Abstract

I write up problems that I didn't know how to do here.

Problem 1. Show that

$$\lim_{n\to\infty} \frac{n}{a^n} = 0$$

if $a > 1$.

Solution. Since $a > 1$, we can write $a = 1 + b$ for $b > 0$. By the binomial theorem we have that

$$a^n = (1+b)^n > \frac{n(n-1)}{2} b^2.$$

It follows that

$$\frac{n}{a^n} < \frac{1}{b^2(n-1)} \to 0,$$

as desired. ■

Problem 2. Discuss the uniform convergence of nz^n for $z \in \mathbb{C}$.

Solution. If $|z| \geq 1$ then nz^n does not converge. If $|z| < 1$ then the considerations of Problem 1 imply that $nz^n \to 0$ as $n \to \infty$. Let $r < 1$. Problem 1 implies that nz^n converges uniformly on $z < r$. ■