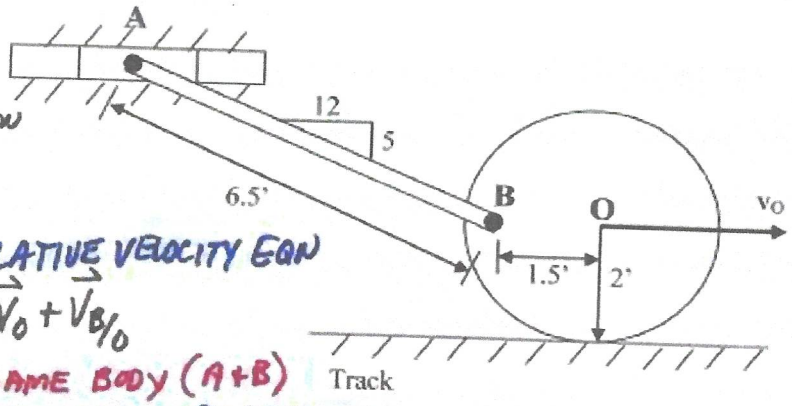
Rigid Body Kinematics II – Problem 3

For the train wheel shown, find the velocity of the piston at A relative to the track and the angular velocity of the push arm AB when the velocity of the train is V_o = 80 fps. Assume no slip.

Scalar Solution



STEP 1: CLASSIFY MOTION

PISTON - TRANSLATION
ARM AB - GPM
WHEEL - GPM

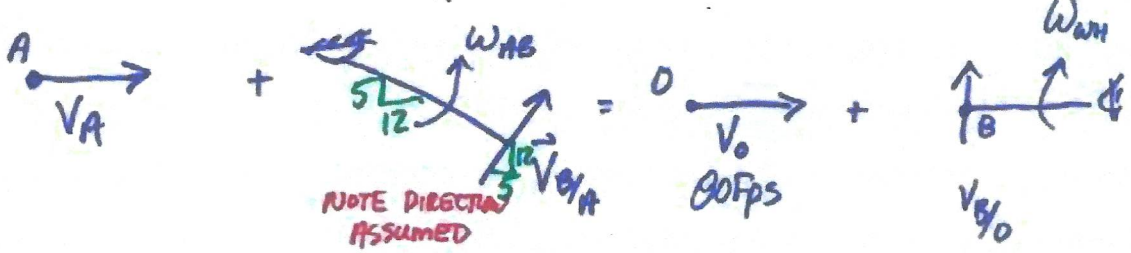
STEP 2: WRITE RELATIVE VELOCITY EQN

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = \vec{V}_O + \vec{V}_{B/O}$

2 POINTS ON SAME BODY (A+B)

STEP 3: DRAW KINEMATIC DIAGRAMS

$\vec{V}_A + \vec{V}_{B/A} = \vec{V}_O + \vec{V}_{B/O}$



STEP 4: WRITE SCALAR EQNS

$(x)\rightarrow \quad V_A + \frac{5}{13} V_{B/A} = 80 + 0$

$(y)\uparrow \quad 0 + \frac{12}{13} V_{B/A} = 0 + V_{B/O}$

2 EQNS 3 UNKS ☹

NO SLIP WHEEL

$V_O = \omega_{WH} r_{WH} \Rightarrow 80 = \omega_{WH}(2) \quad \omega_{WH} = 40 \text{ rps} \circlearrowright$

$(x)\rightarrow \quad V_A + \frac{5}{13}(\omega_{AB})(6.5) \overset{\text{RAFA}}{=} 80$

$(y)\uparrow \quad 12/13(\omega_{AB})(6.5) \overset{\text{RAFA}}{=} \omega_{WH}(r_{WH}) = 40(1.5)$

$\omega_{AB} = 10 \text{ rps} \circlearrowleft$

$V_A = 55 \text{ Fps} \rightarrow$

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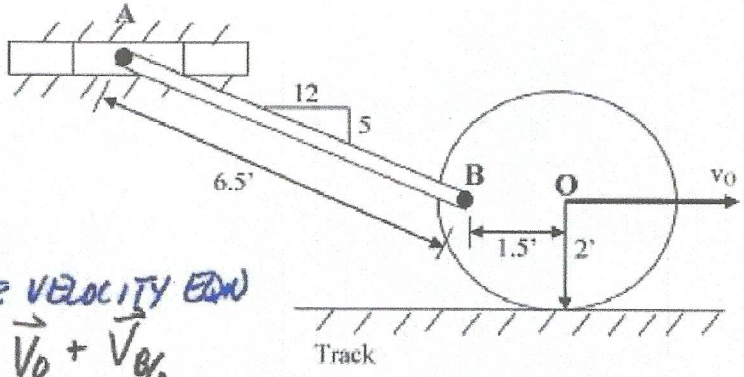
Vector Solution



1. CLASSIFY MOTION
   PISTON - TRANS
   ARM AB - GPM
   WHEEL - GPM
2. WRITE RELATIVE VELOCITY EQN

$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = \vec{V}_O + \vec{V}_{B/O}$$

RAFA $\quad \vec{V}_{B/A} = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$$\vec{V}_{B/O} = \vec{\omega}_{OB} \times \vec{r}_{B/O}$$

3. WRITE VECTOR EQNS

$$[V_A\ 0\ 0] + [0\ 0\ \omega_{AB}] \times [6\ -2.5\ 0] = [80\ 0\ 0] + [0\ 0\ -\omega_{WH}] \times [-1.5\ 0\ 0]$$

$$[V_A\ 0\ 0] + [2.5\omega_{AB}\ \ 6\omega_{AB}\ \ 0] = [80\ 0\ 0] + [0\ \ 1.5\omega_{WH}\ \ 0]$$

$$[2.5\omega_{AB} + V_A,\ 6\omega_{AB},\ 0] = [80\ \ 1.5\omega_{WH}\ \ 0]$$

NEED ω_{WH}

NO SLIP WHEEL

$$\vec{V}_O = \vec{\omega}_{WH} \times \vec{r}_{WH} \Rightarrow [80\ 0\ 0] = [0\ 0\ -\omega_{WH}] \times [0\ 2\ 0]$$

$$[80\ 0\ 0] = [2\omega_{WH}\ 0\ 0]$$

ω_{WH} = 40 rps ↻

$6\omega_{AB} = 1.5\omega_{WH} = 1.5(40)$ $\quad \omega_{AB} =$ 10 rps ↺

$2.5\omega_{AB} + V_A = 80$ $\quad V_A = 80 - (2.5(10)) =$ 55 fps →