STA 2023 SECTION 4.1 PROBABILITY DISTRIBUTIONS

Learning Outcomes:

1) Distinguish between discrete random variables and continuous random variables
2) Construct a discrete probability distribution and its graph and how to determine if a distribution is a probability distribution
3) Find the mean, variance, standard deviation, and expected value of a discrete probability distribution

Notes:

A probability distribution is a statistical function that describes all the possible outcomes and probabilities of a probability experiment.

A random **variable, X,** represents a numerical value associated with each outcome of a probability distribution.

A discrete **random variable** has a finite or countable number of possible outcomes that can be listed.

A continuous **random variable** has an uncountable number of possible outcomes, represented by an interval on the number line.

Example 1: Verify that the discrete probability distribution for the three-day forecast and the number of days of rain, X, is a probability distribution.

Days of Rain, x	0	1	2	3	Total
Probability, $P(x)$	0.216	0.432	0.288	0.064	1

Example 2: The table shows the ages of students in a freshman writing course. Construct a probability distribution.

RF = prob

Age	17	18	19	20	21	22	Total
Students	2	13	4	3	2	1	25

Age, x	17	18	19	20	21	22	Total
P(x)	2/25	13/25	4/25	3/25	2/25	1/25	1

$E(X) = 18.72$ $\sigma = 1.25$ $\sigma^2 = 1.56$

Notes:

The mean of a discrete probability distribution is the sum of all x values multiplied by their corresponding **probability**.

Since the **mean** of a random variable represents the outcome you would **expect** over thousands of trials it is also called the expected value of a discrete random variable. Notation: E(x).

$$\boxed{E(X) = \sum x \,.\, P(X = x)}$$

TI – calculation

The interpretation of the **mean** of a discrete probability distribution can be understood as the most likely or approximately most likely outcome for the discrete random variable.

Variance and Standard Deviation of a Discrete Random Variable

The **variance** of a discrete random variable is

$$\boxed{\sigma^2 = \Sigma(x - \mu)^2 P(x).}$$

The **standard deviation** is

$$\boxed{\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x - \mu)^2 P(x)}.}$$

X TI

You can find the mean and standard deviation in your TI-83/84 calculator.

Step 1: Go to STAT>EDIT. Enter outcomes in L1 and probabilities in L2

Step 2: Go to STAT>CALC. Select 1-var stats and press enter.

Step 3 for TI-83: Type L1, L2 and press enter.

Step 3 for TI-84/89: Set list to L1 and Freqlist to L2 and calculate.

Example 5: Find the mean, variance, and standard deviation for the probability distribution found in example 2.

Bi - 2

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STA 2023 SECTION 4.2 BINOMIAL DISTRIBUTIONS NOTES

X → 0 = failure
X → 1 = Success

Learning Outcomes:

1) Determine whether a probability experiment is a binomial experiment
2) Find binomial probabilities using technology
3) Construct and graph a binomial distribution
4) Find the mean, variance, and standard deviation of a binomial probability distribution

Notes:

A **binomial experiment** is a probability experiment that satisfies these conditions:

a) The experiment has a ___n___ number of trials that are ___independent___ of other trials.

b) There are only ___2___ possible outcomes for each trial. Each outcome can be classified as a **success** (S) or a **failure** (F).

1 0

if not same → Hypergeometric

c) The probability of ___success___ is the **same** for each trial.

failure

d) The random variable x counts for the number of ___success or___ trials.

Notation for binomial experiments:

___n___ represents the number of trials in the experiment

___X___ represents the number of successful trials out of n trials of the experiment.

___P___ represents the probability of success and ___1−P___ represents the probability of failure.

Example 1: Decide whether the experiment is a binomial experiment. If it is, specify the values of *n*, *p*, and *q*, and list the possible values of the random variable *x*. If it is not, explain why.

a) Around 81% of undergraduate college students in the US work while completing their degree. On a particular college campus, ten students are selected at random. The random variable represents the number of students that work.

X = working ? yes = 1, No = 0

Binomial

b) A jar contains five red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, without replacement. The random variable represents the number of red marbles.

X = # of red marbles

not binomial

Notes:

We will now discuss finding the **probability** of obtaining exactly ____ successes in ______ trials of a binomial probability experiment. For binomial experiments, the probability of exactly *x* successes in *n* trials P(x) is found using the formula:

$$\boxed{P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}.}$$

Or **using your TI-83/84:**

TI 84 plus CE: Press 2nd>Vars then select A: binompdf. Enter number of trials n, probability of success p, and number of successes x. Highlight paste and then press enter twice.

TI 84 or 83: Press 2nd>Vars then select A: binompdf. On your screen you will see binompdf(, enter in information in the form: n, p, x, then close the parenthesis and press enter.

Playing with phrases: * * *

At most (≤)	Less than (<)	At least (≥)	more than (>)

In an n=8 number of binomial trials, the number of possible successes is X= {0, 1, 2, 3, 4…….8}

- ✓ At most 4 successes, meaning the number of successes is {0, 1, 2, 3, 4}
- ✓ Less than 4 success, meaning number of successes is {0, 1, 2, 3}
- ✓ At least 4 successes, meaning the number of successes is {4, 5, 6, 7, 8}
- ✓ More than 4 successes meaning the number of successes is {5, 6, 7, 8}

Example 2: Students take a surprise multiple-choice quiz with 4 questions. Each question has 5 choices, one of which is correct. If a student (who did not read the section) guesses at each of the questions, let $x = the\ \#\ correct$. Does this situation satisfy the requirements of a binomial probability distribution? Construct a probability distribution for $x = the\ \#\ correct$.

$P(\text{correct}) = \frac{1}{5} = p$, $P(\text{incorrect}) = \frac{4}{5}$

$n = 4$

$X = \#$ correct answer

$= \{0, 1, 2, 3, 4\}$

✓ binompdf( )

* pdf = prob^y density function

* use: Exact # of
passes

binomcdf( )

* cdf = cumulative density (distribution) function

use: at most ($\leq$)

(a) prob^y of 2 correct answer?

$$P(X=2) = \text{binompdf}\left(4, \frac{1}{5}, 2\right) = 0.1536$$

* binompdf(n, p, x)

(b) prob^y of at most 3 correct answer?

$$P(X \leq 3) = \text{binomcdf}\left(4, \frac{1}{5}, 3\right)$$

$$= \boxed{0.9984}$$

(c) What is the prob^y of **less than** 3 correct answer?

$$P(X<3) = P(X \leq 2)$$

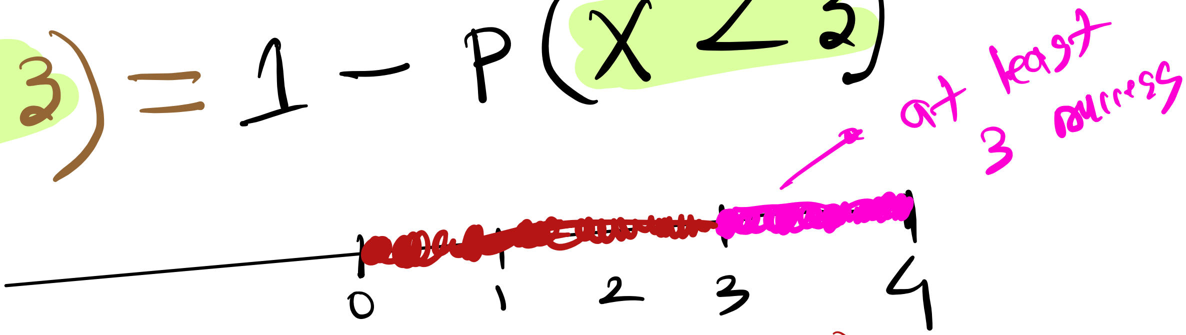
$$= \text{binomcdf}\left(4, \tfrac{1}{5}, 2\right)$$

$$= 0.9728 \checkmark$$

(d) What is the prob^y of "at least" 3 correct answer?

complement

$$P(X \geq 3) = 1 - P(X<3)$$



$$= 1 - P(X<3)$$

$$= 1 - P(X \leq 2)$$

$$= 1 - \text{binomcdf}\left(4, \tfrac{1}{5}, 2\right)$$

$$= 0.0272$$

(e) What is the prob of more than 3 correct answer

$$P(X>3) = P(X=4)$$

$$= \text{binompdf}\left(4, \tfrac{1}{5}, 4\right)$$

$$\boxed{= 0.0016}$$

New

$n = 10$, $p = \frac{1}{5}$, $X = \{0, 1, 2, 3, \ldots\ldots 10\}$

* prob of answering more than 5 correct answer?

$$P(X>5) = 1 - P(X \le 5)$$

$$= 1 - \text{binomcdf}\left(10, \tfrac{1}{5}, 5\right)$$

= Total prob = 1

Notes:
A __discrete__ **probability distribution** lists the possible values of *x* with the corresponding probability of x occurring.

In 4.1 we learned how to find the mean, variance, and standard deviation of discrete probability distributions. While binomial distributions are discrete probability distributions, the formulas for finding the mean, variance, and standard deviation are different from other discrete probability distributions.

The **mean of a binomial probability distribution**:
Formula: $\mu = \text{Mean} = np = E(X)$

Interpretation: Average number of __success__ for n trials

The **variance of a binomial probability distribution**:
Formula: $\text{variance} = np(1-p) = \sigma^2$

The **standard deviation of a binomial probability distribution**:
Formula: $\text{St. dev.} = \sqrt{np(1-p)} = \sigma$

Interpretation: For n trails, you can expect the number of __success__ to vary from the __μ__ by σ on average.

Example 3: There is a 33% chance that South Florida will be hit by a hurricane in a given year. Consider the next three years.

$p = .33$

a) Construct a probability distribution for the number of years South Florida will be hit by a hurricane if we consider the next three years.

x, years	0	1	2	3
P(x)	0.3008	0.444	0.2189	0.0359

X = # of hurricane

$n = 3$

$p = 0.33$

b) Find the mean, variance, and standard deviation for the probability distribution and then interpret the mean and standard deviation.

$\mu = np = 3(.33) = 1$

$\sigma = \sqrt{np(1-p)} = \sqrt{3(.33)(.67)}$

$\sigma^2 = np(1-p)$

$= 3(.33)(.67)$