Chapter 8 Hypothesis Testing with Two Samples

STA 2023 Section 8.2 Testing the Difference Between Means (Independent Samples, $\sigma_1$ and $\sigma_2$ Unknown) NOTES

**Learning Outcomes:**

1) Determine whether two samples are independent or dependent
2) Perform a $t$-test for the difference between two means $\mu_1$ and $\mu_2$ with independent samples with $\sigma_1$ and $\sigma_2$ unknown.

**Notes:**

In chapter 7, we were testing claims about a single population parameter using a sample using a one-sample hypothesis test. A ____________________ **hypothesis test** compares two parameters from two populations by collecting sample statistics from each population.

There are 2 basic design for working with 2 samples:

Section 8.2

1. Independent samples: the observations in one sample do not influence the observations in the other sample.

   Ex: --Give one medication to each of 2 unrelated groups

   --Comparing Test-3 grade points between 2 separate STA-2023 sections.

Section 8.3

2. Paired Samples: each observation in one sample can be paired with an observation in the other sample, such as before and after measurements on the same individual or on related individuals.

   Ex: --Weight before starting Gym and Weight after 2 months of starting Gym

   $d =$ before $-$ after

   --A treatment effect before and after administering it to a specific group of people.

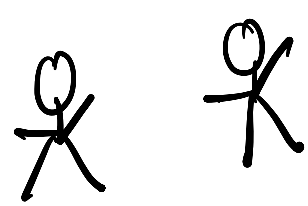
**Example 1:** Classify each pair of random samples as independent or dependent.

a) Sample 1: Triglyceride levels of 70 patients
   Sample 2: Triglyceride levels of the same 70 patients after using a triglyceride-lowering drug for 6 months.

   Dependent

b) Sample 1: Systolic blood pressures of 30 adult women
   Sample 2: Systolic blood pressures of 30 adult men

   Independent



c) Sample 1: One sibling from a set of twins
   Sample 2: The other sibling from a set of twins

   indep.

| **Group 1** | **Group 2** |
|---|---|
| Population mean $\mu_1$<br>Sample mean $\overline{X}_1$<br>Sample size $n_1$<br>Sample standard deviation $S_1$ | Population mean $\mu_2$<br>Sample mean $\overline{X}_2$<br>Sample size $n_2$<br>Sample standard deviation $S_2$ |

~~$\bar{x}_1$ $\bar{x}_2$~~

~~$\mu_1 = \mu_2 = 10$~~

**Notes:**

General Steps in Hypothesis Testing:

Step 1: Write the statistical hypothesis and identify the claim.

The three different hypothesis that can be written when conducting a two-sample hypothesis test for means $\mu_1$ and $\mu_2$ of two populations:

P-value method

$$\begin{cases} H_0\colon \mu_1 = \mu_2 \\ H_a\colon \mu_1 \neq \mu_2 \end{cases} \qquad \begin{cases} H_0\colon \mu_1 \leq \mu_2 \\ H_a\colon \mu_1 > \mu_2 \end{cases} \qquad \begin{cases} H_0\colon \mu_1 \geq \mu_2 \\ H_a\colon \mu_1 < \mu_2 \end{cases}$$

2-tail test     Right tail     left tail

Step 2: Determine level of significance, type of statistical test (left-tailed, right-tailed, or two-tailed), and the distribution of comparison,

a. Level of significance is represented by $\alpha$ and the type of test will be determined by the inequality symbol in the **alternative** hypothesis as we saw in previous sections.

b. If you are testing a claim about the difference between two mean, you do NOT know $\sigma$ for both populations, then you will use the ______________________ as the distribution of comparison

Step 3: Perform the hypothesis test.

Step 4: Interpret the results in the context of the claim.

<u>The Test Statistic</u>: Since a value of $\sigma$ is NOT known for each population, we use the $t - distribution$.

$$\boxed{\cancel{t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}}}}$$

use calculator

<u>The P-Value</u>: This is the probability of getting a test statistic as least as extreme as the one representing the sample data, that is $P - value = P(t \text{ is in an interval}^{**})$ ***Get from the calculator**.

**here interval means to the right of critical value (Right tailed test) or to the left of critical value (left tailed test) or beyond the critical values (2 tailed test)

Decision: Interpret the P-value.

$If\ P - Value \leq \alpha, reject\ the\ null\ hyp. and\ accept\ the\ alternate.$

If P-value > α, Failed to reject $H_0$

Conclusion:

- Reject the null: There is enough evidence to say that the alternate hypothesis is true (the means of the 2 samples are not the same)
- Fail to reject the null: There is not enough evidence to say that the means are different in some way.

**Calculator Steps:**

1. Go to **STAT** and highlight the **Tests**
2. Select [4] **2-SampleTTest** and press Enter
3. Choose either one of the following based on the situation given in question:
   - ✓ If the summary statistics are given, select **Stats** as the **Inpt** option and enter $\bar{x}_1$, $s_1$, $\mu_1$, $\bar{x}_2$, $s_2$, $n_2$.
   - ✓ if the raw data are given, select **Data** as the **Inpt** option and enter the location of the data as the **List1** and **List2** options.
4. Select the form of the alternate hypothesis.
5. Select **No** for the **Pooled** option
6. Highlight **Calculate** and press **ENTER**

Example #1: Low-fat diets or low-carb diets? Which diet is more effective for weight loss? A sample of 77 subjects went on a low-carbohydrate diet for 6 months. At the end of that time, the sample mean weight loss was 4.7 kilograms with a sample standard deviation of 7.16 kilograms. A second sample of 79 subjects went on a low-fat diet for 6 months. Their sample mean weight loss was 2.6 kilograms with a sample standard deviation of 5.90 kilograms. Can you conclude that the mean weight loss differs between the 2 diets? Use a 1% level of significance.

| low-carb ($\mu_1$) | low fat ($\mu_2$) |
|---|---|
| $n_1 = 77$<br>$\bar{x}_1 = 4.7$, $s_1 = 7.16$ | $n_2 = 79$<br>$\bar{x}_2 = 2.6$, $s_2 = 5.9$ |

$\boxed{\alpha = 0.01}$

$H_1: \mu_1 \neq \mu_2$ (claim)

$H_0: \mu_1 = \mu_2$

$t = 1.9964$

$P = 0.0477 > \alpha$, failed to reject $H_0$

At 1% level of sig., we don't have enough evidence to reject Ho; mean weight loss may not be different in these two group.

$\min\{5,7\} = 5$

One sample case, $df = n-1$

* If independent sample $\Rightarrow df = \min\{n_1-1, n_2-1\}$

* If dependent sample $\Rightarrow df = n_1+n_2-2$

* Pop variances not equal $\Rightarrow \sigma_1^2 \neq \sigma_2^2$

↳ pooled = "No"

* Pop variances are equal $\Rightarrow \sigma_1^2 = \sigma_2^2$

↳ pooled = "yes"

**Example 3:** The results of a state mathematics test for random samples of students taught by two different teachers at the same school are reviewed. Teacher 1's 8 students had a mean score of 473 with a standard deviation of 39.7. Teacher 2's 18 students had a mean score of 459 with a standard deviation of 24.5. Can you conclude that Teacher 1's scores are significantly higher? Use $\alpha = 0.10$. Assume the populations are normally distributed and the population variances are not equal.

| T1 ($\mu_1$) | T2 ($\mu_2$) |
| --- | --- |
| $n_1 = 8$ | $n_2 = 18$ |
| $\bar{x}_1 = 473$ | $\bar{x}_2 = 459$ |
| $s_1 = 39.7$ | $s_2 = 24.5$ |

STAT → TESTS → [4]

$H_0: \mu_1 \le \mu_2$

$H_1: \mu_1 > \mu_2$ (claim)

$\boxed{\alpha = 0.10}$

pooled = NO

$t = 0.9224$

$P = 0.1896 > \alpha$, failed to reject Ho

At 10% level of sig., we don't have enough evidence to reject Ho, meaning T1 students score might be lower than T2 student's score.

$df = \min\{8-1, 18-1\}$

$= \min\{7, 17\} = 7$