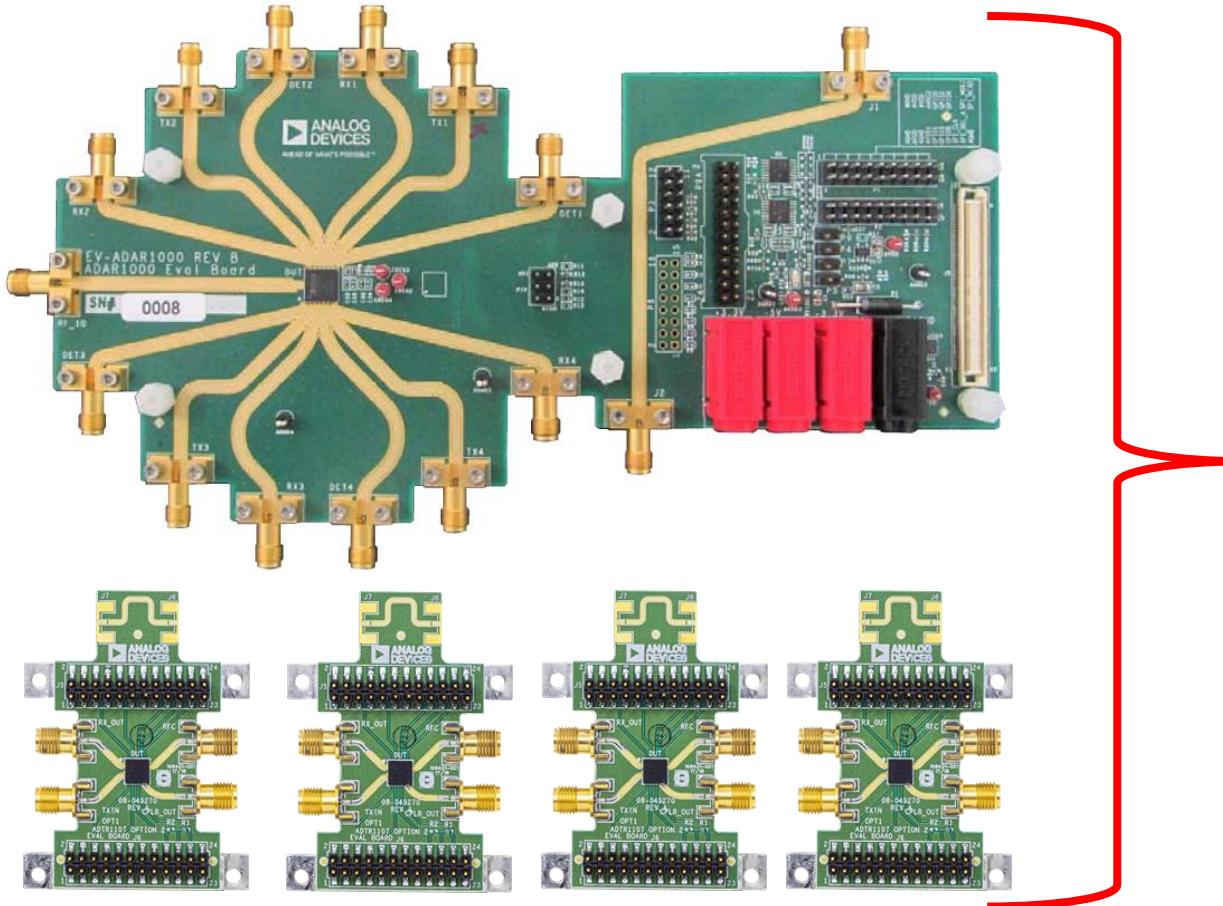
# X Microwave makes things simple and clean:



[https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium\\_Luther2019.pdf](https://www.xmicrowave.com/wp-content/uploads/Texas-Symposium_Luther2019.pdf)

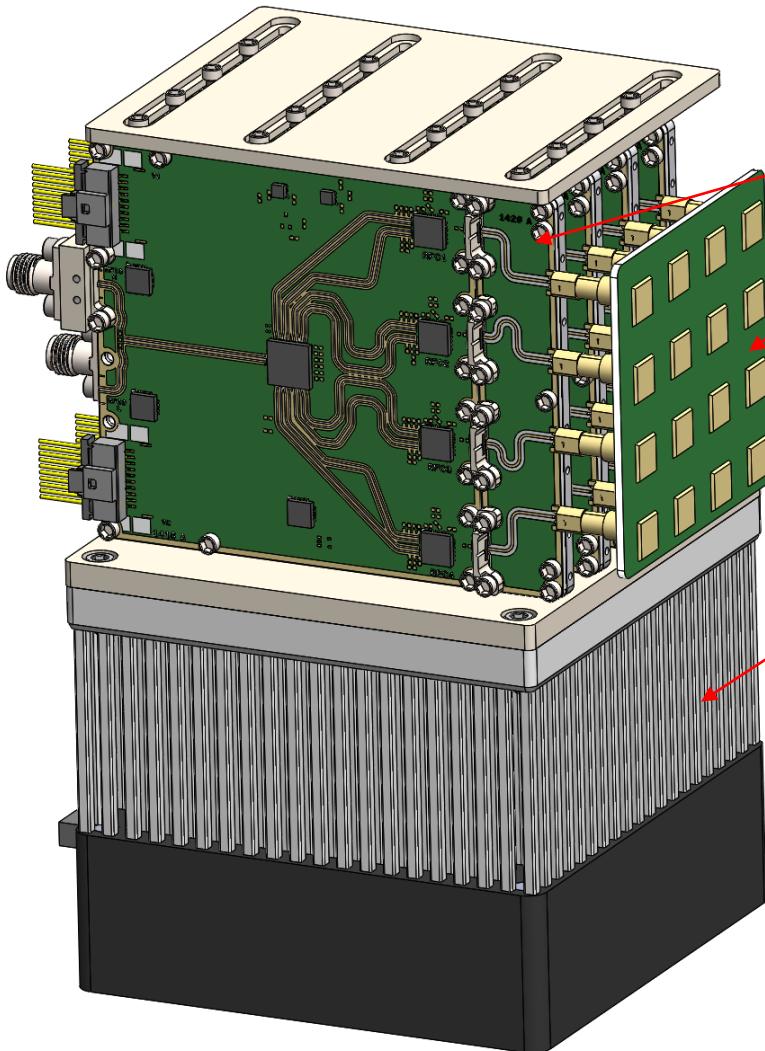
- The XM Prototype is closer to the final product
  - Fewer prototype iterations, faster time to production

# X Microwave ADAR1000 Module



ADAR1000 + 4 ADTR1107 (TR Modules—i.e. PA/LNA/Switch)

# Stack ADAR1000 Modules Together for the Phased Array Cube:

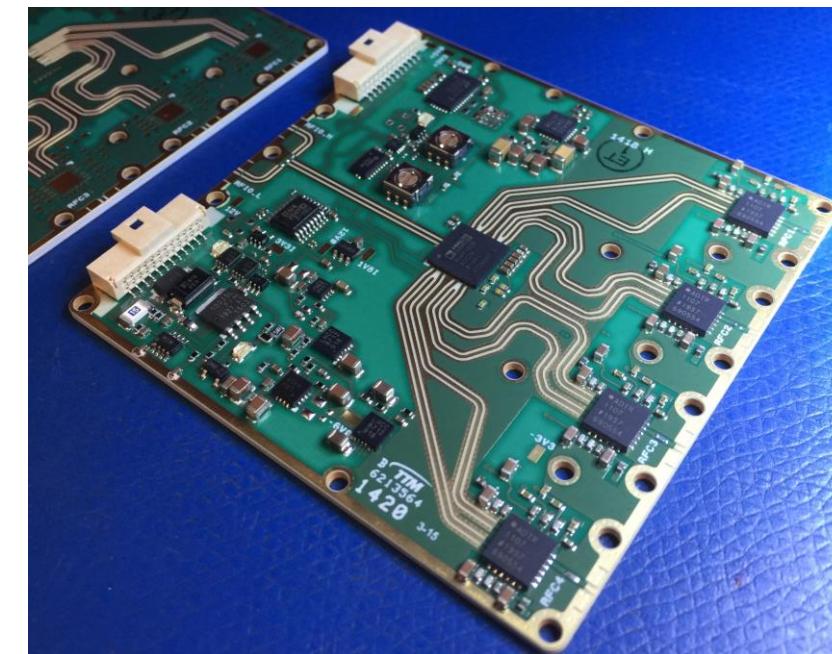


**Stack 4 together to create a 4x4 array**

Interposer board to fit whatever lattice spacing

Antenna snaps on

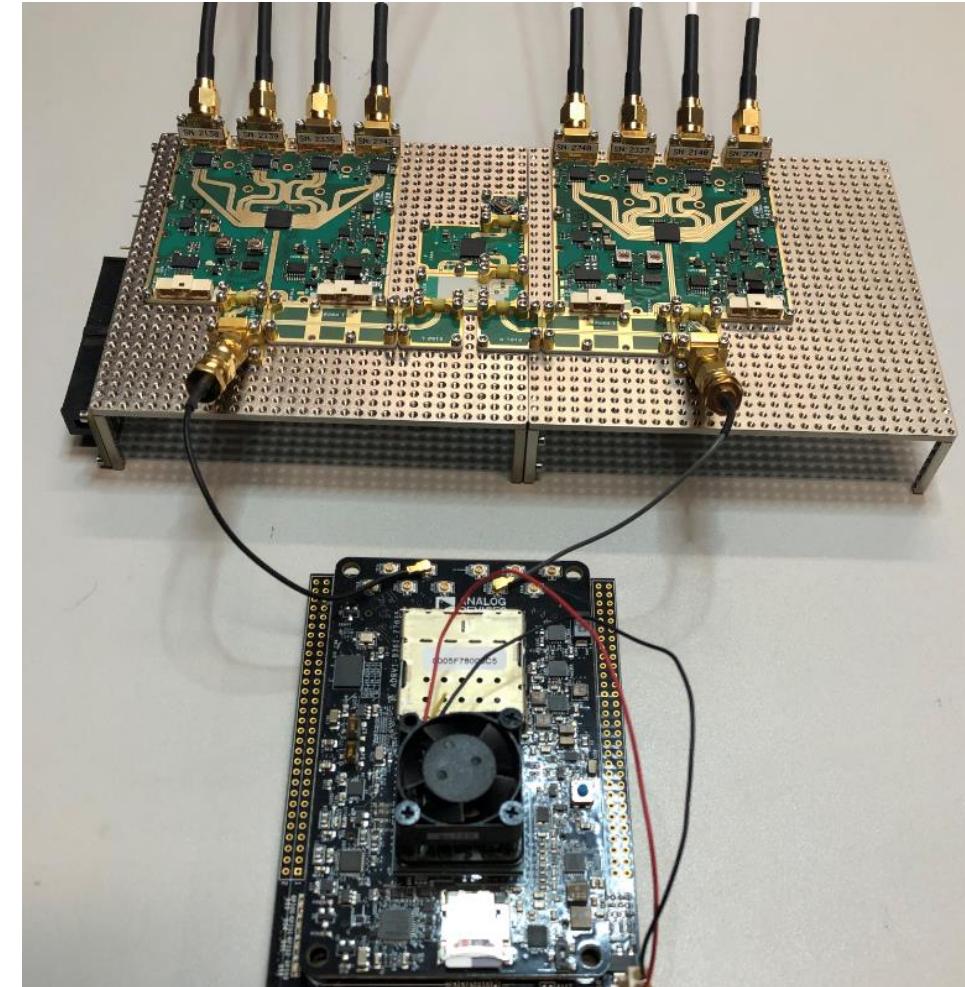
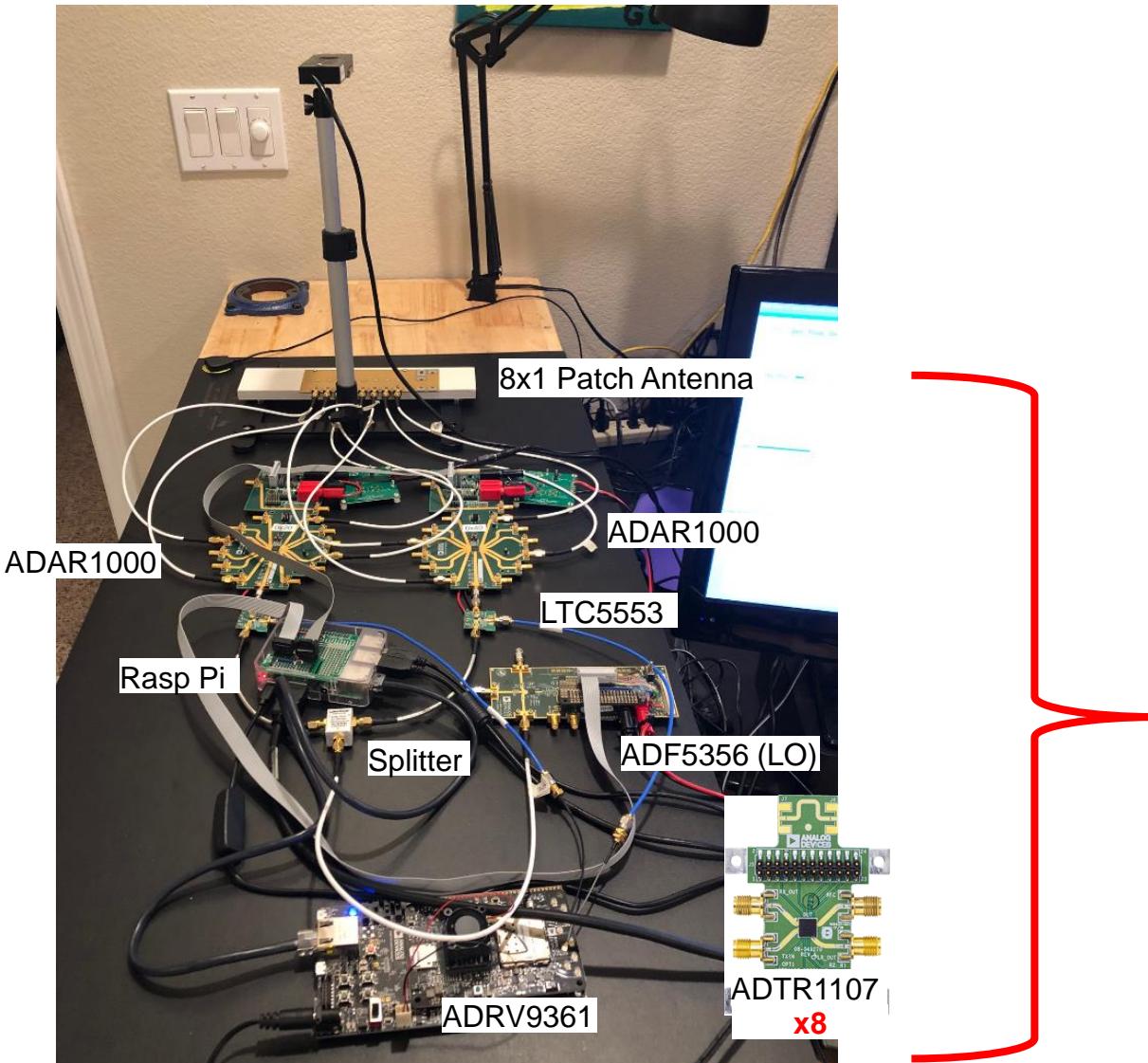
Heatsink



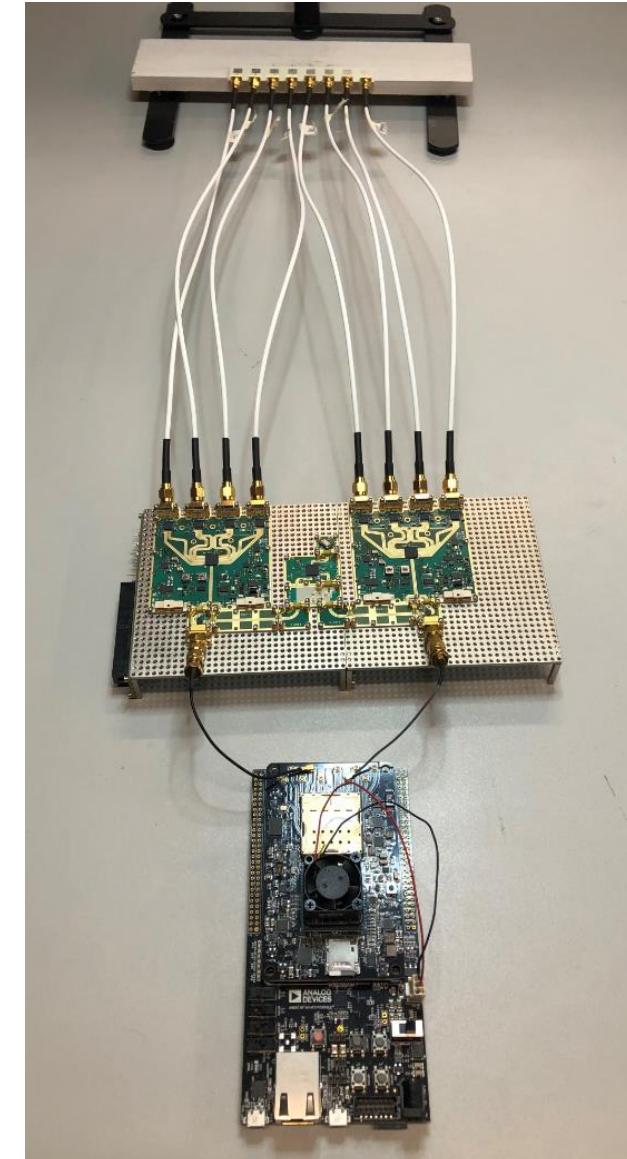
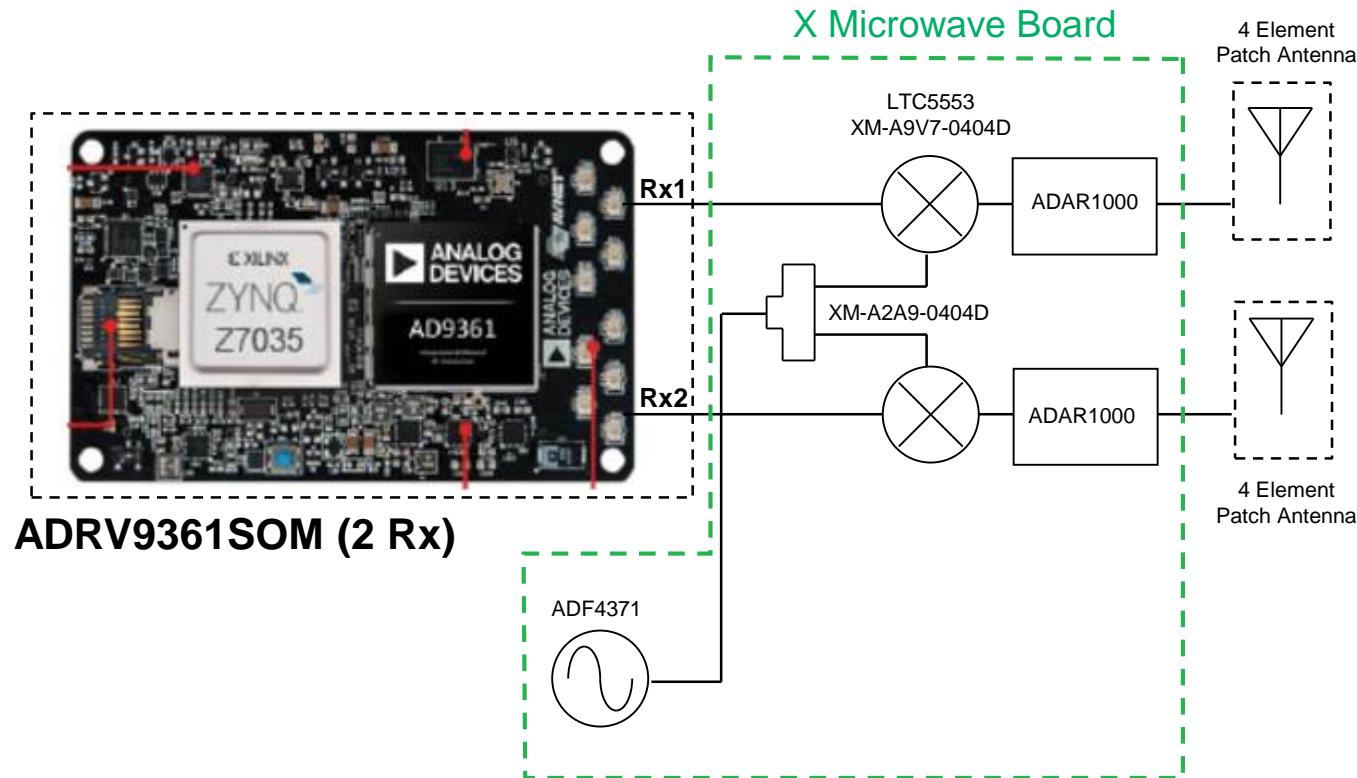
# RF Layout in 60 seconds (time lapse video)

Build the entire prototype in 90 min  
(time lapse video)

# 2 Beam Setup For Monopulse Tracking

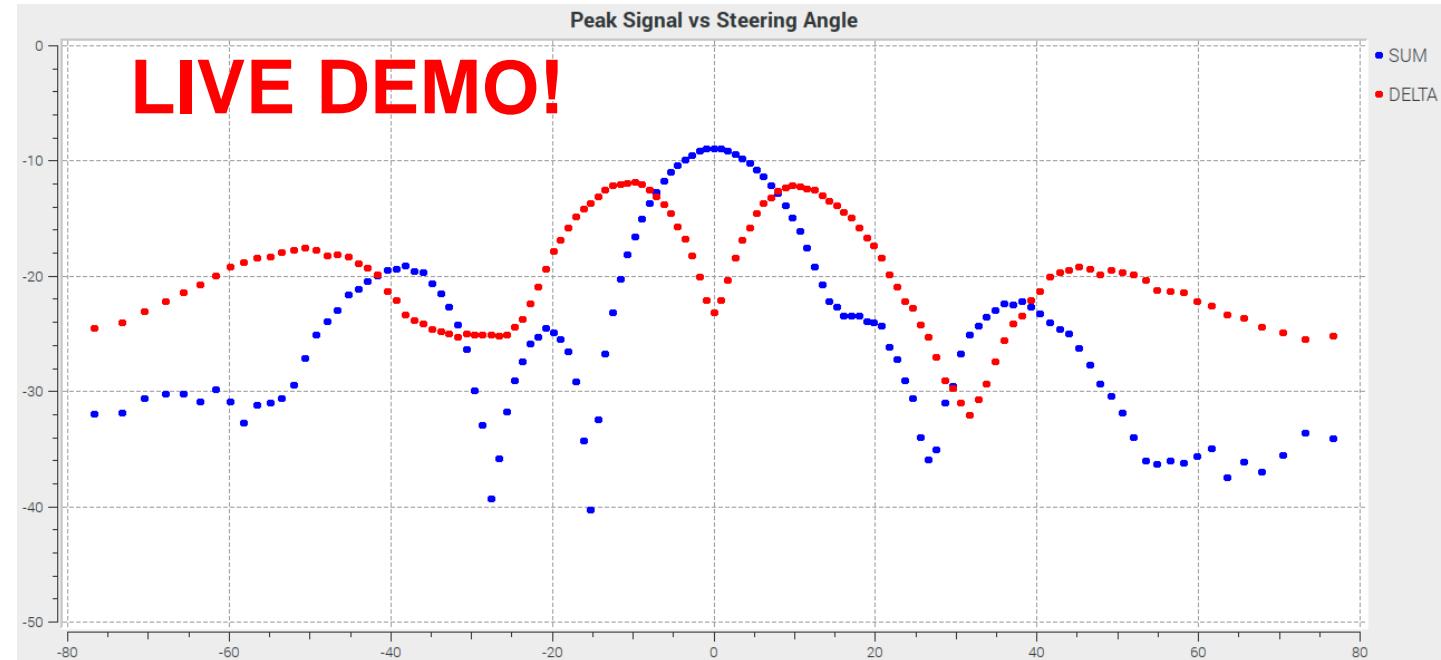


# Hybrid Beamformer with X Microwave



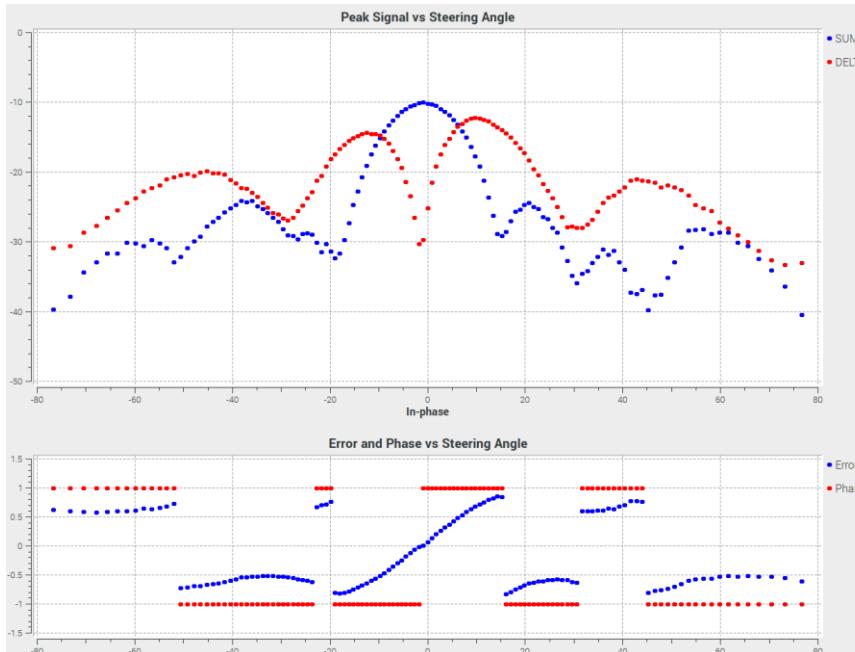
# Understanding Monopulse Tracking: Lab

- ▶ Now let's plot the sum and Delta for our 8 channel array
  - Sum =  $Rx_1 + Rx_2$
  - Delta =  $Rx_1 - Rx_2$
- ▶ Our HPBW is  $\frac{1}{2}$  what our 4 channel array was!
  - Math works!
- ▶ And we see a nice null at the peak of Sum plot.
- ▶ This is all very nice, and as we expected.



# Understanding Monopulse Tracking: Lab

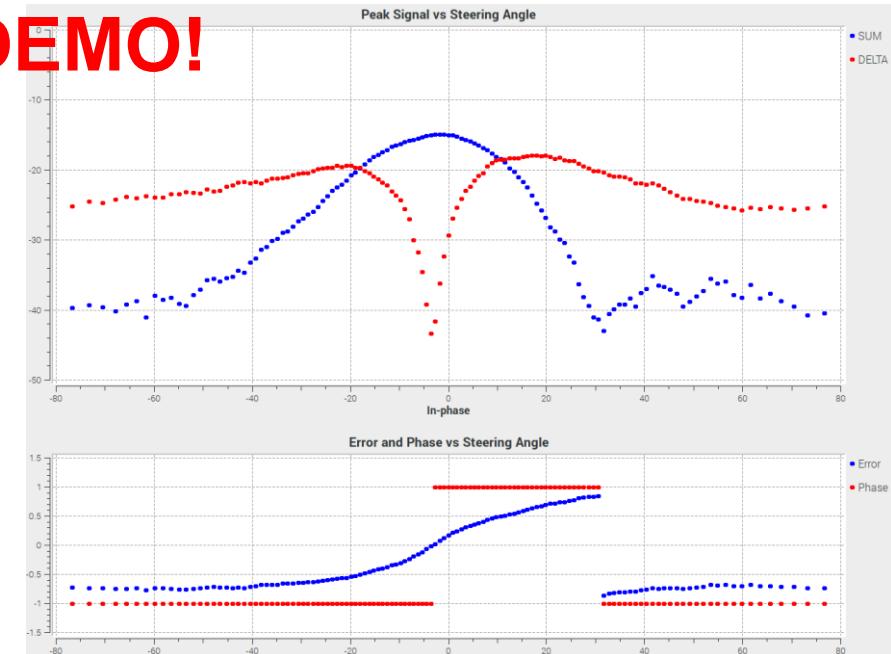
- With 8 elements, we can do a nice taper too!
  - In Matlab, use the blackman function and normalize to gain of 127:



LIVE DEMO!

## Blackman Taper

```
>> blackman(10)/max(blackman(10)) * 127
ans =
    0
    6.7924
   34.4496
   84.1210
 127.0000
 127.0000
   84.1210
   34.4496
    6.7924
    0
```

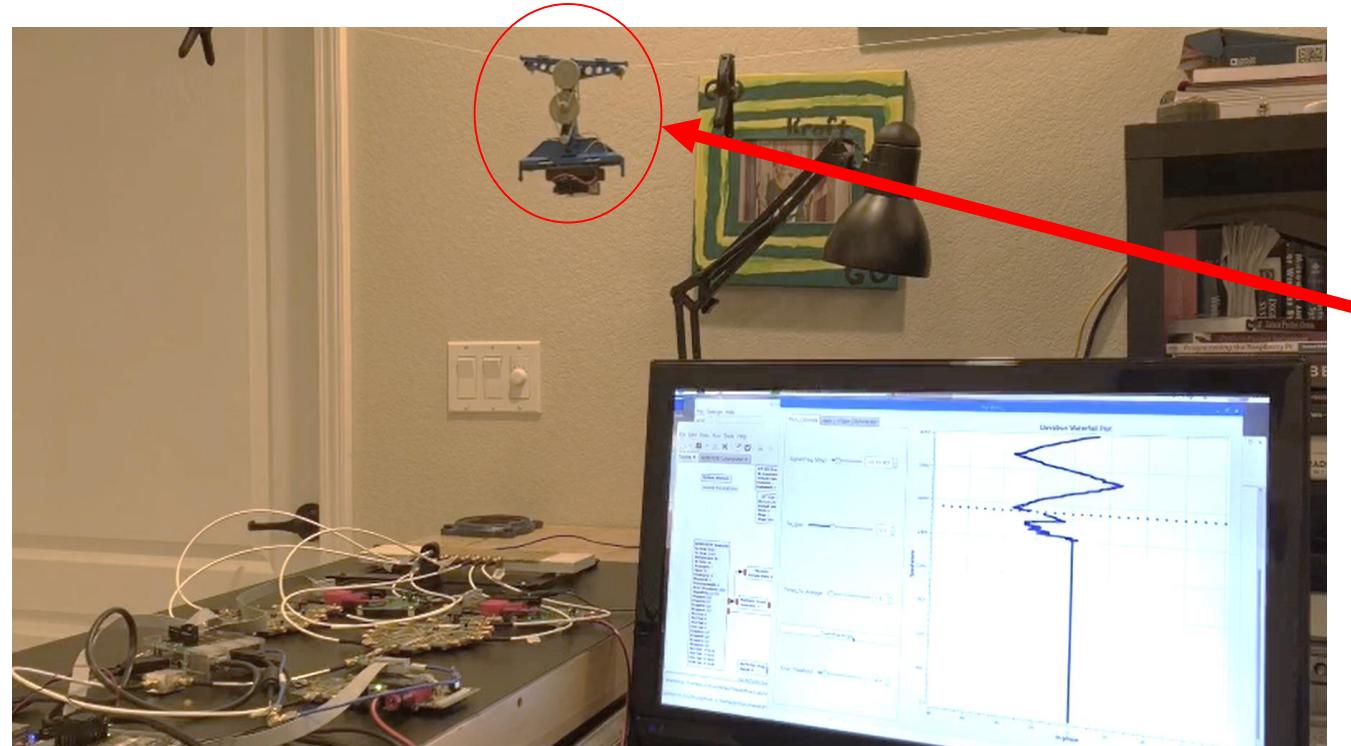


- The tapering reduces the side lobes, but also notice how it also expands the “range” of our error function
- So now, if for some reason, we fall +/-30 deg away, we can still bring it back to tracking lock without rescanning the entire array. Without taper, we could only do +/-15 deg before we lost the null we were tracking.

# FINALLY we can track our RF Source

- ▶ Let's use that "error" function to check out this monopulse tracking in action!
- ▶ We are NOT rescanning the entire beam to find position
- ▶ We are ONLY moving the beam as determined by the error function:

**LIVE DEMO!**



RF Source mounted on  
toy cable car

# Summary

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# Summary

## Workshop Goals:

1. Gain an **intuitive** understanding of beamforming concepts
2. **Hands on** experimenting with these concepts
3. Quickly **prototype** your own phased array system

- We learned about:
  - Steering Angle, Tapering, Grating Lobes, Beam Squint
  - Hybrid Beamforming and Monopulse Tracking
- Detailed Explanation in Analog Dialogue
  - New three part series explaining and deriving all of this
  - <https://www.analog.com/en/analog-dialogue.html>



The image shows a screenshot of the Analog Dialogue website. At the top right is the Analog Dialogue logo. Below it is a graph with multiple colored curves (purple, green, blue) representing antenna patterns. A purple banner at the bottom left of the graph area says "JUN 2020". To the right of the graph, the text reads "Phased Array Antenna Patterns—Part 2: Grating Lobes and Beam Squint". At the bottom, there are three small profile pictures with names: Peter Delos, Bob Broughton, and Jon Kraft.