Chapter 8 Hypothesis Testing with Two Samples

STA 2023 Section 8.2 Testing the Difference Between Means (Independent Samples, σ_1 and σ_2 Unknown) NOTES

Learning Outcomes:

1) Determine whether two samples are independent or dependent
2) Perform a *t*-test for the difference between two means μ_1 and μ_2 with independent samples with σ_1 and σ_2 unknown.

Notes:

In chapter 7, we were testing claims about a single population parameter using a sample using a one-sample hypothesis test. A ____________________ **hypothesis test** compares two parameters from two populations by collecting sample statistics from each population.

There are 2 basic design for working with 2 samples:

Section 8.2

1. Independent samples: the observations in one sample do not influence the observations in the other sample.

   Ex: --Give one medication to each of 2 unrelated groups

   --Comparing Test-3 grade points between 2 separate STA-2023 sections.

Section 8.3

2. Paired Samples: each observation in one sample can be paired with an observation in the other sample, such as before and after measurements on the same individual or on related individuals.

   Ex: --Weight before starting Gym and Weight after 2 months of starting Gym

   --A treatment effect before and after administering it to a specific group of people.

   d = before − after

Example 1: Classify each pair of random samples as independent or dependent.

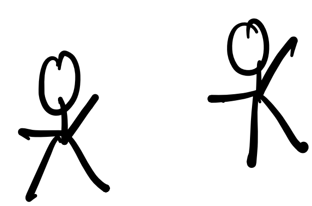
a) Sample 1: Triglyceride levels of 70 patients
   Sample 2: Triglyceride levels of the same 70 patients after using a triglyceride-lowering drug for 6 months.

   Dependent

b) Sample 1: Systolic blood pressures of 30 adult women
   Sample 2: Systolic blood pressures of 30 adult men

   Independent

c) Sample 1: One sibling from a set of twins
   Sample 2: The other sibling from a set of twins



indep.

Group 1	**Group 2**
Population mean μ_1 Sample mean $\overline{X}_1$ Sample size n_1 Sample standard deviation S_1	Population mean μ_2 Sample mean $\overline{X}_2$ Sample size n_2 Sample standard deviation S_2

~~$\bar{x}_1$ $\bar{x}_2$~~

~~$\mu_1 = \mu_2 = 10$~~

Notes:

General Steps in Hypothesis Testing:

Step 1: Write the statistical hypothesis and identify the claim.

The three different hypothesis that can be written when conducting a two-sample hypothesis test for means μ_1 and μ_2 of two populations:

P-value method

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_a: \mu_1 \neq \mu_2 \end{cases} \qquad \begin{cases} H_0: \mu_1 \leq \mu_2 \\ H_a: \mu_1 > \mu_2 \end{cases} \qquad \begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_a: \mu_1 < \mu_2 \end{cases}$$

2-tail test   Right tail   left tail

Step 2: Determine level of significance, type of statistical test (left-tailed, right-tailed, or two-tailed), and the distribution of comparison,

a. Level of significance is represented by α and the type of test will be determined by the inequality symbol in the **alternative** hypothesis as we saw in previous sections.

b. If you are testing a claim about the difference between two mean, you do NOT know σ for both populations, then you will use the ________________________ as the distribution of comparison

Step 3: Perform the hypothesis test.

Step 4: Interpret the results in the context of the claim.

<u>The Test Statistic</u>: Since a value of σ is NOT known for each population, we use the $t - distribution$.

$$\boxed{t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

use calculator

<u>The P-Value</u>: This is the probability of getting a test statistic as least as extreme as the one representing the sample data, that is $P - value = P(t \text{ is in an interval}^{**})$ ***Get from the calculator.**

**here interval means to the right of critical value (Right tailed test) or to the left of critical value (left tailed test) or beyond the critical values (2 tailed test)

Decision: Interpret the P-value.

$If\ P - Value \leq \alpha, reject\ the\ null\ hyp.\ and\ accept\ the\ alternate.$

If P-value > α, Failed to reject H_0

Conclusion:

- Reject the null: There is enough evidence to say that the alternate hypothesis is true (the means of the 2 samples are not the same)
- Fail to reject the null: There is not enough evidence to say that the means are different in some way.

Calculator Steps:

1. Go to **STAT** and highlight the **Tests**
2. Select [4] **2-SampleTTest** and press Enter
3. Choose either one of the following based on the situation given in question:
   - ✓ If the summary statistics are given, select **Stats** as the **Inpt** option and enter $\bar{x}_1$, s_1, μ_1, $\bar{x}_2$, s_2, n_2.
   - ✓ if the raw data are given, select **Data** as the **Inpt** option and enter the location of the data as the **List1** and **List2** options.
4. Select the form of the alternate hypothesis.
5. Select **No** for the **Pooled** option
6. Highlight **Calculate** and press **ENTER**

Example #1: Low-fat diets or low-carb diets? Which diet is more effective for weight loss? A sample of 77 subjects went on a low-carbohydrate diet for 6 months. At the end of that time, the sample mean weight loss was 4.7 kilograms with a sample standard deviation of 7.16 kilograms. A second sample of 79 subjects went on a low-fat diet for 6 months. Their sample mean weight loss was 2.6 kilograms with a sample standard deviation of 5.90 kilograms. Can you conclude that the mean weight loss differs between the 2 diets? Use a 1% level of significance.

low-carb (μ_1)	low fat (μ_2)
$n_1 = 77$	$n_2 = 79$
$\bar{x}_1 = 4.7$, $s_1 = 7.16$	$\bar{x}_2 = 2.6$, $s_2 = 5.9$

$\boxed{\alpha = 0.01}$

$H_1: \mu_1 \neq \mu_2$ (claim)

$H_0: \mu_1 = \mu_2$

$t = 1.9964$

$P = 0.0477 > \alpha$, failed to reject H_0

At 1% level of sig., we don't have enough evidence to reject H_0; mean weight loss may not be different in these two group.

$\min\{5,7\} = 5$

One sample case, $df = n-1$

* If independent sample $\Rightarrow df = \min\{n_1-1, n_2-1\}$

* If dependent sample $\Rightarrow df = n_1+n_2-2$

* Pop variances not equal $\Rightarrow \sigma_1^2 \neq \sigma_2^2$

↳ pooled = "NO"

* Pop variances are equal $\Rightarrow \sigma_1^2 = \sigma_2^2$

↳ pooled = "yes"

Example 3: The results of a state mathematics test for random samples of students taught by two different teachers at the same school are reviewed. Teacher 1's 8 students had a mean score of 473 with a standard deviation of 39.7. Teacher 2's 18 students had a mean score of 459 with a standard deviation of 24.5. Can you conclude that Teacher 1's scores are significantly higher? Use $\alpha = 0.10$. Assume the populations are normally distributed and the population variances are not equal.

T1 (μ_1)	T2 (μ_2)
$n_1 = 8$	$n_2 = 18$
$\bar{x}_1 = 473$	$\bar{x}_2 = 459$
$s_1 = 39.7$	$s_2 = 24.5$

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$ (claim)

$\boxed{\alpha = 0.10}$

STAT → TESTS → [4]

pooled = No

$t = 0.9224$

$P = 0.1896 > \alpha$, failed to reject Ho

At 10% level of sig., we don't have enough evidence to reject Ho, meaning T1 students score might be lower than T2 student's score.

$df = \min\{8-1, 18-1\}$
$= \min\{7, 17\} = 7$

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STA 2023 Section 8.3 Testing the Difference Between Means (Dependent Samples) NOTES

A

T-test

Learning Outcomes:

1) Perform a t-test to test the mean of the difference between population with dependent samples.

d → set of difference

Notes:

Recall: Two samples are **dependent** if each member of one sample corresponds to a member of the other sample.

With dependent samples, every data value in one sample is matched to a data value in the other sample. Therefore, the hypothesis test is based on the difference between each matched pair of data values in the samples.

To perform the hypothesis test, the difference, d, between each matched data pair in the samples is found ($d = x_1 - x_2$). = difference = before − After

Then the sample mean of the differences, denoted $\bar{d}$,and the sample standard deviation of the mean of the differences, denoted s_d, are both calculated. $\{d, \bar{d}, s_d\}$

$\bar{d}$ and s_d are the test statistics used to perform the hypothesis test. The claim we will be testing will be about the *population mean of the differences*, denoted μ_d. = pop mean difference

The three different hypothesis that you could test for dependent samples are shown below: $= \mu_1 - \mu_2$ = before mean − after mean

Choose the correct hypothesis by identifying the claim in the problem statement.

1) Statistical Hypothesis: $\boxed{\begin{cases} H_0: \mu_D \geq 0 \\ H_a: \mu_D < 0 \end{cases}}$ left tail

If the claim is null hypothesis a variation of the statement "Sample 1 data values are _____________ to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are _____________ sample 2's" will be in the problem statement.

2) Statistical Hypothesis: $\boxed{\begin{cases} H_0: \mu_D \leq 0 \\ H_a: \mu_D > 0 \end{cases}}$ Right tail

If the claim is null hypothesis a variation of the statement "Sample 1 data values are ________________ to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are ____________ sample 2's" will be in the problem statement.

$\boxed{df = n - 1}$

3) Statistical Hypothesis: $\boxed{\begin{cases} H_0: \mu_D = 0 \\ H_a: \mu_D \neq 0 \end{cases}}$ 2-tail test

If the claim is null hypothesis a variation of the statement "Sample 1 data values are ____________ to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are _____________ to sample 2's" will be in the problem statement.

P-Value Method for Hypothesis Testing for the Difference between Means, dependent samples (t-test for the mean difference):

1) Verify that you have random samples and normally distributed populations.

2) Find the P-value using the TI-83/84 calculator using the same steps outlined in rejection regions method instructions.

3) Make the decision to reject or fail to reject the null hypothesis
   To make the decision, compare the p-value to the level of significance α
   i. If p-value $\leq$ α, then you reject the null hypothesis
   ii. If p-value $>$ α, then you fail to reject the null hypothesis

$\mu_2 > \mu_1$

$$\mu_D = \mu_1 - \mu_2$$

Example 1: A shoe manufacturer claims that the athletes can increase their vertical jump using their shoes. The vertical jumps of 8 randomly selected athletes are measured. After the athletes have used the shoes for 8 months, their vertical jumps are measured again. At α = 0.10, is there enough evidence to support the manufacturer's claim? Assume the vertical jump heights are normally distributed. Use old heights as sample 1.

Athlete	1	2	3	4	5	6	7	8
Heights (old) before	24	22	25	28	35	32	30	27
Height (new) after	26	25	25	29	33	34	35	30

$H_0: \mu_D \geq 0$

$H_1: \mu_D < 0$ (claim)

left tail

$t = -2.33$

$p = 0.0262 < \alpha$

$df = 7$

Reject H_0

At 10 % level of sig., we have enough evidene to support claim of shoe manufacturer

Example 2: A dietitian wishes to see if a person's cholesterol level will lower if the diet is supplemented by a certain mineral. Six randomly selected subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. Can it be concluded that the cholesterol level has been lowered at $\alpha = 0.10$? Assume the variable is approximately normally distributed. Use before data as sample 1.

Subject	1	2	3	4	5	6
Before	210	235	208	190	172	244
After	190	170	210	188	173	228

μ_1 = pop mean before taking mineral

μ_2 = pop mean after taking "

$\mu_D = \mu_1 - \mu_2$

$H_0: \mu_D \le 0$

$H_1: \mu_D > 0$

$t = 1.611$

$P = 0.084 < \alpha$

$df = 5$

Reject Ho

At 10% level of sig., we have enough evidence to support the dietitician claim.

Indep. Sample $\to$ 2-Sample T-test

Dependent sample $\to$ T-test

pooled = yes	pooled = No
$\sigma_1^2 = \sigma_2^2$	$\sigma_1^2 \neq \sigma_2^2$
$df = n_1 + n_2 - 2$	$df = \min\{n_1 - 1, n_2 - 1\}$

$\boxed{A}$

85%

\$ 100

$\boxed{B}$

87%

\$ 250

Success/ Failure

STA 2023 Section 8.4: Testing the Difference Between Proportions

P = pop prop

In this section, you will learn how to use a *z*-test to test the difference between two population proportions . and . using a sample proportion from each population. If a claim is about two population parameters . and ., then some possible pairs of null and alternative hypotheses are

$$\begin{cases} H_0\colon p_1 = p_2 \\ H_a\colon p_1 \neq p_2 \end{cases}, \quad \begin{cases} H_0\colon p_1 \leq p_2 \\ H_a\colon p_1 > p_2 \end{cases}, \quad \text{and} \quad \begin{cases} H_0\colon p_1 \geq p_2 \\ H_a\colon p_1 < p_2 \end{cases}.$$

2-tail   Right   left

For example, *Are older, more experienced workers more likely to use a computer at work than younger workers?* The General Social Survey took a poll to address this question. They asked 350 employed people aged 25–40 whether they used a computer at work, and 259 said they did. They also asked the same question of 500 employed people aged 41–65, and 384 of them said that they used a computer at work.

Notation:

Population 1	**Population 2**
Population Proportion of the category of interest is = P_1 Sample proportion of the category of interest = $\widehat{P_1}$ Number of individuals in the category of interest = X_1 Sample size = n_1 $\hat{p}_1 = \frac{x_1}{n_1}$	Population Proportion of the category of interest is = P_2 Sample proportion of the category of interest = $\widehat{P_2}$ Number of individuals in the category of interest = X_2 Sample size = n_2 $\hat{p}_2 = \frac{x_2}{n_2}$

Chapter 8 Hypothesis Testing with Two Samples

Assumptions for Performing a Hypothesis Test for the Difference Between Two Proportions

1. There are two simple random samples that are independent of one another.
2. Each population is at least 20 times as large as the sample drawn from it.
3. The individuals in each sample are divided into two categories.
4. Both samples contain at least 10 individuals in each category.

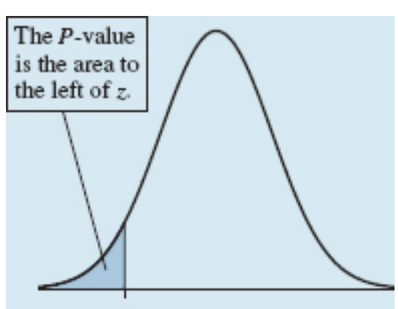
Performing a Hypothesis Test for the Difference Between Two Proportions Using the *P*-Value Method

Check to be sure the assumptions are satisfied. If they are, then proceed with the following steps.

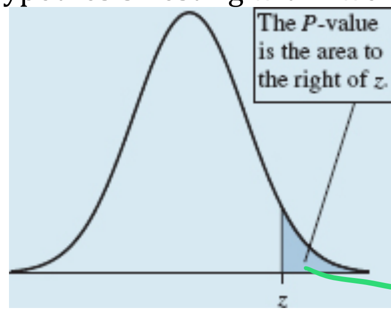
- **Step 1:** State the null and alternate hypotheses. The null hypothesis will have the form H_0: $p_1 = p_2$. The alternate hypothesis will be $p_1 < p_2$, $p_1 > p_2$, or $p_1 \neq p_2$
- **Step 2:** If making a decision, choose a significance level α.
- **Step 3:** Compute the test statistic $\cancel{z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}}$

  where $\hat{p}$ is the pooled proportion: $\cancel{\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}}$
- **Step 4:** Compute the *P*-value. The *P*-value is an area under the normal curve. The *P*-value depends on the alternate hypothesis as follows:

  The *P*-value is the area to the left of z.

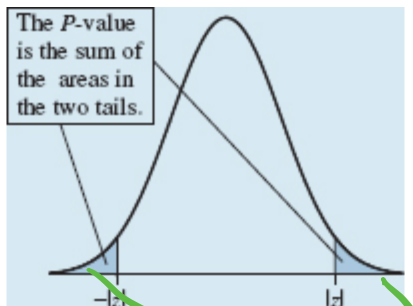


Left-tailed: H_1: $p_1 < p_2$



Right tail
normalcdf( )

Right-tailed: H_1: $p_1 > p_2$



2normalcdf( )

Two-tailed: H_1: $p_1 \neq p_2$

- **Step 5:** Interpret the *P*-value. If making a decision, reject H_0 if the *P*-value is less than or equal to the significance level α.
- **Step 6:** State a conclusion.

Example: Are younger drivers more likely to have accidents in their driveways? Traffic engineers tabulated types of car accidents by drivers of various ages. Out of a total of 82,486 accidents involving drivers aged 15–24 years, 4243 of them, or 5.1%, occurred in a driveway. Out of a total of 219,170 accidents involving drivers aged 25–64 years, 10,701 of them, or 4.9%, occurred in a driveway. Can you conclude that accidents involving drivers aged 15–24 are more likely to occur in driveways than accidents involving drivers aged 25–64? Use the α = 0.05 significance level.

$\hat{p}_1$ $\hat{p}_2$

Solution:

younger driver (p_1)

$x_1 = 4243$

$n_1 = 82486$

adult driver (p_2)

$x_2 = 10701$

$n_2 = 219170$

$\alpha = 0.05$

$H_0: p_1 \leq p_2$

$H_1: p_1 > p_2$ (claim)

$z = 2.9489$

$P = 0.0016 < \alpha$

At 5% level of sig., younger driver are more likely to have acident in their driveway.

- **Step 1.** Press **STAT** and highlight the **TESTS** menu.
- **Step 2.** Select **2-PropZTest** and press **ENTER** (Figure A). The **2-PropZTest** menu appears.

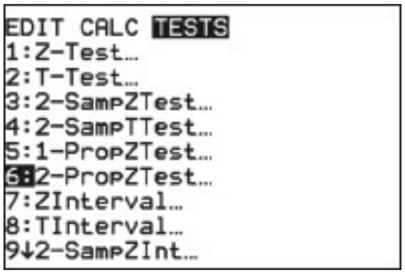


Figure A

- **Step 3.** Enter the values of x_1, n_1, x_2, and n_2
- **Step 4.** Select the form of the alternate hypothesis.

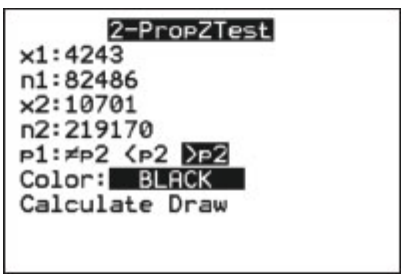


Figure B

- **Step 5.** Highlight **Calculate** and press **ENTER** (Figure C).

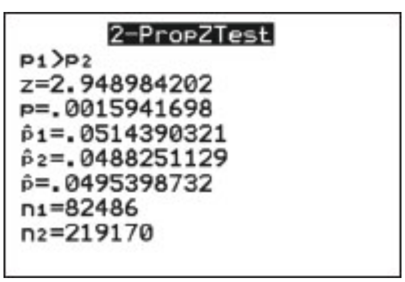


Figure C

Test 3

6.3, Chapt 7

chapt 8