2022 International Conference on Distributed Computing, VLSI, Electrical Circuits and Robotics (DISCOVER) | 978-1-6654-8716-0/22/$31.00 ©2022 IEEE | DOI: 10.1109/DISCOVER55800.2022.9974669

Comparison of Memristor Mathematical Models

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***Abstract*—The major passive electronic circuit element is memristor, that offers changeable CMOS compatibility, resistance, flexibility, and no leakage current. The features of a memristor vary depending on the application. There are broadly five categories of memristor models LIDM, NLIDM, STBM, TEAM, and VTEAM models. This paper provides a basic overview of each of these memristor models. These models have been realized and I-V characteristics are simulated in Cadence Virtuoso tool using Verilog-A code blocks.Finally a comparison of these models is presented.**

***Index Terms*—Memristor Model, ThrEshold Adaptive Memristor model(TEAM), Linear Ionic Drift Model(LIDM), Simmons Tunnel Barrier Model(STBM), Non-Linear Ionic Drift Model(NLIDM), Voltage Threshold Adaptive Memristor model(VTEAM).**

I. Introduction

A Von-Neumann architecture is the most widely utilized architecture in modern processors. Separate structures of memory, logic and other interface circuits are used to connect these two in this architecture.Since there is a temporal lag between memory devices and logic circuits, high parallel processing power and speed ourcomes cannot be obtained.As a result, this architecture has a Von-Neumann bottleneck. A bio-inspired circuit, also known as neuromorphic computing, is the solution which is inspired by the nature.

The VLSI system employs neuromorphic computing, which replicates neurobiological architecture in the nervous system using electronic analog circuits. A device that does computations using physical artificial neurons is known as a neuromorphic computer/chip (made of silicon). Rather than thinking of brains as computers, scientists want to make computers more like brains. There is a probability that neuromorphic computing will overtake today's design in the future [1].

In a neuromorphic computer system, a memristor is used. The memristor is the fourth fundamental element. When he studied non-linear elements like inductors, capacitors, and resistors, the professor named Lean Chua, an IEEE fellow from the University of California, was dubbed the "Father of Nonlinearity." He realized that charge and flux had an undiscovered relationship. After that, he conducted some research. Figure 1 shows the Relation between fundamental variables.

In 1971 [3], he devised the equation that relates Charge and Flux.

$$dQ = mdq \tag{1}$$

Memristors combine the properties of a resistor and non-volatile memory, as its name suggests. Although Lean Chua is a mathematician, he lacks practical proof for this equation. Lean Chua and one of his students, stivan Chang, offered a slight change to this equation.

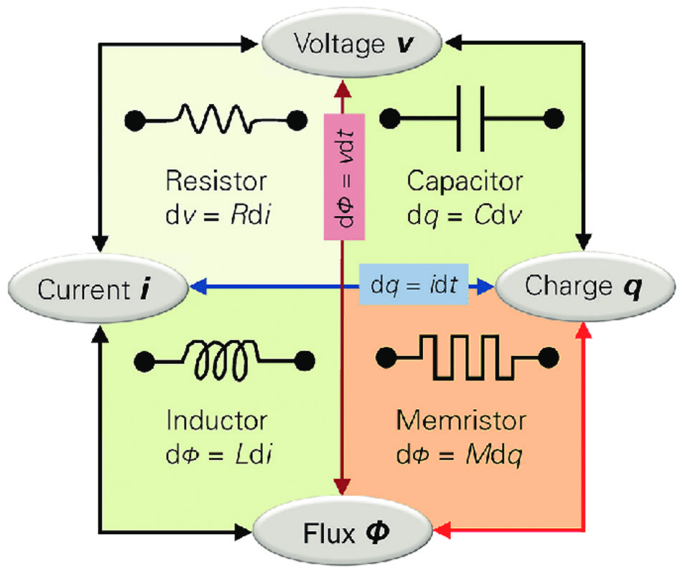

Fig. 1. Relation between fundamental variables [2].

$$V = R(X)I \tag{2}$$

X stands for state variable. Until the previous state is erased, the current resistance of the memristor is also dependent on it.

II. WORKING OF MEMRISTOR.

A memristor is nothing more than resistance with a memory characteristic. A memristor is a two-terminal device with one positive and one negative terminal. It is a non-reactive element that does not store energy. It recalls the amount of current or amount of voltage applied in previous instance. Symbol of memristor is shown in Figure 2. [4].

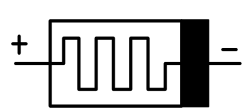

Fig. 2. Symbol [5].

Memory's resistance is regarded as rationality.

- The zero and one logic is represents high and low resistance states of mem-resistance respectively.

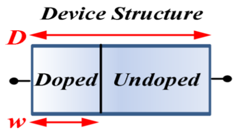

Fig. 3. Memristor Device structure [5].

Memristors have two regions: There is a doped and an undoped zones. Figure 3 shows the device structure of memristor. A greater number of charge carriers are present in the doped area, allowing current to flow more freely and lowering total resistance when the memristor is turned ON. Figure 4 shows the memristor equivalent circuit.

Because there are fewer charge carriers in the undoped layer, currently does not flow as freely as it does in the doped region, causing the overall resistance to rising, indicating that the resistor is in the OF condition.Figure 5 shows the Memristor equivalent circuit.The resistance of current flows from p to n increases, while the resistance from n to p decreases.

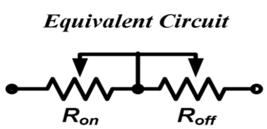

Fig. 4. Memristor equivalent circuit [5].

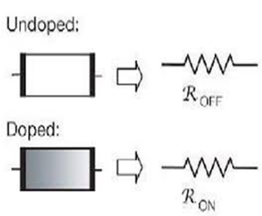

Fig. 5. Memristor equivalent circuit [5].

The operation of a memristor is presented in three steps.

- The current or voltage supplied to the memristor.
- The length of time current flows through the memristor cross-section or crossbar gap, as well as how the titanium cube is transformed from a semi-conducting element to a regular conducting element.
- The titanium cube's recollections.

HP Labs claimed to have a physical functional model of the memristor in 2008. The new element memristor then opens up new forms of electronics with a diverse variety of domains.

The various implemention models of memristor is published by HP Lab's, which are used for researching, designing, and simulating to develop numerous application of memristor. This work presents discussion on various memristor models.

III. DIFFERENT MODELS OF MEMRISTORS

A memristor can be used in several applications. As a result, distinct memristor properties are required for each application; yet, some memristor features are necessary for all applications. It has a lot of scalabilities [6].

Due to the continual shrinking process, The technology of complementary metal-oxide-semiconductor (CMOS) is reaching its limits. This affects various aspects of the gadget, including its size, performance, and power consumption. The memristor is one of the prospective devices under consideration for usage with deep Nano-scale CMOS. [7].

A. Linear Ionic Drift Model(LIDM)

In 2008, R S Williams of HP Labs created the initial physical model of a memristor. The physical component of width D is said to be divided into two zones. One side is doped with positive oxygen ions or has oxygen vacancies and another hand is undoped. A resistor is used to represent each zone. Because the doped patch with width w has less resistance, it is more conductive, while those who haven't been drugged face a lot of resistance. The conductance is also assumed to be ohmic, resulting in a uniform field [6]. Figure 6 shows the the Current voltage relationship of this model.

The ion drift follows a straight line, and the average ion mobility μV is equal for all ions [8].

Equation 3 shows the Current voltage relationship is modeled as.

$$v(t) = \left[Ron \left(\frac{x(t)}{\mathrm{D}} \right) + Roff \left(1 - \frac{x(t)}{\mathrm{D}} \right) \right] i(t) \quad (3)$$

The device's resistance at x(t)=0 is $Roff$ and at x(t)=d is Ron. In this situation, x(t) is contained inside the intervals [0 D].

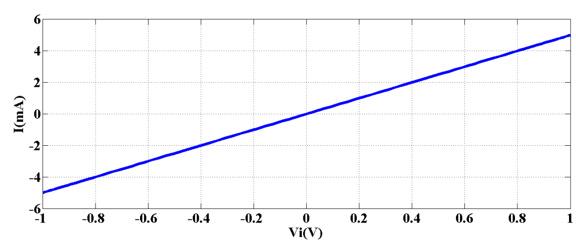

Fig. 6. I-V Characteristics of LIDM.

It refers to symmetric switching times from low to high resistance and vice versa. In partial memregister, both of these properties of linear dependence and symmetric switching durations are absent [9].

As a result, this model is only suitable in situations when highly approximated behavior is required and accuracy can be compromised. What's more, the doped region width w can likely exceed the physical dimensions. As a result, the window function is employed.

Window functions are employed to keep the state variable within the physical constraints of memristor, which poses another issue that is when the state variable approaches the border, the rate of change goes to zero, and no further changes in state variables are possible.

B. Non-Linear Ionic Drift Model(NLIDM)

NLIDM proposes a nonlinear voltage-to-internal state derivative relationship for a voltage administered memristor. This model also assumes asymmetric switching.The LIDM creates the memristor's hysteresis properties, despite the fact that it has a number of flaws in terms of fundamental electrodynamics. Studies and tests show that the behavior of the created memristor is extremely nonlinear Furthermore, This model is insufficient for other applications. such as logic circuits, nonlinear features are necessary, hence more appropriate models are developed [6]. Figure 7 shows the I-V relationship of this model.

Equation 4 The suggested model's voltage and current dependencies are represented as:

$$i(t) = x(t)^n \beta sin\, h(\alpha v(t)) + \chi[e^{(\gamma v(t)-1)}] \tag{4}$$

n determines the form of the x state variable over the currents. where γ, χ, α, and β are experimental device variable parameters. In the interval [0,1], the state variables are standardized.

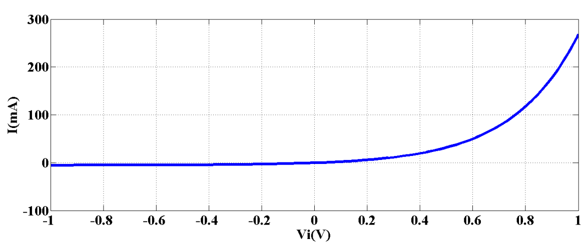

Fig. 7. I-V Characteristics of NLIDM.

C. Simmons Tunnel Barrier Model(STBM)

STBM memristor was proposed by M.D. Pickett and D.B. Strukov in 2008. The undoped area width is decide the net resistance of this model. Non-linearity and anti-symmetric switching features were also considered in this model [6].V-I Characteristics simulation result is shown in Figure 8

Pickett and Strukov claim that, The current most precise physical depiction of a memristor is the STBM.

few flaws are:

- The model is complicated. This model has a very complex equation and is slow to simulate.
- The link between current and voltage isn't specified.
- The model isn't universal, in the sense that it doesn't apply to all sorts of memristors and only suits one type of memristor.

This memristor model control is up to date. As a result, it's commonly employed in digital applications.

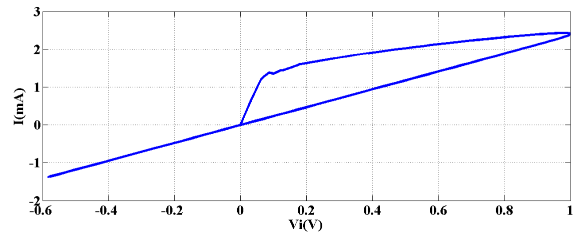

Fig. 8. I-V Characteristics of STBM.

D. ThrEshold Adaptive Memristor model (TEAM)

The TEAM model was developed by S.Kvatinsky is straightforward and broad. [6]. A few assumptions for computing efficiency and analysis simplification:

- The state variable is not modified at a particular threshold level.
- Instead of an exponential relationship, the internal state drift derivative and the memristor current, have a polynomial relationship.

Any memristor model can be utilised with the TEAM model because of the reliance of the internal state derivative. Two separate functions are multiplied, where one is the function of current and the other one is a function of State Variable h [1 0]. One may estimate the current and another estimates the state variable itself.

This state variable's derivative will be:

$$\frac{dw(t)}{\mathrm{dt}} = \begin{cases} D_{off}(h)\ G_{off}\left(\dfrac{i(t)}{i_{off}}-1\right)^{\alpha_{off}} ; 0 < i_{off} < i \\ D_{on}(h)\ G_{on}\left(\dfrac{i(t)}{i_{on}}-1\right)^{\alpha_{on}} ; i < i_{on} < 0 \\ 0; Otherwise \end{cases} \tag{5}$$

Constants of above equations are G_{off} , G_{on} , α_{off} and α_{on} ($G_{off} > 0$, $G_{on} < 0$). i_{off} and i_{on} represent threshold levels of device current. h is the internal state variable. $D_{off}(h)$ & $D_{on}(h)$ act as the window functions, where h is bounded in the interval $[h_{on}\ , h_{off}]$. The equality of this two function need not be true; for example, the STBM has an asymmetric reliance on h.

If we assume the current-voltage characteristics are comparable, V-I Reletionship of this model Shown in Figure 9. the memory resistance changes linearly in x, giving us:

$$v(t) = \left[R_{on} + \frac{R_{off} - R_{on}}{\mathrm{a_{off}} - \mathrm{a_{on}}}\right] i(t) \tag{6}$$

The model equation has been considerably reduced, and the characteristics are comparable to the simultaneous barrier model.

- The T model is extensively used for simulation.
- uses undoped region width as a state variable.
- This model is very flexible and the fitting parameters can be adjusted to different models.
- uses the current threshold
- to provide sufficient accuracy with reduced complexity Uses windows function.

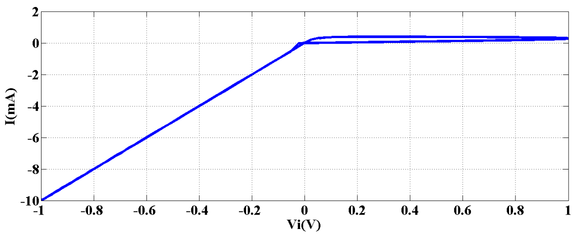

Fig. 9. I-V Characteristics of TEAM.

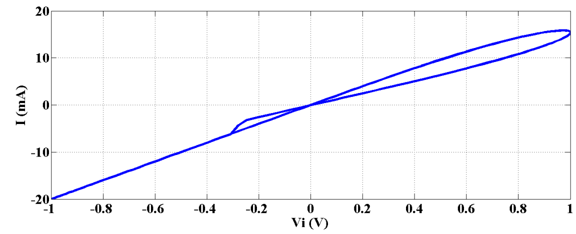

Fig. 10. I-V Characteristics of VTEAM.

E. Voltage Threshold Adaptive Memristor(VTEAM)

The nature of voltage-administered memristor is proposed to be described using this paradigm [6].

- Vteam depends on the derivative quotation of an internal state variable.
- Merits of the Team model are merged with the usage of cut-off voltage rather than cut-off current.
- The I-V Characteristics of the Vteam model is ambiguous and it can be selected from any voltage and current Characteristics. .
- It's a voltage-controlled memristor model.
- Its behavior is asymmetric and switching.
- When compared to other models, This model is both efficient and accurate in terms of calculation [8].

The advantages of the VTEAM model are that it is a simple, general, and flexible model and has sufficient accuracy It is voltage controlled memristor. It is having anti-symmetric switching behavior [10].Figure 10 shows the I-V Characteristics of vteam model.

The Vteam was introduced by Shahar Kvatinsky in 2014, and it features threshold voltage [10]. Furthermore, the threshold voltage above the threshold current is desirable. Despite threshold current, certain memristor tests have revealed the presence of threshold voltage.

The mathematical expression for the VTEAM state variable derivative is the same as for the TEAM model:

$$\frac{dw(t)}{\mathrm{dt}} = \begin{cases} G_{off}\left(\frac{V(t)}{V_{on}}-1\right)^{\alpha_{off}} D_{off}(w); 0<V_{off}<V \\ 0; V_{on}<V<V_{off} \\ G_{on}\left(\frac{V(t)}{V_{on}}-1\right)^{\alpha_{on}} D_{on}(w); 0<V_{on}<V \end{cases} \tag{7}$$

The constants of the above equations are G_{off}, G_{on}, α_{off}, and α_{on}. The V_{off} and V_{on} are voltage threshold. D_{on} and D_{off} represents the window functions. The voltage-current relationship for VTEAM can be written in equation 8.

$$i(t) = \left[R_{on} + \left(\frac{R_{off}-R_{on}}{w_{off}-w_{on}}\right)(w-w_{off})\right]^{-1} v(t) \tag{8}$$

The current in the memristor is denoted by i(t), resistance of the memristor is Roff and Ron, internal state variable's limits are Woff and Won. v(t) is the voltage as a function of time.

IV. CDF PARAMETERS.

The cadence's Component Description Format (CDF) function enables us to input specific parameters from each instantiation symbol to the schematic. The CDF parameters we used are displayed in Table I.

TABLE I
SHOWS THE CDF PARAMETERS FOR VARIOUS MEMRISTOR MODELS.

Parameter	LIDM	NLIDM	STM	TEAM	VTEAM
model	0	3	1	2	4
wndw_typ	0	0	0	0	0
dt	1e-10	1e-10	1e-10	1e-10	1e-10
init_state	0	0	0	0	0
R_off	2000KΩ	1000KΩ	1000KΩ	1000KΩ	1000KΩ
R_on	100	50	50	50	50
D	3n	5e-09	5e-09	5e-09	5e-09
uv	1e-15	1e-15	1e-15	1e-15	1e-15
w_mul	1e+08	1e+08	1e+08	1e+08	1e+08
P_coeff	2	2	2	2	2
J	1	1	1	1	1
P_wndn_n	1e-18	1e-18	1e-18	1e-18	1e-18
T_volt	0	0	0	0	0
c_off	3.5e-06	3.5e-06	3.5e-06	3.5e-06	3.5e-06
i_off	0.00015	0.00015	115e-6	0.00015	0.00015
i_on	-8.9e-06	-8.9e-06	-8.9e-06	-8.9e-06	-8.9e-06
x_c	1.07e-10	1.07e-10	1.07e-10	1.07e-10	1.07e-10
b	0.0005	0.0005	500e-6	0.0005	0.0005
alpha	2	2	2	2	2
a_on	2e-09	2e-09	2e-09	2e-09	2e-09
a_off	1.2e-09	1.2e-09	1.2e-09	1.2e-09	1.2e-09
K_on	-10	-10	-10	-8e-13	-10
K_off	5e-4	5e-4	5e-4	8e-13	5e-4
alpha_on	3	3	3	3	3
alpha_off	3	3	3	3	3
v_on	-0.2	-0.2	-0.2	1.78	-0.2
IV_relt	0	0	0	0	0
x_on	0	0	0	0	0

V. COMPARISON OF MEMRISTOR MODELS.

The first physical memristor was developed by researchers at HP Lab in 2008, and its behavior was found to be in strong accord with that anticipated by Leon Chua back in 1971. Since then, many memristor models have been proposed, along with others, for various applications. Table II Shows the Comparison of memristor models. the VTEAM and TEAM models respectively. These two representations are the more methodical and take the shortest amount of time to process. As a result, they're strong contenders for logic circuits and neuromorphic applications.

TABLE II
SHOWS A COMPARISON OF VARIOUS MEMRISTOR MODELS.

model	LIDM	NLIDM	STM	TEAM	VTEAM
State variable	$0 \leq w \leq D$	$0 \leq w \leq 1$	$A_{off} \leq x \leq a_{on}$	$x_{on} \leq x \leq x_{off}$	$w_{on} \leq w \leq w_{off}$
Control mechanism	current	voltage	current	current	voltage
I-V and mem deduction	Accurate	I-V Accurate, mem deduction ambiguous	Ambiguous	Accurate	Accurate
matching mem system def	Yes	No	No	Yes	Yes
Generic	No	No	No	Yes	Yes
Accuracy	Low	Low	High	Sufficient	High
Threshold exist	No	N0	Practically exist	Yes	Yes

CONCLUSION

Verilog-A code was used to implement the LIDM, NLIDM, STBM, TEAM, and VTEAM memristor models. The behaviour and I-V relationship of memristor models were simulated in Cadence virtuoso 180nm technology and addressed in this paper. Memristors are particularly attractive alternatives for memory compositions because of their tremendous capacity, a rapid changeover period, and minimal power usage. The threshold levels are missing in the memristor models presented so far, omitting TEAM and VTEAM. For any given current and voltage levels, this causes memristance to fluctuate. Threshold current and voltage are used in the VTEAM and TEAM models respectively. These two representations are the more methodical and take the shortest amount of time to process. As a result, they're strong contenders for logic circuits and neuromorphic applications.

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