

# 1-prop Z-interval

## Chapter 6 Confidence Intervals

### STA 2023 SECTION 6.3 Confidence Intervals for Population Proportions NOTES

#### Learning Outcomes:

- Find a point estimate for the population proportion
- Construct and interpret confidence intervals for a population proportion
- Determine the minimum sample size required when estimating a population proportion

proportion CI  
Success/Failure

$p$  = pop proportion

When trying to estimate the value of a population proportion of successes, the best point estimate for the population proportion  $p$  is the sample proportion  $\hat{p}$ , where  $\hat{p} = \frac{x}{n} = \frac{\# \text{ of successes}}{\# \text{ in the sample}}$ .

#### Requirements for constructing a confidence interval estimate:

- A simple random sample of size  $n$  is taken.
- $n$  must not be more than 5% of the population size  $N$  – this allows us to treat each subject as an independent event.
- Outcomes must be classified as a *success or failure*.
- The following conditions must apply  $n\hat{p} \geq 10$  and  $n\hat{q} \geq 10$ , where  $\hat{q} = 1 - \hat{p}$  = the proportion of failures. This allows us to use the normal distribution.

part/whole = proportion

#### Constructing a Confidence Interval Estimate for the Population Proportion $p$ :

Step 1:  $C$  and  $\alpha = 1 - C$

Step 2: critical value,  $Z_{1-\frac{\alpha}{2}} = \text{invNorm}()$

Step 3:  $SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Step 4:  $ME = Z_{1-\frac{\alpha}{2}} \cdot SE$

Step 5: CI  $\Rightarrow$   
 $(\hat{p} - ME, \hat{p} + ME)$

Example 1: Among 271 cell phone owners surveyed, 116 said they had an Android phone.

- a. Find a point estimate for the true proportion of all cell phone owners who have an Android phone.

$$n = 271, x = 116, \hat{p} = \frac{x}{n} = \frac{116}{271} = 0.428$$

- b. Construct a 95% confidence interval estimate for the true proportion of all cell phone owners who have an Android phone.

Step 1:  $C = 0.95, \alpha = 0.05$



$$Z_{0.975} = \text{invNorm}(0.975, 0, 1) = 1.9599$$

Step 3

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.428(1-0.428)}{271}} \\ = 0.0301$$

Step 4

$$ME = (1.9599)(0.0301) \\ = 0.0589$$

Step 5  
95%.

CI is  $(0.428 - 0.0589, 0.428 + 0.0589)$

$\Leftrightarrow (0.3691, 0.4869)$  ✓

We are 95% confident that  
android cell-phone users in the  
pop is bet<sup>n</sup> the prop.  
 $(0.3691, 0.4869)$

- c. If an advertisement for Android phones claims that the majority (more than 50%) of all cell phone owners have an Android phone, does the estimate from part b support or contradict the claim?

contradicting, max % we get from CI is 48.69%

### Finding a Sample Size Necessary to Estimate a Population Proportion

If we wish to estimate a population proportion  $p$ , how many subjects must we sample?

- The sample size  $n$  does NOT depend on the population size.
- The sample size  $n$  does depend on the confidence level and the margin of error needed.
- The sample size  $n$  also depends on if there is a previous estimate for  $\hat{p}$ . (results from other studies or a pilot study)

$$ME = Z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$* ME \propto \frac{1}{\sqrt{n}}$$

$n \uparrow$  ME  $\downarrow$  length of CI shorter

$$* ME \propto Z_{1-\frac{\alpha}{2}}$$

$C \uparrow$  ME  $\uparrow$  length of CI will be wider

**Example 2:** An educator wishes to construct a 98% confidence interval estimate for the true proportion of elementary school children in Florida who are proficient in reading. How many subjects must be included in the study if she wishes to be within 5 percentage points of the true proportion?

- a. Find the needed sample size, assuming no estimate for the proportion of elementary school children in Florida who are proficient in reading.

$$ME = .05$$

$$C = .98$$

$$\alpha = .02$$

Assume  $\hat{p} = .50$ ,  $\hat{q} = 1 - \hat{p} = .50$

$$.05 = 2.3263 \cdot \sqrt{\frac{(.5)(.5)}{n}}$$

$$\boxed{542}$$

- b. Find the sample size needed if the results of a recent study suggest that the proportion of elementary school children in Florida who are proficient in reading is 85%.



$$\hat{p} = .85$$

$$\hat{q} = .15$$

$$Z_{.99} = \text{invNorm}()$$

$$.05 = 2.3263 \sqrt{\frac{(.85)(.15)}{n}}$$

$$\Rightarrow (.05)^2 = (2.3263)^2 \cdot \frac{(.85)(.15)}{n}$$

$$\Rightarrow n = \frac{(2.3263)^2 (.85)(.15)}{(.05)^2}$$

$$= 275.995 \approx \boxed{276}$$