BIBB 585-401—Homework 7

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Problem 1

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\sigma'(z) = -(1+e^{-z})^{-2}\frac{d}{dz}(1+e^{-z})$$

$$\sigma'(z) = \frac{(e^{-z})}{(1+e^{-z})^2}$$

$$\sigma'(z) = \frac{1}{1+e^{-z}}\frac{(e^{-z})}{(1+e^{-z})}$$

$$\sigma'(z) = \sigma(z)\frac{(e^{-z})}{(1+e^{-z})}$$

$$Since \quad 1 - \frac{1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}} = 1 - \sigma(z)$$

$$\sigma'(z) = \sigma(z)(1-\sigma(z))$$

Problem 2

$$\Gamma_j^{\ell-1} = \sum_i \Gamma_i^\ell \frac{\partial y_i^\ell}{\partial y_j^{\ell-1}}$$

$$y_i^\ell = \sigma(\sum_k w_{ik}^\ell y_k^{\ell-1} + b_i^\ell)$$

$$Let\ u = (\sum_k w_{ik}^\ell y_k^{\ell-1} + b_i^\ell)$$

$$\frac{\partial y_i^\ell}{\partial y_j^{\ell-1}} = \frac{\partial \sigma}{\partial u} \quad \frac{\partial u}{\partial y_j^{\ell-1}}$$

$$\frac{\partial \sigma}{\partial u} = \sigma(u)(1-\sigma(u))$$

Realizing that only the jth term in the summation of k in u is dependent on y_j, we can say that:

$$\frac{\partial u}{\partial y_j^{\ell-1}} = w_{ij}^\ell$$

$$\frac{\partial y_i^\ell}{\partial y_j^{\ell-1}} = \sigma(u)(1-\sigma(u))w_{ij}^\ell$$

$$\frac{\partial y_i^\ell}{\partial y_j^{\ell-1}} = y_i^\ell(1-y_i^\ell)w_{ij}^\ell$$

$$\Gamma_j^{\ell-1} = \sum_i \Gamma_i^\ell \ y_i^\ell(1-y_i^\ell)w_{ij}^\ell$$

Problem 3

$$\Gamma_j^K = \frac{\partial L}{\partial y_j^K}$$

$$\Gamma_j^K = \frac{\partial}{\partial y_j^K}(\frac{1}{2}\sum_j (y_j^K - a_j)^2)$$

Again we can pull out just one term of the summation:

$$\Gamma_j^K = \frac{\partial}{\partial y_j^K}(\frac{1}{2}(y_j^K - a_j)^2)$$

$$\Gamma_j^K = y_j^K - a_j$$

Problem 4