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Understanding Phased Array Antenna Concepts

Lecture and Lab

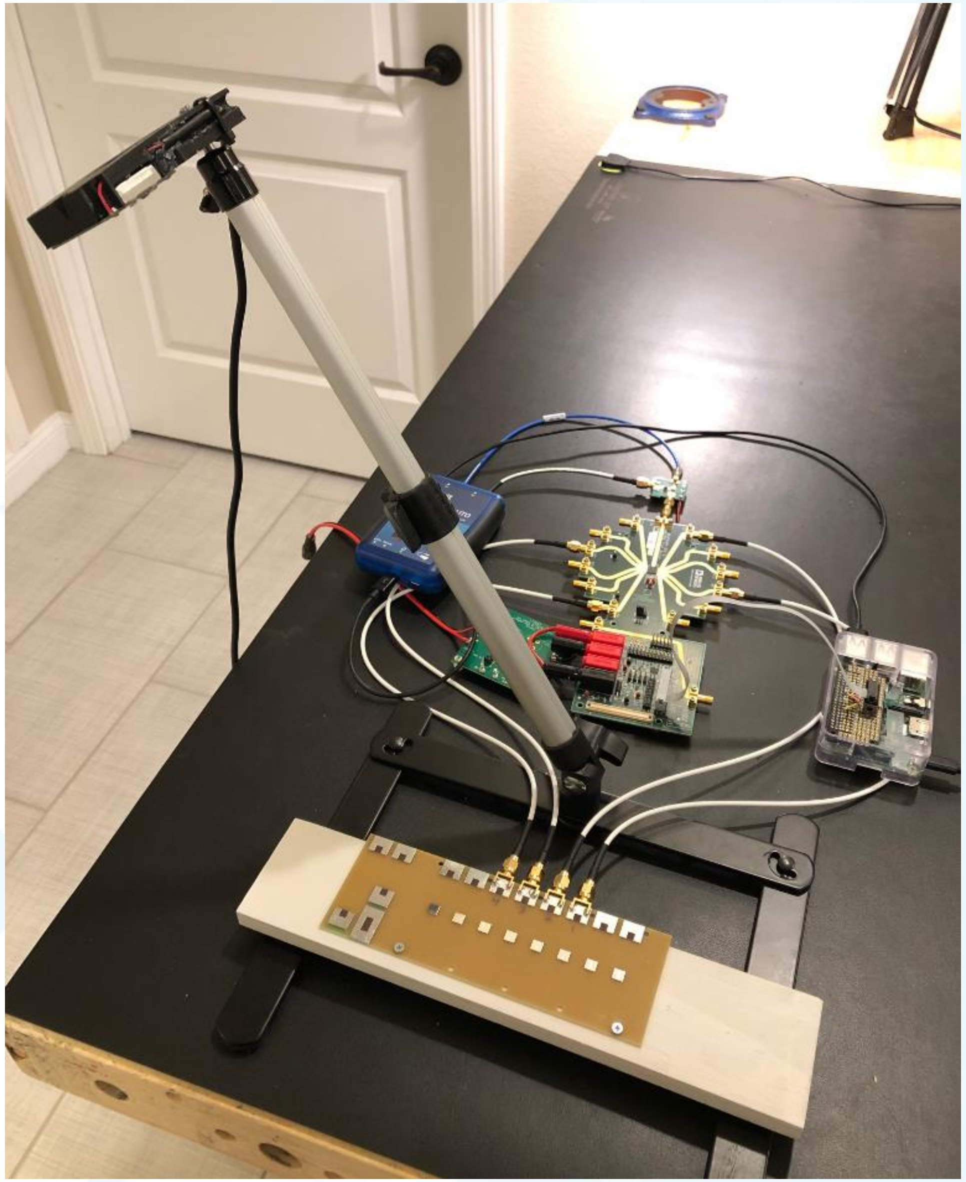
Jon Kraft (FAE)

"Anytime you have to compensate for the speed of light, you're probably doing something cool."

Many thanks to Bob Broughton for his instruction and slide contributions.

Agenda

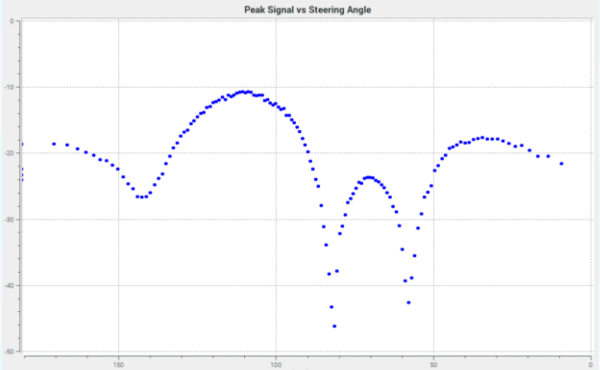
- ADAR1000 Intro and Lab Setup
- Pluto SDR and GNURadio
- Steering Angle
- Beam Width and Null Locations
- Beam Tapering
- Grating Lobes
- Beam Squint
- Where do we go from here?



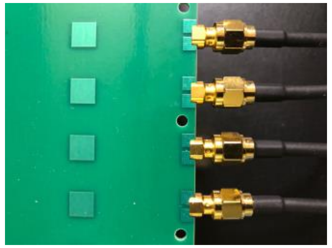
Intro to ADAR1000 and Lab Setup

Phased Array Lab Setup

- The lab consists of:
  - A 10 GHz spacing (d=0.015m) 4 element linear patch antenna
  - ADAR1000 X Band Analog Beamformer
  - LTC5552 2-18 GHz Active Mixer
  - ADALM-Pluto (AD9363) Software Defined Radio
  - LT3045 3.3V LDO (to power the ADAR1000 and LTC5552)
  - Raspberry Pi running GNURadio
    - Controls ADAR1000 via Python SPI commands
    - Controls Pluto via USB (IIOLIB)
    - Displays received data from Pluto

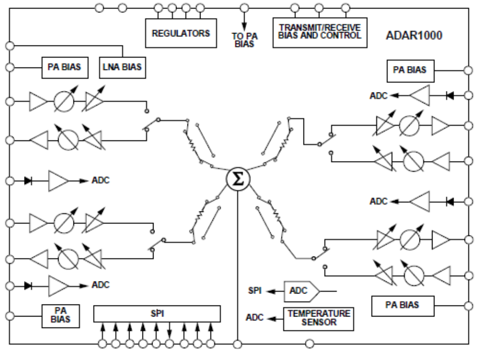


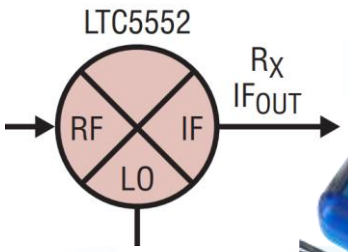
4 Patch Antenna



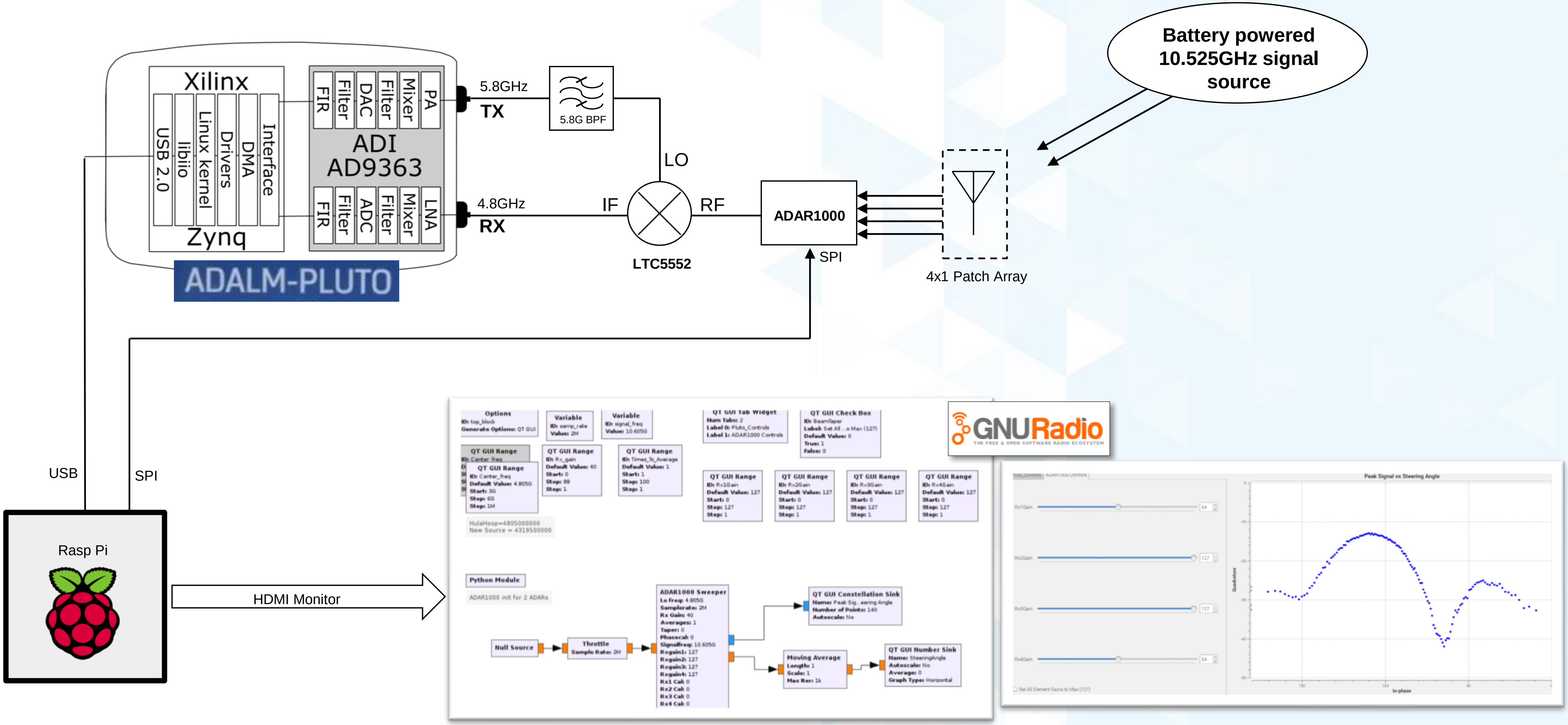
+

ADAR1000





Phased Array Lab Setup: Tracking an RF Source



ADAR1000: 4 Channel Analog Beamformer

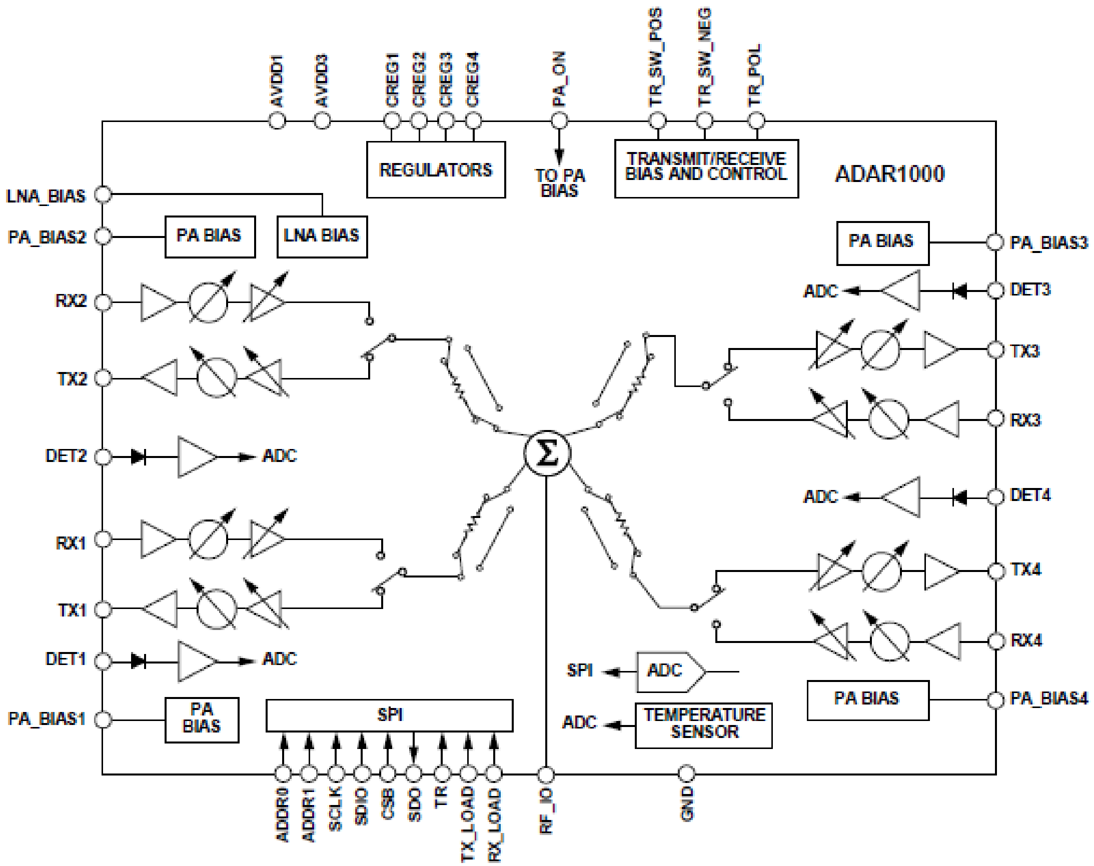
Key Features

- 8 GHz to 16 GHz frequency range
- Single-pin transmit and receive control
- 360° phase adjustment range
- 2.8° phase resolution
- ≥31 dB gain adjustment range
- Bias and control for external transmit and receive modules
- Memory for 121 prestored beam positions
- Four –20 dBm to +10 dBm power detectors
- 88-terminal, 7 mm × 7 mm LGA package

Key Benefits

- Compact form factor for electronically steered analog beamformer
- Negative bias voltage from integrated DAC intended for gate bias of GaAs or GaN amplifier
- Support low power bias mode with 50% reduction in power consumption

FUNCTIONAL BLOCK DIAGRAM



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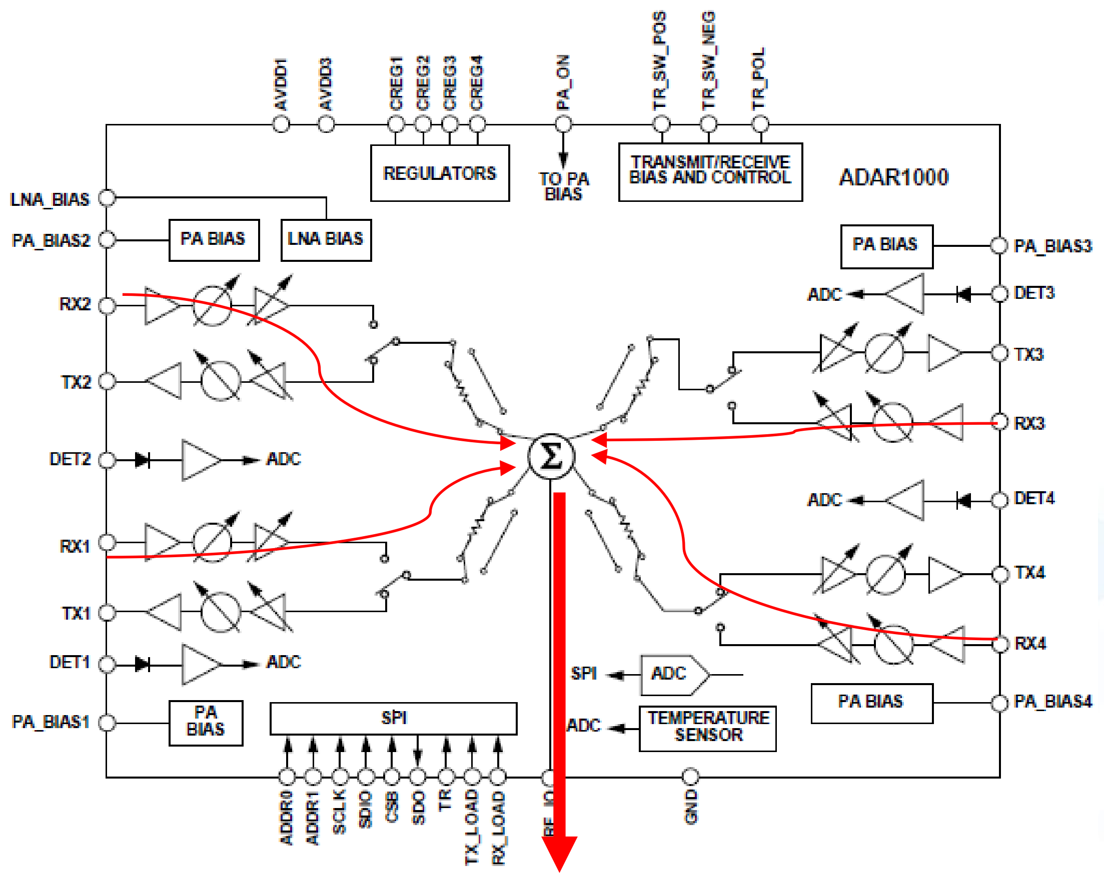
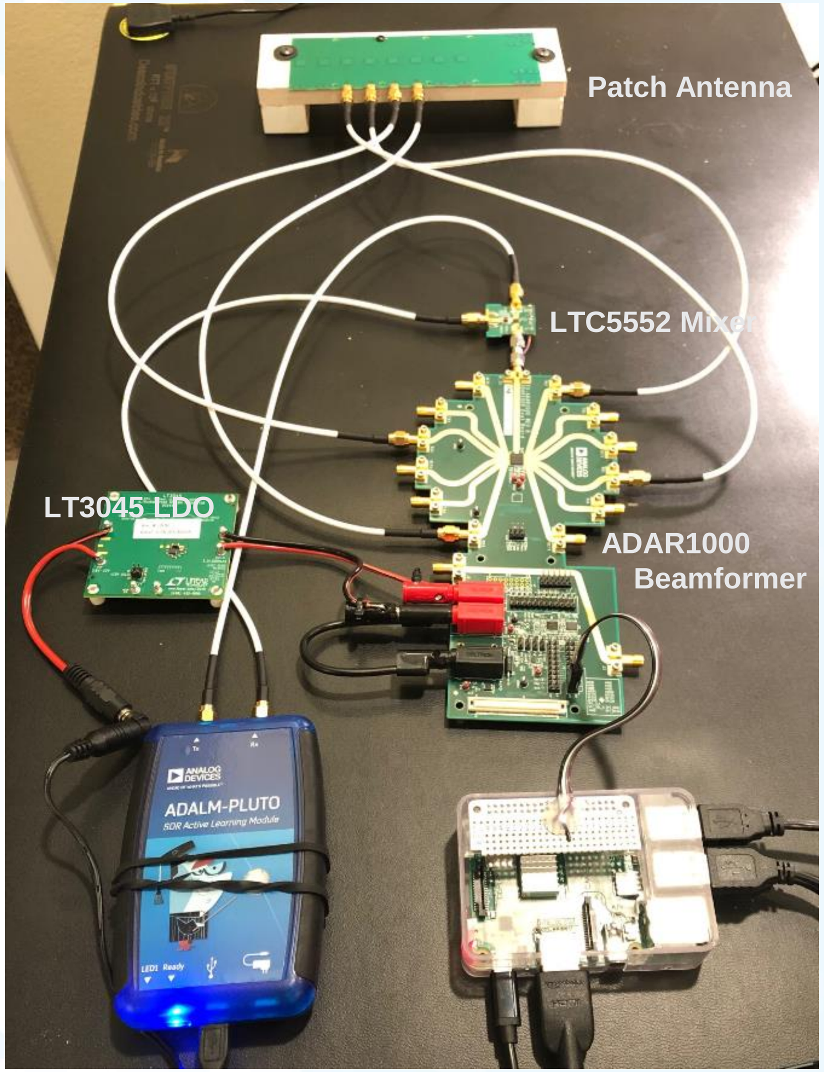
ADAR1000: 4 Channel Analog Beamformer

- We'll only be using the receive path, for this lab

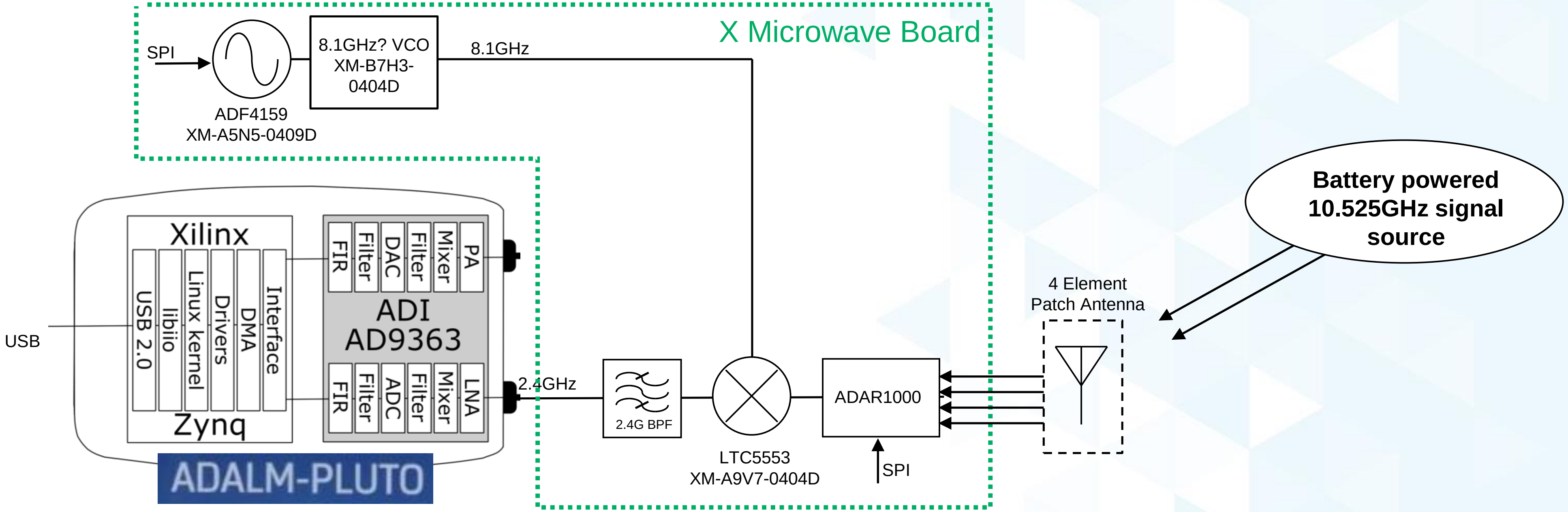
FUNCTIONAL BLOCK DIAGRAM

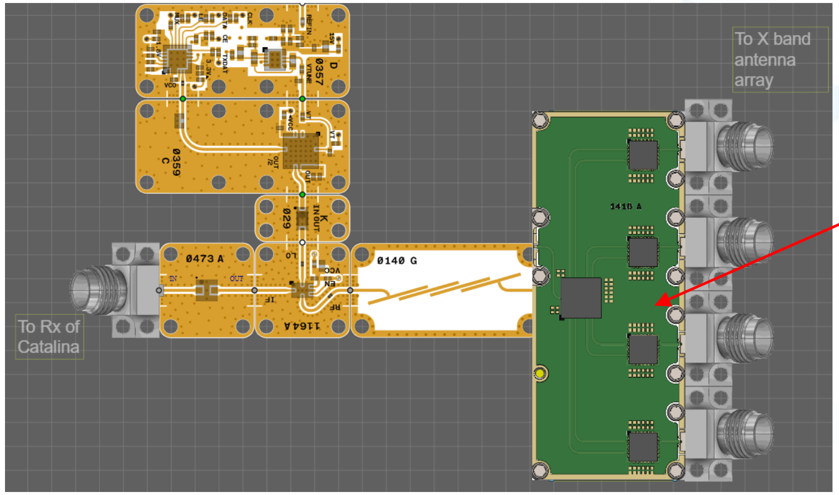




Declutter this Setup with X Microwave!

https://www.xmicrowave.com/

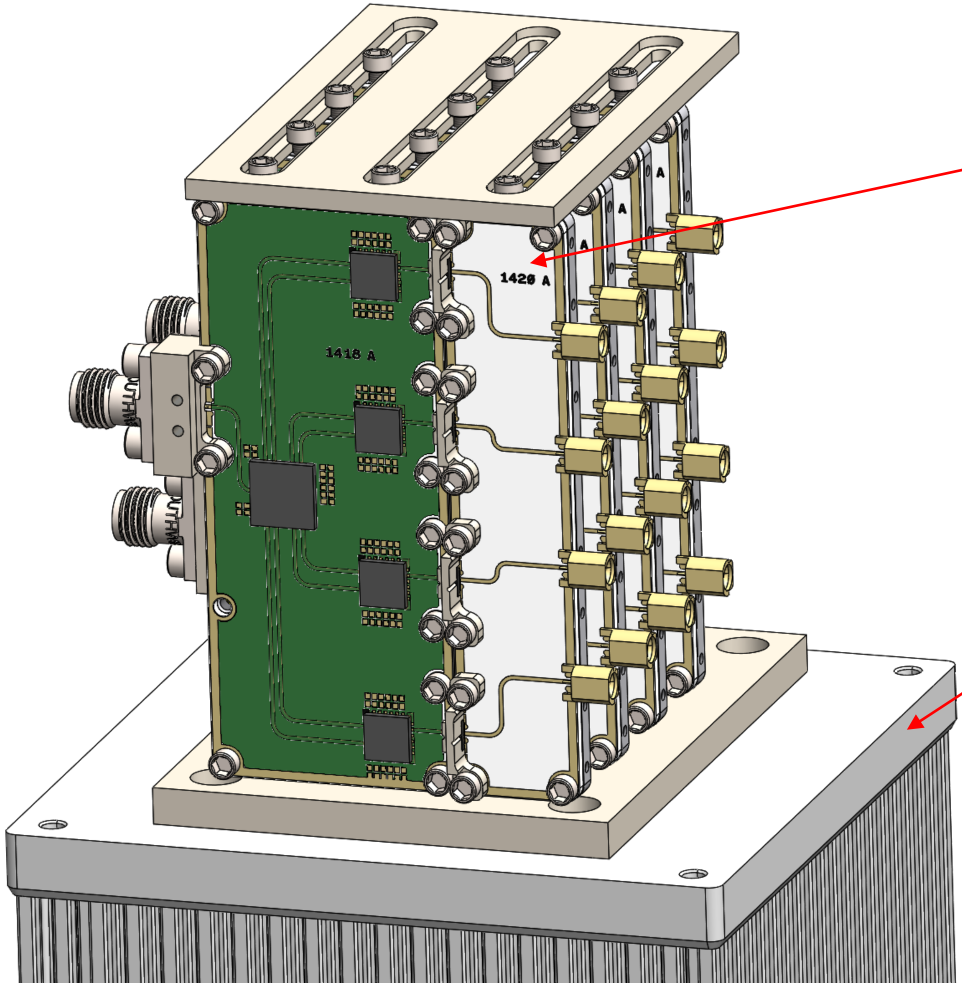




ADAR1000 + 4 ADTR1107 (TR Modules—i.e. PA/LNA/Switch)

Stack ADAR1000 Modules Together for the Phased Array Cube:

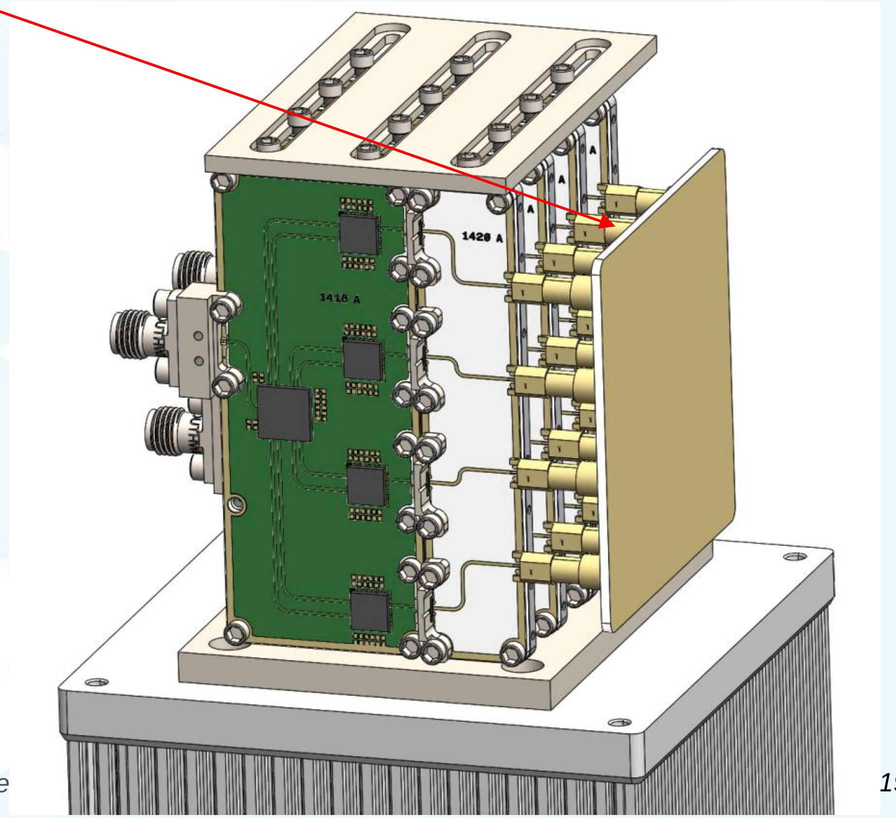
https://www.xmicrowave.com/



Stack 4 together to create a 4x4 array

- Interposer board to fit whatever lattice spacing
- Antenna snaps on

- Heatsink (if needed)

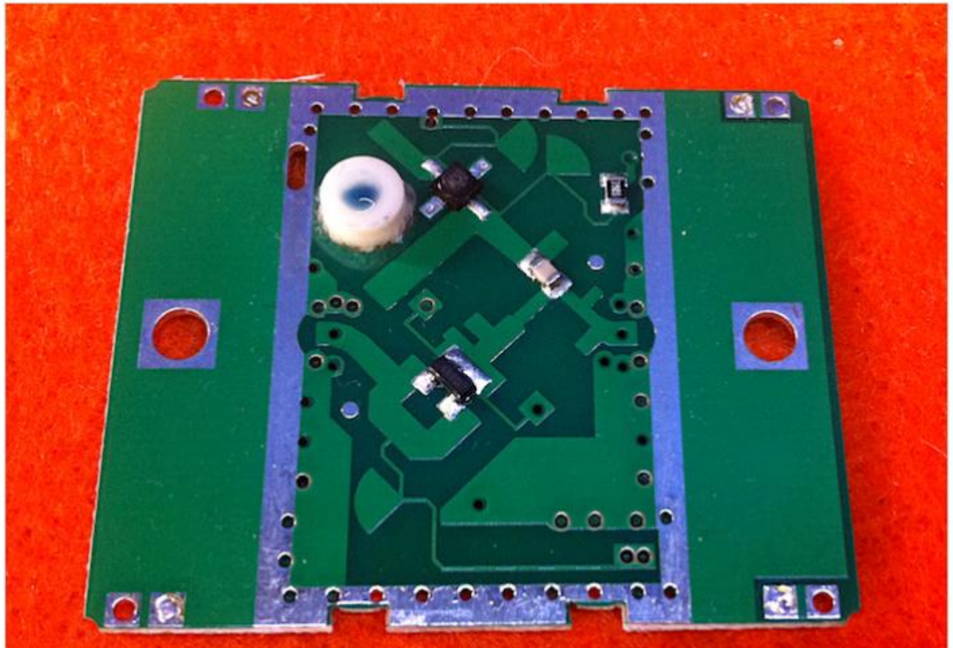


Getting Started with GNURadio and Pluto

GNURadio
THE FREE & OPEN SOFTWARE RADIO ECOSYSTEM

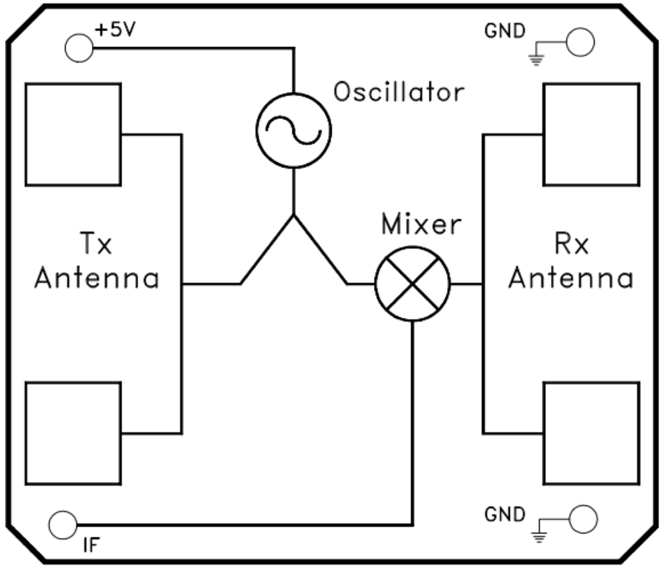
Phased Array Lab Setup: 10.5GHz RF Source

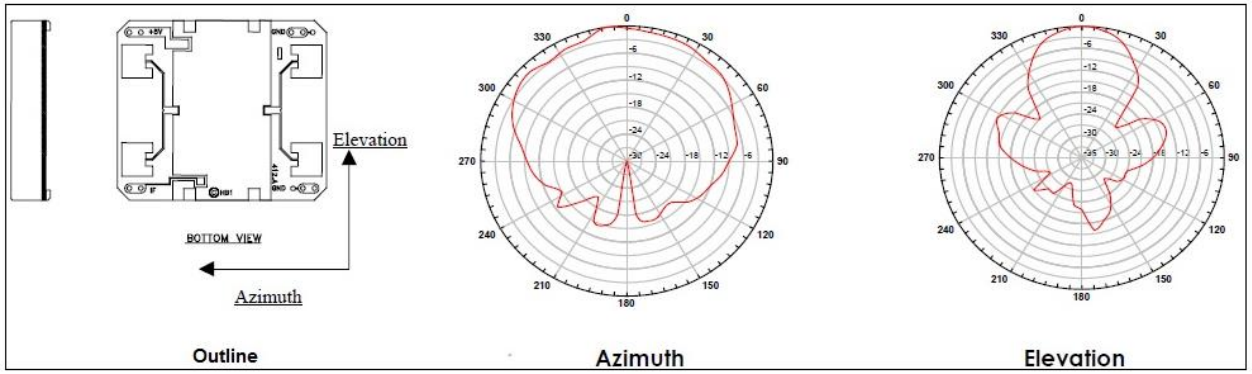
- What is that crazy box??
  - It’s the ultra fun HB100!
    - $3 (includes shipping!) on Ebay
    - Draws 40mA from 5V (it MUST be a **CLEAN** 5V!)
  - A poor quality ~10.5GHz tone
    - It’s good enough for us though!
  - Freq is constant, but unit to unit can vary from 10.1 GHz to 10.9 GHz
  - So the first thing we need to do is find the frequency!



https://www.allaboutcircuits.com/news/teardown-tuesday-hb100-doppler-radar-module/

What sorcery is this?

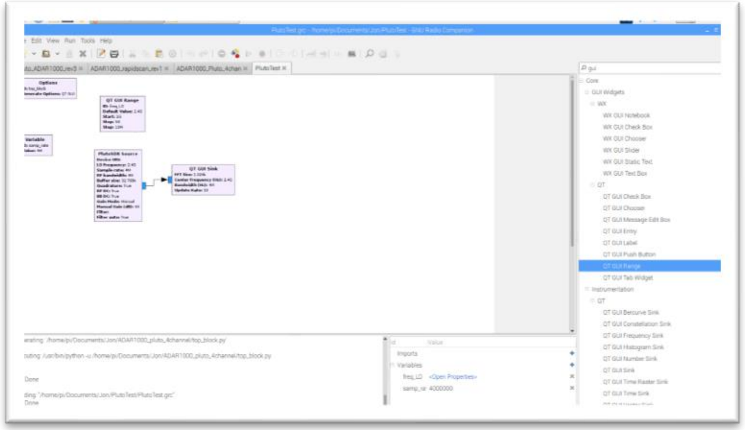


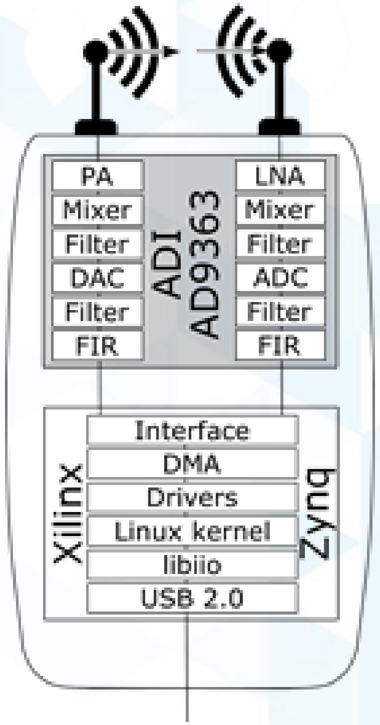


https://www.limpkin.fr/public/HB100/HB100_Microwave_Sensor_Application_Note.pdf

Getting Started with GNURadio and Pluto

- GNU Radio is a free, open source, toolkit that provides signal processing blocks to implement various software defined radio signal chains:
  - https://www.gnuradio.org/
  - It is similar to Matlab Simulink, but its free!
  - It easily incorporates any Python script
- Pluto, and our other TRx products, have plugins:





- More fun labs (and lunch and learns) for GNURadio and Pluto available here:
  - www.github.com/jonkraft

Getting Started with GNU Radio:

Find the RF Source's Frequency

Open "Exercise 1"

- It is not scary! Let us examine it:

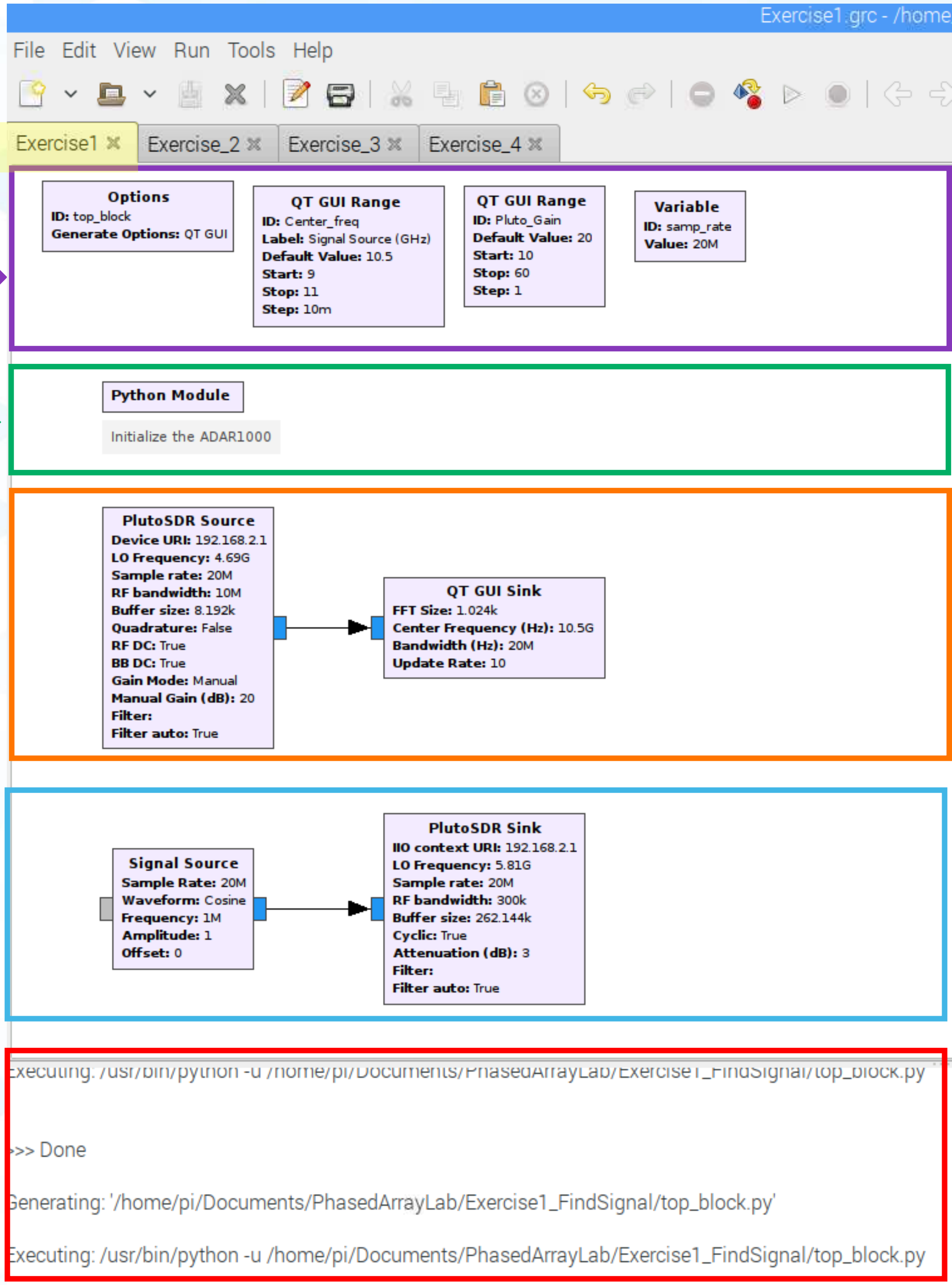
Variables and GUI Objects →

ADAR1000 initialization via SPI →

Grab data from Pluto and Plot →

Set Pluto's Tx to 5.81 GHz (for the LO to the mixer)→

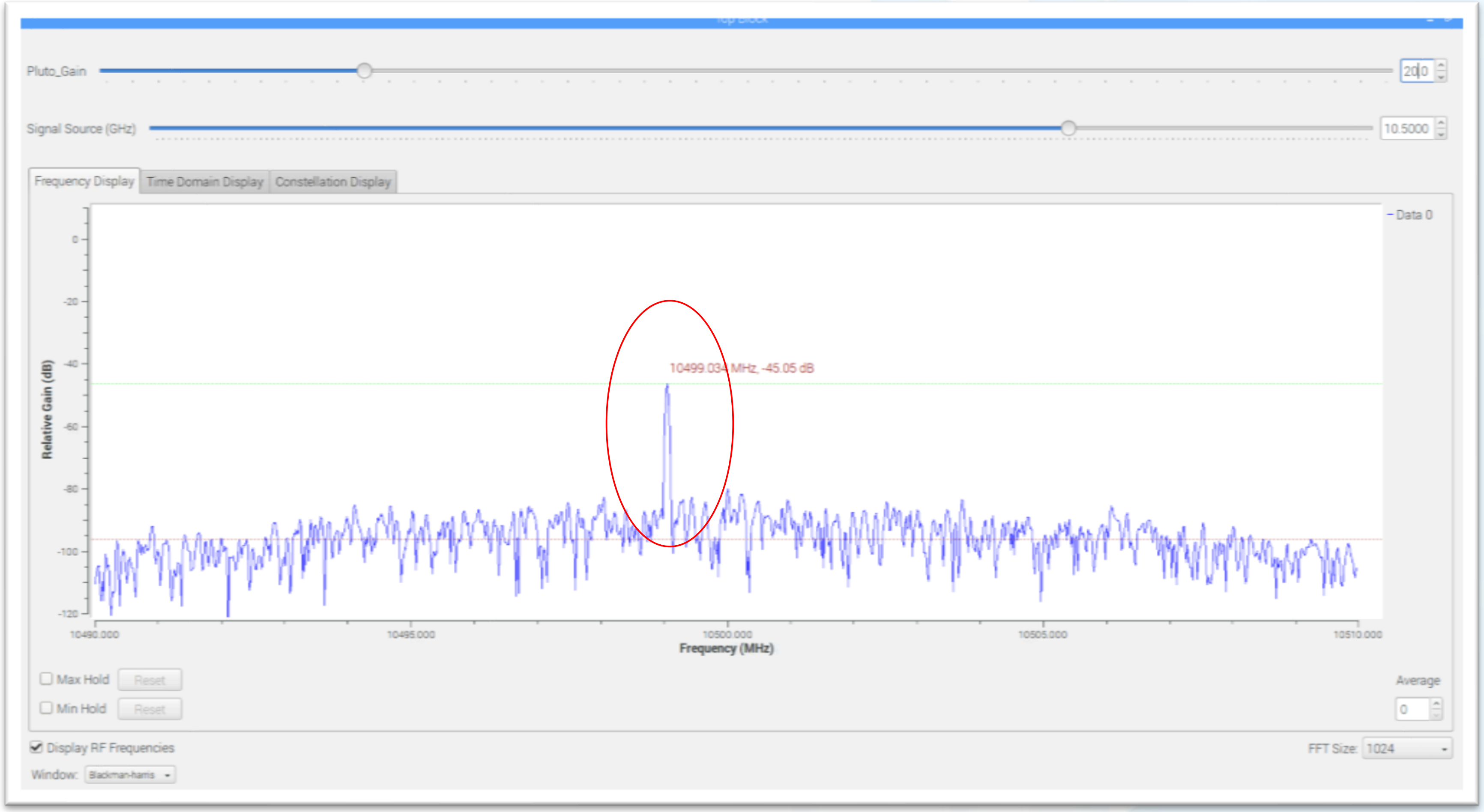
Error Messages will show up here →



Getting Started with GNU Radio:

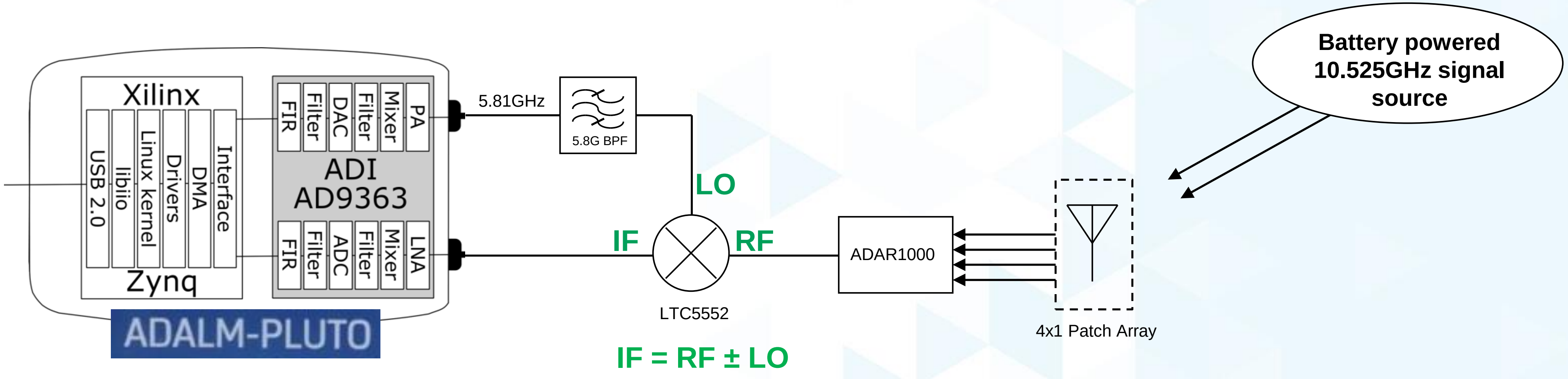
Find the RF Source's Frequency

- Run the GNURadio Flowgraph, "Exercise 1"
  - Adjust the "Signal Source" slider to find the frequency that the X band source is transmitting on



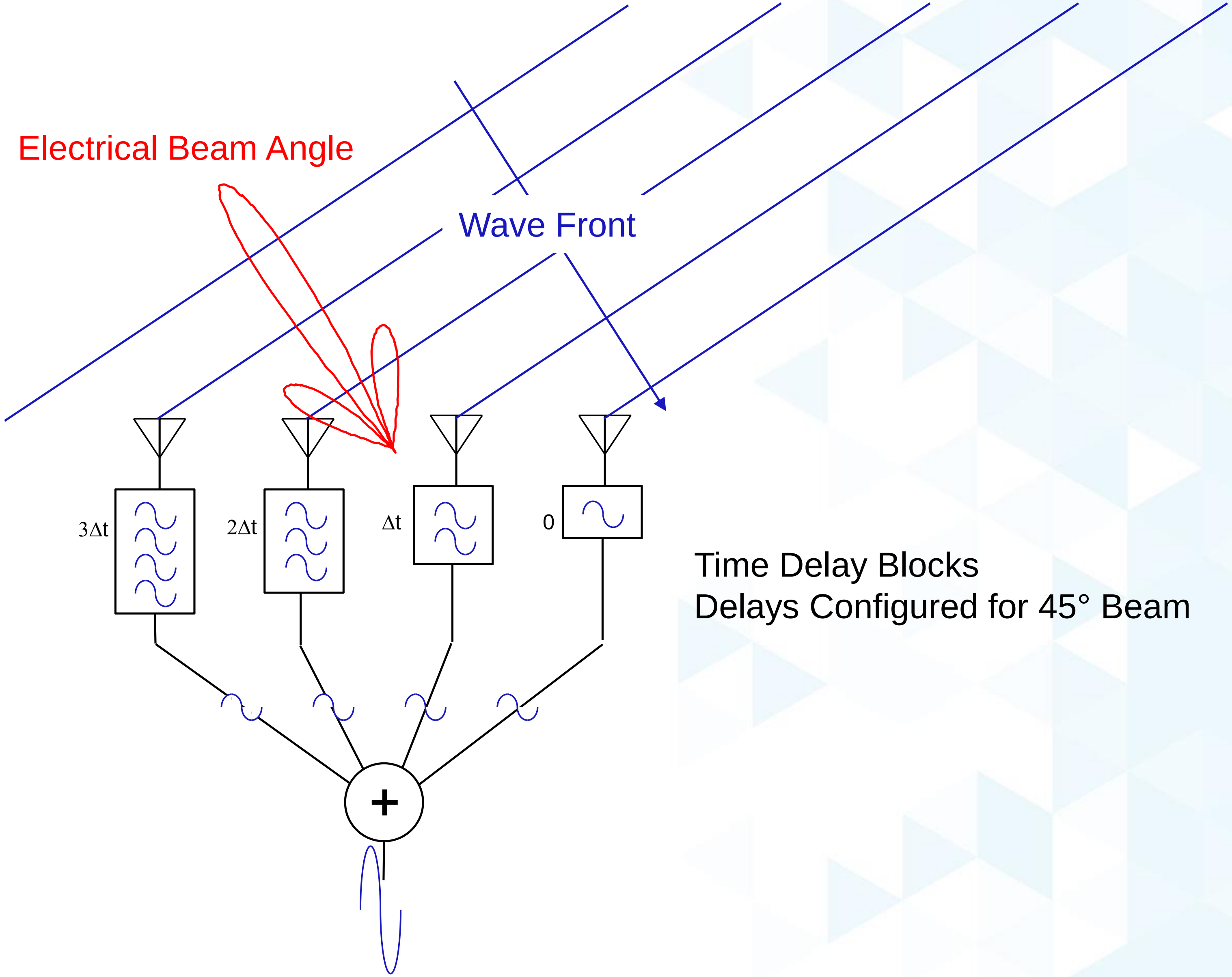
Getting Started with GNU Radio

- Enter the frequency of that tone: ____ GHz
- Remember the Mixer's LO freq is: 5.81 GHz
- Therefore, the frequency at Pluto Rx is: ____ GHz or possibly ____ GHz
  - Why is it the lesser of those two numbers?



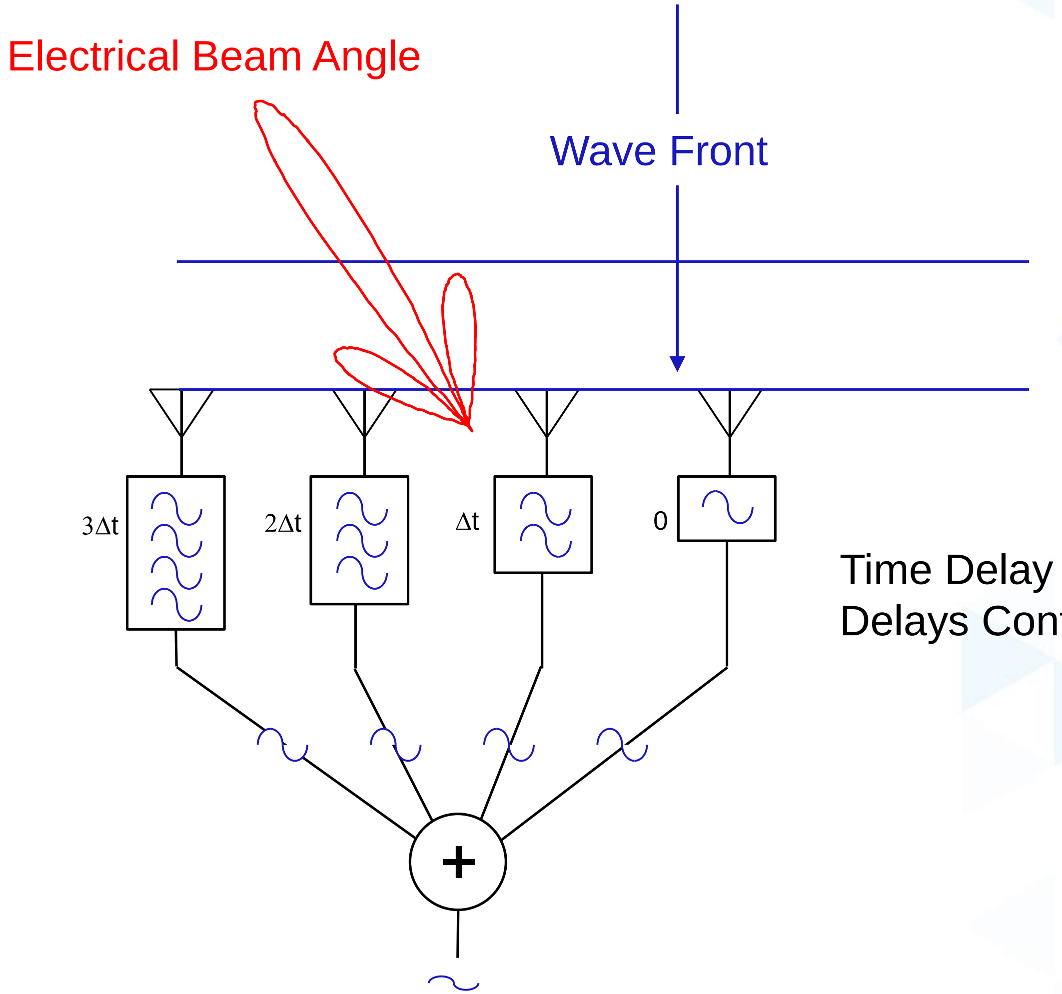
Understanding Steering Angle

Understanding Steering Angle: Math and Theory



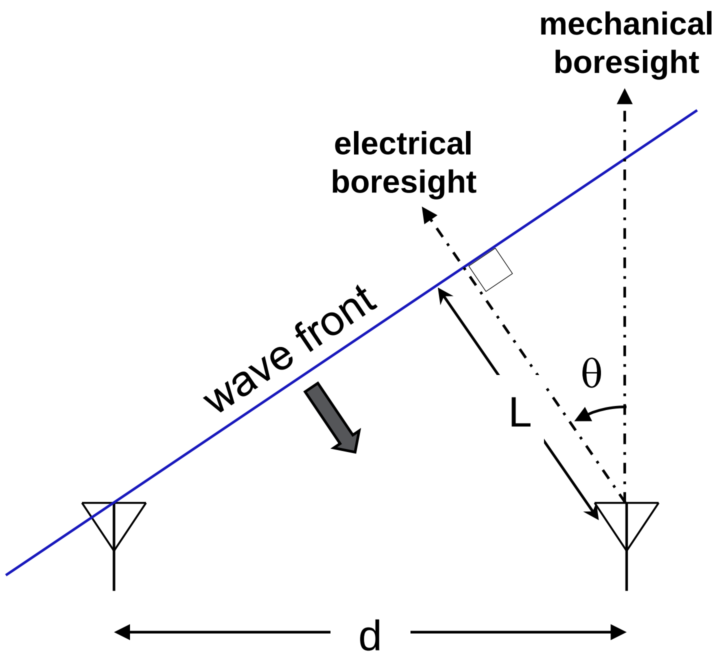
Time Delay Blocks
Delays Configured for 45° Beam

Understanding Steering Angle: Math and Theory



Time Delay Blocks
Delays Configured for 45° Beam

Understanding Steering Angle: Math and Theory



θ - beam electrical angle
Δt - incremental time delay between elements
$\Delta\phi$ - incremental phase shift between elements
L - incremental propagation distance between elements
d - distance between elements
C - speed of light 3×10^8 m/s

From trig:

$L = d \sin \theta$

For phase shift between elements:

$\Delta\phi = 2\pi\, L / \lambda = 2\pi\, f\, L / c$

which can be simplified…

$\Delta\phi = 2\pi\, f\, d \sin \theta / c$

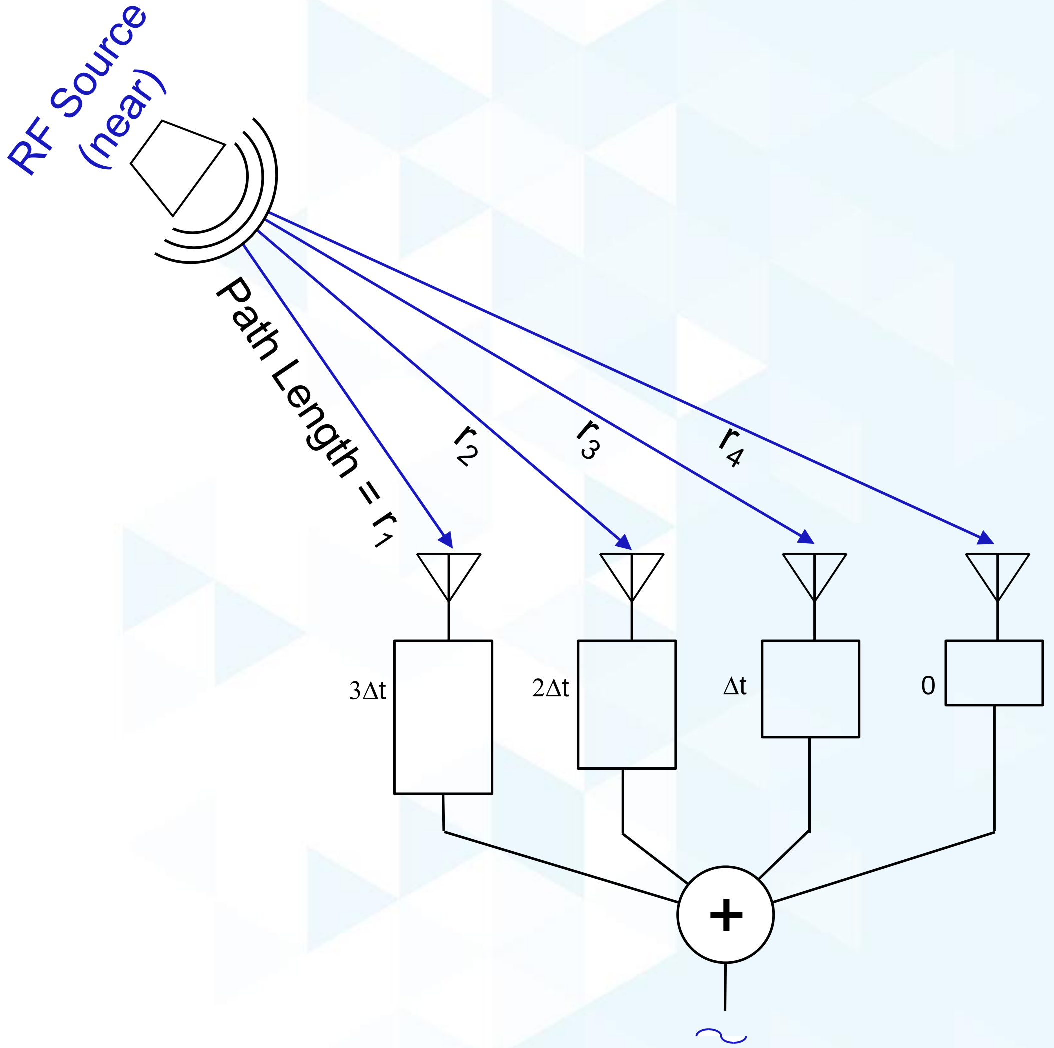
Solving for θ:

$\theta = \sin^{-1}(\Delta\phi\, c / (2\pi\, f\, d))$

Understanding Steering Angle: Math and Theory

- So right now, we have this:
- And each one of those thetas are slightly different.
- And each of the r's hit the circular RF wavefront differently.
- It makes the math hard!
- But if we just assumed that the RF source was far away….

Remember L is the incremental propagation distance between elements :
$L = d \sin \theta$



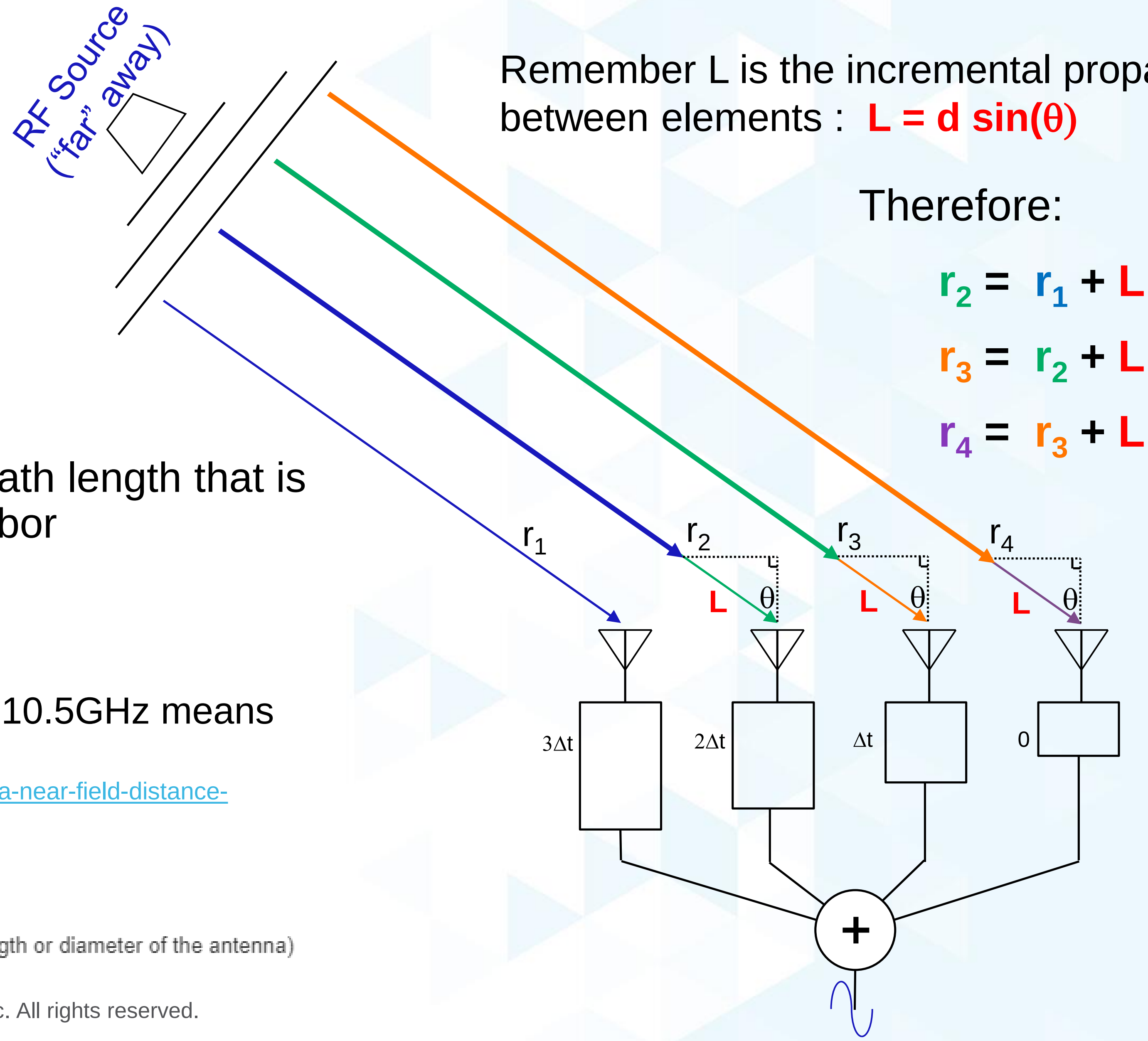
Understanding Steering Angle: Math and Theory

- In the "far field"
  - All of our lines are parallel
  - Therefore all thetas are equal.
- Each element simply has a path length that is **d sin(θ)** longer than its neighbor
- How far is far?
  - 4 elements, 15mm spacing at 10.5GHz means far field is at >142 mm

https://www.everythingrf.com/rf-calculators/antenna-near-field-distance-calculator

$$\text{Far Field} \geq \frac{2D^2}{\lambda}$$

D = Antenna dimensions (Can be the length or diameter of the antenna)



Remember L is the incremental propagation distance between elements : **L = d sin(θ)**

Therefore:

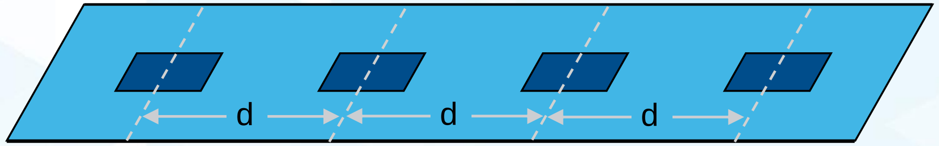
$$r_2 = r_1 + L$$

$$r_3 = r_2 + L = r_1 + 2L$$

$$r_4 = r_3 + L = r_1 + 3L$$

Understanding Steering Angle: Math and Theory

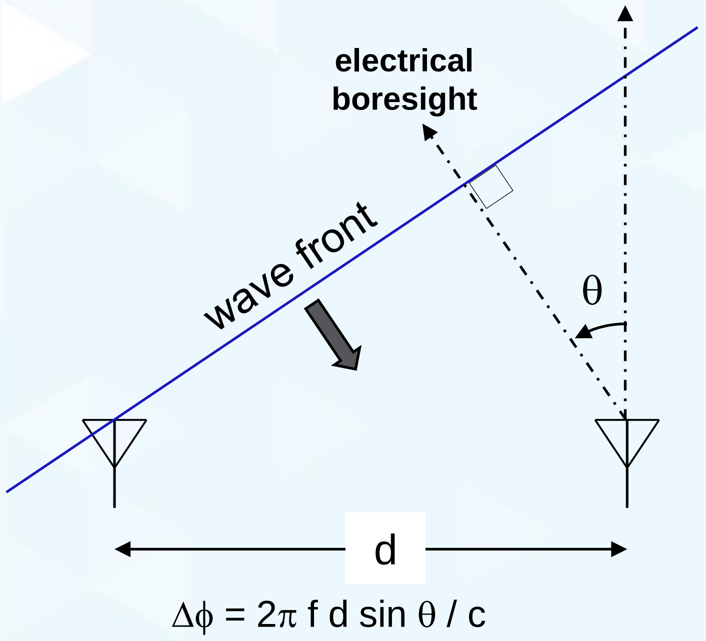
- So let's math it out:
  - Remember: $\Delta\phi = 2\pi\ f\ d\ \sin\theta\ /\ c$
  - If we position our RF source at $\theta = 22.5^o$, then what is $\Delta\phi$?
    - $\theta = 30^o = 0.52$ rad
  - d = 0.015 m (the antenna was designed for d= $\lambda/2$ at 10GHz)
    - $\lambda = c/f = (3\text{x}10^8\ \text{m/s})\ /\ (10\text{GHz}) = 0.03\ \text{m}$
  - f = 10.6 GHz



- Therefore:
  - $\Delta\phi = 2\pi\ f\ d\ \sin\theta\ /\ c = 2\pi * 10.6\text{x}10^9 * 0.015 * \sin(0.52)\ /\ 3\text{x}10^8$

    → $\Delta\phi = 1.67$ rad = 95^o

- What does this mean?
  - If our RF source is at $\theta = 30^o$, then our maximum signal will be realized if we sum all our elements such that each element is shifted by 95^o from its neighbor.
  - But don't take Math's word for it! Let's try it for ourselves!



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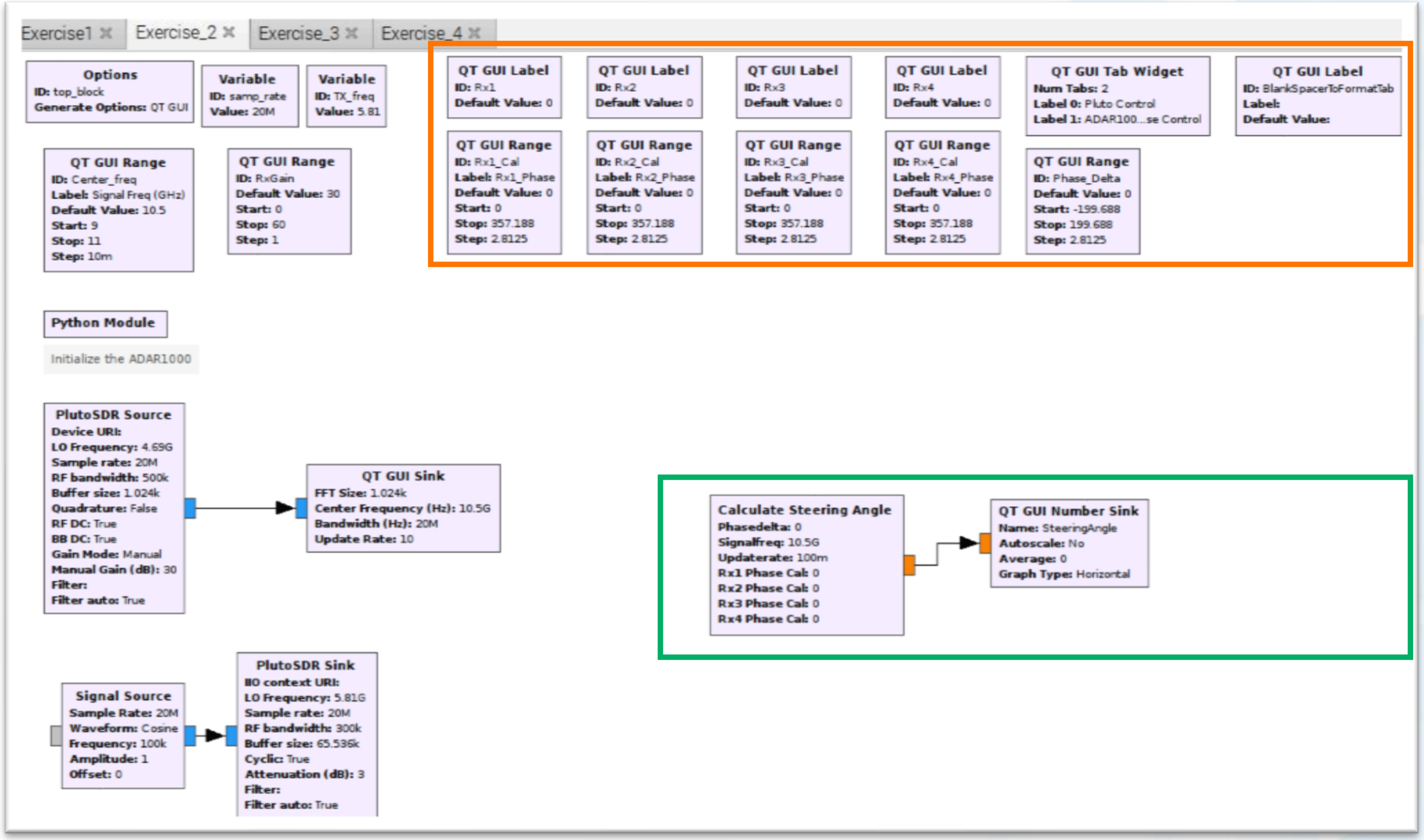
Understanding Steering Angle: Lab Experiments

- Before we can learn how to form a beam, we must first learn how to use a selfie stand!
- Loosen the big nut to move
  - Locks back into increments of about 15º
- Use your iPhone “Measure” app to get the exact angle!

Understanding Steering Angle: Lab Experiments

- **Stop** any running flowgraphs
- **Click** on the “Exercise_2” Tab in GNURadio.
- You will see some new blocks have been added!



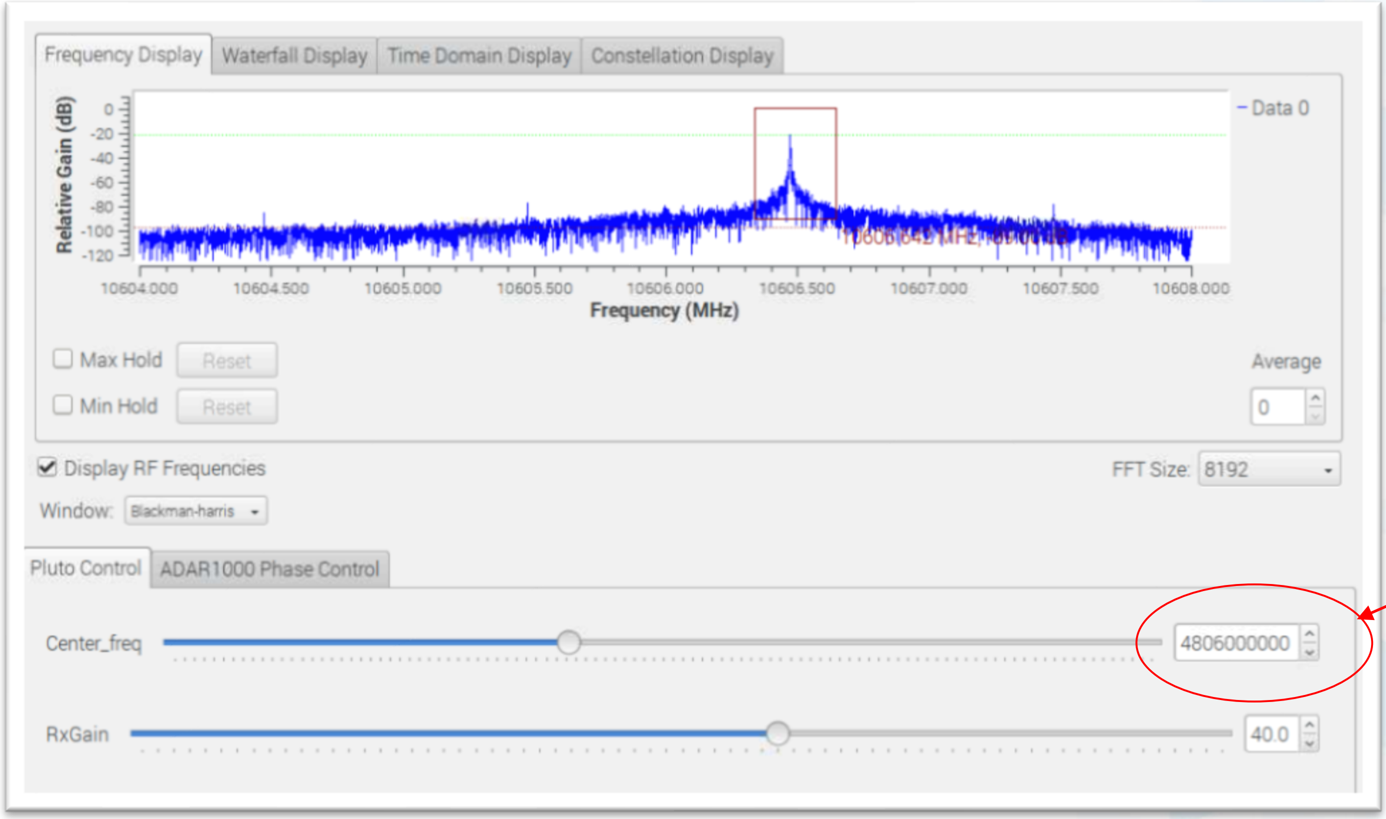
These are new controls for our GUI that will allow us to adjust the phase of each ADAR1000 Receive Channel

This is a Python block which:

1. Programs the ADAR1000 for the phases we set from the controls above.
2. Calculates steering angle from those phases.

Understanding Steering Angle: Lab Experiments

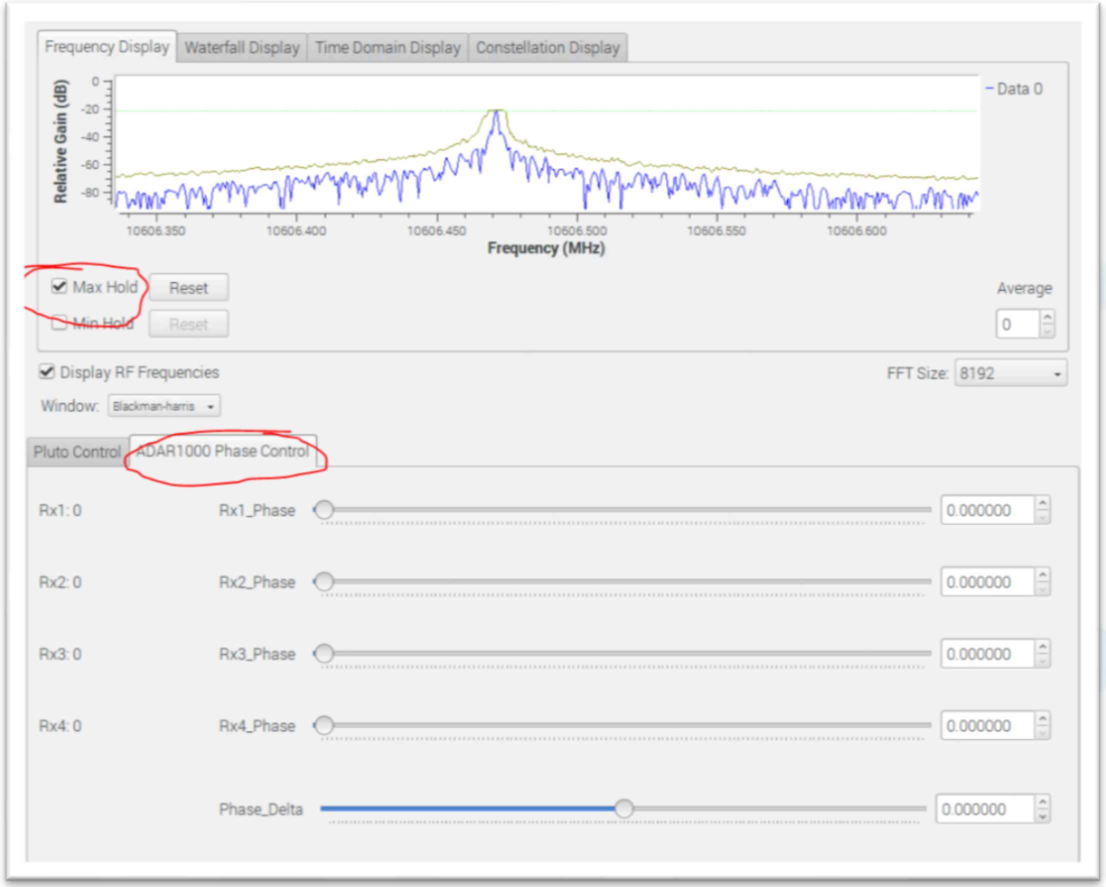
- Press Play on the Exercise_2
  - Adjust the frequency and Gain of Pluto, if needed.
- Zoom in on the peak:
  -



Change this to the frequency you found in Exercise 1

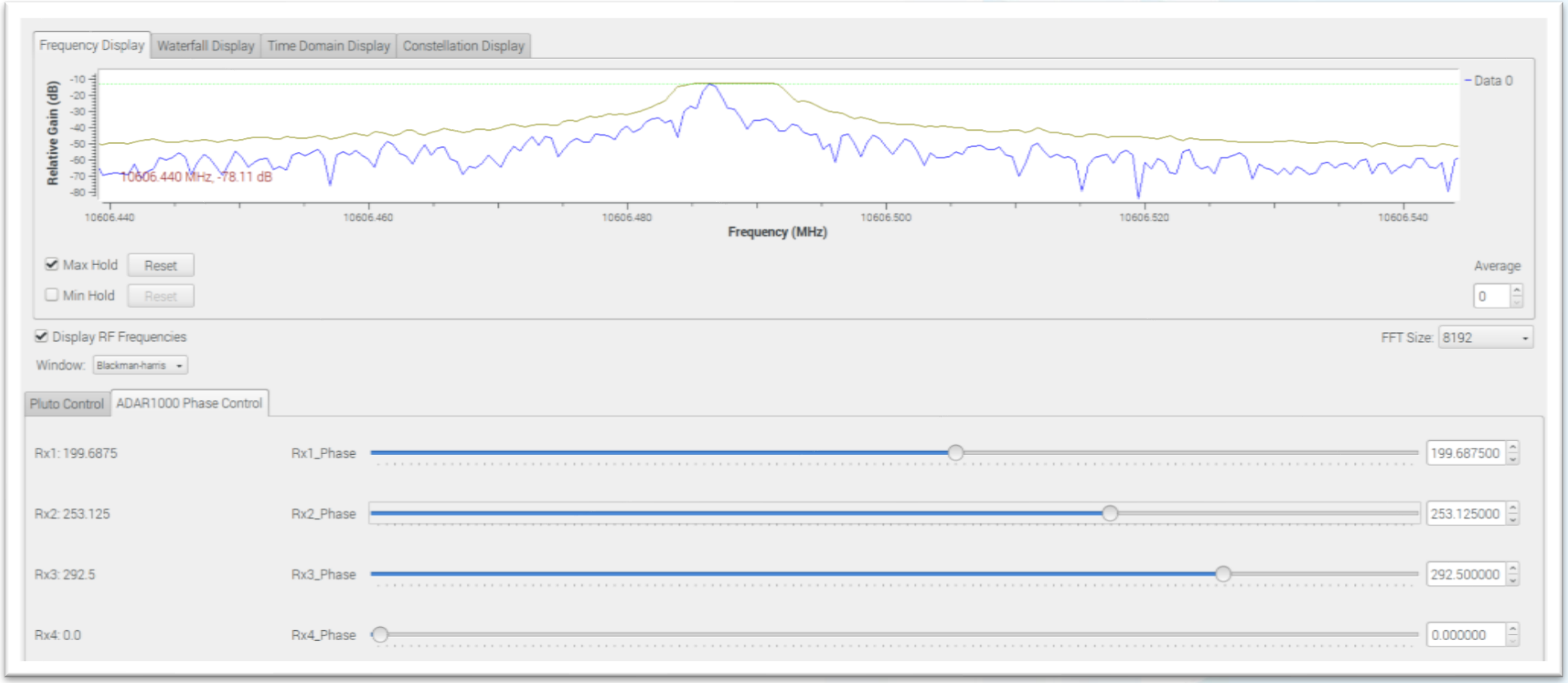
Understanding Steering Angle: Lab Experiments

- Select “Max Hold” and the “ADAR1000 Phase Control” Tab



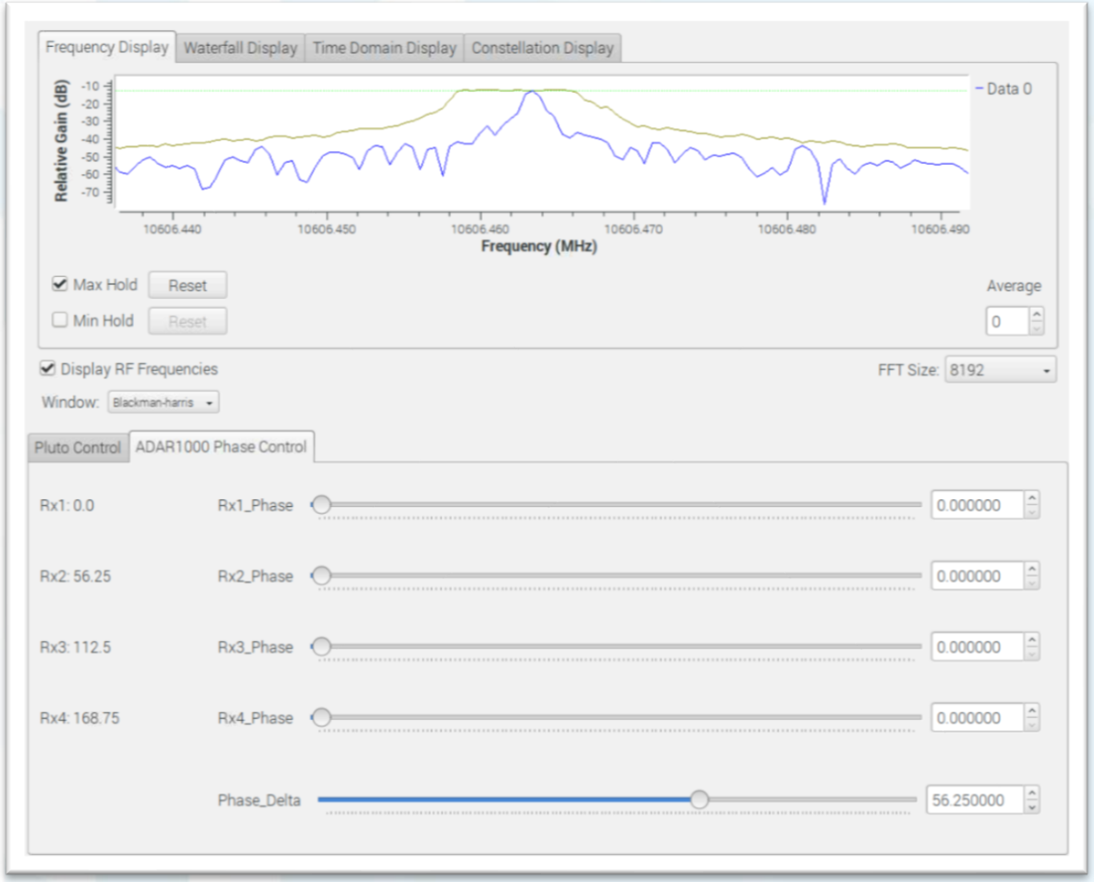
Understanding Steering Angle: Lab Experiments

- Grab the "Rx_Phase" sliders and move them around.
  - You are changing the phase delay of each individual ADAR1000 antenna element
  - You should see the peak FFT response rising and falling
- Why does the peak rise and fall in response to each element's phase?



Understanding Steering Angle: Lab Experiments

- We just learned that uniformly spaced elements should be optimally set with a consistent phase delta between elements. So let’s try changing the phase delta.
- Return the “Rx_Phase” Sliders to 0º
- Now, move the Phase_Delta slider
  - This moves all the element’s phases together. You can see the actual phase of each element on the left column
  - The change in peak will be more dramatic as now all 4 elements are changing phase when the slider moves
- What Phase_Delta gives the largest peak?
  - __________ deg
- How does that phase delta relate to steering angle?



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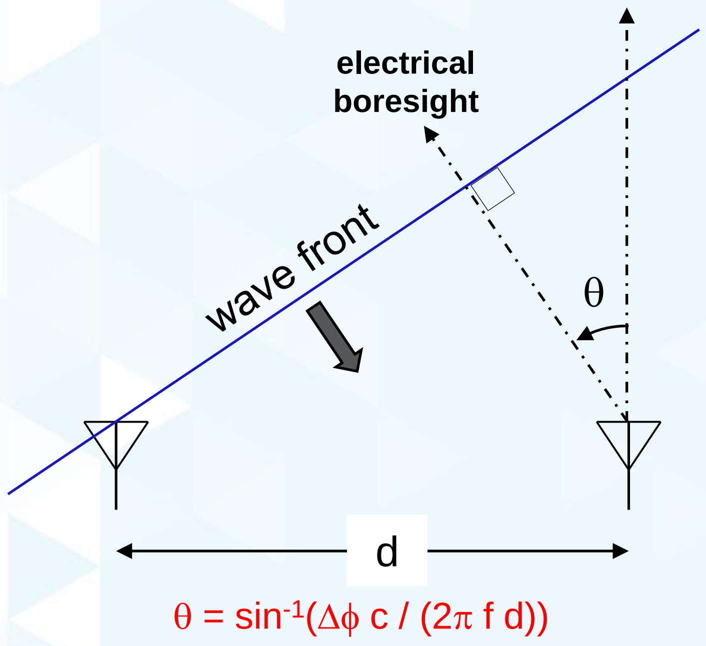
Understanding Steering Angle: Lab Experiments

- Remember: Steering Angle = $\theta = \sin^{-1}(\Delta\phi \; c / (2\pi \; f \; d))$
- In the lab above, I found a phase delta of 56.25º gave the peak response
  - $\Delta\phi = 56.25^{\circ}$ = 0.981 rad (divide degrees by 57.3 to get rad)
  - d = 0.015 m (the antenna was designed for d= $\lambda/2$ at 10GHz)
    - $\lambda = c/f = (3\text{x}10^{8}\ \text{m/s}) / (10\text{GHz}) = 0.03$ m
  - f = 10.6 GHz
- Therefore:
  - $\theta = \sin^{-1}(0.981 * 3\text{x}10^{8} / (2\pi * 10.6\text{x}10^{9} * 0.015)$
    $= 0.299$ rad = 17º
  - And that is about where my RF Source was!



Understanding Beam Width and Null Locations

Understanding Beam Width: Lab Experiments

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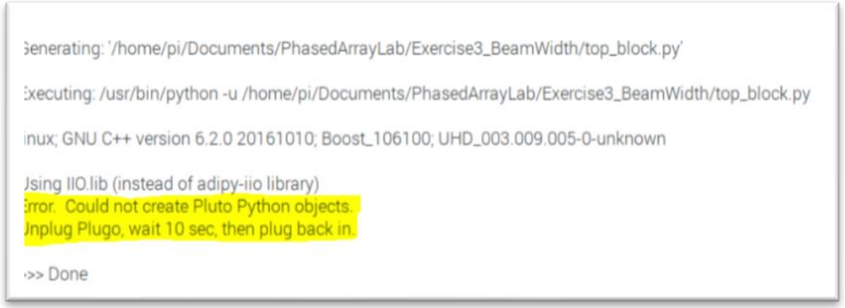
- By manually sweeping the phase delta, we got an idea for where the peak was, and maybe even where some nulls were.
- So let's automate this "slider bar" sweep, and get a better idea of what the beam looks like.

- Before we do this, let's give our hard working Pluto a break. We need to reset it, as we're going to control it in a different way. And its just more robust if we reboot it first.

- So everyone! Please unplug Pluto, wait 10 sec, and plug back in!

- If you ever see this error, now you know what to do:



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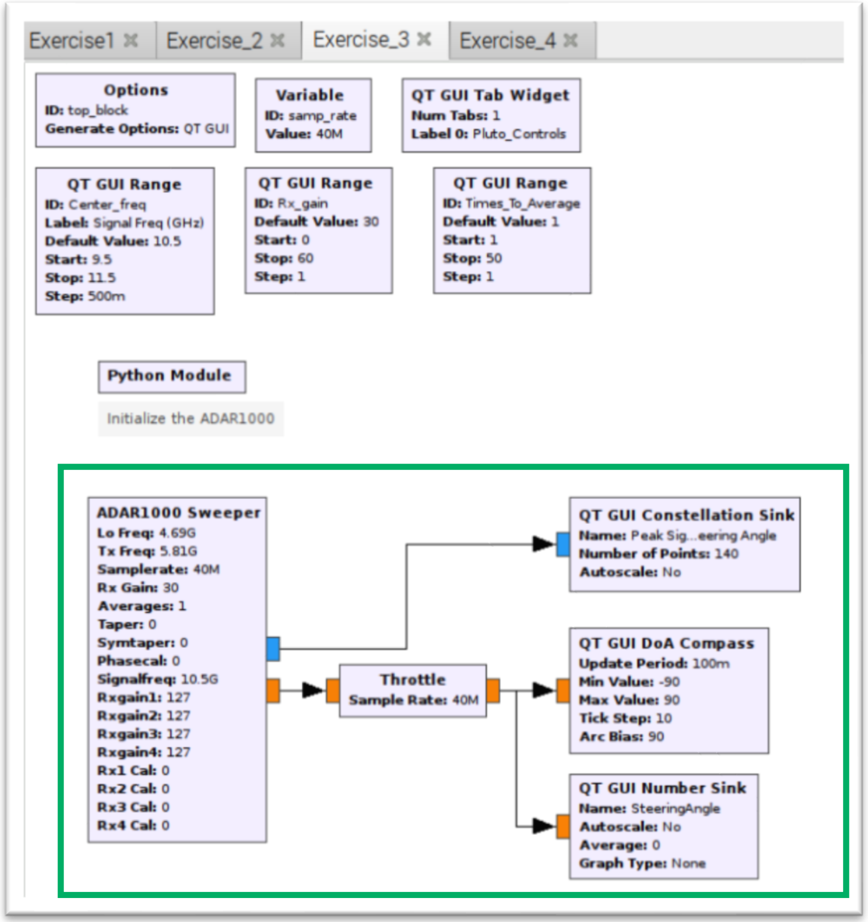
Understanding Beam Width: Lab Experiments

- **Stop** any running flowgraphs
- **Click** on the “Exercise_3” Tab in GNURadio.
- Once again, our GNUradio flowgraph looks different.



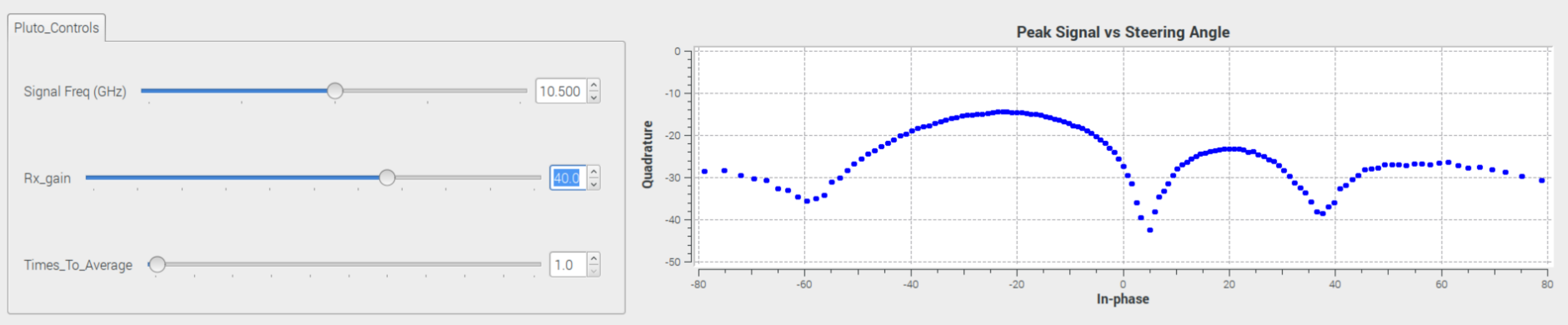
- Don’t be alarmed! Let’s calmly examine the differences:

Nearly everything has been replaced by this mysterious “ADAR1000 Sweeper” block

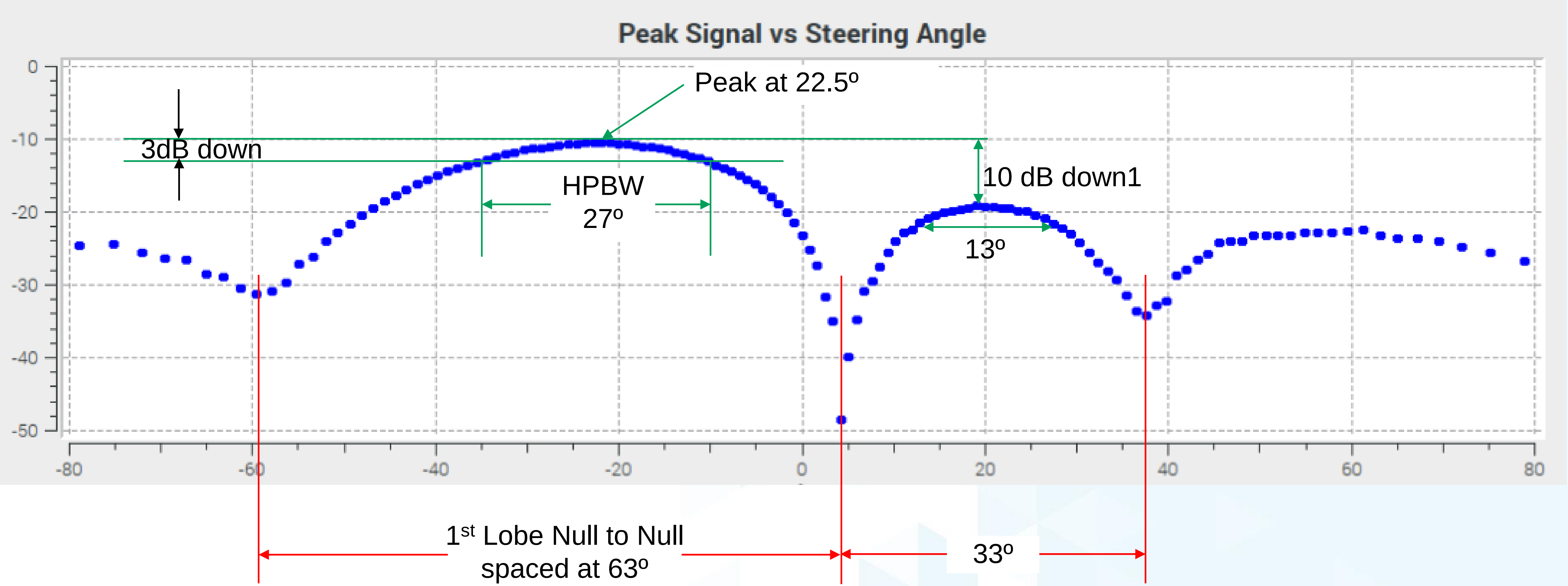
- This is a Python file that controls Pluto and the ADAR1000
- It is much faster to change the ADAR1000 beams and grab Pluto data with this script, then the GNUradio plug ins we used before
- This script also calculates and plots our beam, as we will discover.

Understanding Beam Width: Lab Experiments

- You should see a plot with a peak near the value you found in the previous exercise.
- Take a moment to align the axis of the antenna with the path of the radiator
  - Our patch antenna is spatially very narrow, so if you're "off" the waveform will be somewhat distorted
- Then make observations on:
  - The width of the "main" lobe
  - Number of peaks and nulls
  - Distance between Null to Null
- Ignore the "In-Phase" and "Quadrature" axis labels. These are really "Steering Angle (theta)" and "Peak Signal (dB)"
  - Just an unfortunate side effect of using a constellation plot to view x-y data…. Ahhh GNU Radio.



Understanding Beam Width: Lab Experiments



Understanding Beam Width: Lab Experiments

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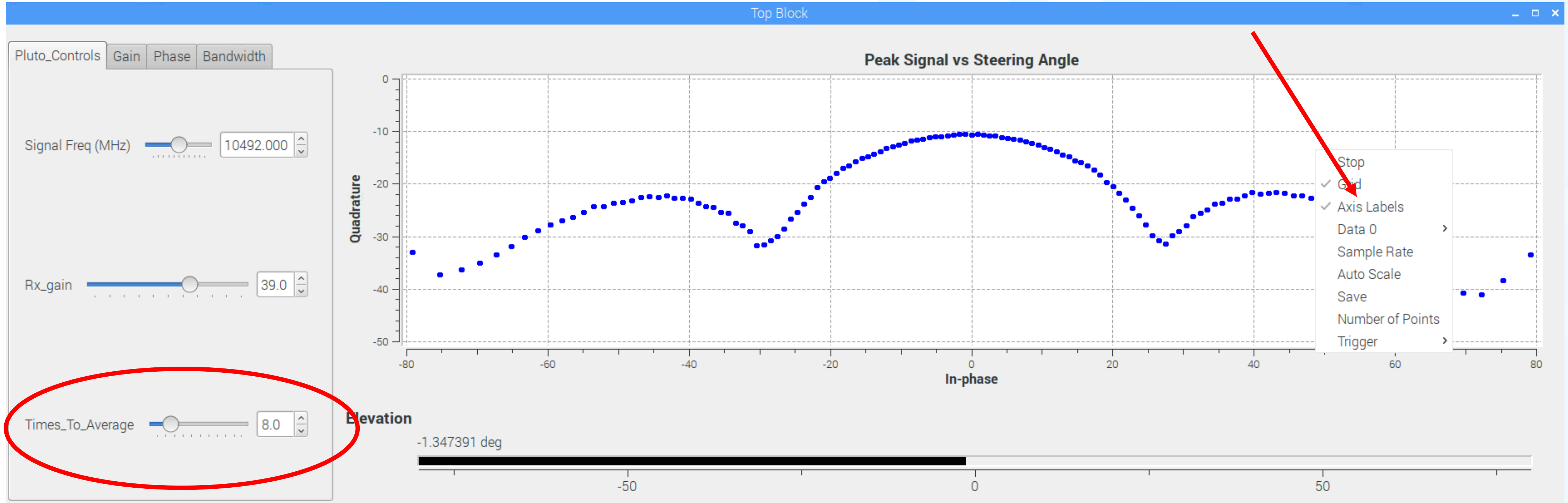
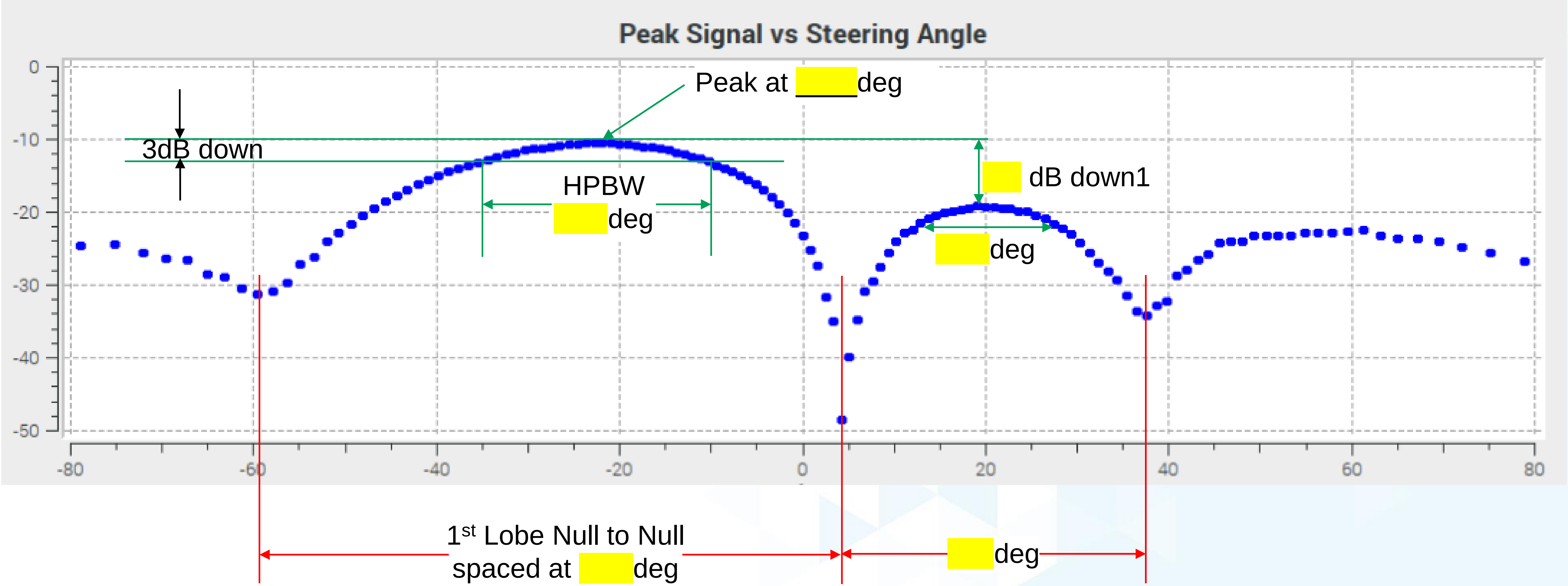
- Tips for Making these measurements:
- "Middle Click" then select "Stop" to stop the graph



- "Times_To_Average" loops the Pluto buffer x times (in this case 8) and reports the average peak FFT from those buffers.

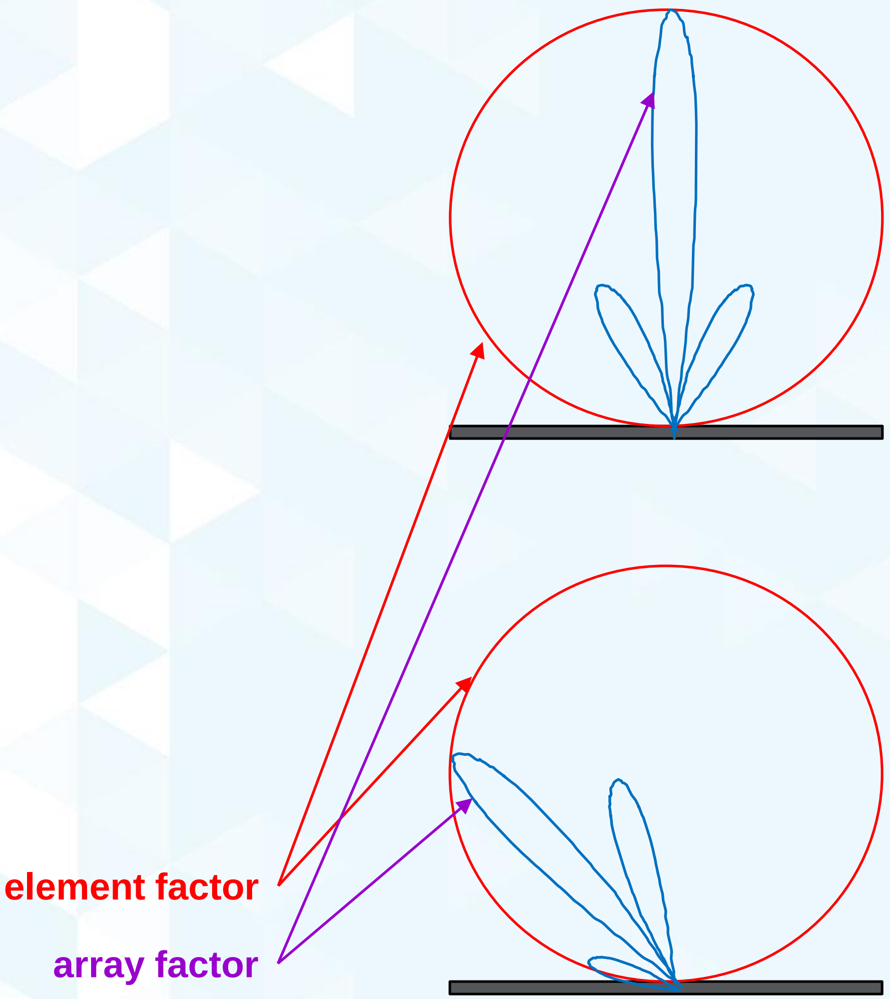
Understanding Beam Width: Lab Experiments

- Now it's your turn! Set the RF source to 0º, or ±20º, etc. And make these measurements:
  - (turn on averaging if it helps)



Understanding Beam Width: Math and Theory

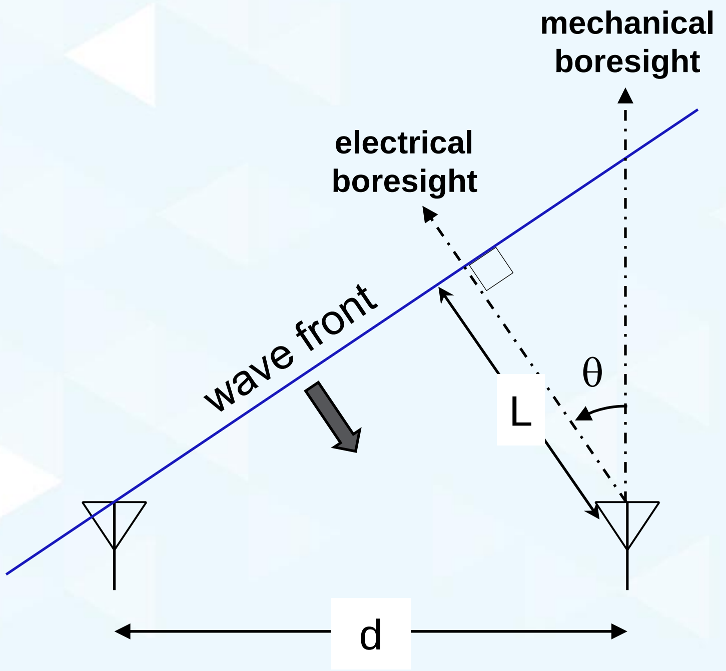
- Is that the right beam shape? Right beam width, null width, # of nulls, etc?
  - Let's understand the math to see if we are measuring it properly.
- All of our analysis, and experiments are for a linear array.
  - Specifically, a 4x1, equally spaced, linear array.
  - A linear array is the building block from which many other array types can spring out of.
- "Element Factor" - $G_E(\theta,\phi)$ is the radiating pattern of a single element in the array
- "Array Factor" - $G_A(\theta,\phi)$ is calculated via array geometry and beam weights (amplitude and phase)
- The Phased Array Gain (dB) → $G(\theta,\phi) = G_A(\theta,\phi) + G_E(\theta,\phi)$
- Assuming all our elements are the same, let's focus on the Array Factor, G_A



Understanding Beam Width: Math and Theory

- Recall that each element receives a signal that may be delayed relative to the element next to it.
- And the array factor is the summation of all those signals. If each signal has the same amplitude, with shifted phases, then, for our 4 element array, the Array Factor is:
  - $G_A = e^{j0\cdot\Delta\phi} + e^{j1\cdot\Delta\phi} + e^{j2\cdot\Delta\phi} + e^{j3\cdot\Delta\phi}$
    - Where the first phase delta is zero, so this simplifies to:
  - $G_A = 1 + e^{j\Delta\phi} + e^{j2\cdot\Delta\phi} + e^{j3\cdot\Delta\phi}$
- $G_A = e^{j(N-1)\cdot\Delta\phi/2} \cdot \dfrac{\sin(N\cdot\Delta\phi/2)}{\sin(\Delta\phi/2)}$
- Array gain is at a maximum when $\Delta\phi = 0$, so for small values of $\Delta\phi$, the magnitude of G_A is:
  - $|G_{A(\Delta\phi = 0)}| = \dfrac{\sin(N\cdot 0)}{\sin(0)} = N$
- Then normalizing that for other steering angles near $\Delta\phi = 0$
  - $\boxed{|G_{A(NORM)}| = \dfrac{\sin(N\cdot\Delta\phi/2)}{N\cdot\sin(\Delta\phi/2)}}$

see http://www.waves.utoronto.ca/prof/svhum/ece422/notes/15-arrays2.pdf

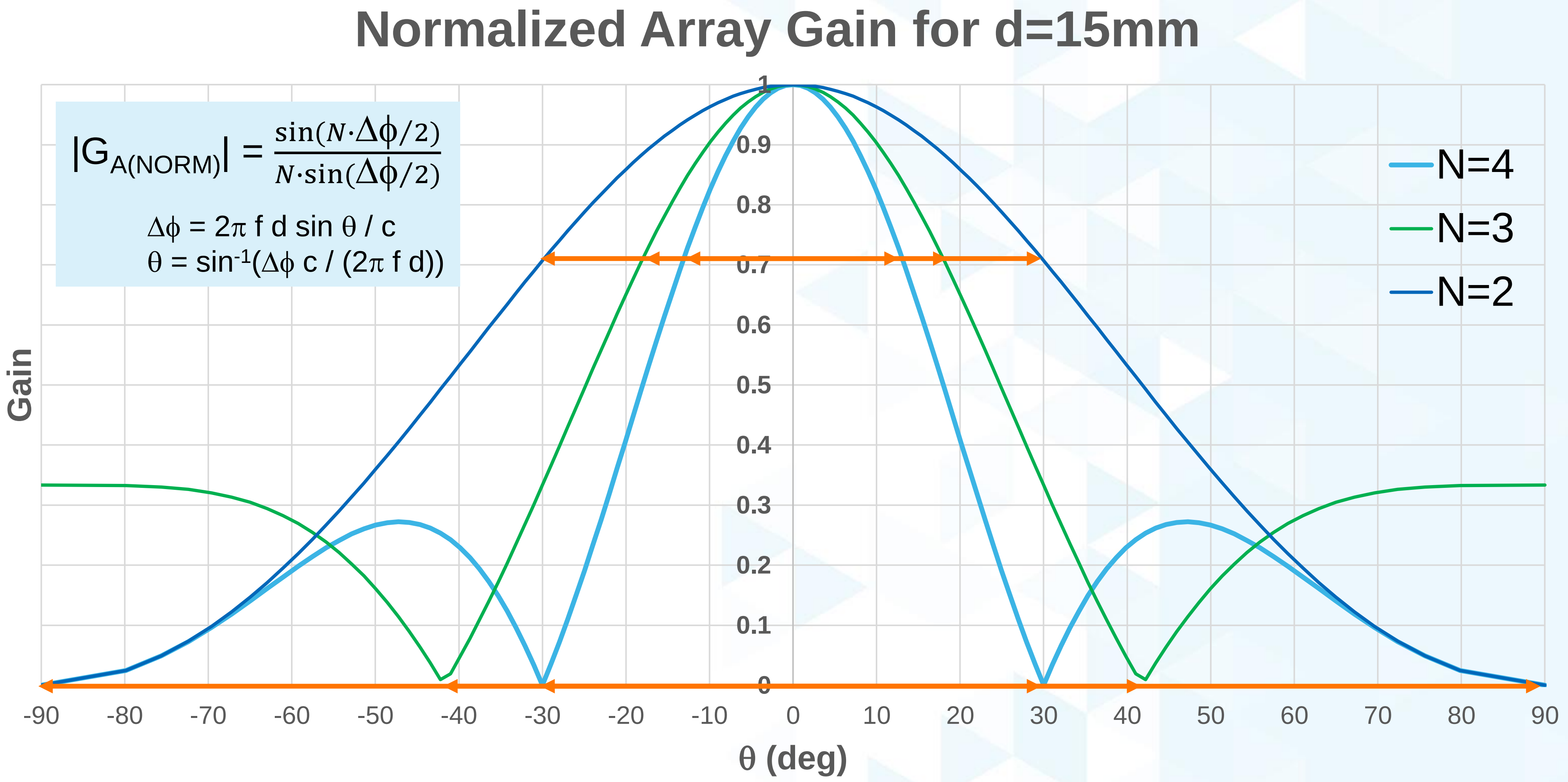


Remember:

$\theta = \sin^{-1}(\Delta\phi\, c / (2\pi f d))$

$\Delta\phi = 2\pi f d \sin\theta / c$

Understanding Beam Widths: Math and Theory



Understanding Beam Width: Math and Theory

- With the normalized array factor, we can find an equation, related to N (the number of elements) to find the beam width.
- The halfpower beam width (HPBW) occurs when f is 3dB down from its peak, which is:
  - HPBW is the steering angle when $G_{A(NORM)} = 1/\sqrt{2}$
    - $\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 1/\sqrt{2}$
    - For N=4 and f=10 GHz, solving for $\Delta\phi$ gives:
      - $\Delta\phi = 0.715$ rad
      - Then $\theta = \sin^{-1}(\Delta\phi\ c\ /\ (2\pi\ f\ d)) = \sin^{-1}(0.715*3E8/(2pi*10E9*0.015)) = 0.23$ rad = 13.16º
      - That is for ½ of the beam width, so the half power beam width is 2x that
      - HPBW = 26.3º (N=4, f=10GHz). HPBW=24.8º at f=10.6GHz
- How does that compare to your measured result?
- We can do similar calcs to arrive at beam widths for N=2 and 3 (f=10GHz)
  - N=3, HPBW = 36.2º (34.0º for f=10.6GHz)
  - N=2, HPBW = 60.0º (56.2º for f=10.6GHz)

Understanding Beam Width: Math and Theory

- We can find the first null beam width (FNBW) by solving the normalized AF for when it equals 0:
  - $\frac{\sin(N \cdot \Delta\phi/2)}{N \cdot \sin(\Delta\phi/2)} = 0$
  - For N=4 and f=10 GHz, solving for $\Delta\phi$ gives:
    - $\Delta\phi$ = 1.57 rad
    - Then $\theta = \sin^{-1}(\Delta\phi\ c\ /\ (2\pi\ f\ d))$ = $\sin^{-1}$(1.57*3E8/(2pi*10E9*0.015)) = 0.52 rad = 29.98º
    - That is for ½ of the width, so the total null to null width is 2x that
    - FNBW = 60.0º (N=4, f=10GHz). HPBW=56.2º at f=10.6GHz

- For N=3, the FNBW = 83.9º (78.2º for f=10.6GHz)

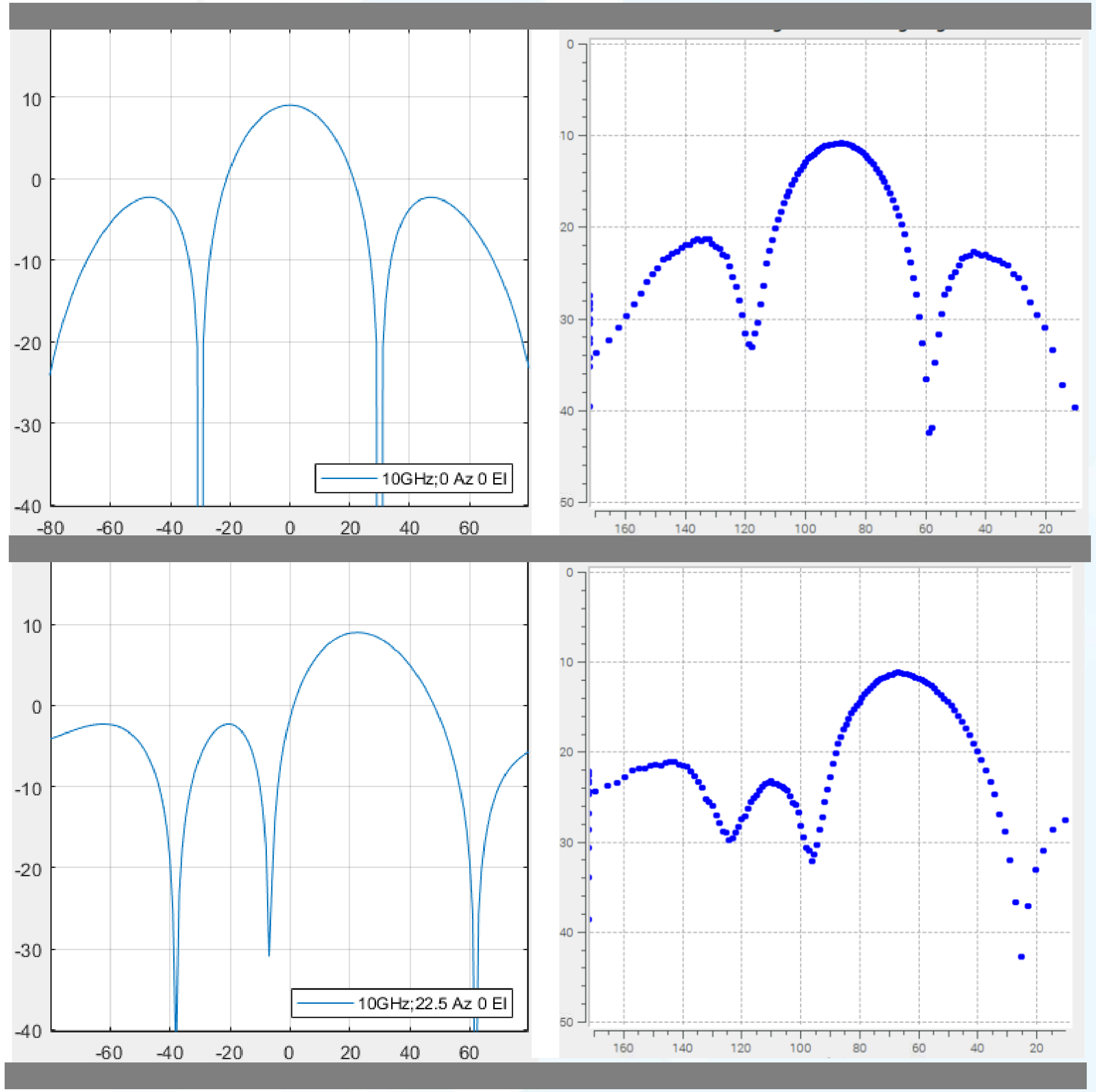
- For N=2, the FNBW = 161º (137º for f=10.6GHz)

Understanding Beam Width: Compare to Ideal

- Let's compare our measured results, with Matlab's Sensor Array Analyzer Results:
  - Shapes are approximately the same
  - Null locations, HPBW match well
  - Number of peaks/nulls match

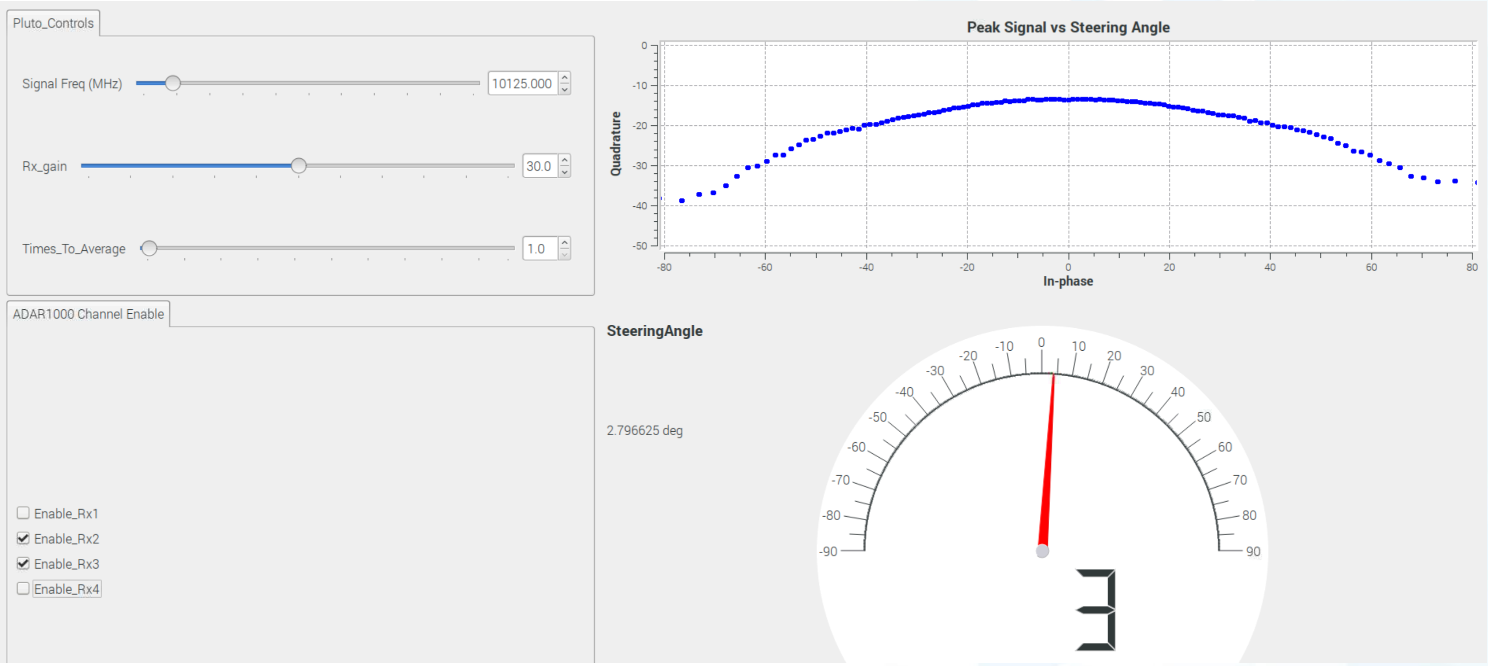
f=10GHz, θ=0°

f=10GHz, θ=22.5°



Understanding Beam Width: Lab

- Click on the ADAR1000 Controls Tab
- We can enable and disable each element
- Observe what happens when our N=4 array becomes N=3 or 2
  - How do these measurements compare with the predicted?
  - Consider the HPBW and the peak gain



Understanding Beam Width: Math and Theory

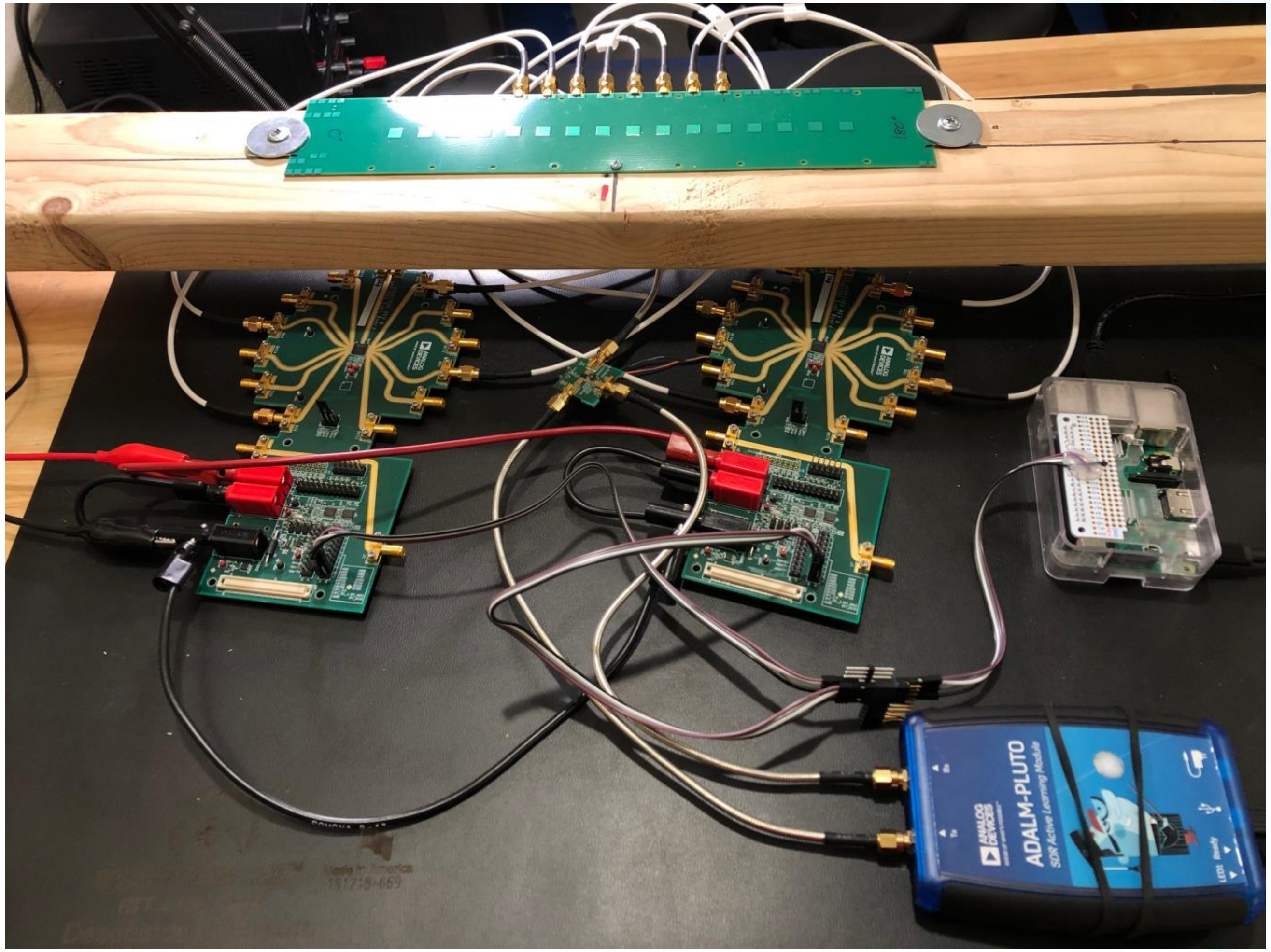
- Summary of key points for a linear, uniformly spaced, array:
  And for close to mechanical boresight ($\Delta\phi = 0$)

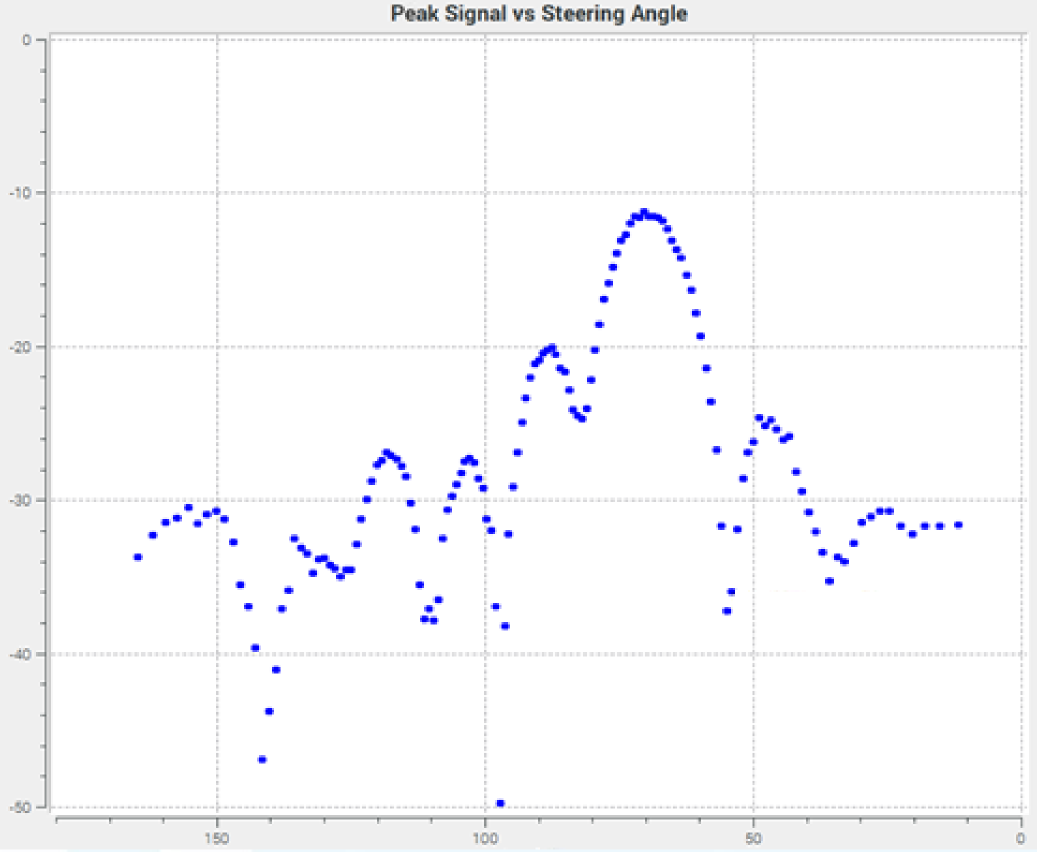
  - As N decreased, the HPBW increased
    - And the width of the sidelobes is ½ the width of the main lobe

  - As N decreased, the number of lobes and nulls also decreased

- We saw dramatic effects when we disabled an element
  - But what if we didn't disable it, we just attenuated it??
  - That is the next exercise: Beam Tapering!
  - And its effect is even more interesting….

Here's what 2 ADAR1000s, for N=8 looks like:

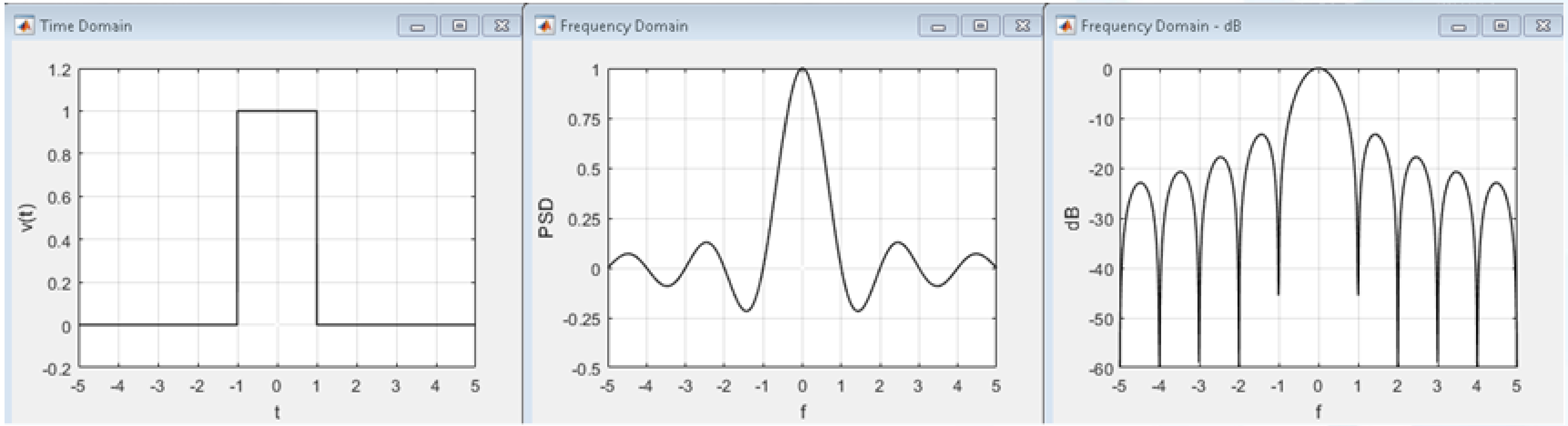




Understanding Beam Tapering

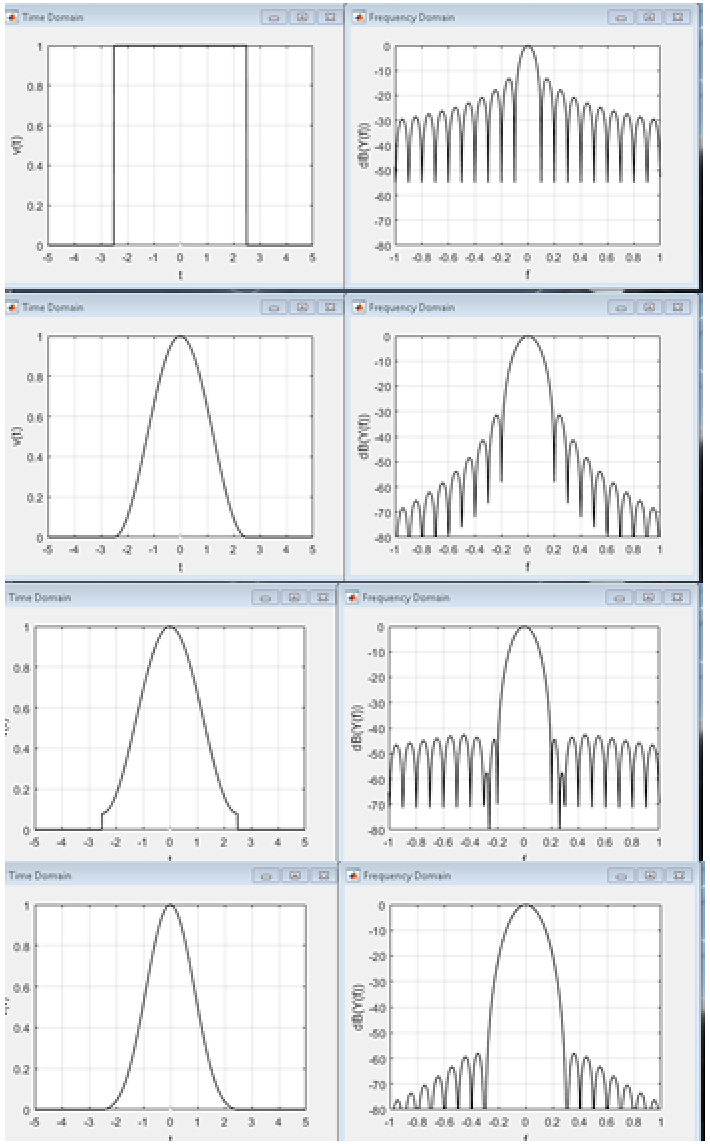
Understanding Beam Tapering: Math and Theory

- With all elements at the same gain, we have effectively have a box car window.
  - This is analogous to rectangular window FFT



  - Time domain pulse → frequency domain sin(x)/x → first sidelobe -13dBc, etc.
  - As pulse becomes wider…
    - Main lobe narrows
    - Sidelobes move in
    - Sidelobe levels remain unchanged

Understanding Beam Tapering: Window Functions



Boxcar – 1st sidelobe @ -13dBc
Narrowest main lobe

Hanning – 1st sidelobe < -30dBc
Main lobe broadens

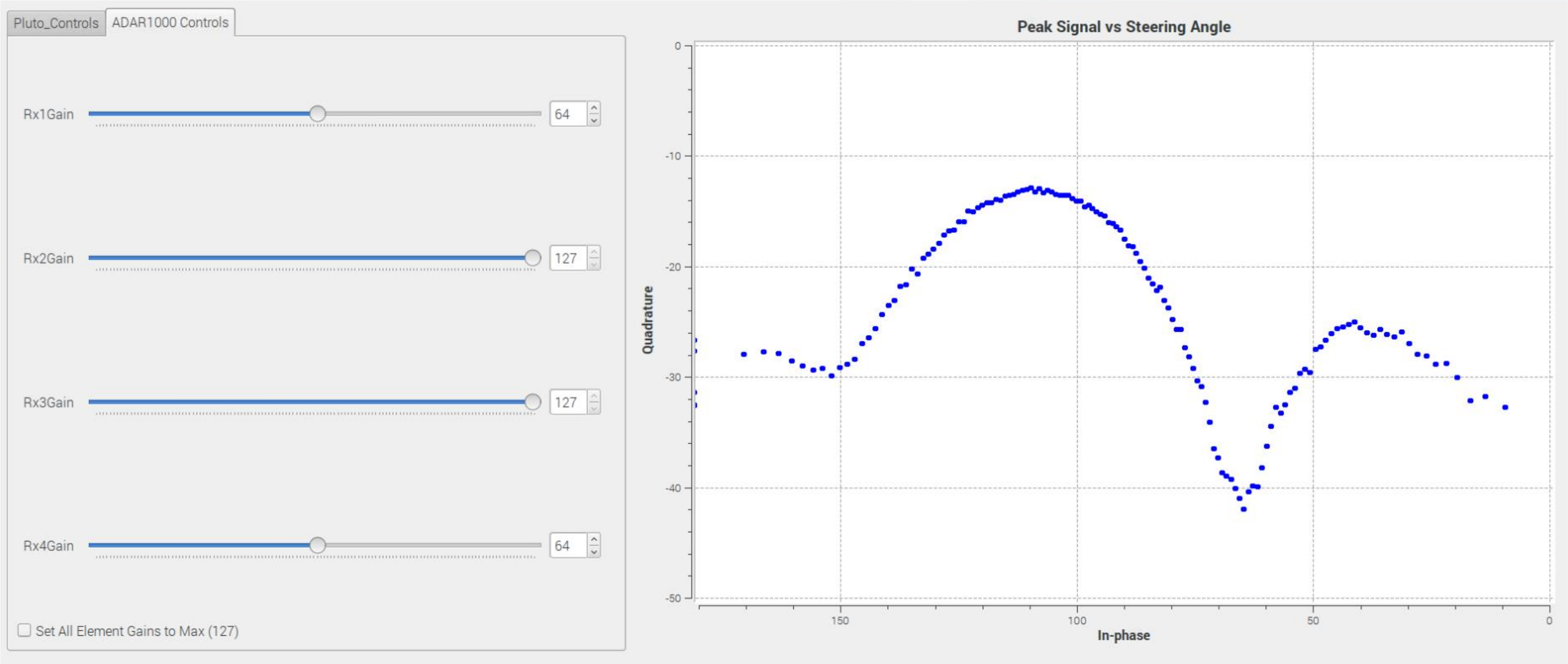
Hamming – Lower close-in sidelobes
Narrower main lobe than Hanning

Blackman – Lowest sidelobes
Broadest main lobe

Note: windowing losses not shown in these examples

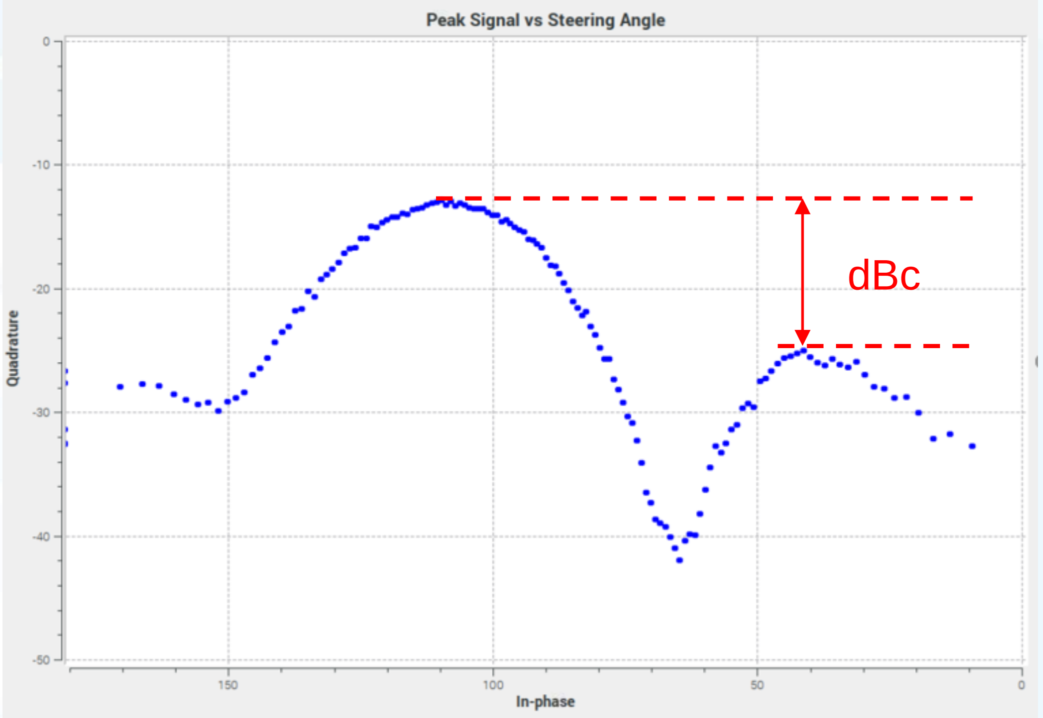
Understanding Beam Tapering: Lab

- Click on the ADAR1000 Controls Tab
- We now have amplitude control of each element
- Now, let's observe the effect if Rx1 and Rx4 are ½ the gain of Rx2 and Rx3:



Understanding Beam Tapering: Lab

- For all gains = 127, how far down, in dBc, are the sidelobes?
  - first side lobe = ________ dBc
- Set Rx1 and Rx4 gain to 55, and Rx2 and Rx3 to 127
  - first side lobe = ________ dBc
- Set Rx1 and Rx4 gain to 30, and Rx2 and Rx3 to 127
  - first side lobe = ________ dBc
- Try varying the gains of each element
  - Observe the impact to the main beam width and side lobe gain



Understanding Beam Tapering: Math and Theory

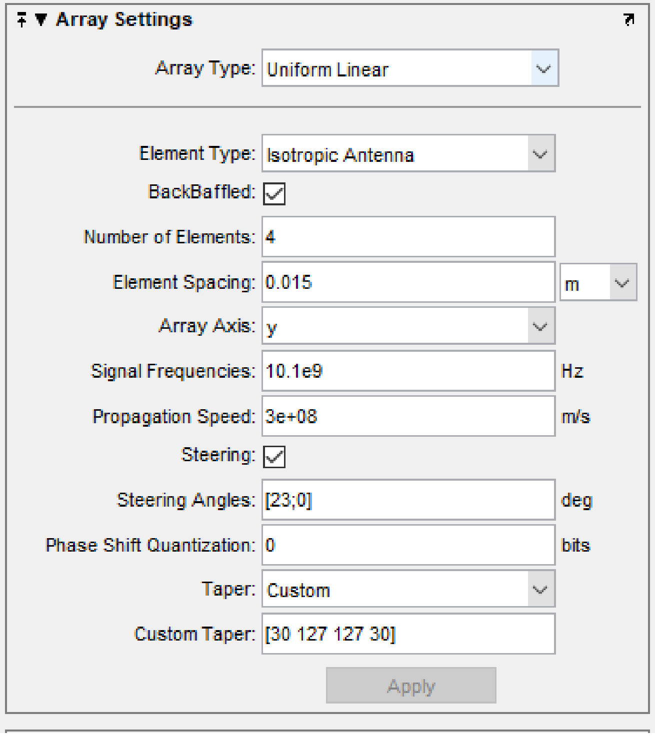
ANALOG DEVICES
AHEAD OF WHAT'S POSSIBLE™

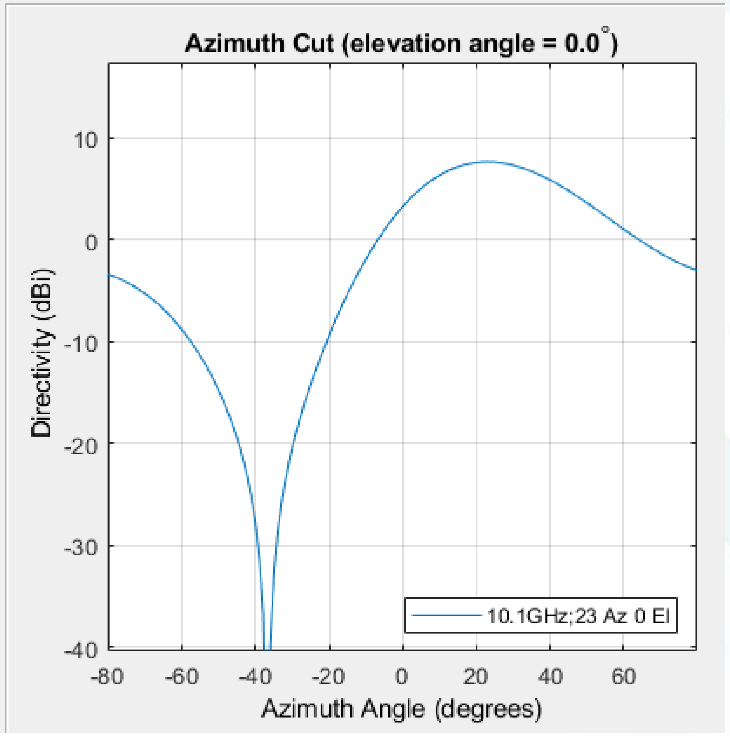
- When we set Rx1 and Rx4 to 30, we were following a "Blackman" Taper:
  - In Matlab, use the blackman function and normalize to gain of 127:

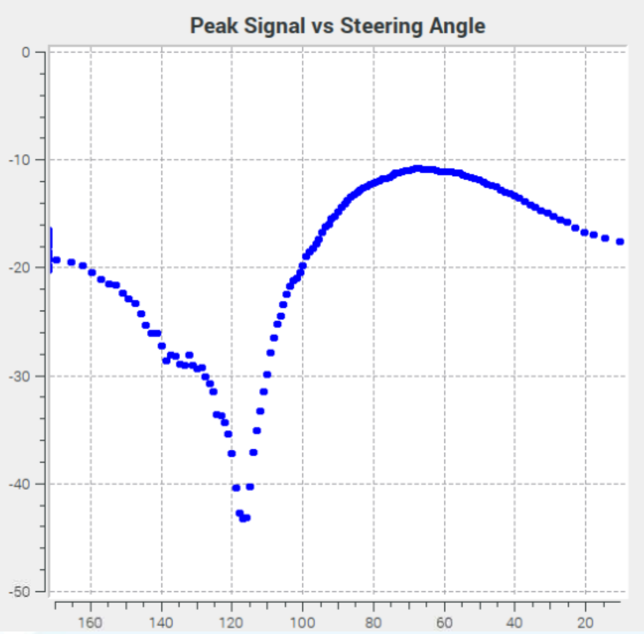
```
>> blackman(6)/max(blackman(6))*127
ans =
         0  → No element
   30.0246  → Rx1
  127.0000  → Rx2
  127.0000  → Rx3
   30.0246  → Rx4
         0  → No element
```

It's a little hard to do much tapering with only 4 elements

But the blackman taper end elements are 0, so if we take some liberties and say we have 6 elements (with the outer ones just gain=0), then we can do a "6 element" blackman taper.





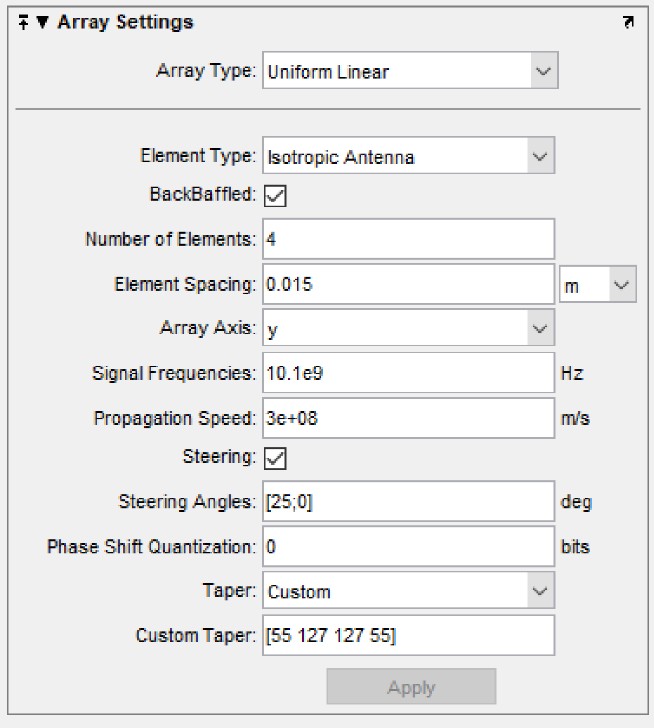


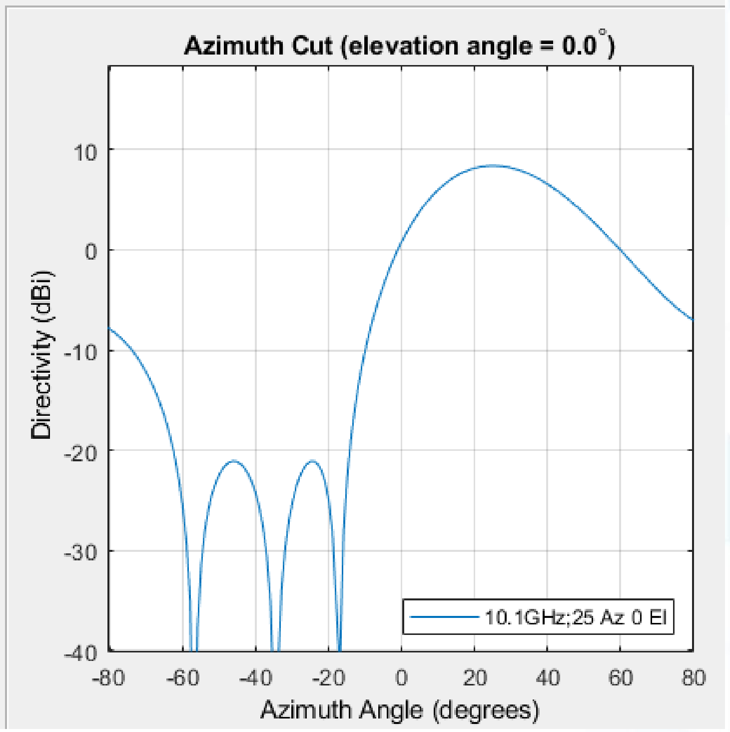
Understanding Beam Tapering: Math and Theory

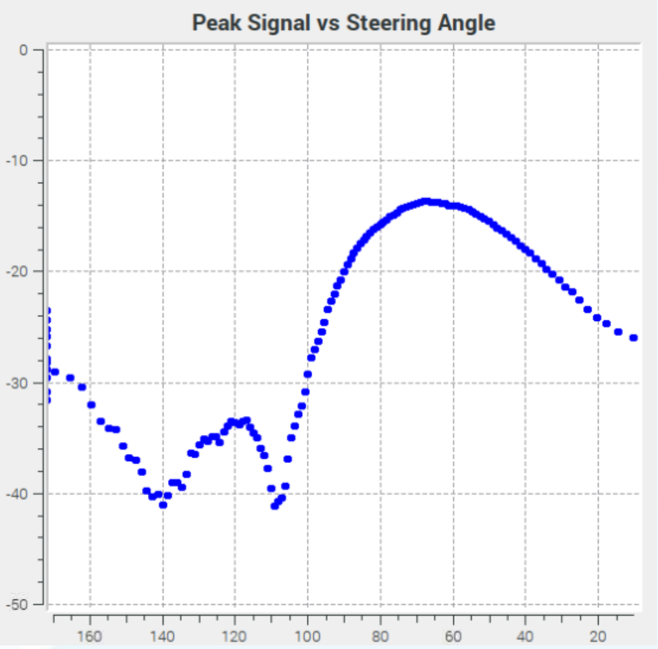
- When we set Rx1 and Rx4 to 55, we were following a “Hamming” Taper:
  - In Matlab, use the hamming function and normalize to gain of 127:

```
>> hamming(6)/max(hamming(6))*127
ans =
    11.1385 ——> No element
    55.3937 ——> Rx1
   127.0000 ——> Rx2
   127.0000 ——> Rx3
    55.3937 ——> Rx4
    11.1385 ——> No element
```

Again, we take some liberties and only use 4 elements



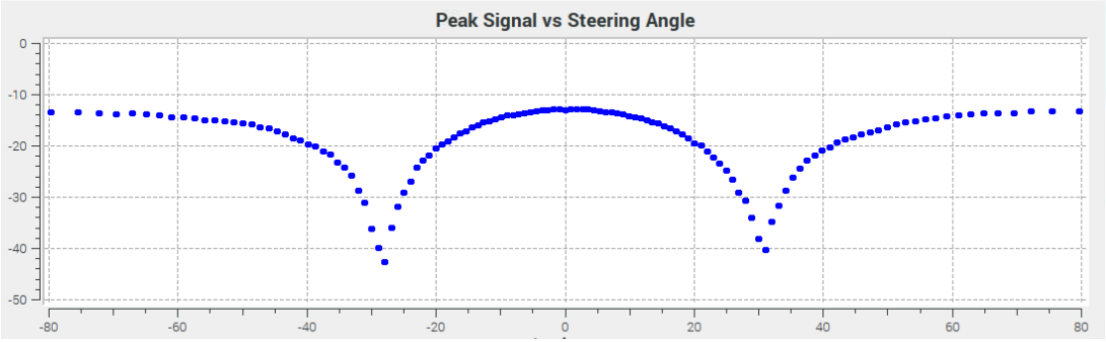




Understanding Grating Lobes

Understanding Grating Lobes: Lab

- In the tapering lab, perhaps you couldn't resist "tapering" the inner elements (Rx2 and Rx3).
  - You may have been surprised with how unrecognizable the beam became!



- Strange new lobes, equal to our peak angle magnitude, start appearing…..

- What's going on???

Understanding Grating Lobes: Math and Theory

ANALOG DEVICES
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- Grating Lobes are analogous to aliasing on an ADC
  - From sampling theory, we need minimum sample rate of 2x frequency of interest

- Grating Lobes are "Spatial Aliasing"
  - Each antenna element is a special "sample"
  - Need two "samples" per wavelength to avoid spatial aliasing (aka Grating Lobes)
    - For $<\lambda/2$ element spacing, no grating lobes occur
    - For $>\lambda/2$ element spacing, grating lobes will appear at the opposite horizon

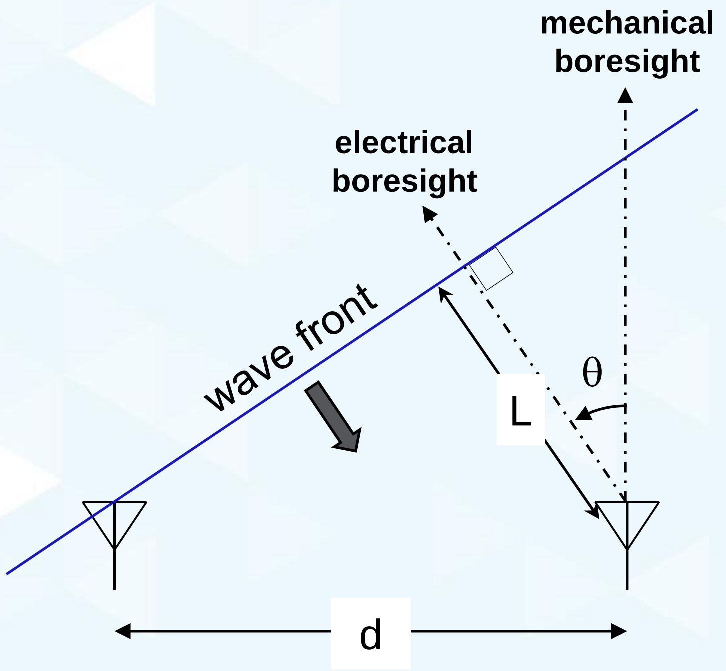
- Let's return to our old friend, Math.

Understanding Grating Lobes: Math and Theory

- Let's start with our familiar equations:
  - $\Delta\phi = (2\pi\ d\ /\ \lambda) \cdot \sin(\theta)$
  - Then our steering angle is
    → $\theta = \sin^{-1}(\Delta\phi/2\pi \cdot \lambda/d)$

- This produces a maxima (main lobe) every $2\ \pi$
  - i.e. a main lobe occurs when $\Delta\phi = m \cdot 2\pi$, for m=0, ±1, ±2, etc.

- For simplicity, start with the case of $\theta = 0$ (full broadside).
  - Then the θ of the main lobe, is:
  - $\theta_{MAIN} = \sin^{-1}(m\ \lambda/d)$, for m=0, ±1, ±2, etc.
  - But remember that the arcsin is a weird function…..



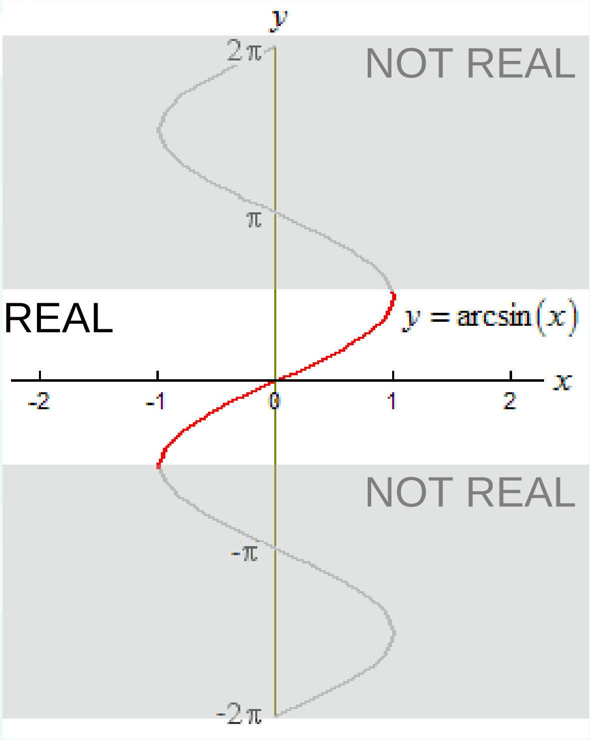
Remember:
$\theta\ = \sin^{-1}(\Delta\phi\ c\ /\ (2\pi\ f\ d))$
$\Delta\phi = 2\pi\ f\ d\ \sin\theta\ /\ c$

Understanding Grating Lobes: Math and Theory

$\theta_{MAIN} = \sin^{-1}(m\,\lambda/d)$, for m=0, ±1, ±2, etc.

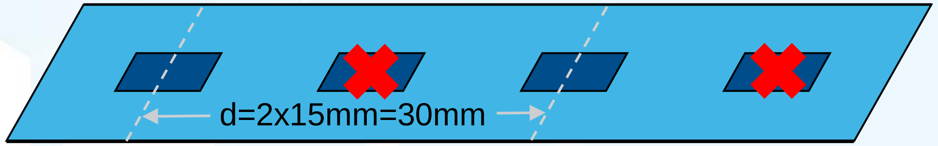
- The arcsin argument must be bounded between -1 and 1
  - So valid (real) solutions depend on that (λ/d) term
  - In our case, λ =30mm, and d=15mm so (λ/d) = 2
  - This yields only one real solution to arcsin. And it occurs at m=0.
  - But what if (λ/d) was less than 1?
  - You would have multiple real solutions! Which means multiple main lobes!!

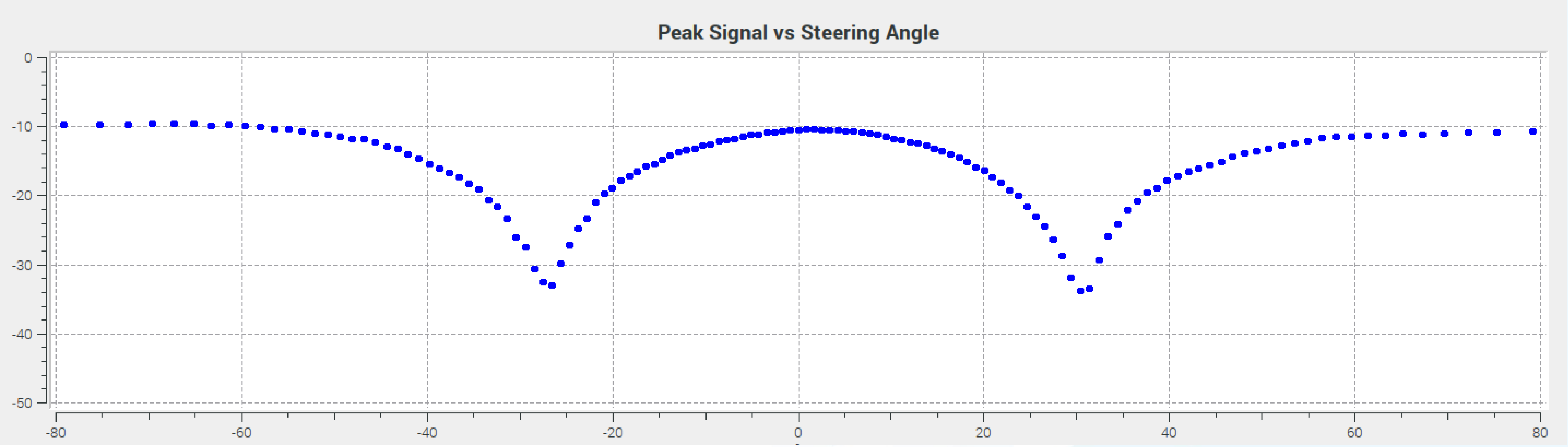
- So if λ =30mm, and d=3*15mm then (λ/d) = 0.66
  - We now have valid arcsin solutions for more than m=0
  - θ(main lobe) = $\sin^{-1}(m * 0.66)$, is valid for m=0 AND ±1
  - We will see 3 main lobes!
    - Located at θ = 0, θ = ±0.72 rad (±41º)



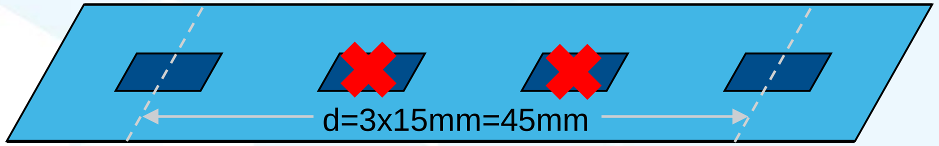
Understanding Grating Lobes: Lab

- Stay in the Exercise 4 GNUradio flowgraph
- Set your radiating element to 0º (full broadside)
- Let's observe some combinations now:
  - Set every other element to 0: Rx1 = 127, Rx2 = 0, Rx3 = 127, and Rx4 = 0
    - Now our d = 2 * 15mm = 30 mm. So lambda/d is about 1
    - We expect to see main lobes at $\sin^{-1}(0 \cdot 1)$, $\sin^{-1}(\pm 1 \cdot 1)$ which is 0º and ±90º

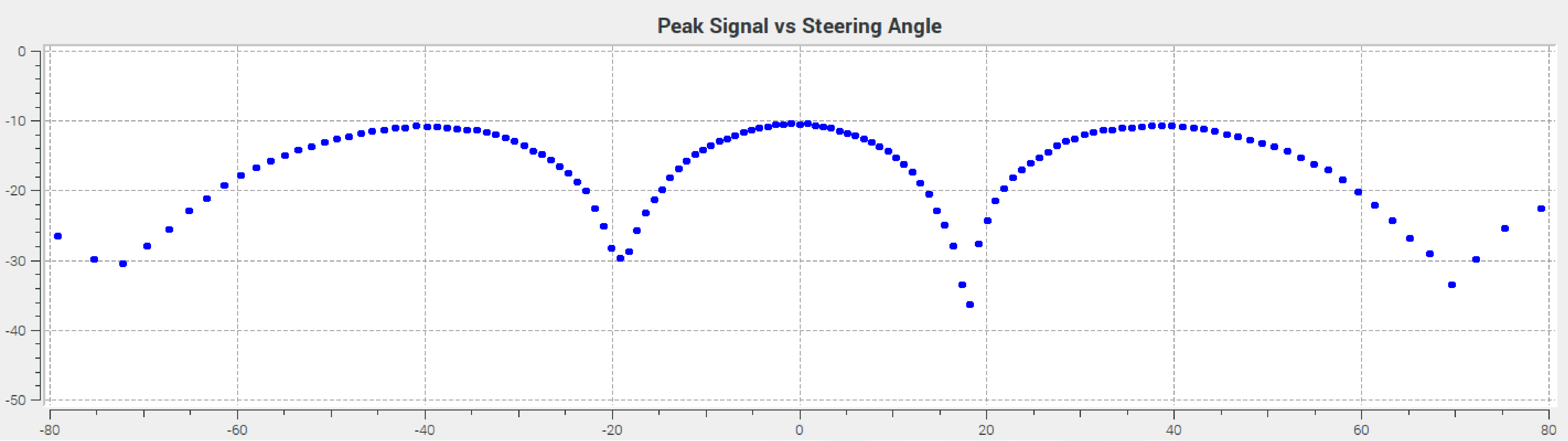




Understanding Grating Lobes: Lab



- Now set Rx1 and Rx4 to 127, and Rx 2 and 3 to 0
  - Now our d = 3 * 15 mm = 45 mm. So lambda/d is about 0.66 (for f=10GHz)
  - If f = 10.5GHz, then we expect to see main lobes at $\sin^{-1}(0)$ and $\sin^{-1}(\pm 1 \cdot 0.0285/0.045)$ which is 0º and ±40º



Understanding Grating Lobes: Math and Theory

- Those were pretty extreme grating lobes. We don't often set our element spacing to >lambda
- More often, the lambda/2 guideline may be violated a little (0.55 lambda spacing)
- In this case, the grating lobes start to appear as we deviate from mechanical boresight (theta=0)
- Require <lambda/2 element spacing to steer beam to horizon without Grating Lobes
- Can tolerate wider element spacing if the max beam angle is constrained

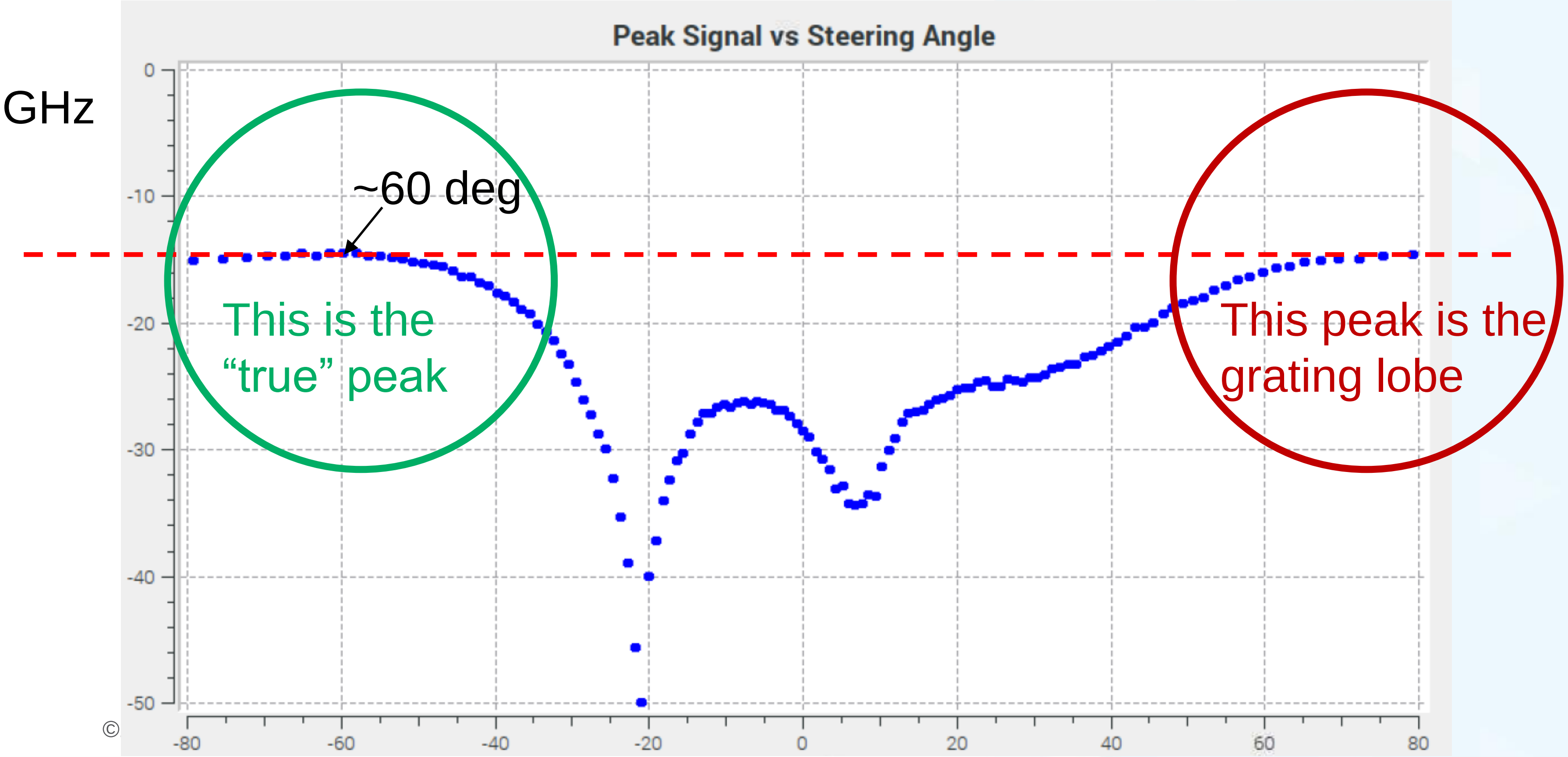
$$d_{MAX} = \frac{\lambda}{1 + sin\theta_{MAX}}$$

- If lambda = 10.5GHz, and d=15mm, then:
  - Theta(max) = arcsin(3E8/(10.5E9*0.015)-1) = 65 deg

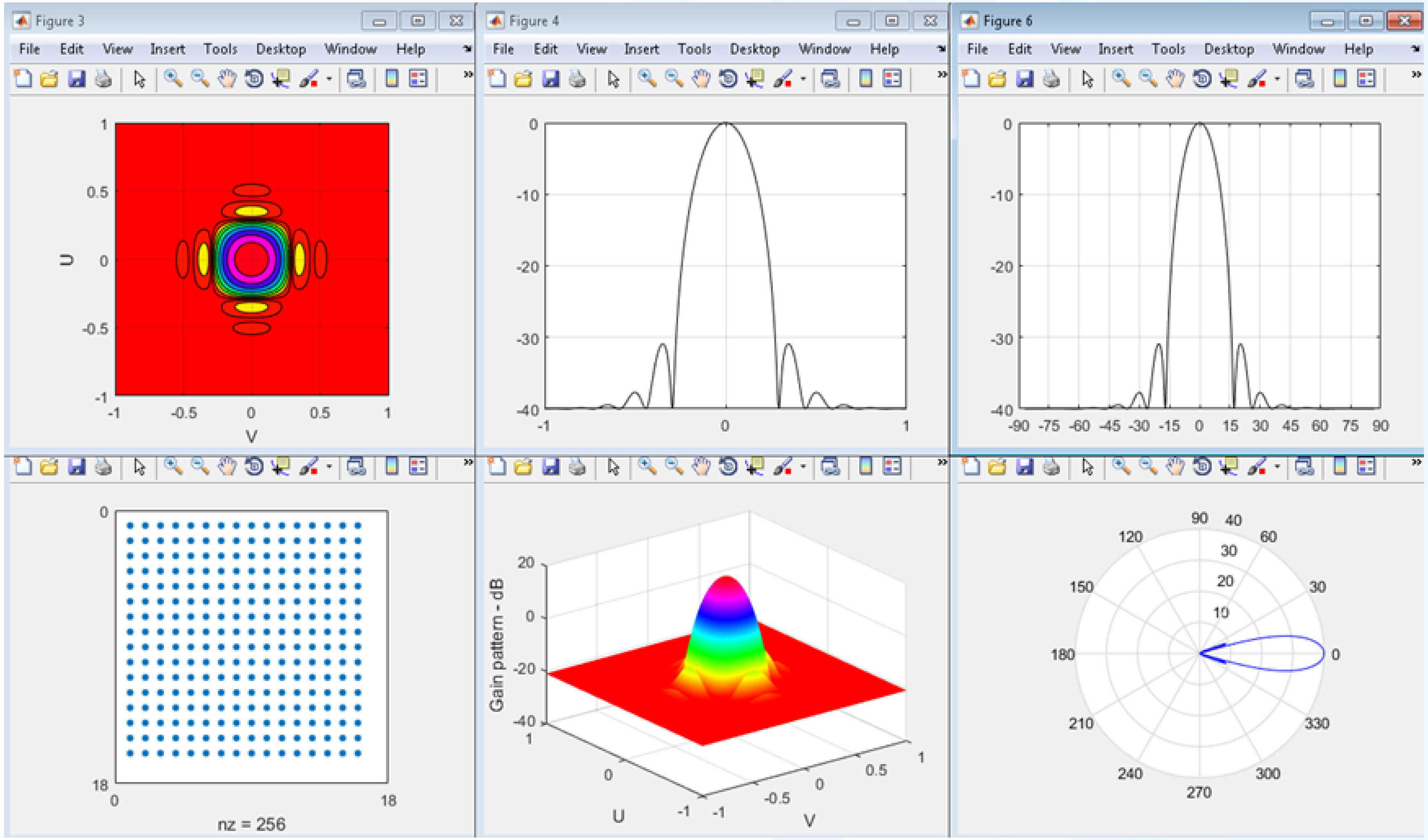
Understanding Grating Lobes: Math and Theory

- Try it on your setup!
- Bring the radiator closer to the horizon and note when you have two equal peaks
- Use iPhone angle measure app!

f=10.492GHz

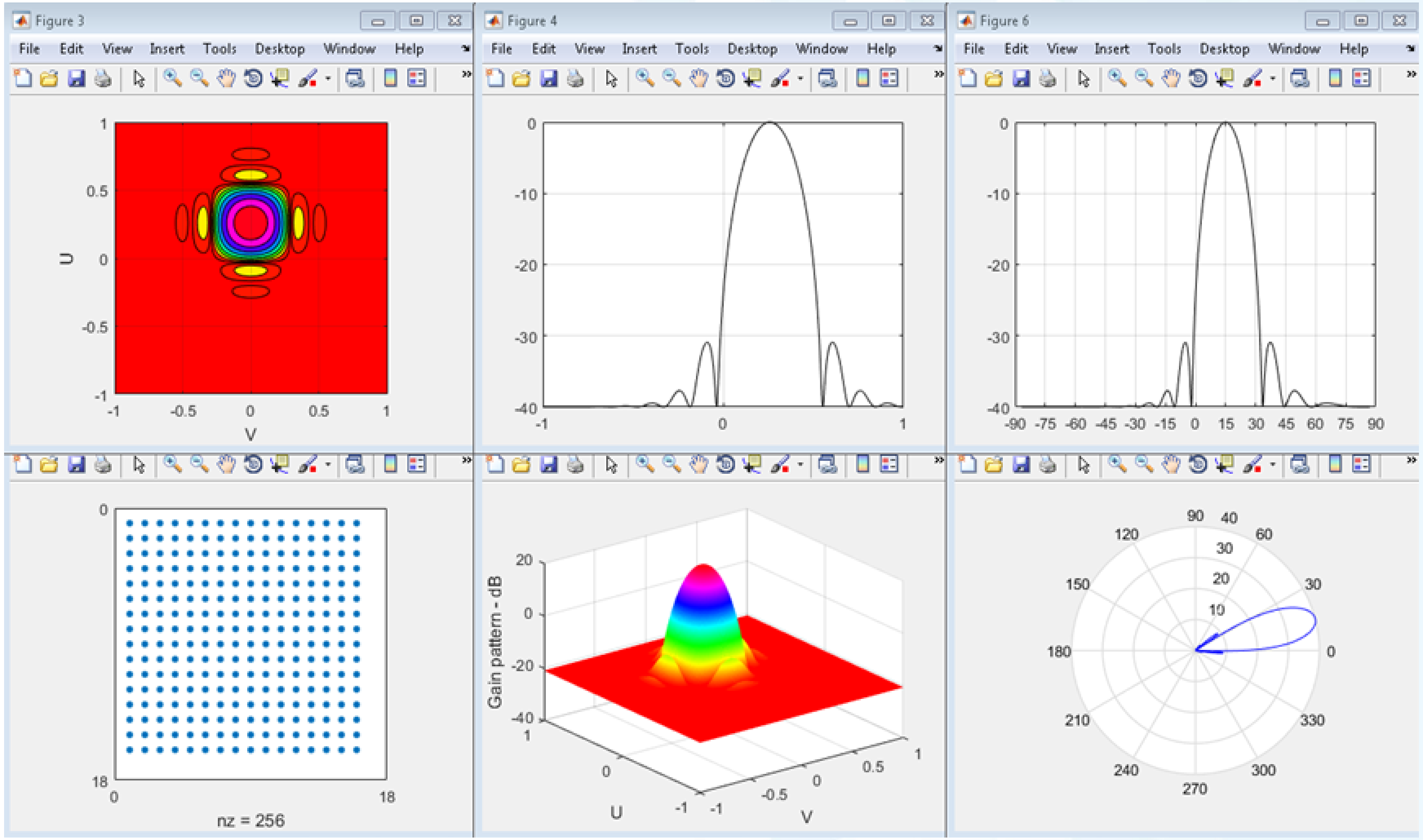


Spatial Aliasing – Zero Grating Lobes - 0°



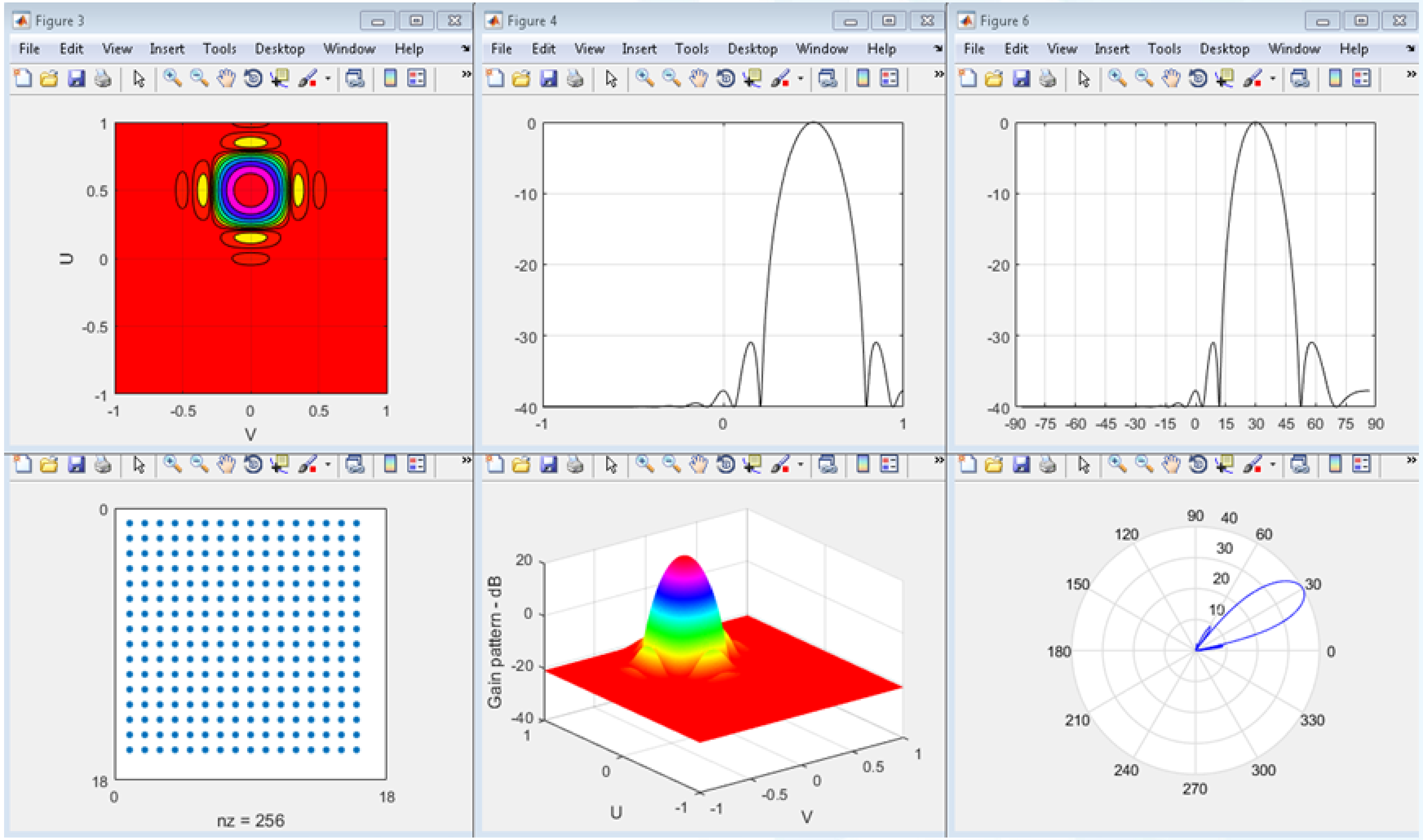
Element spacing = 0.4λ

Spatial Aliasing – Zero Grating Lobes - 15°



Element spacing = 0.4λ

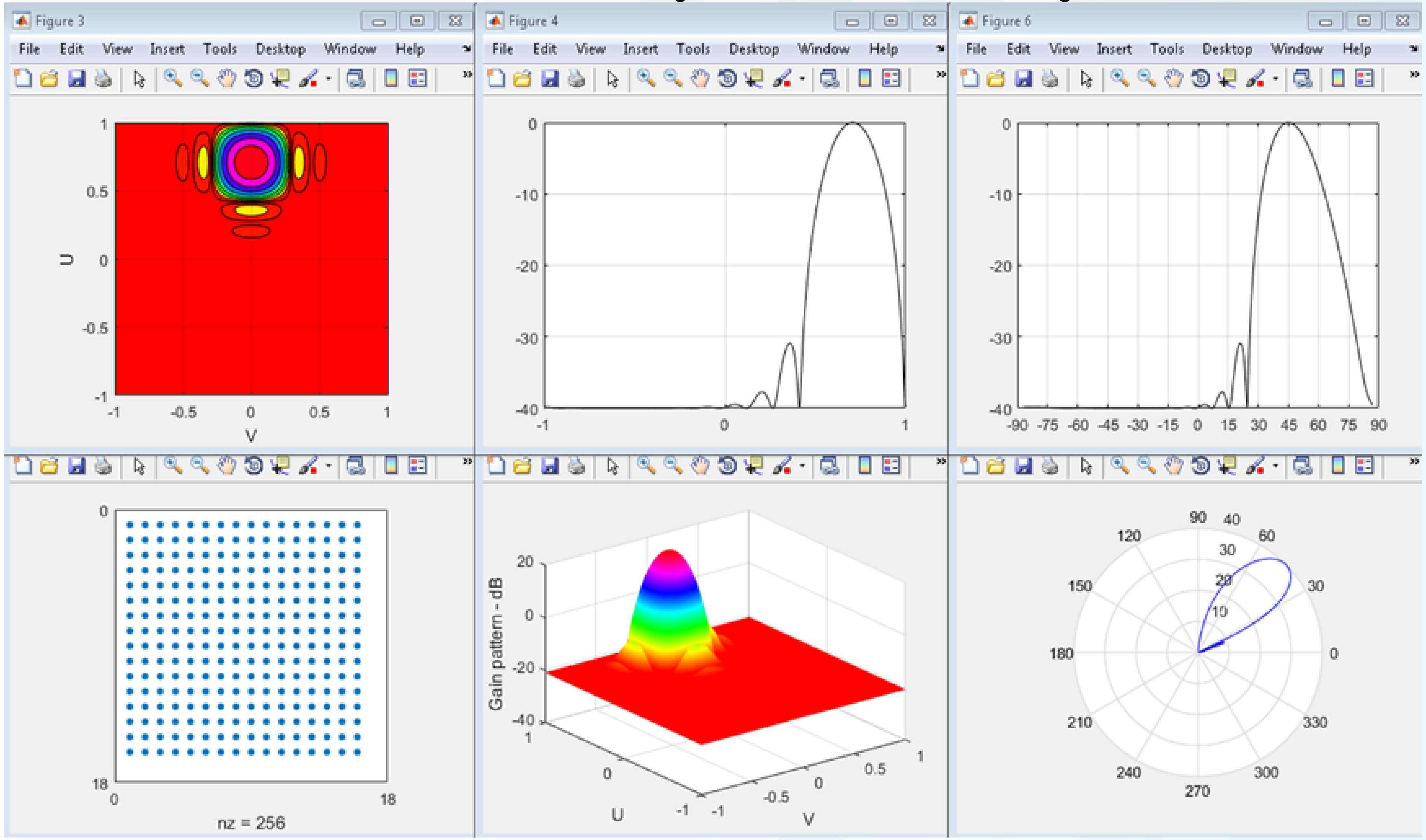
Spatial Aliasing – Zero Grating Lobes - 30°



Element spacing = 0.4λ

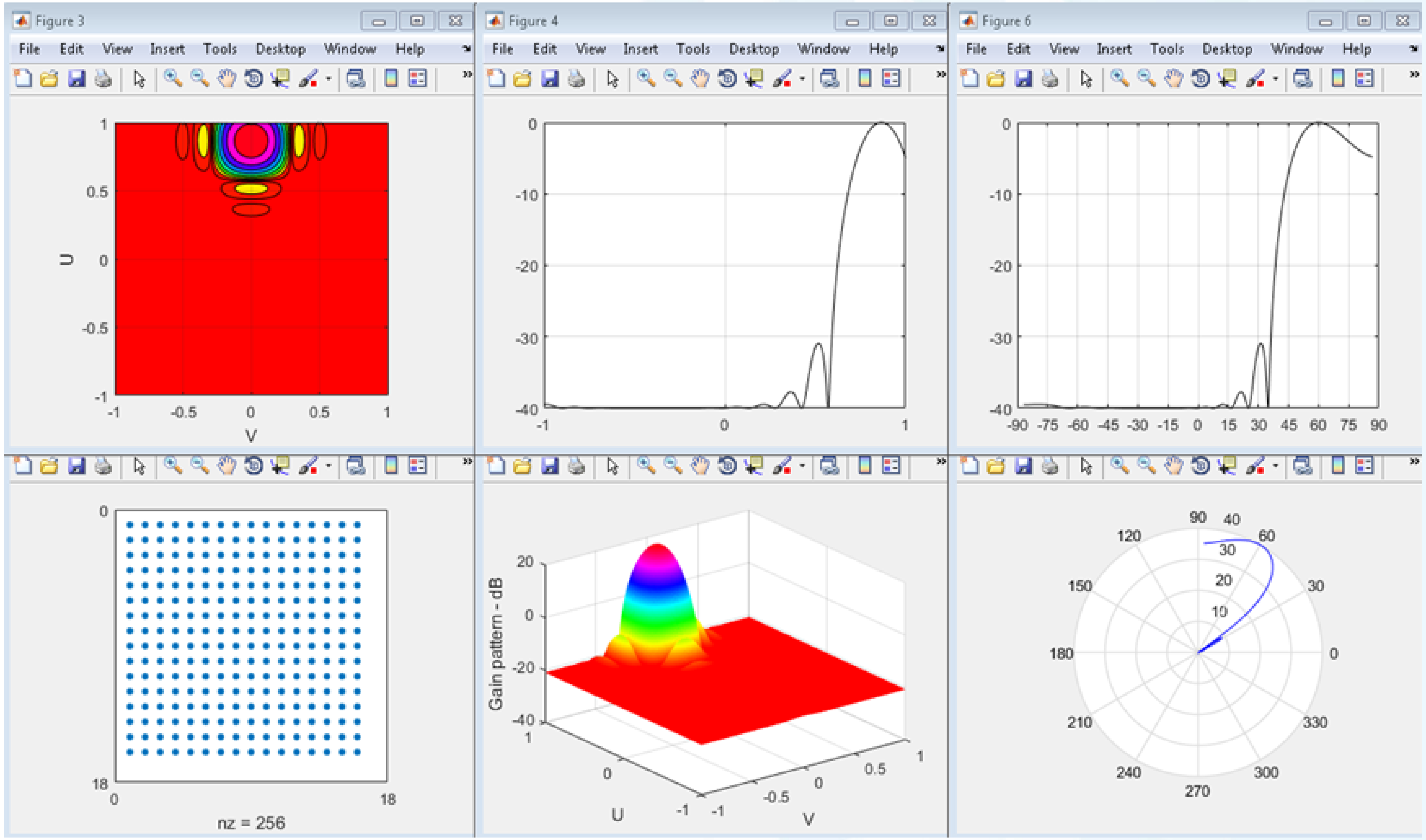
Spatial Aliasing – Zero Grating Lobes - 45°

notice beam width increasing toward horizon → foreshortening



Element spacing = 0.4λ

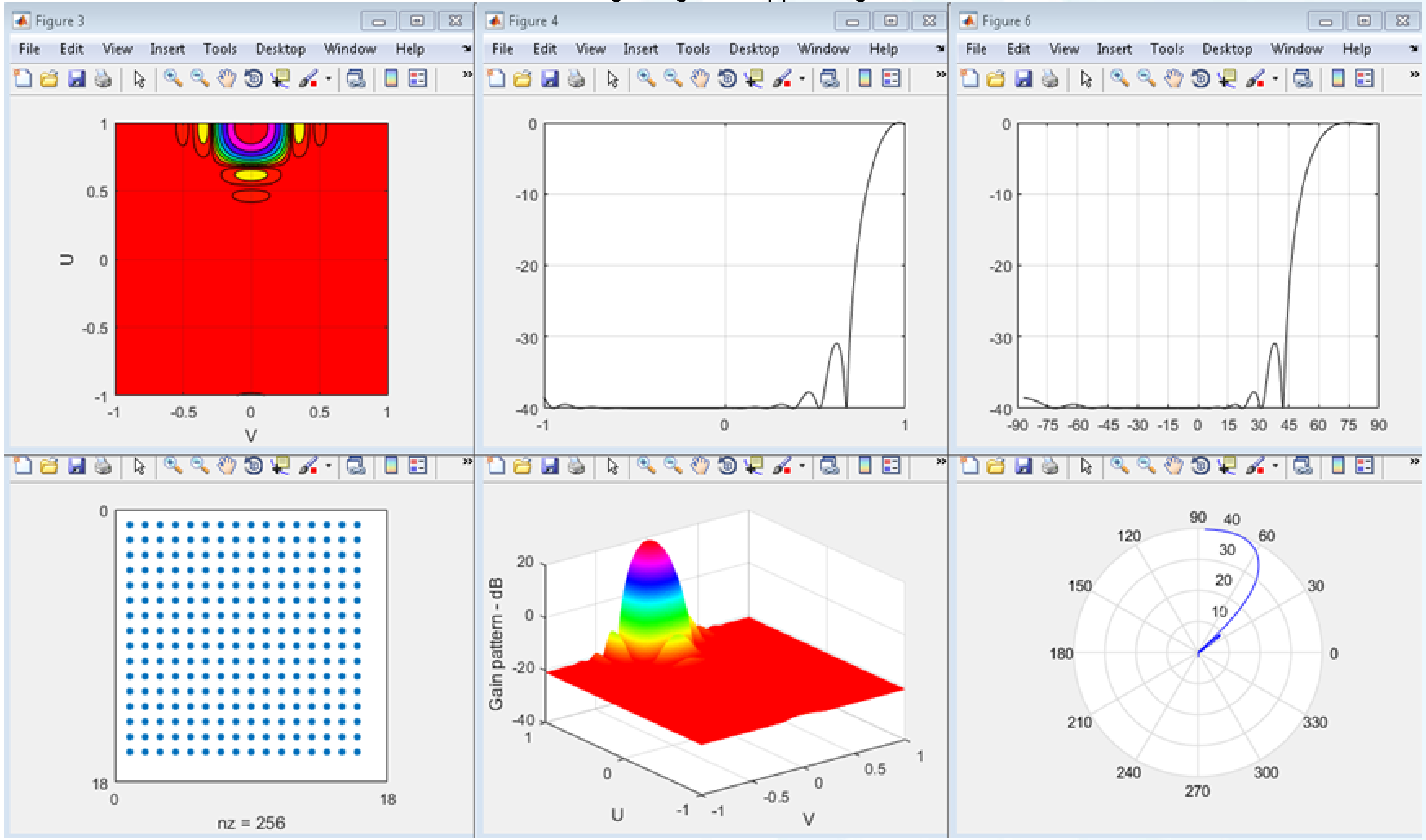
Spatial Aliasing – Zero Grating Lobes - 60°



Element spacing = 0.4λ

Spatial Aliasing – Zero Grating Lobes - 75°

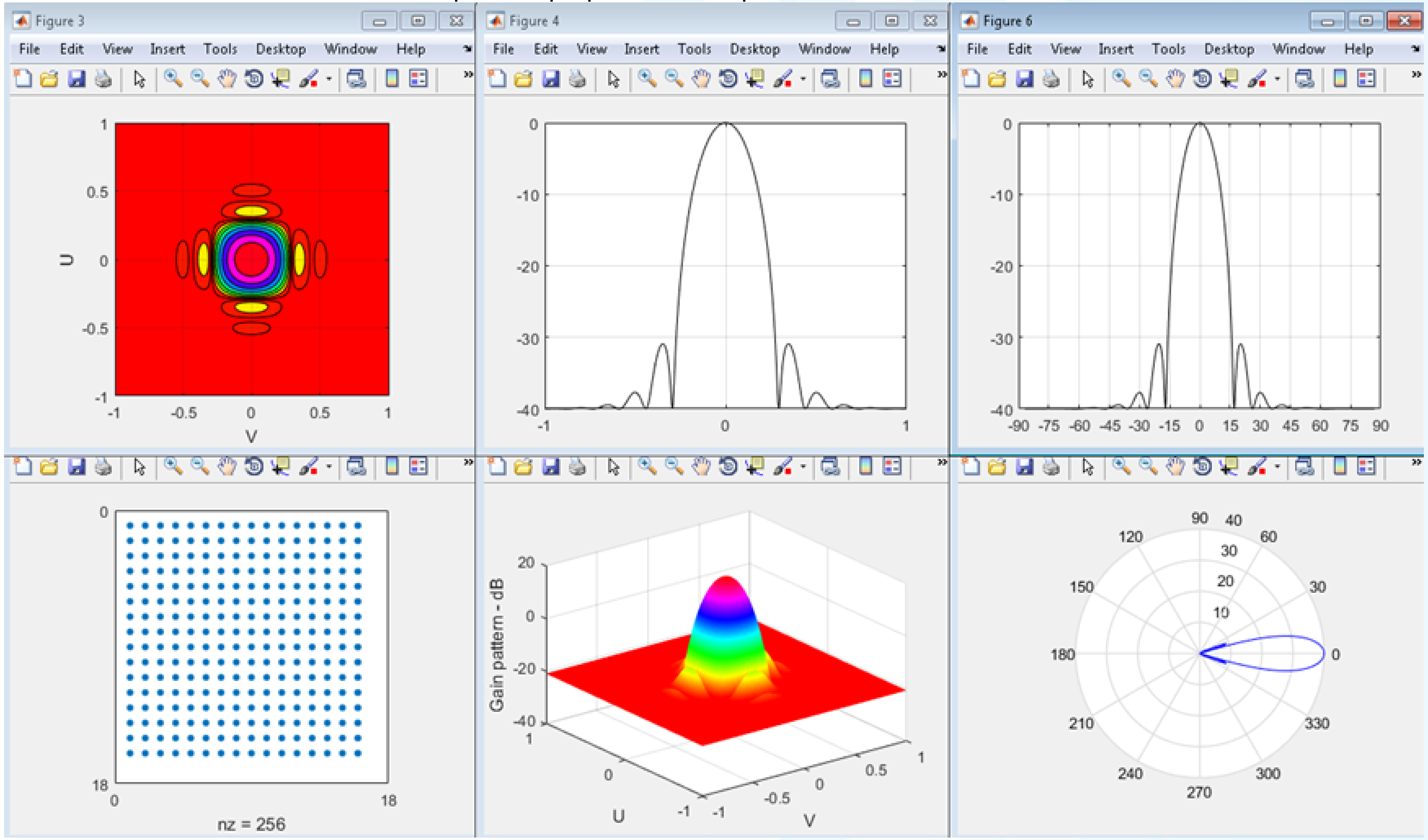
smallest hint of grating lobe appearing on horizon



Element spacing = 0.4λ

Spatial Aliasing – Zero Grating Lobes - 0° (repeat)

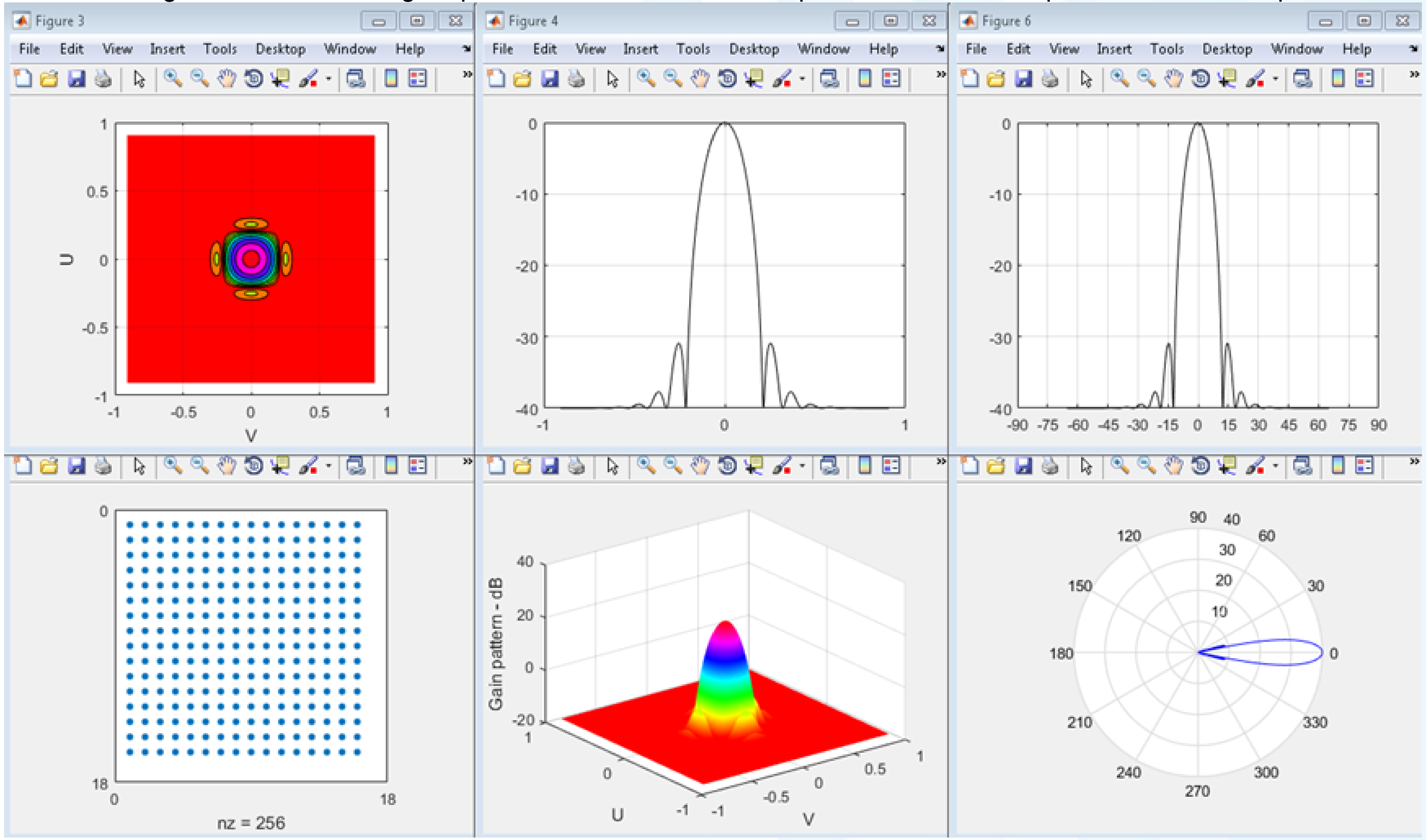
repeat for purposes of comparison to next slide…



Element spacing = 0.4λ

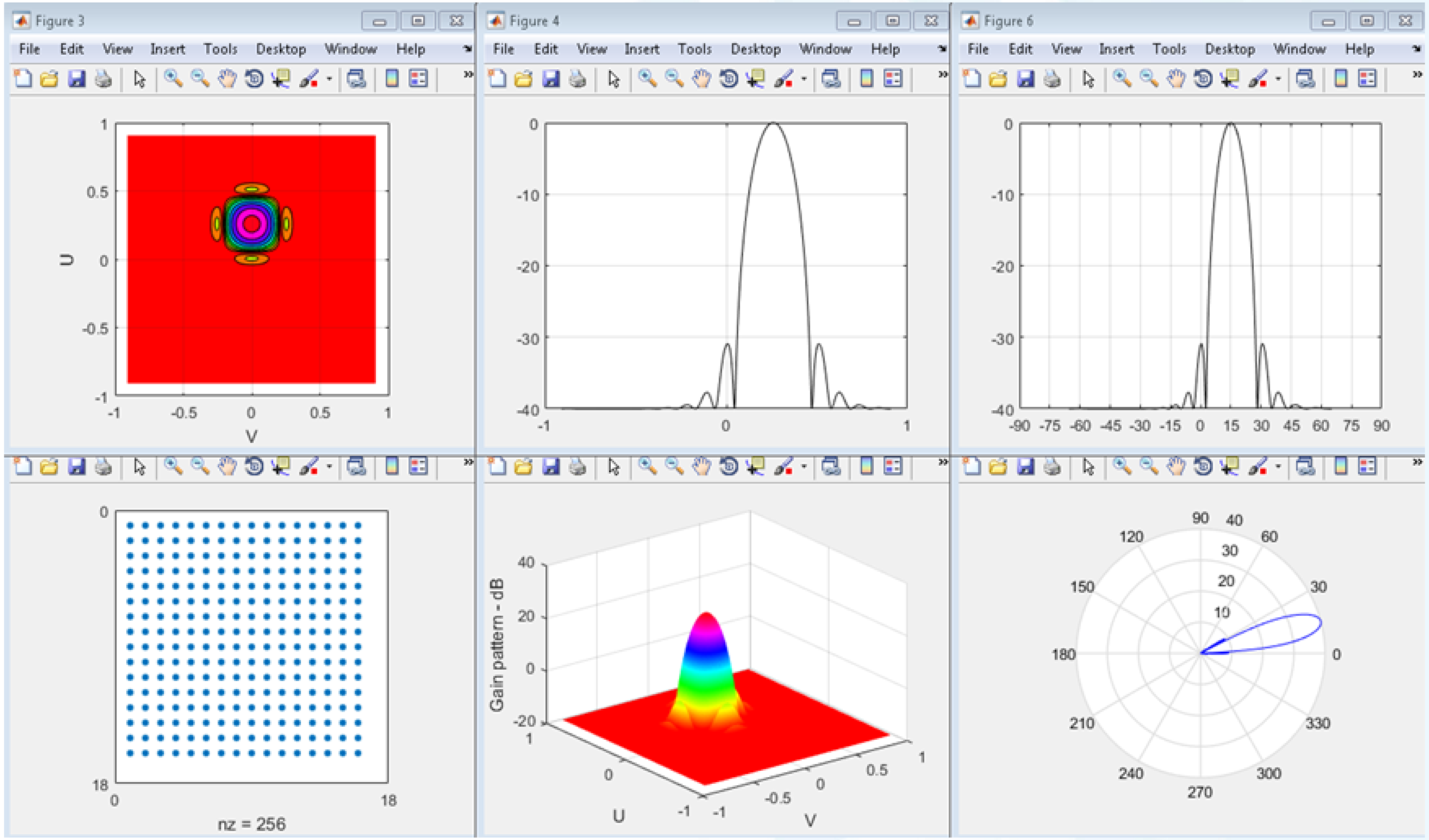
Spatial Aliasing – Grating Lobes - 0°

note: gain increased → larger aperture due to wider element placement, but now aperture is undersampled



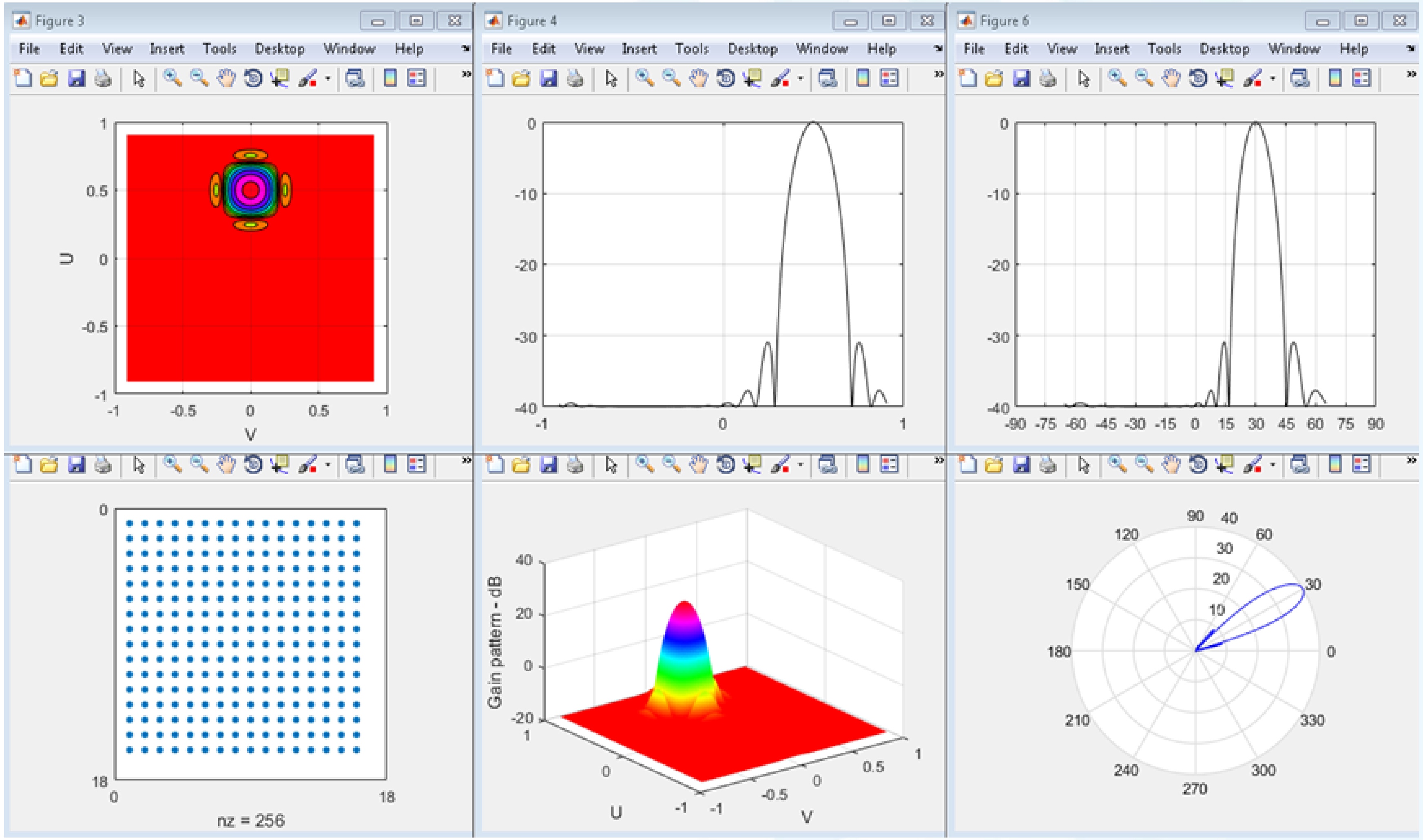
Element spacing = 0.55λ

Spatial Aliasing – Grating Lobes - 15°



Element spacing = 0.55λ

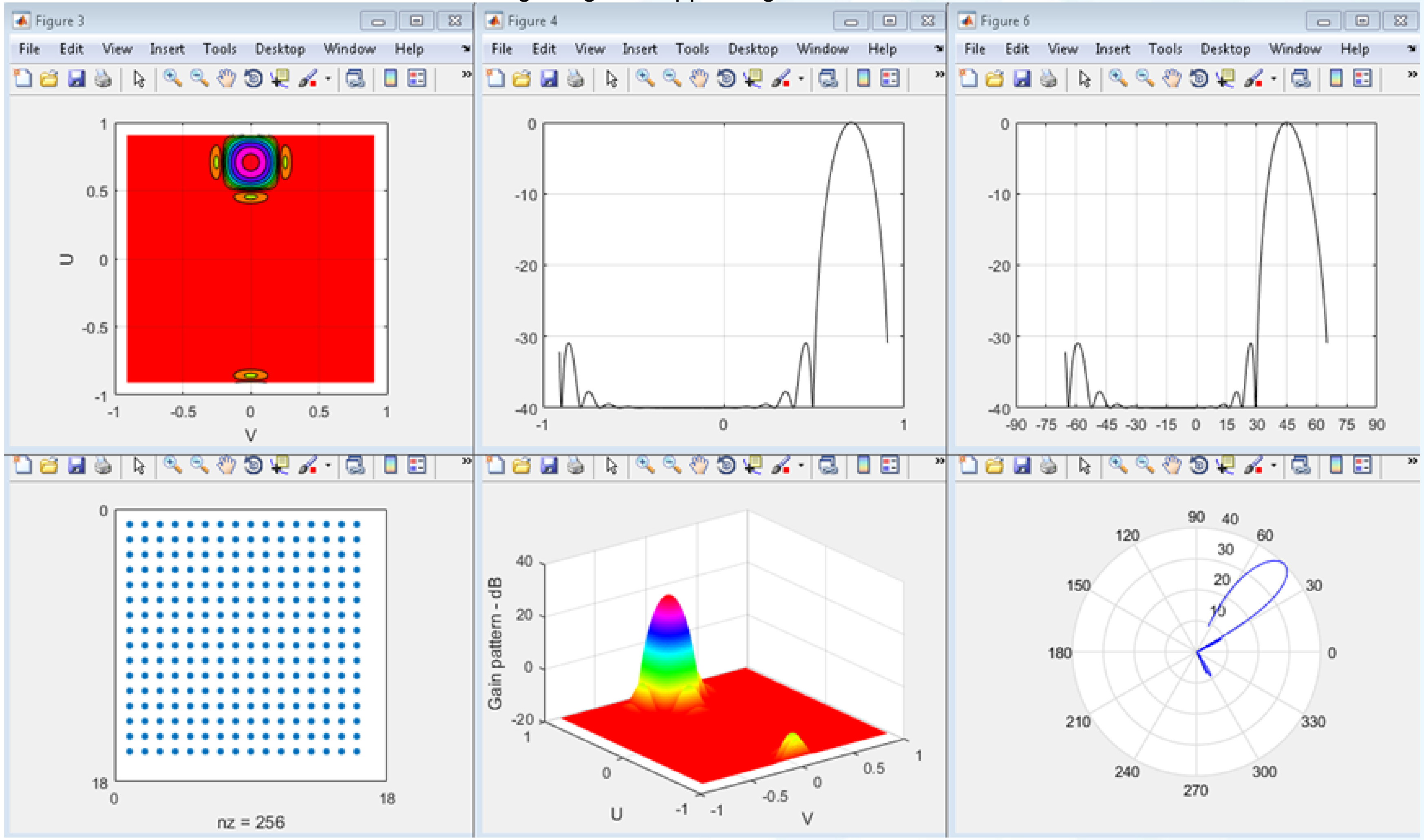
Spatial Aliasing – Grating Lobes - 30°



Element spacing = 0.55λ

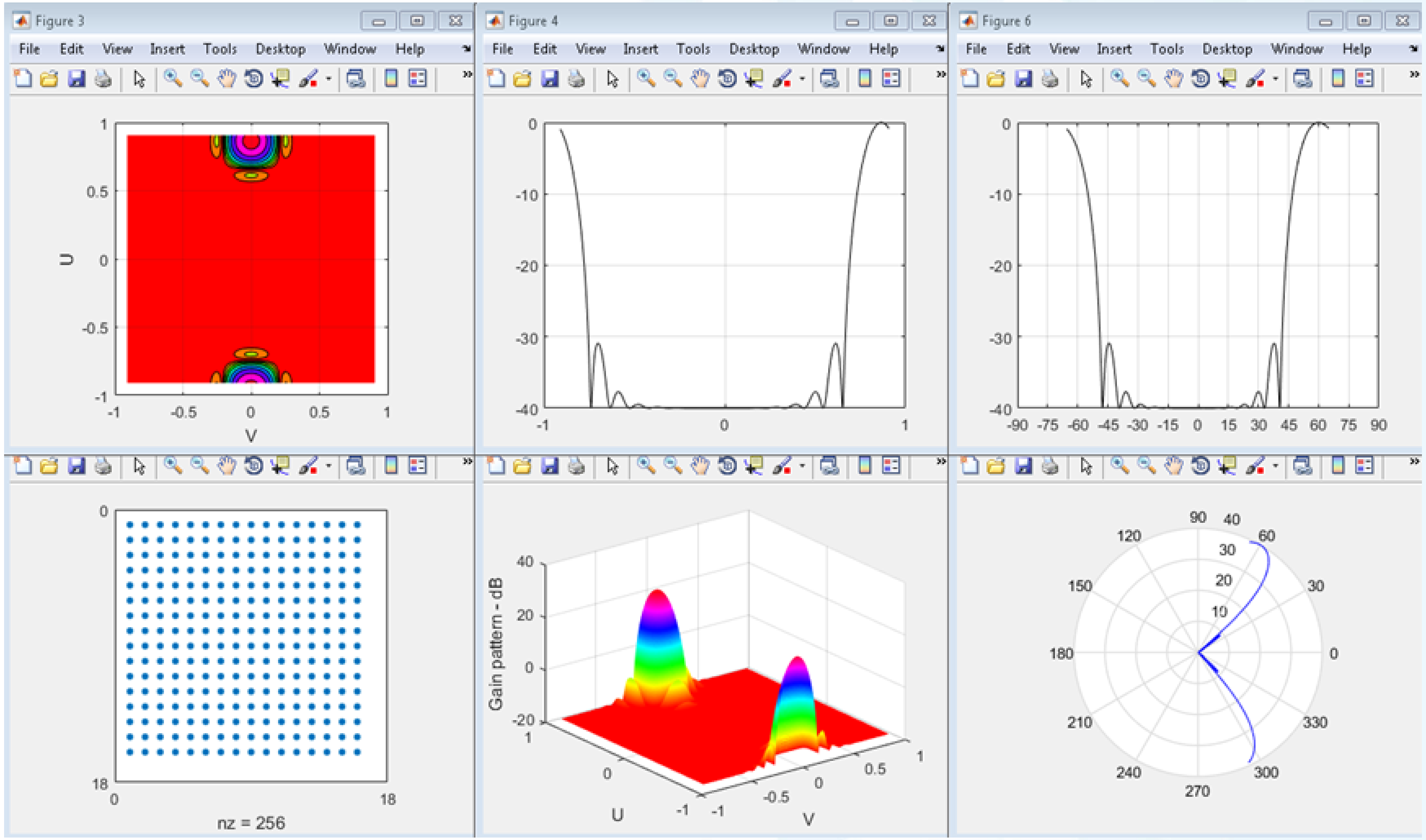
Spatial Aliasing – Grating Lobes - 45°

grating lobe appearing on horizon



Element spacing = 0.55λ

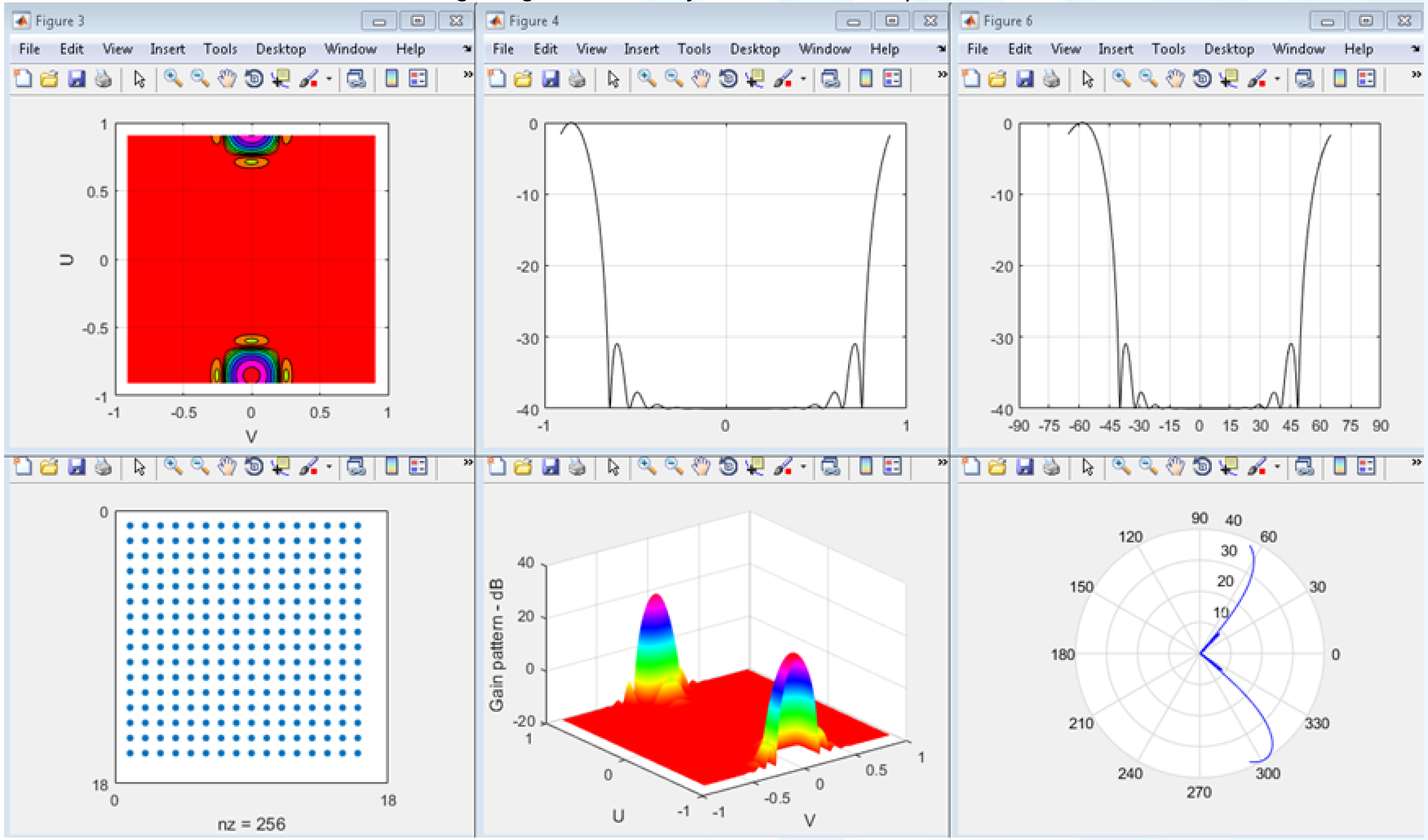
Spatial Aliasing – Grating Lobes - 60°



Element spacing = 0.55λ

Spatial Aliasing – Grating Lobes - 75°

grating lobe now fully in antenna hemisphere

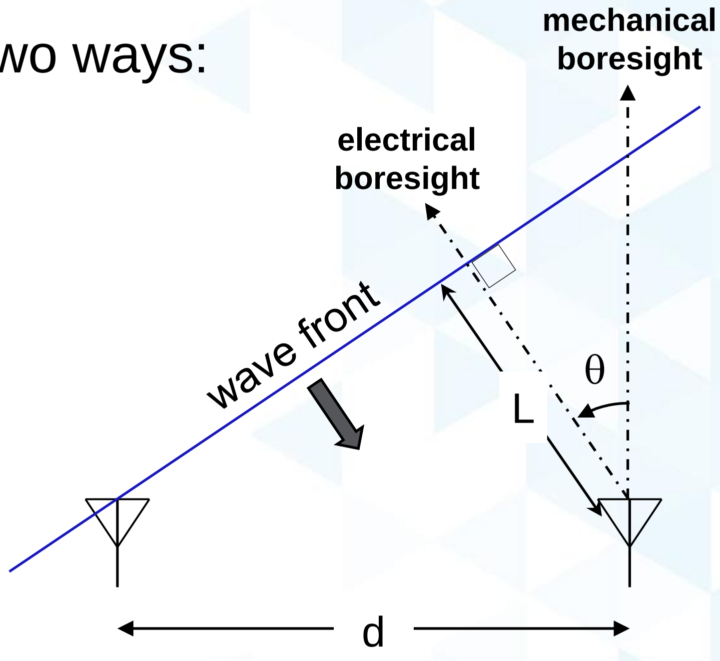


Element spacing = 0.55λ

Understanding Beam Squint

Understanding Beam Squint: Math and Theory

- The same delay can be solved for in two ways:
  1. As a time delay:
     $\theta = \sin^{-1}(\Delta t\, c / d)$
     This **is not** a function of frequency
  2. As a phase delay:
     $\theta = \sin^{-1}(\Delta\phi\, c / (2\pi f d))$
     This **is** a function of frequency



From trig:
$L = d \sin\theta$

For time delay between elements:
$\Delta t = L / c$
which can be simplified…
$\Delta t = d \sin\theta / c$
Solving for θ:
$\theta = \sin^{-1}(\Delta t\, c / d)$

For phase shift between elements:
$\Delta\phi = 2\pi L / \lambda = 2\pi f L / c$
which can be simplified…
$\Delta\phi = 2\pi f d \sin\theta / c$
Solving for θ:
$\theta = \sin^{-1}(\Delta\phi\, c / (2\pi f d))$

θ - beam electrical angle
Δt - incremental time delay between elements
Δϕ - incremental phase shift between elements
L - incremental propagation distance between elements
d - distance between elements
C - speed of light 3×10^{8} m/s

Note that the electrical steering angle is not a function of frequency when time delay is employed.

If phase shift is employed, the beam moves and de-focusses, or "squints", as a function of frequency

Understanding Beam Squint: Math and Theory

- Let's consider an example:
  - 10 GHz Array, with d=15mm
  - Array is 100 by 100 elements
  - Therefore the distance between two elements on opposite sides is 1.485m
  - We want to steer the beam to +/- 60º from mechanical boresight
- What is the delay between those two elements?
  - $\Delta t = d \sin(\theta) / c = 1.486\text{m} \sin(60º) / (3\text{E}8\text{m/s}) = 4.29$ ns
  - $\Delta\phi = 2\pi f d \sin(\theta) / c = 2\pi\ 10\text{E}9\text{Hz}\ 1.486\text{m} \sin(60) / (3\text{E}8\text{m/s}) = 270$ rad
- Now calculate the beam angle:
  - $\theta = \sin^{-1}(\Delta t\, c / d) = \sin^{-1}(4.29\text{E-}9\ 3\text{E}8 / 1.486\text{m}) = 60º$
  - $\theta = \sin^{-1}(\Delta\phi\, c / (2\pi f d)) = \sin^{-1}(270 \text{ rad } 3\text{E}8 / (2\pi\ 10\text{E}9\ 1.486\text{m}) = 60º$
- They match! As expected. But what if our signal BW is 1 GHz (9-10GHz signal)?
- Change f to 9 GHz, but keep the same beam weights (phase delta):
  - $\theta = \sin^{-1}(270 \text{ rad } 3\text{E}8 / (2\pi\ 9\text{E}9\ 1.486\text{m}) =$ **74.6º !!!!**
- That is beam squint:
  - the defocusing of the beam across frequency as you move away from mechanical boresight
  - A true time delay system doesn't have that problem because it doesn't have that frequency dependence.

Understanding Beam Squint: Math and Theory

ANALOG DEVICES
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Management of Beam Squint

- In an array employing phase shifters, there is a tradeoff between
  - $\Delta\theta$ – maximum tolerable squint angle (radians)
  - θ_{MAX} – maximum beam angle (radians)
  - f_0 – carrier frequency (Hz)
  - Δf – instantaneous bandwidth (Hz)

such that

$$\Delta f = \frac{\Delta\theta\, f_0}{\tan\theta_{MAX}}$$

System	$f(\Delta\theta\,,$	$\theta_{MAX}\,,$	$f_0\,)$ =	Δf	Comments
Ground Radar	0.2°	80°	10GHz	6.2MHz	OK for beam, TTD enables WB null steering
Airborne Radar	0.3°	80°	38GHz	35.1MHz	TTD enables advanced waveforms
5G	3°	60°	30GHz	907MHz	PS OK
GEO	0.2°	8°	30GHz	745MHz	TTD enables ↑ digital beams, capacity
LEO	1°	70°	30GHz	190MHz	TTD enhances data capacity
Airborne SatCom	0.6	70°	30GHz	114MHz	TTD enhances data capacity
EW	1°	60°	10GHz	100MHz	Need >>1GHz, not possible with PS

Understanding Beam Squint: Math and Theory

- In our lab, we have 4 elements
  - so max distance is 3*15mm = 0.045 m
  - Let's set our nominal frequency to be 10 GHz
  - We want to steer the beam to +/- 45º from mechanical boresight

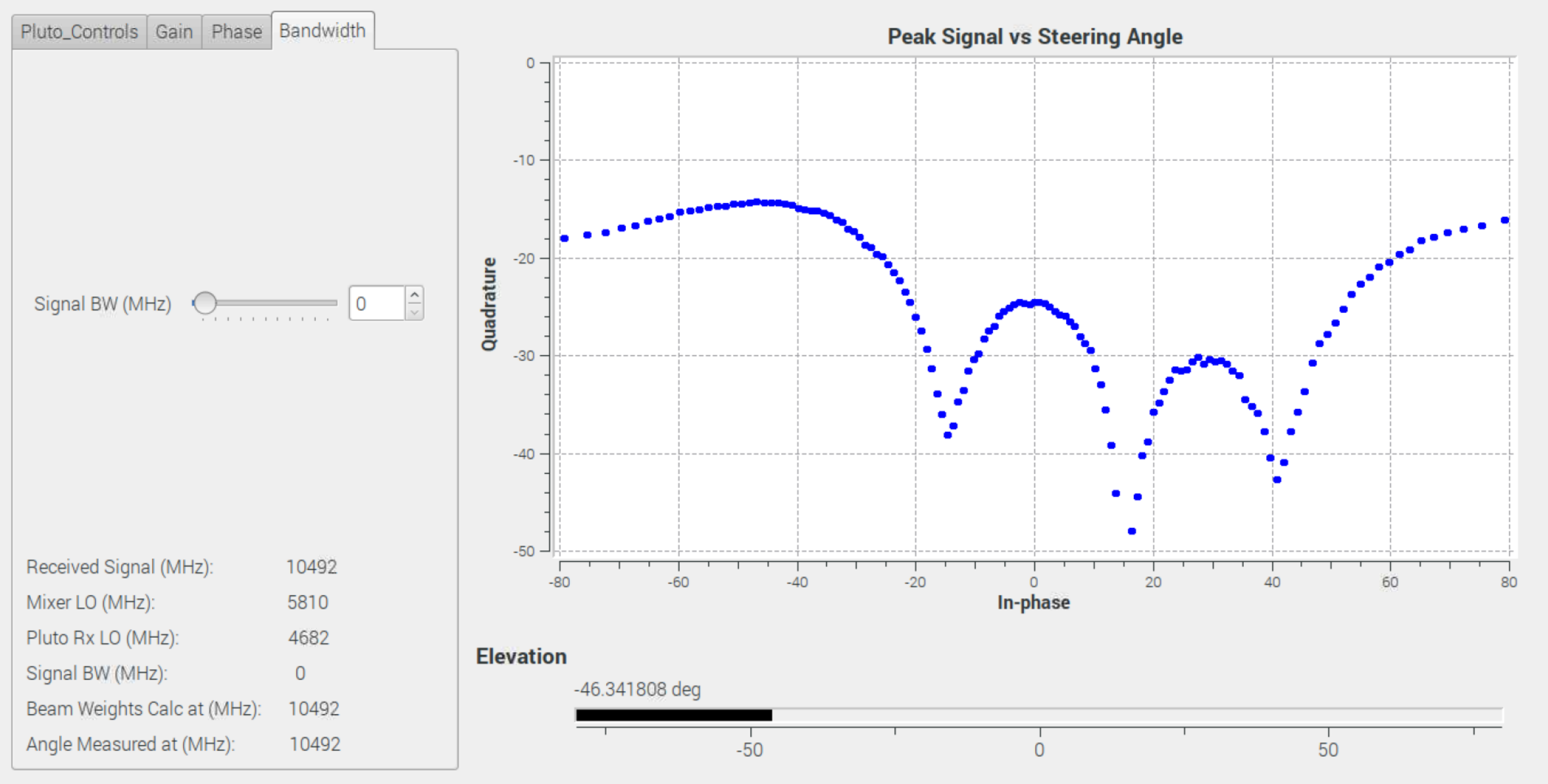
  - $\Delta\phi = 2\pi$ f d sin(θ) / c = 2π 10E9Hz 0.045m sin(45) / (3E8m/s) = 6.66 rad

- For f=9.5GHz:
  - θ = $\sin^{-1}$(6.66 rad 3E8 / (2π 9.5E9 0.045) = 0.839 rad = 48.06º (the beam shifts by 3.06º)

- Or: $\Delta \boldsymbol{f} = \dfrac{\Delta\boldsymbol{\theta}\,\boldsymbol{f_0}}{\tan \boldsymbol{\theta_{MAX}}}$ → 500MHz = $\dfrac{\Delta\boldsymbol{\theta}\ \mathbf{9.5}\ \boldsymbol{GHz}}{\tan(45\boldsymbol{deg})}$ → $\Delta\boldsymbol{\theta}$ = 0.0526 rad = 3.02º

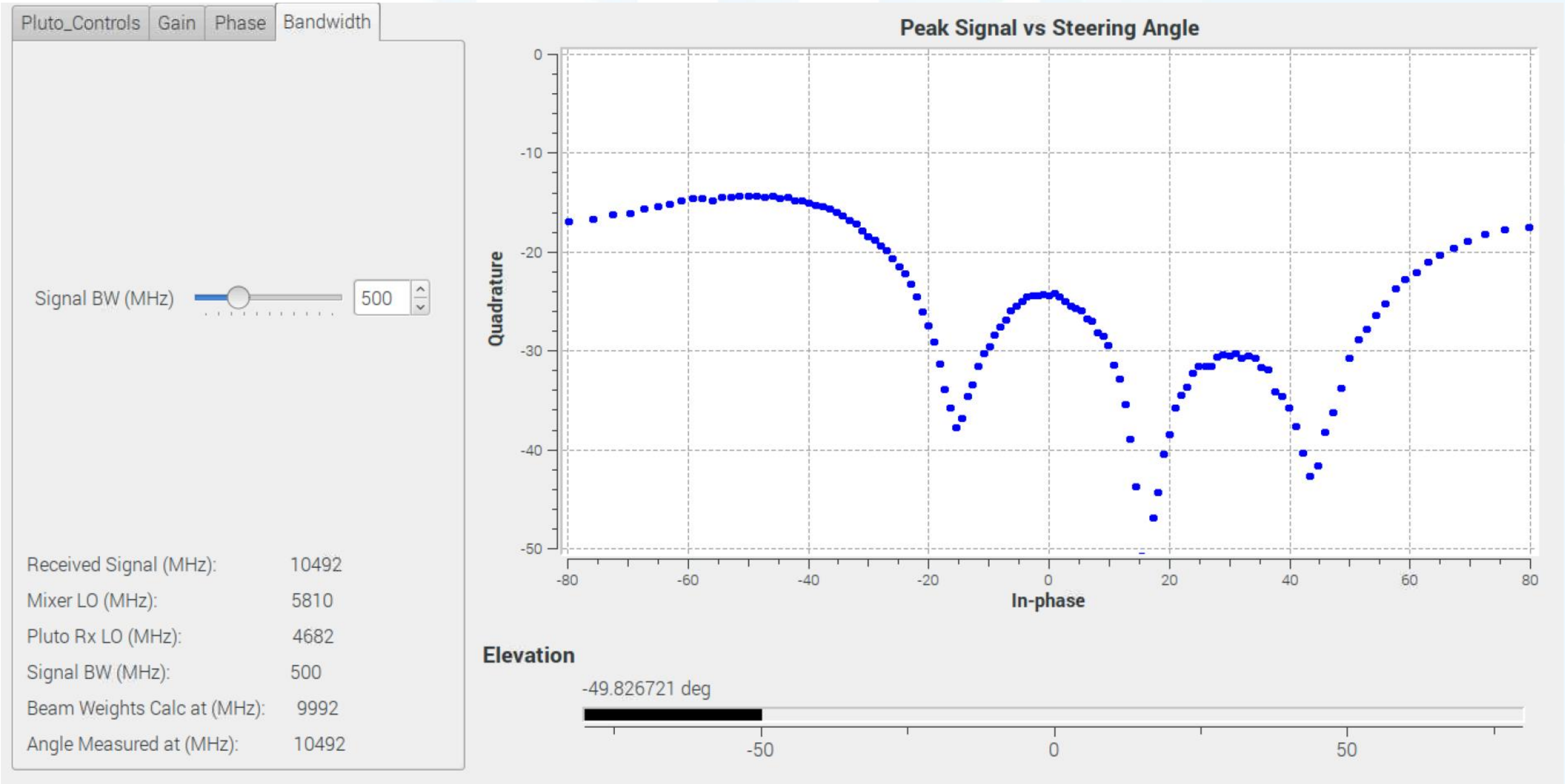
Understanding Beam Squint: Lab

- Return to “Exercise 4”
- Set theta to about 45 deg
- Click on the “Bandwidth” tab
- With BW = 0 MHz:
  - We are calculating the angle for just a narrow band sine wave



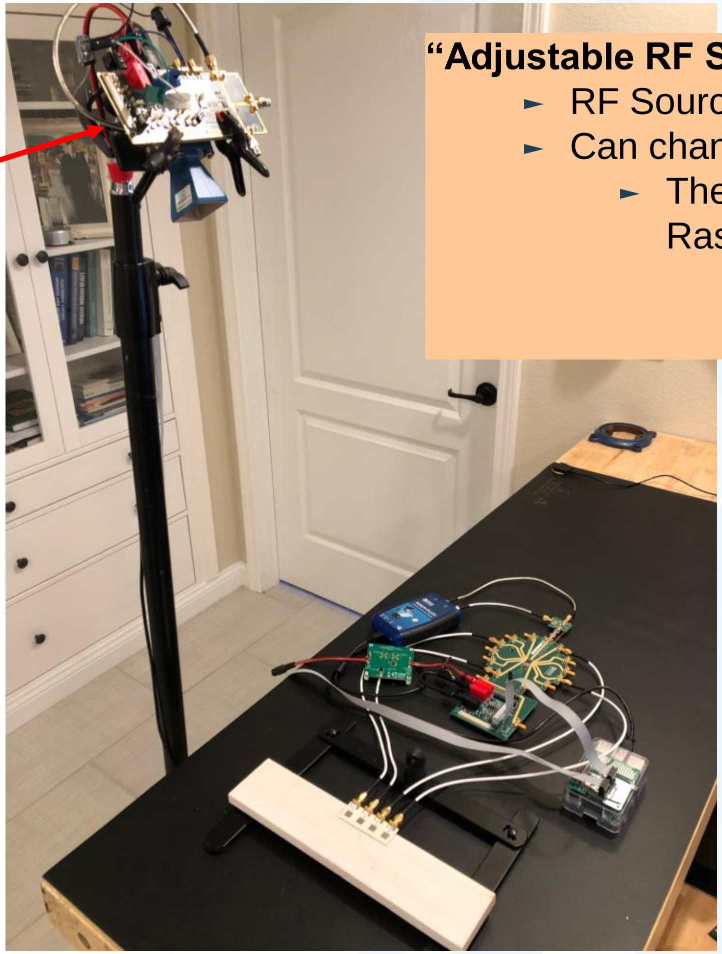
Understanding Beam Squint: Lab

- Now change the signal BW to 500MHz
- This recalculates the angles for:
  - Signal – 500MHz
  - We are now looking at the furthest corner of our bandwidth
- The new angle has changed!
  - The change is:
  - -46.3 – (-49.8) = 3.5º
  - Just about what we calculated!



Understanding Beam Squint: Lab

- Here's another way to measure it:
- We could also look at beam squint if we had a programmable frequency source.
- The ADF5356 is just such a source!

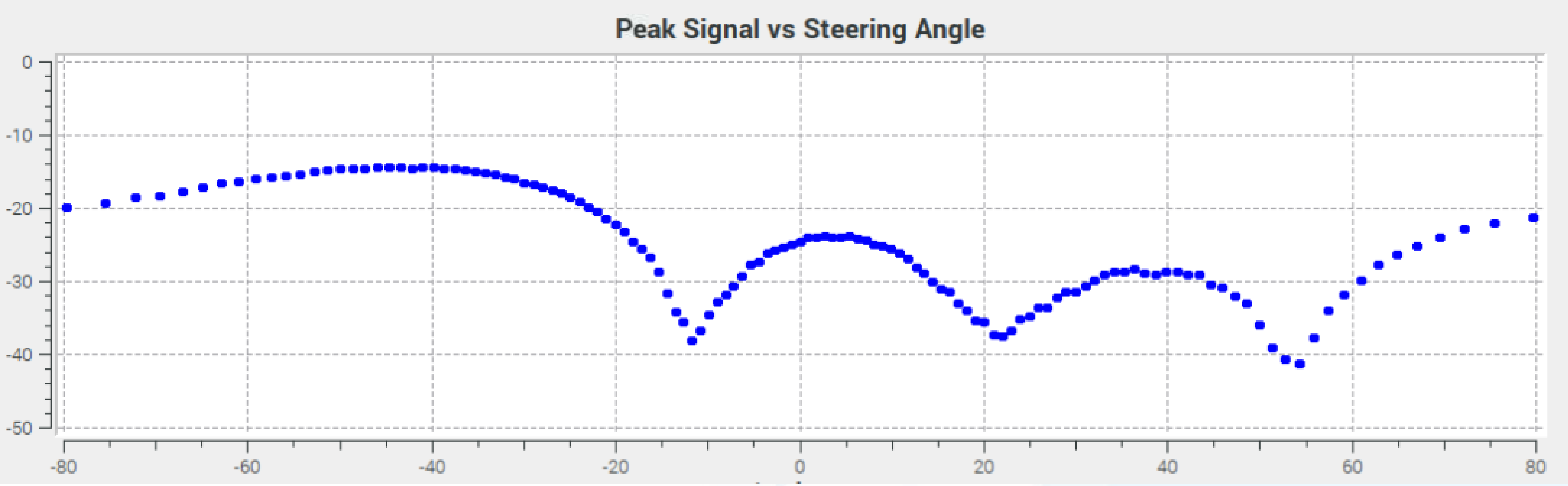


"Adjustable RF Source" Setup

- RF Source is the ADF5356
- Can change freq from 9.5 to 11 GHz
  - The second ribbon cable from Rasp Pi controls the ADF5356 SPI

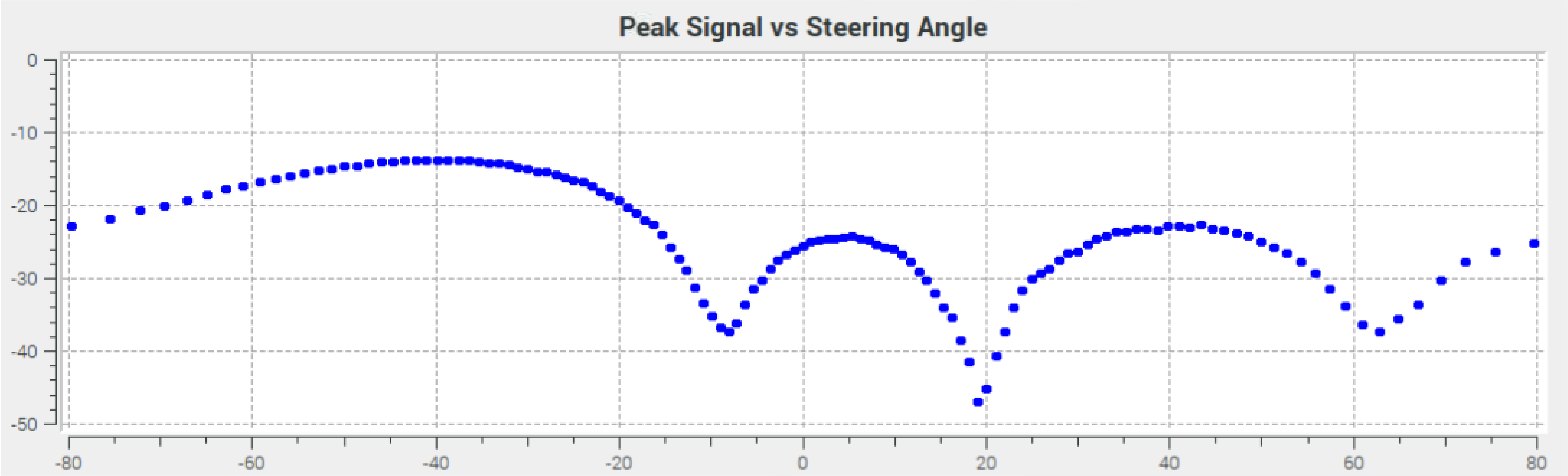
Understanding Beam Squint: Lab

- f = 10.5 GHz, 43.8º (at 10 GHz):
  - Peak at 44º (delta of 4º)



Understanding Beam Squint: Lab

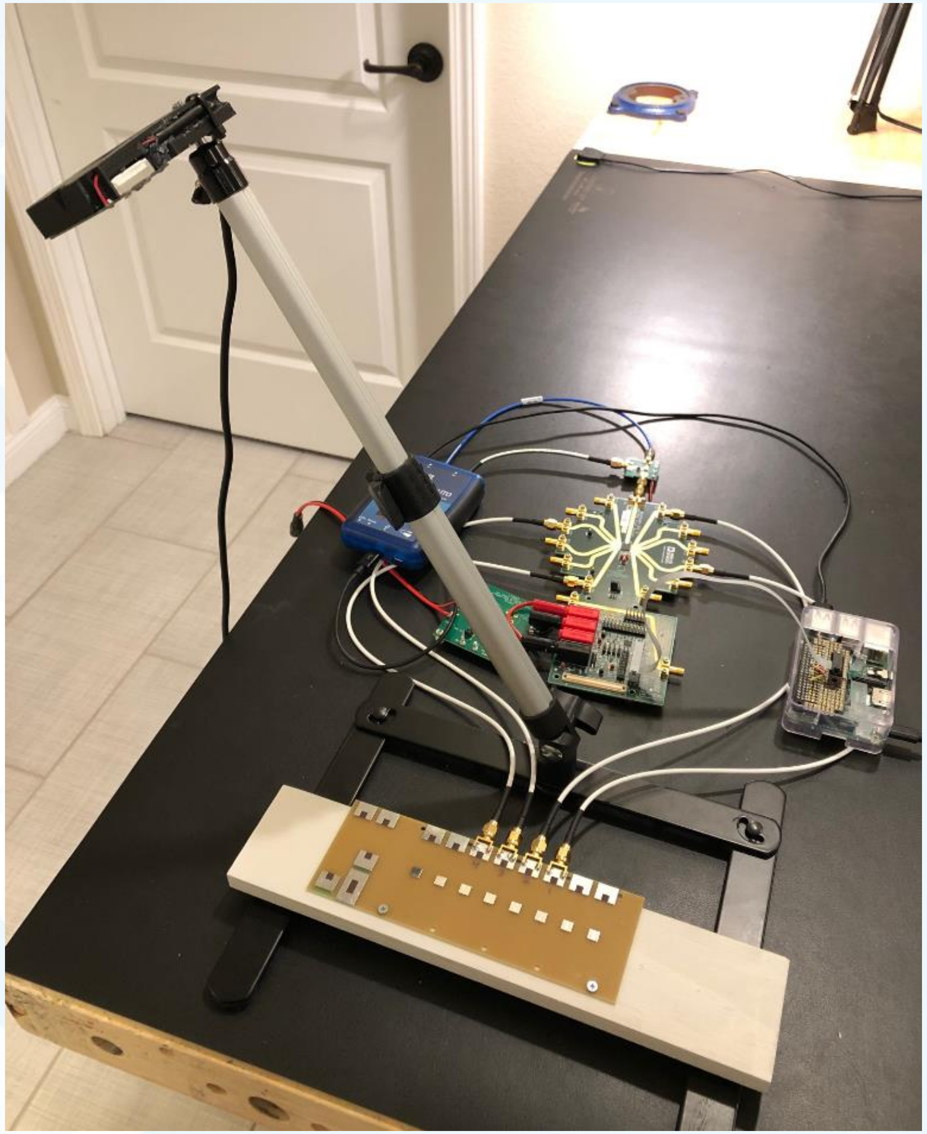
- f = 10 GHz, 40º:
  - Peak at 40º



Lab Wrap Up

How to Duplicate this Setup

- You can make your own setup
  - Keep learning and experimenting!
- Complete instructions, BOM, etc. are here:
  - www.github.com/JonKraft/PhasedArray
- The Antenna is the hardest part to find.
  - Ozan Gurbuz made an antenna just for this lab
    - Gerber files at the link above
  - You can also order the layout directly from PCBWAY.com
    - https://www.pcbway.com/project/shareproject/10_5GHz__X_Band__Patch_Antenna.html



Sources for More Information

- Phased Array Antenna Handbook
  - Robert J. Mailloux
  - https://pdfs.semanticscholar.org/2a93/5a6beae90d9f30e1cf1ef5c17b168456e1b0.pdf

- ECE422, Radio and Microwave Wireless Systems
  - Dr. Sean Hum, Univ of Toronto
  - http://www.waves.utoronto.ca/prof/svhum/ece422/notes/15-arrays2.pdf

- ECE753, Modern Antennas in Wireless Telecommunications
  - Dr. Natalia Nikolova, McMaster University
  - https://www.ece.mcmaster.ca/faculty/nikolova/antenna_dload/current_lectures/L13_Arrays1.pdf