One-Way Analysis of Variance

Comparing Two Groups

- The ***t*** test in the previous section showed how to compare two group means of an independent variable.

$$H_o : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

- For example, the effect on mullen of two different debonders.
- What would we do if there were three or more debonders we wanted to compare?

Comparing 2, 3 or more Groups

- Analysis of Variance (ANOVA) is used when you are interested in comparing ***two, three, or more group means*** of one or more independent variables (factors) *in an experiment*.

$$H_o : \mu_1 = \mu_2 = \ldots = \mu_n$$

$$H_a : \text{at least one mean different}$$

ANOVA

- ANOVA decomposes the total variation in an experiment into meaningful components.
- The important components of variation are the variation *between the levels of a factor* and the variation *within each level*.
  - The within group variation is also called *noise*, random error or experimental error.

ANOVA

- The ANOVA test compares the *between group variation* to the *within group variation* as a ratio. This ratio is then compared to an ***F***-distribution.

$$\frac{between\ group\ variation}{within\ group\ variation} \sim F(df_1, df_2)$$

ANOVA

- If the between variation is due only to random variation then the ***F-ratio*** will be ***small***, and the corresponding ***p-value*** may be ***large*** (***p*** > 0.05 or 0.10).
  - If ***p-value*** is **large**, we conclude that there are ***no differences*** in the mean between group sample levels.
  - This is the same conclusion if a ***p-value*** was large for a ***t*** test

ANOVA

- If the between variation is not random and due to difference in the group levels then the ***F-ratio*** will be ***large*** and the corresponding ***p-value*** would be small (***p*** < 0.05 or 0.10).
  - If the ***p-value*** is **small**, we would conclude that there **<u>are</u>** ***differences*** between group levels.

Completely Randomized Design Experiment

- The best way to understand ANOVA is to use an example.
- We will start with the simplest experiment:
- A one-way or single factor ***c***ompletely ***r***andomized ***d***esign (***CRD***).

Completely Randomized Design Experiment

- The ***CRD*** assumes you can ***randomly*** assign any levels of a factor to any run of the experiment.
  - If you can't randomly assign the levels due to some **restriction**, this method is <u>not</u> correct.

Completely Randomized Design Experiment

- The ***major*** assumptions of the ***CRD***:
  1. The experimental error is: independent, equal and normally distributed.
     - This is by far the <u>most</u> important.
  2. The response variable is normally distributed for each group level.
  3. The statistical model is additive.

Completely Randomized Design Experiment

- The statistical model is the general linear model that relates an observed measurement (**response**) to underlying parameters (**factors**) and random variables (errors).
  - Similar to the multiple regression model.

One-Way ANOVA

- The ***CRD*** design with <u>one factor</u> is analyzed by the ***one-way ANOVA***.
- We will follow the same outline in the ***t*** test section for the steps of a statistical test.
- The hypothesis is different (comparing more than two means) and the test statistic is ***F***.
  - We will discuss the steps later.

One-Way ANOVA

- The general linear model for the ***one-way ANOVA*** is:

$$Y_{ij} = \mu + X_i + \varepsilon_{(i)j}$$

$i = 1 \ldots k$ (number of factor levels)

$j = 1 \ldots n$ (subgroup sample size for each factor level)

$\mu =$ overall average

$X_i =$ average effect of the ith factor

$\varepsilon_{(i)j} =$ error for the *j* individual values for the *i* factors.

One-Way ANOVA

- The general linear model in plain English:

  Response = overall average
  + difference of each subgroup average from the overall average
  + difference in individual values from their subgroup averages

One-Way ANOVA

- The general linear model example for mullen and debonders in plain English:

  Mullen = overall average for Mullen

  + difference of each debonder type average Mullen from the overall average

  + difference in individual Mullen values from their debonder group average Mullen

Steps in a Statistical Test: ANOVA

1. State the null, $\boldsymbol{H_o}$, and alternative, $\boldsymbol{H_a}$, hypothesis.
2. Collect a representative sample from the population of interest.
3. Compute a test statistic (***F***) from sample values, ***assuming H_o is true***.

Steps in a Statistical Test: ANOVA

4. Determine the probability of getting a more extreme value of the ***F*** statistic. (The ***p-value***.)
5. If the ***p-value*** is <u>small</u> (e.g. ***p* < *.05***), the result is unlikely if H_o is true, so we reject H_o. Otherwise, we <u>fail to reject</u> H_o.

ANOVA

- Step 1: Suppose a researcher thinks that varying the amount of cellulose fiber in the furnish of a pad will effect the wicking rate. Pads are made with 50%, 60%, 70% and 80% fiber addition.

One-Way ANOVA

- The factor (variable) is fiber addition amount. The levels of the fiber % addition are 50%, 60%, 70% and 80%.

  The hypothesis test for this ***one-way ANOVA***:

$$H_o : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_a : \text{at least one mean is different}$$

One-Way ANOVA

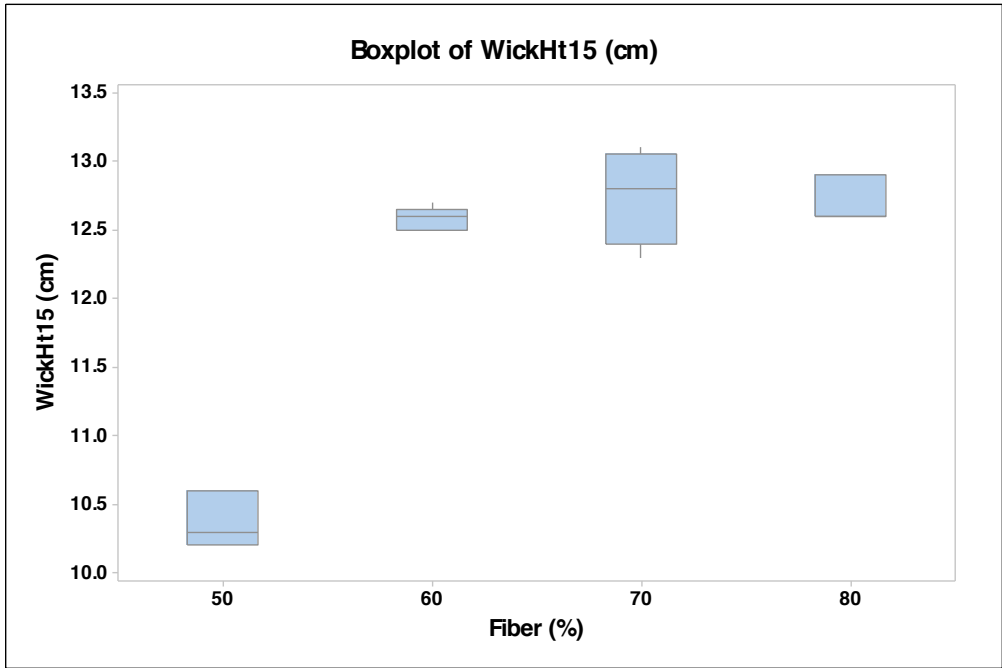
- Step 2: An experiment is run in which hand sheets are made with 50%, 60%, 70% and 80% fiber additions to the furnish, with adequate sample size.

One-Way ANOVA

- Step 3 & 4: There are more than **2 groups** (there are 4) to the fiber levels: use ***one-way ANOVA***.
- The data is in *Minitab Dataset 11.xls*
- ***WickHt15 (cm)*** is the wicking height response.
- ***Fiber (%)*** is the factor for percent fiber added.

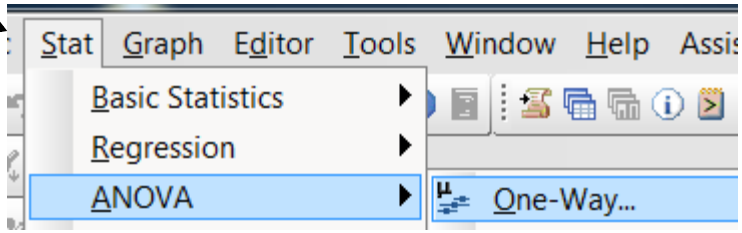
Plot the Data

- Create a ***Box Plot*** of ***WickHt15 (cm)*** with ***Fiber (%)*** as the **Categorical variable**.



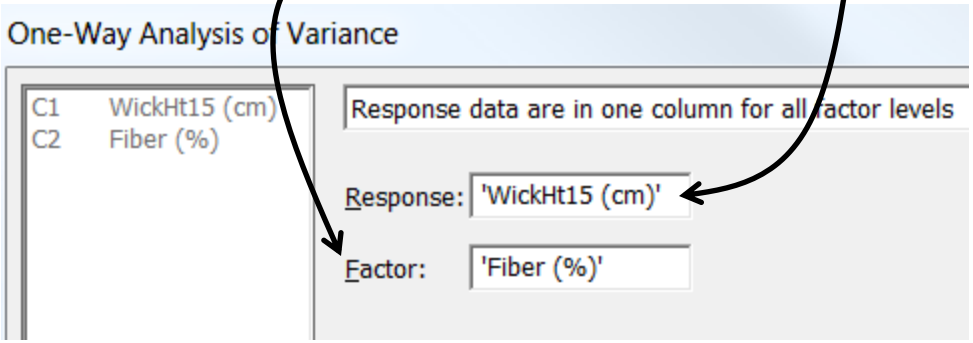
One-Way ANOVA

- Minitab's ***One-Way ANOVA*** procedure can be accessed by selecting **Stat** → **ANOVA** → **One-Way…**



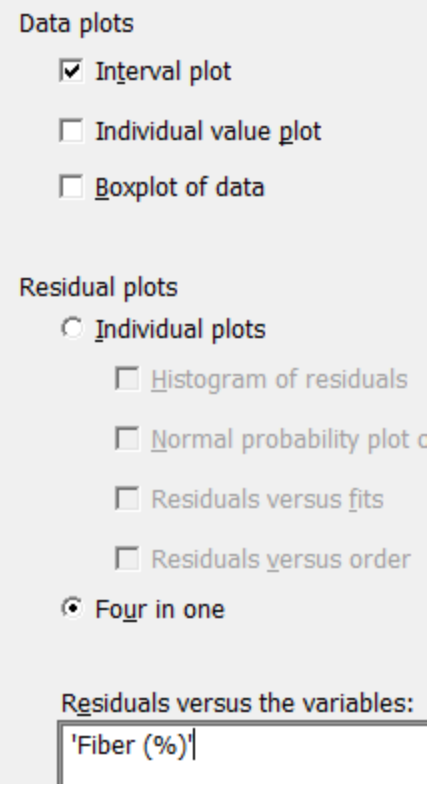
One-Way ANOVA

- Select '***WickHt15 (cm)***' as the **Response** and '***Fiber (%)***' as the **Factor**.



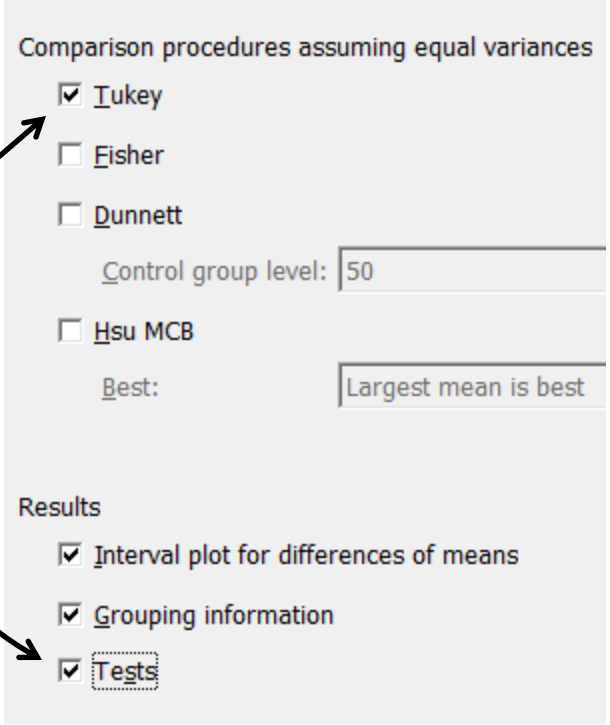
One-Way ANOVA

- Click the **Graphs…** button → select **Interval Plot** for the **Data plots** → select the **Four in one** for the **Residual Plots** → select ***Fiber (%)*** for the **Residuals versus the variables** → click **OK**.



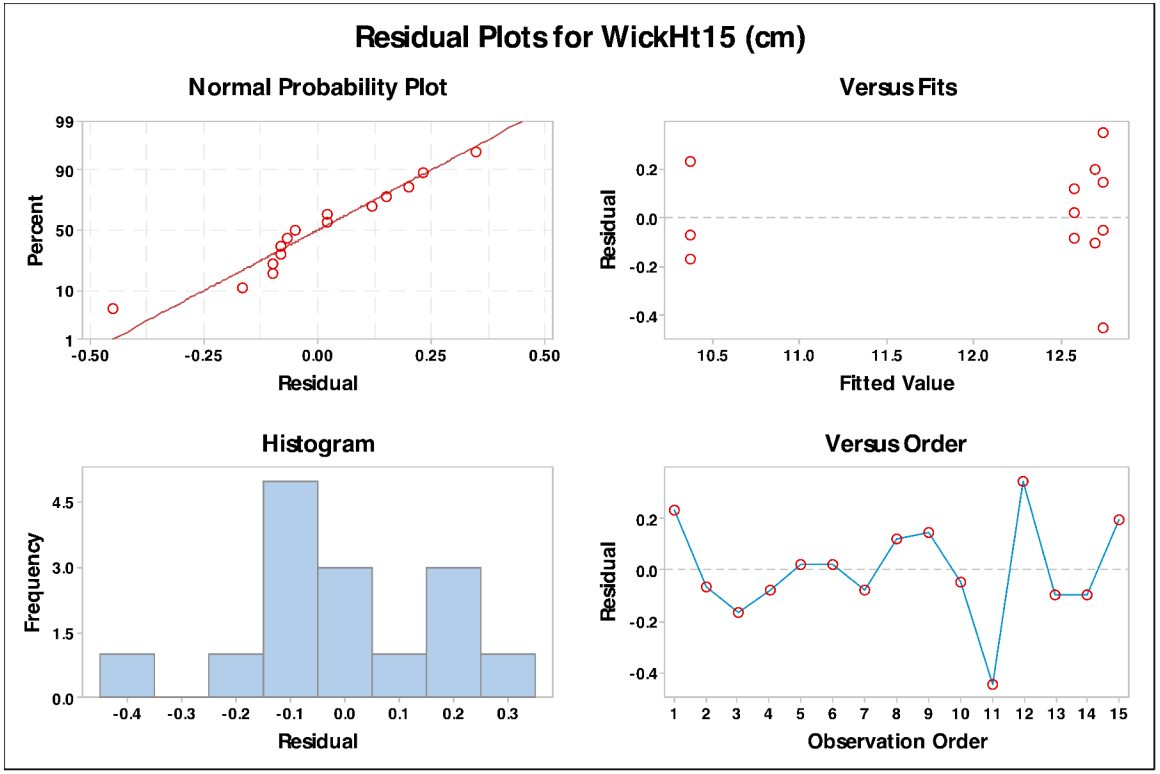
One-Way ANOVA

- Click the **Comparisons…** button → select **Tukey** as the **Comparison procedure**.
- Select **Tests**, in addition to all the other results → click **OK** twice.



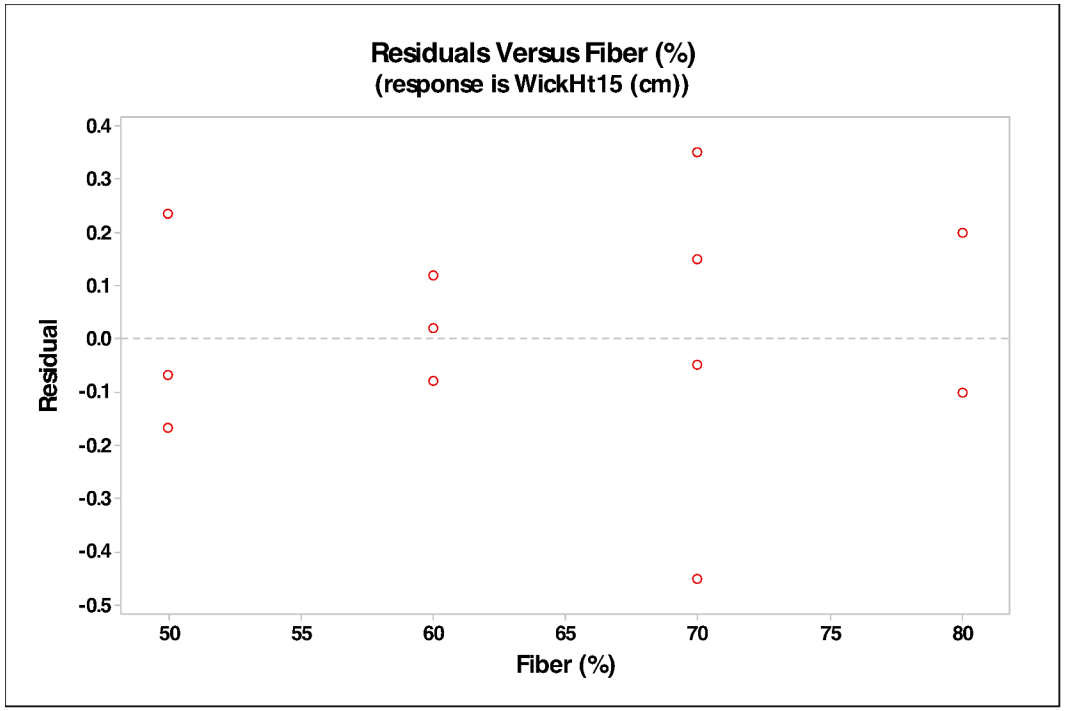
One-Way ANOVA

- How do the residual plots look?



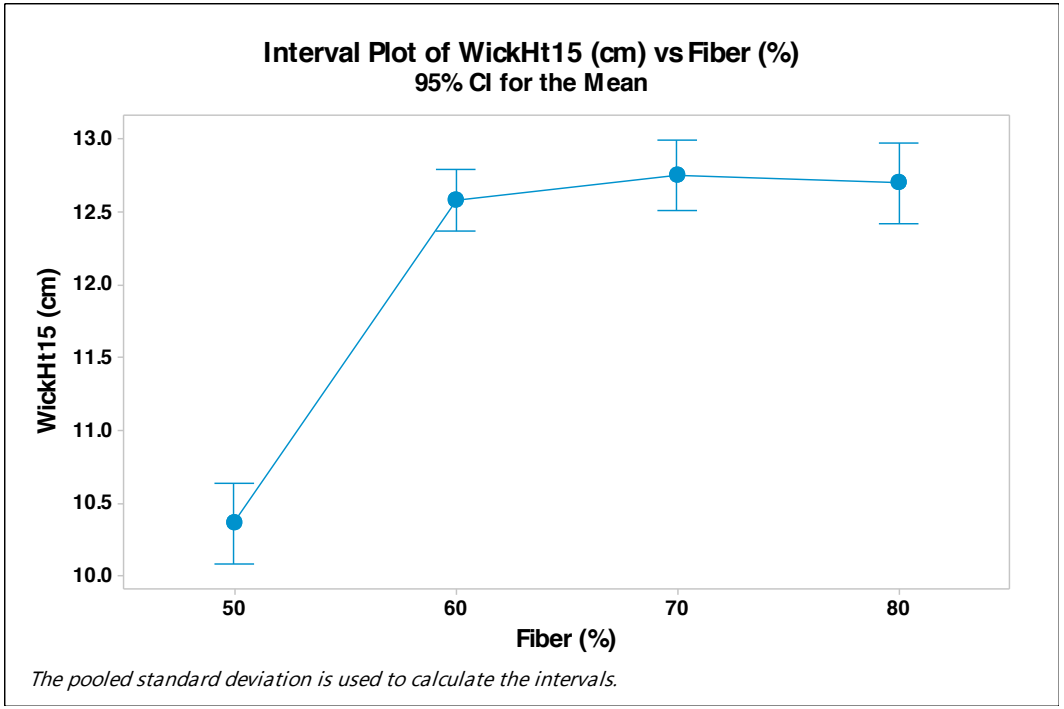
One-Way ANOVA

- How does the Residual versus ***Fiber*** plot look?



One-Way ANOVA

- The **Interval Plot** shows that the 50% ***fiber*** addition wicks at a lower height than the other levels.



One-Way ANOVA

- Steps **3** & **4**: Minitab results

Analysis of Variance

Source	DF	Adj SS	Adj MS	**F-Value**	**P-Value**
Fiber (%)	3	12.7647	4.25489	**89.21**	**0.000**
Error	11	0.5247	0.04770		
Total	14	13.2893			

p-value < .05

F-value is large

One-Way ANOVA

- Step 5 (con't): The ***p-value*** < 0.05 indicates there are ***significant differences*** between the levels of fiber addition.

One-Way ANOVA

Other statistical results:

Good adjusted and predicted R^2

```
Model Summary

       S     R-sq  R-sq(adj)  R-sq(pred)
0.218396  96.05%     94.98%      92.51%
```

Mean for each level of fiber % addition

```
Means

Fiber
(%)     N     Mean    StDev        95% CI
50      3   10.367    0.208  ( 10.089,  10.644)
60      5  12.5800   0.0837  (12.3650, 12.7950)
70      4   12.750    0.342  ( 12.510,  12.990)
80      3   12.700    0.173  ( 12.422,  12.978)

Pooled StDev = 0.218396
```

One-Way ANOVA

- The multiple comparison procedure ***Tukey*** is recommended for any planned ***comparisons*** with a significant ***F*** test.
- We selected the ***Tukey*** multiple comparison test already on slide 11-26 of the One-Way ANOVA instructions.

One-Way ANOVA

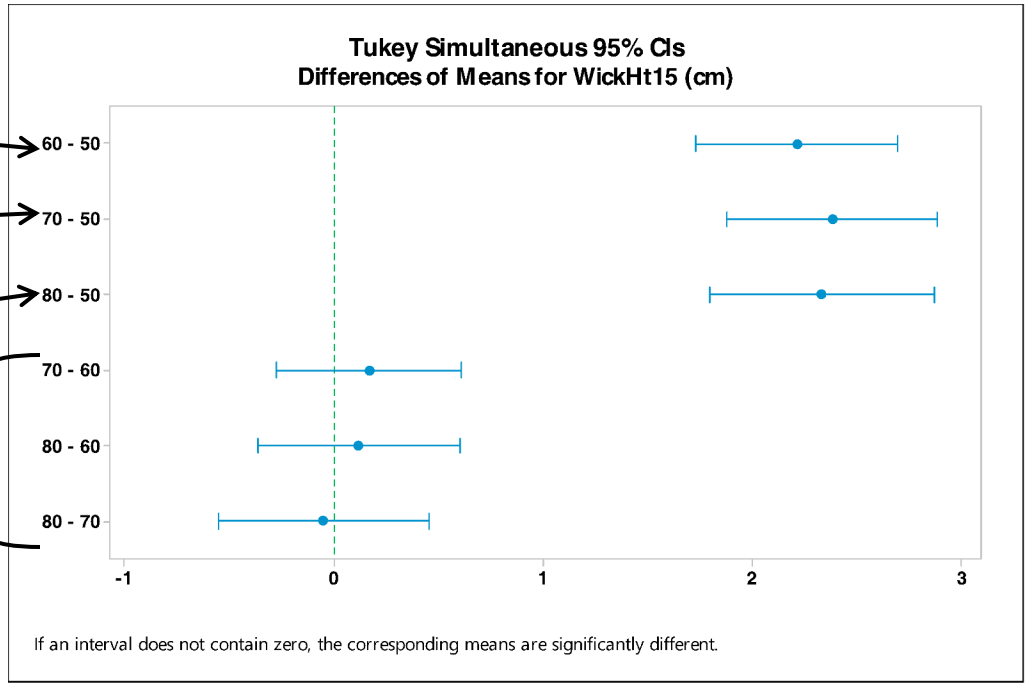
- The ***Tukey*** test results are in the plot '**Tukey Simultaneous 95% CIs**'.

60% has higher wicking compared to 50%

70% has higher wicking compared to 50%

80% has higher wicking compared to 50%

60%, 70% and 80% do not have different wicking heights.



One-Way ANOVA

Tukey Pairwise Comparisons

Grouping Information Using the Tukey Method and 95% Confidence

Fiber (%)	N	Mean	Grouping	
70	4	12.750	A	
80	3	12.700	A	
60	5	12.5800	A	
50	3	10.367		B

Means that do not share a letter are significantly different.

If 95% CI does not include zero (0) then levels are different

Tukey Simultaneous Tests for Differences of Means

Difference of Levels	**Difference of Means**	SE of Difference	**95% CI**	T-Value	Adjusted P-Value
60 - 50	2.213	0.159	**(1.733, 2.694)**	13.88	0.000
70 - 50	2.383	0.167	**(1.881, 2.886)**	14.29	0.000
80 - 50	2.333	0.178	**(1.796, 2.870)**	13.09	0.000
70 - 60	0.170	0.147	(-0.271, 0.611)	1.16	0.662
80 - 60	0.120	0.159	(-0.360, 0.600)	0.75	0.874
80 - 70	-0.050	0.167	(-0.552, 0.452)	-0.30	0.990

Individual confidence level = 98.82%

One-Way ANOVA

Conclusions:

- If the goal is maximize wicking rates, then the 60%, 70% and 80% fiber levels have significantly higher (better) wicking rates compared to 50%.
- If 60% fiber in the furnish is the low cost solution, this would be the addition level to run.

EXERCISE

- Suppose it was desired to test the effect on Mullen of four different debonder suppliers. The suppliers are labeled A, B, C and D in the experiment.
- The trial results are in the dataset '***Exercise 6a.xls***'.
- Do a One-Way ANOVA on the data.
  - Please interpret your results.