Problem 3: Particle Kinematics I

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)$ m/s², where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18$ m starting from rest. Neglect the size of the car.

$$v = \int a_t \, dt = \int 0.5e^t \, dt$$

$$v = 0.5e^t + C_1$$

at $t = 0$, $v = 0$ $\Rightarrow$ $C_1 = -0.5$ m/s

$$v = 0.5e^t - 0.5 \quad \text{------ (1)}$$

$$s = \int v \, dt = \int (0.5e^t - 0.5) \, dt$$

$$s = 0.5e^t - 0.5t + C_2$$

at $t = 0$, $s = 0$ $\Rightarrow$ $0 = 0.5 + C_2$ $\Rightarrow$ $C_2 = -0.5$ m

$$s = 0.5e^t - 0.5t - 0.5$$

at $s = 18$ m $\Rightarrow$ $18 = 0.5e^t - 0.5t - 0.5$

$$\therefore t = 3.705 \text{ s}$$

From (1) $\Rightarrow$ $v = 0.5e^{3.705} - 0.5 = \underline{\underline{19.83 \text{ m/s}}}$ at $s = 18$ m

$$a_N = \frac{v^2}{\rho} = \frac{(19.83)^2}{30} = 13.11 \text{ m/s}$$

$$a_T = \frac{dv}{dt} = \frac{d}{dt}(0.5e^t - 0.5) = 0.5e^t = 0.5e^{3.705} = 20.33 \text{ m/s}^2$$

$$a = \sqrt{(13.11)^2 + (20.33)^2} = \underline{\underline{24.2 \text{ m/s}}} \quad \text{at } s = 18 \text{ m}$$