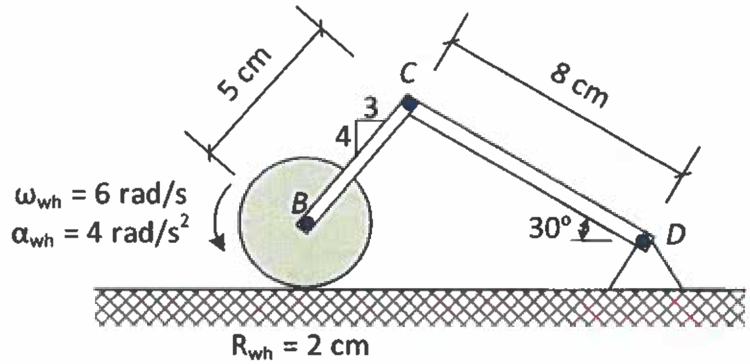
Rigid Body Kinematics II – Problem 1

The no-slip wheel at *B* has an angular velocity of 6 rad/s and an angular acceleration of 4 rad/s^2 (both CCW) as shown. Determine the angular velocity and acceleration of bars *BC* and *CD* as well as the acceleration of point *C* at this instant.



CLASSIFY MOTION

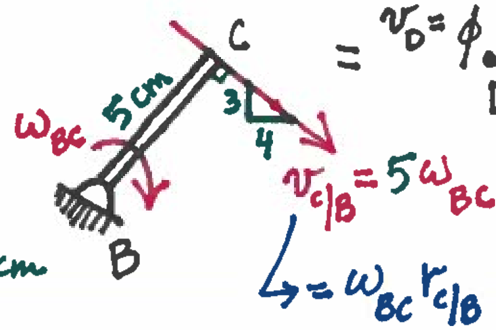
WHEEL B – GPM
BAR BC – GPM
BAR CD – RAFA

RELATIVE VELOCITY EQN:

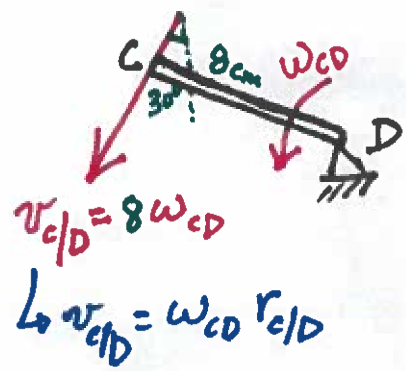
$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} = \vec{v}_D + \vec{v}_{C/D}$$

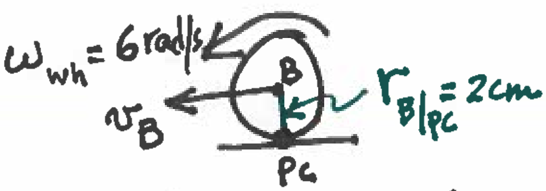
KINEMATIC DIAGRAMS:

$v_B = 12\text{ cm/s} \leftarrow$ B $\quad + \quad$



$= v_D = \phi$ D $+$





No Slip wheel $\Rightarrow v_B = \omega_{wh} r_{B/PC}$
$= 6\frac{\text{rad}}{\text{s}}(2\text{cm}) = 12\text{ cm/s} \leftarrow$

$\overset{+}{\rightarrow} x: \quad -12 + 5\omega_{BC}\left(\frac{4}{5}\right) = \phi - 8\omega_{CD}\sin 30$

$-12 + 4\omega_{BC} = -4\omega_{CD}$ ---- (1)

$+\uparrow y: \quad 0 - 5\omega_{BC}\left(\frac{3}{5}\right) = \phi - 8\omega_{CD}\cos 30$

$-3\omega_{BC} = -6.9282\,\omega_{CD}$ --- (2)

2 Eqs 2 unk. ✓

Solve Eqs (1) & (2) to find ω_{BC} and ω_{CD}

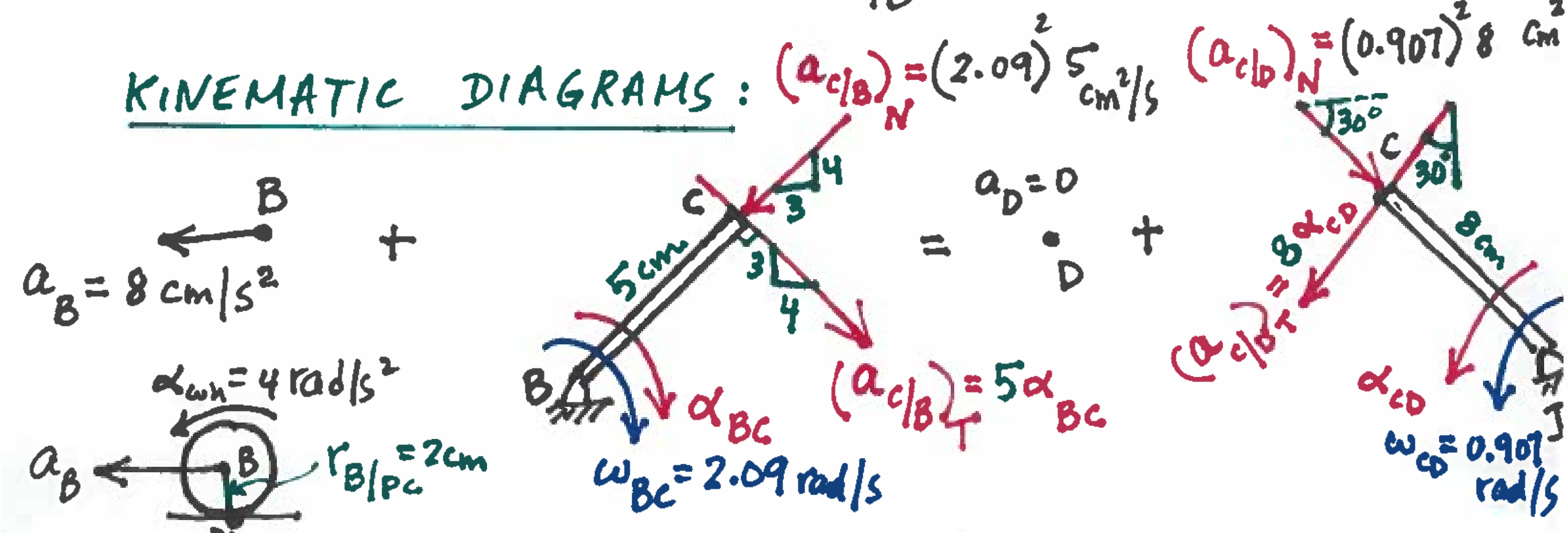
$\omega_{BC} = \underline{\underline{2.09 \text{ rad/s}}} \circlearrowright$

$\omega_{CD} = \underline{\underline{0.907 \text{ rad/s}}} \circlearrowleft$

RELATIVE ACCELERATION EQN:

$$\vec{a}_C = \vec{a}_B + \vec{a}_{C/B} = \vec{a}_D + \vec{a}_{C/D}$$

KINEMATIC DIAGRAMS:



$$a_B = \alpha_{wh}\, r_{B/Pc} = 4(2) = 8\ \text{cm/s}^2$$

$$(a_{C/B})_T = \alpha_{BC}\, r_{C/B} = 5\alpha_{BC}$$

$$(a_{C/B})_N = \omega_{CB}^2\, r_{C/B} = (2.09)^2 5\ \text{cm}^2/\text{s}$$

$$(a_{C/D})_T = \alpha_{CD}\, r_{C/D} = 8\alpha_{CD}$$

$$(a_{C/D})_N = \omega_{CD}^2\, r_{C/D} = (0.907)^2 8\ \text{cm}^2/\text{s}$$

$\overset{+}{\rightarrow} x$: $-8 + 5\alpha_{BC}\left(\frac{4}{5}\right) - (2.09)^2 5\left(\frac{3}{5}\right) = \phi - 8\alpha_{CD}\sin 30 + (0.907)^2 8 \cos 30$

$$\alpha_{BC} + \alpha_{CD} = 6.701 \quad \text{---- (1)}$$

$+\uparrow y$: $\cancel{\phi} - 5\alpha_{BC}\left(\frac{3}{5}\right) - (2.09)^2 5\left(\frac{4}{5}\right) = \phi - 8\alpha_{CD}\cos 30 - (0.907)^2 8 \sin 30$

$$-3\alpha_{BC} + 6.9282\,\alpha_{CD} = 14.1818 \quad \text{--- (2)}$$

2 Eqs, 2 Unk. ✓ Solve Eqs (1) & (2):

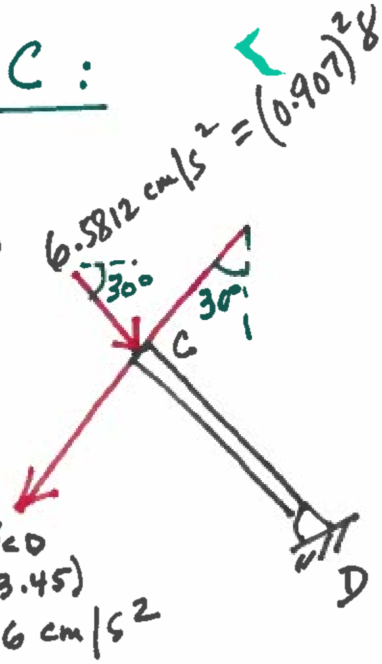
$$\underline{\underline{\alpha_{BC} = 3.25\ \text{rad/s}^2 \circlearrowright}}$$

$$\underline{\underline{\alpha_{CD} = 3.45\ \text{rad/s}^2 \circlearrowleft}}$$

ACCELERATION OF POINT C:

$$\vec{a}_C = \vec{a}_D + \vec{a}_{C/D}$$

$= \quad a_D = 0 \quad \overset{\bullet}{D} \quad +$



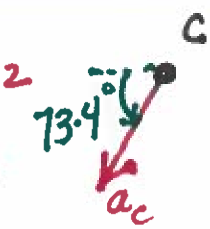
$\xrightarrow{+} x: \quad (a_c)_x = 0 - 27.6 \sin 30 + 6.5812 \cos 30$

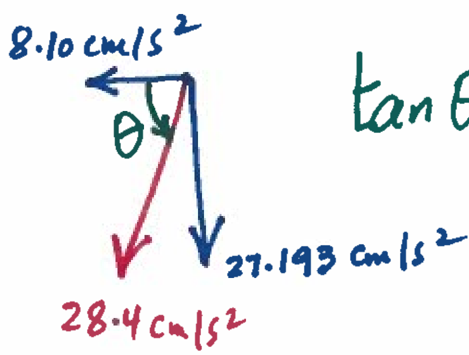
$= -8.10 = 8.10\ cm/s^2 \leftarrow$

$+\uparrow y: \quad (a_c)_y = 0 - 27.6 \cos 30 - 6.5812 \sin 30$

$= -27.193 = 27.193\ cm/s^2 \downarrow$

$$a_c = \sqrt{(8.10)^2 + (27.193)^2} = \underline{\underline{28.4\ cm/s^2}}$$





$$\tan\theta = \frac{27.193}{8.10} \Rightarrow \theta = 73.4°$$

Note: One may also calculate a_c using $\vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$ to obtain same result

Try it!