Work Energy II - Problem 2

At the instant shown, the unbalanced wheel has a clockwise angular velocity of 4 rad/sec. Determine the angular velocity of the wheel after it has displaced 3π feet to the right.

CLASSIFY MOTION

WHEEL — GPM

BLOCK — TRANS

PROPERTIES

DATA

Wheel: Wgt = 322 lbs → M = 10 SLUGS

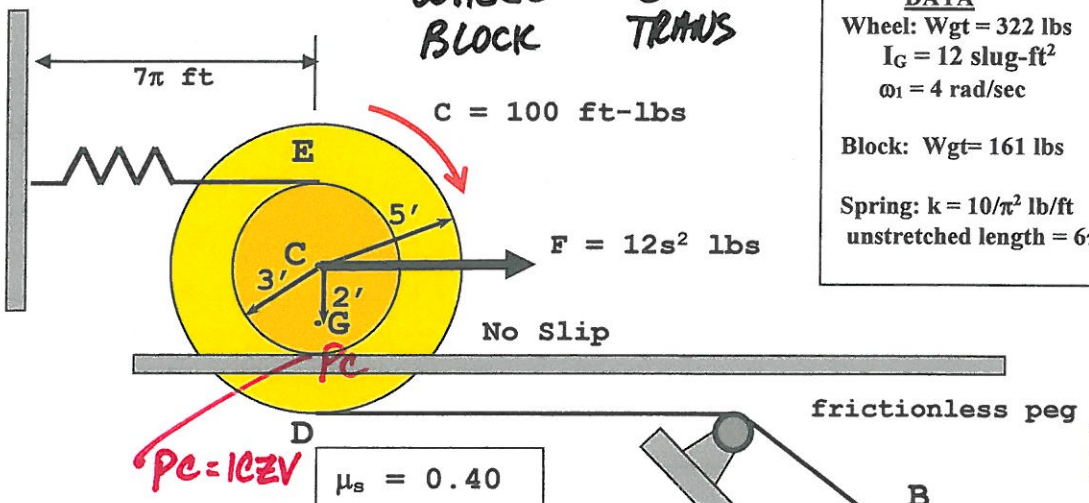
$I_G = 12$ slug-ft^2

$\omega_1 = 4$ rad/sec

Block: Wgt = 161 lbs → M = 5 SLUGS

Spring: $k = 10/\pi^2$ lb/ft

unstretched length = 6π ft = l_0



$\mu_s = 0.40$

$\mu_k = 0.2$

$\mu_s = 0.60$

$\mu_k = 0.518$

DISPLACEMENTS

$S_{WH} = S_C = 3\pi$ Ft

$S_C = \theta_{WH}\, r_{C/IC} \Rightarrow 3\pi = \theta_{WH}(3)$

$\theta_{WH} = \pi$ RAD ↻

$S_E = \theta_{WH}\, r_{E/IC} = \pi(6) = 6\pi$ Ft

$\Delta y_G = \pi$ rad $\Rightarrow$ 4 Ft

DISPLACEMENTS

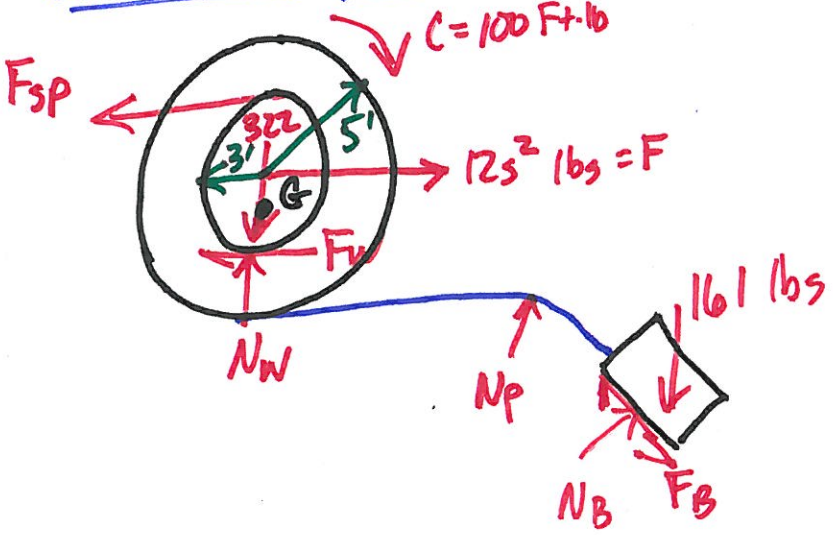
$S_B = S_D = \theta_W r_{D/IC}$

$= \pi(2) = 2\pi$ Ft

$\frac{\Delta y}{2\pi} = \frac{4}{5}$

$\Delta y = \frac{4}{5}(2\pi)$

CALCULATE WORK



WHAT DOES WORK?

→ SPRING

→ COUPLE-MOMENT

→ VARIABLE FORCE

→ WHEEL WEIGHT

→ BLOCK FRICTION

→ BLOCK WEIGHT

Work from spring

$$U_{sp} = -\frac{1}{2}k(s_2^2 - s_1^2)$$

$$s_1 = l_1 - l_0 = (7\pi - 6\pi) = \pi$$

$$s_2 = l_2 - l_0 = (13\pi - 6\pi) = 7\pi$$

$$U_{sp} = -\frac{1}{2}\left(\frac{10}{\pi^2}\right)((7\pi)^2 - (\pi)^2)$$

$$= -240 \text{ ft lbs}$$

Work from coupled moment

$$U_c = \bar{m}(\theta_2 - \theta_1)$$

$$U_c = 100(\pi - 0)$$

$$= 314 \text{ ft·lbs}$$

Work from variable force

$$U_p = \int_{s_1}^{s_2} F\cos\theta \, ds$$

$$= \int_0^{3\pi} 12s^2 ds = 4s^3 \Big|_0^{3\pi}$$

$$= 3349 \text{ ft·lbs}$$

Work from wheel weight

$$U_{w_{wh}} = -w\Delta y$$

$$= -322(4)$$

$$= -1288 \text{ ft·lbs}$$

Work from block weight

$$U_{w_b} = -w\Delta y$$

$$= -161\left(\frac{4}{5}(2\pi)\right)$$

$$= -809 \text{ ft·lbs}$$

Work from block friction

$$U_{fr} = -\mu_k N d$$

$$= -(.518)\left(\frac{3}{5}(161)\right)(2\pi)$$

$$= -314 \text{ ft·lbs}$$

Total Work

$$U_{1\text{-}2} = \sum U$$

$$= -240 + 314 + 3349 - 1288 - 809 - 314$$

$$= 1012 \text{ ft·lbs}$$

ENERGY

$$T_1 = \tfrac{1}{2} m_w V_{G_1}^2 + \tfrac{1}{2} I_G \omega_1^2 + \tfrac{1}{2} m_B V_{B_1}^2$$

$\omega_1 = 4$ rps ↻

$V_{G1} = \omega_1 r_{G/pc} = 4(1) = 4$ Fps

$V_{B1} = V_D = \omega_1 r_{D/pc} = 4(2) = 8$ Fps

$$T_1 = \tfrac{1}{2}(10)(4)^2 + \tfrac{1}{2}(12)(4)^2 + \tfrac{1}{2}(5)(8)^2 = 336 \text{ Ft·lbs}$$

$$T_2 = \tfrac{1}{2} m_w V_{G_2}^2 + \tfrac{1}{2} I_G \omega_2^2 + \tfrac{1}{2} m_B V_{B_2}^2$$

$\omega_2 = ?$

$V_{G2} = \omega_2 r_{G/IC} = 5\omega_2$

$V_{B2} = \omega_2 r_{D/IC} = 2\omega_2$

$$T_2 = \tfrac{1}{2}(10)(5\omega_2)^2 + \tfrac{1}{2}(12)\omega_2^2 + \tfrac{1}{2}(5)(2\omega_2)^2 = 141\omega_2^2$$

WORK-ENERGY

$$T_1 + U_{1\text{-}2} = T_2$$

$$336 + 1012 = 141\omega_2^2$$

$$\omega_2 = 3.09 \text{ rps} \circlearrowright$$