ECON899: Computational Economics
Fall 2022, University of Wisconsin
Instructor: Dean Corbae

Problem Set #1 - Goal Due Date 9/14/22

This problem introduces you to dynamic programming in an infinite horizon growth model on the computer. In particular, you are to modify the programs that I have placed on my website called:

- vfigrowth07.m (matlab code),
- hw2ns_pablo.f95 (fortran code),
- HW2ns_parallel.f90 (fortran code to do parallel processing),
- 02Growth_compute.jl and 02Growth_model.jl (julia code - see Readme_Garrett).

These programs use value function iteration to solve for the decision rule $K_{t+1}(K_t)$ in a non-stochastic setting. You are to modify these programs to add uncertainty over technology shocks in matlab or julia and in fortran. Then you are to compare the savings in speed between matlab or julia, fortran, and parallel processing. An introduction to these computer languages will be given by the teaching assistant, as well as a discussion of how to write programs that do parallel programming. A particularly helpful site for fortran is: http://www.cs.mtu.edu/~shene/COURSES/cs201/NOTES/fortran.html

Specifically, assume that households have log preferences, the production technology satisfies $Y_t = Z_t K_t^{\theta}$ where $\theta = 0.36$, and capital depreciates at rate $\delta = 0.025$. We will assume technology shocks follow a 2 state Markov Process. The transition matrix is calibrated to NBER business cycle data where we take an expansion to be an instance of a positive technology shock and recession to be an instance of a negative technology shock. The transition matrix is given by

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}.$$

where, for instance, $prob(Z_{t+1} = Z^g | Z_t = Z^g) = 0.977$.

As for modifying the program, you must expand the state space to add technology shocks from the set $\mathfrak{Z} = \{Z^g = 1.25, Z^b = 0.2\}$. I chose these values in order to satisfy that $\overline{Z} = 1$.To see this, note that Π implies an invariant distribution over the two states of $\overline{p}^g = 0.763$ and $\overline{p}^b = 0.237$ (check me). In that case, I chose $Z^g = 1.25$ and solved for Z^b in $\overline{Z} = \overline{p}^g Z^g + \overline{p}^b Z^b$.

1. State the dynamic programming problem.
2. Plot the value function over K for each state Z. Is it increasing (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$ for $K_{i+1} > K_i$)? Is it "concave" (in the sense that $V(K_{i+1}, Z) - V(K_i, Z)$ is decreasing)?
3. Is the decision rule increasing in K and Z (i.e. is $K'(K_{i+1}, Z) \geq K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z^g) \geq K'(K, Z^b)$)? Is saving increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)?