

**Part 1.** Write your answers to each of the multiple-choice problems below on the designated lines. Use only CAPITAL LETTERS for your responses. If you change your answer, strike out the old answer and write your new answer next to it. Write clearly. (2 pts each problem, 30 pts total)

1) E

5) A

9) D

13) B

2) A

6) E

10) C

14) A

3) B

7) D

11) B

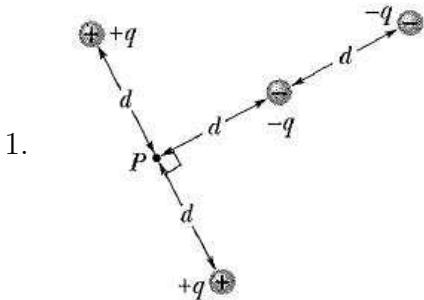
15) E

4) A

8) C

12) A

**Part 2.** Select **THREE** problems as mandatory, any other extra will give you bonus points. You must show **EACH STEP** in your solution **AND** the **UNITS** for the full credit consideration. **If you will write only the numbers I will not give you any credit.** (70 pts total as a minimum)



1.

What is the net electric potential at point P due to the four particles if  $V = 0$  at infinity,  $q = 5.00 \text{ fC}$ , and  $d = 4.00 \text{ cm}$ ?

**Solution:**

$$V = \frac{q}{4\pi\epsilon_0} = \left( -\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d} \right) = \frac{q}{8\pi\epsilon_0 d} = 5.62 \cdot 10^{-4} \text{ V}$$

2. Two particles are fixed to an  $x$  axis: particle 1 of charge  $q_1 = 2.1 \cdot 10^{-8} \text{ C}$  at  $x = 20 \text{ cm}$  and particle 2 of charge  $q_2 = -4.00 q_1$  at  $x = 70 \text{ cm}$ . At what coordinate on the axis is the net electric field produced by the particles equal to zero?

**Solution:**

At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge  $q_2 = -4.00 q_1$  located at  $x_2 = 70 \text{ cm}$  has a greater magnitude than  $q_1 = 2.1 \cdot 10^{-8} \text{ C}$  located at  $x_1 = 20 \text{ cm}$ , a point of zero field must be closer to  $q_1$  than to  $q_2$ . It must be to the left of  $q_1$ .

Let  $x$  be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\epsilon_0} \left( \frac{|q_1|}{(x - x_2)^2} - \frac{|q_2|}{(x - x_1)^2} \right)$$

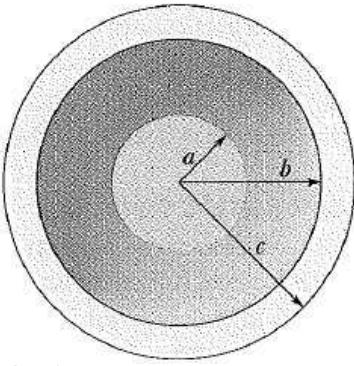
If the field is to vanish, then

$$\frac{|q_1|}{(x - x_2)^2} = \frac{|q_2|}{(x - x_1)^2}$$

$$\frac{|q_2|}{|q_1|} = \frac{(x - x_2)^2}{(x - x_1)^2}$$

From here  $x = -30 \text{ cm}$ .

3.



A solid sphere of radius  $a = 2.00 \text{ cm}$  is concentric with a spherical conducting shell of inner radius  $b = 2a$  and outer radius  $c = 2.4a$ . The sphere has a net uniform charge  $q_1 = +5.00 \text{ fC}$ ; the shell has a net charge  $q_2 = -q_1$ . What is the magnitude of the electric field at radial distances (a)  $r = 0$ , (b)  $r = a/2$ , (c)  $r = a$ , (d)  $r = 1.50a$ , (e)  $r = 2.30a$ , and (f)  $r = 3.50a$ ?

**Solution:**

At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found.

For  $r < a$ , the charge enclosed by the Gaussian surface is  $q_1(r/a)^3$ . Gauss law yields

$$4\pi r^2 E = \left(\frac{q_1}{\epsilon_0}\right) \left(\frac{r}{a}\right)^3$$

$$E = \frac{q_1 r}{4\pi \epsilon_0 a^3}$$

(a)

For  $r = 0$ , the above equation implies  $E = 0$ .

(b)

For  $r = a/2$ , we have

$$E = \frac{q_1 a/2}{4\pi \epsilon_0 a^3} = 5.62 \cdot 10^{-2} \text{ N/C}$$

(c)

For  $r = a$ , we have

$$E = \frac{q_1}{4\pi \epsilon_0 a^2} = 0.112 \text{ N/C}$$

In the case where  $a < r < b$ , the charge enclosed by the Gaussian surface is  $q_1$ , so Gauss law leads to

$$4\pi r^2 E = \frac{q_1}{\epsilon_0}$$

$$E = \frac{q_1}{4\pi \epsilon_0 r^2}$$

(d)

For  $r = 1.50a$ , we have

$$E = \frac{q_1}{4\pi \epsilon_0 (1.50a)^2} = 0.0499 \text{ N/C}$$

(e)

In the region  $b < r < c$ , since the shell is conducting, the electric field is zero. Thus, for  $r = 2.30a$ , we have  $E = 0$ .

(f)

For  $r > c$ , the charge enclosed by the Gaussian surface is zero. Gauss law yields

$$4\pi r^2 E = 0$$

Thus,  $E = 0$  at  $r = 3.50a$ .

4. Two electrons are fixed 2.0 cm apart. Another electron is shot from infinity and stops midway between the two. What is its initial speed?

**Solution:**

We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then

$$U_f = \frac{2e^2}{4\pi\epsilon_0 d},$$

where  $d$  is half the distance between the fixed electrons. The initial kinetic energy is

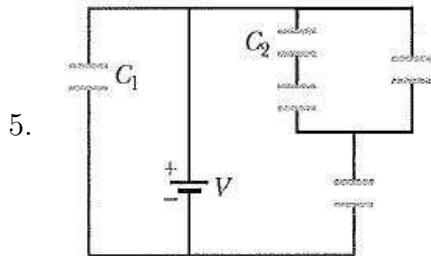
$$K_i = \frac{1}{2}mv^2,$$

where  $m$  is the mass of an electron and  $v$  is the initial speed of the moving electron. The final kinetic energy is zero. Thus,

$$\begin{aligned} K_i &= U_f \\ \frac{1}{2}mv^2 &= \frac{2e^2}{4\pi\epsilon_0 d} \end{aligned}$$

Hence,

$$v = \sqrt{\frac{4e^2}{4\pi\epsilon_0 dm}} = \sqrt{\frac{9 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \cdot 4 \cdot (1.6 \cdot 10^{-19} \text{ C})^2}{0.01 \text{ m} \cdot 9.11 \cdot 10^{-31} \text{ kg}}} = 320 \text{ m/s}$$



The battery has a potential difference of  $V = 10.0 \text{ V}$  and the five capacitors each have a capacitance of  $10.0 \mu\text{F}$ . What is the charge on (a) capacitor 1 and (b) capacitor 2?

**Solution:**

(a)

The potential difference across  $C_1$  is  $V_1 = 10.0 \text{ V}$ . Thus,

$$q_1 = C_1 V_1 = 10 \mu\text{F} \cdot 10 \text{ V} = 1 \cdot 10^{-4} \text{ C}$$

(b)

We first consider the three-capacitor combination consisting of  $C_2$  and its two closest neighbors, each of capacitance  $C$ . The equivalent capacitance of this combination is

$$C_{eq} = C + \frac{C_2 C}{C + C_2} = 1.5C$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.5C} = 0.4V_1$$

Since this voltage difference is divided equally between  $C_2$  and the one connected in series with it, the voltage difference across  $C_2$  satisfies  $V_2 = V/2 = V_1/5$ . Thus

$$q_2 = C_2 V_2 = 10 \mu\text{F} \cdot \left( \frac{10 \text{ V}}{5} \right) = 2 \cdot 10^{-5} \text{ C}$$