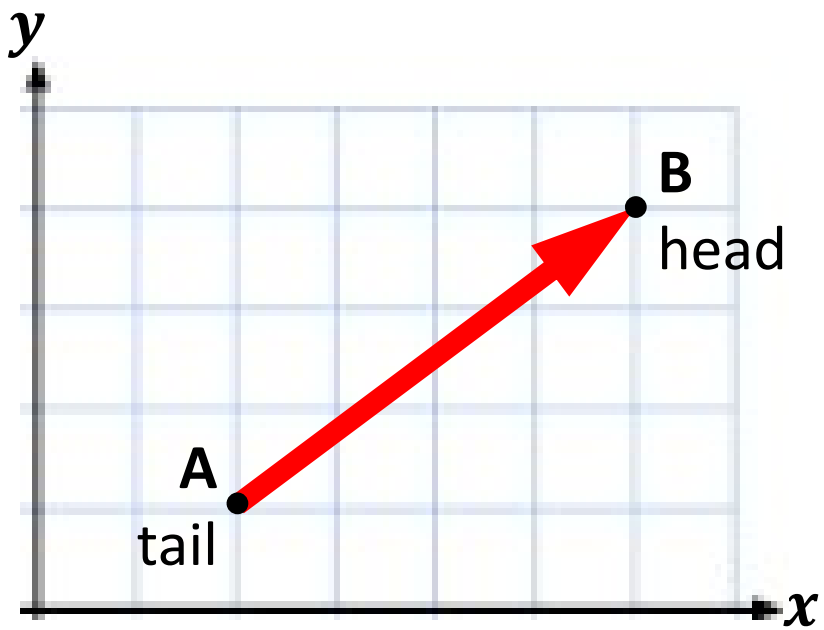
3D Forces by Unit Vector

Hibbeler 14e: Sec 2.5-2.8

REVIEW: VECTOR NOTATION



Vector notation:

$\boldsymbol{AB}$, $\underline{AB}$, $\overrightarrow{AB}$, $\overline{AB}$

Force vector:

$\vec{F}_{AB}$ "force AB"

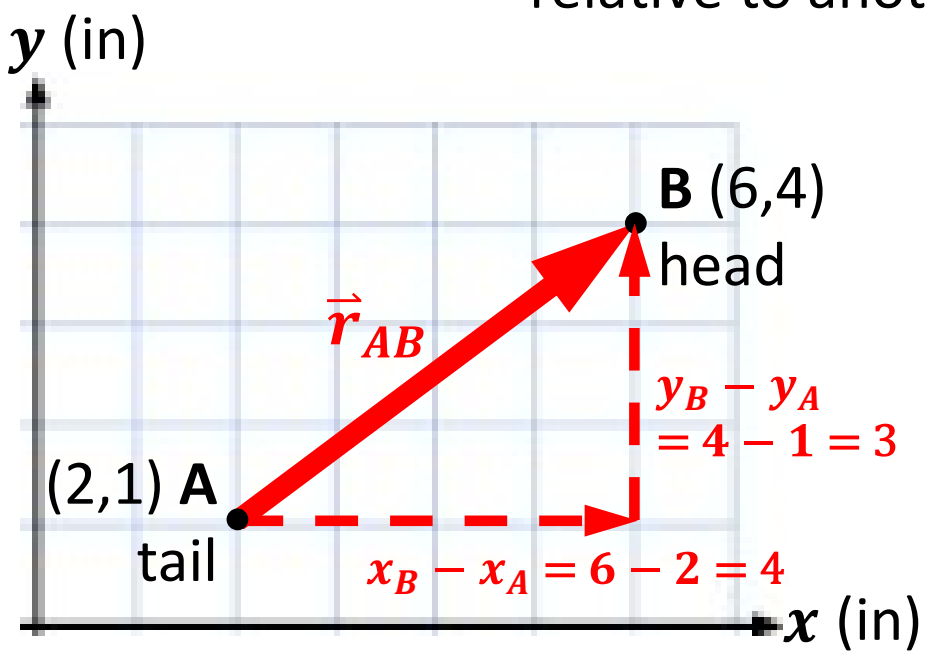
Position vector:

$\vec{r}_{AB}$ "from A to B"

$\vec{r}_{B/A}$ "B w.r.t. A"

POSITION VECTOR

Locates a point in space relative to another point



Position vector: $\vec{r}_{AB}$

$$\vec{r}_{AB} = (6-2)\hat{\imath} + (4-1)\hat{\jmath}$$

$$= (4\hat{\imath} + 3\hat{\jmath})\ in$$

$$|\vec{r}_{AB}| = \sqrt{(6-2)^2 + (4-1)^2}$$

$$= \sqrt{4^2 + 3^2} = \sqrt{16+9} = 5\ in$$

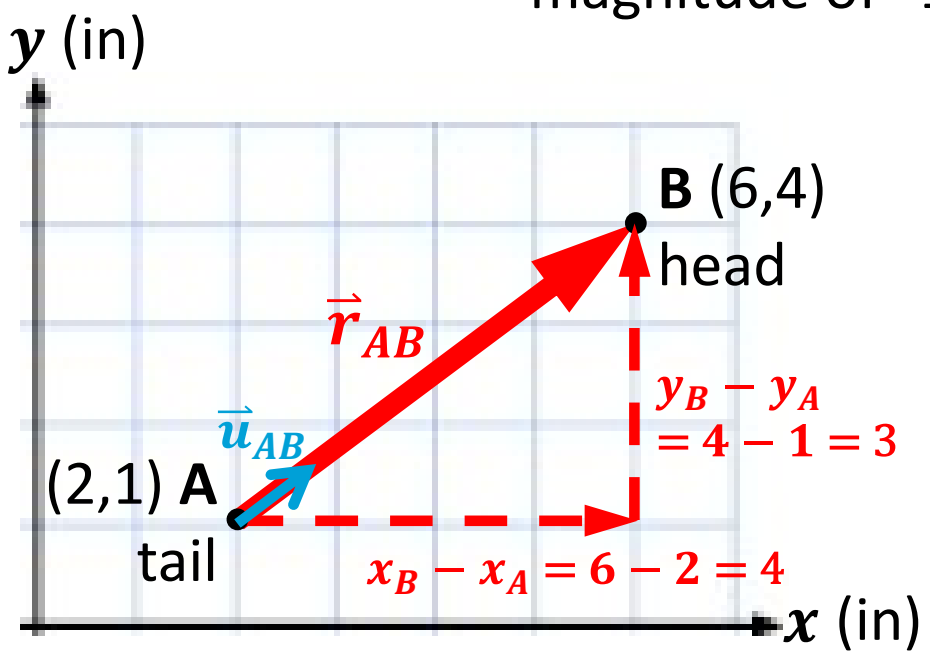
BASIC EQUATION FOR POSITION VECTOR

$$\vec{r}_{AB} = (x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

UNIT VECTOR

In direction of A to B with magnitude of "1" and dimensionless



Unit vector: $\vec{u}_{AB} = \dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$

$$\vec{u}_{AB} = \frac{(4\hat{\imath} + 3\hat{\jmath})\ in}{5\ in} = 0.8\hat{\imath} + 0.6\hat{\jmath}$$

Check $|\vec{u}_{AB}| = \sqrt{0.8^2 + 0.6^2}$

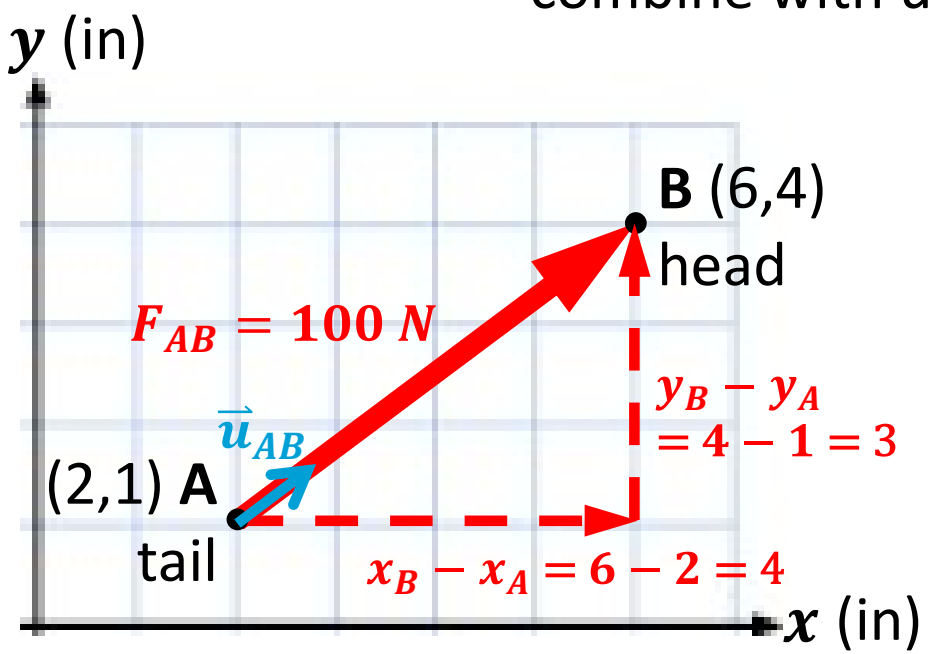
$$= \sqrt{0.64 + 0.36} = 1$$

BASIC EQUATION FOR UNIT VECTOR

$$\vec{u}_{AB} = \frac{(x_B - x_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

FORCE VECTOR

If force has line of action directed from A to B, combine with unit vector in direction of A to B



Force vector:

$$\vec{F}_{AB} = \left|\vec{F}_{AB}\right| \vec{u}_{AB} = F_{AB} * \vec{u}_{AB}$$

$$\vec{F}_{AB} = (100\ N)(0.8\hat{\imath} + 0.6\hat{\jmath})$$

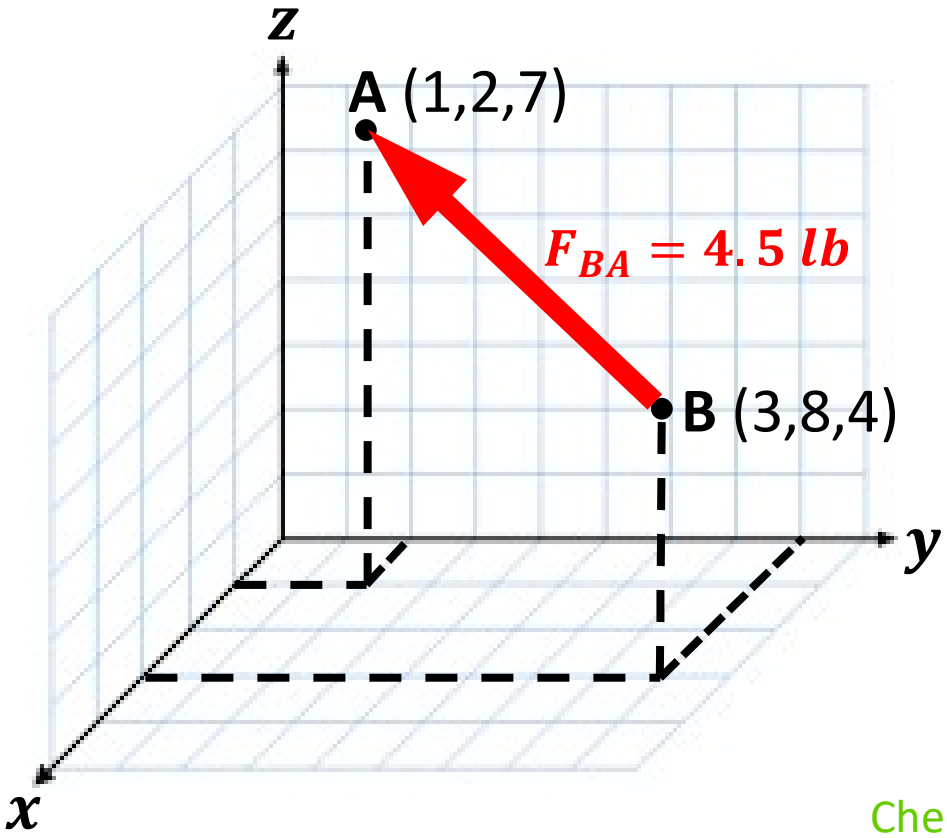
$$= (80\hat{\imath} + 60\hat{\jmath})\ N$$

Check $\left|\vec{F}_{AB}\right| = \sqrt{80^2 + 60^2}$

$$= \sqrt{6400 + 3600}$$

$$= \sqrt{10000} = 100\ N$$

3D FORCE VECTOR



Step 1: Position vector

$$\vec{r}_{BA} = (1-3)\hat{\imath} + (2-8)\hat{\jmath} + (7-4)\hat{k}$$
$$= -2\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$$

Step 2: Magnitude of position vector

$$|\vec{r}_{BA}| = \sqrt{(-2)^2 + (-6)^2 + (3)^2} = 7$$

Step 2: Unit vector

$$\vec{u}_{BA} = \frac{\vec{r}_{BA}}{|\vec{r}_{BA}|} = \frac{-2\hat{\imath} - 6\hat{\jmath} + 3\hat{k}}{7}$$
$$= -\frac{2}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{3}{7}\hat{k}$$

Check $|\vec{u}_{BA}| = \sqrt{\left(-\frac{2}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{3}{7}\right)^2} = 1$

Step 4: Force vector

$$\vec{F}_{BA} = \left|\vec{F}_{BA}\right| \vec{u}_{BA} = (4.5\ lb)\left(-\frac{2}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{3}{7}\hat{k}\right) = \left(-1.29\hat{\imath} - 3.86\hat{\jmath} + 1.93\hat{k}\right) lb$$