Chapter 8 Hypothesis Testing with Two Samples

STA 2023 Section 8.2 Testing the Difference Between Means (Independent Samples, σ_1 and σ_2 Unknown)
NOTES

2- Sample T-test

Learning Outcomes:

1) Determine whether two samples are independent or dependent
2) Perform a t-test for the difference between two means μ_1 and μ_2 with independent samples with σ_1 and σ_2 unknown.

Notes:

In chapter 7, we were testing claims about a single population parameter using a sample using a one-sample hypothesis test. A two sample **hypothesis test** compares two parameters from two populations by collecting sample statistics from each population.

There are 2 basic design for working with 2 samples:

1. Independent samples: the observations in one sample do not influence the observations in the other sample.

   Ex: --Give one medication to each of 2 unrelated groups

   --Comparing Test-3 grade points between 2 separate STA-2023 sections.

Dependent

2. Paired Samples: each observation in one sample can be paired with an observation in the other sample, such as before and after measurements on the same individual or on related individuals.

   Ex: --Weight before starting Gym and Weight after 2 months of starting Gym

   --A treatment effect before and after administering it to a specific group of people.

Example 1: Classify each pair of random samples as independent or dependent.

a) Sample 1: Triglyceride levels of 70 patients
   Sample 2: Triglyceride levels of the same 70 patients after using a triglyceride-lowering drug for 6 months.

   Dependent

b) Sample 1: Systolic blood pressures of 30 adult women
   Sample 2: Systolic blood pressures of 30 adult men

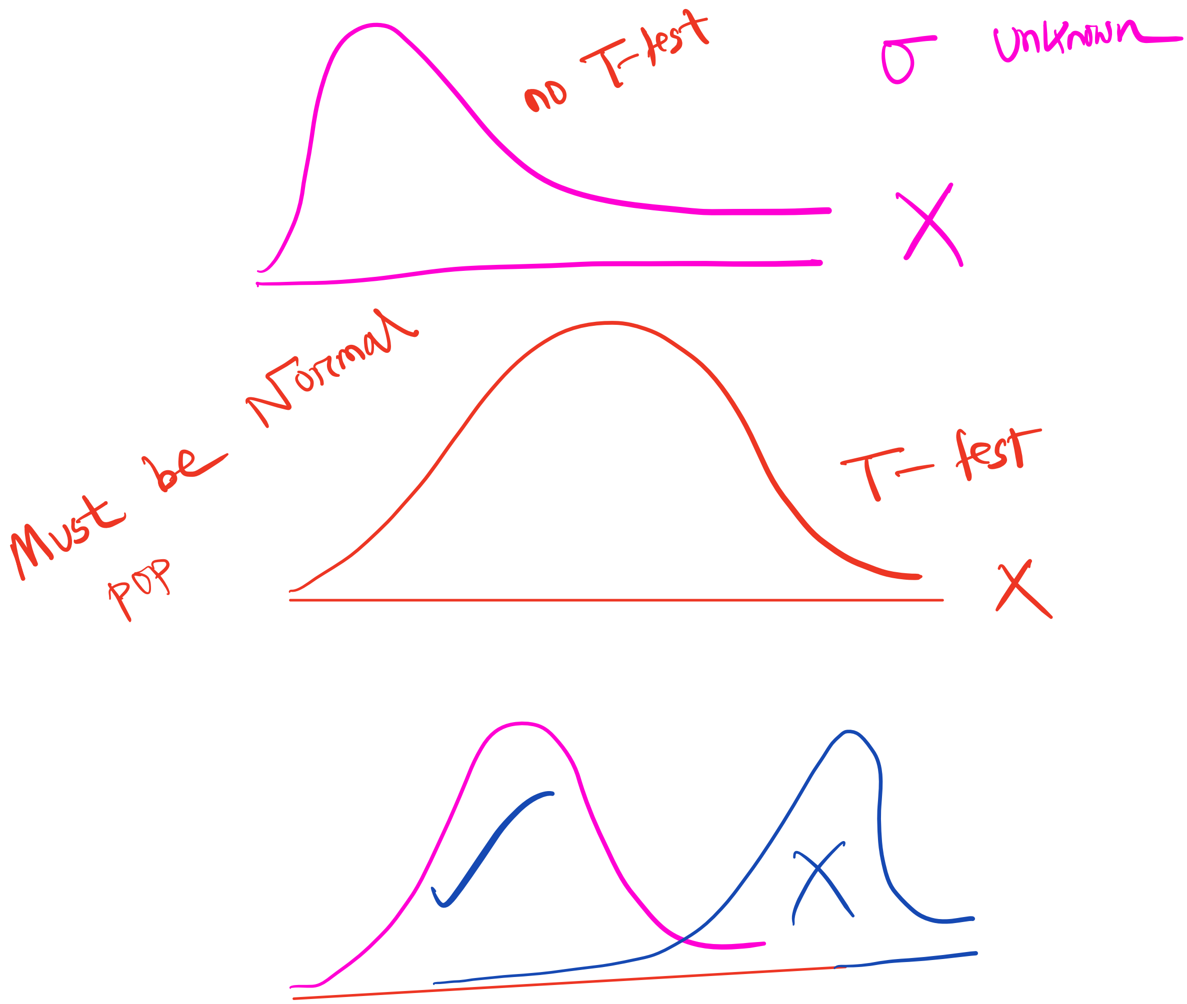
   indep.

c) Sample 1: One sibling from a set of twins
   Sample 2: The other sibling from a set of twins

   indep.

Indep Sample → 2 Sample T-test

Paired sample → T-test


no T-test
σ unknown
X
Must be Normal
pop
T-test
X
X

Chapter 8 Hypothesis Testing with Two Samples

Group 1	**Group 2**
Population mean μ_1 Sample mean $\overline{X_1}$ Sample size n_1 Sample standard deviation S_1	Population mean μ_2 Sample mean $\overline{X_2}$ Sample size n_2 Sample standard deviation S_2

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

Notes:

General Steps in Hypothesis Testing:

Step 1: Write the statistical hypothesis and identify the claim.

The three different hypothesis that can be written when conducting a two-sample hypothesis test for means μ_1 and μ_2 of two populations:

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_a: \mu_1 \neq \mu_2 \end{cases} \qquad \begin{cases} H_0: \mu_1 \leq \mu_2 \\ H_a: \mu_1 > \mu_2 \end{cases} \qquad \begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_a: \mu_1 < \mu_2 \end{cases}$$

2-tail test Right tail Left tail

Step 2: Determine level of significance, type of statistical test (left-tailed, right-tailed, or two-tailed), and the distribution of comparison,

a. Level of significance is represented by α and the type of test will be determined by the inequality symbol in the **alternative** hypothesis as we saw in previous sections.

b. If you are testing a claim about the difference between two mean, you do NOT know σ for both populations, then you will use the ________________________ as the distribution of comparison

Step 3: Perform the hypothesis test.

Step 4: Interpret the results in the context of the claim.

<u>The Test Statistic</u>: Since a value of σ is NOT known for each population, we use the $t - distribution$.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Calculator

<u>The P-Value</u>: This is the probability of getting a test statistic as least as extreme as the one representing the sample data, that is $P - value = P(t\ is\ in\ an\ interval^{**})$ ***Get from the calculator**.

**here interval means to the right of critical value (Right tailed test) or to the left of critical value (left tailed test) or beyond the critical values (2 tailed test)

We don't have enough evidence to support Ho

Decision: Interpret the P-value.

$If\ P - Value \leq \alpha, reject\ the\ null\ hyp.\ and\ accept\ the\ alternate.$

If P-value > α, Failed to reject H_0

We don't have enough evidence to reject Ho

Conclusion:

- Reject the null: There is enough evidence to say that the alternate hypothesis is true (the means of the 2 samples are not the same)
- Fail to reject the null: There is not enough evidence to say that the means are different in some way.

Calculator Steps:

1. Go to **STAT** and highlight the **Tests**
2. Select [4] **2-SampleTTest** and press Enter
3. Choose either one of the following based on the situation given in question:
   - ✓ If the summary statistics are given, select **Stats** as the **Inpt** option and enter $\bar{x}_1$, s_1, μ_1, $\bar{x}_2$, s_2, n_2.
   - ✓ if the raw data are given, select **Data** as the **Inpt** option and enter the location of the data as the **List1** and **List2** options.
4. Select the form of the alternate hypothesis.
5. Select **No** for the **Pooled** option
6. Highlight **Calculate** and press **ENTER**

Example #2: Low-fat diets or low-carb diets? Which diet is more effective for weight loss? A sample of 77 subjects went on a low-carbohydrate diet for 6 months. At the end of that time, the sample mean weight loss was 4.7 kilograms with a sample standard deviation of 7.16 kilograms. A second sample of 79 subjects went on a low-fat diet for 6 months. Their sample mean weight loss was 2.6 kilograms with a sample standard deviation of 5.90 kilograms. Can you conclude that the mean weight loss $\boxed{\text{differs}}$ between the 2 diets? Use a 1% level of significance.

low carb (μ_1)

$n_1 = 77$

$\bar{x}_1 = 4.7$

$s_1 = 7.16$

low-fat (μ_2)

$n_2 = 79$

$\bar{x}_2 = 2.6$

$s_2 = 5.9$

$\alpha = 0.01$

Indep. Sample.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

2-tail-test

$t = 1.9965$

$P = 0.0477 > \alpha$

$df = \min\{n_1 - 1, n_2 - 1\}$

$= \min\{76, 78\} = 76$ ✓

Failed to reject Ho

What if $\alpha = 0.10$,

$P = 0.0477 < \alpha \rightarrow$ Reject H_0

At 1% level of sig., we don't have enough evidence to reject H_0, both diets might have equal impact on the weight loss program.

Example #3: In a survey of adults with diabetes, the average body mass index (BMI) in a sample of 1924 women was 31.1 with a standard deviation of 0.2. The average BMI in a sample of 1559 men was 30.4 with a standard deviation of 0.6. Test the claim that the average BMI of women with diabetes is larger than the average BMI for men with diabetes. Use a level of significance of $\alpha = 1\%$.

Women

$n_1 = 1924$

$\bar{x}_1 = 31.1$

$s_1 = 0.2$

men

$n_2 = 1559$

$\bar{x}_2 = 30.4$

$s_2 = 0.6$

$\alpha = 0.01$

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$

Right tail

$t = 44.1215$

$P = 0 < \alpha$

Decision Reject Ho

At 1% level of sig., we don't have enough evidence to support Ho.

$\therefore$ Avg. BMI in women > Avg. BMI men

* If variance $\boxed{\text{are equal}}$ → pooled "yes"
(unlikely)

* If " $\boxed{\text{are not}}$ equal → pooled "No"

Example 4: A manufacturer claims that the mean driving cost per mile of its sedans is less than that of its leading competitor. You conduct a study using 30 randomly selected sedans from the manufacturer and 32 from the leading competitor. The results are shown below. At *α* = 0.05, can you support the manufacturer's claim? Assume the populations are normally distributed and the variances are equal. *(Adapted from American Automobile Association)*

μ_1 **Manufacturer**	**Competitor** μ_2
$\bar{x}_1 = \$0.48/\text{mi}$	$\bar{x}_2 = \$0.51/\text{mi}$
$s_1 = \$0.05/\text{mi}$	$s_2 = \$0.07/\text{mi}$
$n_1 = 30$	$n_2 = 32$

$H_0: \mu_1 \geq \mu_2$

$H_1: \mu_1 < \mu_2$ (Claim)

STAT → TESTS → [4] 2-SampleT-test → pooled=yes

$t = -1.9303$

$P = 0.0291 < \alpha$

Reject Ho

At 5% level of sig, we have enough evidence to support the claim that the mean driving cost per mile of its sedans is less than that of its leading competitor.

Example 5: The results of a state mathematics test for random samples of students taught by two different teachers at the same school are reviewed. Teacher 1's 8 students had a mean score of 473 with a standard deviation of 39.7. Teacher 2's 18 students had a mean score of 459 with a standard deviation of 24.5. Can you conclude that Teacher 1's scores are significantly higher? Use α = 0.10. Assume the populations are normally distributed and the population variances are not equal.

Teacher-1 (μ_1)

$\bar{x}_1 = 473$

$s_1 = 39.7$

$n_1 = 8$

Teacher-2 (μ_2)

$\bar{x}_2 = 459$

$s_2 = 24.5$

$n_2 = 18$

$\alpha = 0.10$

$H_0: \mu_1 \le \mu_2$

$H_1: \mu_1 > \mu_2$ (Claim), Right tail

STAT → TEST → [4] 2 Sample T-test → ← Stats

$t = 0.9224$

$p = 0.1896 > \alpha$

Failed to Reject Ho

At 10% level of sig., we don't have enough evidence to reject Ho, that means teacher-1 scores might be less or equal to teach-2 scores.

STA 2023 Section 8.3 Testing the Difference Between Means (Dependent Samples) NOTES

Learning Outcomes:

1) Perform a t-test to test the mean of the difference between population with dependent samples.

Notes:

Recall: Two samples are **dependent** if each member of one sample corresponds to a member of the other sample.

With dependent samples, every data value in one sample is matched to a data value in the other sample. Therefore, the hypothesis test is based on the difference between each matched pair of data values in the samples.

To perform the hypothesis test, the difference, d, between each matched data pair in the samples is found ($d = x_1 - x_2$). = before − after

Then the sample mean of the differences, denoted $\bar{d}$,and the sample standard deviation of the mean of the differences, denoted s_d, are both calculated.

$\bar{d}$ and s_d are the test statistics used to perform the hypothesis test. The claim we will be testing will be about the *population mean of the differences*, denoted μ_d. $= \mu_1 - \mu_2$

The three different hypothesis that you could test for dependent samples are shown below:

Choose the correct hypothesis by identifying the claim in the problem statement.

1) Statistical Hypothesis: $\begin{cases} H_0: \mu_D \geq 0 \\ H_a: \mu_D < 0 \end{cases}$ $\mu_d = \mu_1 - \mu_2$

If the claim is null hypothesis a variation of the statement "Sample 1 data values are bigger or equal to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are smaller sample 2's" will be in the problem statement.

2) Statistical Hypothesis: $\begin{cases} H_0: \mu_D \leq 0 \\ H_a: \mu_D > 0 \end{cases}$ $\mu_d = \mu_1 - \mu_2$

If the claim is null hypothesis a variation of the statement "Sample 1 data values are smaller or equal to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are bigger than sample 2's" will be in the problem statement.

3) Statistical Hypothesis: $\begin{cases} H_0: \mu_D = 0 \\ H_a: \mu_D \neq 0 \end{cases}$, 2-tail - test

If the claim is null hypothesis a variation of the statement "Sample 1 data values are equal to sample 2's" will be in the problem statement.

If the claim is alternative hypothesis a variation of the statement "Sample 1 data values are different to sample 2's" will be in the problem statement.

P-Value Method for Hypothesis Testing for the Difference between Means, dependent samples (t-test for the mean difference):

1) Verify that you have random samples and normally distributed populations.

2) Find the P-value using the TI-83/84 calculator using the same steps outlined in rejection regions method instructions.

3) Make the decision to reject or fail to reject the null hypothesis
   To make the decision, compare the ____________ to the level of significance α
   i. If p-value ________ α, then you reject the null hypothesis
   ii. If p-value ________ α, then you fail to reject the null hypothesis

P-value $\leq \alpha$, Reject Ho

$P > \alpha$, Failed to reject Ho

Example 1: A shoe manufacturer claims that the athletes can increase their vertical jump using their shoes. The vertical jumps of 8 randomly selected athletes are measured. After the athletes have used the shoes for 8 months, their vertical jumps are measured again. At $\alpha = 0.10$, is there enough evidence to support the manufacturer's claim? Assume the vertical jump heights are normally distributed. Use old heights as sample 1.

Athlete	1	2	3	4	5	6	7	8
Heights (old) Before, μ_1	24	22	25	28	35	32	30	27
Height (new) After, μ_2	26	25	25	29	33	34	35	30
d	-2	-3	0	-1	2	-2	-5	-3

d = before − after

μ_d = before μ_1 − after μ_2

$H_0: \mu_d \geq 0$

$H_1: \mu_d < 0$ (Claim)

$\alpha = 0.10$

$t = -2.33333$

$p = 0.0262 < \alpha$

Reject H_0

If $\alpha = 0.01$

At 10% level of sig., we have enough evidence to support the claim, new shoes helping to jump more of vertical height.

$\mu_d = \mu_1 - \mu_2$

Example 2: A dietitian wishes to see if a person's cholesterol level will lower if the diet is supplemented by a certain mineral. Six randomly selected subjects were pretested, and then they took the mineral supplement for a 6-week period. The results are shown in the table. Can it be concluded that the cholesterol level has been lowered at $\alpha = 0.10$? Assume the variable is approximately normally distributed. Use before data as sample 1.

	Subject	1	2	3	4	5	6
μ_1	**Before**	210	235	208	190	172	244
μ_2	**After**	190	170	210	188	173	228

$H_0: \mu_d \le 0$

$H_1: \mu_d > 0$ (claim)

$\alpha = 0.10$

$$\boxed{\begin{aligned} t &= 1.6079 \\ P &= 0.0844 < \alpha \end{aligned}}$$

Reject H_0

At 10% level of sig., we have enough evidence to support the claim.

Example 3: A golf instruction and club fitting company claims that people who play golf can improve (decrease) their average golf scores by taking lessons from one of their coaches. The average golf scores of eight randomly selected people who play golf are determined before and after taking lessons from one of the company's coaches. The sample statistics for the data were calculated to be $\bar{d} = -1.625$ and $s_d = 4.627$. At $\alpha = 0.10$, is there enough evidence to support the company's claim? Assume scores are normally distributed and use before scores as sample 1.