La función primitiva

$$\int \sqrt{1 - k^2 \operatorname{sen}^2 x}\, dx + C \quad (k < 1)$$

que se anula cuando x es igual a cero, se llama «integral elíptica» y se designa por $E(x)$,

$$E(x) = \int \sqrt{1 - k^2 \operatorname{sen}^2 x}\, dx + C_2 \text{ con } E(0) = 0.$$

Existen también tablas de los valores que toma esta función para los diferentes valores de x.

Ejercicios

I. Calcular las integrales:

1. $\int x^5\, dx$. Resp.: $\frac{x^6}{6} + C$.

2. $\int (x + \sqrt{x})dx$. Resp.: $\frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + C$.

3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4}\right)dx$. Resp.: $6\sqrt{x} - \frac{1}{10}x^2\sqrt{x} + C$.

4. $\int \frac{x^2\, dx}{\sqrt{x}}$. Resp.: $\frac{2}{5}x^2\sqrt{x} + C$.

5. $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2\right)dx$. Resp.: $-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + C$.

6. $\int \frac{dx}{\sqrt[4]{x}}$. Resp.: $\frac{4}{3}\sqrt[4]{x^3} + C$.

7. $\int \left(x^2 + \frac{1}{\sqrt[3]{x}}\right)^2 dx$. Resp.: $\frac{x^5}{5} + \frac{3}{4}x^2\sqrt[3]{x^2} + 3\sqrt[3]{x} + C$.

Integración por sustitución:

8. $\int e^{5x}\, dx$. Resp.: $\frac{1}{5}e^{5x} + C$.

9. $\int \cos 5x\, dx$. Resp.: $\frac{\operatorname{sen} 5x}{5} + C$.

10. $\int \operatorname{sen} ax\, dx$. Resp.: $-\frac{\cos ax}{a} + C$.

11. $\int \frac{\ln x}{x} dx$. Resp.: $\frac{1}{2}\ln^2 x + C$.

12. $\int \frac{dx}{\operatorname{sen}^2 3x}$. Resp.: $-\frac{\operatorname{cotg} 3x}{3} + C$.

13. $\int \frac{dx}{\cos^2 7x}$. Resp.: $\frac{\operatorname{tg} 7x}{7} + C$.

14. $\int \frac{dx}{3x - 7}$. Resp.: $\frac{1}{3}\ln|3x - 7| + C$.

15. $\int \frac{dx}{1 - x}$. Resp.: $-\ln|1 - x| + C$.

16. $\int \frac{dx}{5 - 2x}$. Resp.: $-\frac{1}{2}\ln|5 - 2x| + C$.

17. $\int \operatorname{tg} 2x\, dx$. Resp.: $-\frac{1}{2}\ln|\cos 2x| + C$.

18. $\int \operatorname{cotg} (5x - 7)dx$. Resp.: $\frac{1}{5}\ln|\operatorname{sen} (5x - 7)| + C$.

19. $\int \frac{dy}{\operatorname{cotg} 3y}$. Resp.: $-\frac{1}{3}\ln|\cos 3y| + C$.

20. $\int \operatorname{cotg} \frac{x}{3}\, dx$. Resp.: $3 \ln\left|\operatorname{sen} \frac{x}{3}\right| + C$.

21. $\int \operatorname{tg} \varphi \cdot \sec^2 \varphi\, d\varphi$. Resp.: $\frac{1}{2}\operatorname{tg}^2 \varphi + C$.

22. $\int (\operatorname{cotg} e^x)e^x\, dx$. Resp.: $\ln|\operatorname{sen} e^x| + C$.

23. $\int \left(\operatorname{tg} 4S - \operatorname{cotg} \frac{S}{4}\right) dS$. Resp.: $-\frac{1}{4}\ln|\cos 4S| - 4 \ln\left|\operatorname{sen} \frac{S}{4}\right| + C$.

24. $\int \operatorname{sen}^2 x \cos x\, dx$. Resp.: $\frac{\operatorname{sen}^3 x}{3} + C$.

25. $\int \cos^3 x \operatorname{sen} x\, dx$. Resp.: $-\frac{\cos^4 x}{4} + C$.

26. $\int \sqrt{x^2 + 1}\, x\, dx$. Resp.: $\frac{1}{3}\sqrt{(x^2 + 1)^3} + C$.

27. $\int \frac{x\, dx}{\sqrt{2x^2 + 3}}$. Resp.: $\frac{1}{2}\sqrt{2x^2 + 3} + C$.

28. $\int \frac{x^2\, dx}{\sqrt{x^3 + 1}}$. Resp.: $\frac{2}{3}\sqrt{x^3 + 1} + C$.

29. $\int \frac{\cos x\, dx}{\operatorname{sen}^2 x}$. Resp.: $-\frac{1}{\operatorname{sen} x} + C$.

30. $\int \frac{\operatorname{sen} x\,dx}{\cos^3 x}$. Resp.: $\frac{1}{2\cos^2 x} + C$.

31. $\int \frac{\operatorname{tg} x}{\cos^2 x}\,dx$. Resp.: $\frac{\operatorname{tg}^2 x}{2} + C$.

32. $\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x}\,dx$. Resp.: $-\frac{\operatorname{cotg}^2 x}{2} + C$.

33. $\int \frac{dx}{\cos^2 x\sqrt{\operatorname{tg} x - 1}}$. Resp.: $2\sqrt{\operatorname{tg} x - 1} + C$.

34. $\int \frac{\ln(x+1)}{x+1}\,dx$. Resp.: $\frac{\ln^2(x+1)}{2} + C$.

35. $\int \frac{\cos x\,dx}{\sqrt{2\operatorname{sen} x + 1}}$. Resp.: $\sqrt{2\operatorname{sen} x + 1} + C$.

36. $\int \frac{\operatorname{sen} 2x\,dx}{(1+\cos 2x)^2}$. Resp.: $\frac{1}{2(1+\cos 2x)} + C$.

37. $\int \frac{\operatorname{sen} 2x\,dx}{\sqrt{1+\operatorname{sen}^2 x}}$. Resp.: $2\sqrt{1+\operatorname{sen}^2 x} + C$.

38. $\int \frac{\sqrt{\operatorname{tg} x + 1}}{\cos^2 x}\,dx$. Resp.: $\frac{2}{3}\sqrt{(\operatorname{tg} x + 1)^3} + C$.

39. $\int \frac{\cos 2x\,dx}{(2+3\operatorname{sen} 2x)^3}$. Resp.: $-\frac{1}{12}\frac{1}{(2+3\operatorname{sen} 2x)^2} + C$.

40. $\int \frac{\operatorname{sen} 3x\,dx}{\sqrt[3]{\cos^4 3x}}$. Resp.: $\frac{1}{\sqrt[3]{\cos 3x}} + C$.

41. $\int \frac{\ln^2 x\,dx}{x}$. Resp.: $\frac{\ln^3 x}{3} + C$.

42. $\int \frac{\operatorname{arcsen} x\,dx}{\sqrt{1-x^2}}$. Resp.: $\frac{\operatorname{arcsen}^2 x}{2} + C$.

43. $\int \frac{\operatorname{arctg} x\,dx}{1+x^2}$. Resp.: $\frac{\operatorname{arctg}^2 x}{2} + C$.

44. $\int \frac{\arccos^2 x}{\sqrt{1-x^2}}\,dx$. Resp.: $-\frac{\arccos^3 x}{3} + C$.

45. $\int \frac{\operatorname{arccotg} x}{1+x^2}\,dx$. Resp.: $-\frac{\operatorname{arccotg}^2 x}{2} + C$.

46. $\int \frac{x\,dx}{x^2+1}$. Resp.: $\frac{1}{2}\ln(x^2+1) + C$.

47. $\int \frac{x+1}{x^2+2x+3}\,dx$. Resp.: $\frac{1}{2}\ln(x^2+2x+3) + C$.

48. $\int \frac{\cos x\,dx}{2\operatorname{sen} x + 3}$. Resp.: $\frac{1}{2}\ln(2\operatorname{sen} x + 3) + C$.

49. $\int \frac{dx}{x\ln x}$. Resp.: $\ln|\ln x| + C$.

50. $\int 2x(x^2+1)^4\,dx$. Resp.: $\frac{(x^2+1)^5}{5} + C$.

51. $\int \operatorname{tg}^4 x\,dx$. Resp.: $\frac{\operatorname{tg}^3 x}{3} - \operatorname{tg} x + x + C$.

52. $\int \frac{dx}{(1+x^2)\operatorname{arctg} x}$. Resp.: $\ln|\operatorname{arctg} x| + C$.

53. $\int \frac{dx}{\cos^2 x(3\operatorname{tg} x + 1)}$. Resp.: $\frac{1}{3}\ln(3\operatorname{tg} x + 1) + C$.

54. $\int \frac{\operatorname{tg}^3 x}{\cos^2 x}\,dx$. Resp.: $\frac{\operatorname{tg}^4 x}{4} + C$.

55. $\int \frac{dx}{\sqrt{1-x^2}\operatorname{arcsen} x}$. Resp.: $\ln|\operatorname{arcsen} x| + C$.

56. $\int \frac{\cos 2x}{2+3\operatorname{sen} 2x}\,dx$. Resp.: $\frac{1}{6}\ln|2+3\operatorname{sen} 2x| + C$.

57. $\int \cos(\ln x)\frac{dx}{x}$. Resp.: $\operatorname{sen}(\ln x) + C$.

58. $\int \cos(a+bx)\,dx$. Resp.: $\frac{1}{b}\operatorname{sen}(a+bx) + C$.

59. $\int e^{2x}\,dx$. Resp.: $\frac{1}{2}e^{2x} + C$.

60. $\int e^{\frac{x}{2}}dx$. Resp.: $3e^{\frac{x}{3}} + C$.

61. $\int e^{\operatorname{sen} x}\cos x\,dx$. Resp.: $e^{\operatorname{sen} x} + C$.

62. $\int a^{x^2}x\,dx$. Resp.: $\frac{a^{x^2}}{2\ln a} + C$.

63. $\int e^{\frac{x}{a}}dx$. Resp.: $ae^{\frac{x}{a}} + C$.

64. $\int (e^{2x})^2\,dx$. Resp.: $\frac{1}{4}e^{4x} + C$.

65. $\int 3^x e^x\,dx$. Resp.: $\frac{3^x e^x}{\ln 3 + 1} + C$.

66. $\int e^{-3x}\,dx$. Resp.: $-\frac{1}{3}e^{-3x} + C$.

67. $\int (e^{5x} + a^{5x})\,dx$. Resp.: $\frac{1}{5}\left(e^{5x} + \frac{a^{5x}}{\ln a} + C\right)$

68. $\int e^{x^2+4x+3}(x+2)\,dx$. Resp.: $\frac{1}{2}e^{x^2+4x+3} + C$.

69. $\int \frac{(a^x - b^x)^2}{a^x b^x}\,dx$. Resp.: $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln a - \ln b} - 2x + C$.

70. $\int \frac{e^x\,dx}{3 + 4e^x}$. Resp.: $\frac{1}{4}\ln(3 + 4e^x) + C$.

71. $\int \frac{e^{2x}\,dx}{2 + e^{2x}}$. Resp.: $\frac{1}{2}\ln(2 + e^{2x}) + C$.

72. $\int \frac{dx}{1 + 2x^2}$. Resp.: $\frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}x) + C$.

73. $\int \frac{dx}{\sqrt{1 - 3x^2}}$. Resp.: $\frac{1}{\sqrt{3}}\operatorname{arcsen}(\sqrt{3}x) + C$.

74. $\int \frac{dx}{\sqrt{16 - 9x^2}}$. Resp.: $\frac{1}{3}\operatorname{arcsen}\frac{3x}{4} + C$.

75. $\int \frac{dx}{\sqrt{9 - x^2}}$. Resp.: $\operatorname{arcsen}\frac{x}{3} + C$.

76. $\int \frac{dx}{4 + x^2}$. Resp.: $\frac{1}{2}\operatorname{arctg}\frac{x}{2} + C$.

77. $\int \frac{dx}{9x^2 + 4}$. Resp.: $\frac{1}{6}\operatorname{arctg}\frac{3x}{2} + C$.

78. $\int \frac{dx}{4 - 9x^2}$. Resp.: $\frac{1}{12}\ln\left|\frac{2 + 3x}{2 - 3x}\right| + C$.

79. $\int \frac{dx}{\sqrt{x^2 + 9}}$. Resp.: $\ln|x + \sqrt{x^2 + 9}| + C$.

80. $\int \frac{dx}{\sqrt{b^2x^2 - a^2}}$. Resp.: $\frac{1}{b}\ln|bx + \sqrt{b^2x^2 - a^2}| + C$.

81. $\int \frac{dx}{\sqrt{b^2 + a^2x^2}}$. Resp.: $\frac{1}{a}\ln|ax + \sqrt{b^2 + a^2x^2}| + C$.

82. $\int \frac{dx}{a^2x^2 - c^2}$. Resp.: $\frac{1}{2ac}\ln\left|\frac{ax - c}{ax + c}\right| + C$.

83. $\int \frac{x^2\,dx}{5 - x^6}$. Resp.: $\frac{1}{6\sqrt{5}}\ln\left|\frac{x^3 + \sqrt{5}}{x^3 - \sqrt{5}}\right| + C$.

84. $\int \frac{x\,dx}{\sqrt{1 - x^4}}$. Resp.: $\frac{1}{2}\operatorname{arcsen} x^2 + C$.

85. $\int \frac{x\,dx}{x^4 + a^4}$. Resp.: $\frac{1}{2a^2}\operatorname{arctg}\frac{x^2}{a^2} + C$.

86. $\int \frac{e^x\,dx}{\sqrt{1 - e^{2x}}}$. Resp.: $\operatorname{arcsen} e^x + C$.

87. $\int \frac{dx}{\sqrt{3 - 5x^2}}$. Resp.: $\frac{1}{\sqrt{5}}\operatorname{arcsen}\sqrt{\frac{5}{3}}x + C$.

88. $\int \frac{\cos x\,dx}{a^2 + \operatorname{sen}^2 x}$. Resp.: $\frac{1}{a}\operatorname{arctg}\left(\frac{\operatorname{sen} x}{a}\right) + C$.

89. $\int \frac{dx}{x\sqrt{1 - \ln^2 x}}$. Resp.: $\operatorname{arcsen}(\ln x) + C$.

90. $\int \frac{\arccos x - x}{\sqrt{1 - x^2}}\,dx$. Resp.: $-\frac{1}{2}(\arccos x)^2 + \sqrt{1 - x^2} + C$.

91. $\int \frac{x - \operatorname{arctg} x}{1 + x^2}\,dx$. Resp.: $\frac{1}{2}\ln(1 + x^2) - \frac{1}{2}(\operatorname{arctg} x)^2 + C$.

92. $\int \frac{\sqrt{1 + \ln x}}{x}\,dx$. Resp.: $\frac{2}{3}\sqrt{(1 + \ln x)^3} + C$.

93. $\int \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}}\,dx$. Resp.: $\frac{4}{3}\sqrt{(1 + \sqrt{x})^3} + C$.

94. $\int \frac{dx}{\sqrt{x}\sqrt{1 + \sqrt{x}}}$. Resp.: $4\sqrt{1 + \sqrt{x}} + C$.

95. $\int \frac{e^x\,dx}{1 + e^{2x}}$. Resp.: $\operatorname{arctg} e^x + C$.

96. $\int \frac{\cos x\,dx}{\sqrt[3]{\operatorname{sen}^2 x}}$. Resp.: $3\sqrt[3]{\operatorname{sen} x} + C$

97. $\int \sqrt{1 + 3\cos^2 x}\,\operatorname{sen} 2x\,dx$. Resp.: $-\frac{2}{9}\sqrt{(1 + 3\cos^2 x)^3} + C$.

98. $\int \frac{\operatorname{sen} 2x\,dx}{\sqrt{1 + \cos^2 x}}$. Resp.: $-2\sqrt{1 + \cos^2 x} + C$.

99. $\int \frac{\cos^3 x}{\operatorname{sen}^4 x}\,dx$. Resp.: $\frac{1}{\operatorname{sen} x} - \frac{1}{3\operatorname{sen}^3 x} + C$.

100. $\int \frac{\sqrt[3]{\operatorname{tg}^2 x}}{\cos^2 x}\,dx$. Resp.: $\frac{3}{5}\sqrt[3]{\operatorname{tg}^5 x} + C$.

101. $\int \frac{dx}{2\operatorname{sen}^2 x + 3\cos^2 x}$. Resp.: $\frac{1}{\sqrt{6}}\operatorname{arctg}\left(\sqrt{\frac{2}{3}}\operatorname{tg} x\right) + C$.

Integrales del tipo $\int \frac{Ax + B}{ax^2 + bx + c}\,dx$:

102. $\int \frac{dx}{x^2 + 2x + 5}$. Resp.: $\frac{1}{2}\operatorname{arctg}\frac{x + 1}{2} + C$.

103. $\int \frac{dx}{3x^2-2x+4}$. Resp.: $\frac{1}{\sqrt{11}} \operatorname{arctg} \frac{3x-1}{\sqrt{11}} + C$.

104. $\int \frac{dx}{x^2+3x+1}$. Resp.: $\frac{1}{\sqrt{5}} \ln \left| \frac{2x+3-\sqrt{5}}{2x+3+\sqrt{5}} \right| + C$.

105. $\int \frac{dx}{x^2-6x+5}$. Resp.: $\frac{1}{4} \ln \left| \frac{x-5}{x-1} \right| + C$.

106. $\int \frac{dz}{2z^2-2z+1}$. Resp.: $\operatorname{arctg}(2z-1) + C$.

107. $\int \frac{dx}{3x^2-2x+2}$. Resp.: $\frac{1}{\sqrt{5}} \operatorname{arctg} \frac{3x-1}{\sqrt{5}} + C$.

108. $\int \frac{(6x-7)\,dx}{3x^2-7x+11}$. Resp.: $\ln |3x^2-7x+11| + C$.

109. $\int \frac{(3x-2)\,dx}{5x^2-3x+2}$. Resp.: $\frac{3}{10} \ln (5x^2-3x+2) - \frac{11}{5\sqrt{31}} \operatorname{arctg} \frac{10x-3}{\sqrt{31}} + C$.

110. $\int \frac{3x-1}{x^2-x+1}\,dx$. Resp.: $\frac{3}{2} \ln (x^2-x+1) + \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{2x-1}{\sqrt{3}} + C$.

111. $\int \frac{7x+1}{6x^2+x-1}\,dx$. Resp.: $\frac{2}{3} \ln (3x-1) + \frac{1}{2} \ln (2x+1) + C$.

112. $\int \frac{2x-1}{5x^2-x+2}\,dx$. Resp.: $\frac{1}{5} \ln (5x^2-x+2) + \frac{8}{5\sqrt{39}} \operatorname{arctg} \frac{10x-1}{\sqrt{39}} + C$.

113. $\int \frac{6x^4-5x^3+4x^2}{2x^2-x+1}\,dx$. Resp.: $x^3 - \frac{x^2}{2} + \frac{1}{4} \ln |2x^2-x+1| + \frac{1}{2\sqrt{7}} \operatorname{arctg} \frac{4x-1}{\sqrt{7}} + C$.

114. $\int \frac{dx}{2\cos^2 x + \operatorname{sen} x \cos x + \operatorname{sen}^2 x}$. Resp.: $\frac{2}{\sqrt{7}} \operatorname{arctg} \frac{2 \operatorname{tg} x + 1}{\sqrt{7}} + C$.

Integrales del tipo $\int \frac{Ax+B}{\sqrt{ax^2+bx+C}}\,dx$:

115. $\int \frac{dx}{\sqrt{2-3x-4x^2}}$. Resp.: $\frac{1}{2} \operatorname{arcsen} \frac{8x+3}{\sqrt{41}} + C$.

116. $\int \frac{dx}{\sqrt{1+x+x^2}}$. Resp.: $\ln \left| x + \frac{1}{2} + \sqrt{x^2+x+1} \right| + C$.

117. $\int \frac{dS}{\sqrt{2aS+S^2}}$. Resp.: $\ln |S+a+\sqrt{2aS+S^2}| + C$.

118. $\int \frac{dx}{\sqrt{5-7x-3x^2}}$. Resp.: $\frac{1}{\sqrt{3}} \operatorname{arcsen} \frac{6x+7}{\sqrt{109}} + C$.

119. $\int \frac{dx}{\sqrt{x(3x+5)}}$. Resp.: $\frac{1}{\sqrt{3}} \ln |6x+5+\sqrt{12x(3x+5)}| + C$.

120. $\int \frac{dx}{\sqrt{2-3x-x^2}}$. Resp.: $\operatorname{arcsen} \frac{2x+3}{\sqrt{17}} + C$.

121. $\int \frac{dx}{\sqrt{5x^2-x-1}}$. Resp.: $\frac{1}{\sqrt{5}} \ln |10x-1+\sqrt{20(5x^2-x-1)}| + C$.

122. $\int \frac{2ax+b}{\sqrt{ax^2+bx+C}}\,dx$. Resp.: $\sqrt{ax^2+bx+c} + C$.

123. $\int \frac{(x+3)\,dx}{\sqrt{4x^2+4x+3}}$.

Resp.: $\frac{1}{4}\sqrt{4x^2+4x+3} + \frac{5}{4} \ln |2x+1+\sqrt{4x^2+4x+3}| + C$.

124. $\int \frac{(x-3)\,dx}{\sqrt{3+66x-11x^2}}$. Resp.: $-\frac{1}{11}\sqrt{3+66x-11x^2} + C$.

125. $\int \frac{(x+3)\,dx}{\sqrt{3+4x-4x^2}}$. Resp.: $-\frac{1}{4}\sqrt{3+4x-4x^2} + \frac{7}{4} \operatorname{arcsen} \frac{2x-1}{2} + C$.

126. $\int \frac{3x+5}{\sqrt{x(2x-1)}}\,dx$. Resp.: $\frac{3}{2}\sqrt{2x^2-x} + \frac{23}{4\sqrt{2}} \ln (4x-1+\sqrt{8(2x^2-x)}) + C$.

II. Integración por partes:

127. $\int xe^x\,dx$. Resp.: $e^x(x-1) + C$.

128. $\int x \ln x\,dx$. Resp.: $\frac{1}{2}x^2 \left(\ln x - \frac{1}{2} \right) + C$.

129. $\int x \operatorname{sen} x\,dx$. Resp.: $\operatorname{sen} x - x \cos x + C$.

130. $\int \ln x\,dx$. Resp.: $x(\ln x - 1) + C$.

131. $\int \operatorname{arcsen} x\,dx$. Resp.: $x \operatorname{arcsen} x + \sqrt{1-x^2} + C$.

132. $\int \ln (1-x)\,dx$. Resp.: $-x-(1-x)\ln(1-x) + C$.

133. $\int x^n \ln x\,dx$. Resp.: $\frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1} \right) + C$.

134. $\int x \operatorname{arctg} x\,dx$. Resp.: $\frac{1}{2}[(x^2+1) \operatorname{arctg} x - x] + C$.

135. $\int x \operatorname{arcsen} x\,dx$. Resp.: $\frac{1}{4}[(2x^2-1)\operatorname{arcsen} x + x\sqrt{1-x^2}] + C$.

136. $\int \ln(x^2+1)\,dx$. Resp.: $x\ln(x+1) - 2x + 2\operatorname{arctg} x + C$.

137. $\int \operatorname{arctg}\sqrt{x}\,dx$. Resp.: $(x+1)\operatorname{arctg}\sqrt{x} - \sqrt{x} + C$.

138. $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}\,dx$. Resp.: $2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2\sqrt{1-x} + C$.

139. $\int \operatorname{arcsen}\sqrt{\frac{x}{x+1}}\,dx$. Resp.: $x\operatorname{arcsen}\sqrt{\frac{x}{x+1}} - \sqrt{x} + \operatorname{arctg}\sqrt{x} + C$.

140. $\int x\cos^2 x\,dx$. Resp.: $\frac{x^2}{4} + \frac{1}{4}x\operatorname{sen} 2x + \frac{1}{8}\cos 2x + C$.

141. $\int \frac{x\operatorname{arcsen} x}{\sqrt{1-x^2}}\,dx$. Resp.: $x - \sqrt{1-x^2}\operatorname{arcsen} x + C$.

142. $\int \frac{x\operatorname{arctg} x}{(x^2+1)^2}\,dx$. Resp.: $\frac{x}{4(1+x^2)} + \frac{1}{4}\operatorname{arctg} x - \frac{1}{2}\frac{\operatorname{arctg} x}{1+x^2} + C$.

143. $\int x\operatorname{arctg}\sqrt{x^2-1}\,dx$. Resp.: $\frac{1}{2}x^2\operatorname{arctg}\sqrt{x^2-1} - \frac{1}{2}\sqrt{x^2-1} + C$.

144. $\int \frac{\operatorname{arcsen} x}{x^2}\,dx$. Resp.: $\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| - \frac{1}{2}\operatorname{arcsen} x + C$.

145. $\int \ln(x+\sqrt{1+x^2})\,dx$. Resp.: $x\ln|x+\sqrt{1+x^2}| - \sqrt{1+x^2} + C$.

146. $\int \operatorname{arcsen} x\frac{x\,dx}{\sqrt{(1-x^2)^3}}$. Resp.: $\frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} + \frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|$.

Utilizar sustituciones trigonométricas en los ejemplos siguientes:

147. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx$. Resp.: $-\frac{\sqrt{a^2-x^2}}{x} - \operatorname{arcsen}\frac{x}{a} + C$.

148. $\int x^2\sqrt{4-x^2}\,dx$. Resp.: $2\operatorname{arcsen}\frac{x}{2} - \frac{1}{2}x\sqrt{4-x^2} + \frac{1}{4}x^3\sqrt{4-x^2} + C$.

149. $\int \frac{dx}{x^2\sqrt{1+x^2}}$. Resp.: $-\frac{\sqrt{1+x^2}}{x} + C$.

150. $\int \frac{\sqrt{x^2+a^2}}{x}\,dx$. Resp.: $\sqrt{x^2-a^2} - a\operatorname{arccos}\frac{a}{x} + C$.

151. $\int \frac{dx}{(\sqrt{a^2+x^2})^3}$. Resp.: $\frac{x}{a^2}\frac{1}{\sqrt{a^2+a^2}} + C$.

Integración de funciones racionales:

152. $\int \frac{2x-1}{(x-1)(x-2)}\,dx$. Resp.: $\ln\left|\frac{(x-2)^1}{x-1}\right| + C$.

153. $\int \frac{x\,dx}{(x+1)(x+3)(x+5)}$. Resp.: $\frac{1}{8}\ln\frac{(x+3)^6}{(x+5)^5(x+1)}$.

154. $\int \frac{x^5+x^4-8}{x^3-4x}\,dx$. Resp.: $\frac{x^3}{3} + \frac{x^2}{2} + 4x + \ln\left|\frac{x^2(x-2)^5}{(x+2)^3}\right| + C$.

155. $\int \frac{x^4\,dx}{(x^2-1)(x+2)}$. Resp.: $\frac{x^2}{2} - 2x + \frac{1}{6}\ln\frac{(x-1)}{(x+1)^3} + \frac{16}{3}\ln(x+2) + C$.

156. $\int \frac{dx}{(x-1)^2(x-2)}$. Resp.: $\frac{1}{x-1} + \ln\frac{x-2}{x-1} + C$.

157. $\int \frac{x-8}{x^3-4x^2+4x}\,dx$. Resp.: $\frac{3}{x-2} + \ln\frac{(x-2)^2}{x^2} + C$.

158. $\int \frac{3x+2}{x(x+1)^3}\,dx$. Resp.: $\frac{4x+3}{2(x+1)^2} + \ln\frac{x^2}{(x+1)^2} + C$.

159. $\int \frac{x^2\,dx}{(x+2)^2(x+4)^2}$. Resp.: $-\frac{5x+12}{x^2+6x+8} + \ln\left(\frac{x+4}{x+2}\right)^2 + C$.

160. $\int \frac{dx}{x(x^2+1)}$. Resp.: $\ln\frac{x}{\sqrt{x^2+1}} + C$.

161. $\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}\,dx$. Resp.: $\ln\frac{(x^2-2x+5)^{\frac{3}{2}}}{x-1} + \frac{1}{2}\operatorname{arctg}\frac{x-1}{2} + C$.

162. $\int \frac{x^3-6}{x^4+6x^2+8}\,dx$. Resp.: $\ln\frac{x^2+4}{\sqrt{x^2+2}} + \frac{3}{2}\operatorname{arctg}\frac{x}{2} - \frac{3}{\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}} + C$.

163. $\int \frac{dx}{x^3+1}$. Resp.: $\frac{1}{6}\ln\frac{(x+1)^2}{x^2-x+1} + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} + C$.

164. $\int \frac{3x-7}{x^3+x^2+4x+4}$. Resp.: $\ln\frac{x^2+4}{(x+1)^2} + \frac{1}{2}\operatorname{arctg}\frac{x}{2} + C$.

165. $\int \frac{4dx}{x^4+1}$. Resp.: $\frac{1}{\sqrt{2}}\ln\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1} + \sqrt{2}\operatorname{arctg}\frac{x\sqrt{2}}{1-x^2} + C$.

166. $\int \frac{x^5}{x^3-1}\,dx$. Resp.: $\frac{1}{3}[x^3 + \ln(x^3-1)] + C$.

167. $\int \frac{x^3+x-1}{(x^2+2)^2}\,dx$. Resp.: $\frac{2-x}{4(x^2+2)} + \ln (x^2+2)^{\frac{1}{2}} - \frac{1}{4\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}} + C$.

168. $\int \frac{(4x^2-8x)\,dx}{(x-1)^2(x^2+1)^2}$. Resp.: $\frac{3x^2-1}{(x-1)(x^2+1)} + \ln\frac{(x-1)^2}{x^2+1} + \operatorname{arctg} x + C$.

169. $\int \frac{dx}{(x^2-x)(x^2-x+1)^2}$. Resp.: $\ln\frac{x-1}{x} - \frac{10}{3\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}} - \frac{2x-1}{3(x^2-x+1)} + C$.

Integración de funciones irracionales:

170. $\int \frac{\sqrt{x}}{\sqrt[3]{x^3}+1}\,dx$. Resp.: $\frac{4}{3}[\sqrt[4]{x^3} - \ln(\sqrt[4]{x^3}+1)] + C$.

171. $\int \frac{\sqrt{x^3}-\sqrt[3]{x}}{6\sqrt[6]{x}}\,dx$. Resp.: $\frac{2}{27}\sqrt[6]{x^9} - \frac{2}{13}\sqrt[6]{x^{13}} + C$.

172. $\int \frac{\sqrt[6]{x}+1}{\sqrt[6]{x^7}+\sqrt[6]{x^5}}\,dx$. Resp.: $-\frac{6}{\sqrt[6]{x}} + \frac{12}{\sqrt[3]{x}} + 2\ln x - 24\ln(\sqrt[6]{x}+1) + C$

173. $\int \frac{2+\sqrt[3]{x}}{\sqrt[6]{x}+\sqrt[3]{x}+\sqrt{x}+1}\,dx$. Resp.: $\frac{6}{5}\sqrt[6]{x^5} - \frac{3}{2}\sqrt[3]{x^4} + 4\sqrt[6]{x^3} - 6\sqrt[3]{x^2} + 6\sqrt[6]{x} - 9\ln(\sqrt[6]{x}+1) + \frac{3}{2}\ln(\sqrt[3]{x^2}+1) + 3\operatorname{arctg}\sqrt[6]{x} + C$.

174. $\int \sqrt{\frac{1-x}{1+x}}\frac{dx}{x^2}$. Resp.: $\ln\left|\frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}}\right| - \frac{\sqrt{1-x^2}}{x} + C$.

175. $\int \sqrt{\frac{1-x}{1+x}}\frac{dx}{x}$. Resp.: $2\operatorname{arctg}\sqrt{\frac{1-x}{1+x}} + \ln\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} + C$.

176. $\int \frac{\sqrt[3]{x}+\sqrt{x}}{\sqrt[3]{x^8}+\sqrt[6]{x^{15}}}\,dx$. Resp.: $14\left[\sqrt[6]{x} - \frac{1}{2}\sqrt[3]{x} + \frac{1}{3}\sqrt[6]{x^3} - \frac{1}{4}\sqrt[3]{x^2} + \frac{1}{5}\sqrt[6]{x^5}\right] + C$.

177. $\int \sqrt{\frac{2+3x}{x-3}}\,dx$. Resp.: $\sqrt{3x^2-7x-6} + \frac{11}{2\sqrt{3}}\ln\left(x - \frac{7}{6} + \sqrt{x^2-\frac{7}{3}x-2}\right) + C$.

Integrales del tipo $\int R(x, \sqrt{ax^2+bx+c})\,dx$:

178. $\int \frac{dx}{x\sqrt{x^2-x+3}}$. Resp. $\frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{x^2-x+3}-\sqrt{3}}{x} + \frac{1}{2\sqrt{3}}\right| + C$.

179. $\int \frac{dx}{x\sqrt{2+x-x^2}}$. Resp. $\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2+x-x^2}+\sqrt{2}}{x} + \frac{1}{2\sqrt{2}}\right| + C$.

180. $\int \frac{dx}{x\sqrt{x^2+4x-4}}$. Resp.: $\frac{1}{2}\operatorname{arcsen}\frac{x-2}{x\sqrt{2}} + C$.

181. $\int \frac{\sqrt{x^2+2x}}{x}\,dx$. Resp.: $\sqrt{x^2+2x} + \ln|x+1+\sqrt{x^2+2x}| + C$.

182. $\int \frac{dx}{\sqrt{(2x-x^2)^3}}$. Resp.: $\frac{x-1}{\sqrt{2x-x^2}} + C$.

183. $\int \sqrt{2x-x^2}\,dx$. Resp.: $\frac{1}{2}[(x-1)\sqrt{2x-x^2} + \operatorname{arcsen}(x-1)] + C$.

184. $\int \frac{dx}{x-\sqrt{x^2-1}}$. Resp.: $\frac{x^2}{2} + \frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\ln|x+\sqrt{x^2-1}| + C$.

185. $\int \frac{dx}{(1+x)\sqrt{1+x+x^2}}$. Resp.: $\ln\left|\frac{x+\sqrt{1+x+x^2}}{2+x+\sqrt{1+x+x^2}}\right|$

186. $\int \frac{(x+1)}{(2x+x^2)\sqrt{2x+x^2}}\,dx$. Resp.: $\frac{1}{\sqrt{2x+x^2}} + C$.

187. $\int \frac{1-\sqrt{1+x+x^2}}{x\sqrt{1+x+x^2}}\,dx$. Resp.: $\ln\left|\frac{2+x-2\sqrt{1+x+x^2}}{x^2}\right| + C$.

188. $\int \frac{\sqrt{x^2+4x}}{x^2}\,dx$. Resp.: $-\frac{8}{x+\sqrt{x^2+4x}} + \ln|x+2+\sqrt{x^2+4x}| + C$.

Integrales binomias:

189. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx$. Resp.: $2\left(1+x^{\frac{1}{3}}\right)^{\frac{3}{2}} + C$.

190. $\int x^{\frac{1}{3}}\left(2+x^{\frac{2}{3}}\right)^{\frac{1}{4}}dx$. Resp.: $\frac{10x^{\frac{2}{3}}-16}{15}\left(2+x^{\frac{2}{3}}\right)^{\frac{5}{4}} + C$.

191. $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$. Resp.: $\frac{x}{\sqrt{1+x^2}} + C$.

192. $\int \frac{dx}{x^2(1+x^2)^{\frac{3}{2}}}$. Resp.: $-(1+x^2)^{-\frac{1}{2}}\left(2x+\frac{1}{x}\right)+C.$

193. $\int \sqrt[4]{\left(1+x^{\frac{1}{2}}\right)^3}\,dx$. Resp.: $\frac{8}{77}(7\sqrt{x}-4)(1+\sqrt{x})^{\frac{7}{4}}+C.$

194. $\int \frac{\sqrt{2-\sqrt[3]{x}}}{\sqrt[3]{x}}\,dx$. Resp.: $\frac{2(4+3\sqrt[3]{x})\ (2-\sqrt[3]{x})^{\frac{3}{2}}}{5}$.

195. $\int x^5\sqrt[3]{(1+x^3)^2}\,dx$. Resp.: $\frac{5x^3-3}{40}(1+x^3)^{\frac{5}{3}}$.

Integración de funciones trigonométricas:

196. $\int \operatorname{sen}^3 x\,dx$. Resp.: $\frac{1}{3}\cos^3 x-\cos x+C.$

197. $\int \operatorname{sen}^5 x\,dx$. Resp.: $-\cos x+\frac{2}{3}\cos^3 x-\frac{\cos^5 x}{5}+C.$

198. $\int \cos^4 x \operatorname{sen}^3 x$. Resp.: $-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C.$

199. $\int \frac{\cos^3 x}{\operatorname{sen}^4 x}\,dx$. Resp.: $\operatorname{cosec} x-\frac{1}{3}\operatorname{cosec}^3 x+C.$

200. $\int \cos^2 x\,dx$. Resp.: $\frac{x}{2}+\frac{1}{4}\operatorname{sen} 2x+C.$

201. $\int \operatorname{sen}^4 x\,dx$. Resp.: $\frac{3}{8}x-\frac{\operatorname{sen} 2x}{4}+\frac{\operatorname{sen} 4x}{32}+C.$

202. $\int \cos^6 x\,dx$. Resp.: $\frac{1}{16}\left(5x+4\operatorname{sen} 2x-\frac{\operatorname{sen}^3 2x}{3}+\frac{3}{4}\operatorname{sen} 4x\right)+C.$

203. $\int \operatorname{sen}^4 x\cos^4 x\,dx$. Resp.: $\frac{1}{128}\left(3x-\operatorname{sen} 4x+\frac{\operatorname{sen} 8x}{8}\right)+C.$

204. $\int \operatorname{tg}^3 x\,dx$. Resp.: $\frac{\operatorname{tg}^2 x}{2}+\ln|\cos x|+C.$

205. $\int \operatorname{cotg}^5 x\,dx$. Resp.: $-\frac{1}{4}\operatorname{cotg}^4 x+\frac{1}{2}\operatorname{cotg}^2 x+\ln|\operatorname{sen} x|+C.$

206. $\int \operatorname{cotg}^3 x\,dx$. Resp.: $-\frac{\operatorname{cotg}^2 x}{2}-\ln|\operatorname{sen} x|+C.$

207. $\int \sec^8 x\,dx$. Resp.: $\frac{\operatorname{tg}^7 x}{7}+\frac{3\operatorname{tg}^5 x}{5}+\operatorname{tg}^3 x+\operatorname{tg} x+C.$

208. $\int \operatorname{tg}^4 x\sec^4 x\,dx$. Resp.: $\frac{\operatorname{tg}^7 x}{7}+\frac{\operatorname{tg}^5 x}{5}+C.$

209. $\int \frac{dx}{\cos^4 x}$. Resp.: $\operatorname{tg} x+\frac{1}{3}\operatorname{tg}^3 x+C.$

210. $\int \frac{\cos x}{\operatorname{sen}^2 x}\,dx$. Resp.: $C-\operatorname{cosec} x.$

211. $\int \frac{\operatorname{sen}^3 x\,dx}{\sqrt[3]{\cos^4 x}}$. Resp.: $\frac{3}{5}\cos^{\frac{5}{3}} x+3\cos^{-\frac{1}{3}} x+C.$

212. $\int \operatorname{sen} x\operatorname{sen} 3x\,dx$. Resp.: $-\frac{\operatorname{sen} 4x}{8}+\frac{\operatorname{sen} 2x}{4}+C.$

213. $\int \cos 4x\cos 7x\,dx$. Resp.: $\frac{\operatorname{sen} 11x}{22}+\frac{\operatorname{sen} 3x}{6}+C.$

214. $\int \cos 2x\operatorname{sen} 4x\,dx$. Resp.: $-\frac{\cos 6x}{12}-\frac{\cos 2x}{4}+C.$

215. $\int \operatorname{sen}\frac{1}{4}x\cos\frac{3}{4}x\,dx$. Resp.: $-\frac{\cos x}{2}+\cos\frac{1}{2}x+C.$

216. $\int \frac{dx}{4-5\operatorname{sen} x}$. Resp.: $\frac{1}{3}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-2}{2\operatorname{tg}\frac{x}{2}-1}\right|+C.$

217. $\int \frac{dx}{5-3\cos x}$. Resp.: $\frac{1}{2}\operatorname{arctg}\left|2\operatorname{tg}\frac{x}{2}\right|+C.$

218. $\int \frac{\operatorname{sen} x\,dx}{1+\operatorname{sen} x}$. Resp.: $\frac{2}{1+\operatorname{tg}\frac{x}{2}}+x+C.$

219. $\int \frac{\cos x\,dx}{1+\cos x}$. Resp.: $x-\operatorname{tg}\frac{x}{2}+C.$

220. $\int \frac{\operatorname{sen} 2x}{\cos^4 x+\operatorname{sen}^4 x}\,dx$. Resp.: $\operatorname{arctg}(2\operatorname{sen}^2 x-1)+C.$

221. $\int \frac{dx}{(1+\cos x)^2}$. Resp.: $\frac{1}{2}\operatorname{tg}\frac{x}{2}+\frac{1}{6}\operatorname{tg}^3\frac{x}{2}+C.$

222. $\int \frac{dx}{\operatorname{sen}^2 x+\operatorname{tg}^2 x}$. Resp.: $-\frac{1}{2}\left[\operatorname{cotg} x+\frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right)\right]+C.$

223. $\int \frac{\operatorname{sen}^2 x}{1+\cos^2 x}\,dx$. Resp.: $\sqrt{2}\operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right)-x+C.$

$\int \frac{4\cos^2 x}{\operatorname{tg}^2 x\,(1+\cos^2 x)^2}\,dx$