AA103 Final Project

0.1 Project Description

For our final project, you will be designing, building and launching a water rocket!

Water rockets are generally made from 2 liter soda bottles filled partially with water and pressurized with air. The water is exhausted through a nozzle creating thrust and launching the bottle into the sky. There is a vibrant group of garage builders on the internet and some hava achieved amazing performance, flying rockets to over 2000 feet in altitude! (albeit not with soda bottles...)

While without smoke and fire, water rockets are a lot of fun.

This project is broken into two parts:

1. Pre-work problem set covering the analysis and design of the rocket
2. The rocket build and altitude contest

Rules and Constraints

Since we will be holding a contest, there are some rules and constraints that must be followed:

- Everyone must use a standard 2 liter soda bottle. The beverage choice is up to you...
- All rockets will be pressurized to 7 bar (7×10^5 Pa)
- All rockets will carry a 10 g altimeter payload on top
- All rockets must use the provided bottle cap / nozzle. However, the nozzle internal hole may be modified by the teams

0.2 Pre-work - Analysis and Design

Problem 1 - Δ-V analysis

Firs we will ask you to compute the Δ-V your rocket is capable of delivering. A water rocket is referred to as a "blow-down" system as there is no external reservoir of propellant maintaining constant pressure in the rocket. So the pressure inside will decay as water is expelled.

In general terms, a gas expansion process can be represented as

$$pV^n = \text{const} \tag{1}$$

a) What is n for the three following processes?

   a) An Isobaric (constant pressure) process
   b) An Isothermal (constant temperature) process
   c) An Isentropic (constant entropy) process

b) Given that a water rocket expels the water in a very short period of time (a second or two) and that the gas must perform work on the water in this expulsion, which type of process above do you think most accurately models the expansion process of the gas?

c) Given an initial internal pressure, $p_0 = 7$ bar, an ambient atmospheric pressure, $p_a = 1$ bar, and the density of water $rho_w = 1000kg/m^3$ what is the initial water exit velocity?

d) If the expulsion process were isobaric, derive the rocket ΔV as a function of *water fill fraction*, $\alpha = \frac{V_w}{V_t}$ where V_w is the volume of the bottle initially filled with water and V_t is the total volume of the bottle. You may assume the mass of your bottle is 50g plus 15g for the altimeter payload.

e) Since the explusion process is not really isobaric, we would like to derive the ΔV for the isothermal and isentropic cases as well. Unfortunately, there is not a closed form solution for this so we will need to compute a differential equation for velocity increment and then integrate numerically. The jupyter notebook linked gives the background necessary to do this integration.

f) Given the results from this analysis, how would you pick the fill fraction α? How different is your answer for the isothermal and isentropic cases?

Problem 2 - Ballistic simulation and optimization

In the previous problem we looked at ΔV potential of your rocket. This is interesting and informative but it neglects to consider two unfortunate facts:

1. Your rocket must fight gravity
2. Your rocket will be slowed by aerodynamic drag

In this problem you will develop a very simple numerical simulation of the rocket's flight trajectory and use this to interogate its performance sensitivity to some design parameters.