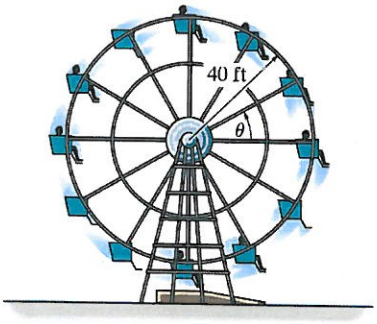
Problem 4: Particle Kinematics I

The Ferris wheel turns such that the speed of the passengers is increased by $\dot{v} = (4t)$ ft/s^2, where t is in seconds. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = 30°$.



$\dot{v} = \frac{dv}{dt} = 4t$

$v = \int 4t\,dt = 2t^2 + C_1$

at $t=0$, $v=0 \Rightarrow C_1 = 0$

$v = 2t^2$ ---- (1)

$s = \int v\,dt = \int 2t^2\,dt = \frac{2t^3}{3} + C_2$

at $t=0$, $s=0 \Rightarrow C_2 = 0$

$s = \frac{2t^3}{3}$ ---- (2)

$\theta = 30° \Rightarrow s = r\theta = (40\text{ ft})\frac{\pi}{6} = 20.94\text{ ft}$

$\hookrightarrow = \frac{\pi}{6}$ rad

From (2) $\Rightarrow$ $s = 20.94\text{ ft} = \frac{2t^3}{3}$ $\therefore t = 3.1554\text{ s}$

From (1) $\Rightarrow$ $v = 2t^2 = 2(3.1554)^2 = 19.91\text{ ft/s}$

$a_T = \frac{dv}{dt} = 4t = 4(3.1554) = 12.62\text{ ft/s}$

$a_N = \frac{v^2}{\rho} = \frac{(19.91)^2}{40} = 9.91\text{ ft/s}^2$

$a = \sqrt{a_N^2 + a_T^2} = \sqrt{(9.91)^2 + (12.62)^2} = 16.0\text{ ft/s}^2$