# ECON 899b Problem Set 5

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There are two attached Julia files:

- "ps05_functions.jl" contains all functions needed to solve the firms' static and dynamic problems, compute the transition matrix $Q(\omega'|\omega)$, and simulate the evolution of the industry.
- "ps05_script.jl" solves the model, simulates the evolution of the industry, and plots the results.

## 1 Question 1

**Define the relevant state space assuming that firms use symmetric strategies.**

Assume firms play symmetric strategies:

$$x_i(\omega_i, \omega_{-i}) = x_j(\omega_i, \omega_{-i})$$
$$q_i(\omega_i, \omega_{-i}) = q_j(\omega_i, \omega_{-i})$$

This implies payoffs are symmetric:

$$\pi_i(\omega) = (a - bq_i(\omega) - bq_j(\omega))q_i(\omega) = (a - bq_j(\omega) - bq_i(\omega))q_j(\omega) = \pi_j(\omega)$$

Since there are only $n = 2$ players, strategies and payoffs are also anonymous (trivially).

Therefore, the relevant industry state space is

$$S = \{(\omega_1, \omega_2) : \omega_i \in \Omega, \omega_1 > \omega_2\}$$

where $\Omega$ is the productivity grid. In other words, WLOG, we can consider all ordered 2-tuples of productivity levels, where the first element is the higher productivity level.

## 2 Question 2

**Assume that firms are competing in quantity in the product market (i.e. Cournot). Solve the Nash equilibrium profit and quantities for each state $\omega$. Tabulate the profits.**

See Table 1 and Figure 1.

*Referenced code from Alex von Hafften and John Higgins.

## 3 Question 3

**Define the Markov perfect equilibrium.**

A Markov perfect equilibrium is a pair of policy functions $(x_1(\omega), x_2(\omega))$ such that

1. The functions solve each firm's dynamic programming problem, given the perceived industry state transition probabilities $Q(\omega'|\omega)$
2. The perceived probabilities $Q(\omega'|\omega)$ are consistent with both firms' optimal actions

## 4 Question 4

**Solve the Markov-perfect equilibrium investment strategies. Plot the strategy surface (see Figure 2 in paper). Use the equilibrium strategy to construct the transition probability matrix for the industry state: $Q(\omega'|\omega)$.**

Figure 2 plots the equilibrium investment strategies and value functions.

To construct the aggregate transition probability matrix, note that

$$\begin{aligned} Q(\omega'|\omega) &= \Pr\Big((\bar{q}_1', \bar{q}_2')\Big|(\bar{q}_1, \bar{q}_2)\Big) \\ &= \Pr\Big(\bar{q}_1'\Big|\bar{q}_1, x_1^*(\bar{q}_1, \bar{q}_2)\Big) \times \Pr\Big(\bar{q}_2'\Big|\bar{q}_2, x_2^*(\bar{q}_1, \bar{q}_2)\Big) \end{aligned}$$

## 5 Question 5

**Starting from state $\omega = (0, 0)$, simulate the evolution of the industry over 25 years by sampling states from $Q(\omega'|\omega)$. Repeat this process 1,000 times, and record the frequency of each state at $T = 25$. Plot this joint probability distribution over $\bar{q}_{1,25}$ and $\bar{q}_{2,25}$ (as in Figure 4 in the paper).**

Figure 3 plots the joint distribution over $(\bar{q}_{1,25}, \bar{q}_{2,25})$.

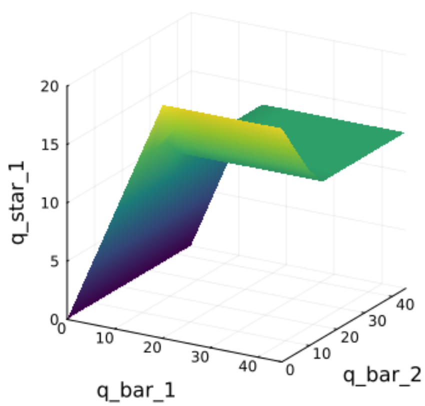
The long-run industry structure tends to be symmetric: both firms always end with capacity of 20 or less, and the mass is concentrated where firms have similar capacity levels. This matches the results in Besanko and Doraszelski (2004).

## Table 1: Cournot-Nash Quantities and Static Payoffs

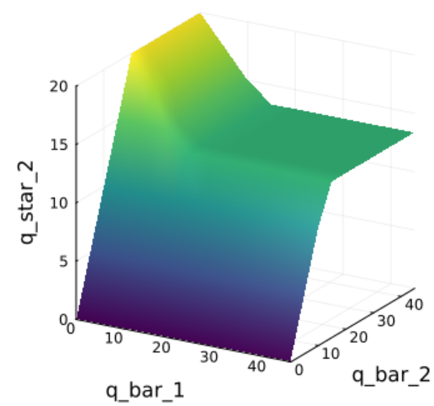
| Industry State $\omega$ | | N.E. Strategies | | Static Payoffs | |
|---|---|---|---|---|---|
| $\bar{q}_1$ | $\bar{q}_2$ | $q_1^\star$ | $q_2^\star$ | $\pi_1$ | $\pi_2$ |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5.0 | 0.0 | 5.0 | 0.0 | 17.5 | 0.0 |
| 5.0 | 5.0 | 5.0 | 5.0 | 15.0 | 15.0 |
| 10.0 | 0.0 | 10.0 | 0.0 | 30.0 | 0.0 |
| 10.0 | 5.0 | 10.0 | 5.0 | 25.0 | 12.5 |
| 10.0 | 10.0 | 10.0 | 10.0 | 20.0 | 20.0 |
| 15.0 | 0.0 | 15.0 | 0.0 | 37.5 | 0.0 |
| 15.0 | 5.0 | 15.0 | 5.0 | 30.0 | 10.0 |
| 15.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 15.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 20.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 20.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 20.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 20.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 20.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 25.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 25.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 25.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 25.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 25.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 25.0 | 25.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 30.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 30.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 30.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 30.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 30.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 30.0 | 25.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 30.0 | 30.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 35.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 35.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 35.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 35.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 35.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 35.0 | 25.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 35.0 | 30.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 35.0 | 35.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 40.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 40.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 40.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 25.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 30.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 35.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 40.0 | 40.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 0.0 | 20.0 | 0.0 | 40.0 | 0.0 |
| 45.0 | 5.0 | 17.5 | 5.0 | 30.625 | 8.75 |
| 45.0 | 10.0 | 15.0 | 10.0 | 22.5 | 15.0 |
| 45.0 | 15.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 20.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 25.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 30.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 35.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 40.0 | 13.333 | 13.333 | 17.778 | 17.778 |
| 45.0 | 45.0 | 13.333 | 13.333 | 17.778 | 17.778 |

**Figure 1:** Cournot Equilibria

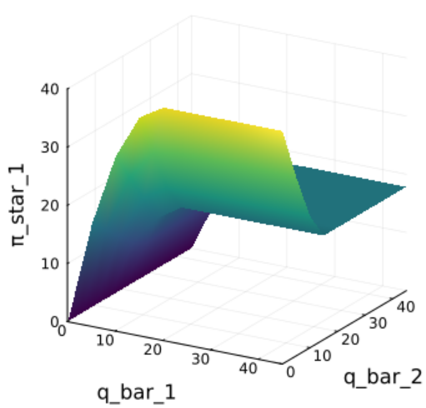
**(a)** Firm 1's Cournot Quantity $q_1^*(\omega)$




**(b)** Firm 2's Cournot Quantity $q_2^*(\omega)$




**(c)** Firm 1's Static Payoff $\pi_1^*(\omega)$




**(d)** Firm 2's Static Payoff $\pi_2^*(\omega)$

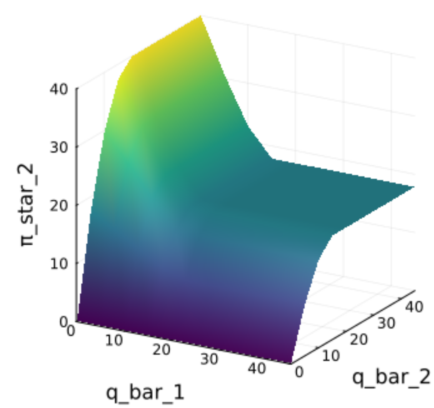
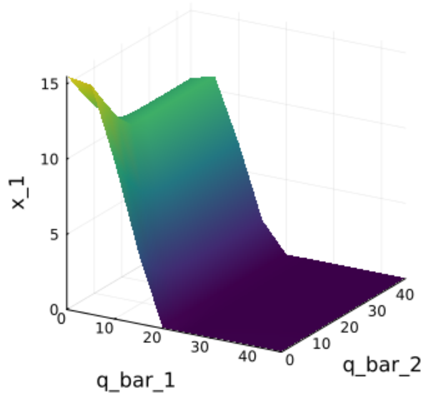
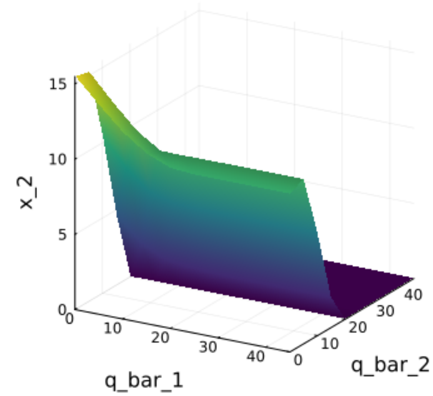


**Figure 2:** Equilibrium Investment Strategies and Value Functions

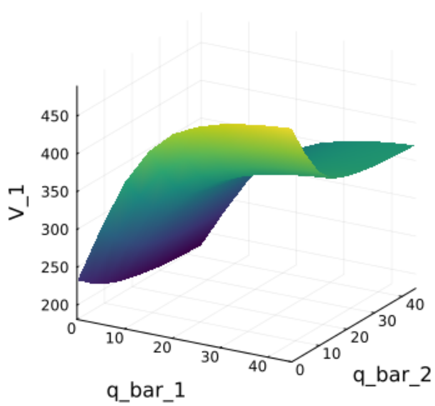
**(a)** Firm 1's Investment Strategy $x_1^*(\omega)$




**(b)** Firm 2's Investment Strategy $x_2^*(\omega)$




**(c)** Firm 1's Value Function $V_1(\omega)$




**(d)** Firm 2's Value Function $V_2(\omega)$

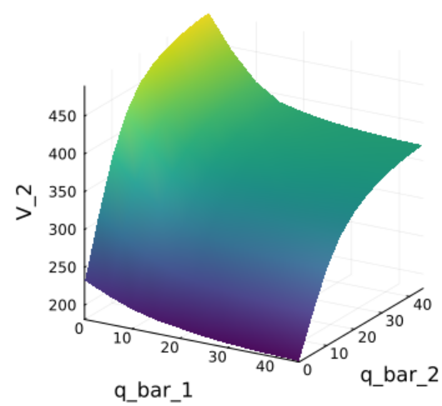


**Figure 3:** Distribution of Final Industry State $\omega_{25}$