Dynamic Models in Biology
Lab 6
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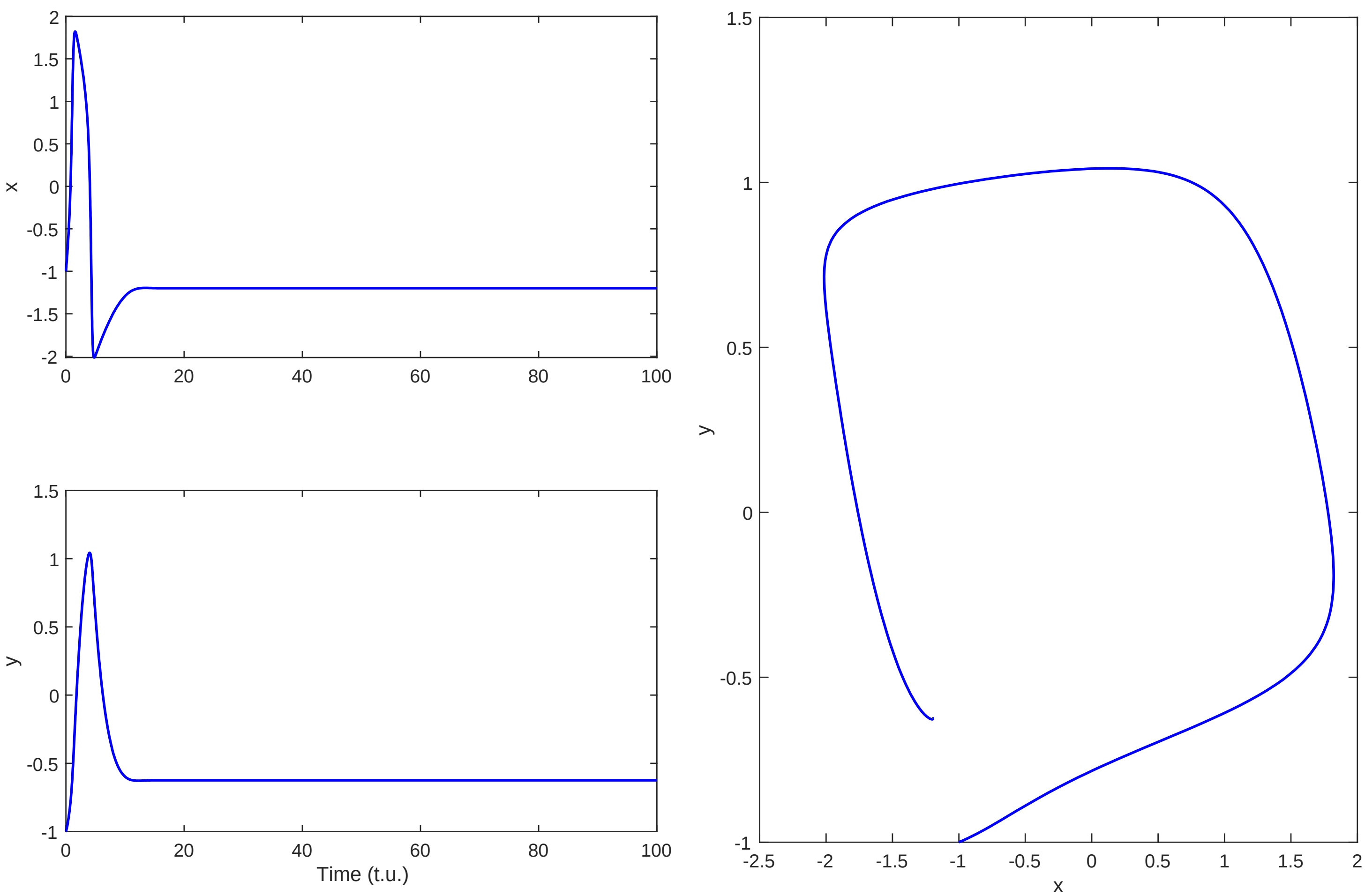
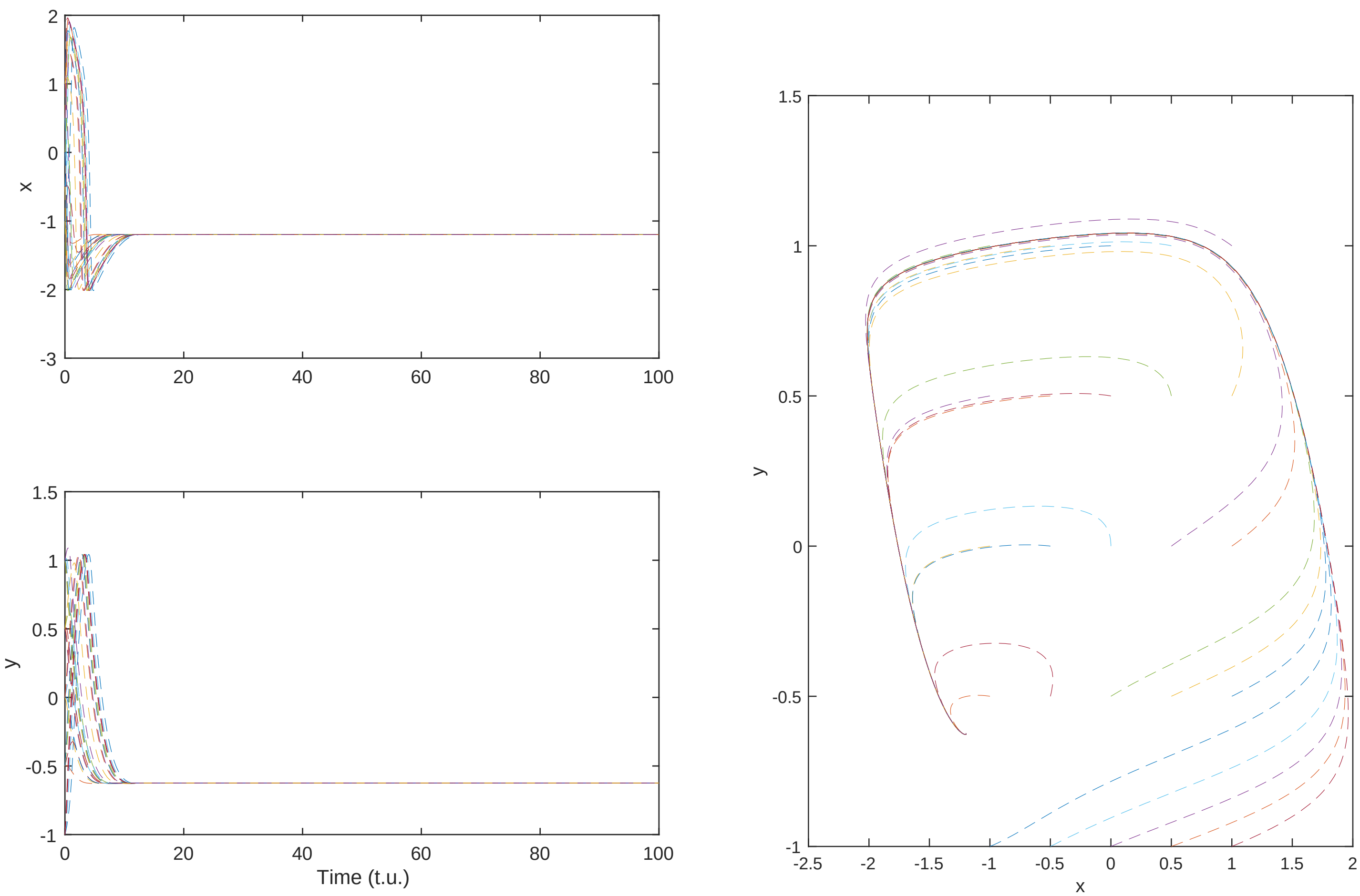
Baseline Run



Varying the initial conditions, we can see that no matter the starting conditions, both x and y reach a stable fixed point, in this case, a spiral it seems (based on the trajectories).



Fixed Point

We can find this exact fixed point and classify it:

$$\dot{x} = 3\left(x - \frac{x^3}{3} - y\right)$$

$$\dot{x} = 0 => y = \frac{-x^3}{3} + x$$

$$\dot{y} = \frac{1}{3}(x + 0.7 - 0.8y)$$

$$\dot{y} = 0 => y = \frac{1}{0.8}(x + 0.7)$$

These two nullclines intersect (intersection point found via wolframalpha) at the stable fixed point which is approximately at **(-1.2, -0.62)**, which I consistent with the traces shown on the previous page.

To classify it using stability analysis via the Jacobian:

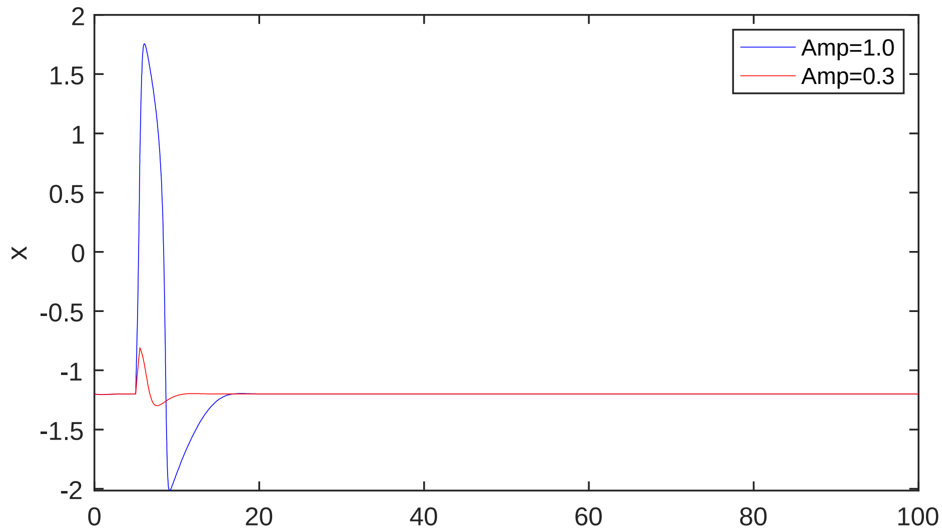
$$J = \begin{bmatrix} 3 - 3x^2 & -3 \\ \frac{1}{3} & \frac{-0.8}{3} \end{bmatrix}$$

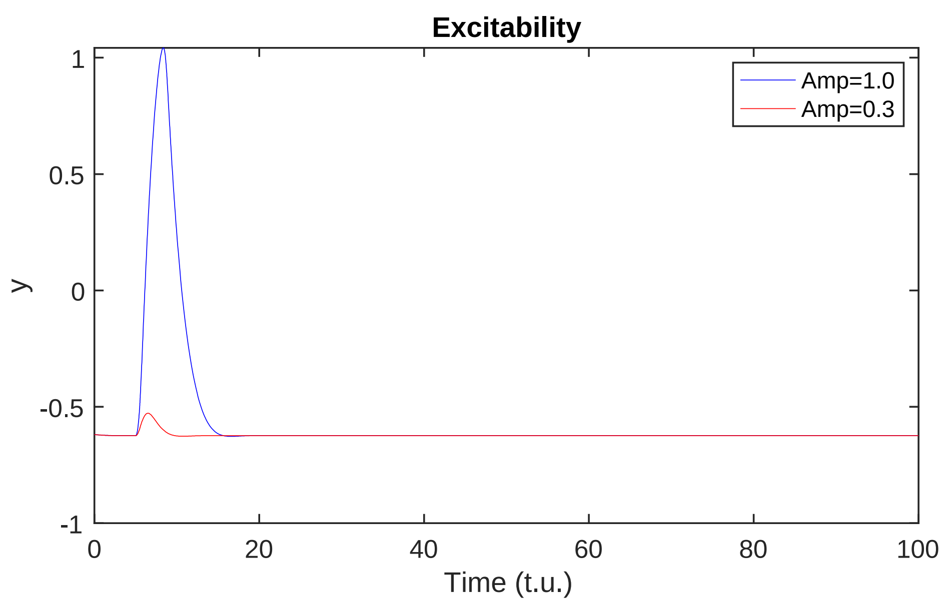
$$J(-1.2, -0.62) = \begin{bmatrix} -1.32 & -3 \\ \frac{1}{3} & \frac{-0.8}{3} \end{bmatrix}$$

Calculating the eigenvalues for this matrix (calculation not shown), one gets **a complex conjugate pair with negative real part**, corresponding to **a stable spiral point**, consistent with the simulation figures above.

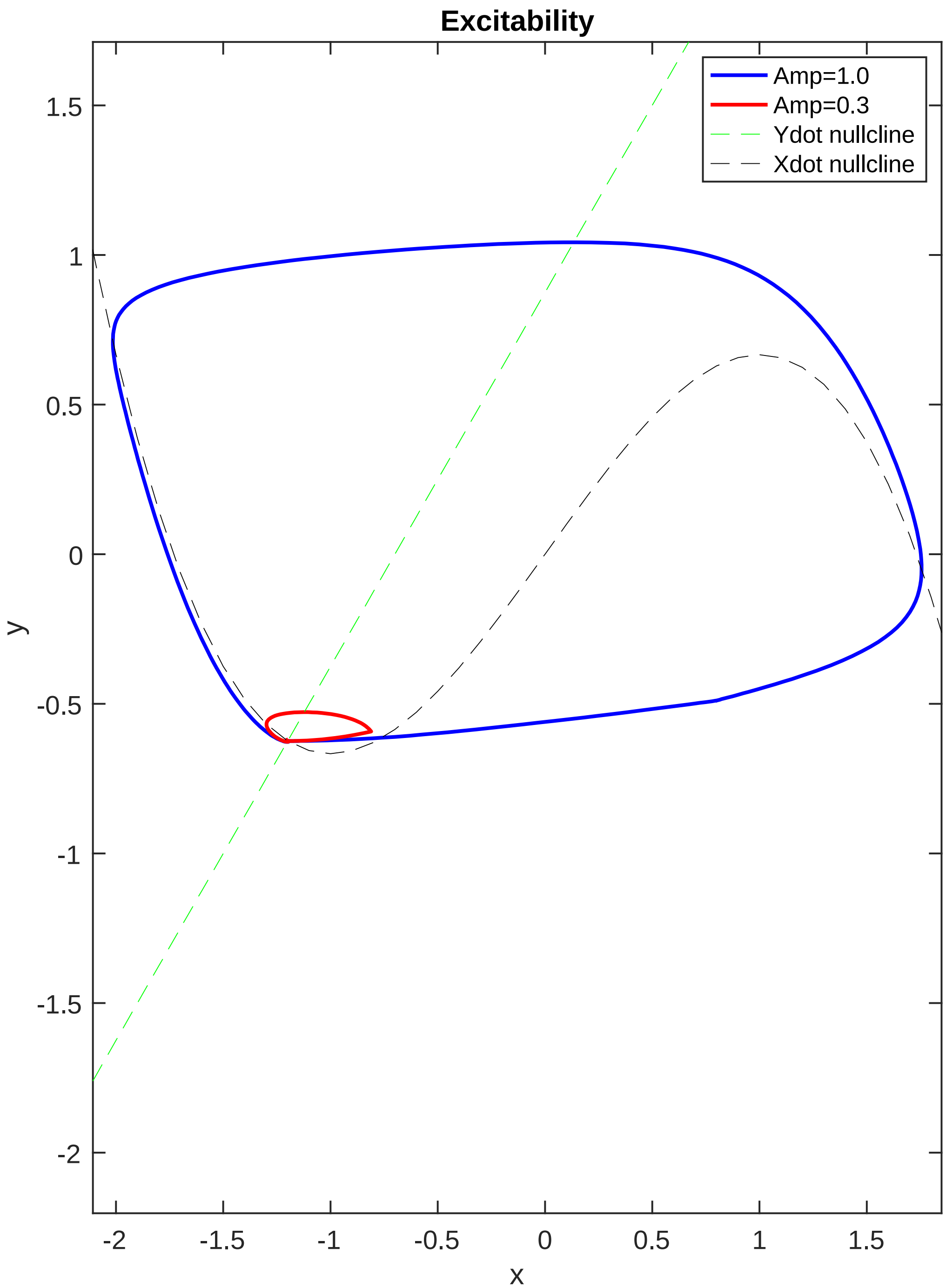
Excitability

You can see that different amplitude stimulations can lead to subthreshold or suprathreshold responses:

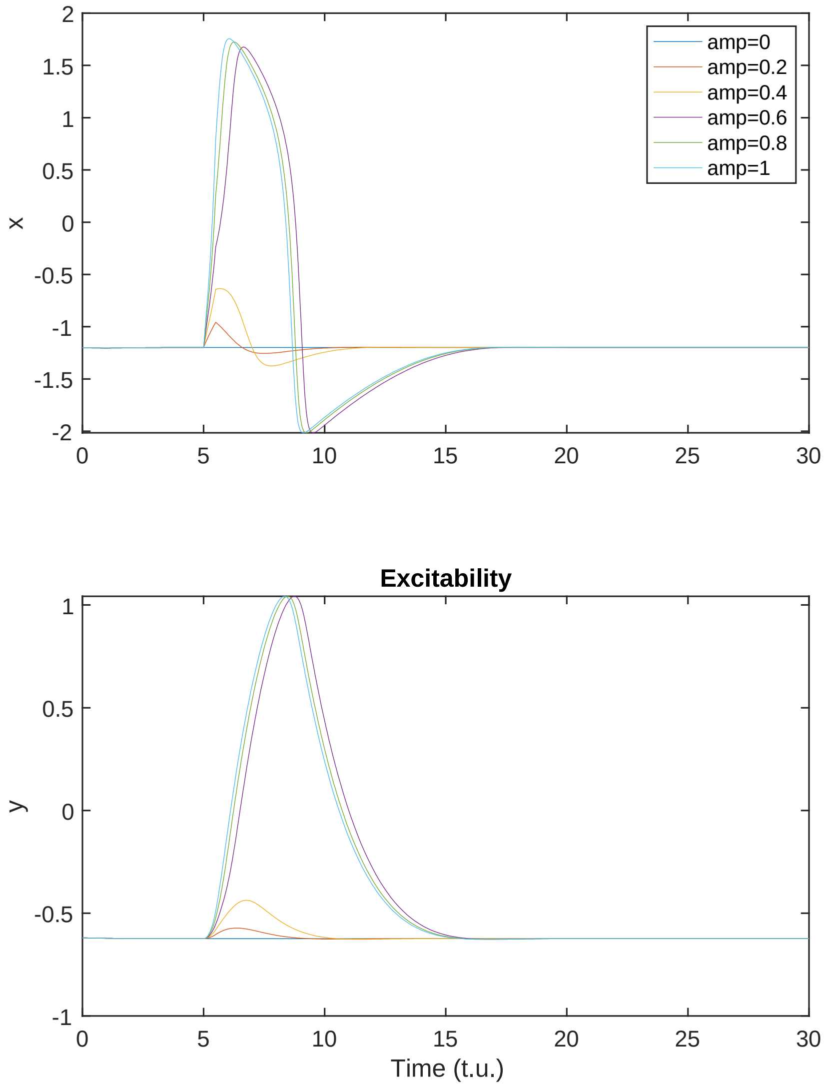




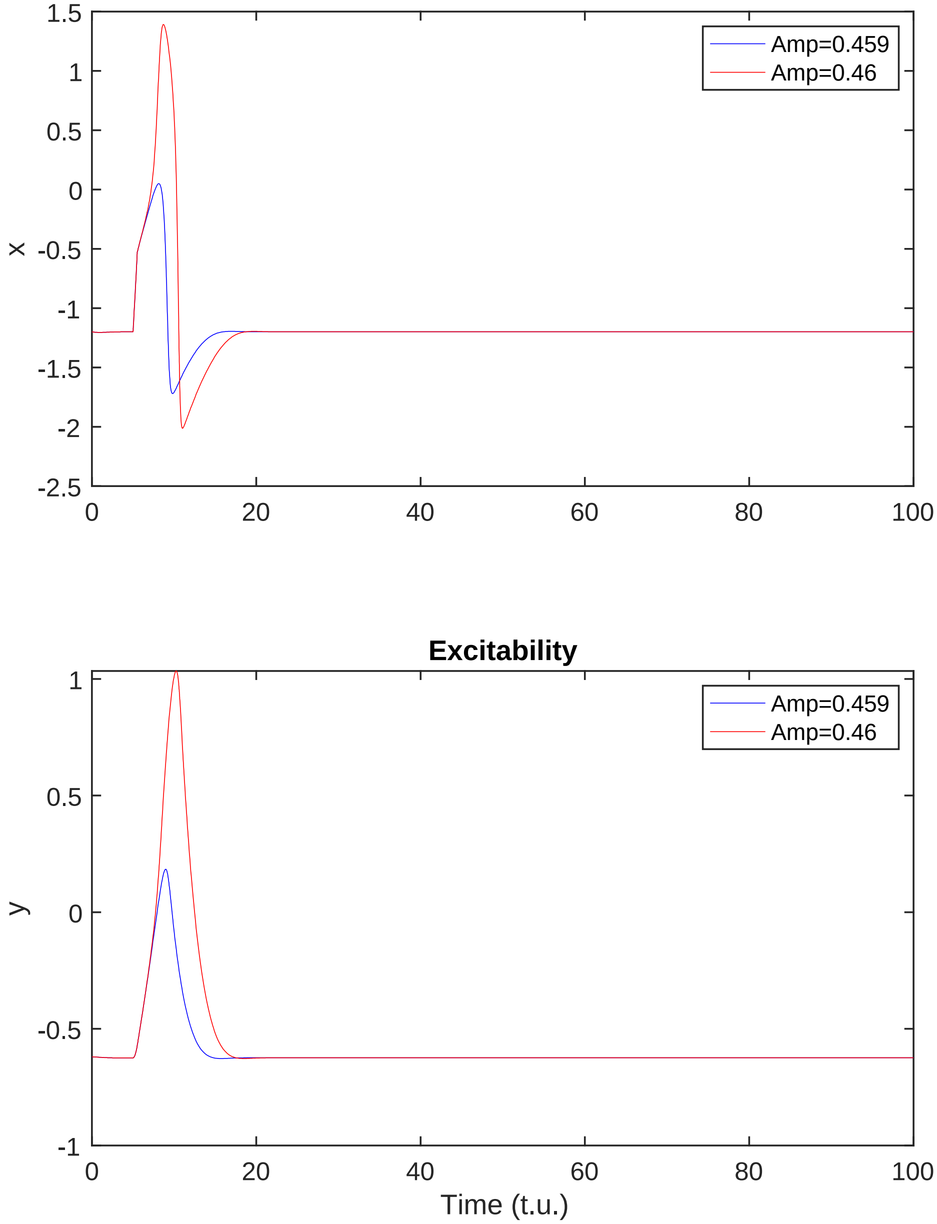
We can try to analyze this in phase space. You can see that there is a sort of barrier with the Xdot nullcline necessitating a stimulation amplitude above a certain threshold to get the flow in the x direction moving enough to not just return right back to the fixed point. Presumably if the stimulation moves x past that nullcline then x will travel around the entire trace shown in blue, whereas with a subthreshold stimulation it doesn't make it past the nullcline



We can vary ranges of stimulation amplitudes to see that there is some threshold for getting the action potential response:

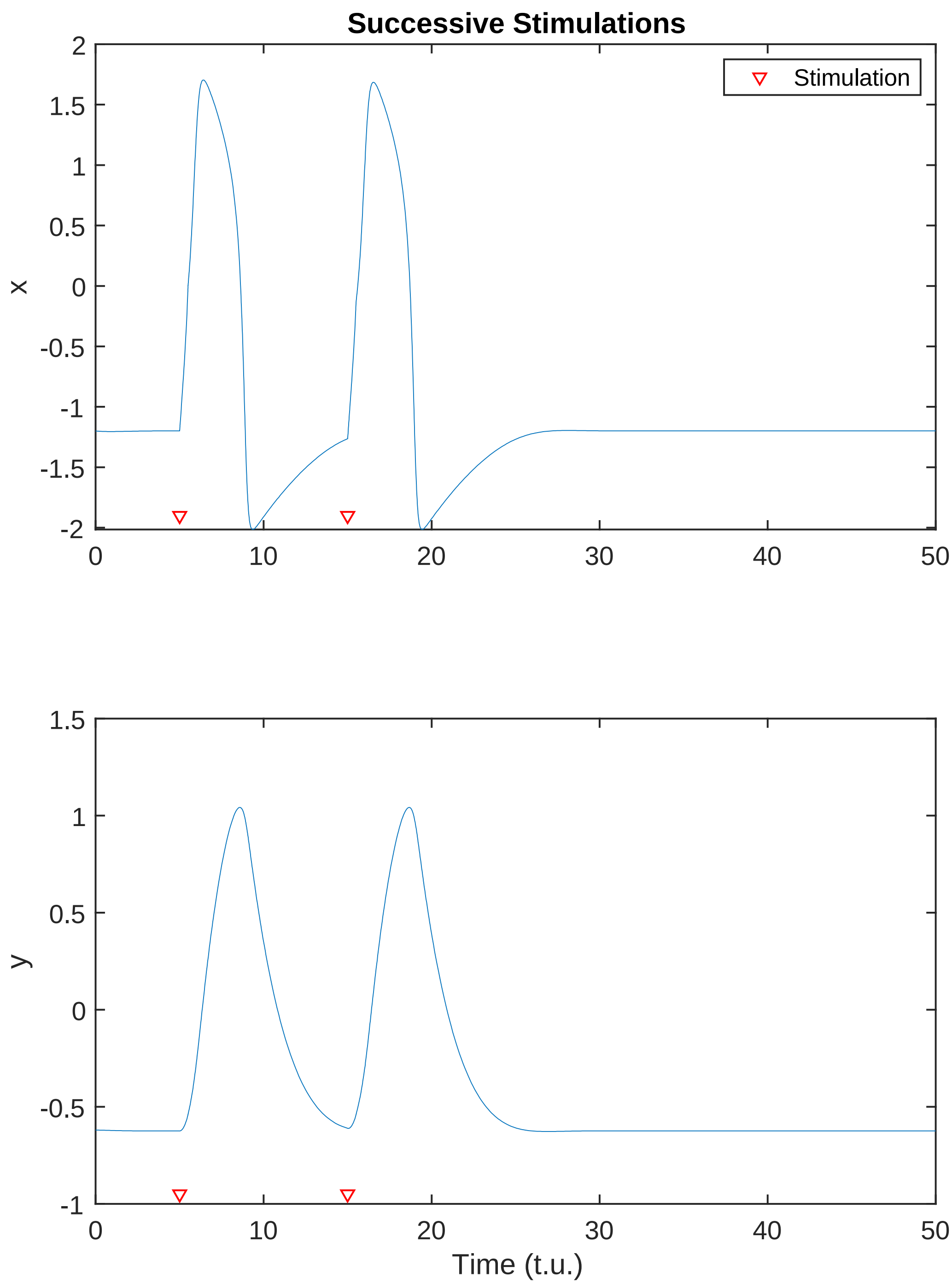


Via simulation, it becomes clear that this threshold is right at 0.46:

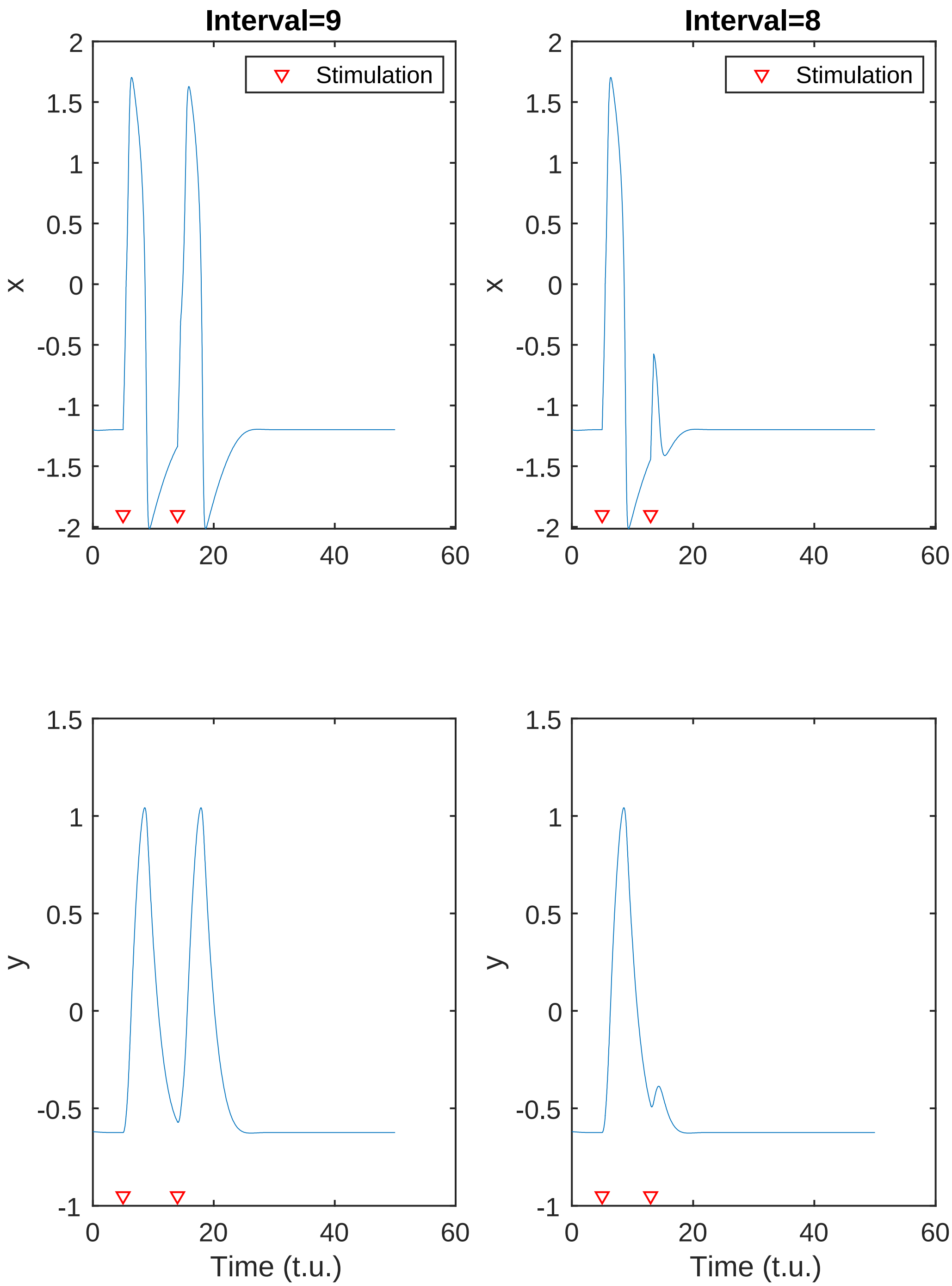


Refractoriness

With 10 time units between stimulations, you get two full action potentials

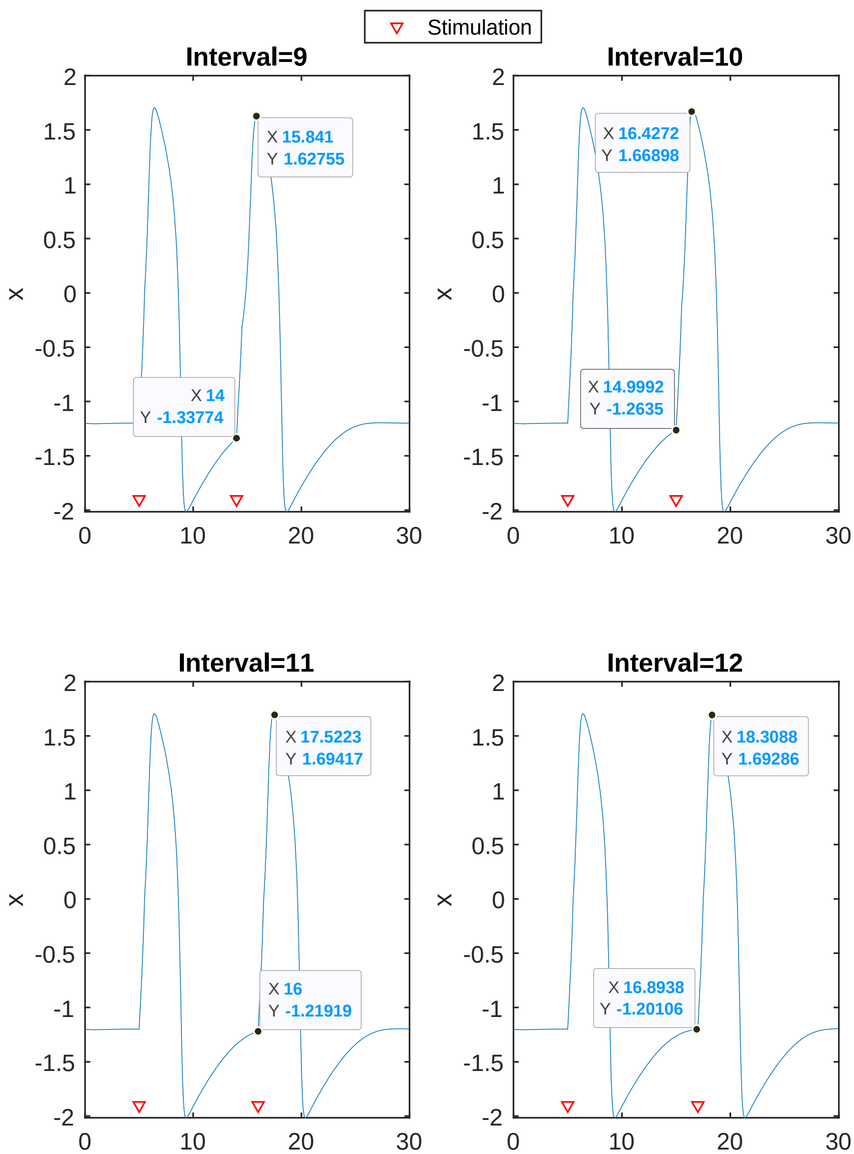


As that interval between stimulations gets smaller, we see a new action potential cannot form too close to the first. This interval threshold is between 8 and 9 time units between stimulations, and indicates an absolute refractory period.



Latency

In addition to an absolute refractory period, there is a “relative” refractory period, where the first action potential is still in a state of hyperpolarization and is not yet back to the baseline resting potential. One way to capture this phenomenon is to measure the latency between the second stimulus and the second action potential peak, for varying intervals between stimulations. You can see below that the latency is related to the intervals. As the interval decreases (i.e. action potentials closer together), the first action potential affects the second, and it takes longer for the second action potential to peak (increased latency).



As mentioned before, this occurs because the first action potential has this hyperpolarization where it dips below resting potential, and thus the second action potential has to make up extra ground so-to-speak before it can hit the threshold voltage. After a certain point though, the resting potential goes back to normal, and so increasing the intervals should no longer affect the latency of the second action potential. If we plot the latency between the second stimulus as a function of the interval, we do in fact see that latency decreases as the action potentials are separated. This effect flattens out as the interval gets larger, since the first action potential has had enough time to go back to resting potential and longer intervals no longer have additional effect