Chapter 3 Probability

STA 2023 SECTION 3.1 BASICS OF PROBABILITY AND COUNTING NOTES

Learning Outcomes:

1) Identify the sample space of a probability experiment
2) Use the Fundamental Counting Principle to find the number of ways two or more events can occur
3) Distinguish among classical probability, empirical probability, and subjective probability
4) Find the probability of the complement of an event

Notes:

Probability - the chance or likelihood of an event occurring Examples:

Probability Experiment – a procedure that produces outcomes Examples:

Event – an outcome **or** collection of outcomes from a sample space Examples:

Simple Event –

Compound Event –

Sample Space – a list that contains **all** of the possible outcomes of a probability experiment
Examples:

*When we look at a sample space, there is a probability associated with each event. The list of each event along with the associated probability is called a _____.
**Notation for probability: P(*event*)

The Fundamental Counting Principle:
If one event can occur in *m* ways and a second event can occur in *n* ways, then the number of ways the two events can occur in sequence is $m \times n$. This rule can be extended to any number of events occurring in sequence. Example:

If a probability experiment can be accomplished in k-stages, each stage can be performed in n_i, *i*=1, 2, 3,….k possible ways, then the experiment itself can be performed as

$$n_1 \times n_2 \times n_3 \times \ldots\ldots \times n_k$$

Possible ways.

Finding sample space: (1) Tree diagram and (2) Cross-tabulation

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Example 1: Identify the sample space and determine the number of outcomes.

a) A survey consists of asking people for their blood types (O, A, B, and AB), including whether they are Rh-positive or Rh-negative. Determine the number of outcomes and identify the sample space.

b) Two coin tossed together with a six-sided dice.

c) 3-coin tossed together:

d) A box contains five balls labeled as {1, 2, 3, 4, 5}. Suppose we randomly select two balls from the box by (i) with replacement and (ii) without replacement. How many possible outcomes in both cases?

e) A license plate number consists of a combination of 3 letters followed by 4-digits, such as ABC-1234.
(i)How many such license plates are possible if there is no restriction on the choices for the letters or digits?

(ii) How many such license plates are possible if 3 letters must be distinct but have no number restriction?

(iii) How many such licenses are possible if all letters and numbers are distinct even numbers?

There are 2 basic **approaches** to finding the probability of an event:

1. Classical Approach – occurs with equally likely outcomes – each outcome has the same chance of occurring

   $P(A) =$

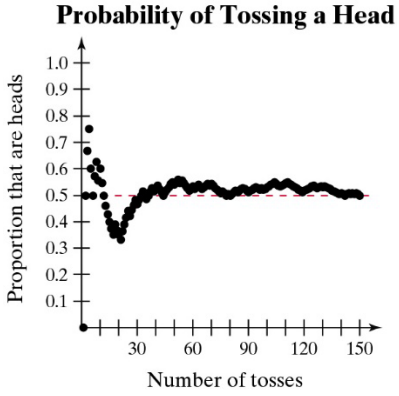
   Examples:

2. Relative Frequency Approach – the probability of an event occurring is based on the **relative frequency** of the event (also known as the Empirical Method)
   - **Law of Large Numbers** – As a probability experiment is repeated a large number of times, the proportion of times that a given event occurs will approach the true probability of that event occurring.

     $P(A) \approx$

Examples:

Suppose you want to determine the probability of tossing a head with a fair coin. You toss the coin 10 times and get 3 heads, obtaining an empirical probability of 3/10. Because you tossed the coin only a few times, your empirical probability is not representative of the theoretical probability, which is 1/2. The law of large numbers tells you that the empirical probability after tossing the coin several thousand times will be very close to the theoretical or actual probability.



Properties of Probability:

- $0 \leq P(A) \leq 1 \quad for\ any\ event\ A$
- $\sum P(x) = 1 \quad where\ x\ assumes\ all\ events\ in\ a\ sample\ space$

If $P(A) = 0$, then event A cannot occur – we call this an _____.

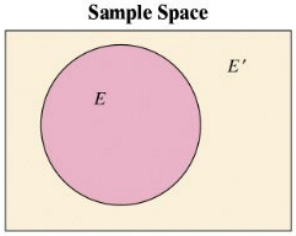
If $P(A) = 1$, then event A is certain to occur – we call this a_____.

Unusual Event – any event that has a small probability of occurring

- the standard is $P(A) \leq$ _____
- the closer a probability is to _____, the more unusual is the event.

Complementary Events

The **complement of event *E*** is the set of all outcomes in a sample space that is not included in event *E*. The complement of event *E* is denoted by E′ and is read as "*E* prime." The Venn diagram at the left illustrates the relationship between the sample space, event *E*, and its complement E′.



$$P(E) + P(E') = 1$$
$$P(E) = 1 - P(E')$$
$$P(E') = 1 - P(E)$$

STA 2023 SECTION 3.2 CONDITIONAL PORBABILITY AND THE MULTIPLICATION RULE NOTES

Learning Outcomes:

1) Calculate conditional probability
2) Distinguish between independent and dependent events
3) Use the Multiplication Rule to find the probability of two events occurring in sequence and to find conditional probabilities

Notes:

If the occurrence of one event does not affect the probability of the occurrence of the other event, the events are _____.

If the occurrence of one event does affect the probability of the occurrence of another event, then the events are _____.

Example 1: Determine if the events are independent or dependent.

a) Selecting a king (event *A*) from a standard deck of 52 playing cards and then selecting a queen (event *B*) from the deck without replacement.

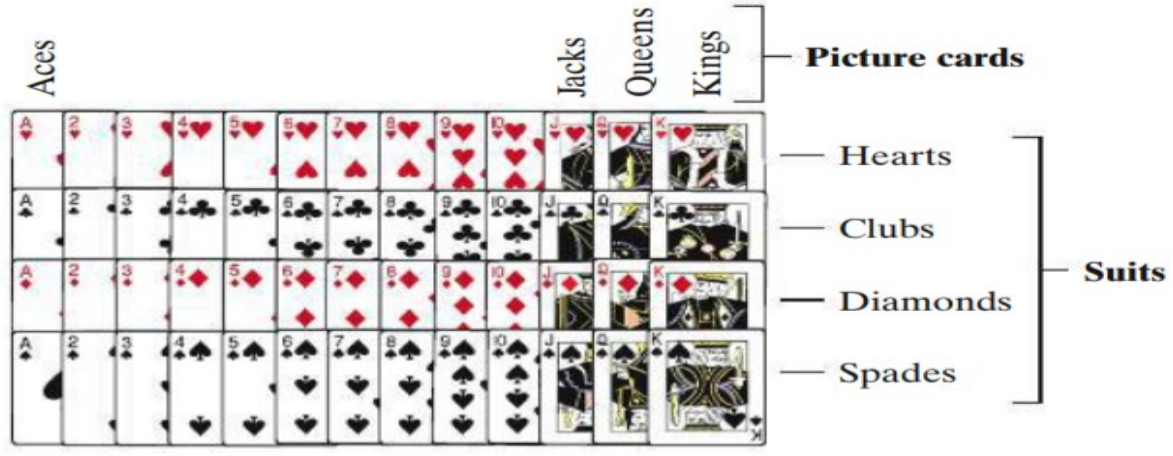
b) Rolling a six-side die and getting a 4 (event *A*), and then rolling the six-sided die again and obtaining a 6 (event *B*).

Notes:

If we are considering two events, A and B, occurring in a sequence and the two events are _____, then the probability that the second event will occur is called **conditional probability**.

Notation: If A is the first event and B is the second event then **the conditional probably of event B** is written as $\boldsymbol{P(B|A)}$ (read as "probability of event *B*, _____ event *A* occurred").

Example 2: Calculate conditional probability.



a) Two cards are selected in sequence from a standard deck without replacement. Find the probability that the second card is a queen, given that the first card is a king.

b) Two cards are selected in sequence from a standard deck without replacement. Find the probability that the second card is a heart, given that the first card is a heart.

c) The table below shows the results of a survey in which 2276 social media users were asked whether they have ever been offended by something they saw on social media. Find the probability that a user is male, given that the user was offended by something on social media. *(Adapted from The Harris Poll)*

	Have you ever been offended by something on social media?		
Gender	**Yes**	**No**	**Total**
Female	619	549	1168
Male	532	576	1108
Total	1151	1125	2276

d) The table above a shows the results of a survey in which 2276 social media users were asked whether they have ever been offended by something they saw on social media. Find the probability that a user is not offended, given that the user was female. *(Adapted from The Harris Poll)*

Notes:
We have been considering the probabilities of single events up until now. We will now consider finding the probabilities for more than one event occurring in a sequence.

The probability that two events *A* and *B* will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

For independent events the rule can be simplified to

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

This rule can be extended for any number of independent events.

Example 3: Find each probability.

a) Two cards are selected, without replacement, from a standard deck of 52 playing cards. Find the probability of selecting a king and then selecting a queen.

b) Two cards are selected, without replacement, from a standard deck of 52 playing cards. Find the probability of selecting two hearts in a row.

c) Two cards are selected, without replacement, from a standard deck of 52 playing cards. Find the probability of selecting a diamond and then a spade.

Example 4: The probability that a rotator cuff surgery is successful is 0.9.

a) Find the probability that four rotator cuff surgeries are successful.

b) Find the probability that none of four rotator cuff surgeries are successful.

c) Find the probability that the first two are successful and the second two are not.

d) Find the probability that at least one of four rotator cuff surgeries is successful.

STA 2023 SECTION 3.3 THE ADDITION RULE NOTES

Learning Outcomes:

1) Determine whether two events are mutually exclusive
2) Use the Addition Rule to find the probability of two events

The Addition Rule

Compound Event –

The Addition Rule will address situations where compound events may occur **in a single trial.** $P(A \ \boldsymbol{or} B) -$

General Addition Rule – For any two events *A and B,* in a single trial,

$$P(A \ \boldsymbol{or} B) = P(A) + P(B) - P(A \ and \ B)$$

Example: For the class, if one person is chosen at random, find $P(have \ a \ dog \ \boldsymbol{or} \ cat)$.

Example: In a class, there are 31 students. Seventeen of them are freshmen and the rest are sophomores. Of the freshmen, 10 are female and of the sophomores, 5 are female. If one student is chosen at random from the class, find the following:

a. P(sophomore)
b. P(sophomore or male)
c. P(female freshman)
d. P(female **or** freshman)

Mutually Exclusive Events (also called _____ events) – arise in a situation when the occurrence of one event excludes the others in a sample space from occurring. Therefore, events *A and B* **cannot** occur in the same trial. If events $A \text{ and } B$ are mutually exclusive, then $P(A \text{ and } B) =$_____.

The Addition Rule for Mutually Exclusive Events – If events *A and B* are mutually exclusive events, then $\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$ in a single trial.

Examples:

Example 2: Use the Addition Rule to Find Probabilities

a) You select a card from a standard deck. Find the probability that the card is a red 4 or an ace.

b) You select a card from a standard deck. Find the probability that the card is a spade or a queen.

c) You roll a six-sided die. Find the probability of rolling a 2 or an odd.

d) You roll a six-sided die. Find the probability of rolling a number less than 5 or rolling an odd number.

Chapter 3 Probability

A Summary of Probability:

Type of probability and probability rules	In words	In symbols
Classical Probability	The number of outcomes in the sample space is known and each outcome is equally likely to occur.	$P(E) = \frac{\text{Number of outcomes in event } E}{\text{Number of outcomes in sample space}}$
Empirical Probability	The frequency of each outcome in the sample space is estimated from experimentation.	$P(E) = \frac{\text{Frequency of event } E}{\text{Total frequency}} = \frac{f}{n}$
Range of Probabilities Rule	The probability of an event is between 0 and 1, inclusive.	$0 \le P(E) \le 1$
Complementary Events	The complement of event E is the set of all outcomes in a sample space that are not included in E, and is denoted by E'.	$P(E') = 1 - P(E)$
Multiplication Rule	The Multiplication Rule is used to find the probability of two events occurring in sequence.	$P(A \text{ and } B) = P(A) \cdot P(B\|A)$ Dependent events $P(A \text{ and } B) = P(A) \cdot P(B)$ Independent events
Addition Rule	The Addition Rule is used to find the probability of at least one of two events occurring.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = P(A) + P(B)$ Mutually exclusive events