7. Gaia – Zirkuitu elektrikoen egoera iragankorra eta korronte alternoa

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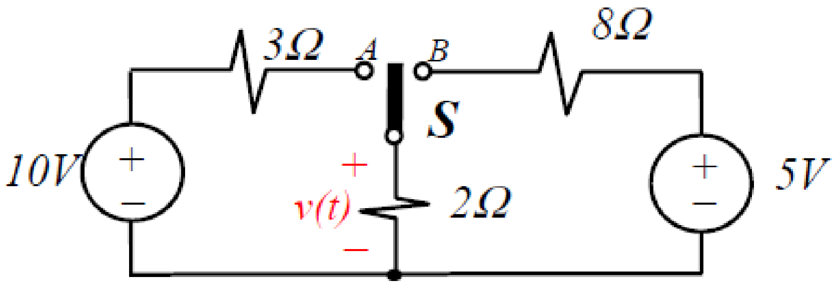
Gaiaren gai-zerrenda

1. Egoera iragankorra zirkuitu linealetan
2. RC zirkuitua
    - Karga prozesua eta denbora konstantea
    - Deskarga prozesua eta denbora konstantea
    - Erantzuna seinale karratuei
3. RL zirkuitua
4. Zirkuitu linealak korronte alternoan

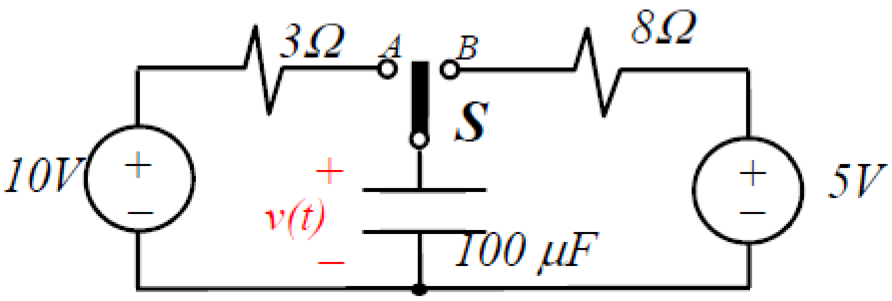
1. Egoera iragankorra zirkuitu linealetan

- Zirkuitu **erresistibo batean**, zirkuituan edozein aldaketak **berehalako aldaketa** sortarazten du zirkuituaren egoeran



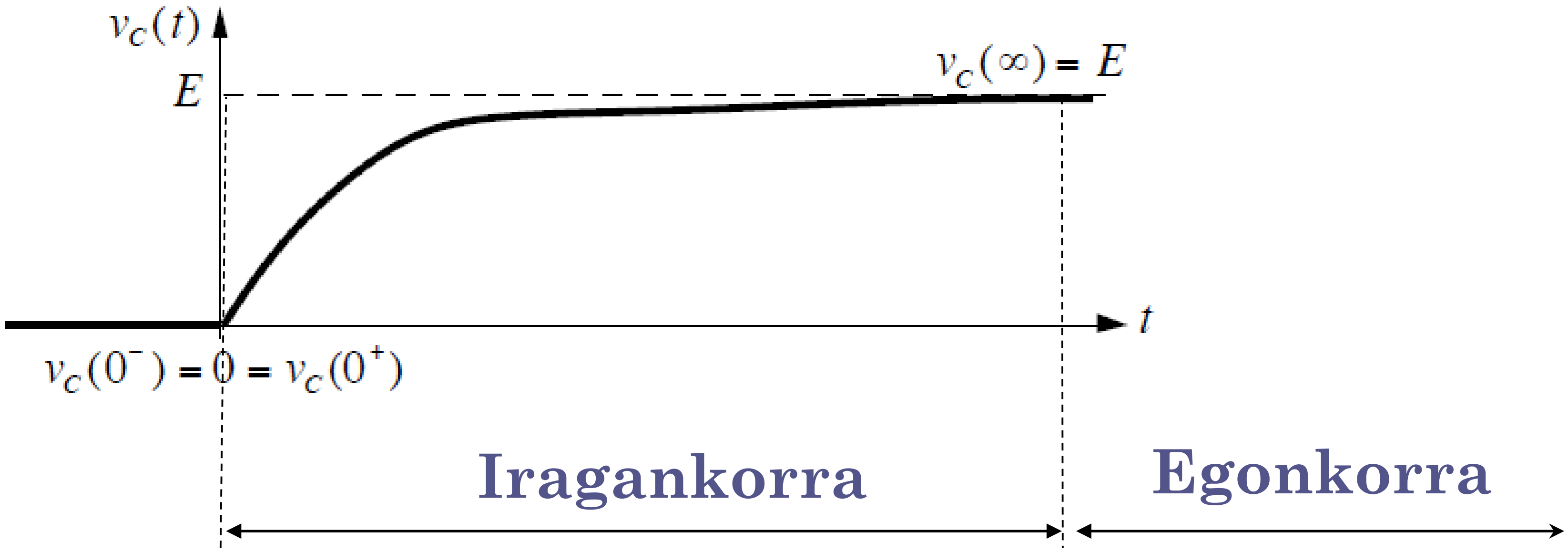
$$v(t) = R \cdot i(t)$$

- **Kondentsadore** bat badago, **oreka** egoerara (egoera egonkorra) heltzeko **denbora bat** (egoera iragankorra) behar da, kondentsadorearen portaera dela eta

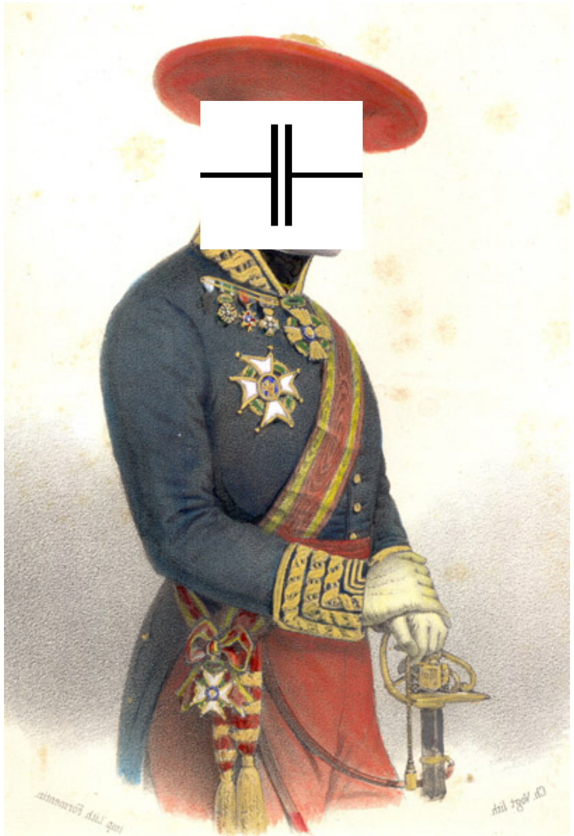


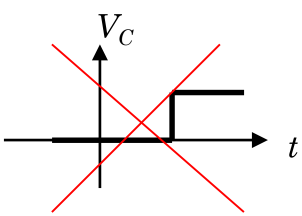
$$i(t) = C\frac{dv(t)}{dt}$$

1. Egoera iragankorra zirkuitu linealetan

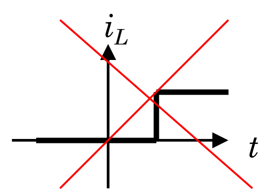


2. RC ZIRKUITUA





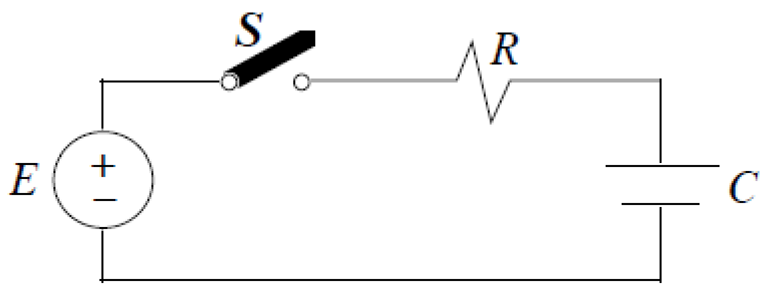
$$\boxed{i(t) = C\frac{dv_C(t)}{dt}}$$



$$\boxed{v(t) = L\frac{di_L(t)}{dt}}$$

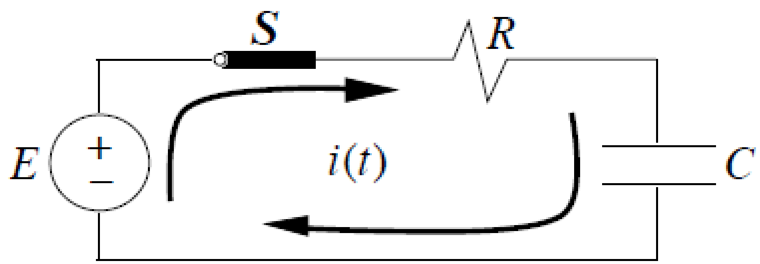
2. RC Zirkuitua (DC)

- **Zirkuitua:**



$$V_C = 0 \rightarrow Q_c = 0$$

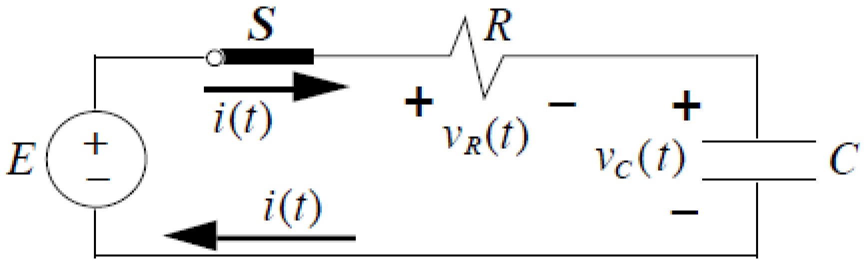
- **Karga prozesua:**



  - Etengailua itxi → Aldaketa: t = 0
  - Kondentsadorea kargatzen hasi → Egoera iragankorra

2. RC Zirkuitua (DC)

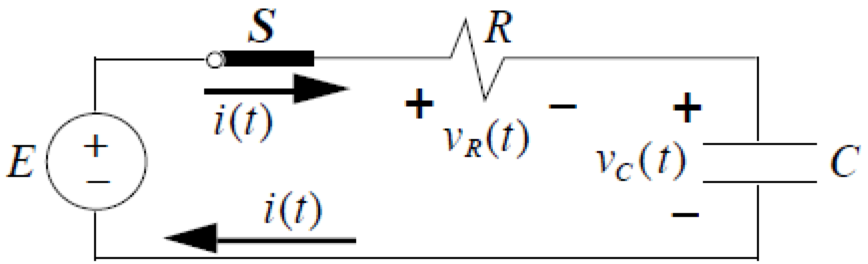
- **Karga prozesua:**



- Portaera ekuazioak:
  - Erresistentzia: $v_R(t) = Ri(t)$
  - Kondentsadorea: $i(t) = C\dfrac{dv_C(t)}{dt}$
- Ondorioa: $v_R(t) = RC\dfrac{dv_C(t)}{dt}$
- KVL (KTL) $E = v_R(t) + v_C(t)$

2. RC ZIRKUITUA (DC)

- **Karga prozesua:**



- Ekuazio diferentziala:

$$E = v_R(t) + v_C(t)$$

$$E = RC\frac{dv_C(t)}{dt} + v_C(t)$$

$$\frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = \frac{E}{RC}$$

- Soluzio orokorra:

$$v_C(t) = K_1 \cdot e^{-\frac{t}{RC}} + K_2$$

  - K_1 eta K_2 konstanteak zirkuituaren hasierako (t=0) eta bukaerako (t=∞) egoeren menpekoak dira.

2. RC ZIRKUITUA (DC)

- **Karga prozesua:** $v_C(t) = K_1 e^{-\frac{t}{RC}} + K_2$

  - Hasierako egoera egonkorra (t=0):

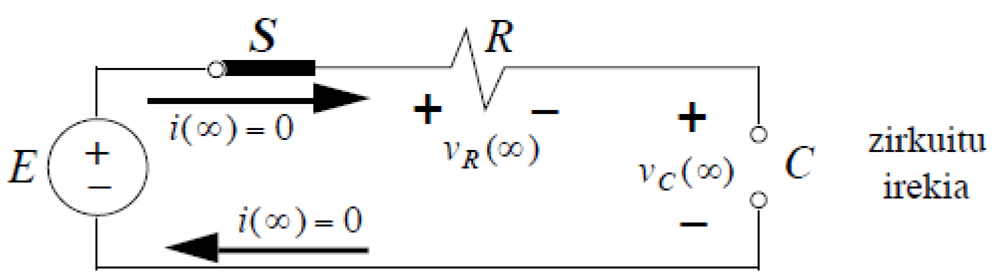
$$v_C(0^-) = 0V$$

$$v_C(0^+) = K_1 + K_2$$

$$v_C(t^-) = v_C(t^+) \rightarrow v_C(0^-) = v_C(0^+)$$

$$K_1 + K_2 = 0$$

  - Bukaerako egoera egonkorra (t=∞)



$$E = v_R(\infty) + v_C(\infty) = Ri(\infty) + v_C(\infty) = v_C(\infty) \rightarrow v_C(\infty) = E$$

$$v_C(\infty) = K_1 e^{-\infty} + K_2 = K_1 \cdot 0 + K_2 = K_2 \rightarrow K_2 = E$$

2. RC ZIRKUITUA (DC)

$$K_1 + K_2 = 0$$

- **Karga prozesua:**

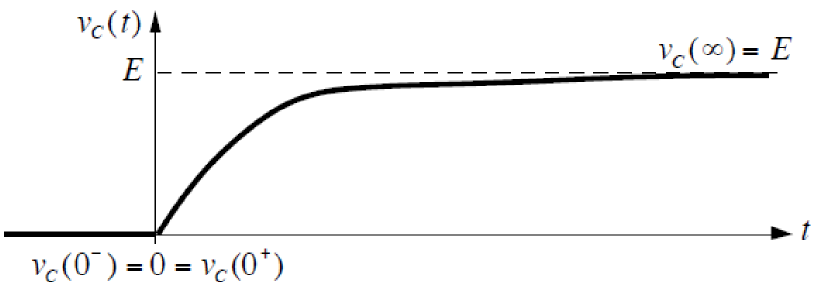
$$K_2 = \mathrm{E} \quad \text{eta} \quad K_1 = -E$$

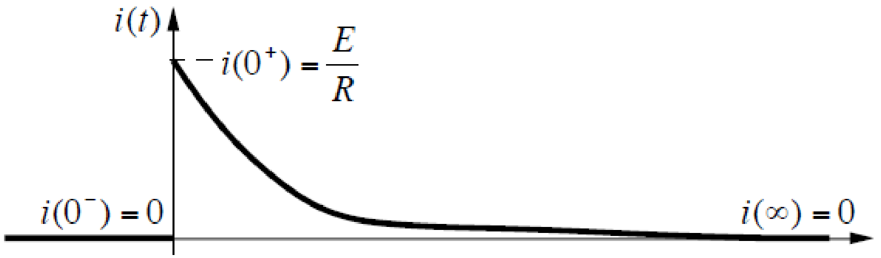
- Tentsioa: $$v_C(t) = E \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$
- Korrontea: $$i(t) = C\frac{dv_C(t)}{dt} \rightarrow i(t) = \frac{E}{R} \cdot e^{-\frac{t}{RC}}$$
- Tentsio erresistentzian $$v_R(t) = E \cdot e^{-\frac{t}{RC}}$$
- Kondentsadorean metaturiko karga

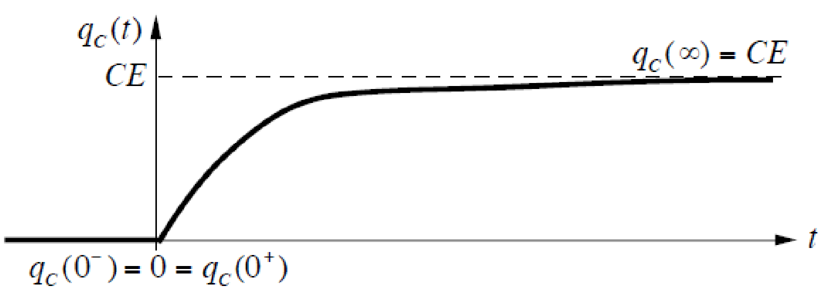
$$q_C(t) = CE \cdot \left(1 - e^{-\frac{t}{RC}}\right)$$

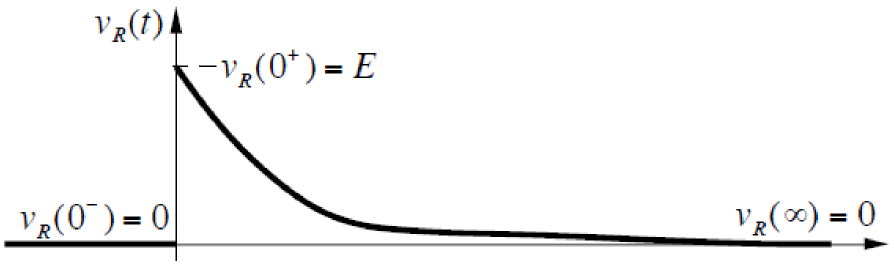
2. RC ZIRKUITUA (DC)

- **Karga prozesua:**









2. RC ZIRKUITUA (DC)

Karga prozesua – Denbora konstantea:

$$\tau = RC$$

$$v_C(t=\tau) = E\left(1-e^{-\frac{\tau}{RC}}\right) = E\cdot\left(1-e^{-1}\right) = 0.63E;\ q_C(t=\tau) = 0.63CE$$

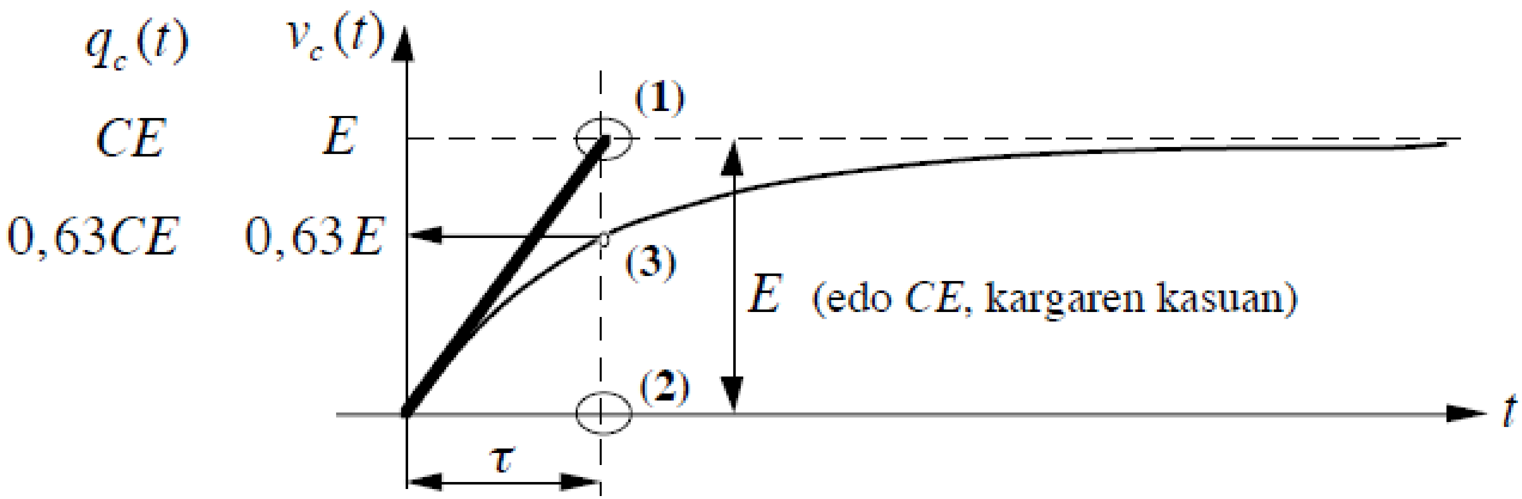
$$i(t=\tau) = \frac{E}{R}\cdot e^{-\frac{\tau}{RC}} = \frac{E}{R}\cdot e^{-1} = 0.37\cdot\frac{E}{R};\ \mathrm{v}_R(t=\tau) = 0.37\cdot E$$

- **Definizioa:** RC zirkuitu baten denbora-konstantea, hasierako unetik kondentsadoreak orekan izango duen tentsioaren (kargaren) % 63ko tentsioa (karga) lortu arte igarotzen den denbora-tartea da.

2. RC ZIRKUITUA (DC)

- **Karga prozesua – Denbora konstantea:**
  - Tentsioa:

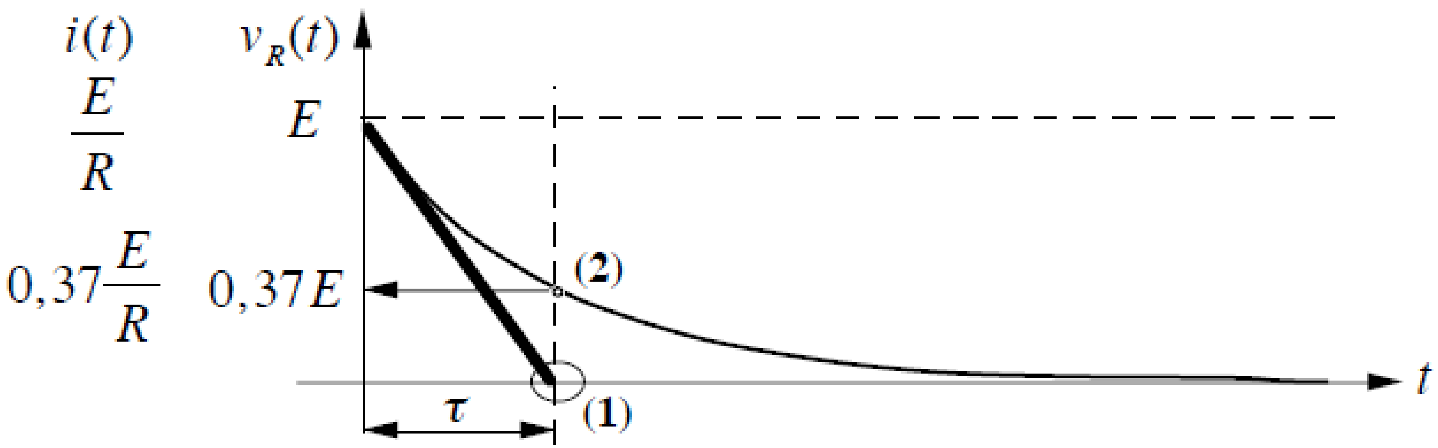
$$\left[\frac{dv_C(t)}{dt}\right]_{t=0} = \left[\frac{d}{dt}\left[E\left(1-e^{-\frac{t}{RC}}\right)\right]\right]_{t=0} = \frac{E}{RC} = \frac{E}{\tau}$$



2. RC Zirkuitua (DC)

- **Karga prozesua – Denbora konstantea:**
  - Korrontea:

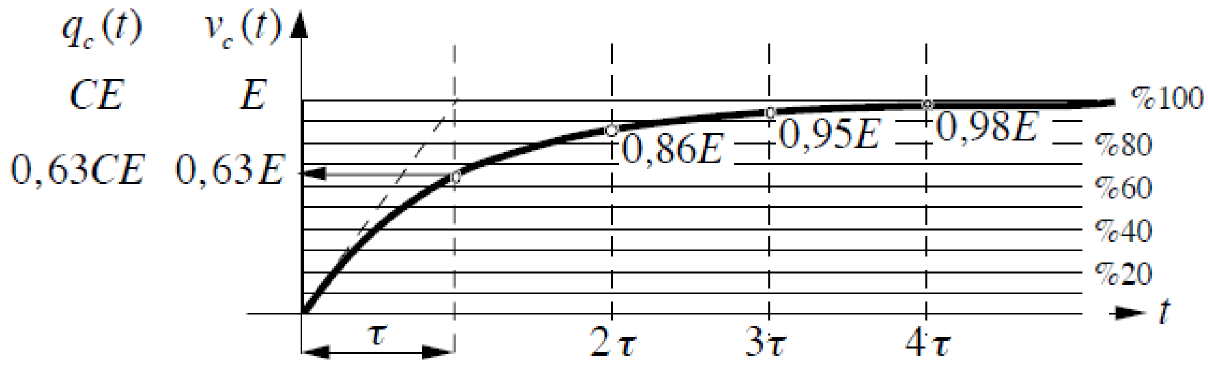
$$\left[\frac{dv_R(t)}{dt}\right]_{t=0} = \left[\frac{d}{dt}\left(E\cdot e^{-\frac{t}{RC}}\right)\right]_{t=0} = -\frac{E}{RC} = -\frac{E}{\tau}$$

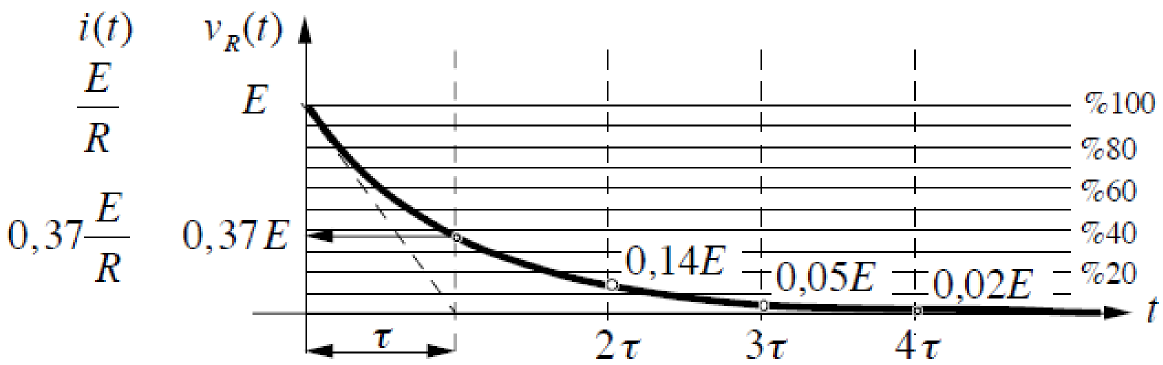


2. RC ZIRKUITUA (DC)

- **Karga prozesua – Denbora konstantea:**

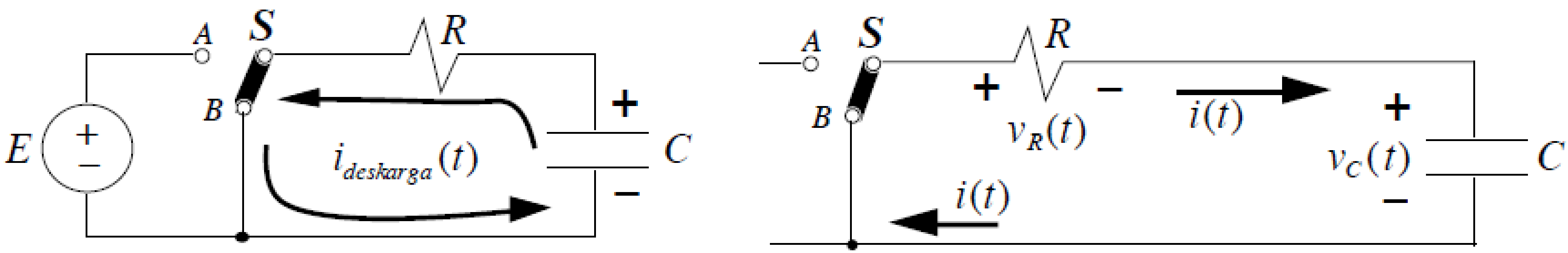
$t=\tau$	$v_C(\tau)=0,63E$	$q_C(\tau)=0,63CE$	$i(\tau)=0,37\frac{E}{R}$	$v_R(\tau)=0,37E$
$t=2\tau$	$v_C(2\tau)=0,86E$	$q_C(2\tau)=0,86CE$	$i(2\tau)=0,14\frac{E}{R}$	$v_R(2\tau)=0,14E$
$t=3\tau$	$v_C(3\tau)=0,95E$	$q_C(3\tau)=0,95CE$	$i(3\tau)=0,05\frac{E}{R}$	$v_R(3\tau)=0,05E$
$t=4\tau$	$v_C(4\tau)=0,98E$	$q_C(4\tau)=0,98CE$	$i(4\tau)=0,02\frac{E}{R}$	$v_R(4\tau)=0,02E$





2. RC Zirkuitua (DC)

- **Deskarga prozesua:**

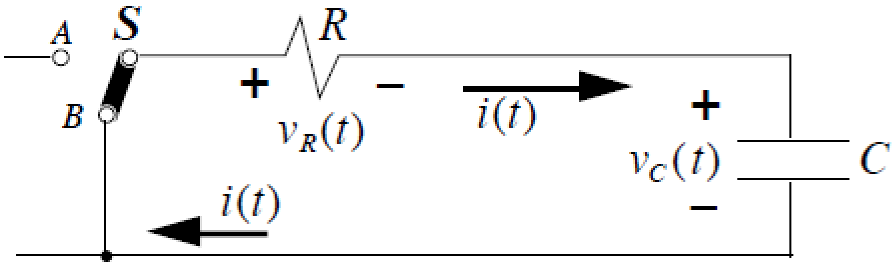


- Portaera ekuazioak:
  - Erresistentzia: $v_R(t) = Ri(t)$
  - Kondentsadorea: $i(t) = C\dfrac{dv_C(t)}{dt}$
- Ondorioa: $v_R(t) = RC\dfrac{dv_C(t)}{dt}$
- KVL (KTL) → HEMEN DESBERDINA

$$0 = v_R(t) + v_C(t)$$

2. RC ZIRKUITUA (DC)

Deskarga prozesua:



- Ekuazio diferentziala:

$$0 = v_R(t) + v_C(t)$$

$$0 = RC\frac{dv_C(t)}{dt} + v_C(t)$$

$$\frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = 0$$

- Soluzio orokorra:

$$v_C(t) = K_1 \cdot e^{-\frac{t}{RC}} + K_2$$

  - K_1 eta K_2 konstanteak zirkuituaren hasierako (t=0) eta bukaerako (t=∞) egoeren menpekoak

2. RC Zirkuitua (DC)

- **Deskarga prozesua:** $v_C(t) = K_1 \cdot e^{-\frac{t}{RC}} + K_2$

  - Hasierako egoera egonkorra (t=0):

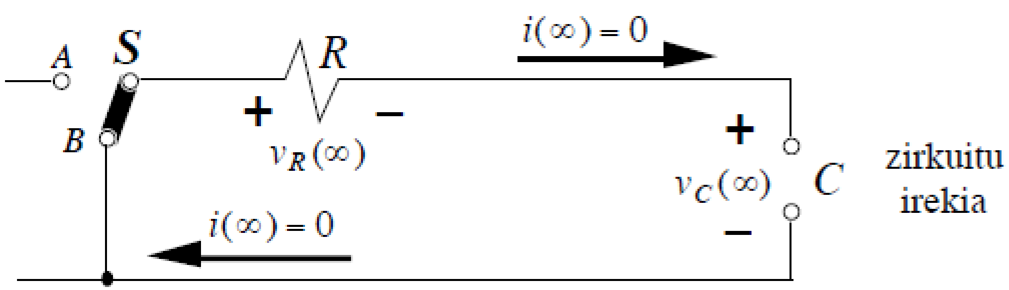
$$v_C(0^-) = E\ V$$

$$v_C(0^+) = K_1 + K_2$$

$$v_C(t^-) = v_C(t^+) \rightarrow v_C(0^-) = v_C(0^+)$$

$$K_1 + K_2 = E$$

  - Bukaerako egoera egonkorra (t=∞)



$$0 = v_R(\infty) + v_C(\infty) = Ri(\infty) + v_C(\infty) = v_C(\infty) \rightarrow v_C(\infty) = 0$$

$$v_C(\infty) = K_1 e^{-\infty} + K_2 = K_1 \cdot 0 + K_2 = K_2 \rightarrow K_2 = 0$$

2. RC ZIRKUITUA (DC)

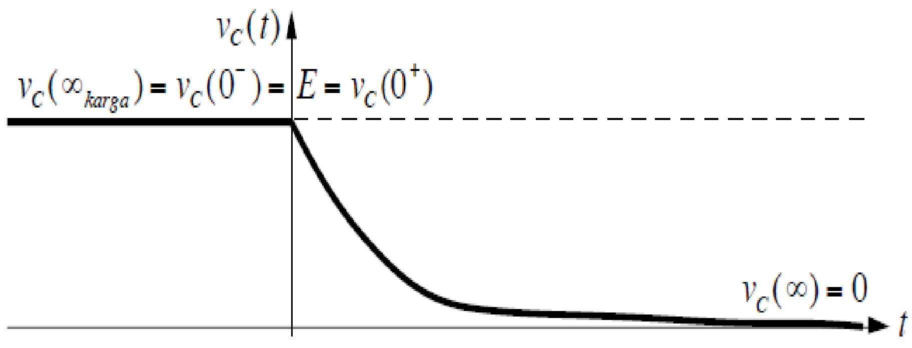
- **Deskarga prozesua:** $K_1 + K_2 = 0$

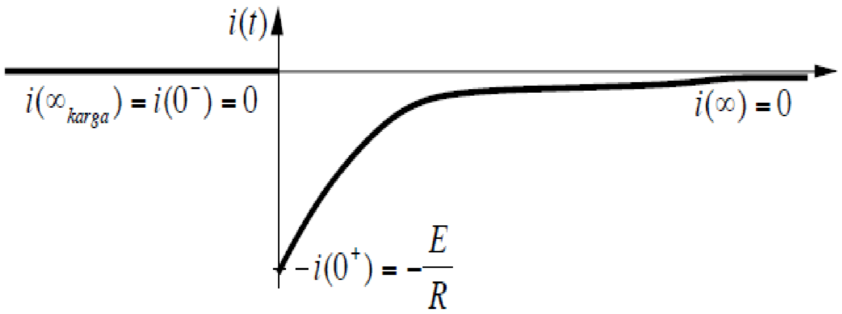
  $K_2 = 0 \quad \text{eta} \quad K_1 = E$

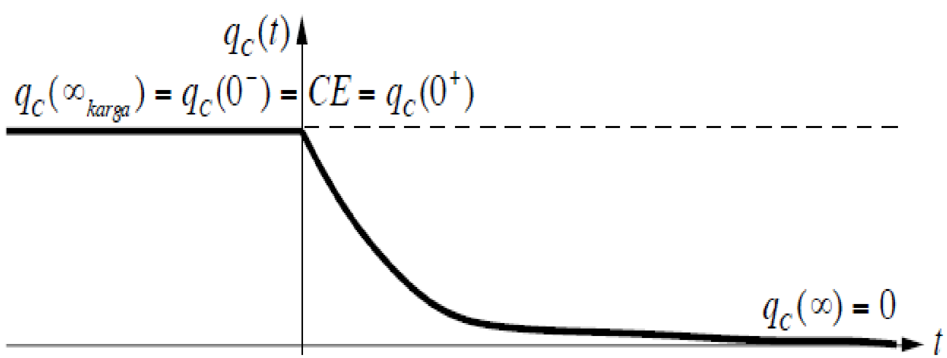
  - Tentsioa: $v_C(t) = E \cdot e^{-\frac{t}{RC}}$
  - Korrontea: $i(t) = C \cdot \dfrac{dv_C(t)}{dt} \rightarrow i(t) = -\dfrac{E}{R} \cdot e^{-\frac{t}{RC}}$
  - Tentsio erresistentzian $v_R(t) = -E \cdot e^{-\frac{t}{RC}}$
  - Kondentsadorean metaturiko karga $q_C(t) = CE \cdot e^{-\frac{t}{RC}}$

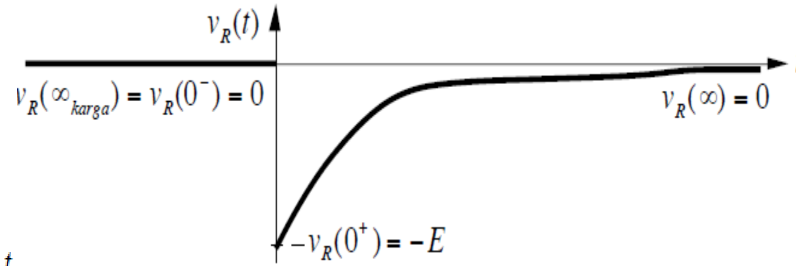
2. RC ZIRKUITUA (DC)

- **Deskarga prozesua:**







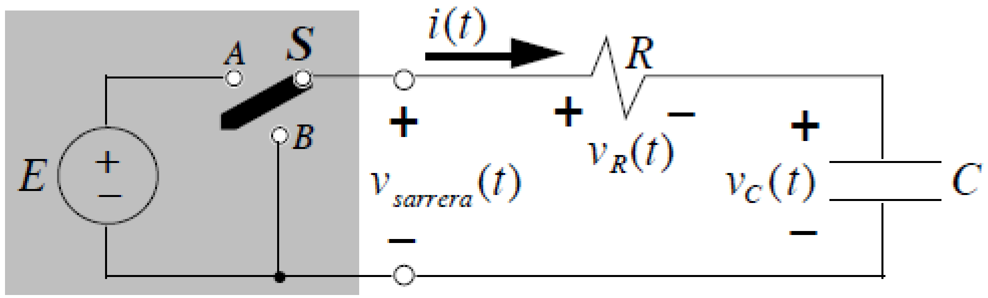


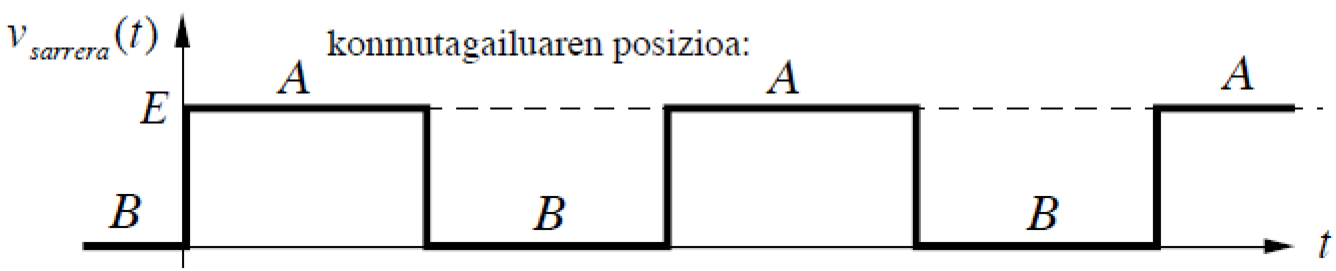
- **Denbora konstantea:**

$$\tau_{karga} = R_{karga}C \quad \text{eta} \quad \tau_{deskarga} = R_{deskarga}C$$

2. RC Zirkuitua

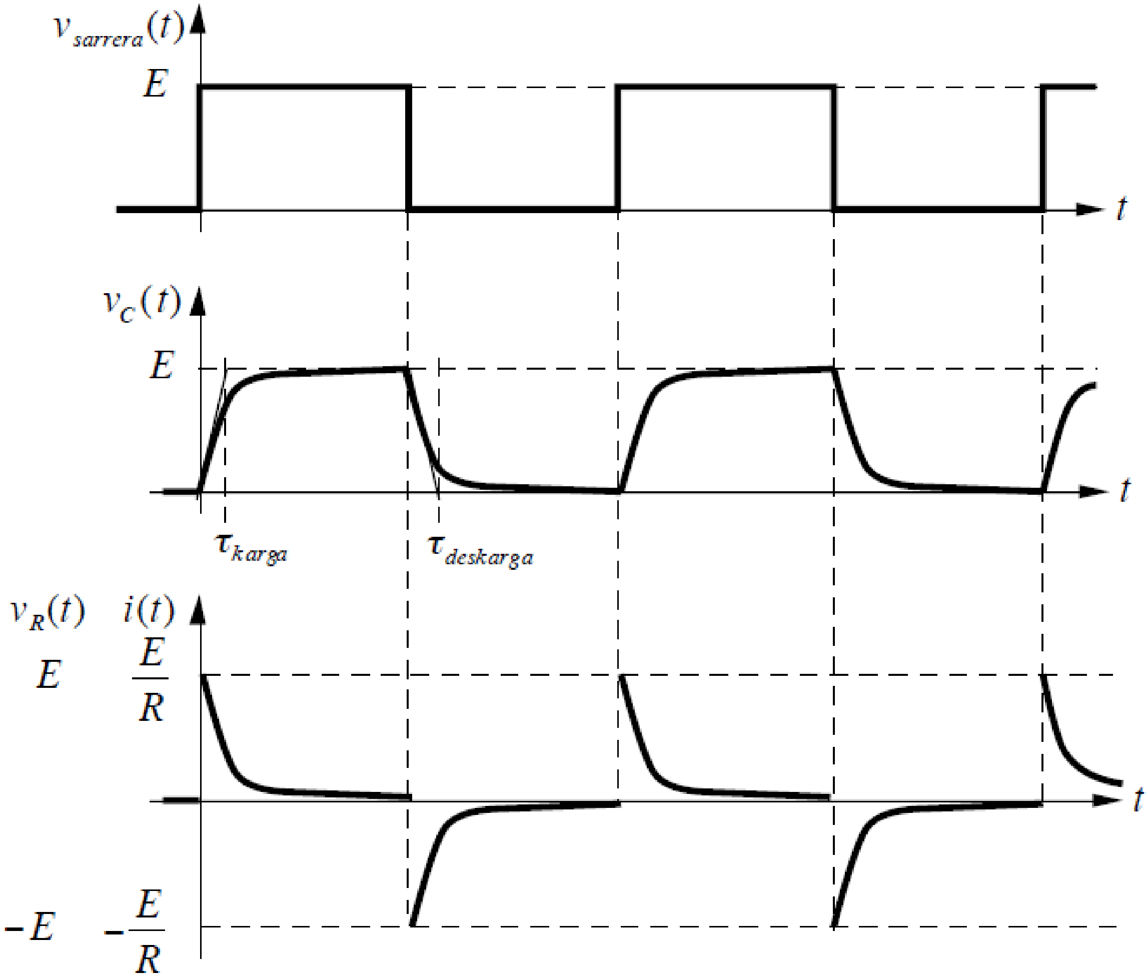
- **Erantzuna seinale karratuei:**





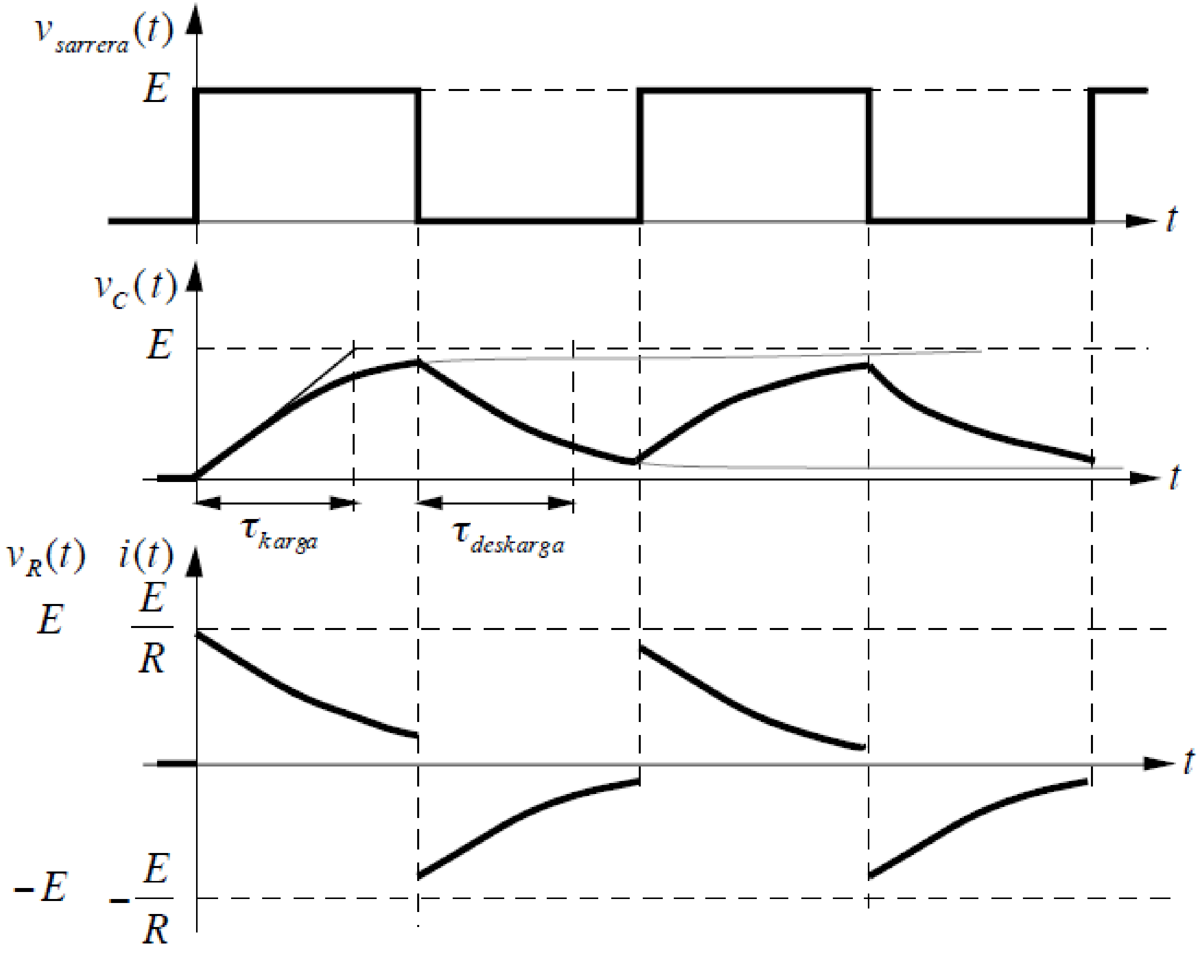
2. RC ZIRKUITUA

- **Erantzuna seinale karratuei:**
  - 1. kasua: (T/2)>>4τ



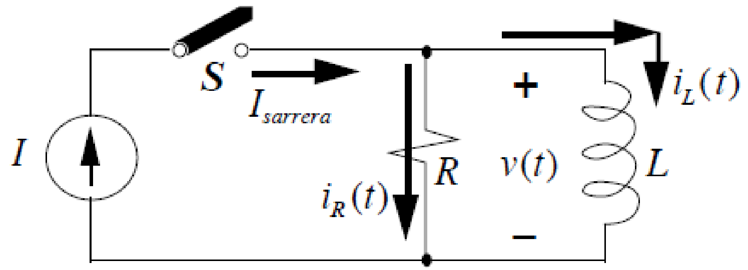
2. RC ZIRKUITUA

- **Erantzuna seinale karratuei:**
  - 2. kasua: (T/2)<<4τ



3. RL ZIRKUITUA (DC)

- **Zirkuitua:**



$Q_c = 0$

- Portaera ekuazioak:
  - Erresistentzia: $v(t) = Ri_R(t)$
  - Harila: $$v(t) = L\frac{di_L(t)}{dt}$$
  - Ondorioa: $$i_R(t) = \frac{L}{R}\frac{di_L(t)}{dt}$$

- KCL (KKL)

$$I_{sarrera} = i_R(t) + i_L(t) \rightarrow I_{sarrera} = \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t)$$

3. RL Zirkuitua (DC)

- **Karga prozesua:**

Etengailua itxita: $I_{sarrera} = I$

Ekuazioa: $\dfrac{di_L(t)}{dt} + \dfrac{R}{L} i_L(t) = \dfrac{R}{L} \cdot I$

- **Deskarga prozesua:**

Etengailua irekita: $I_{sarrera} = 0$

Ekuazioa: $\dfrac{di_L(t)}{dt} + \dfrac{R}{L} i_L(t) = 0$

Soluzio orokorra: $i_L(t) = K_1 \cdot e^{-\frac{R}{L}t} + K_2$

Denbora konstantea: $\tau = \dfrac{L}{R}$

Karga prozesua:

Hasierako egoera:

$$0 = i_L(0^-) = i_L(0^+) = K_1 + K_2$$

Bukaerako egoera:

$$I = i_L(\infty) = K_2$$

Konstanteen balioak:

$$K_2 = I \quad \text{eta} \quad K_1 = -I$$

Soluzio partikularra:

$$i_L(t) = I \cdot \left(1 - e^{-\frac{R}{L}t}\right)$$

Deskarga prozesua:

Hasierako egoera:

$$I = i_L(0^-) = i_L(0^+) = K_1 + K_2$$

Bukaerako egoera:

$$0 = i_L(\infty) = K_2$$

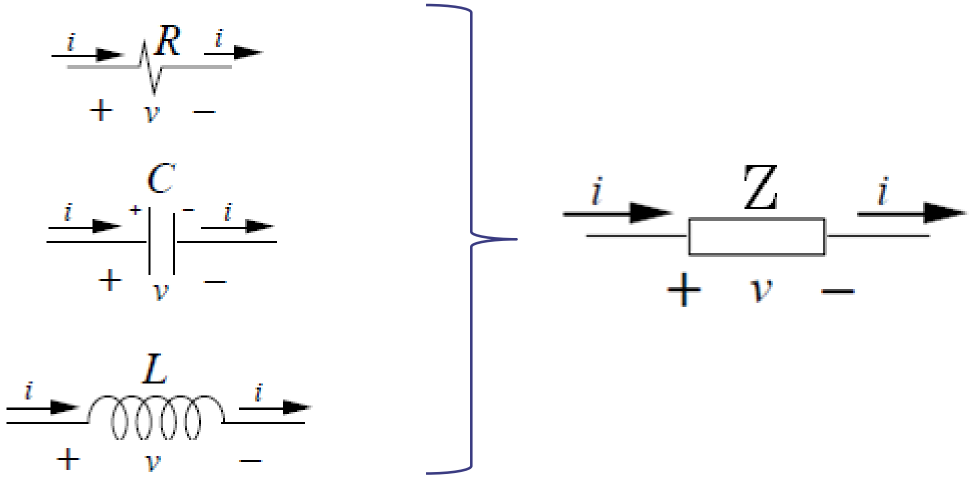
Konstanteen balioak:

$$K_2 = 0 \quad \text{eta} \quad K_1 = I$$

Soluzio partikularra:

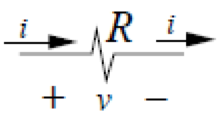
$$i_L(t) = I \cdot e^{-\frac{R}{L}t}$$

- **Inpedantzia kontzeptua**

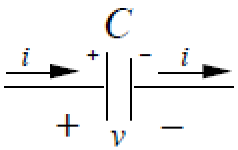


$$Z = R + jX\left[\Omega\right]$$

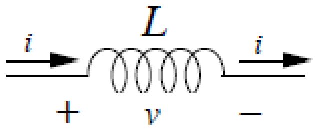
$$G = Y + jB\left[s\right]$$



$$Z_R = R$$



$$Z_c = -jX_c \rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$



$$Z_L = jX_L \rightarrow X_L = \omega L = 2\pi fL$$

4. ZIRKUITU LINEALAK KORRONTE ALTERNOAN

- **Erregimen sinusoidala:** Sarrerako seinale sinusoidalak direnean, zirkuitu elektrikoek lortzen duten egoera geldikorra

$$v_{in} = V_{in} \cos(\omega_0 t + \theta_{in})$$

$$v_{out} = V_{out} \cos(\omega_0 t + \theta_{out})$$

  - Zirkuitu linealek ez dute sarrerako seinalearen harmonikorik sortzen ezta beste edozein maiztasuneko seinalerik ere ez.
- Ekuazio diferentzialak → Zenbaki konplexuak
  - × Zirkuituen egoera geldikorra baino ez da lortzen
  - ✓ Kalkuluak asko errazten dira → **Ekuazio aljebraiko lineala**

4. Zirkuitu linealak korronte alternoan

Fasoreak:

- Tentsio edo korronte sinusoidal bat denboraren eremuan:

$$x(t) = X_m \cos(\omega_0 t + \theta)$$

- Bere $\tilde{X}$ fasorea zenbaki konplexu bat da

$$\tilde{X} = X_m e^{j\theta} = X_m \lfloor\theta$$

  - Normalean polarretan baina koordenatu kartesiarretan ere

$$\tilde{X} = X_m e^{j\theta} = X_{RE} + jX_{im} \qquad X_m = \sqrt{X_{\mathrm{Re}}^2 + X_{\mathrm{Im}}^2} \qquad \theta = \arctan\left(\frac{X_{\mathrm{Im}}}{X_{\mathrm{Re}}}\right)$$

  - Euler-en formularen arabera

$$\tilde{X} = X_m e^{j\theta} = X_m(\cos\theta + \mathrm{j} \cdot \sin\theta)$$

- $\tilde{X}$ fasoretik abiatuz x(t) denboraren eremuko seinalea erraz berreskura daiteke

$$x(t) = Re\left(\tilde{X} e^{jw_0 t}\right)$$

4. Zirkuitu linealak korronte alternoan

- **Fasoreak:**
  - Fasoreekin lan egitean zirkuitu-teorian azaldutako axioma eta emaitza guztiak aplikagarriak dira
    - Zirkuitu dinamikoak erregimen sinusoidalean ebazteko zirkuitu erresistiboak ebazteko erabiltzen den metodologia berbera erabil daiteke
    - Alde bakarra zenbaki konplexuekin lan egin beharko dugula izango da
    - Inpedantzia konplexua: Tentsio fasore bat eta korronte fasore baten arteko erlazioa

$$Z(jw) = \frac{\tilde{V}}{\tilde{I}} = R + jX$$

      - Ohm-etan neurtzen da
      - Ez da fasorea (ez da denboraren menpekoa)
      - Orokorrean maiztasunaren menpekoa

7. GAIA – ZIRKUITU ELEKTRIKOEN EGOERA IRAGANKORRA ETA KORRONTE ALTERNOA

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