2D Equilibrium

Hibbeler 14e: Ch 3, Sec 5.1-5.4

EQUILIBRIUM

$\sum \vec{F} = 0 \quad \sum \vec{M} = 0$

Remains at rest if originally at rest (i.e. static), or has constant velocity if originally in motion

Supports --- must know for solving problems!

- Prevent *translation* by exerting a force in the opposite direction
- Prevent *rotation* by exerting a moment in the opposite direction

Free-body diagram (FBD) --- must draw for solving problems!

- Equilibrium application tool to account for forces and moments
- Sketch which shows body *FREE* from its surroundings with *ALL* the known and unknown *forces* and *moments* exerted on the body

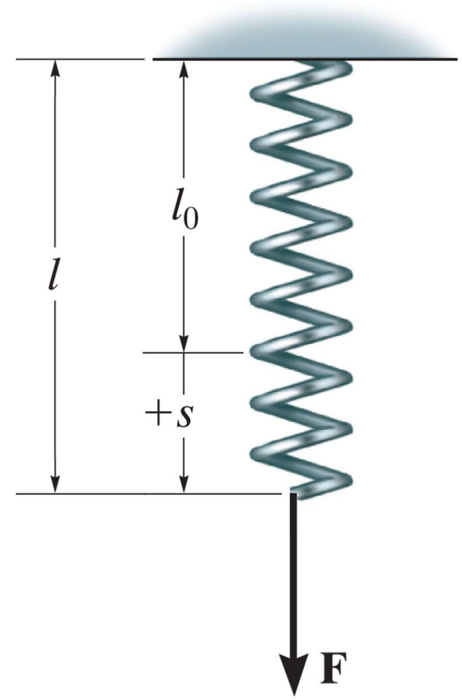
Equations of equilibrium for 2D --- must write for solving problems!

- PARTICLE --- $\sum F_x = 0\,,\ \sum F_y = 0$
- BODY --- $\sum F_x = 0\,,\ \sum F_y = 0\,,\ \sum M_z = 0$

SUPPORTS: PARTICLE EQUILIBIRUM

Springs --- linearly elastic (follows Hooke's Law)

Force is *proportional* to the spring *deformation (s)* and stiffness or *spring constant (k)*

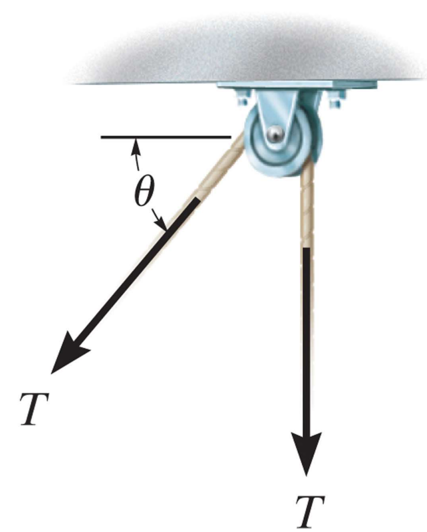


Cables --- negligible weight and cannot stretch

Can only carry "pulling" or *tension force (**T**)* that always acts in the *direction of the cable*

Pulleys --- frictionless

Changes direction of the *tension force (**T**)* in a continuous cable with *constant magnitude* of tension throughout its length



SUPPORTS: RIGID-BODY EQUILIBRIUM

Smooth contact

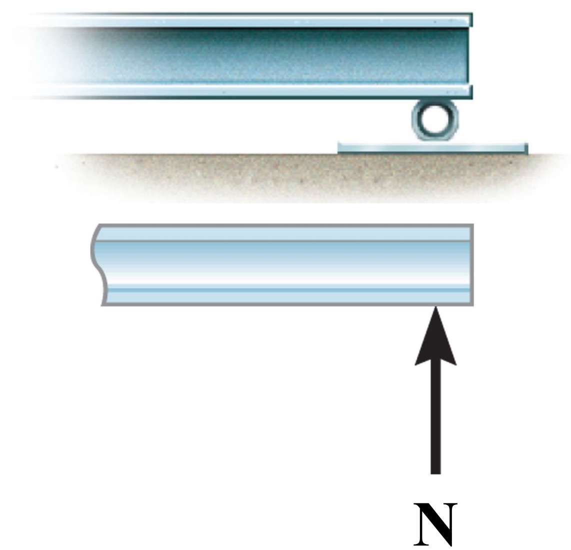
Surfaces exert a *normal force (**N**)* on the body, *perpendicular* to the surface at the point of contact

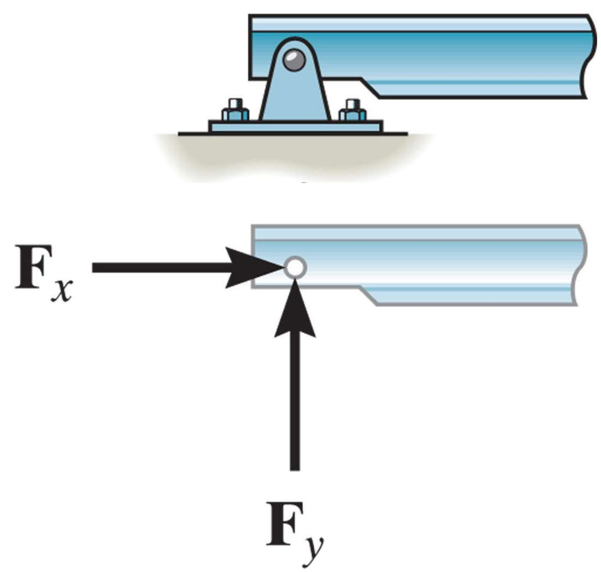
(e.g. surface, roller, rocker, slot)

Support reactions

Various types of supports exert *reaction forces*, restrict or prevent translation and rotation

(e.g. link, pin or hinge, fixed)





PROCEDURE FOR DRAWING FBD

Draw outlined shape

- Identify and isolate PARTICLE or BODY from its surroundings; cut *"free"* from all supports, constraints, connections

Show all forces and moments

- Applied loads are *active*; tend to set body in motion
- Reactions at supports and points of contact are *reactive*; tend to *restrict or prevent* motion (translation, rotation)
- Weight acts *down* through the *center of gravity (G)* point

Identify each loading and give dimensions

- *Known* --- label with proper magnitude and direction
- *Unknown* --- use letter to represent magnitude and direction angle
- Include coordinate axes and indicate dimensions and geometry

EQUATIONS OF EQUILIBRIUM

Concurrent force system

Particle equilibrium

$$\sum \vec{\boldsymbol{F}} = \boldsymbol{0}$$

2D: $\sum F_x = 0, \sum F_y = 0$

Can solve for *2 unknowns*

3D: $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$

Can solve for *3 unknowns*

Force in spring known via $F = ks$

Nonconcurrent force system

Rigid-body equilibrium

$$\sum \vec{\boldsymbol{F}} = \boldsymbol{0} \quad \sum \vec{\boldsymbol{M}} = \boldsymbol{0}$$

2D: $\sum F_x = 0, \sum F_y = 0, \sum M_z = 0$

Can solve for *3 unknowns*

Most often for coplanar equilibrium

3D: $\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$

$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$

Can solve for *6 unknowns*

Use vector analysis!

PROCEDURE FOR ANALYSIS --- 2D EQUILIBRIUM, COPLANAR FORCES

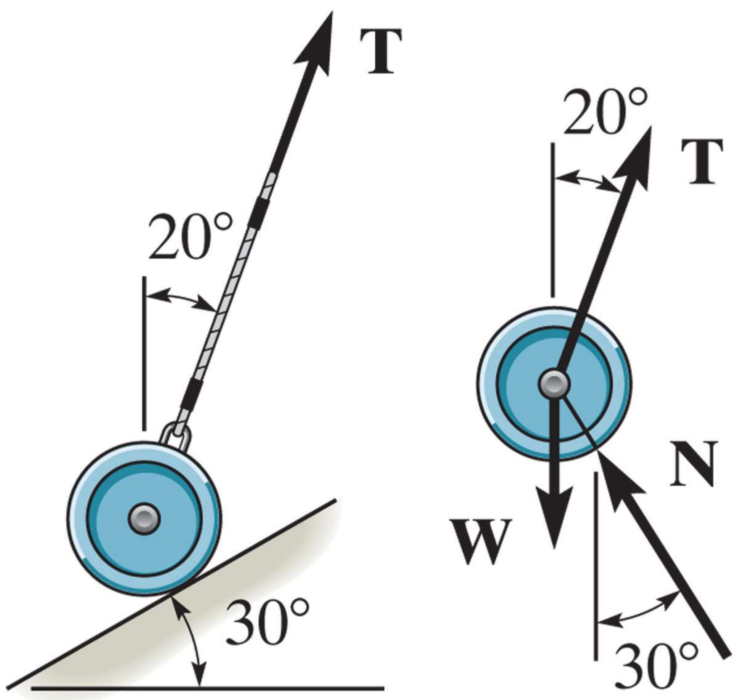
Select a PARTICLE or BODY, isolate from its surroundings, **draw FBD**

- Coordinate axes --- establish in any suitable orientation
- Free the body --- remove all supports and sketch outlined shape
- Label all --- known and unknown forces and moments acting on the body with “assumed” sense for unknowns

Classify force system, **write equilibrium equations**, solve for unknowns

- Concurrent --- 2 equations can only solve for 2 unknowns
- Non-concurrent --- 3 equations can only solve for 3 unknowns
- Reference arrows --- next to equations to define positive directions
- Negative solution --- sense is reverse of that assumed on the FBD

EXAMPLE: PARTICLE EQUILIBRIUM



Sum force in the x direction

$$+\rightarrow \sum F_x = 0$$

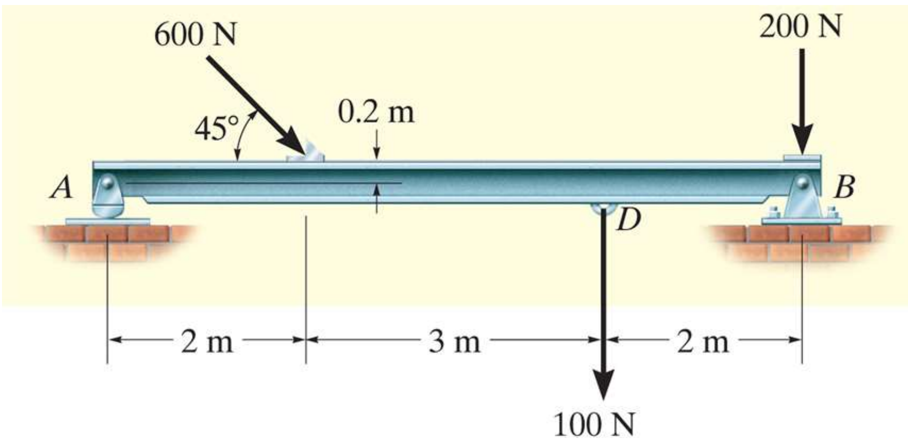
$$T sin20° - N sin30° = 0$$

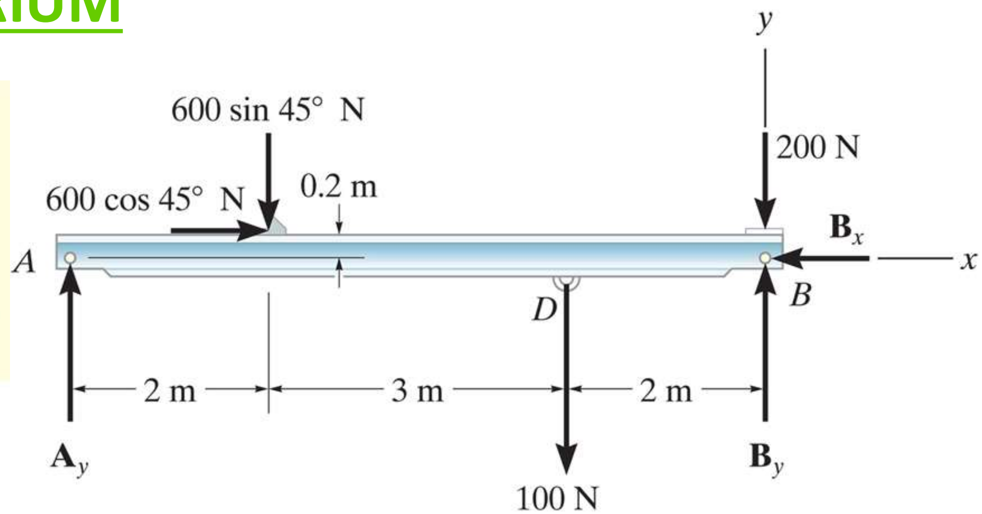
Sum forces in the y direction

$$+\uparrow \sum F_y = 0$$

$$T cos20° - N cos30° - W = 0$$

EXAMPLE: RIGID-BODY EQUILIBRIUM





Sum forces in the x direction to solve directly for B_x

$$+\rightarrow \sum F_x = 0; \quad 600cos45°N - B_x = 0$$

Sum moments about B to solve directly for A_y

$$+ccw \sum M_B = 0; \quad 100N(2m) + 600sin45°(5m) - A_y(7m) = 0$$

Sum forces in the y direction and plug in A_y to solve for B_y

$$+\uparrow \sum F_y = 0; \quad A_y - 600sin45°N - 100N - 200N - B_y = 0$$

HINTS, TIPS, SIMPLIFICATIONS

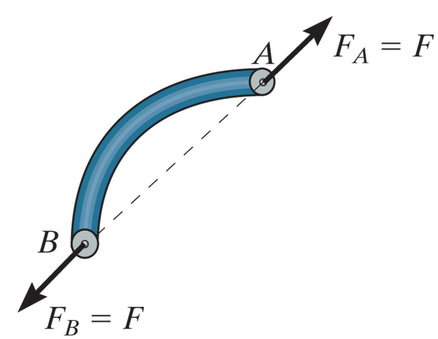
Non-concurrent system

- $\sum M$ about point where LOA of multiple unknown forces *intersect*

More unknowns than equations

- *Draw additional FBD*, look for relationships, apply geometry

Two-force members



- Forces applied at *2 points or pins only* (no other external forces, neglect weight); shape of member is not a factor!
- Equilibrium requires that the forces must be *equal magnitude, opposite direction, collinear* sharing same LOA

Three-force members

- Forces applied at *3 points* form *concurrent* or *parallel* system