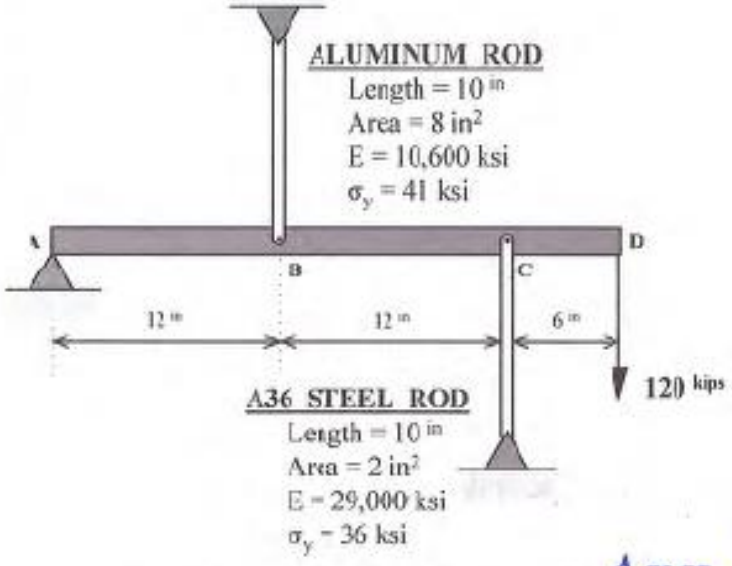
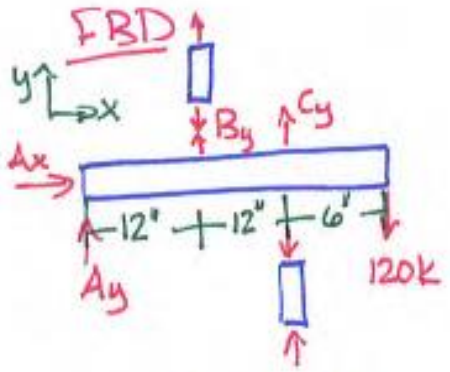
Worksheet 6A
Statically Indeterminate Structures

The rigid bar ABCD is suppored by 2014-T4 aluminum and structural steel rods ard must support a 120 kip load at D. Find the axial stress is the two rods and the deflecton at point D.





Eqns of Equil:

$\rightarrow\Sigma F_x = 0 = A_x$

$\uparrow \Sigma F_y = 0 = A_y + B_y + C_y - 120$

$\circlearrowleft \Sigma M_A = 0 = 120(30) - C_y(24) - B_y(12)$

3 eqns, 4 unks

statically indeterminate

Axial Deformation

$\Delta B = \delta_{AL} = \left(\frac{PL}{AE}\right)_{AL}$

$\Delta C = \delta_{ST} = \left(\frac{PL}{AE}\right)_{ST}$

$\frac{\Delta B}{12"} = \frac{\Delta C}{24"}$

$\frac{B_y(10")}{12"(8in^2)(10600ksi)} = \frac{C_y(10")}{24"(2in^2)(29000ksi)}$

$B_y - 0.731 C_y = 0$

4 eqns
4 unks

using ΣF_y, ΣM_A + Axial Deformation:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -12 & -24 \\ 0 & 1 & -.731 \end{bmatrix} \begin{bmatrix} A_y \\ B_y \\ C_y \end{bmatrix} = \begin{bmatrix} 120 \\ -3600 \\ 0 \end{bmatrix}$$

$A_y = -70.15k$ $B_y = 80.3k$ $C_y = 109.85k$

Axial Stress: $\sigma_{AL} = \frac{B_y}{A_{AL}} = \frac{80.3}{8} = 10.04 ksi$ (T)

$\sigma_{ST} = \frac{C_y}{A_{ST}} = \frac{109.85}{2} = 59.9 ksi$ (C)

Check Assumptions!

Elastic $\Rightarrow \sigma_{ACT} \leq \sigma_y$

$\sigma_{AL} = 10.04 \leq 41$ OK!

$\sigma_{ST} = 59.9 \not\leq 36$ NO!

Steel rod is plastic

Steel rod is plastic!

$\therefore \sigma_{ST} = \sigma_y = 36\ ksi$

$C_y = (\sigma_{ST})(A_{ST}) = (36)(2) = 72k$

$\Sigma M_A \quad B_y = \dfrac{(30")(120k) - (24")(72k)}{12"}$

$= 156k$

$\sigma_{AL} = \dfrac{156}{8} = 19.5\ ksi \leq 41\ ksi$ OK!

Deflection @ D

$\dfrac{\Delta B}{12"} = \dfrac{\Delta D}{30"}$

$\Delta D = \dfrac{30}{12}\delta_{AL} = \dfrac{30}{12}\left(\dfrac{PL}{AE}\right)_{AL}$

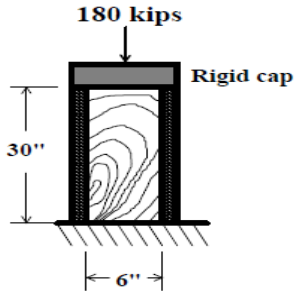
$= \dfrac{30}{12}\left(\dfrac{(156)(10)}{(8)(10600)}\right)$

$= \underline{0.046\ in \downarrow}$

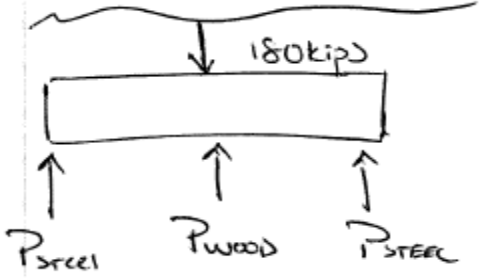
Note: can not solve using Δ_C since steel rod is plastic!

Worksheet 6B
Statically Indeterminate Structures

A 6"x6"x30" southern pine block is subjected to an axial load of 180 kips. Use a modulus of elasticity of 1,800,000 psi and a yield stress 2000 psi. The block is reinforced with 2 A36 steel plates which are each .25" thick. Determine the actual stresses in the wood and steel members.



FBD ABOUT RIGID CAP



$\Sigma F_y = 0 \Rightarrow 2P_{ST} + P_W - 180 = 0$

FROM COMPATABILITY DIAGRAM

$\delta_{ST} = \delta_W$ (WOOD & STEEL DEFORM THE SAME AMOUNT)

$\therefore \frac{P_{ST} L_{ST}}{AE_{ST}} = \frac{P_W L_W}{AE_W}$

$\left(\frac{PL}{AE}\right)_{ST} = \left(\frac{PL}{AE}\right)_W$ $\quad E_{ST} = 29 \times 10^6$ psi

$$\frac{(P_{ST})(30")}{(.25")(6")(29\times10^6)} = \frac{(P_W)(30")}{(6")(6")(1.8\times10^6)}$$

$1.48966\, P_{ST} = P_W$

SUB INTO ΣF_y EQN.

$2P_{ST} + 1.48966\, P_{ST} = 180K$

$P_{ST} = 51.58K$ (per Steel Bar)

$P_W = 180 - 2P_{ST} = 76.84K$

CHECK ELASTIC ASSUMPTION

$$\sigma_{ST} = \frac{P}{A} = \frac{51.58K}{(0.25)(6)} \leq \sigma_y = 36ksi$$

$$34.39\ ksi \leq 36\ ksi \quad \checkmark$$

$$\sigma_w = \frac{P}{A} = \frac{76.84K}{(6)(6)} \leq \sigma_y = 2ksi$$

$$2.1344\ ksi \leq 2ksi \quad \times \text{ No!!}$$

SET $\sigma_w = \sigma_y = 2ksi$

$$P_w = 2ksi\,(36in^2) = 72K$$

$$2P_{ST} + P_w = 180K$$

$$2P_{ST} = 108$$

$$P_{ST} = 54K$$

$$\sigma_{ST} = \frac{P}{A} = \frac{54}{(.25)(6)} = 36\ ksi$$

$$\sigma_{ST} \leq \sigma_y = 36\ ksi$$

$$36\ ksi \leq 36\ ksi \quad \checkmark$$

$\therefore$ $\sigma_{ST} = 36ksi$, $\sigma_w = 2ksi$