Problem Set 5: Computation of Dynamic Games

Due date: December 22nd, 2021

Consider the following dynamic game of capacity accumulation analyzed in Besanko and Doraszelski (RJE, 2004). Firms use Markov investment strategies in order to stochastically accumulate production capacity in order to improve their position in the product market. Let $\tilde{x}(\omega)$ denotes the investment strategy of firm 2 in state $\omega = (\bar{q}_1, \bar{q}_2)$. Firm 1 solves the following stochastic dynamic programing problem:

$$V_1^{\tilde{x}}(\omega) = \pi_1(\omega) + \max_{x_1 \geq 0} \left\{ -x_1 + \beta \sum_{\bar{q}_1'} W_1^{\tilde{x}}(\bar{q}_1'|\omega) \Pr(\bar{q}_1'|\bar{q}_1, x_1) \right\}$$

where $W_1^{\tilde{x}}(\bar{q}_1) = \sum_{\bar{q}_2'} V_1^{\tilde{x}}(\bar{q}_1, \bar{q}_2) \Pr(\bar{q}_2'|\bar{q}_2, \tilde{x}_2)$. Capacity evolves over time as a controlled markov process:

$$\Pr(\Delta\bar{q}_{1t}|\bar{q}_{1t}, x_{1t}) = \begin{cases} \frac{(1-\delta)\alpha x_{1t}}{1+\alpha x_{1t}} & \text{If } \Delta q_{1t} = 1, \\ \frac{(1-\delta)}{1+\alpha x_{1t}} + \frac{\delta \alpha x_{1t}}{1+\alpha x_{1t}} & \text{If } \Delta q_{1t} = 0, \\ \frac{\delta}{1+\alpha x_{1t}} & \text{If } \Delta q_{1t} = -1 \end{cases}$$

where δ is the capital depreciation parameter. Firm 2 faces a symmetric markov process. The model is parametrized as follows:

- Homogenous products: $Q(P) = a - bQ$.
- Zero marginal cost
- Heterogeneous (discrete) capacities:
$$0 \leq q_i \leq \bar{q}_i$$
and industry structure: $\omega_t = \{\bar{q}_{1t}, \bar{q}_{2t}\}$.
- Capacity grid: $\bar{q} \in \{0, 5, 10, \ldots 45\}$
- Parameter values:
$$\delta = 0.1, \beta = 1/1.05, \alpha = 0.06, a = 40, b = 10$$

1. Define the relevant state space assuming that firms use symmetric strategies.
2. Assume that firms are competing in quantity in the product market (i.e. Cournot). Solve the Nash equilibrium profit and quantities for each state ω. Tabulate the profits.

3. Define the Markov-perfect equlibrium.

4. Solve the Markov-perfect equilibrium investment strategies. Plot the strategy surface (see Figure 2 in paper). Use the equilibrium strategy to construct the transition probability matrix for the industry state: $Q(\omega'|\omega)$.

5. Starting from state $\omega = (0, 0)$, simulate the evolution of the industry over 25 years by sampling states from $Q(\omega'|\omega)$. Repeat this process $1,000$ times, and record the frequency of each state at $T = 25$. Plots this joint probability distribution over $\bar{q}_{1,25}$ and $\bar{q}_{2,25}$ (as in Figure 4 in the paper).