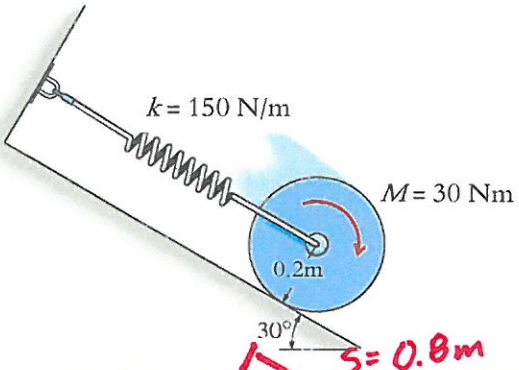
Work Energy II - Problem 1

The 20-kg disk is originally at rest, and the spring holds it in equilibrium. A couple moment of $M = 30$ N·m is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved $s = 0.8$m down along the inclined plane. The disk rolls without slipping.



CLASSIFY MOTION

DISK GPM

PROPERTIES

$m_D = 20kg$

$W_D = 20(9.81) = 196.2\ N$

$I_D = \frac{1}{2}mr^2 = \frac{1}{2}(20)(.2)^2 = .04\ kg\cdot m^2$

CALCULATE WORK

WHAT DOES WORK?

WEIGHT

MOMENT

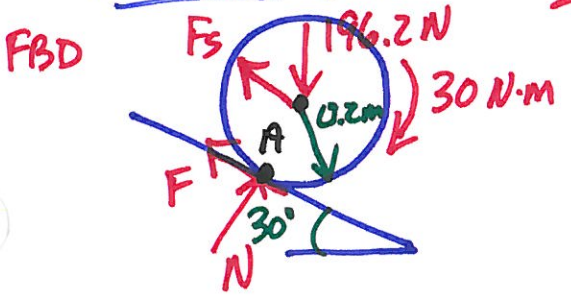
SPRING

INITIAL SPRING TENSION

$F_s = -KS_1 \quad \circlearrowleft \Sigma M_A = 0$

@ REST BEFORE MOMENT IS APPLIED

FBD



$F_s(.2) - 196.2\sin 30(.2) = 0$

$F_s = 98.1\ N$

$S_1 = \frac{F_s}{K} = \frac{98.1}{150} = 0.654m$

WORK FROM WEIGHT

$U_w = -WA_Y = -196.2(-.8\sin 30°)$

$U_w = 78.48\ J$

WORK FROM COUPLE MOMENT

$U_c = M(\theta_2 - \theta_1)$

$U_c = 30(\frac{.8}{.2} - 0)$

$= 120J$

WORK FROM SPRING

$U_{sp} = -\frac{1}{2}K(S_2^2 - S_1^2)$

$S_2 = 0.8 + .654 = 1.454m$

$U_{sp} = -\frac{1}{2}(150)(1.454^2 - .654^2)$

$= -126.8\ J$

TOTAL WORK

$U_{1-2} = \Sigma U$

$= 78.48 + 120 - 126.8 = 71.68J$

KINETIC ENERGY

NO SLIP WHEEL $V_2 = \omega_2 r = .2\omega_2$

$T_1 = 0$ (@ REST)

$T_2 = \frac{1}{2}mv_2^2 + \frac{1}{2}I_G\omega_2^2$

$= \frac{1}{2}(20)(.2\omega_2)^2 + \frac{1}{2}(.4)(\omega_2)^2 = 0.6\omega_2^2$

$T_1 + U_{1-2} = T_2$

$0 + 71.68 = 0.6\omega_2^2$

$\omega_2 = 11\ rps \circlearrowright$