



$$a_G = \alpha R / 6$$

kinematic relation

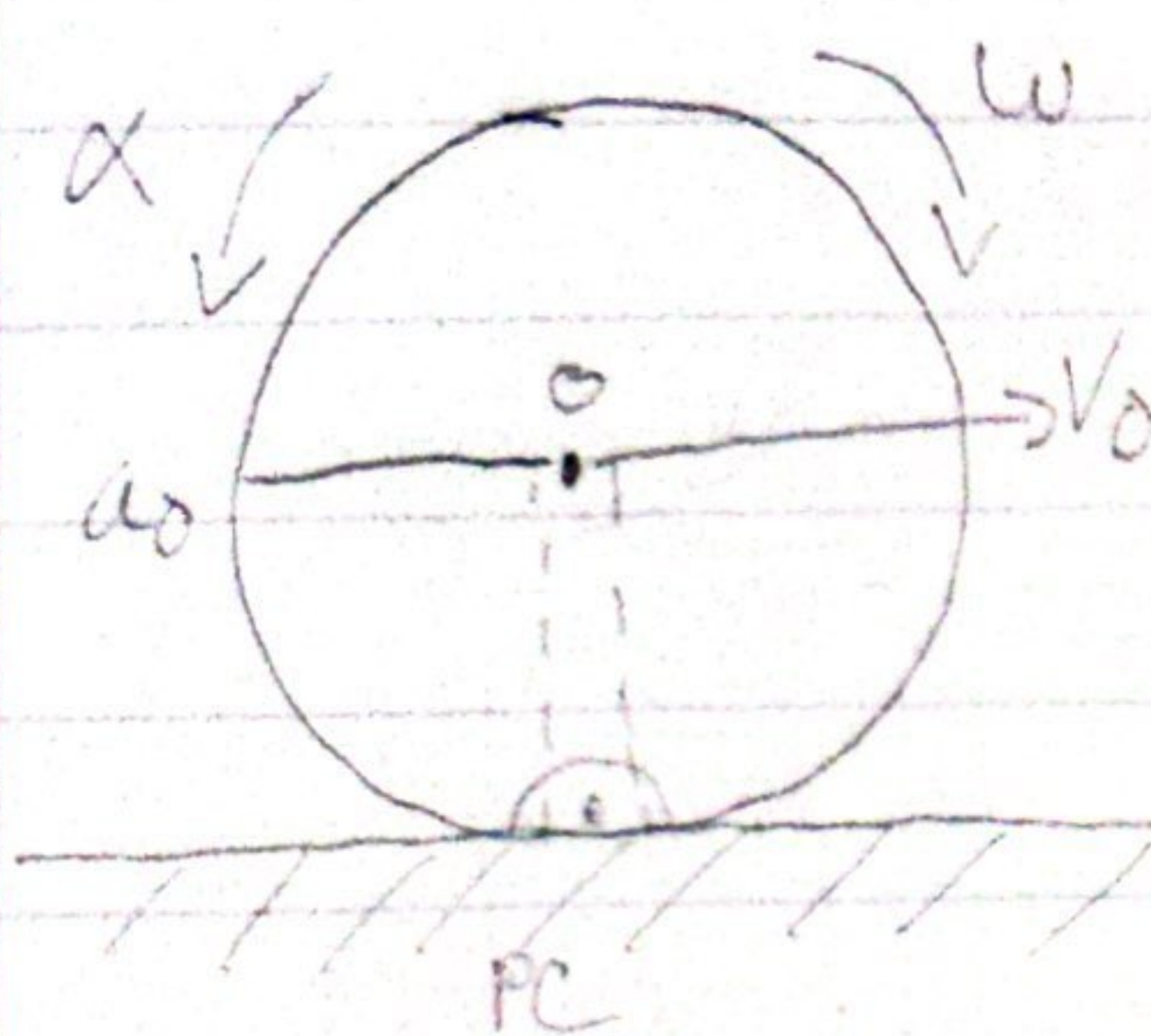
$$a_B = 1.5\alpha \quad a_B = a_{By}$$

$$a_{By} = 1.5\alpha$$

$$v_B = \omega R / 6$$

No slip: kinematic

Slipping:

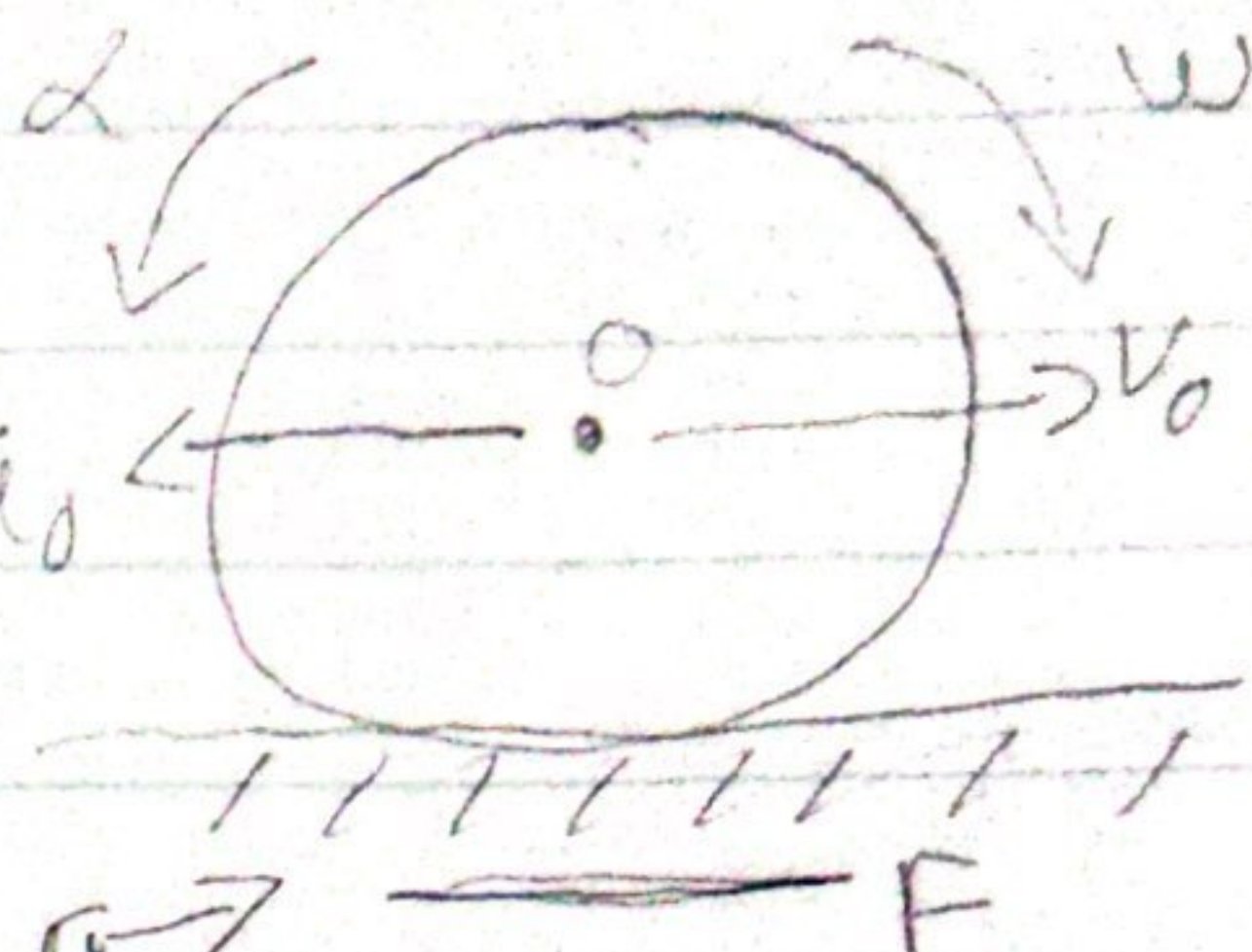


\* assume no

slip first &

\* solve for

friction force  $a_0$



opposite motion

of P.C. solve  $a_0 \neq \omega R$

$(v_0 \neq \omega R)$  two objects move over each other

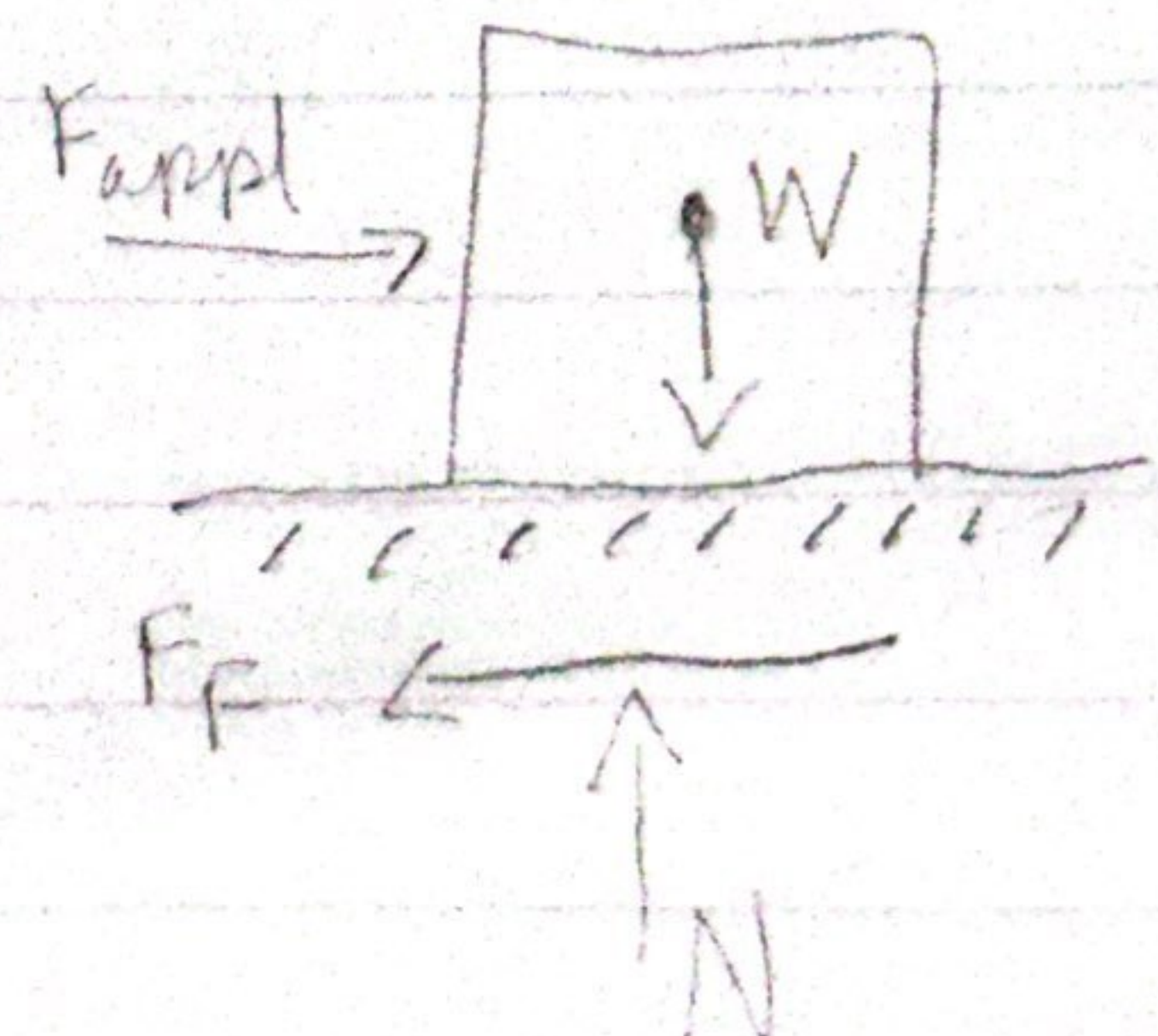
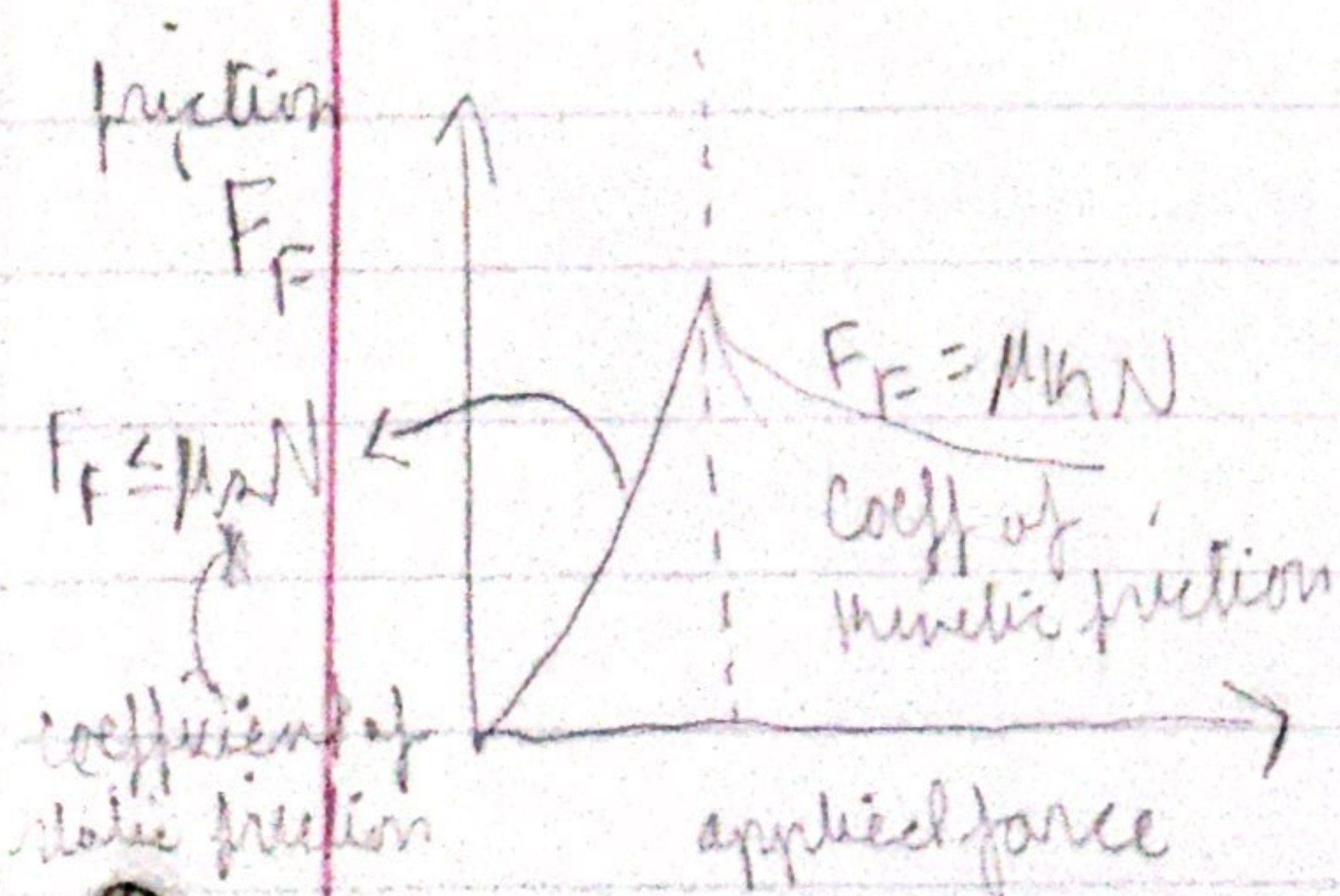
$$F_f = \mu_k N$$

if no, then use this

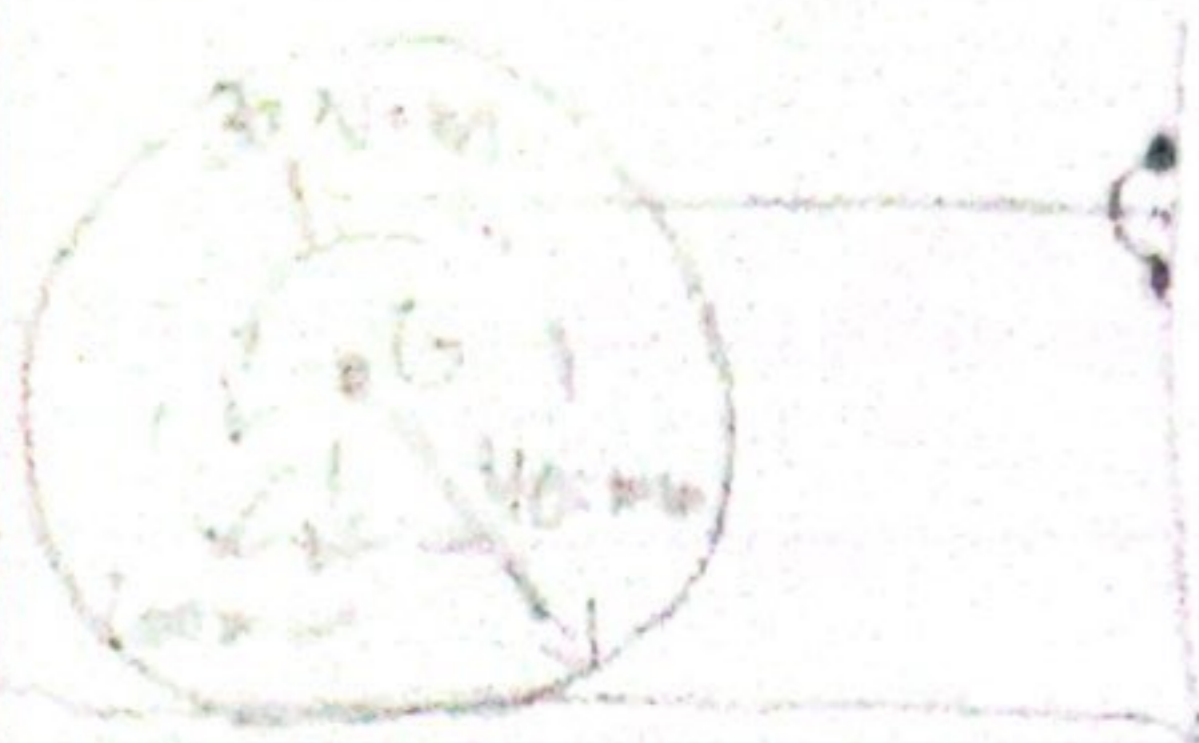
$$\checkmark * a_0 = \alpha R$$

$$v_0 = \omega R$$

solve  $F \leq \mu_s N$  if yes use this



## force acceleration II: problem 5



two objects moving  
over each other  
 $F = \mu_k N$

make an assumption to start  
the problem, always assume no slips

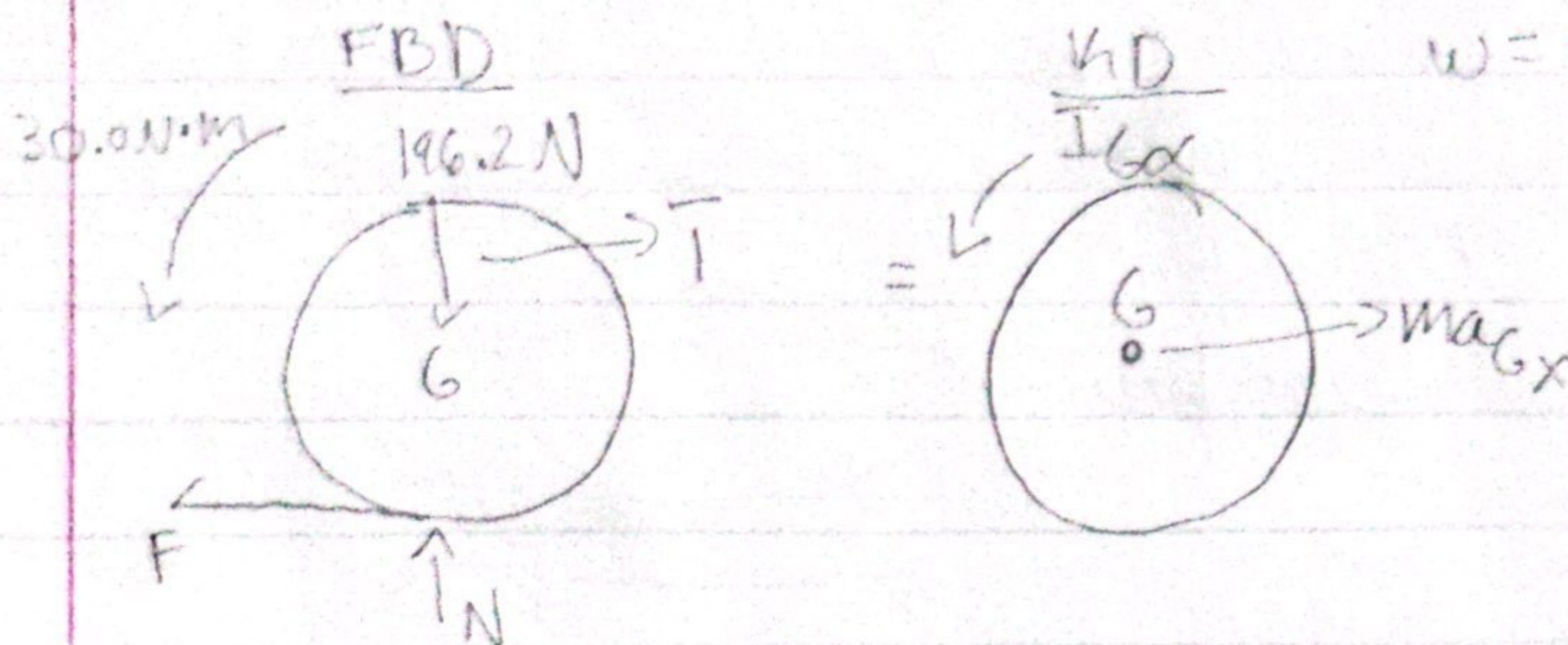
radius of gyration  $k = 0.25 \text{ m}$   
 $\hookrightarrow I_G = mk^2$

Clarify motion:

$$m = 20 \text{ kg}$$

$$I_G = mk^2 = (20)(0.25)^2 = 1.25$$

$$W = Mg = (20)(9.81) = 196.2$$



$$\Rightarrow \Sigma F_x: -F + T = Ma_{Gx} = 20 a_{Gx}$$

must be slipping  $\therefore F = \mu_k N = 0.1(196.2) = 19.62 \text{ N}$   
at ground

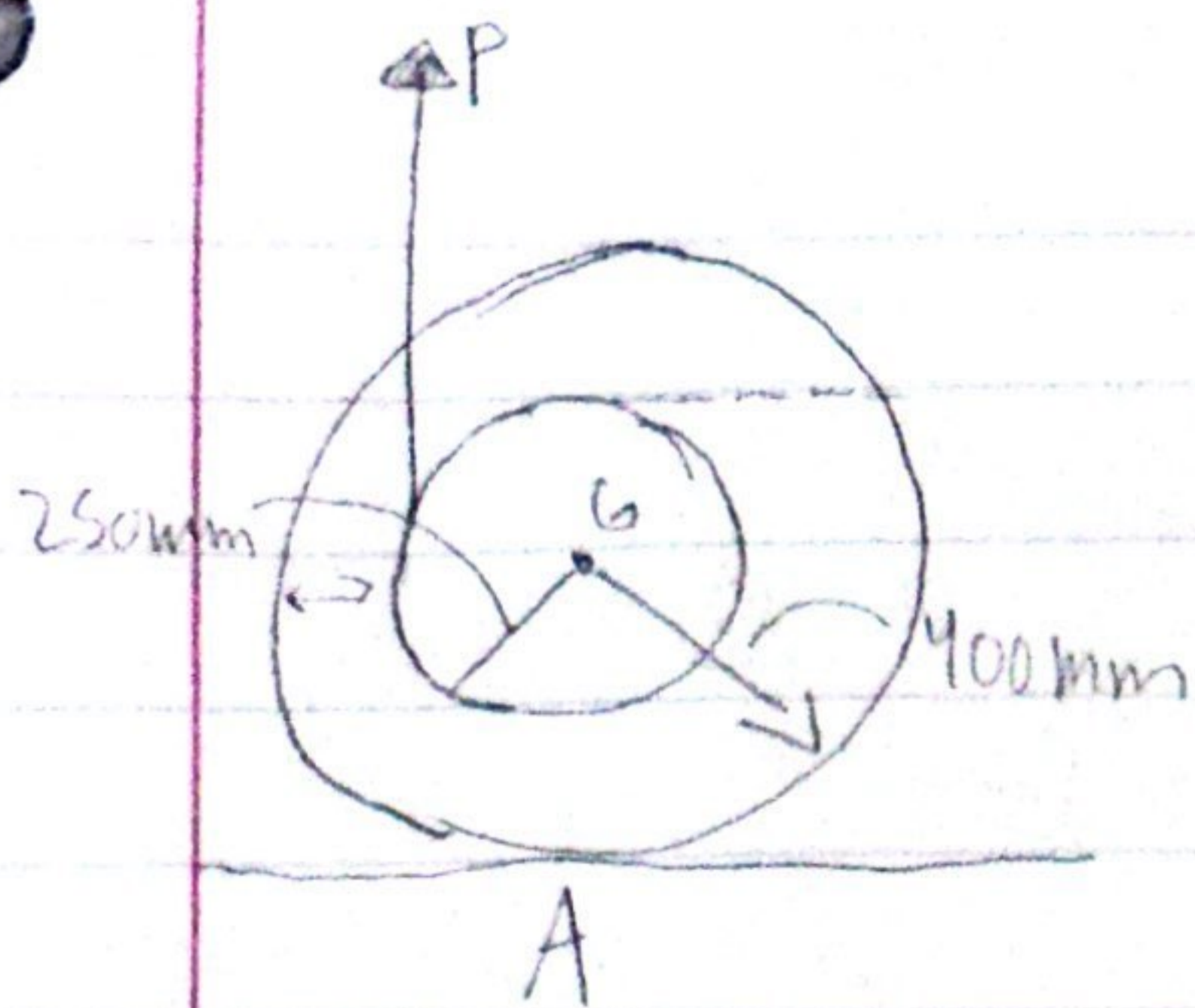
Statics:

$$\begin{aligned} \rightarrow \Sigma F_x &= 0 \\ \uparrow \Sigma F_y &= 0 \\ \curvearrowright \Sigma M_G &= 0 \end{aligned}$$

Dynamics:

$$\begin{aligned} \rightarrow \Sigma F_x &= ma_{Gx} \\ \uparrow \Sigma F_y &= ma_{Gy} \\ \curvearrowright \Sigma M_G &= I_G \alpha \end{aligned}$$

# force acceleration III: problem I



$$m = 10 \text{ kg}$$

$$\text{radius of gyration } I_G = 0.3 \text{ m} \quad d_1 = 0.25 \text{ m}$$

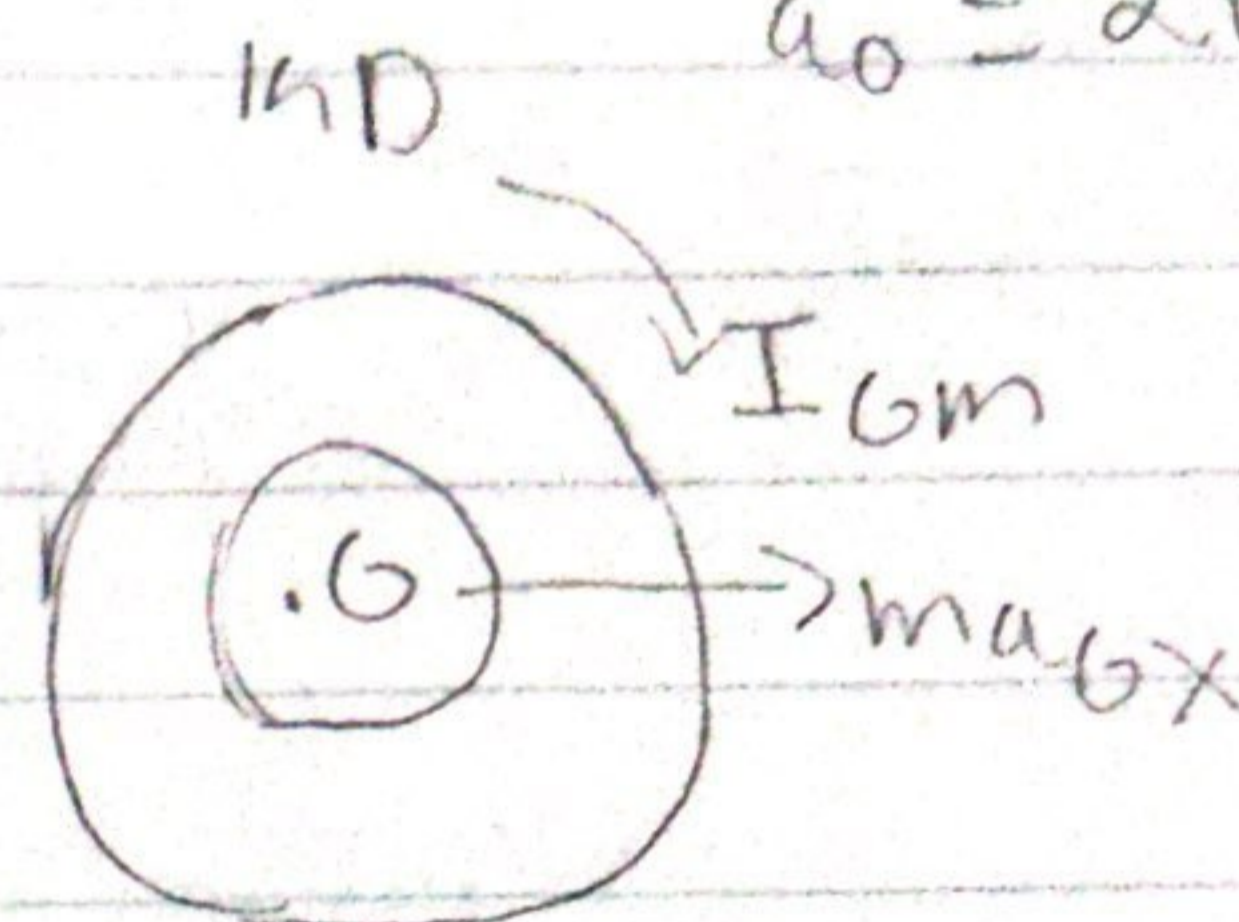
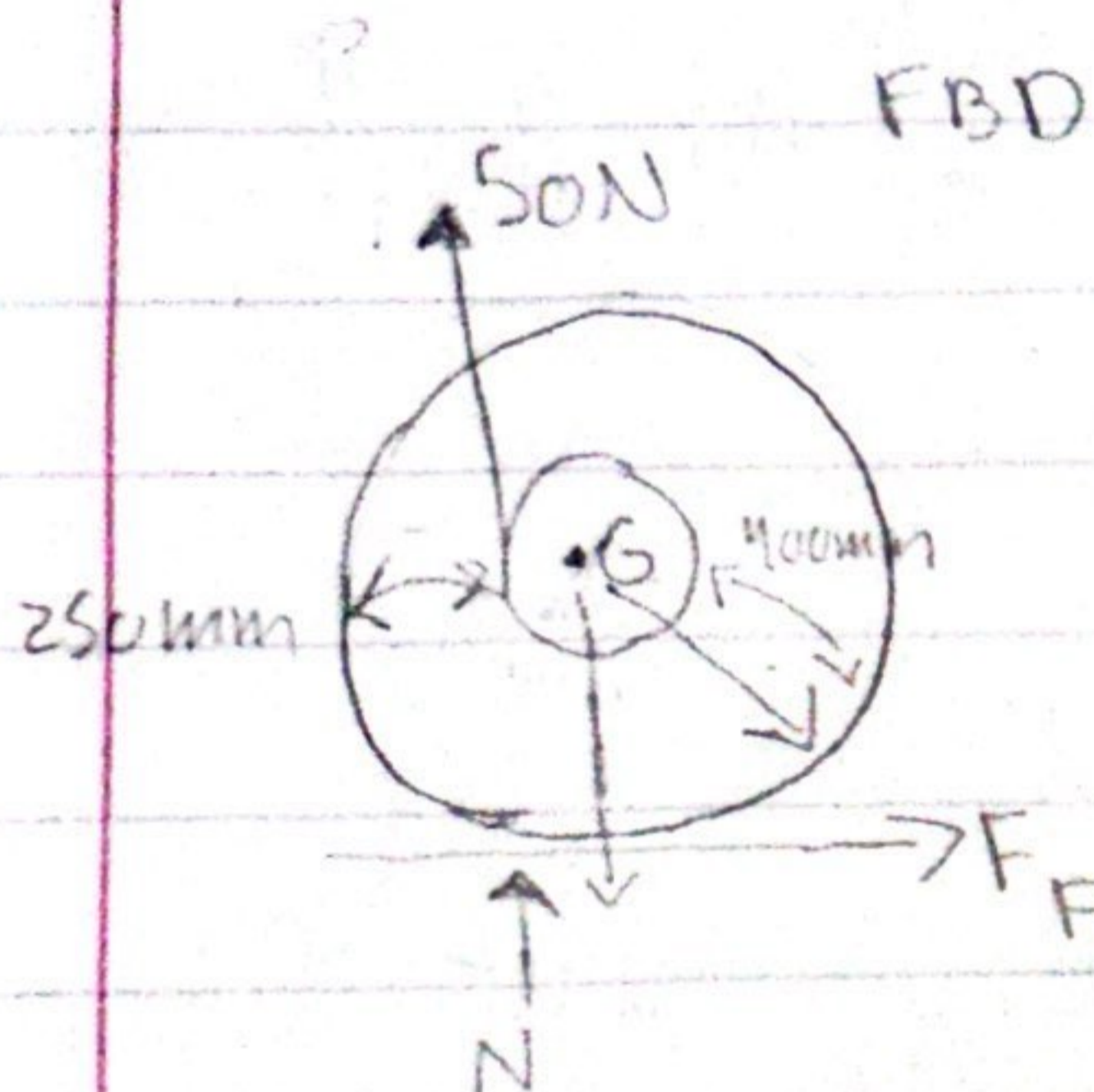
$$\mu_s = 0.2 \quad \mu_k = 0.15 \quad P = 50 \text{ N} \quad d_2 = 0.4 \text{ m}$$

rpm

$$I_G = mk^2 = (10 \text{ kg})(0.3)^2 = 0.9$$

$$W = mg = (10)(9.81) = 98.1$$

$$a_0 = \alpha r \quad (9.81)(0.4) = 3.924$$



$$1) \uparrow \sum F_y \Rightarrow 50 - 98.1 + N = 0 \quad N = 48.1$$

$$2) \rightarrow \sum F_x \Rightarrow F = m a_{Gx} = 10 a_{Gx} = 10(0.4\alpha)$$

$$3) \sum M_G \Rightarrow -50(0.25) + F(0.4) = -0.9\alpha$$

assume no slip

$$F = 4\alpha$$

$$a_{Gx} = \alpha r = 0.4\alpha$$

$$-50(0.25) + (0.4\alpha)(0.4) = -0.9\alpha$$

$$\alpha = 5.0 \text{ rad/s}^2$$

$$a_{Gx} = 0.4\alpha = 2.0 \text{ m/s}^2 \quad F = 10 a_{Gx} = 20.0 \text{ N}$$

$$F_{\max} = \mu_s N = 0.2(48.1 \text{ N}) = 9.62 \text{ N}$$

$F > F_{\max} \therefore$  slipping

$$F = \mu_k N = 0.15(48.1 \text{ N}) = 7.21 \text{ N}$$

$$-50(0.25) + F(0.4) = -0.9\alpha$$

$$-50(0.25) + 7.21(0.4) = -0.9\alpha$$

$$\boxed{\alpha = 10.68 \text{ rad/s}^2}$$

# Force acceleration



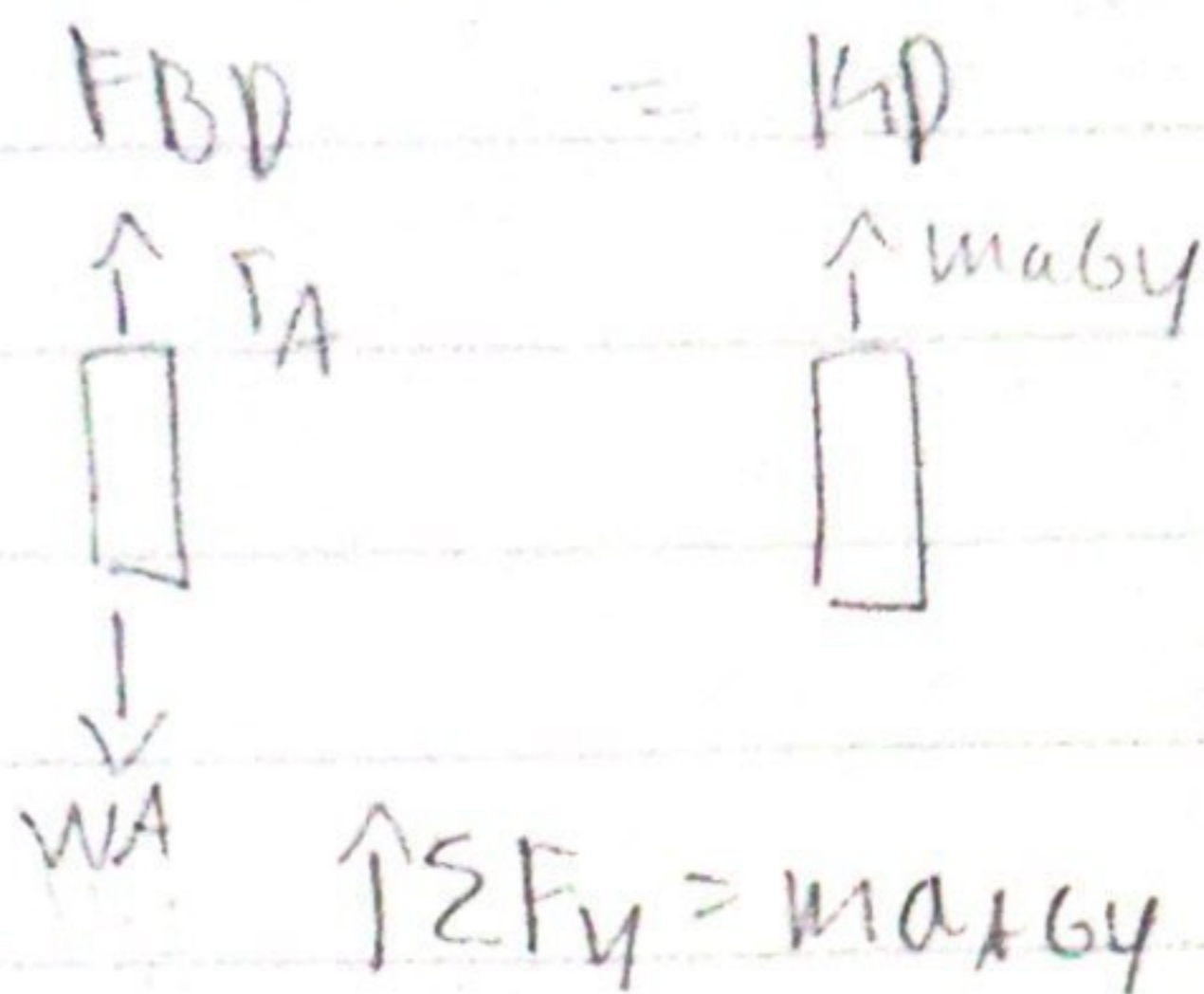
$$m_A = 5 \text{ kg}$$

$$m_B = 10 \text{ kg}$$

$$m_{\text{pulley}} = 3 \text{ kg}$$

$$r = 0.15 \text{ m}$$

Block A:



$$a = \alpha r$$

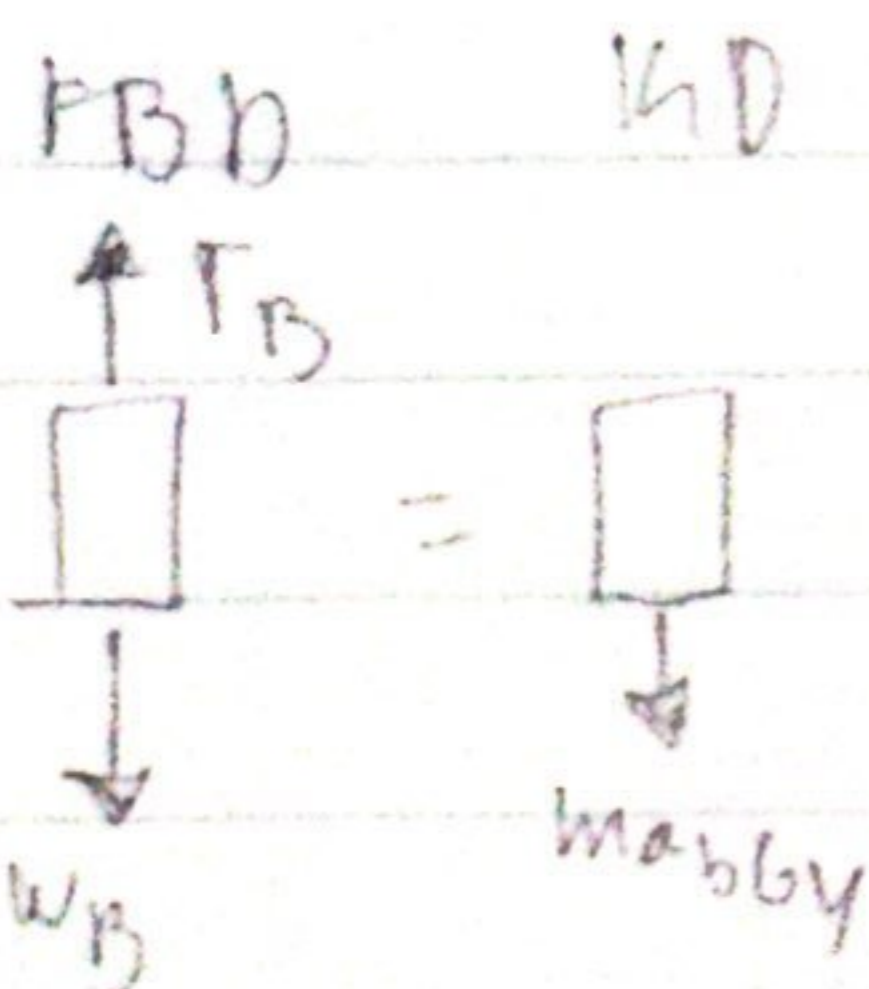
$$T_A - W_A = m_A a_B y \quad \alpha r$$

$$T_A - (5 \text{ kg})(9.81) = 5 \text{ kg} \alpha r$$

$$T_A - 49 = (5 \text{ kg})(0.15 \text{ m}) \alpha$$

$$\textcircled{1} T_A = 0.75 \alpha + 49$$

Block B:



$$\uparrow \Sigma F_y = m a_B y$$

$$T_B - W_B = -m a_B y$$

$$T_B - (10 \text{ kg})(9.8) = -(10 \text{ kg}) \alpha r$$

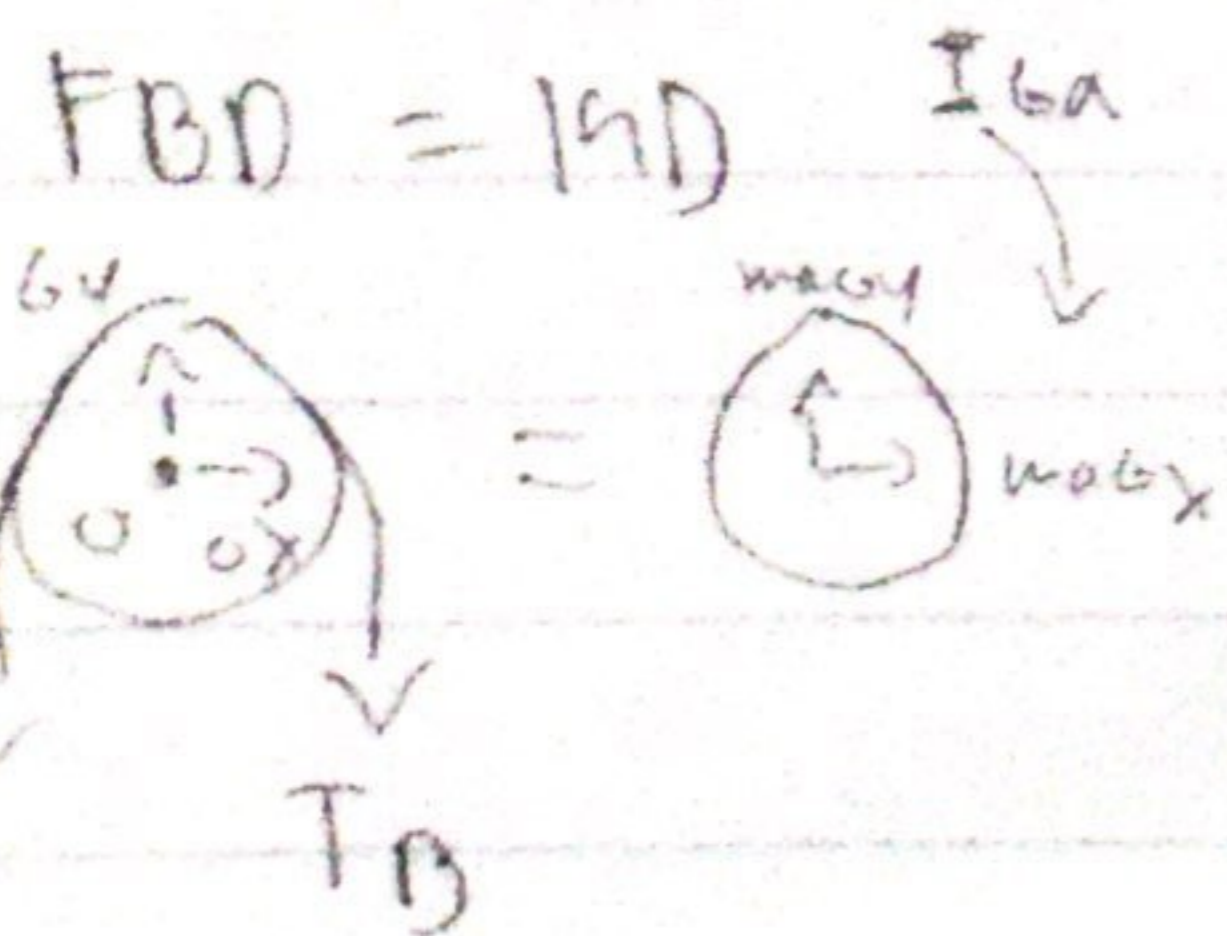
$$T_B - (10)(9.8) = -(10 \text{ kg}) \alpha (0.15 \text{ m})$$

$$T_B = -1.5 \alpha + 98$$

$$I_0 = \frac{1}{2} m r^2$$

$$\frac{1}{2} (3) (0.15)^2$$

Drum



$$\uparrow \Sigma M_O = I_0 \alpha \quad T_B(0.15 \text{ m}) - T_A(0.15 \text{ m}) = I_0 \alpha$$

$$T_B - T_A = \frac{1}{2} (3) (0.15) \alpha$$

$$T_B - T_A = 0.225 \alpha$$

$$T_B - T_A = 0.225 \alpha$$

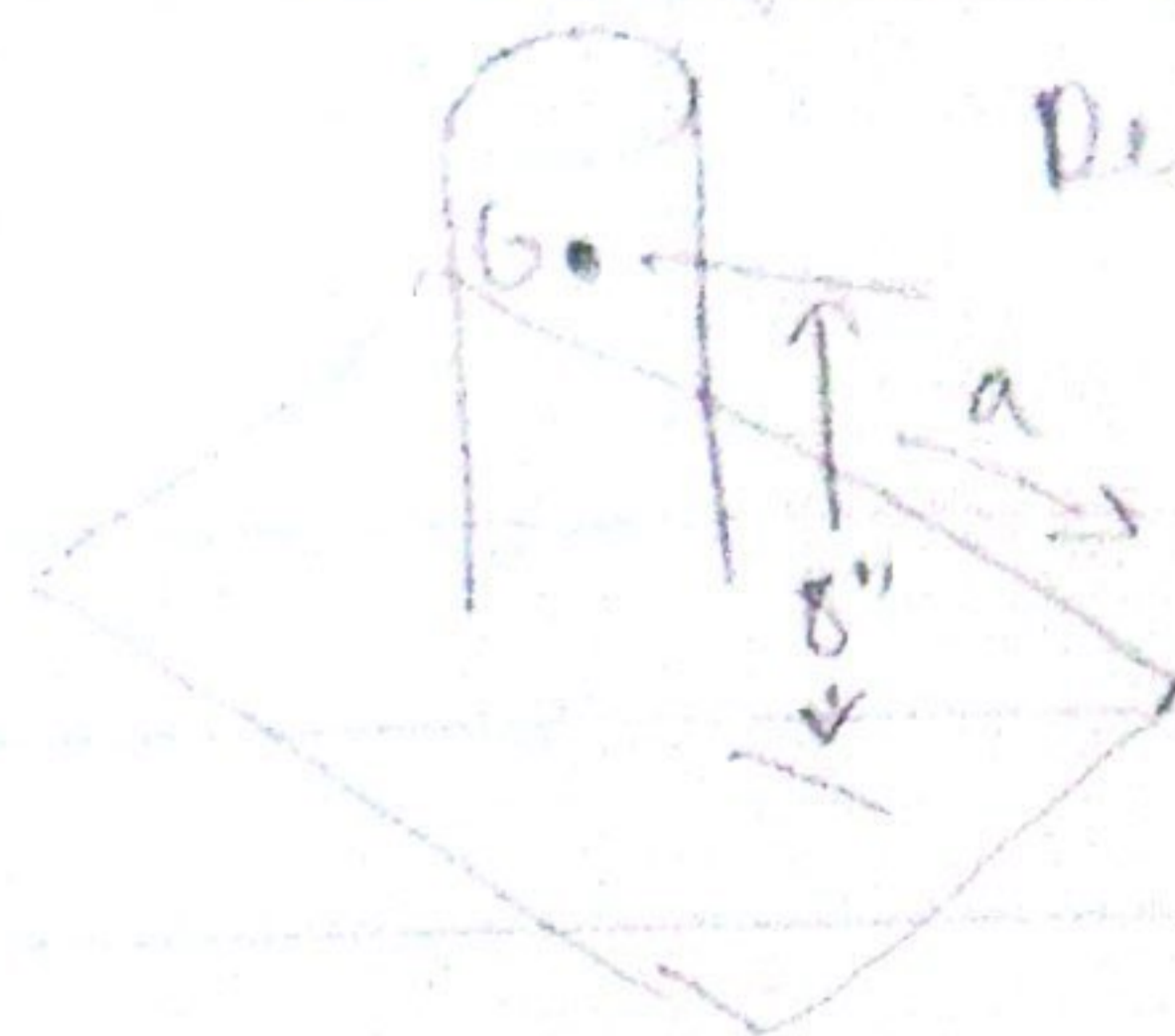
$$-1.5 \alpha + 98 - (0.75 \alpha + 49) = 0.225 \alpha$$

$$\boxed{\alpha = 19.798}$$

$$a = \alpha r = a = (19.798)(0.15 \text{ m})$$

$$\boxed{a = 2.97 \text{ m/s}^2}$$

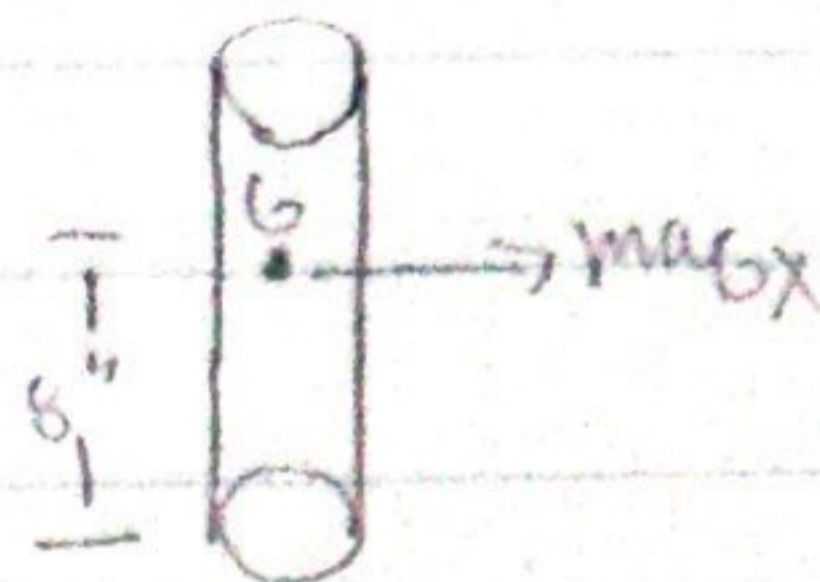
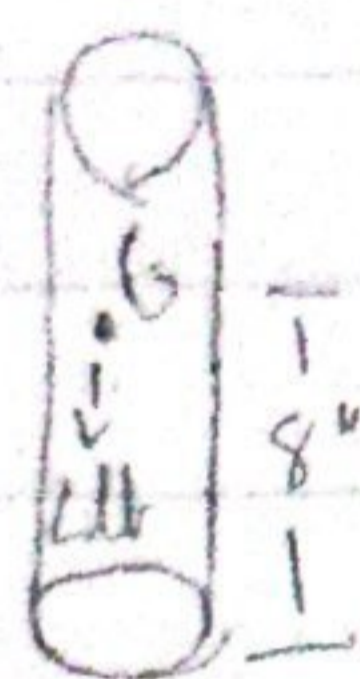
# Force acceleration 1. Problem 11:



$$\text{Diameter} = 1.5" \quad \mu = 0.2$$

$$m = \frac{2 \text{ lb}}{g} = 0.0621 \quad \frac{2}{9.81}$$

FBD = I.D



$$\rightarrow \sum F_x = m a_{Gx} \Rightarrow 0.0621 a_{Gx}$$

$$\uparrow \sum F_y = m a_{Gy} = N - 2 = 0$$

$$N = 2 \text{ lbs}$$

$$\sum M_G = I_G \alpha \Rightarrow \theta F - 2x = 0$$

$$F = \mu N = 0.2(2) = 0.4 \text{ lb}$$

$$\theta F - 2x = 0 \Rightarrow 8(0.4) = 2x \quad x = 1.6" > 0.75"$$

assume tipping

$$x = 0.75$$

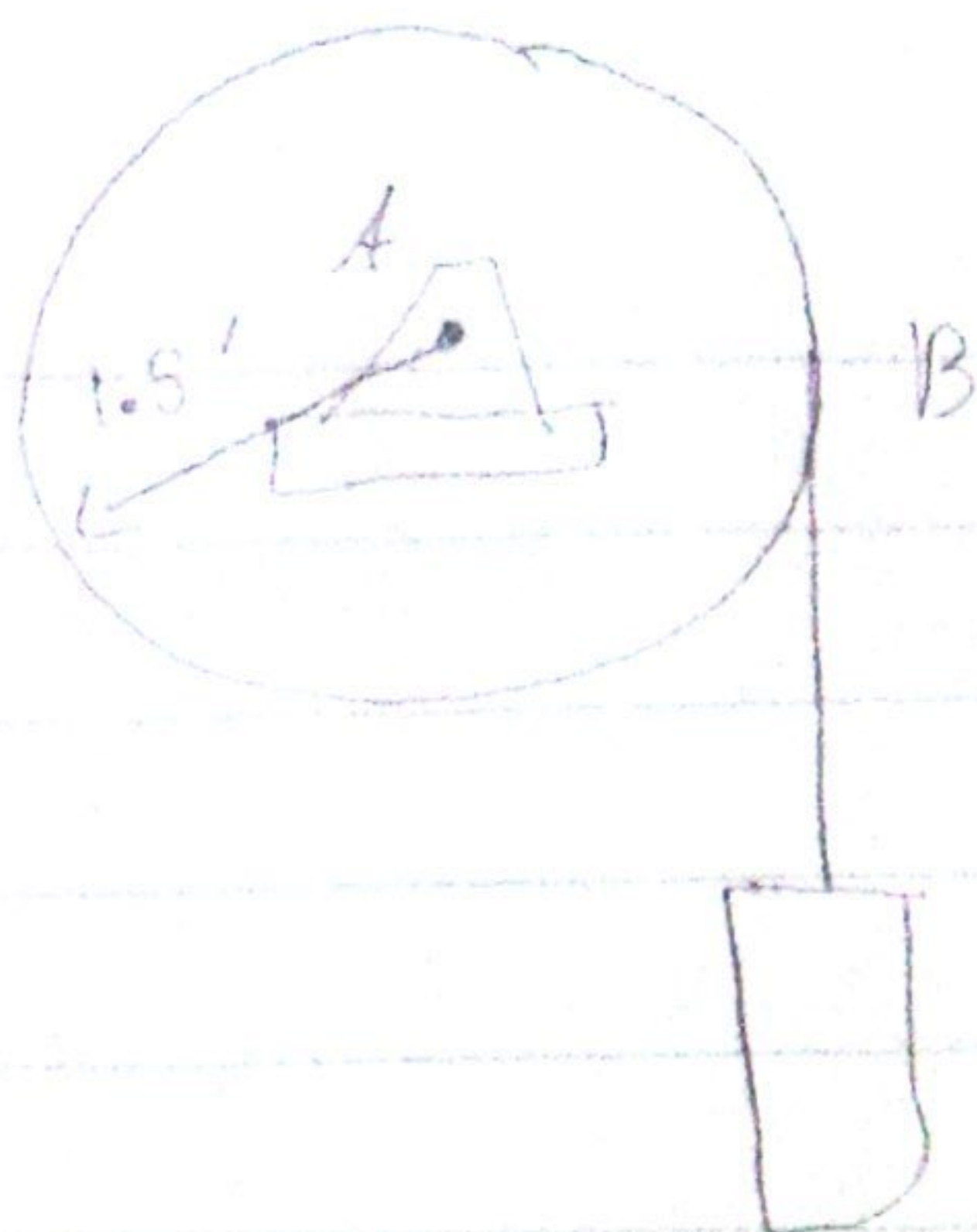
$$\theta F = 2x \quad \theta F = 2(0.75) \quad F = 0.0621 a_{Gx}$$

$$F = 0.1875$$

$$0.1875 = 0.0621 a_{Gx}$$

$$a_{Gx \text{ max}} = 3.02 \text{ ft/s}^2$$

## Force acceleration:



$$m_{\text{drum}} = \frac{20}{32.2} = 0.621$$

$$I_G = 0.8 \text{ ft} \cdot (0.621)(0.8^2) = 0.397$$

$$m_b = 12 \text{ lb} = \frac{12}{32.2} = 0.3726$$

## Properties:

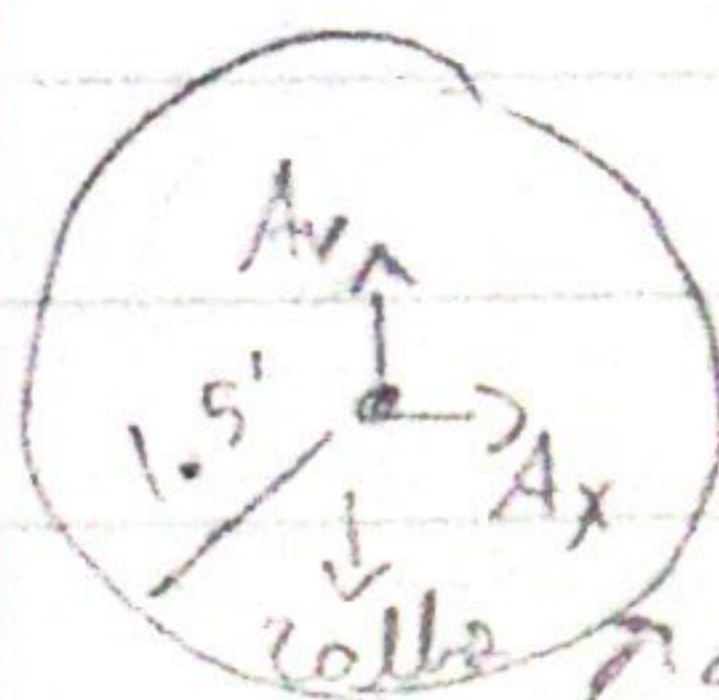
$$I_G = m h^2$$

$$m_d = \frac{20}{32.2} = 0.621 \quad w_B = 12 \text{ lbs}$$

$$w_d = 20 \text{ lbs}, \quad h = 0.8 \text{ ft}$$

$$m_B = \frac{12}{32.2} = 0.373$$

## FBD Wheel:



$$\sum F_x = A_x = 0$$

$$\sum F_y = A_y - 20 - T = 0$$

$$\sum M = 1.5T = I_G \alpha$$

$$\begin{cases} A_y - 20 = T \\ 1.5T = 0.397\alpha \end{cases} \quad \begin{cases} 1.5(A_y - 20) = 0.397\alpha \end{cases}$$

## FBD Block:



$$\sum F_y = T - 12 = m_a g_y$$

$$A_y - 20 - 12 = 0.373 a_{Gx}$$

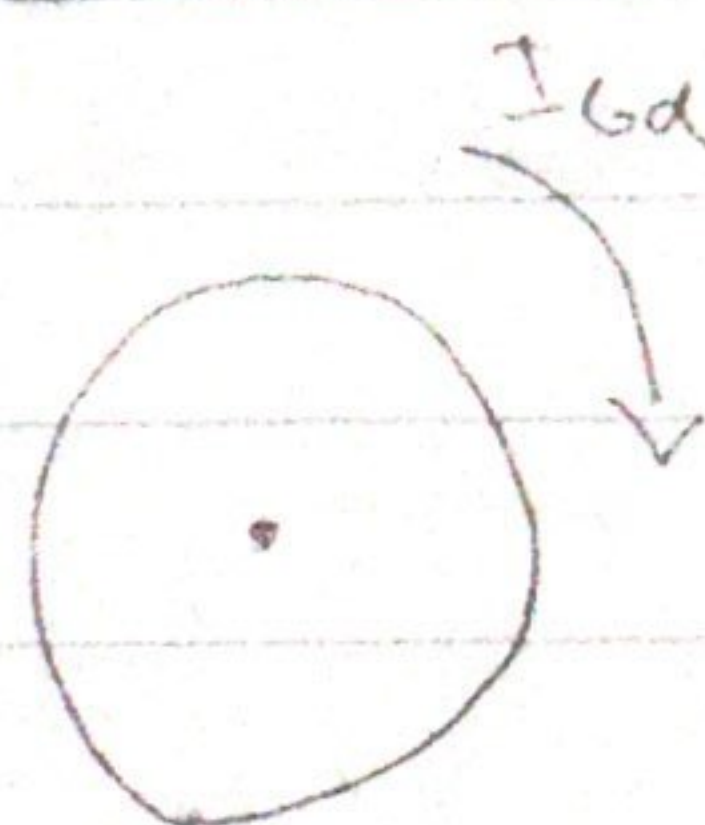
$$A_y - 8 = 0.373 \times 1.5 \alpha$$

## Motion:

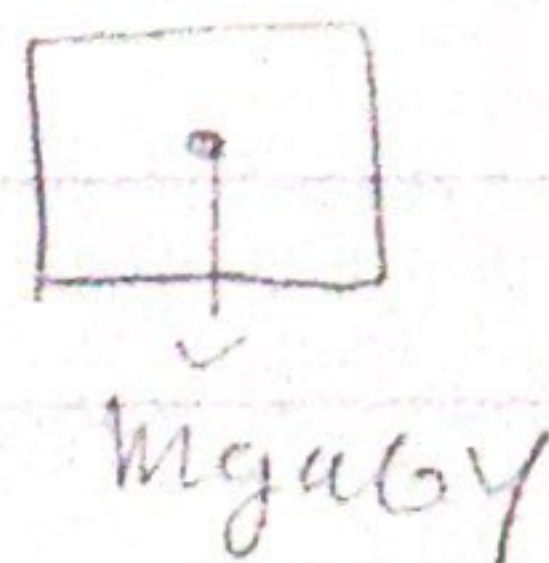
Drum: Rotate

Block: Translation

## 14D wheel

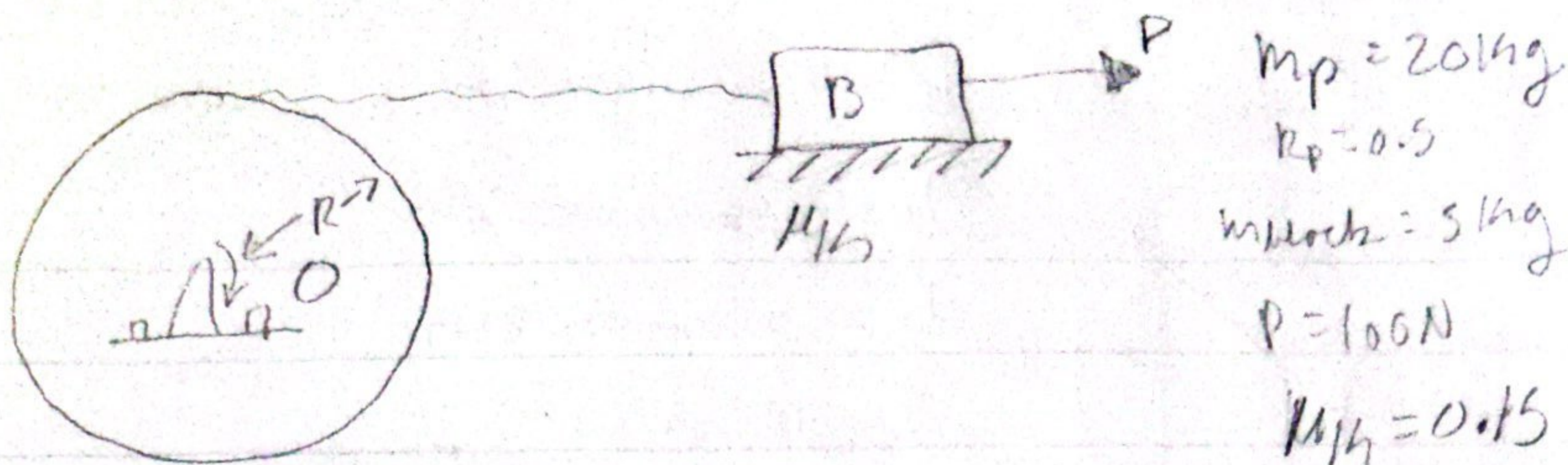


## 14D Block:



$$\frac{A_y - 8}{0.56} = \frac{A_y - 14.29}{0.56} = \alpha$$

$$\frac{A_y - 8}{0.56} = \alpha$$



$$\Sigma F_x = m_B a \quad P - T - \mu_k m_p g = m_p a$$

$$100 - T - (0.15 \times 5 \times 9.81) = 5a$$

$$100 - T - 7.3575 = 5a$$

$$92.6425 - T = 5a \quad \text{eqn 1}$$

$$\Sigma M_O = I_O \alpha$$

$$TR = I_O \alpha$$

$$I_O = \frac{1}{2} m_p R^2$$

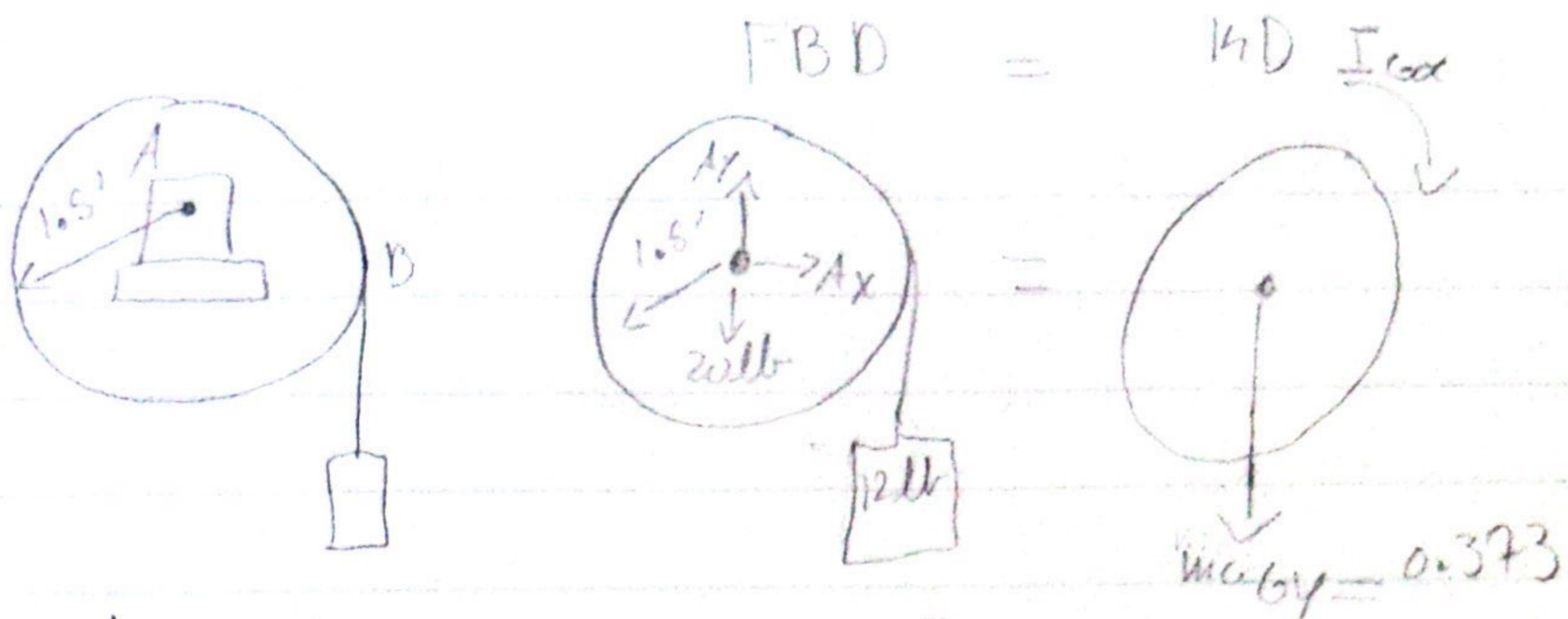
$$T(0.5) = \left( \frac{1}{2} \times 20 \times (0.5)^2 \right) \alpha = 0.5T = (2.5) \alpha \quad T = 5\alpha$$

$$T = 5\left(\frac{a}{0.5}\right) = 10a = T \quad \text{eqn 2}$$

$$92.6425 - 10a = 5a \quad 92.6425 = 15a \quad a = \frac{92.6425}{15} = \boxed{6.175 \text{ m/s}^2}$$

$$T = 10a = 10(6.175) = \boxed{61.75 \text{ N}}$$

# Force acceleration I: Problem 3



$$W_D = 20 \text{ lb}$$

$$F = 12 \text{ lb}$$

$$r = 0.8 \text{ ft}$$

$$F = ma$$

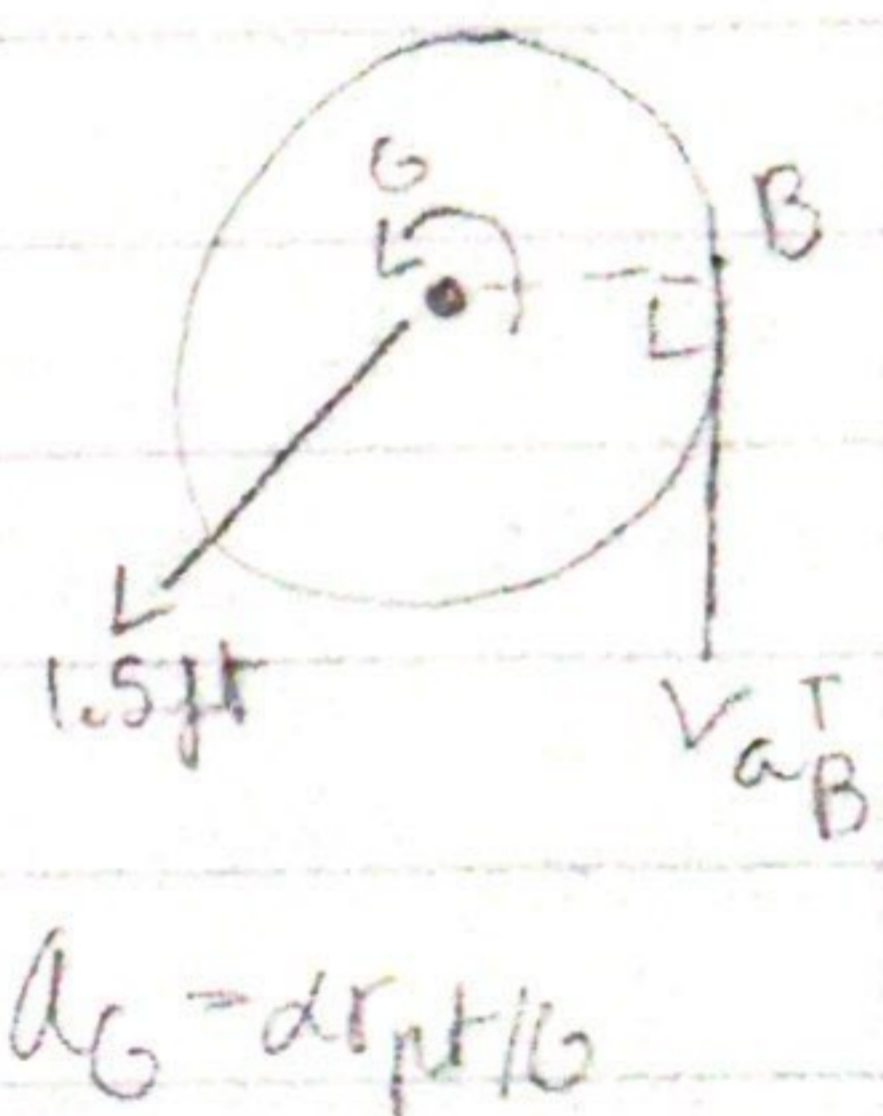
$$\frac{12 \text{ lb}}{32.2} = \frac{m_B (32.2)}{32.2}$$

$$m_B = 0.373$$

$$m_D = \frac{20}{32.2} = 0.621$$

$$I_G = mk^2$$

$$I_G = (0.621)(0.8)^2 = 0.39744$$



$$\uparrow \sum F_y = ma_{Gy}$$

$$A_y = 0$$

$$\uparrow \sum F_y = ma_{Gy}$$

$$A_y - 20 - T = 0$$

$$T - 12 = -(0.373)a_y$$

$$\sum M_G = I_G \alpha = 1.5T = -0.39744 \alpha$$

$$\uparrow \sum F_y = m_B a_{Gy}$$

$$T - 12 = 0.373 a_y$$

$$a_B = 1.5 \alpha \quad a_B = a_{Gy}$$

$$a_{Gy} = 1.5 \alpha$$

$$a_T = \alpha r \quad a_{\text{Block}} = a_B = a_{Gy} = 1.5 \alpha$$

$$T - 12 = -(0.373)(1.5 \alpha) \quad T + 0.56 \alpha = 12$$

$$A_y - 20 - T = 0 \quad \Rightarrow \quad -T + A_y = 20$$

$$1.5T = 0.39744 \alpha$$

$$1.5T - 0.39744 \alpha = 0$$

$$\begin{bmatrix} 1 & 0.56 & 0 \\ -1 & 0 & 1 \\ 1.5 & 0.397 & 0 \end{bmatrix} \begin{Bmatrix} T \\ \alpha \\ A_y \end{Bmatrix} = \begin{Bmatrix} 12 \\ 20 \\ 0 \end{Bmatrix}$$

$$T = 3.85 \text{ lbs}$$

$$\alpha = 14.55 \text{ rad/s}^2$$

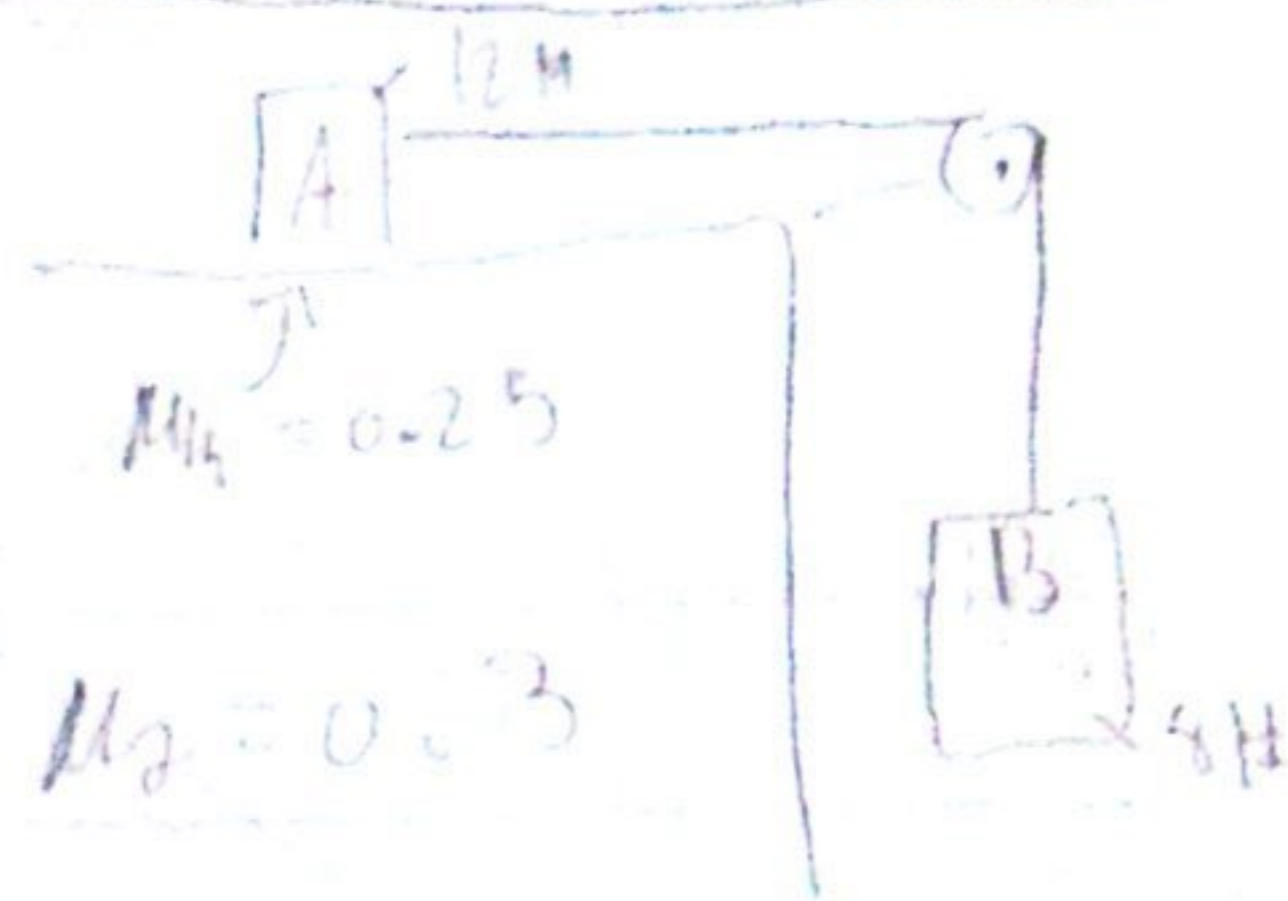
$$A_y = 23.85 \text{ lb}$$

$$\sum M_G = I_G \alpha$$

$$1.5(12) = -0.39744 \alpha$$

$$\alpha = 45.3 \text{ rad/s}^2$$

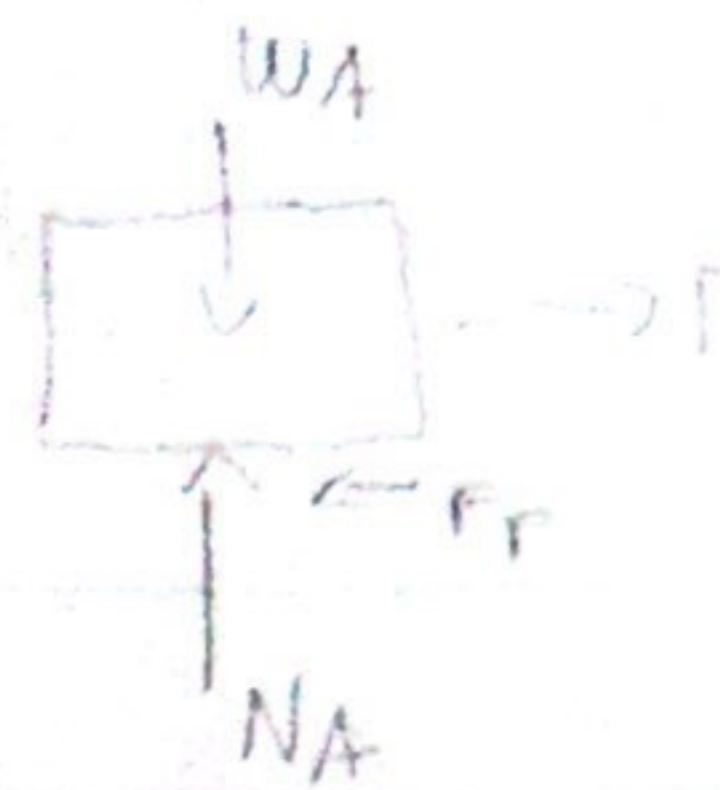
Force acceleration:



FBD

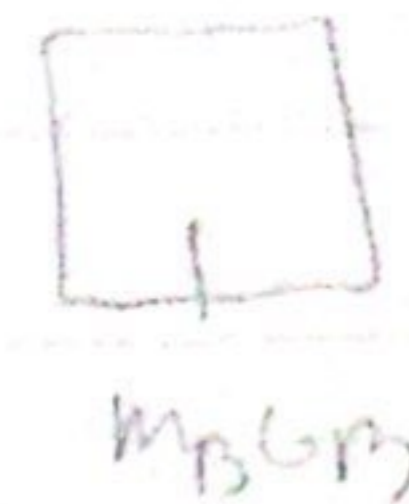
14.17

A:



$$a = \frac{12}{32.2} = 0.373$$

B:



$$B = \frac{8}{32.2} = 0.248$$

$$A: \sum x \Rightarrow T - F_f = m_a a \quad (1)$$

$$B: \sum x \Rightarrow 0$$

$$\sum y \Rightarrow N_A - W_A = 0$$

$$F_f = \mu N_A = (0.25)(12)$$

$$F_f = 3 \text{ H}$$

$$\sum y \Rightarrow T - 8 \text{ H} = m_B a_B$$

$$\frac{T - 8}{-0.248} = a \quad (2)$$

$$\sum x \Rightarrow T - F_f = m_a a \quad T - 3 = 0.373 \left( \frac{T - 8}{-0.248} \right) \quad \frac{0.373}{0.248} = 1.504(T - 8)$$

$$T - 3 = 1.504T + 12.03$$

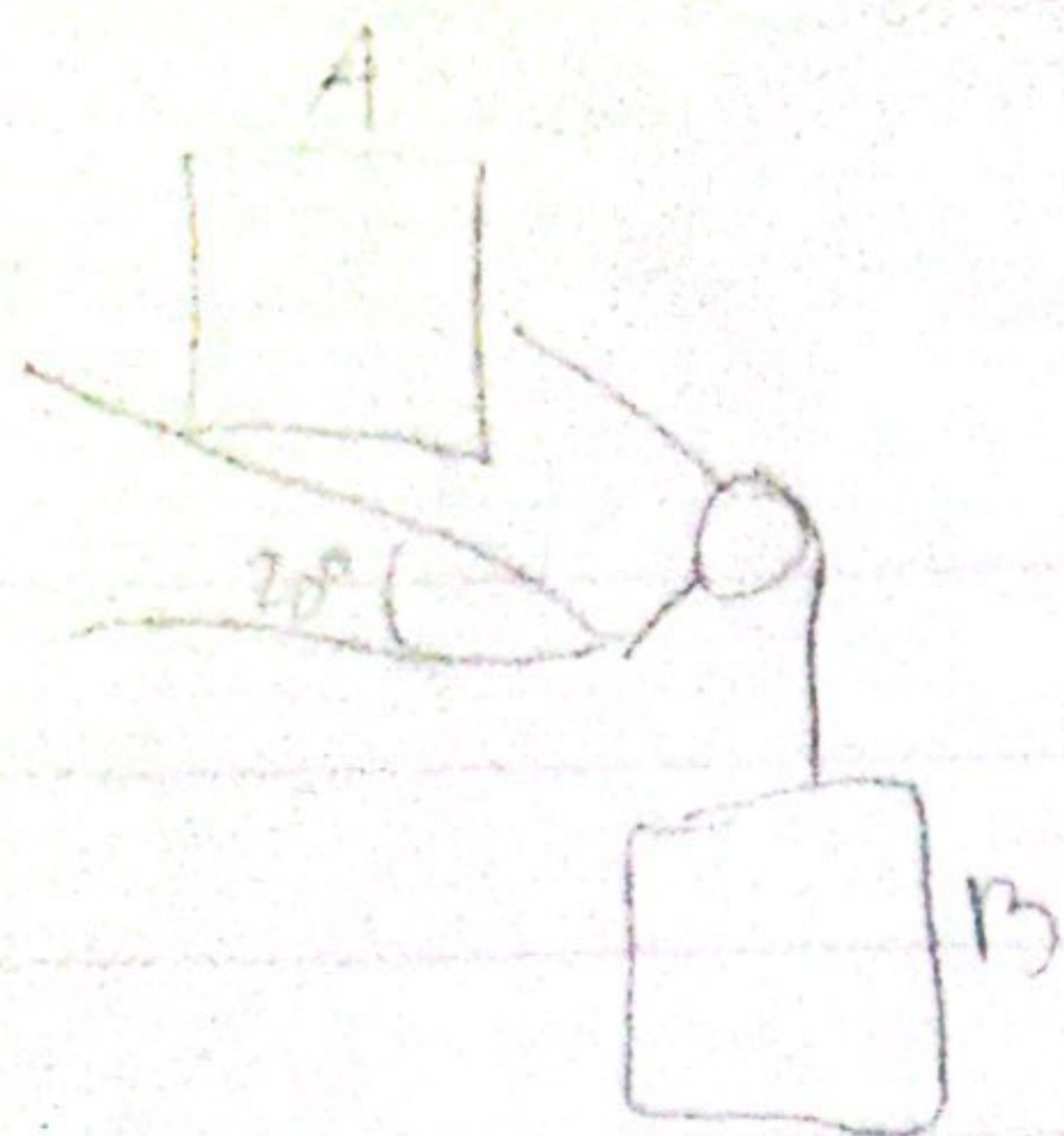
$$2.504T = 15.03$$

$$T = 6 \text{ H}$$

$$1.504T - 1.504 \times 8$$

$$1.504T - 12.032$$

## Force acceleration:



## Classify motion:

Blocks A & B: Translation

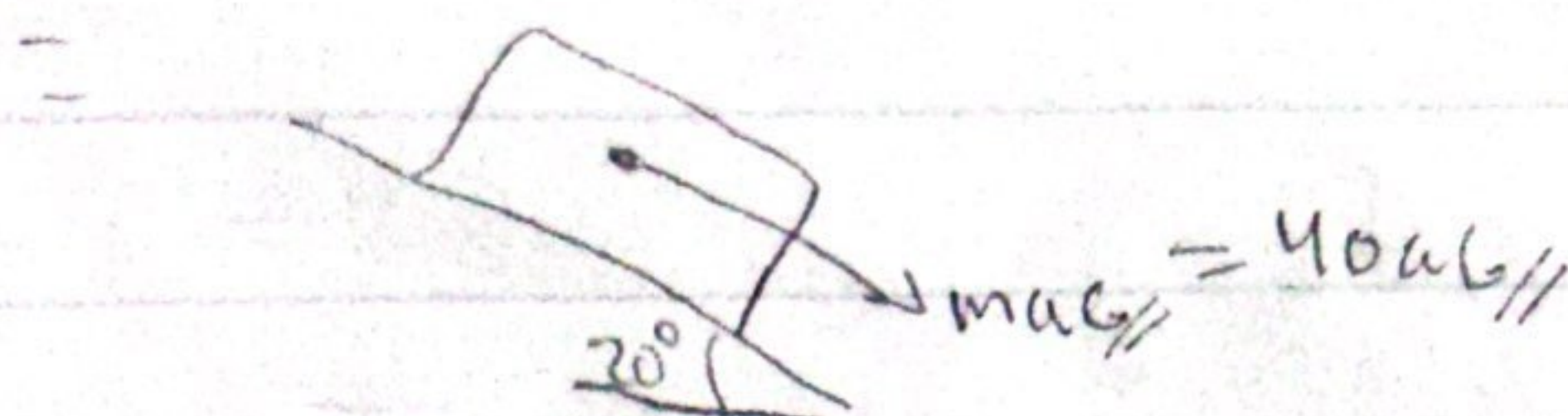
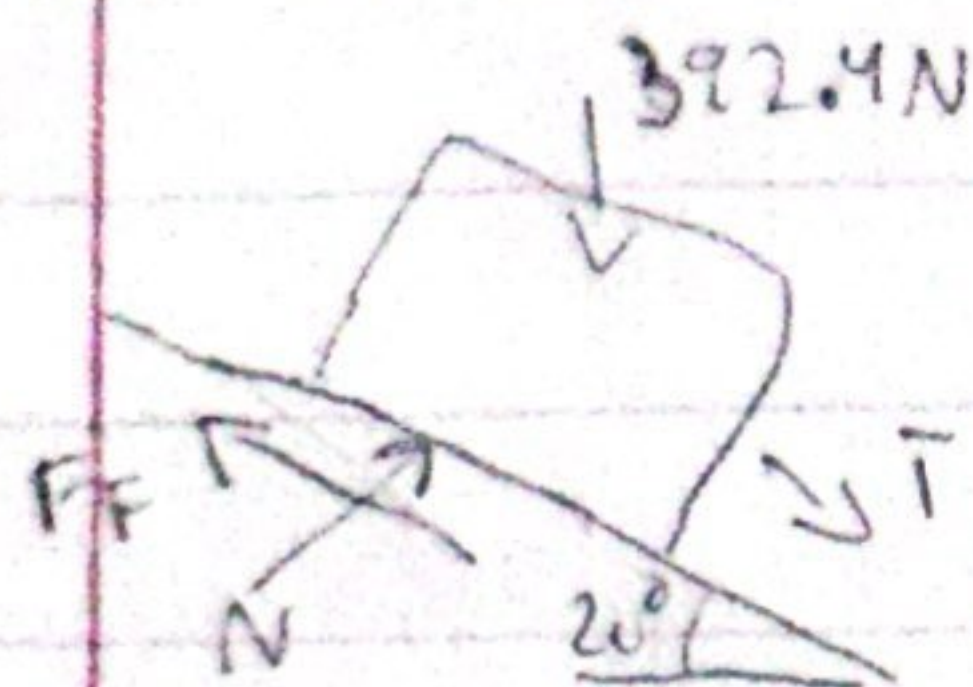
## Properties:

$$m_A = 40 \text{ kg} \quad W_A = 40(9.8) = 392.4 \text{ N}$$

$$m_B = 30 \text{ kg} \quad W_B = 30(9.8) = 294.3 \text{ N}$$

$$\mu_s = 0.2 \quad \mu_k = 0.15$$

FBD = KD



A:

$$F_f = \mu_k N = 0.15N = 0.15(369) = 55.4 \text{ N}$$

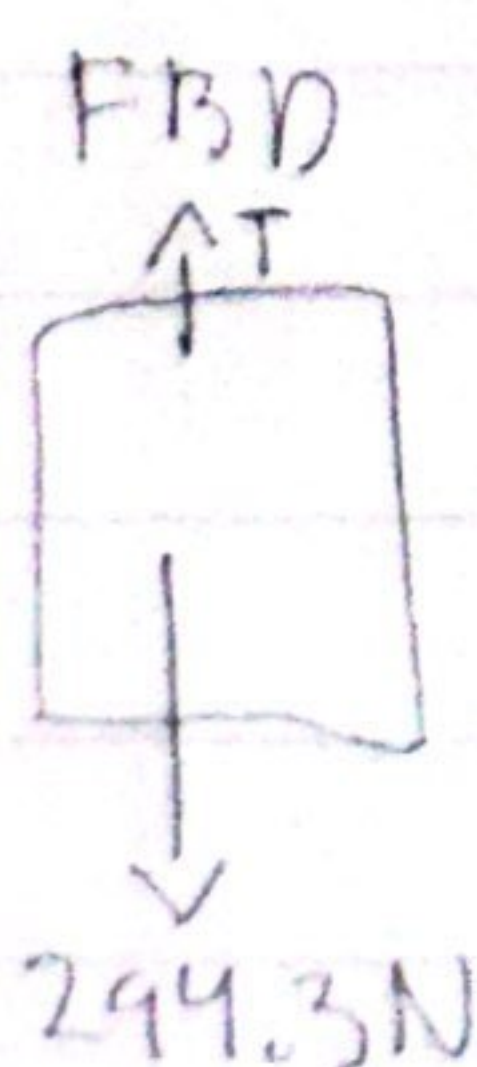
$$\sum F_{\perp} = ma_{g\perp} \Rightarrow N - 392.4 \cos 20^\circ = 0 \Rightarrow N = 369 \text{ N}$$

$$\sum F_{\parallel} = ma_{g\parallel} \Rightarrow T - F_f + 392.4 \sin 20^\circ = 40a_{g\parallel}$$

$$T - 55.4 \text{ N} + 392.4 \sin 20^\circ = 40a_{g\parallel}$$

$$(1): T - 40a_{g\parallel} = -78.86$$

B:



= KD



$$m_B a_y = 30a_y$$

$$(2) \sum F_y = ma_{g_y} \Rightarrow 294.3 - T = 30a_y$$

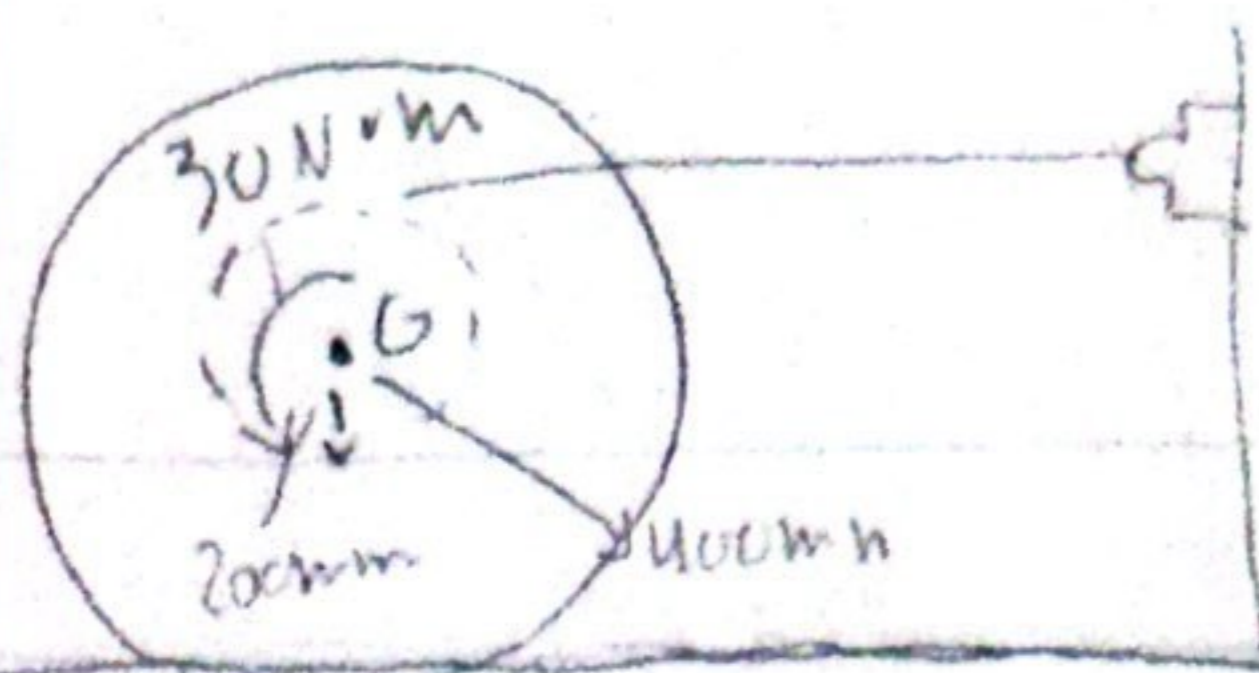
$$\text{kinematic: } a_{g\parallel} = a_y$$

$$(1) T - 40a_{g\parallel} = -78.86$$

$$(2) T + 30a_{g\parallel} = 294.3$$

$$\boxed{T = 134 \text{ N} \quad a_{g\parallel} = 5.32}$$

Force acceleration:



$$M = 20 \text{ kg} \quad R = 250 \text{ mm} \quad \mu = 0.1$$

$$C = 300 \text{ mm}$$

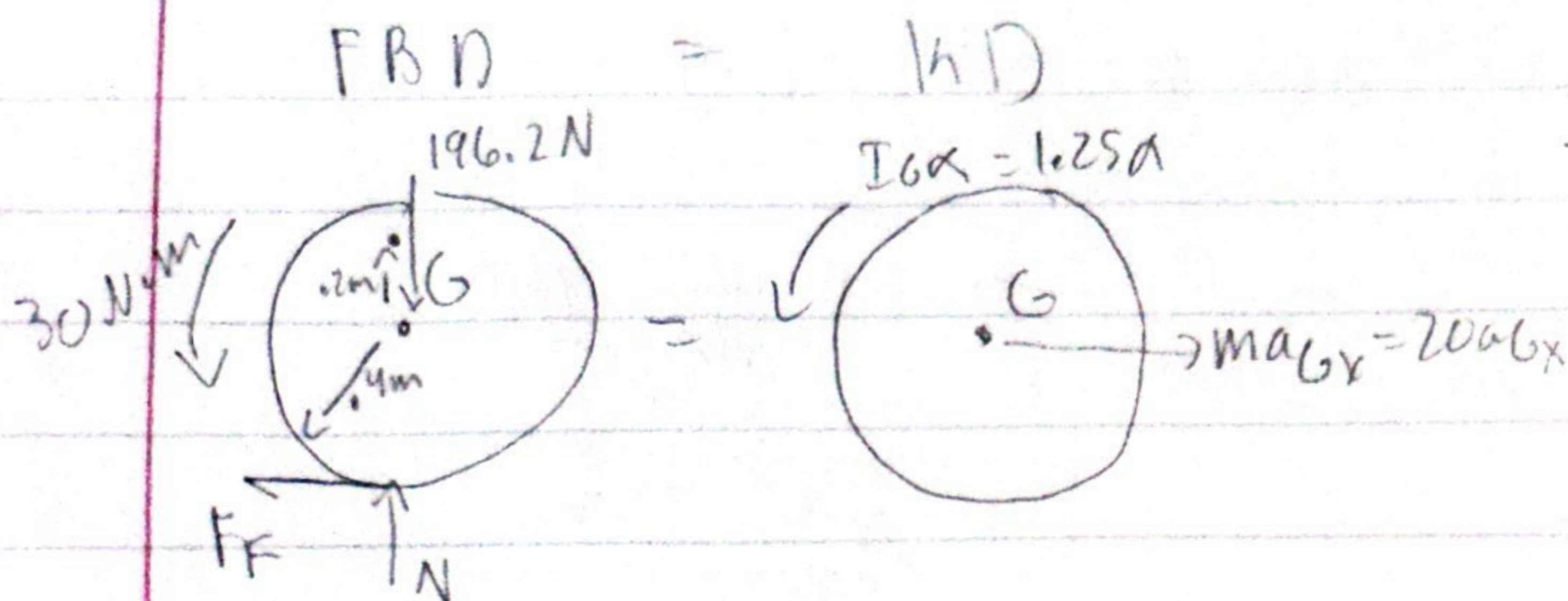
Classify motion

Gpm

properties

$$M = 20 \text{ kg} \quad W = 20(9.81) = 196.2 \text{ N}$$

$$I_G = m r_G^2 = 20(0.25^2) = 1.25 \text{ kg} \cdot \text{m}^2$$



$$\uparrow \sum F_y = m a_{Gy} \Rightarrow N - 196.2 = 0 \quad N = 196.2$$

$$\rightarrow \sum F_x = m a_{Gx} \Rightarrow T - F_f = 20 a_{Gx} \quad (1)$$

$$\sum M_G = I_G \alpha \Rightarrow 30 - F_f(0.4) - T(0.2) = 1.25 \alpha$$

$$2 \text{ Eqn 4 unknown must solve } F_f = \mu N = 0.1(196.2) = \boxed{F_f = 19.26 \text{ N}}$$

$$a_{Gx} = r \alpha \quad a_{Gy} = r \alpha = 0.2 \alpha$$

$$(1): T - 19.26 = 20(0.2 \alpha)$$

$$T - 19.26 = 4 \alpha \Rightarrow T = 19.26 + 4 \alpha$$

$$(2): 30 - 19.26(0.4) - T(0.2) = 1.25 \alpha$$

$$30 - 7.68 - 0.2T = 1.25 \alpha \Rightarrow 22.32 - 0.2T = 1.25 \alpha$$

$$T = 55.2 \text{ N} \quad \alpha = 8.89$$

$$22.32 - 0.2(19.26 + 4 \alpha) = 1.25 \alpha$$

$$22.32 - 3.852 - 0.8 \alpha = 1.25 \alpha \Rightarrow 18.468 = 2.05 \alpha$$

$$\boxed{\alpha = \frac{18.468}{2.05} = 9.0088}$$

$$\boxed{T = 19.26 + 4(9.0088) = 55.2951}$$