Internal Forces

Hibbeler 14e: Sec 7.1

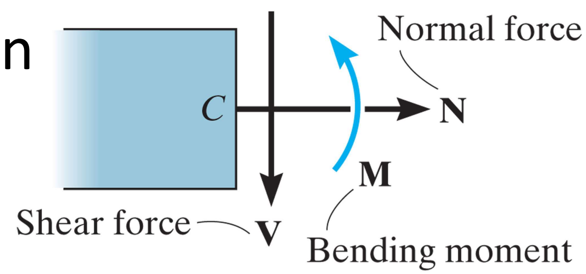
INTERNAL FORCES

Method of sections

Develop *within* structural or mechanical members due to external loads

Internal loadings --- prevent relative translation and rotation

- Normal force $\mathbf{N}_i$ acts perpendicular to section
- Shear force $\mathbf{V}_i$ acts tangent to section
- Bending moment $\mathbf{M}_i$ is a couple moment



Free-body diagram (FBD) --- must draw for solving problems!

- Remove from supports, *entire structure*, solve support reactions
- Pull pins at joints, *disconnect members*, solve pin reactions
- Section through member, *isolate segment*, solve internal forces

Equations of equilibrium for 2D --- must write for solving problems!

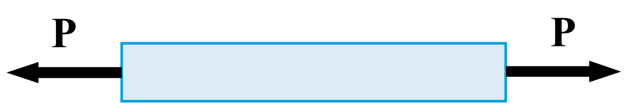
- NON-CONCURRENT --- $\sum F_x = 0\,,\ \sum F_y = 0\,,\ \sum M_z = 0$

INTERNAL FORCES: RIGID-BODY EQUILIBRIUM

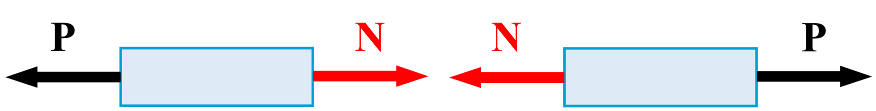
Axial loaded members

Forces applied *parallel* to the longitudinal axis of the member; results in *tension* or *compression*

(e.g. truss members)



Forces are equal, opposite, collinear

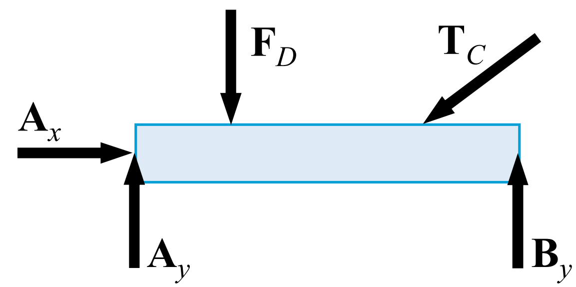


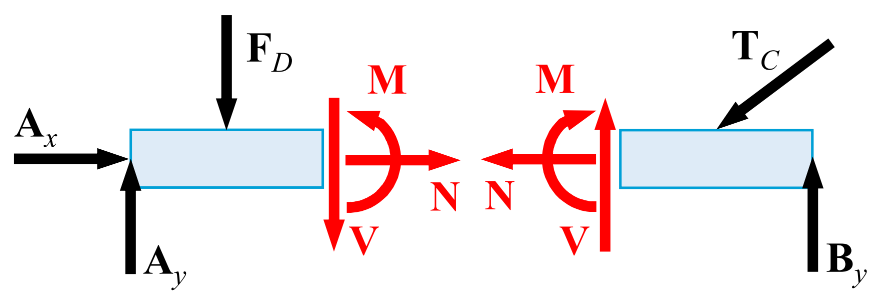
Draw *normal force (**N**)* in tension; compression if negative solution

Axial and transverse loaded

Forces applied have components *perpendicular* to member's axis; results in *shear* and *bending*

(e.g. frames, machines, beams)

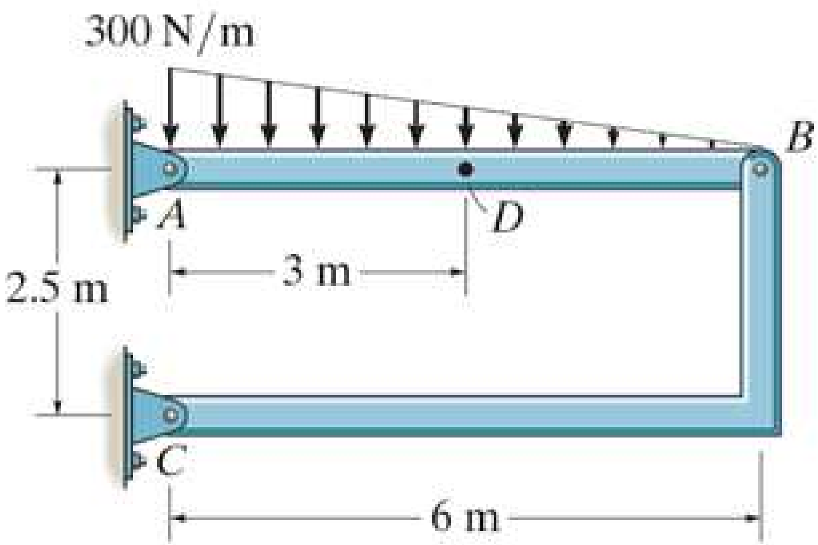


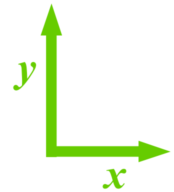


Draw **N, V, M** in positive directions; opposite sense if negative solution

EXAMPLE: EXTERNAL REACTIONS

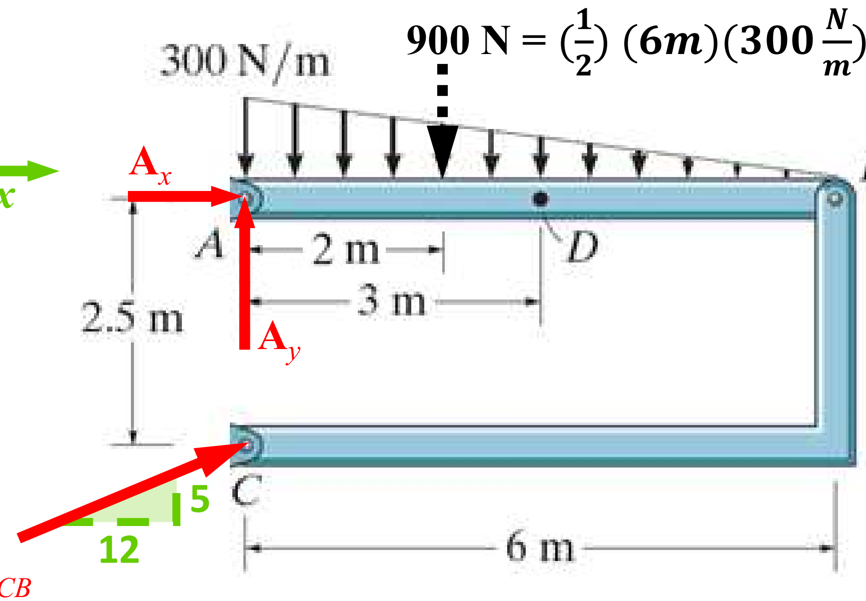
$\sum F_x = 0, \;\; \sum F_y = 0, \;\; \sum M_z = 0$





FBD of entire structure

consider as one solid body



Sum moments about A to solve directly for F_{CB}

$$+ccw \sum M_A = 0; \quad \frac{12}{13} F_{CB}(2.5m) - 900N(2m) = 0$$

$$+ccw \sum M_A = 0; \quad (2.3077m)F_{CB} - 1800Nm = 0$$

$$+ccw \sum M_A = 0; \quad (2.3077m)F_{CB} = 1800Nm \qquad F_{CB} = \frac{1800Nm}{2.3077m} = 780N$$

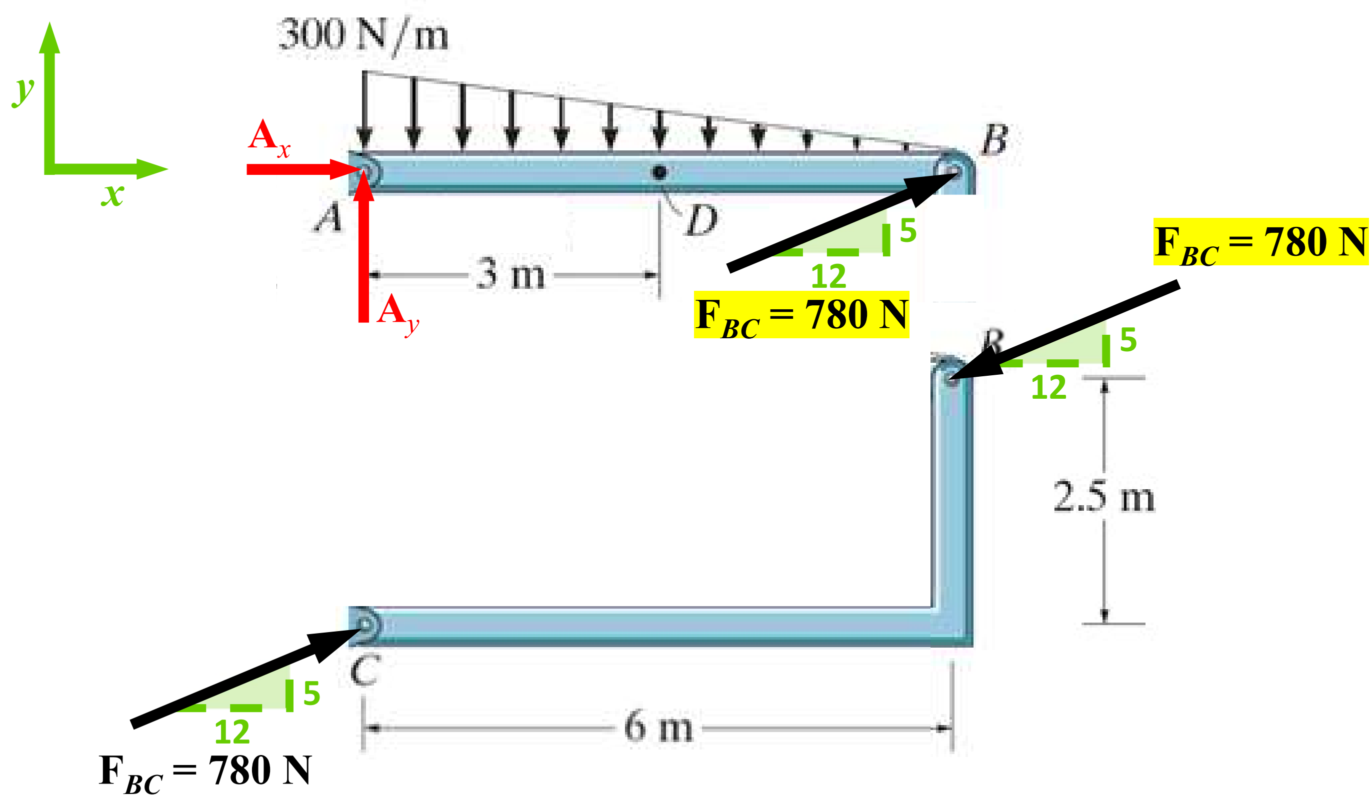
EXAMPLE: DISASSEMBLE

Pull pins at joints to disconnect members

FBD of each member

principle of action-reaction

Forces common to two members which are in contact act with equal magnitude but opposite sense on the respective FBD of the members



EXAMPLE: METHOD OF SECTIONS

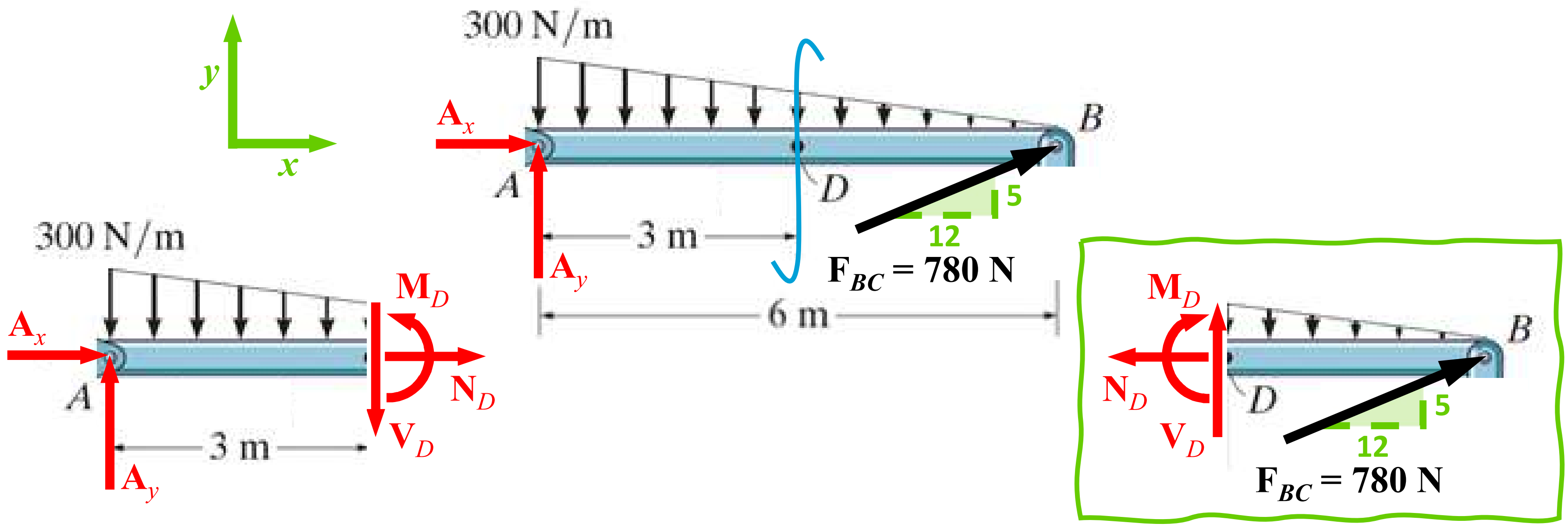
Cut through member at point of interest

FBD of cut section

Segment with least number of loads

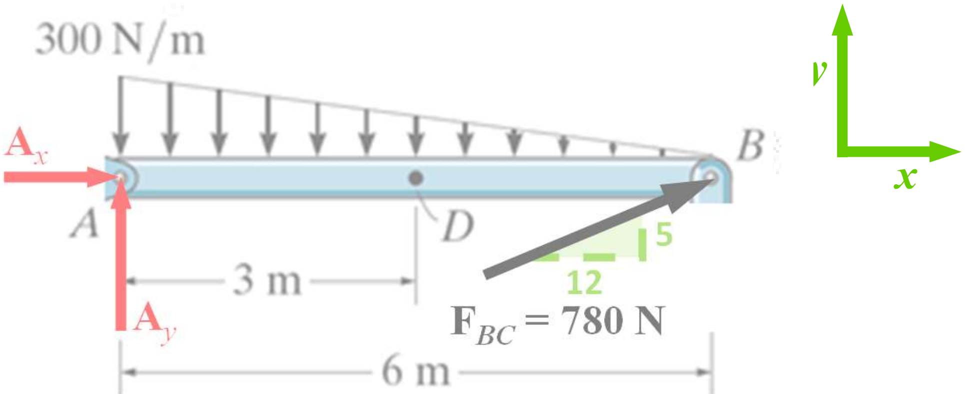
Keep all distributed loadings, couple moments, and forces acting on the member in their *exact locations*

Then pass an imaginary section through member, perpendicular to its axis at the point where the internal loadings are to be determined



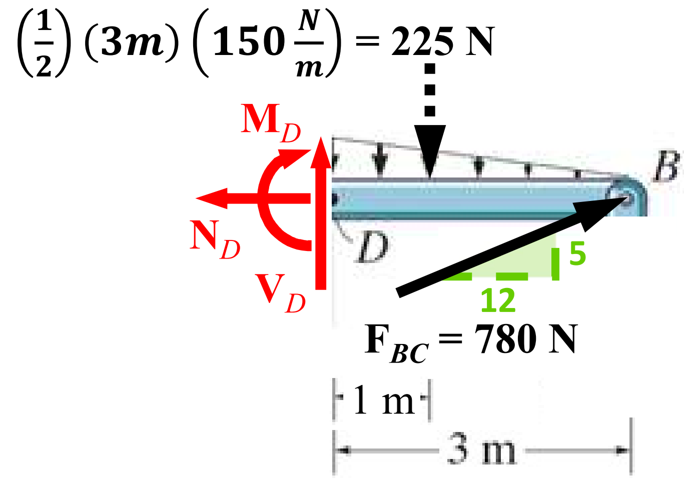
EXAMPLE: INTERNAL FORCES

Normal force, shear force, bending moment



FBD of cut section

expose internal forces as external



Sum forces in the x-direction to solve directly for N_D

$+\!\rightarrow \sum F_x = 0;\quad -N_D + \frac{12}{13}780N = 0 \qquad N_D = 720\ N\ \leftarrow on\ DB$

Sum forces in the y direction to solve directly for V_D

$+\!\uparrow \sum F_y = 0;\quad V_D - 225N + \frac{5}{13}780N = 0 \qquad V_D = -75 = 75\ N\ \downarrow on\ DB$

Sum moments at the section about D to solve directly for M_D

$+ccw \sum M_D = 0;\quad -M_D - 225N(1m) + \frac{5}{13}780N(3m) = 0 \qquad M_D = 675\ Nm\ \ on\ DB$

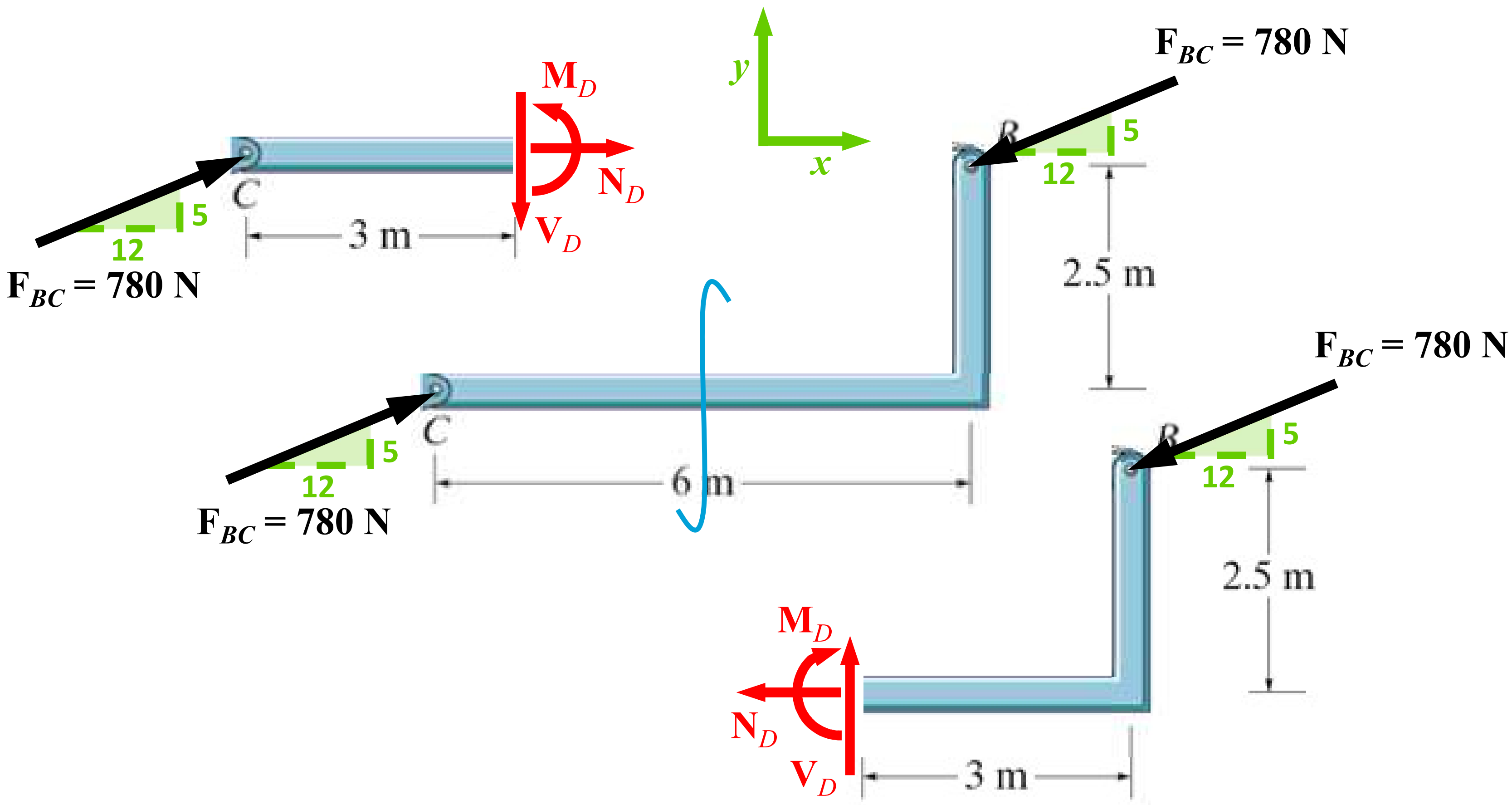
EXAMPLE: INTERNAL FORCES

Normal force, shear force, bending moment

FBD of cut section

expose internal forces as external

Forces acting on a two-force member may result in *shear* and *bending*



PROCEDURE FOR ANALYSIS --- INTERNAL FORCES, 2D EQUILIBRIUM

Only if needed

- Entire structure FBD to solve for support reactions
- Pull pins, action-reaction, member FBDs to solve for pin reactions

Method of sections and FBD of segment with least number of loads

- *Keep* all loadings acting on the member in their *exact locations*, then make a perpendicular "cut" through the member
- Draw FBD of segment with exposed internal loadings ($\mathbf{N}_i$ $\mathbf{V}_i$ $\mathbf{M}_i$) acting in their "positive" directions at the cross section

Apply equations of equilibrium to solve for internal loadings

- $\sum M$ at the section about the point where normal and shear forces *intersect* to obtain direct solution for *bending moment*

HINTS, TIPS, SIMPLIFICATIONS

- 2-force members equal opposite collinear with known direction
- If cut goes through distributed load find *resultant load* after cut
- *Negative* solution indicates *opposite* sense to that drawn on FBD