STA 2023 SECTION 7.1 Introduction to Hypothesis Testing NOTES

H_0: Null

H_1 or H_a

Learning Outcomes:

1) State a null hypothesis and an alternative hypothesis
2) Identify type I and type II errors and interpret the level of significance
3) Know whether to use a one-tailed or two-tailed statistical test.
4) Summarize the results of a hypothesis test in the context of the claim

The purpose is to test a hypothesis or claim about a population parameter (a population mean μ or a population proportion p)

A <u>hypothesis test</u> or <u>test of significance</u> is a procedure (a set of steps) for testing a claim about a property of a population. We test a claim by analyzing sample data to distinguish between results that can quickly occur by chance and those that are highly unlikely to occur by chance.

Setting Up A Test:

- **The *Null Hypothesis* (H_0)**: This <u>statement of equality</u> is the opposite of the alternative hypothesis. Examples: $=$ or $\geq$ or $\leq$

- **The *Alternate Hypothesis* (H_1)** is a statement that the value of the parameters <u>differs</u> from the value specified by the null hypothesis. This is called the <u>statement of strict inequality or difference.</u> Examples: $\neq$ or $<$ or $>$

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.* *. . . not shorter than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$ Left tail test	*. . . less than k.* *. . . below k.* *. . . fewer than k.* *. . . shorter than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.* *. . . not longer than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$ Right tail test	*. . . greater than k.* *. . . above k.* *. . . more than k.* *. . . longer than k.*
. . . equal to k. *. . . k.* *. . . exactly k.* *. . . the same as k.* *. . . not changed from k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$ 2-tail test	*. . . not equal to k.* *. . . different from k.* *. . . not k.* *. . . different from k.* *. . . changed from k.*

One-tail test

These same statements can apply to population proportions by using p instead of µ

H_0: Defendent in innocent

H_1: Defendent in guilty

We begin a test by **assuming that the null hypothesis is true**.

- If the data provide **strong evidence** against the null hypothesis, we reject the null (assume it is not true) and accept the alternate hypothesis to be true (the value is <, >, or $\neq$). We are suggesting that the sample values did NOT happen by chance, they happened because the true value is different from the claimed value.
- If the data does **NOT** provide strong evidence against the null hypothesis, we conclude that it might be true, but **cannot** say it is true (must check **every subject** in the population to say for sure), so we say that there is **NOT ENOUGH** evidence to reject the null hypothesis. We could also say that there is **NOT enough** evidence to say that the alternate is true. We are suggesting that the sample values happened by chance.

Decision → Reject Ho → H_1 is true

Decision → Failed to reject Ho → Ho might true

Example 1: Write the statistical hypothesis and identify the claim.

a) A company publicizes that their employees contribute an average of \$2000 annually to charity. → $\boxed{\mu = 2000}$

H_0: $\mu = 2000$ (Claim)

H_1: $\mu \neq 2000$

2-tail

b) A company advertises that the mean life of its product is more than 20 years → $\mu > 20$

H_0: $\mu \leq 20$

H_1: $\mu > 20$ (Claim)

Right

c) A car dealership announces that the mean time for an oil change is less than 10 minutes. $\mu < 10$

H_0: $\mu \geq 10$

H_1: $\mu < 10$ (Claim)

Left tail

Next monday, July - 22nd → Quiz - Section 6.3, 7.1, 7.2, 7.3

Reject H_0

or

Fail to Reject H_0

There is no accept H_0

P

d) A school publicizes that the proportion of its students who are involved in extracurricular activities is 61%.

$P = 0.61$ (Claim)

$H_0: P = 0.61$

$H_1: P \neq 0.61$

H_0: Defendent in innocent

H_1: Defendent in guilty

Notes:

Whenever a decision is made, there is a possibility that it is a wrong decision. There are 2 types of error:

α

- Type I Error – we reject the null hypothesis H_0 when it is actually true.
- Type II Error – we fail to reject the null hypothesis H_0 when it is actually false.

Type-I: punishing an innocent person

Type-II: Releasing a guilty person

	Truth of H_0	
Decision	H_0 **is true.**	H_0 **is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

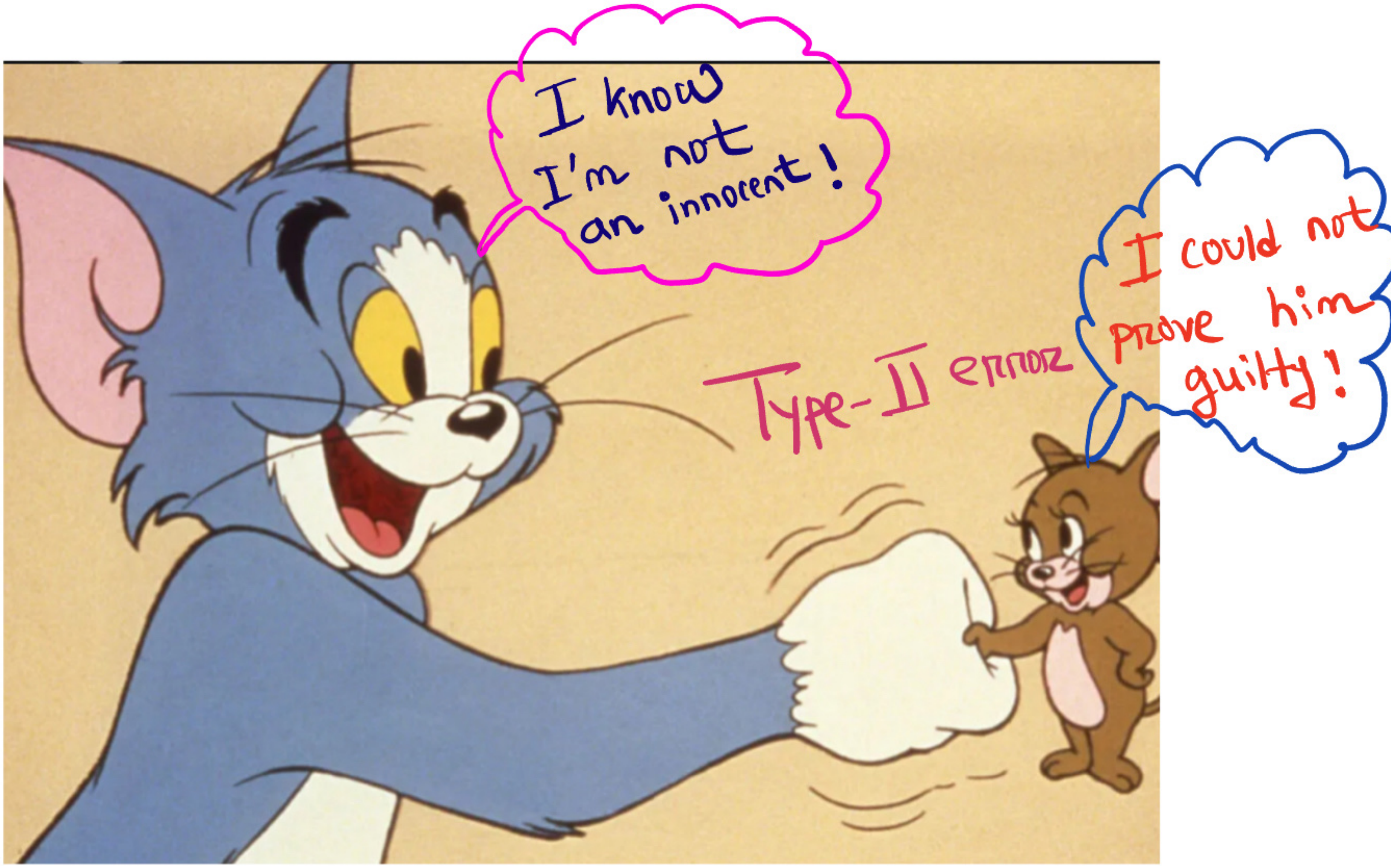
Which type of error has been made?

left tail

1. A test is made of $H_0: \mu = 5$ and $H_1: \mu < 5$. The true value of μ is five and the null hypothesis is rejected. $\boxed{\mu = 5}$

Type-I:


I Love
Type-I
error!


I know I'm not an innocent!
Type-II error
I could not prove him guilty!

2. A restaurant owner thinks that the mean amount spent by diners is greater than $30. The sample data does not provide enough evidence to say that $\mu > \$30$ but it truly is greater than $30.

$\mu > 30$

H_0: $\mu \leq 30$
H_1: $\mu > 30$
Right tail

Type-II

Example 2: The USDA upper limit for salmonella contamination for ground beef is 7.5%. A meat inspector reports that the ground beef produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. *(Source: U. S. Department of Agriculture).*

$p > 0.075$

a) State the hypothesis and identify the claim.

H_0: $p \leq .075$
H_1: $p > .075$
Right tail

b) Determine when will type I error will occur.

Type-I: Actual % was below 7.5% but inspector reported it more than 7.5%

c) Determine when a type II error will occur.

Type-II: Actual % was greater than 7.5% but the inspector reported it below 7.5%

7.2	7.3	7.4
σ known	σ unknown	Success/Failure
Z-test	T-test Sample SD is known (S)	1-prop Z-test (proportion test)
Tiger	Cat	

Chapter 7 Hypothesis Testing

STA 2023 SECTION 7.2 Hypothesis Testing for the Mean (σ Known) NOTES

(1) Critical value method

(2) p-value method

Learning Outcomes:

1) Perform a hypothesis test for the mean μ when σ is known

Recall: The *z*-score tells us how many standard deviations $\bar{x}$ is from μ

Recall: When the sample size is large (n > 30), the sample mean $\bar{x}$ is approximately normally distributed with mean μ and standard deviation $\sigma/\sqrt{n}$.

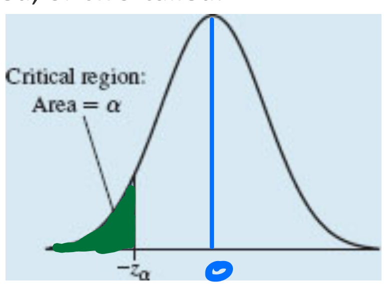
DEFINITION

If we reject H_0 after choosing a significance level α, we say that the result is **statistically significant** at the α level.

We also say that H_0 *is rejected at the* α *level*.

Critical Values for Hypothesis Tests

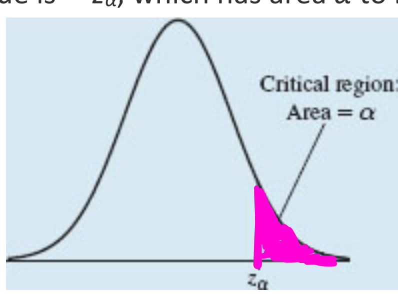
Let α denote the chosen significance level. The critical value depends on whether the alternate hypothesis is left-tailed, right-tailed, or two-tailed.



$H_1: \mu < \mu_0$

$z_\alpha = \text{invNorm}(\alpha, 0, 1)$

For left-tailed H_1: The critical value is $-z_\alpha$, which has area α to its left. Reject H_0 if $z \le -z_\alpha$.



$H_1: \mu > \mu_0$

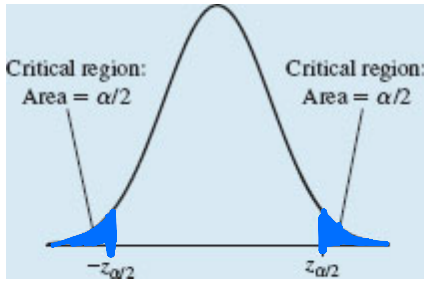
$z_{1-\alpha} = \text{invNorm}(\;)$

For right-tailed H_1: The critical value is z_α, which has area α to its right. Reject H_0 if $z \ge z_\alpha$.

* If Test Statistic value (z) is falling in a rejection region, Reject Ho, otherwise, failed to reject Ho

H_1: $\mu \neq \mu_0$

$Z_{1-\frac{\alpha}{2}}$ = invNorm( )



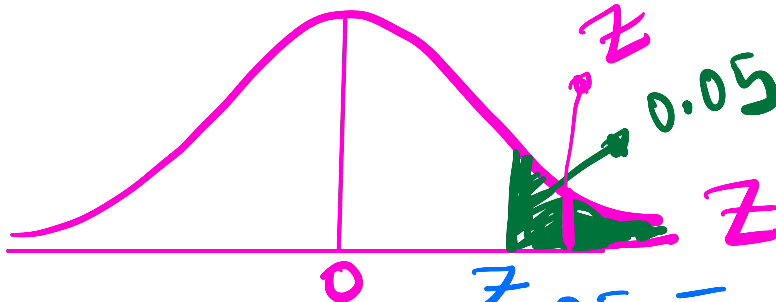
For two-tailed H_1: The critical values are $z_{\alpha/2}$, which has area $\alpha/2$ to its right, and $-z_{\alpha/2}$, which has area $\alpha/2$ to its left. Reject H_0 if $z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$.

H_1: $\mu > 25$, Right tail

A test is made of H_0: $\mu = 25$ versus H_1: $\mu > 25$. The value of the test statistic is $z = 1.84$.

a. Find the critical value and the critical region for a significance level of $\alpha = 0.05$.

Sub=AL



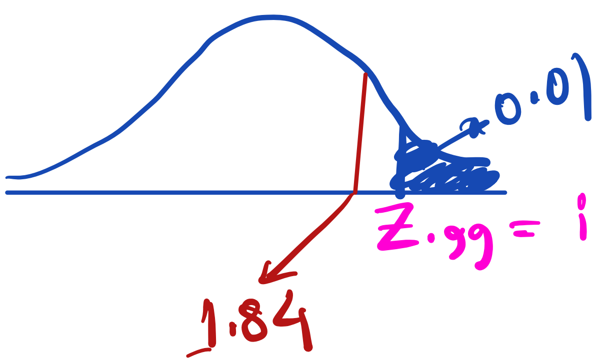
$Z_{.95}$ = invNorm(.95,0,1)

= 1.645

b. Do you reject H_0 at the $\alpha = 0.05$ level?

$Z = 1.84 > Z_{.95}$, Reject Ho

c. Find the critical value and the critical region for a significance level of $\alpha = 0.01$.



$Z_{.99}$ = invNorm(.99,0,1) = 2.326

Failed to reject Ho

d. Do you reject H_0 at the $\alpha = 0.01$ level?

The method we have described requires certain assumptions, which we now state.

Assumptions for Performing a Hypothesis Test About μ When σ Is Known

1. We have a simple random sample.
2. The sample size is large ($n > 30$), or the population is approximately normal.

When these assumptions are met, a hypothesis test can be performed using the following steps.

Performing a Hypothesis Test for a Population Mean with σ Known Using the Critical Value Method

Check to be sure the assumptions are satisfied. If they are, then proceed with the following steps.

- **Step 1:** State the null and alternate hypotheses. The null hypothesis specifies a value for the population mean μ. We will call this value μ_0. So the null hypothesis is of the form H_0: $\mu = \mu_0$. The alternate hypothesis can be stated in one of three ways:

  Left-tailed: H_1: $\mu < \mu_0$
  Right-tailed: H_1: $\mu > \mu_0$
  Two-tailed: H_1: $\mu \neq \mu_0$
- **Step 2:** Choose a significance level α and find the critical value or values.
- **Step 3:** Compute the test statistic $\cancel{z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$. TI 84
- **Step 4:** Determine whether to reject H_0, as follows:

  Left-tailed: H_1: $\mu < \mu_0$ Reject if $z \leq -z_\alpha$.
  Right-tailed: H_1: $\mu > \mu_0$ Reject if $z \geq z_\alpha$.
  Two-tailed: H_1: $\mu \neq \mu_0$ Reject if $z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$.
- **Step 5:** State a conclusion.

Example 1: The American Automobile Association reported that the mean price of a gallon of regular-grade gasoline in Los Angeles in July 2013 was \$4.04. A recently taken simple random sample of 50 gas stations in Los Angeles had an average price of \$3.99 for a gallon of regular-grade gasoline. Assume that the standard deviation of prices is \$0.15. An economist is interested in determining whether the mean cost is less than \$4.04. Use the critical value method to perform a hypothesis test at the $\alpha = 0.05$ significance level.

$\mu < 4.04$

$\sigma = 0.15$

$\alpha = 0.05$

$n = 50$

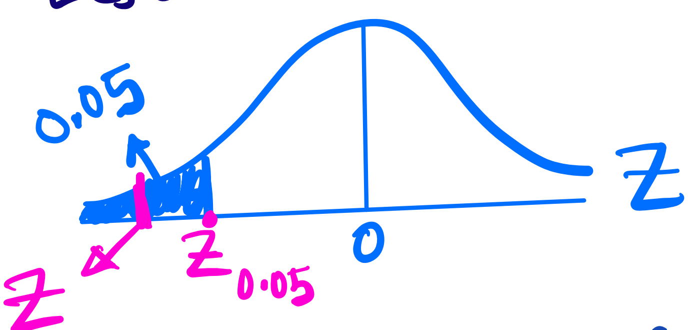
$\bar{x} = 3.99$

Step-1

$H_0: \mu \geq 4.04$

$H_1: \mu < 4.04$ (claim)

Left tail test



Step-2

$Z_{.05} = \text{invNorm}(.05, 0, 1) = -1.6449$

Step-3

TS, $Z = -2.3570$

Step-4

As $Z < Z_{0.05}$, Reject Ho

At 5% level of sig., the economist claim was true, mean gasoline price in Los Angeles is less than $4.04.

The *P*-value Method

- The *P*-value is the probability, assuming that H_0 is true, of observing a value for the test statistic that is as extreme as or more extreme than the value actually observed.
- The smaller the *P*-value, the stronger the evidence against H_0.

* If P-value is low, Ho must go;

Performing a Hypothesis Test for a Population Mean with σ Known Using the *P*-Value Method

Check to be sure the assumptions are satisfied. If they are, then proceed with the following steps. If P-value is high, Ho will fly.

- **Step 1:** State the null and alternate hypotheses. The null hypothesis specifies a value for the population mean μ We will call this value μ_0. So the null hypothesis is of the form H_0: $\mu = \mu_0$. The alternate hypothesis can be stated in one of three ways:
  Left-tailed: H_1: $\mu < \mu_0$
  Right-tailed: H_1: $\mu > \mu_0$
  Two-tailed: H_1: $\mu \neq \mu_0$

Unusual event → prob < 0.05

- **Step 2:** If making a decision, choose a significance level α.
- **Step 3:** Compute the test statistic $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$.
- **Step 4:** Compute the *P*-value of the test statistic. The *P*-value is the probability, assuming that H_0 is true, of observing a value for the test statistic that is as extreme or more extreme than the value actually observed. The *P*-value is an area under the standard normal curve; it depends on the type of alternate hypothesis. Note that the inequality in the alternate hypothesis points in the direction of the tail that contains the area for the *P*-value.

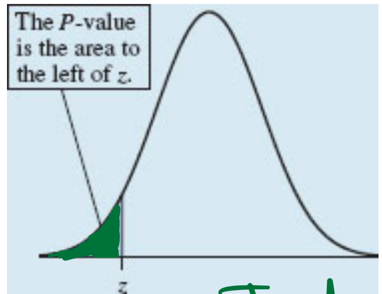
If P-value ≤ α, Reject Ho

If P-value > α, Failed to reject Ho

$\boxed{\text{* P-value} > 0.50 \rightarrow \text{failed to reject Ho}}$

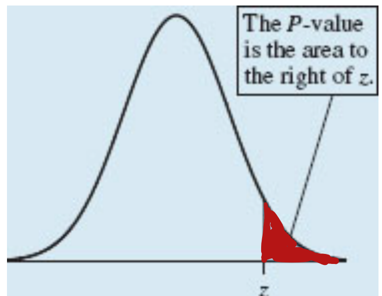
→ Business / Industry / corporation ...

P-value = normalcdf(−1E99, Z, 0,1)



→ Test Statistic value

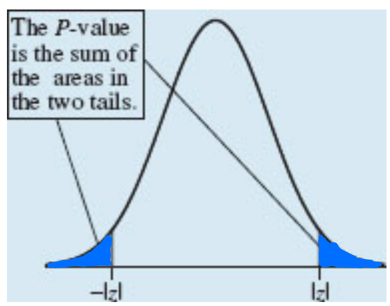
Left-tailed: H_1: $\mu < \mu_0$



→ TS

P-value = normalcdf(Z, 1E99, 0,1)

Right-tailed: H_1: $\mu > \mu_0$



−Z Z

P-value = 2normalcdf(Z, 1E99, 0,1)

= 1 − normal cdf (−Z, Z, 0,1)

Two-tailed: H_1: $\mu \neq \mu_0$

- **Step 5:** Interpret the *P*-value. If making a decision, reject H_0 if the *P*-value is less than or equal to the significance level α.
- **Step 6:** State a conclusion.

$\mu \geq 135$

Example 2: A manufacturer of sprinkler systems designed for fire protection claims that the average activating temperature is at least 135 degrees Fahrenheit. To test this claim, you randomly select a sample of 32 systems and find the mean activation temperature to be 133 degrees Fahrenheit. Assume the population standard deviation is 3.3 degrees Fahrenheit and the population is normally distributed. Is there enough evidence to reject the claim at $\alpha = 0.1$? Use a p-value method.

Step 1: Write the statistical hypothesis and identify the claim.

$H_0: \mu \geq 135$ (Claim)

$H_1: \mu < 135$

Left tail

$n = 32$

$\bar{x} = 133$

$\sigma = 3.3$

$\alpha = .1$

Z-test $\Rightarrow \sigma$ known

$Z = -3.4284$

$P = 3.0363 \times 10^{-4} = 0.00030363$

$P < \alpha$, Reject Ho, H_1 is true

At 10% level of sig., mean activating time for sprinkler is less than 135.

$\alpha = 0.01$, $P < \alpha$, Reject Ho

STA 2023 SECTION 7.3 Hypothesis Testing for the Mean (σ Unknown) NOTES → T- test

Learning Outcomes:

1) Perform a hypothesis test for the mean μ when σ is unknown

If a value of the population standard deviation σ is NOT known, we CANNOT use the normal distribution and MUST use the $t - distribution\ with\ df = n - 1$.

STEPS:

- Decide on the sampling distribution to use:
  - If the population standard deviation σ IS known → use a $z - test\ (normal\ dist.)$
  - If the population standard deviation σ is NOT known → use a $t - test\ (t - dist.)$
  - If testing proportions and $x \geq 10\ and\ n - x \geq 10$, →use a $z - test\ (normal\ dist.)$ → Sucess / Failure → 1-prop-Z-test
- Set up the test for a population mean:

Step-1: State Ho and H1

Step 2: Find critical value → invT(AL, df)

Step-3: Find $\boxed{\text{TS value}}$ and $\boxed{\text{p-value}}$

Step-4: make decision and conclusion

- Calculate the value of the $test\ statistic$ given a claim and the sample data: $\boxed{t = \frac{(\bar{x} - \mu)}{\left(\frac{s}{\sqrt{n}}\right)}}$ → TI calculator

- Decide on the method for testing : Critical Region using Critical Values of t (the traditional method) or $P - value$ (a more current method)
- Make a decision:
  - reject the null hypothesis and accept the alternate → if p-value $\leq \alpha$
  - fail to reject the null hypothesis → if p-value $> \alpha$

Ho "might" be true

* H_0 is rejected
  - We don't have enough evidence to support $\underline{H_0}$
  - We have enough evidence to support $\underline{H_1}$

* H_0 failed to reject
  - We don't have enough evidence to support $\underline{H_1}$
  - We don't have evidence to reject $\underline{H_0}$

- State the conclusion about the claim in terms of the problem:
  - There IS enough evidence to say that the <u>alternate</u> is true.
  - There IS NOT enough evidence to say that the <u>alternate</u> is true.

*The idea behind a hypothesis test is to see if the sample value is "far enough" from the claim value to say there is a significant difference (reject the null hypothesis) OR there is no significant difference in the values (do not reject the null hypothesis). Did we get sample values by chance or because the null hypothesis is wrong?

<u>2 Methods for Testing a Claim:</u>

1. ~~Critical Value Method – the critical region (also called the rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis and accept the alternate. This critical value depends on the significance level $(\alpha = 1 - c)$. The significance level is the probability used to determine if a test statistic is unusual.~~

- If the test statistic falls in the critical region, we reject the null hypothesis and accept the alternate. There is a significant difference between the sample value and the claim value. The original assumption that the null hypothesis is true is probably not correct.
- If the test statistic does NOT fall in the critical region, we fail to reject the null hypothesis. There is NOT a significant difference between the sample value and the claim value. There is NOT enough evidence to say that the alternate is true.

2. **P-Value Method**: The P-value is the probability of getting a test statistic that is at least as extreme as the one representing the sample data.
   - If $P - value \ \leq \alpha$, we reject the null hypothesis and accept the alternate.
   - If $P - value > \alpha$, we fail to reject the null hypothesis.
   - For a $T - Test$, we will get the $P - value$ from the calculator.

$\mu > 110$

Example #1: Use a 5% level of significance to test the claim that statistics students have a mean IQ score greater than 110. A random sample of 25 students had a mean score of 118.0 with standard deviation of 10.7.

H_0: $\mu \leq 110$ H_1: $\mu > 110$, Right tail

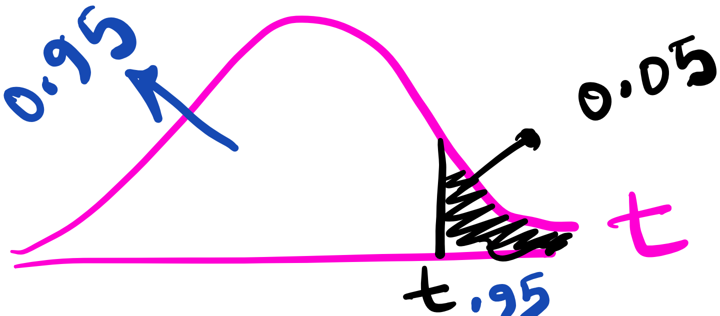
Find the critical value of t and sketch the critical region.

$n = 25$

$\bar{x} = 118$

$s = 10.7$

$\boxed{\alpha = 0.05}$



Find the value of the test statistic: $t =$____________ $\cancel{t = \frac{(\bar{x}-\mu)}{\left(\frac{s}{\sqrt{n}}\right)}}$

$t = 3.7383$

Find the $P - value$: $p =$____________

$p = 5.08 \times 10^{-4} = \boxed{.000508}$

Decision: p-value $< \alpha$, Reject Ho

At 5% level of significant,

Conclusion: we have enough evidence to say mean IQ score of Statistic students more than 110.

→ $\mu = 800$

Example #2: A rental agency claims that the average rent that small business establishments pay in Lafayette is \$800 per month. A random sample of 10 establishments shows an average rent of \$785 with standard deviation of \$20. At $\alpha = 2\%$, is there enough evidence to reject the agency's claim?

H_0: $\mu = 800$ H_1: $\mu \neq 800$, 2 tail test

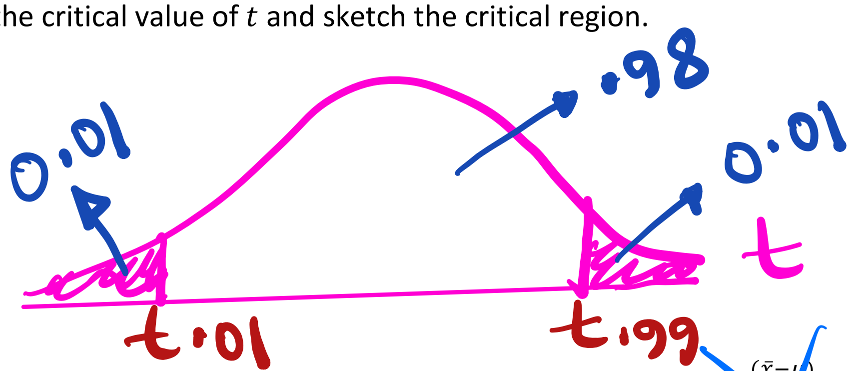
$\bar{x} = 785$

$s = 20$

$n = 10$

$\boxed{\alpha = .02}$

Find the critical value of t and sketch the critical region.



Find the value of the test statistic: $t =$____________ $\cancel{t = \frac{(\bar{x}-\mu)}{\left(\frac{s}{\sqrt{n}}\right)}}$

$t = -2.3717$

Find the $P - value$: $p =$____________

$P = 0.0418$ $> \alpha$, Failed to reject

Decision:

What if $\alpha = .05$, $P < .05$, Reject Ho

Conclusion:

What if $\alpha = .10$, Reject Ho

**The $P - value$ is the probability of getting a test statistic falling in an interval. For a $t - test$, you would find the $P - value$ using $P(t\ is\ in\ an\ interval) = tcdf(LB, UB, df)$. This button is under 2nd DISTR.