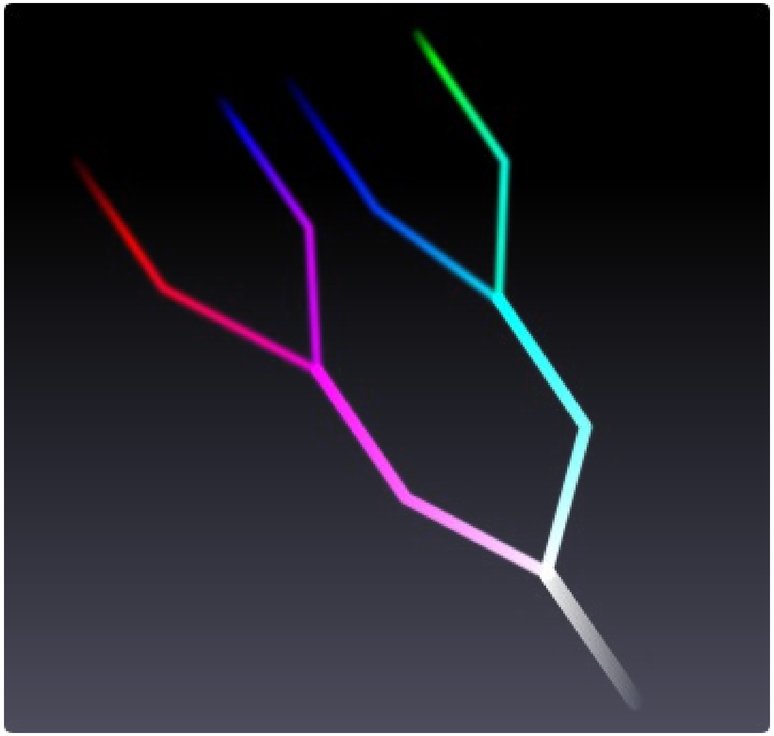
Running Coupling in the Standard Model and Beyond



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Project Aims

“The three coupling constants of the Standard Model vary (run) with energy, admitting the possibility of unifying at a single point (a *Grand Unified Theory*, or GUT).

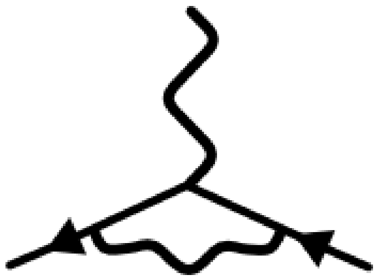
Determine – by analysing these running couplings – whether models admit a Grand Unified Theory, and if so at what scale this unification occurs.”

Outline of this Presentation

- Why Do They Run?
  - Ultraviolet Divergences and Renormalisation
- How Do We Quantify Unification?
- How Do They Run?
  - Standard Model
  - Minimal Supersymmetric Standard Model
    - Fixing Unification in the MSSM
  - (Compactified) Extra Dimensions

Ultraviolet Divergences

- QED (and other quantum field theories) admit amplitude integrals that diverge at high energies/short distances:



$\to \infty$ as $\mu \to \infty$

- To avoid this, we introduce:
  - An ultraviolet cutoff Λ *(regularisation)*
  - Bare and physical mass/coupling *(renormalisation)*

Divergences are 'absorbed' into these terms

Renormalisation

- The process of renormalisation makes parameters of the theory *scale-dependent*
- This scale-dependence is characterised by the *beta-function:*

$$\beta(g)=\mu\frac{\partial g}{\partial\mu}$$

- The coupling 'constant' therefore varies *(runs)* with energy: *'running coupling'*

The Standard Model

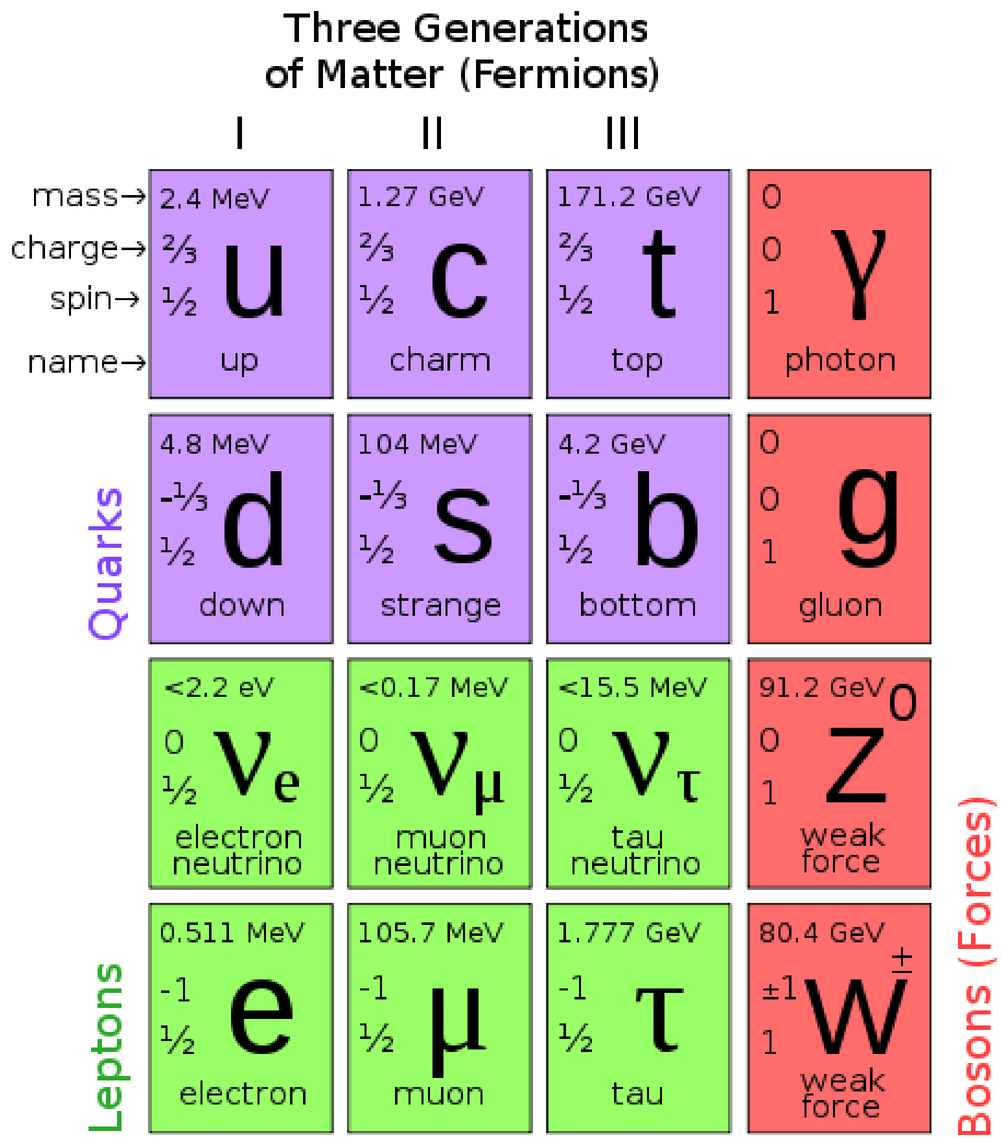


Image Source: wikipedia.org, 'Standard Model'

Beta-Functions of the Standard Model

- Considering one- and two-loop corrections,

$$\mu \frac{d}{d\mu}\alpha_i(\mu) = \frac{1}{2\pi}\left[b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu)\right]\alpha_i^2(\mu)$$

where $\alpha_i(\mu) = g_i^2/4\pi$ gives the coupling strength at an energy scale μ, and $i, j = 1,2,3$ denote the U(1), SU(2) and SU(3) sectors of the Standard Model.

- Alternatively,

$$\mu \frac{d}{d(\ln\mu)}\alpha_i^{-1}(\mu) = \frac{-1}{2\pi}\left[b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu) + O(\alpha_j^2)\right]$$

Quantifying Unification

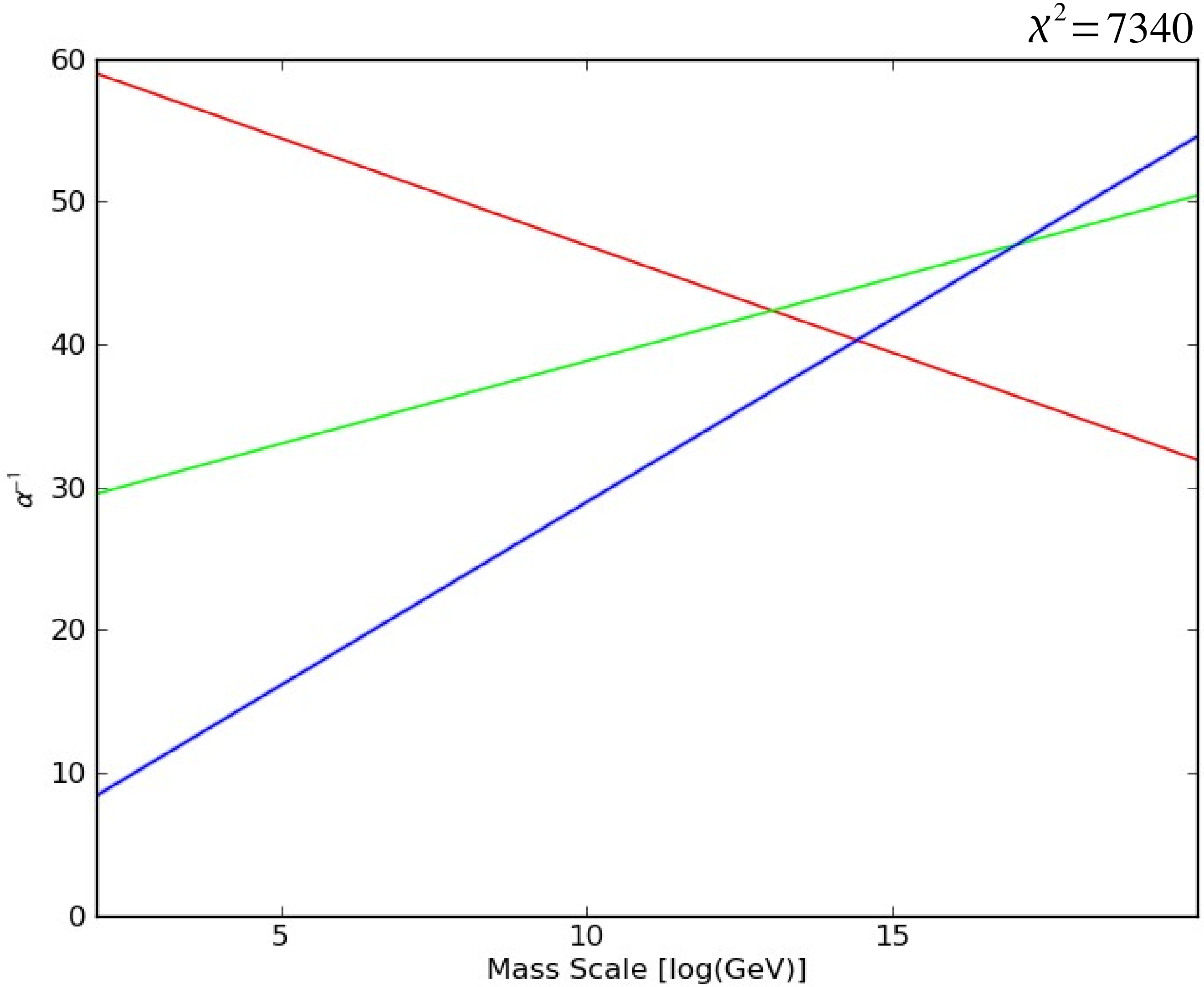
- We can quantify unification:

$$\chi^2 = \min_\mu \left\{ \sum_{i=1}^{3} \left(\frac{\alpha_i^{-1}(\mu) - \overline{\alpha^{-1}}(\mu)}{\sigma[\alpha_i^{-1}(M_Z)]} \right)^2 \right\}$$

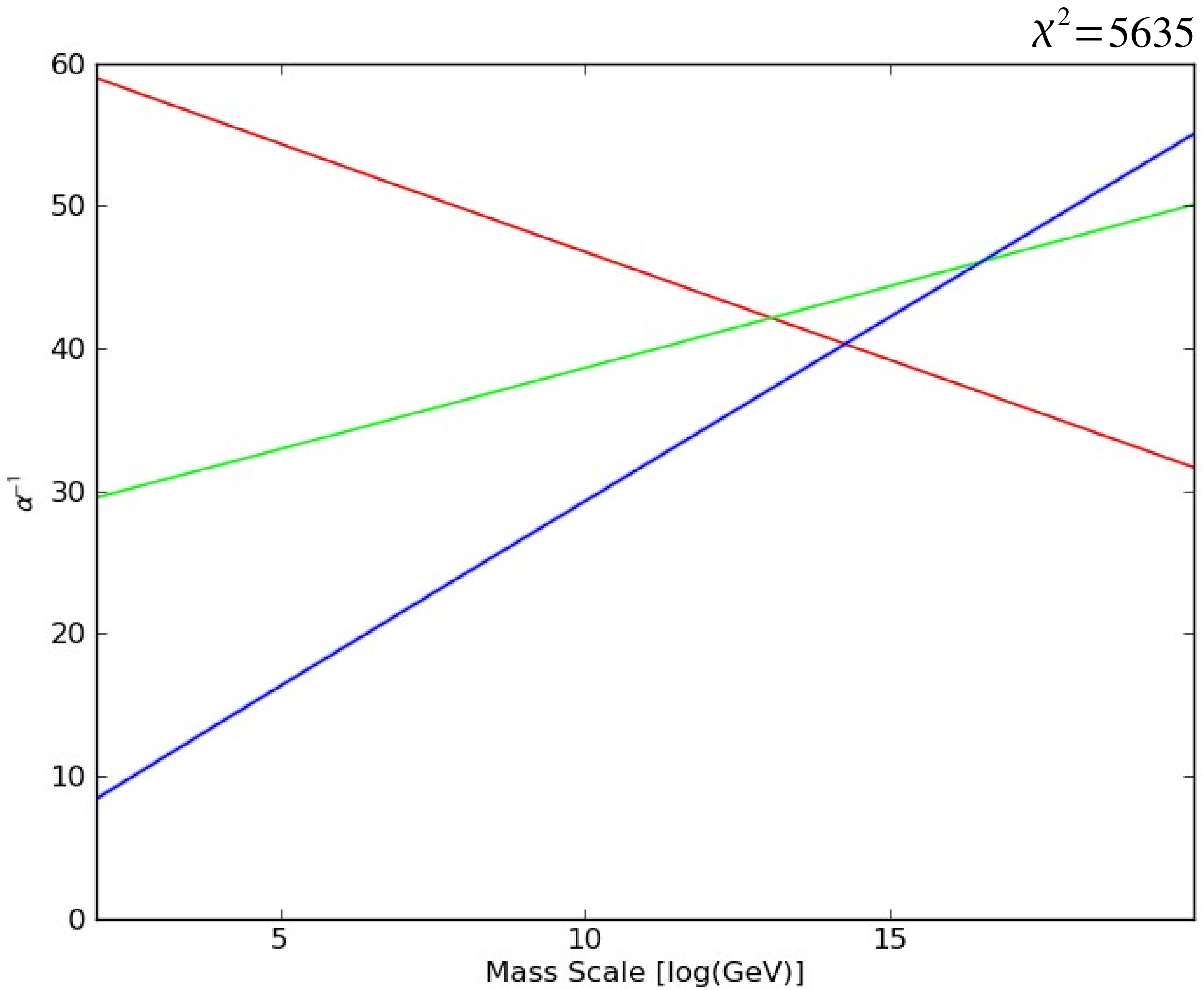
  where $\overline{\alpha^{-1}}(\mu)$ is the mean of $\alpha_{1,2,3}^{-1}$ at μ, and $\sigma[x]$ denotes the error in x

- The value of μ for which χ^2 is a minimum is the *unification energy* M_{GUT}, with associated *unification coupling* $\alpha_{\mathrm{GUT}}^{-1}$

Running Coupling in the Standard Model (1-loop Corrections)



Running Coupling in the Standard Model (2-loop Corrections)



The Minimal Supersymmetric Standard Model (MSSM)

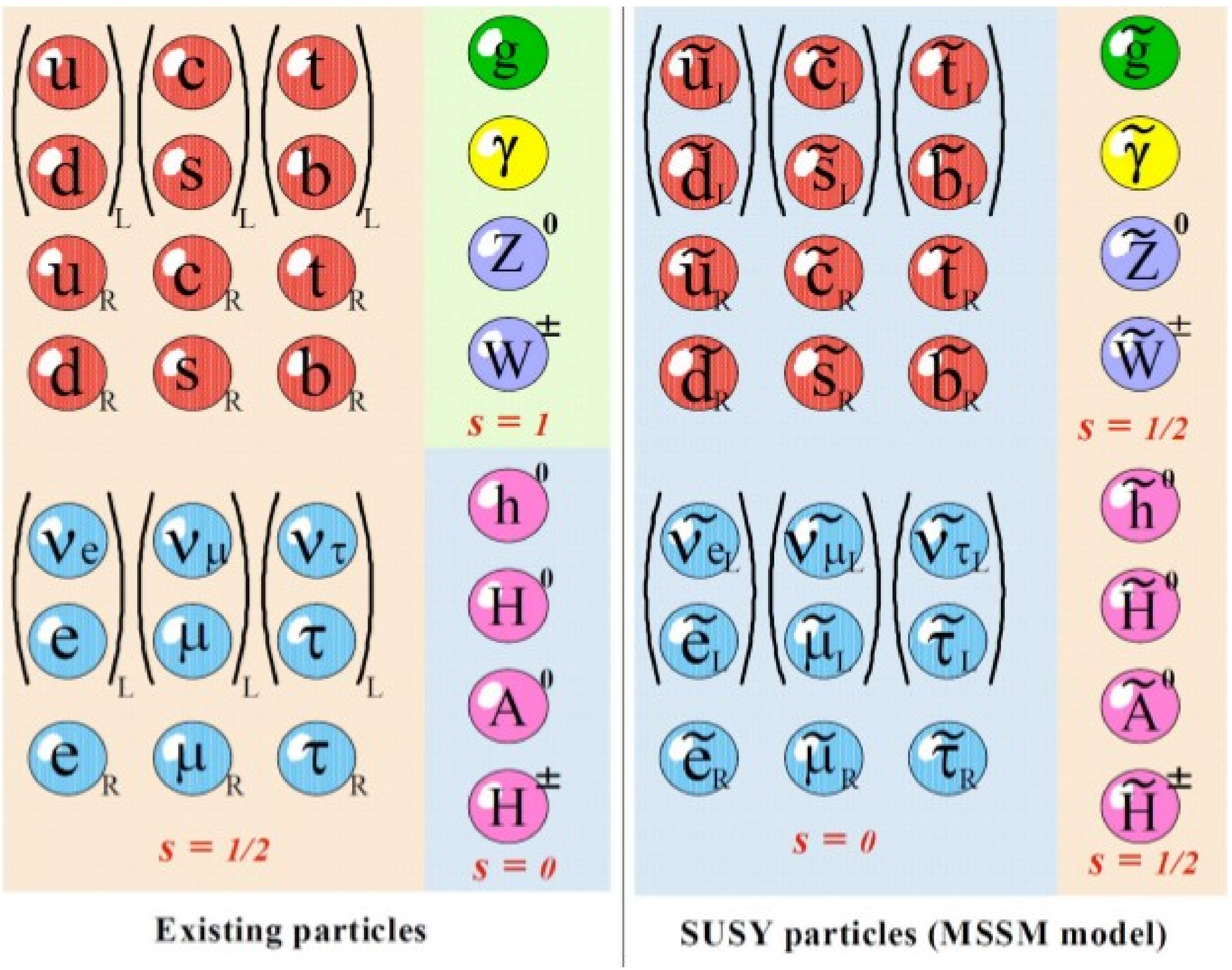


Image Source: “Looking for SUSY at the LHC”, KEK, April 2010

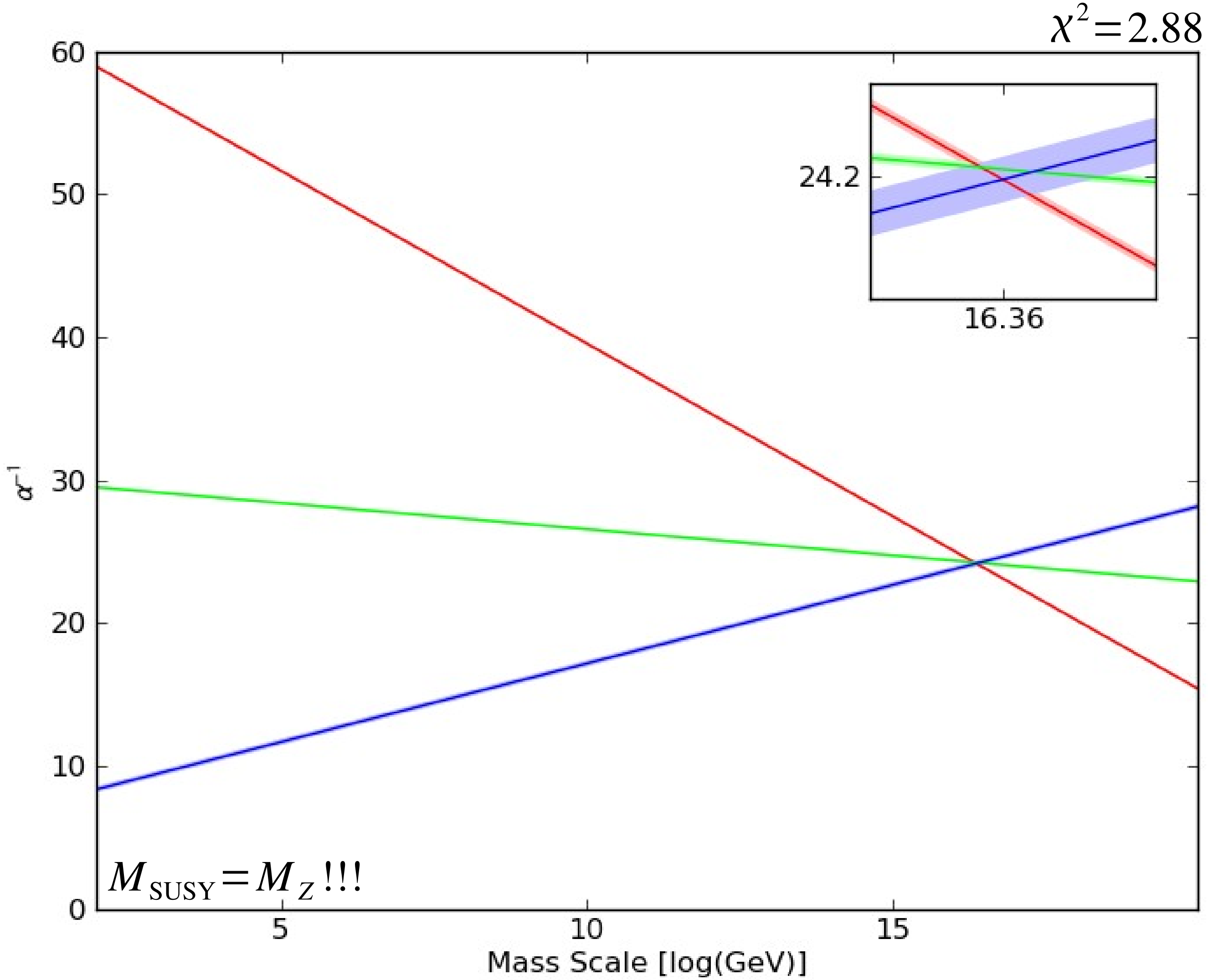
Beta-Functions of the MSSM

- The form of the beta-function remains the same:

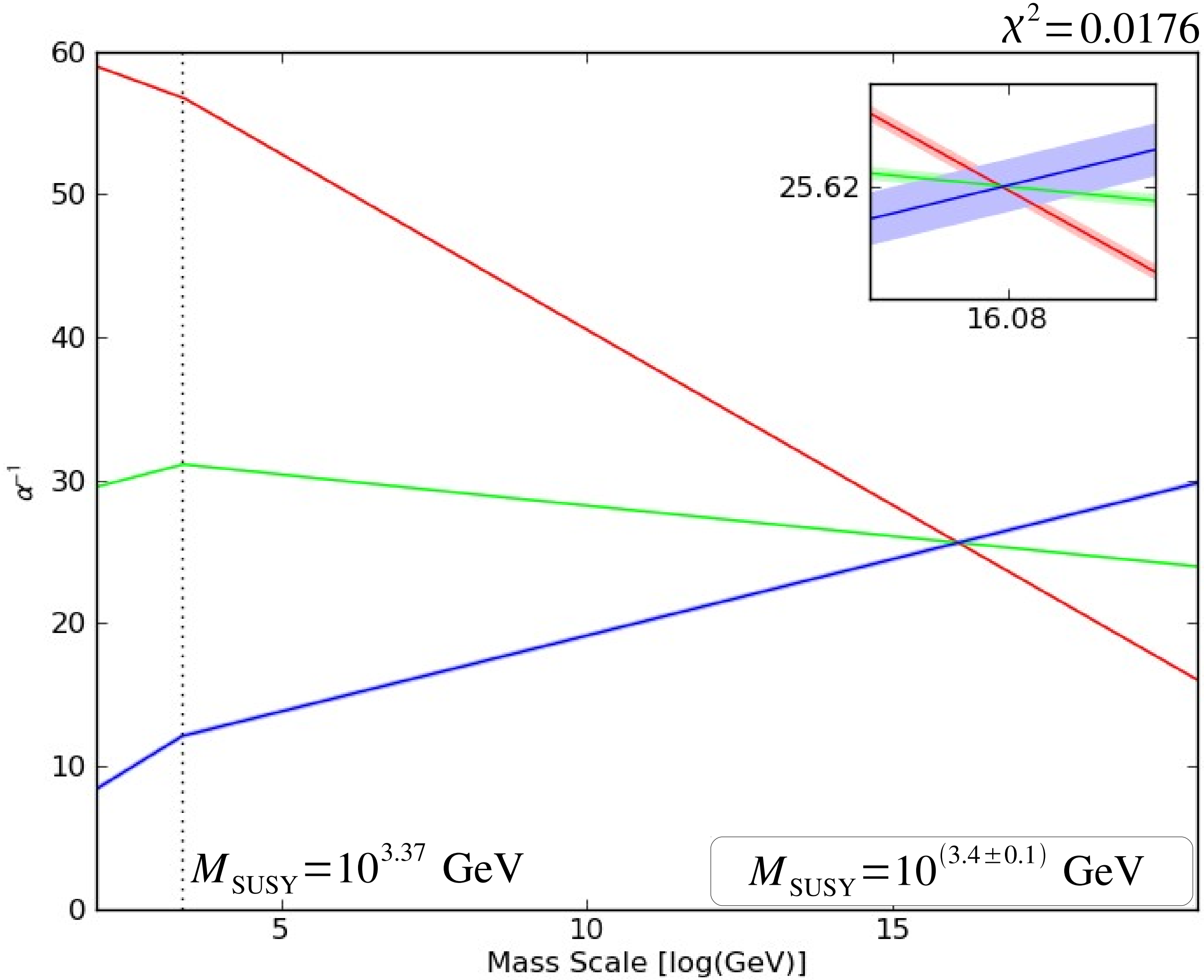
$$\mu \frac{d}{d(\ln \mu)} \alpha_i^{-1}(\mu) = \frac{-1}{2\pi}\left[b_i + \sum_{j=1}^{3} \frac{b_{ij}}{4\pi} \alpha_j(\mu) + O(\alpha_j^2) \right]$$

- We introduce the *SUSY breaking scale* M_{SUSY}
- At energies $\mu \leq M_{\mathrm{SUSY}}$, we use SM values for b_i and b_{ij}
- When $\mu > M_{\mathrm{SUSY}}$, we use MSSM values
- M_{SUSY} is a free parameter!

Running Coupling in the MSSM (1-loop Corrections)



Running Coupling in the MSSM (2-loop Corrections)



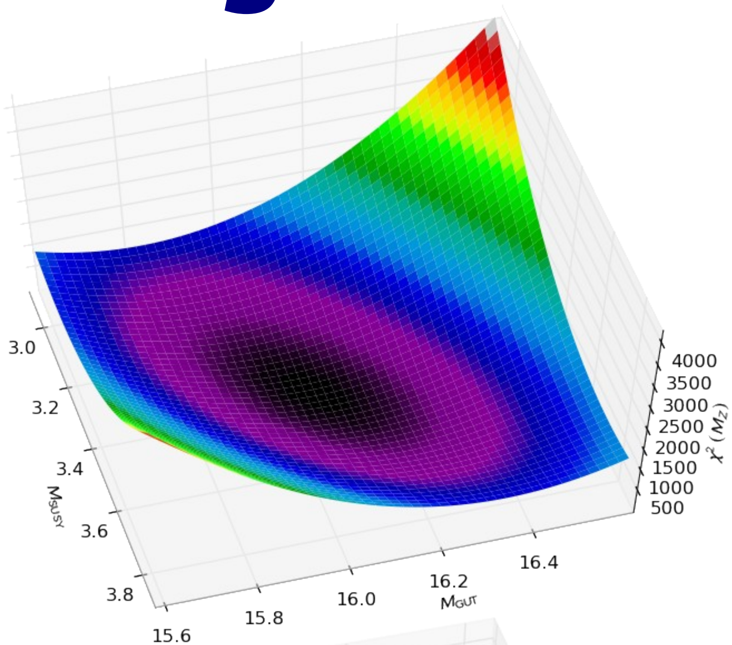
Fixing Unification

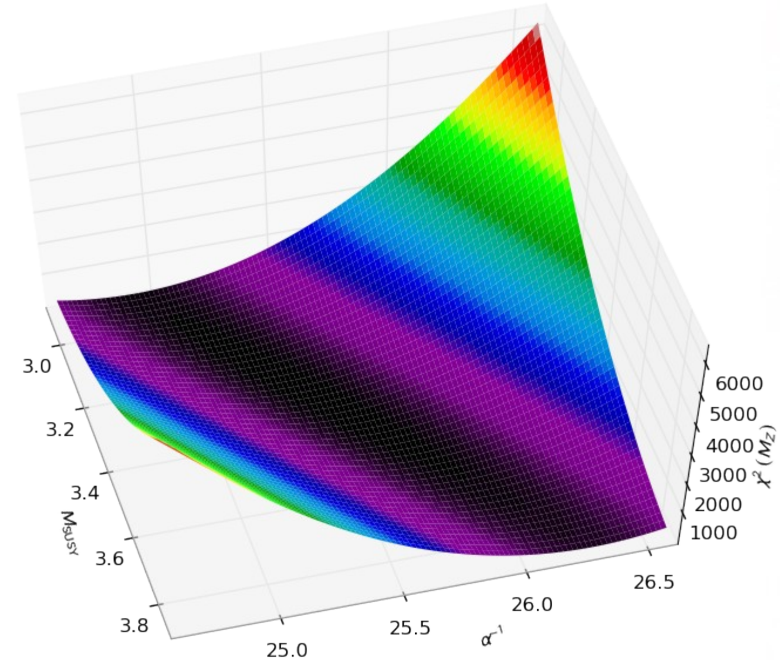
- Idea: Instead of measuring unification based on experimental parameters, we do the reverse: *fix unification* ($\alpha_{\mathrm{GUT}}^{-1}, M_{\mathrm{GUT}}$) *and extrapolate backwards,* then determine how close we are to experimental values
  - No need to propagate errors
- We define a new measure of 'goodness of fit':

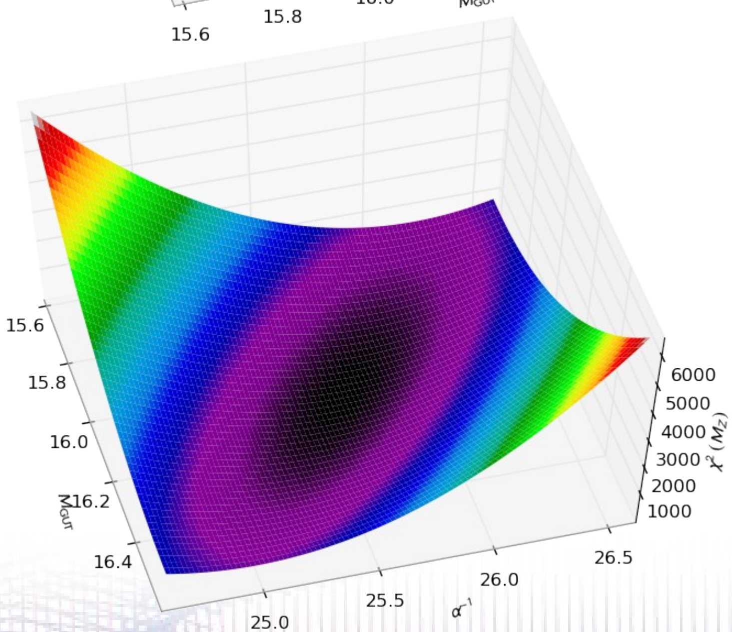
$$\chi_{M_Z}^2 = \sum_{i=1}^{3} \left(\frac{\alpha_{i,\mathrm{calc}}^{-1}(M_Z) - \alpha_{i,\mathrm{exp}}^{-1}(M_Z)}{\sigma[\alpha_{i,\mathrm{exp}}^{-1}(M_Z)]} \right)^2$$

- 3 free parameters in MSSM: $\alpha_{\mathrm{GUT}}^{-1}, M_{\mathrm{GUT}}, M_{\mathrm{SUSY}}$

Fixing Unification in the MSSM







$$M_{\mathrm{GUT}} = 10^{(16.09 \pm 0.01)}\ \mathrm{GeV}$$

$$M_{\mathrm{SUSY}} = 10^{(3.39 \pm 0.01)}\ \mathrm{GeV}$$

$$\alpha_{\mathrm{GUT}}^{-1} = 25.59 \pm 0.02$$

$$\chi_{M_Z}^2 = 6.6 \times 10^{-12}$$

(Compactified) Extra Dimensions

- Extra dimensions have not yet been observed – if they exist, they must be currently out of observational bounds
- Idea: extra dimensions are *small* and 'curled up' (compactified) into circles of radius R
  - Implies that we expect effects of extra dimensions at an energy $\mu_0 = 1/R$

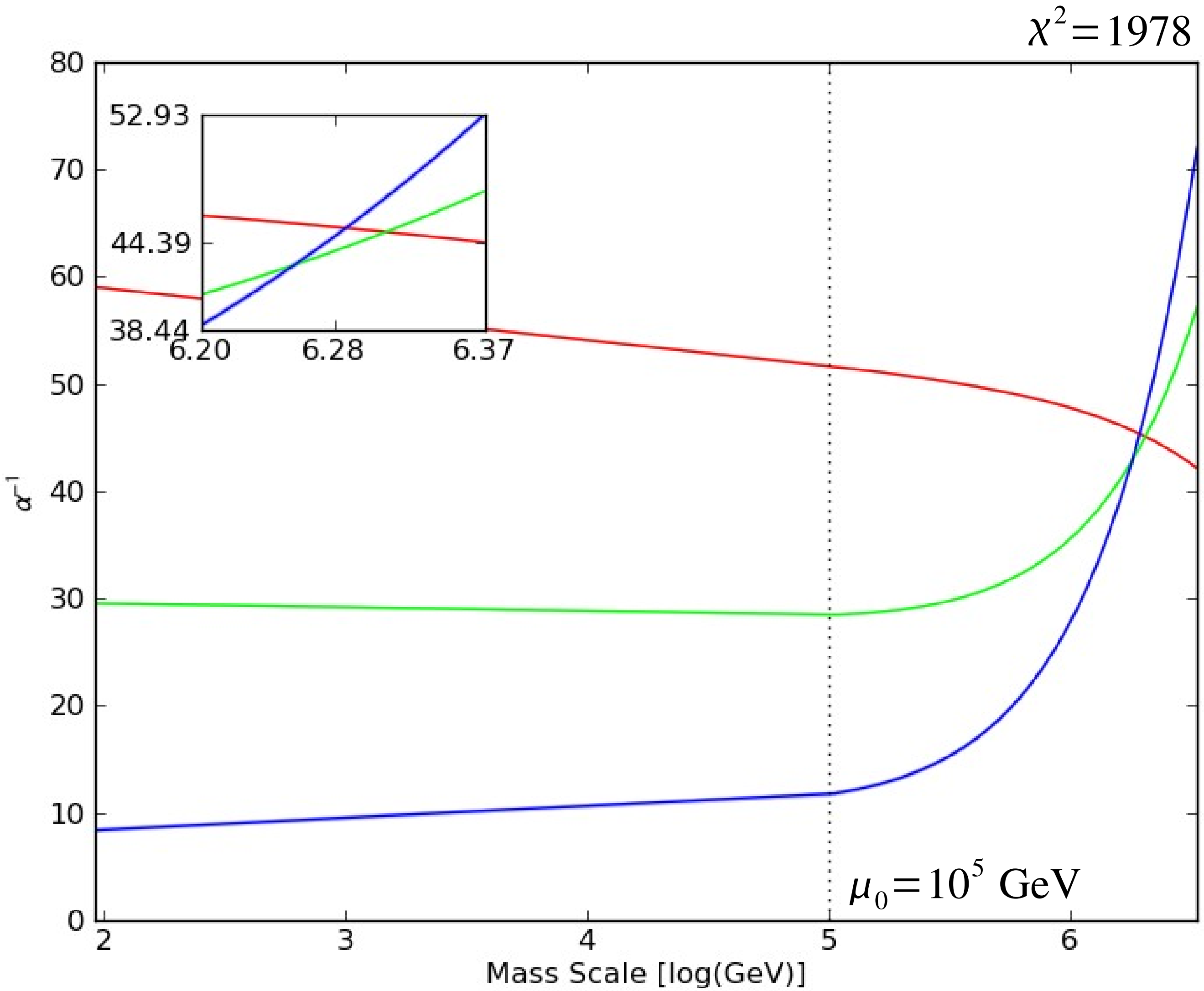
Running Coupling in the MSSM with Extra Dimensions (EDMSSM)

- If we add δ extra dimensions, compactified on a radius $1/\mu_0$, the coupling constants run as

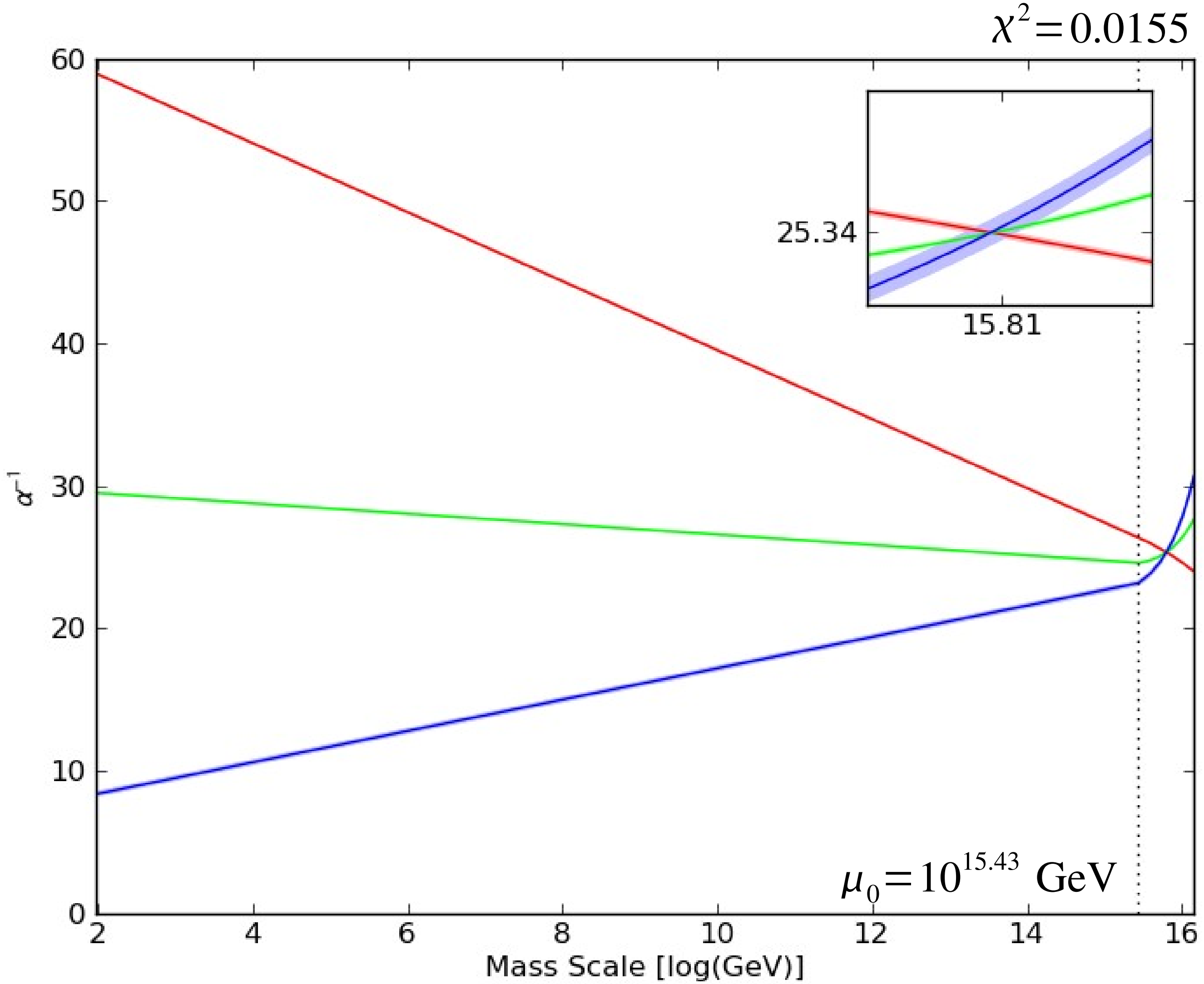
$$\alpha_i^{-1}(\mu)=\alpha_i^{-1}(M_Z)-\frac{b_i}{2\pi}\ln\left(\frac{\mu}{M_Z}\right)+\frac{\tilde{b}_i}{2\pi}\ln\left(\frac{\mu}{\mu_0}\right)-\frac{\tilde{b}_i X_\delta}{2\pi\delta}\left[\left(\frac{\mu}{\mu_0}\right)^\delta-1\right]$$

  when $\mu\geq\mu_0$,where $X_\delta=(2\pi^{\delta/2})/(\delta\Gamma(\delta/2))$ and $\Gamma(x)$ is the Euler gamma function
- μ_0 and δ are free parameters

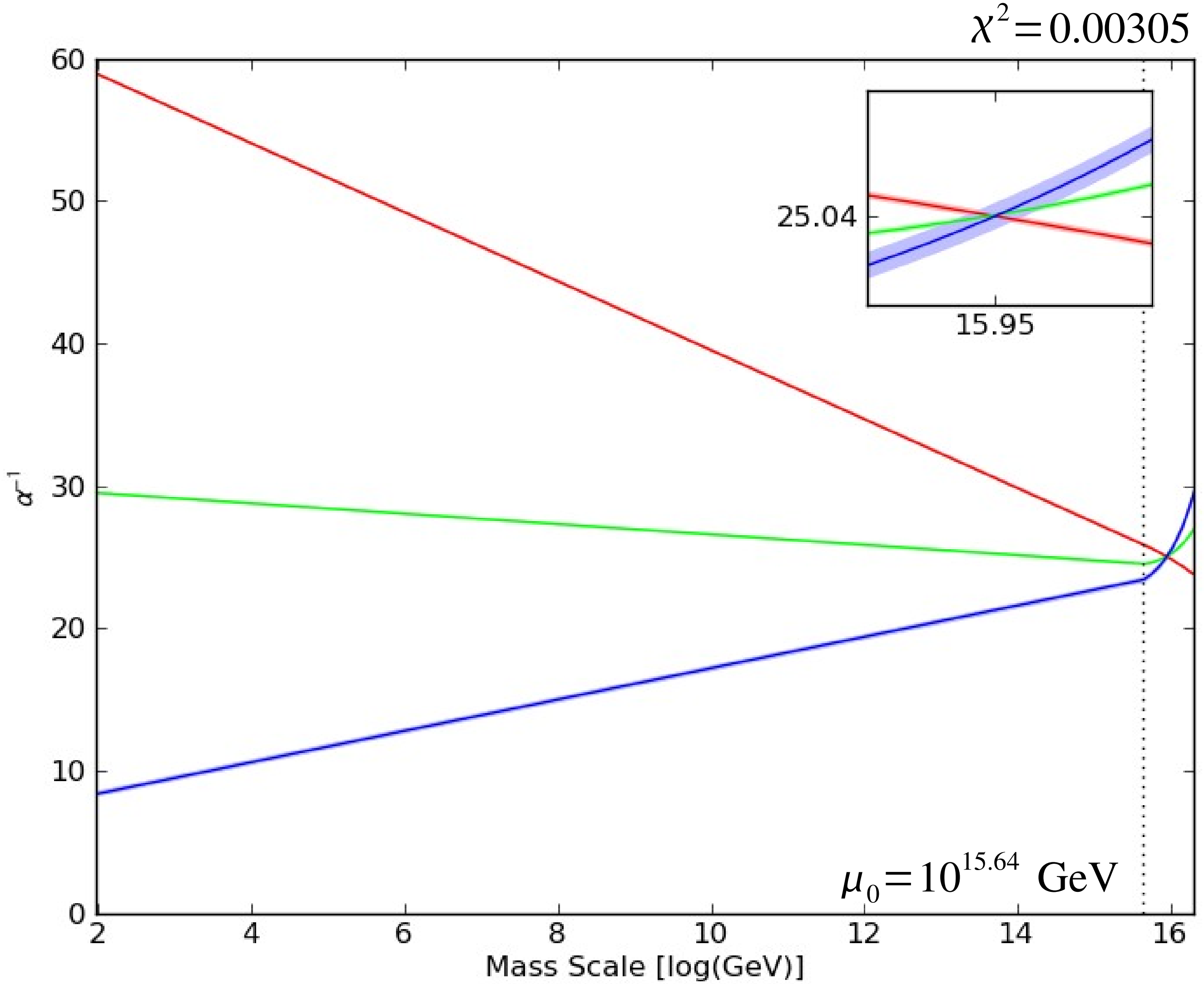
Running Coupling in the MSSM with 1 Extra Dimension



Running Coupling in the MSSM with 1 Extra Dimension



Running Coupling in the MSSM with 2 Extra Dimensions



Conclusions

- Coupling constants vary ('run') due to renormalisation, allowing the possibility for the couplings to unify
- The Standard Model does not admit unification
- The Minimal Supersymmetric Standard Model (MSSM) admits unification best at $M_{\mathrm{SUSY}} = 10^{(3.39 \pm 0.01)}\ \mathrm{GeV} \approx 2.4\ \mathrm{TeV}$
- Admitting extra dimensions to the MSSM lowers the unification scale (but not by much!)

Thanks for Listening!

References and Further Reading

- M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Perseus Books (1995)
- Amaldi et al, *Consistency Checks of Grand Unified Theories*, Phys. Lett. B **281** (1992), pp. 374-382
- Dienes, Dudas and Gherghetta, *Extra Spacetime Dimensions and Unification,* Phys. Lett. B **436**, 55 (1998)