ECE212 Exam Formula Sheet

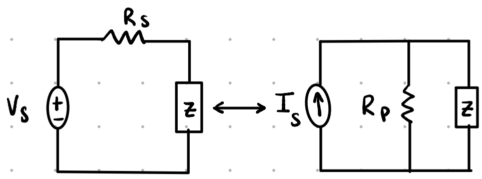
① Basic Techniques / Fundamentals

1. Voltage and Current Division:

$$V_x = \frac{R_x}{R_{eq}} V_s$$

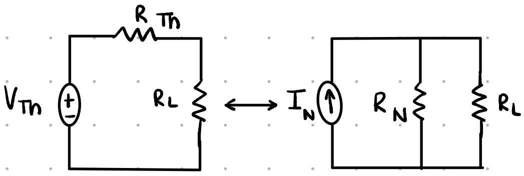
$$I_x = \frac{R_{eq}}{R_x + R_{eq}} I_s$$

2. Source Transformation



$$V_s = I_s R_p \text{ and } R_s = R_p$$

3. Thevenin and Norton



$$V_{Th} = V_{OC} = I_N R_{Th}$$

$$R_N = R_{Th} = \frac{V_{Th}}{I_{SC}}$$

$$I_{SC} = I_N$$

→ Max Power: $R_L = R_{Th}$

$$P_{max} = \frac{(V_{Th})^2}{4R_{Th}}$$

4. Power / Capacitor / Inductor

$$P = VI = \frac{V^2}{R} = I^2 R$$

- Capacitor → goes to open circuit

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

→ in parallel: $C_{eq} = C_1 + C_2 + \dots$

→ in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

- Inductor → goes to short circuit

$$v_L(t) = L\frac{di_L(t)}{dt}$$

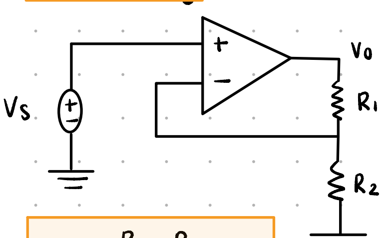
→ in parallel: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots$

→ in series: $L_{eq} = L_1 + L_2 + \dots$

② Operational Amplifier

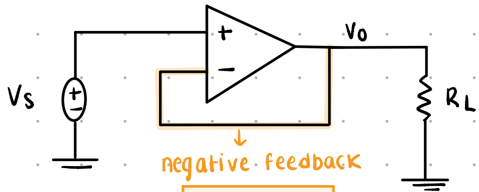
- ideal op amp → 0 input current; $V_P = V_N$; infinite gain

1. Non-inverting



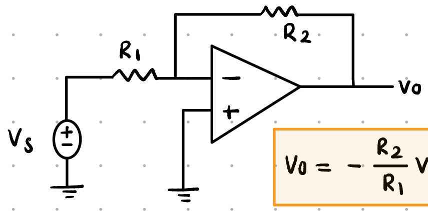
$$\boxed{V_o = \frac{R_1 + R_2}{R_2} V_s}$$

2. Voltage follower / Buffer



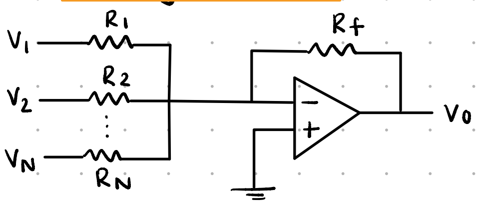
$$\boxed{V_o = V_s}$$

3. Inverting amplifier



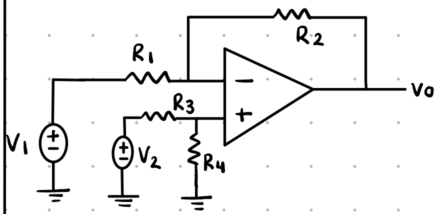
$$\boxed{V_o = -\frac{R_2}{R_1} V_s}$$

4. Summing amplifier



$$\boxed{V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots \frac{V_N}{R_N}\right)}$$

5. Differential Amplifier

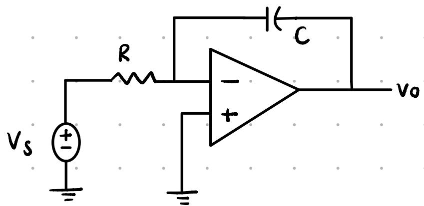


$$\boxed{V_o = \frac{-R_2}{R_1} V_1 + \frac{R_4}{R_3 + R_4}\frac{R_1 + R_2}{R_1} V_2}$$

→ when $\frac{R_3}{R_1} = \frac{R_4}{R_2}$:

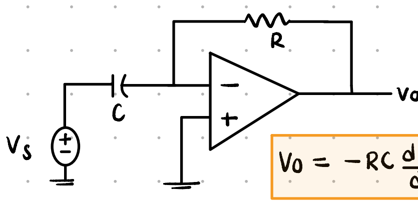
$$\boxed{V_o = \frac{R_2}{R_1}(V_2 - V_1)}$$

6. Inverting Integrator



$$\boxed{V_o(t) = V_o(t_0) - \frac{1}{RC}\int V_s(t)\,dt}$$

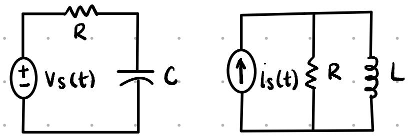
7. integrating differentiator



$$\boxed{V_o = -RC\frac{dV_s}{dt}}$$

③ 1st and 2nd Order Circuits

1. First Order: RC and RL circuits



RC: $\frac{dv(t)}{dt} + \frac{1}{RC}v(t) = \frac{1}{RC}V_s(t)$

RL: $\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{R}{L}i_s(t)$

2. First order: Step Response

RC: $V_C(t) = V_C(\infty) + e^{-t/\tau}[V_C(0) - V_C(\infty)]$

RL: $i_L(t) = i_L(\infty) + e^{-t/\tau}[i_L(0) - i_L(\infty)]$

where $\tau = R_{Th}C$ or $\tau = \frac{L}{R_{Th}}$

3. Second Order: Series RLC circuit

capacitor voltage:

$$V_C'' + \frac{R}{L}V_C' + \frac{1}{LC}V_C = \frac{V_s}{LC}$$

inductor current:

$$i_L'' + \frac{R}{L}i_L' + \frac{1}{LC}i_L = 0$$

4. Second order: Parallel RLC circuit

capacitor voltage

$$V_C'' + \frac{1}{RC}V_C' + \frac{1}{LC}V_C = 0$$

inductor current

$$i_L'' + \frac{1}{RC}i_L' + \frac{1}{LC}i_L = \frac{i_s}{LC}$$

5. Second Order: Steps to solve

1. find initial and final conditions

steady state: $x(\infty)$, $x(0)$, $x'(0)$

→ capacitor: $V_C(0^+) = V_C(0^-)$

$$i_C(0) = C\frac{dV_C(0)}{dt}$$

→ inductor: $i_L(0^+) = i_L(0^-)$

$$V_L(0)\ L\frac{di_L(0)}{dt}$$

2. find form of transient response

→ zero all independent sources

→ solve characteristic eqn.

i) overdamped (two roots):

$$x(t) = x(\infty) + K_1e^{s_1t} + K_2e^{s_2t}$$

ii) critically damped (one root)

$$x(t) = x(\infty) + K_1e^{st} + K_2te^{st}$$

iii) underdamped (complex roots):

$$x(t) = x(\infty) + e^{\alpha t}(K_1\cos\gamma t + K_2\sin\gamma t)$$

3. write eqn. and solve for coeff.

④ AC Circuit Analysis

1. Phasors: phasor representation of a sinusoid $v(t) = V_A \cos(\omega t + \phi)$

$$\boxed{\bar{V} = V_A e^{j\phi} = V_A \angle \phi = V_A \cos\phi + j V_A \sin\phi}$$

2. Phasor Circuit Analysis

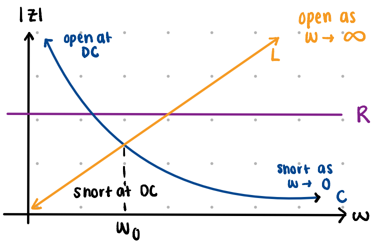
· Impedences (phasor domain) [Ω]

$$\boxed{\begin{aligned} Z_R &= R \\ Z_L &= j\omega L \\ Z_C &= \frac{1}{j\omega C} = \frac{-j}{\omega C} \end{aligned}}$$

→ use same techniques (i.e superposition, Thevenin, etc.) to solve circuit

→ convert back to time domain (if needed)

3. Effects of ω on Impedance



⑤ Mutual Inductance

inductor 1 and inductor 2:

$$V_1(t) = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

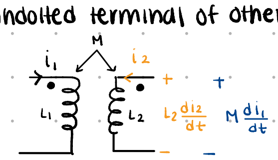
$$V_2(t) = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

Dot convention (to determine sign of second term):

i) current entering dotted terminal → positive voltage at dotted terminal of other coil

ii) current entering undotted terminal → positive voltage at undotted terminal of other coil

ex:



$L_2 \frac{di_2}{dt}$ $\quad M\frac{di_1}{dt}$

· coupling coefficient

$$K = \frac{M}{\sqrt{L_1 L_2}} \leq 1$$

→ $K = 1$ is perfect coupling

⑥ Transformers

$$\boxed{\begin{aligned} \text{eq 1:} \quad & \frac{V_2(t)}{V_1(t)} = \pm\frac{N_2}{N_1} = \pm n \\ \text{eq 2:} \quad & \frac{i_2(t)}{i_1(t)} = \pm\frac{1}{n} \end{aligned}}$$

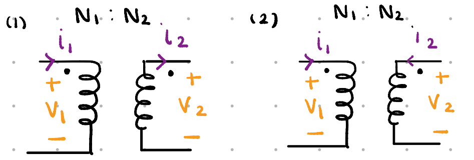
→ $n > 1$: step up transformer

→ $n < 1$: step down transformer

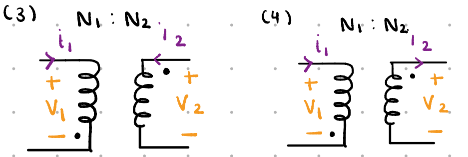
to determine sign:

i) if V_1 and V_2 are both +ve or −ve at dotted terminal use +n for eq. 1 (otherwise −n)

ii) if i_1 and i_2 both enter into or leave the dotted terminal, use −n for eq. 2 (otherwise +n)



(1) $\frac{V_2}{V_1} = n$, $\frac{i_2}{i_1} = \frac{1}{n}$ $\qquad$ (2) $\frac{V_2}{V_1} = n$, $\frac{i_2}{i_1} = -\frac{1}{n}$



(3) $\frac{V_2}{V_1} = -n$, $\frac{i_2}{i_1} = \frac{1}{n}$ $\qquad$ (4) $\frac{V_2}{V_1} = -n$, $\frac{i_2}{i_1} = -\frac{1}{n}$

· reflected impedance

$$\boxed{Z_{in} = \frac{Z_L}{n^2}}$$

Z_L → equivalent impedance on secondary side

⑦ Complex Power

1. instantaneous power

$$\boxed{\begin{aligned} p(t) &= v(t)\, i(t) \\ &= V_A \cos(\omega t + \theta_v)\, I_A \cos(\omega t + \theta_i) \end{aligned}}$$

(time dependent)

2. Average Power

$$\boxed{P_{avg} = \frac{V_A I_A}{2} \cos(\theta_v - \theta_i)}$$

→ P_{avg} for capacitors and inductors is 0

3. RMS value

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\, dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{P_{avg} = V_{rms} \cdot I_{rms} \cos(\theta_v - \theta_i)}$$

· for resistor: $P_{avg} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$

4. Apparent Power and Power factor

$$\boxed{\begin{aligned} S &= V_{rms} I_{rms} \\ pf &= \cos(\theta_v - \theta_i) = \frac{P}{S} \end{aligned}}$$

purely reactive (i.e. capacitors / inductors only) ← $0 \leq pf \leq 1$ → purely resistive

5. Complex Power

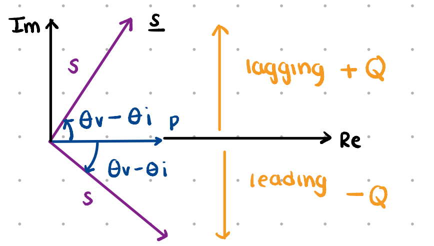
complex conjugate

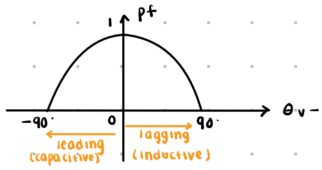
$$\underline{S} = \bar{V}_{rms} \bar{I}_{rms}^{*} = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$= I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*} = P + jQ$$

→ where $Z = (R + jX)$

· power triangle





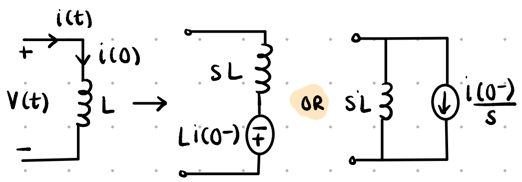
6. Max Power Transfer

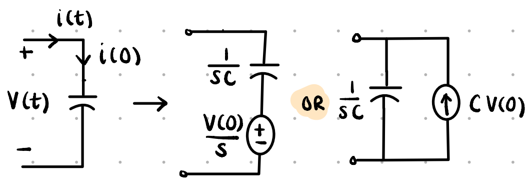
$$Z_L = Z_{th}^* = R_{th} - jX_{th}$$

$$\boxed{P_{max} = \frac{1}{4}\frac{|V_{th,rms}|^2}{R_{th}} = \frac{|V_m|^2}{8 R_{th}}}$$

⑧ Laplace

· convert to laplace domain:





Resistor: $V(s) = R\, I(s)$

Inductor: $V(s) = sL\, I(s)$

Capacitor: $V(s) = \frac{1}{sC} I(s)$

→ $\mathcal{L}\{u(t)\} = \frac{1}{s}$

· use regular techniques to solve, $\mathcal{L}^{-1}$ to switch back to time domain

⑨ Transfer Function and Bode Plots

1. Types of Transfer Functions

voltage gain: $H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$

current gain: $H(\omega) = \frac{I_o(\omega)}{I_i(\omega)}$

transfer impedance: $H(\omega) = \frac{V_o(\omega)}{I_i(\omega)}$

transfer admittance: $H(\omega) = \frac{I_o(\omega)}{V_i(\omega)}$

2. Steps to Draw Bode Plot given $H(j\omega)$

i) write $H(j\omega)$ in standard form,
→ i.e. all terms in the form
$$\left(\frac{j\omega}{x} + 1\right)$$

ii) determine break frequencies

iii) write the magnitude and phase equations using components of the equation

iv) plot for different values of ω

ex

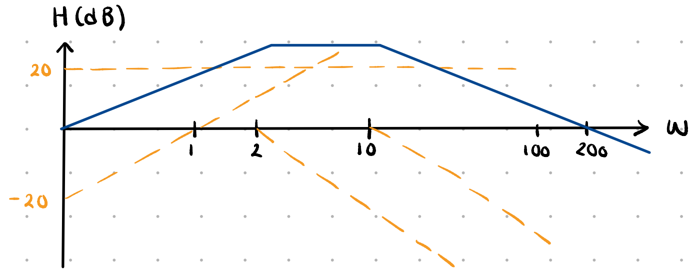
$$H(\omega) = \frac{200 j\omega}{(j\omega+2)(j\omega+10)}$$

i) $H(\omega) = \frac{10 j\omega}{\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{10}+1\right)}$

ii) $\omega = 2$ and $\omega = 10$ are break frequencies

iii) $H_{dB} = 20\log(10) + 20\log|j\omega| - 20\log\left|1+\frac{j\omega}{2}\right| - 20\log\left|1+\frac{j\omega}{10}\right|$

$$\phi = 90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$



3. Types of Filters

TABLE 14.5

Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Low-pass	1	0	$1/\sqrt{2}$
High-pass	0	1	$1/\sqrt{2}$
Band-pass	0	0	1
Band-stop	1	1	0

ω_c is the cutoff frequency for low-pass and high-pass filters; ω_0 is the center frequency for band-pass and band-stop filters.

TABLE 14.3

Summary of Bode straight-line magnitude and phase plots.

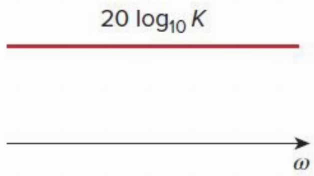
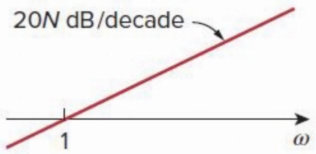
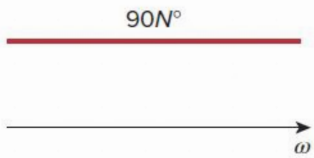
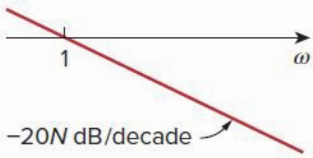
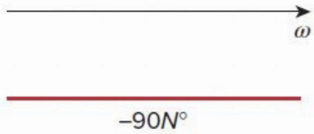
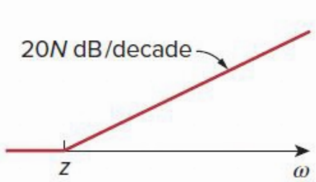
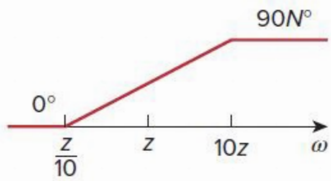
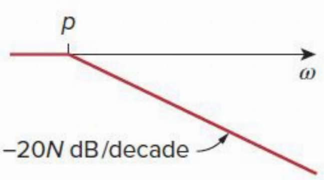
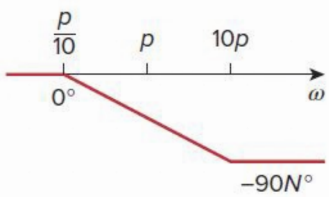
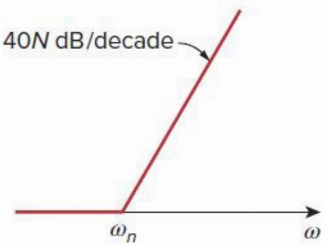
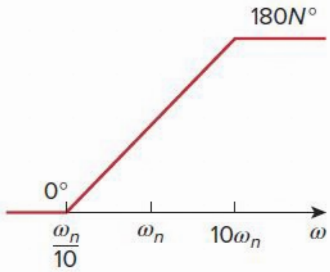
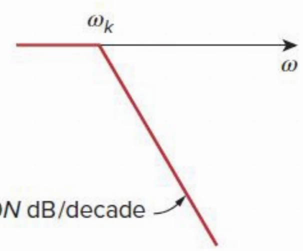
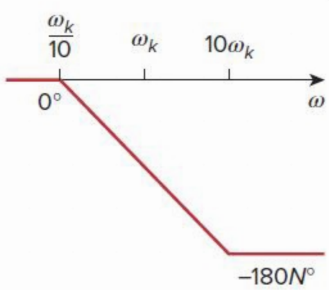
Factor	Magnitude	Phase
K		
$(j\omega)^N$		
$\dfrac{1}{(j\omega)^N}$		
$\left(\dfrac{1+j\omega}{z}\right)^N$		
$\dfrac{1}{(1+j\omega/p)^N}$		
$\left[1+\dfrac{2j\omega\zeta}{\omega_n}+\left(\dfrac{j\omega}{\omega_n}\right)^2\right]^N$		
$\dfrac{1}{[1+2j\omega\zeta/\omega_k+(j\omega/\omega_k)^2]^N}$		

Table of Laplace Transforms

f(t)	F(s)
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$

f(t)	F(s)
$\cos kt$	$\frac{s}{s^2+k^2}$
$\sin kt$	$\frac{k}{s^2+k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$

Table of Properties of the Laplace Transform

f(t)	F(s)
$\alpha f(t)+\beta g(t)$	$\alpha F(s)+\beta G(s)$
$f^{(n)}(t)$	$s^n F(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{at}f(t)$	$F(s-a)$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$f(t)u(t-a)$	$e^{-as}\mathcal{L}\{f(t+a)\}$
$f(t)=f(t+T)$	$\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$