

Appendix I

Important Equations

1. Definition of Flux and Flow:

The flux is given as:

$$\begin{aligned} [1.2.1] \quad J_V &= \frac{Q_V}{A} \\ J_S &= \frac{Q_S}{A} \end{aligned}$$

where the volume flux, J_V , is defined as the volume flow, Q_V , per unit cross-sectional area (the area element being normal to the direction of flow), and the solute flux, J_S , is the flow of solute per unit cross-sectional area. The volume flow is the volume per unit time; the solute flow is the amount of solute (expressed as grams or moles) per unit time.

2. The Continuity Equation:

The Continuity Equation is given as:

$$[1.2.9] \quad \frac{\partial C(x,t)}{\partial t} = -\frac{\partial J(x)}{\partial x}$$

where C is the concentration and J is the flux. What this equation says is that if the flux is not the same everywhere, concentration must be changing—either building up or being depleted.

3. Pressure of a Column of Fluid:

The pressure of a column of fluid on the surface of the earth is given by:

$$[1.2.16] \quad P = \frac{F}{A} = \rho gh$$

where P is the pressure, ρ is the density of the fluid, g is the acceleration due to gravity, and h is the height of the column.

4. Poiseuille's Law:

Poiseuille's Law states:

$$[1.2.17] \quad Q_V = \frac{\pi a^4}{8\eta} \left(\frac{\Delta P}{\Delta x} \right)$$

The equation describes the relationship between pressure and laminar flow through a narrow and long tube. Q_V is the flow through the tube, a is the inner radius of the tube, π is the geometric ratio of the diameter to the

circumference of a circle, ΔP is the pressure difference between the ends of the tube, Δx is the length of the tube, and η is the viscosity of the medium.

5. Definition of Viscosity:

Viscosity is defined from:

$$[1.2.18] \quad \frac{F}{A} = \eta \frac{dv}{dy}$$

where F is the force, A is the area parallel to the force (F/A is the shear stress), η is the viscosity (usually in Pa s), and dv/dy is the transverse gradient of velocity.

6. Law of Laplace:

The Law of Laplace for a cylinder is:

$$[1.2.20] \quad \Delta P = \frac{T}{r}$$

where ΔP is the pressure across the wall, T is the wall tension, and r is the radius. For a sphere, this is

$$[1.2.22] \quad \Delta P = \frac{2T}{r}$$

7. Coulomb's Law:

$$\begin{aligned} [1.3.1] \quad \mathbf{F} &= \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \\ \mathbf{F}_{1 \text{ on } 2} &= \frac{q_1 q_2 r_{12}}{4\pi\epsilon_0 r^3} = -\mathbf{F}_{2 \text{ on } 1} \end{aligned}$$

where \mathbf{F} is the electrostatic force between two point charges of magnitude q_1 and q_2 , r is the distance separating the two charges, π is the geometric ratio, and ϵ_0 is a physical constant, the electrical permittivity of space. The top equation gives the magnitude of the force; the bottom equation gives its direction.

8. Definition of Electrical Potential:

$$[1.3.5] \quad U_A = - \int_{\infty}^A \frac{\mathbf{F}_{\text{int}} \cdot d\mathbf{s}}{q_{\text{test}}}$$

where U_A is the potential at point A , \mathbf{F}_{int} is the interaction force acting on a positive test charge of magnitude q_{test} , and $d\mathbf{s}$ is the distance increment. Electrical potential is expressed in volts.

9. Definition of Electric Field Intensity:

The electric field intensity \mathbf{E} is given by:

$$[1.3.9][1.3.11] \quad \mathbf{E} = \frac{\mathbf{F}_{\text{int}}}{q_{\text{test}}} = -\nabla U$$

where \mathbf{F} is the electrical interaction force on a charge of magnitude q_{test} , U is the potential, and ∇ is the gradient operator that converts the scalar function U into a vector with components equal to the partial derivative of U along each axis.

10. Electrical Capacitance:

Electrical capacitance is defined as:

$$[1.3.16] \quad C = \frac{Q}{V}$$

where C is the capacitance, Q is the charge, and V is the voltage difference across the capacitor.

11. Capacitance of Two Parallel Plates:

The capacitance between two parallel plates is given as:

$$[1.3.21] \quad C = \frac{\kappa \varepsilon_0 A}{\delta}$$

where C is the capacitance (in farads = C V^{-1}), κ is the dielectric constant characteristic of the material between the plates (a dimensionless ratio), ε_0 is the electrical permittivity of the vacuum = $8.85 \times 10^{-12} \text{ C}^2 \text{ J}^{-1} \text{ m}^{-1}$, A is the area (in m^2), and δ is the distance between the plates (m).

12. Drag Force and Velocity:

$$[1.3.23] \quad \mathbf{F}_d = -\beta \mathbf{v}$$

where \mathbf{F}_d is the drag force on a particle moving at constant velocity, \mathbf{v} , and β is the drag coefficient.

13. Relation Between Flux and Velocity:

When there is a flux of material, the average velocity of the particles is determined by:

$$[1.3.24] \quad J_s = vC$$

where J_s is the flux of solute, v is the average velocity, and C is the concentration.

14. The Dipole Moment:

The dipole moment is defined as:

$$[1.4.5] \quad \mathbf{p} = q\mathbf{d}$$

where \mathbf{p} is the dipole moment, q is the charge separated in the dipole, and \mathbf{d} is the distance vector pointing from q_- to q_+ .

15. Fick's Dilution Principle:

$$[1.5.3] \quad \text{Volume} = \frac{\text{amount}}{\text{concentration}}$$

$$V = \frac{m}{C}$$

where V is the volume of distribution, m is the amount (either in mass units or in mole units), and C is the concentration, either in mass per unit volume or moles per unit volume.

16. The Arrhenius Equation:

The Arrhenius equation describes the dependence of a rate constant on the temperature. It is given as:

$$[1.5.16] \quad k = A e^{-E_a/RT}$$

where k is the rate constant, A is the preexponential factor, E_a is the activation energy, R is the ideal gas constant, and T is the absolute temperature. The equation in Chapter 1.5 takes the natural logarithm of the rate constant.

17. The Michaelis–Menten Equation:

The Michaelis–Menten equation describes a particular class of enzymatic reactions that obey simple saturation behavior. Not all enzyme reactions have kinetics that can be described in this way. The equation is:

$$[1.5.26] \quad J = \frac{J_{\max}[S]}{K_m + [S]}$$

where J is the enzyme reaction rate, J_{\max} is the maximum reaction rate, $[S]$ is the substrate concentration, and K_m is the Michaelis–Menten constant, which is equal to the substrate concentration when the enzyme is one-half maximal.

18. Fick's First Law of Diffusion:

Fick's First Law of Diffusion is given as:

$$[1.6.3] \quad J_s = -D \frac{\partial C}{\partial x}$$

where J_s is the solute flux, D is the diffusion coefficient, C is the concentration, and x refers to distance along the line parallel to the flux. This equation is valid only when there is no other force on the diffusing solute such as an electric field if the solute is charged, or solvent drag. Strictly speaking, J is a vector and so is the gradient. In three dimensions, the equation can be written as:

$$\mathbf{J} = -D \nabla C = -D \left(\frac{\partial C}{\partial x} \mathbf{i} + \frac{\partial C}{\partial y} \mathbf{j} + \frac{\partial C}{\partial z} \mathbf{k} \right)$$

19. Fick's Second Law of Diffusion:

Fick's Second Law of Diffusion is derived from the First Law and the Continuity Equation. It is

$$[1.6.6] \quad \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

20. Time of Diffusion:

The time for one-dimensional diffusion is estimated as:

$$[1.6.33] \quad x = \sqrt{\bar{x}^2} = \sqrt{2D\Delta t}$$

where x is the distance, D is the diffusion coefficient, and Δt is the time.

21. Concentration Profile for an Initially Sharp Concentration:

For an initially narrow distribution, the distribution after time t and as a function of distance away from the initial source is given as:

$$[1.6.34] \quad C(x, t) = C_0 \sqrt{\frac{1}{4\pi Dt}} e^{\frac{-x^2}{4Dt}}$$

where $C(x, t)$ is the concentration at x after time t , C_0 is the initial concentration at position $x = 0$, D is the diffusion coefficient, and t is the elapsed time.

22. Convection–Diffusion Equation:

Diffusive flux can be augmented by the movement of solvent in solvent drag. The solute flux is given by where the first term on the right-hand side is due to diffusion and the second term is solvent drag.

$$[1.6.36] \quad J_s(x, t) = -D \frac{\partial C(x, t)}{\partial x} + J_v(t)C(x, t)$$

23. Fick's First Law of Diffusion with an External Force:

When a solute is subject to a force, f , per particle, along one dimension, the one-dimensional Fick's equation is modified to:

$$[1.6.42] \quad J = -D \frac{\partial C}{\partial x} + \frac{D}{kT} f C$$

where J is the flux, D is the diffusion coefficient, k is Boltzmann's constant, T is the absolute temperature, f is the force per particle, and C is the concentration.

24. Drag Force and Einstein's Frictional Coefficient:

Particles subject to an external force are accelerated until they reach a terminal velocity. At this time the drag force, F_d , balances the external force, f . It is given as:

$$[1.6.37] \quad F_d = -f = -\beta v$$

where v is the terminal velocity and β is the frictional coefficient. It is given as:

$$[1.6.38] \quad \beta = \frac{kT}{D}$$

where k is Boltzmann's constant, T is the absolute temperature, and D is the diffusion coefficient.

25. Stokes–Einstein Equation:

For a spherical particle, Stokes determined the frictional coefficient as a function of particle size and medium viscosity. Combining it with the Einstein's frictional coefficient, we derive

$$[1.6.45] \quad 6\pi\eta a_s = \frac{kT}{D}$$

where D is the diffusion coefficient, k is Boltzmann's constant, T is the absolute temperature, π is the geometric ratio, η is the fluid viscosity, and a_s is the radius of the spherical particle.

26. The Electrochemical Potential:

The electrochemical potential of a charged species is given by:

$$[1.7.13] \quad \mu = \mu^0 + RT \ln C + z\mathfrak{F}\Psi$$

where μ^0 sets the zero of the function at some standard state; the second term, $RT \ln C$, is the work of concentrating the material to the concentration, C ; the last term is the work necessary to bring the charged species from zero potential to Ψ . The electrochemical potential can be expanded to include other work terms such as pressure–volume work.

27. Gibbs Free Energy:

There are several thermodynamic functions called "free energy." The Gibbs free energy is the free energy under conditions of constant temperature and pressure. You need to know two things about the Gibbs free energy. First, it depends on the composition of a mixture:

$$[1.7.24] \quad G = \sum_i \mu_i n_i$$

where G is the Gibbs free energy, μ_i is the chemical potential of species i in the mixture, and n_i is the number of moles of species i . Second, all spontaneous reactions occur with a decrease in free energy. Thus some reactions with $\Delta G > 0$ can occur by being linked to other reactions so that the overall $\Delta G < 0$.

$\Delta G < 0 \rightarrow$ spontaneous reaction

$\Delta G = 0 \rightarrow$ reaction is at equilibrium

$\Delta G > 0 \rightarrow$ opposite reaction occurs spontaneously

$$[1.7.25]$$

28. Gibbs Free Energy of ATP Hydrolysis:

The free energy of ATP hydrolysis is given as:

$$[1.7.30] \quad \Delta G = \Delta G^0 + RT \ln \left(\frac{[\text{ADP}][\text{Pi}]}{[\text{ATP}]} \right)$$

where ΔG^0 is the standard free energy change under standard conditions of 1 M [ADP], [Pi], and [ATP], 250°C, and 1 atm P.

29. Relation Between Gibbs Free Energy and K_{eq} :

Equilibrium occurs when $\Delta G = 0$. This fact can be used in Eqn [1.7.30] to find

$$[1.7.32] \quad \Delta G^0 = -RT \ln K_{\text{eq}}$$

where K_{eq} is the equilibrium constant. Note that K_{eq} depends on the absolute temperature, T .

30. Absorbance:

Absorbance in spectrophotometry is defined as:

$$A = \log \frac{I_0}{I}$$

where A is the absorbance, I_0 is the intensity of incident light, and I is the intensity of transmitted light.

31. The Beer–Lambert Law:

This relates absorbance to concentration:

$$A = \varepsilon Cd$$

where A is the absorbance, ε is the molar extinction coefficient that depends on the wavelength of the incident light and the chemical nature of the absorbing material, C is the concentration of the absorbing chemical, and d is the path length. The molar extinction coefficient is usually determined with a path length $d = 1\text{ cm}$. Often the equation is given with d omitted, with the assumption that it is 1 cm and the molar extinction coefficient is in units of M^{-1} .

32. Microscopic Resolution:

The resolution of a light microscope is given by:

$$[2.1.A1.1] \quad \text{Resolution} = \frac{0.61 \lambda}{\eta \sin \alpha}$$

where λ is the wavelength of the light, η is the refractive index of the medium, and α is the angle of the cone of light collected by the objective lens. The combined terms $\eta \sin \alpha$ are collectively known as the **numerical aperture**. Note that the resolution has the units of distance and refers to the separation of just discernable circular disks. Thus high resolution is associated with small distances.

33. Relative Centrifugal Force:

Relative centrifugal force is given by the formula:

$$[2.1.A1.15] \quad \text{RCF} = \frac{\omega^2 r}{g}$$

where RCF is given in multiples of the acceleration due to gravity at the surface of the earth, ω is the angular velocity ($= 2\pi \times$ revolutions per second), and r is the distance from the center of rotation.

34. Rate of Centrifugal Sedimentation:

The rate of centrifugal sedimentation is given by:

$$[2.1.A1.17] \quad m(1 - \bar{v}\rho) = \beta \left[\frac{dr/dt}{\omega^2 r} \right]$$

where m is the mass of the sedimenting particle, \bar{V} is the partial specific volume, ρ is the density of the medium through which the particle is sedimenting, β is the drag or frictional coefficient, dr/dt is the radial velocity of sedimentation, and $\omega^2 r$ is the magnitude of the centripetal acceleration.

35. Svedberg:

The **Svedberg** is a unit of sedimentation per unit centrifugal force and is defined as:

$$[2.1.A1.18] \quad s = \left[\frac{dr/dt}{\omega^2 r} \right]$$

where s is the Svedberg, dr/dt is the radial velocity of sedimentation, and $\omega^2 r$ is the magnitude of centripetal acceleration.

36. Stokes' Equation:

This gives the frictional or drag coefficient for a spherical particle as:

$$[2.1.A1.21] \quad \beta = 6\pi\eta a_s$$

where η is the viscosity of the medium and a_s is the radius of the sphere.

37. The Partition Coefficient:

The partition coefficient describes the distribution of a material between the watery phase and an immiscible organic phase. It is defined as:

$$[2.3.1] \quad k_s = \frac{[X]_{\text{organic phase}}}{[X]_{\text{water phase}}}$$

where k_s is the partition coefficient and $[x]$ is the concentration of substance X either in the organic phase or in the aqueous phase.

38. Surface Tension:

It is defined by:

$$[2.4.1] \quad dG = \gamma dA$$

where G is the Gibbs free energy, A is the area, and γ is the surface tension. Because G is in units of J , or N m , and A is in units of m^2 , it follows that γ is in units of N m^{-1} .

39. Permeability:

The permeability refers to the ability of materials to penetrate membranes. It is operationally defined as:

$$[2.5.9] \quad p = \frac{J_s}{\Delta C}$$

where p is the permeability, J_s is the solute flux in the absence of J_v , and ΔC is the concentration difference across the membrane that drives the passive solute flux.

The permeability depends on the characteristics of the membrane–solute pair. For a solute that penetrates the membrane through microscopic pores, the permeability is given as:

$$[2.5.10] \quad p = \frac{n\pi a^2 D}{\delta}$$

where p is the permeability, n is the number of pores per unit area, π is the geometric ratio, a is the radius of the pore, D is the diffusion coefficient of the solute within the pore, and δ is the thickness of the membrane. For a solute that penetrates through the membrane by dissolving in the lipid phase of the membrane, the permeability is given as:

$$[2.5.14] \quad p = \frac{k_s D_{s,\text{lipid}}}{\delta}$$

where p is the permeability, k_s is the partition coefficient of the solute between the aqueous and lipid phases, D is the diffusion coefficient of the solute within the lipid phase, and δ is the thickness of the membrane.

40. Hydraulic Conductivity:

Pressure-driven laminar flow through a membrane is described phenomenologically as:

$$[2.7.22] \quad J_v = L_p \Delta P$$

where J_v is the volume flux, in $\text{mL cm}^{-2} \text{s}^{-1}$, ΔP is the pressure difference that drives bulk flow, and L_p is the hydraulic conductivity, filtration coefficient, or hydraulic permeability.

41. van't Hoff's Law:

The osmotic pressure of an ideal, dilute solution is given as:

$$[2.7.15] \quad \pi = RTC_s$$

where π is the osmotic pressure, R is the gas constant, T is the absolute temperature, and C is the concentration of impermeant solute, in molar. This is a limiting law which is approximately true only for dilute ideal solutions.

42. Osmotic Coefficient:

The osmotic coefficient is defined as:

$$[2.7.16] \quad \varphi = \frac{\pi_{\text{observed}}}{RTC_s}$$

It corrects for the nonideality of solutions.

43. Reflection Coefficient:

The reflection coefficient is a characteristic of a membrane and is defined as:

$$[2.7.24] \quad \sigma = \frac{\pi_{\text{eff}}}{\varphi RTC}$$

where π_{eff} is the effective osmotic pressure observed with a real membrane which is not perfectly semipermeable.

44. Volume Flux in the Presence of ΔP and $\Delta\pi$:

The volume flux across a real membrane which is not perfectly semipermeable to solute is given as:

$$[2.7.42] \quad J_v = L_p[\Delta P - \sigma\Delta\pi]$$

where L_p is the hydraulic permeability, ΔP is the pressure difference across the membrane, σ is the reflection coefficient, and $\Delta\pi$ is the osmotic pressure difference across the membrane.

45. Volume Response of Cells to Osmotic Changes:

When a cell having an initial volume V_0 is in contact with a solution of osmolarity π_{isotonic} and then the bathing solution is changed to one of osmolarity π , it alters its volume according to:

$$[2.7.48] \quad \frac{V_c}{V_0} = \left(1 - \frac{V_b}{V_0}\right) \frac{\pi_{\text{isotonic}}}{\pi} + \frac{V_b}{V_0}$$

where V_c is the new cell volume at equilibrium and V_b is a volume taken to be the osmotically inactive volume of the cell.

46. Proton Motive Force:

The proton motive force is the electrochemical potential difference of H^+ across the inner mitochondrial membrane. It is given as:

$$[2.10.10] \quad \Delta\mu_{\text{out} \rightarrow \text{in}} = RT \ln \frac{[\text{H}^+]_{\text{in}}}{[\text{H}^+]_{\text{out}}} + \mathfrak{F}(\psi_{\text{in}} - \psi_{\text{out}})$$

Here it can be seen that the concentration contribution can be expressed in volts.

47. The Nernst Equation:

At equilibrium, the membrane potential across a membrane permeable to only one ion is given as:

$$[3.1.5] \quad \frac{RT}{z\mathfrak{F}} \ln \frac{[X^+]_{\text{o}}}{[X^+]_{\text{i}}} = \psi_{\text{i}} - \psi_{\text{o}}$$

where R is the gas constant ($= 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$), T is the absolute temperature, z is the valence (\pm integer), \mathfrak{F} is the Faraday ($= 96,500 \text{ C mol}^{-1}$), \ln is the natural logarithm, $[X]$ is the concentration of ion X , ψ is the membrane potential inside or outside of the membrane. Subscript i indicates inside the membrane, o indicates outside of the membrane.

48. Goldman–Hodgkin–Katz Current Equation:

The current across a membrane that is carried by a single ion is given as:

$$[3.1.14] \quad I_{\text{ion}} = \frac{(D/\delta)[(z^2 \mathfrak{F}^2 / RT)E_m][C_i - C_o e^{(-z\mathfrak{F}/RT)E_m}]}{[1 - e^{(-z\mathfrak{F}/RT)E_m}]}$$

where z is the valence of the ion, E_m is the membrane potential, \mathfrak{F} is the Faraday ($= 96,500 \text{ C mol}^{-1}$), R is the gas constant ($= 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$), and T is the absolute temperature; C_i and C_o are the inside and outside concentrations of the ion, respectively.

49. Goldman–Hodgkin–Katz Equation:

The potential across a membrane which is permeable to a variety of ions is given as:

$$[3.1.16] \quad E_m = \frac{RT}{\mathfrak{F}} \ln \left[\frac{P_K[K^+]_{\text{o}} + P_{\text{Na}}[\text{Na}^+]_{\text{o}} + P_{\text{Cl}}[\text{Cl}^-]_{\text{i}}}{P_K[K^+]_{\text{i}} + P_{\text{Na}}[\text{Na}^+]_{\text{i}} + P_{\text{Cl}}[\text{Cl}^-]_{\text{o}}} \right]$$

where E_m is the electrical potential across the membrane, R is the gas constant ($8.314 \text{ J mol}^{-1} \text{ K}^{-1}$), \mathfrak{F} is the Faraday ($9.65 \times 10^4 \text{ C mol}^{-1}$), P_i is the permeability of the i -th ion.

50. Chord Conductance:

The chord conductance is defined as:

$$[3.1.18] \quad I_i = g_i(E_m - E_i)$$

where g_i is the chord conductance of ion i , E_m is the membrane potential, E_i is the equilibrium potential of ion i , and I_i is the current carried by the ion.

51. Slope Conductance:

The slope conductance is defined as:

$$[3.1.19] \quad g_i = \frac{dI_i}{dE_m}$$

where g_i is the conductance of ion i , I_i is the current carried by the ion, and E_m is the membrane potential.

52. The Chord Conductance Equation:

The membrane potential at steady-state or resting conditions is given as:

$$E_m = \frac{g_{Na}}{(g_{Na} + g_K + g_{Cl})} E_{Na} + \frac{g_K}{(g_{Na} + g_K + g_{Cl})} E_K \\ + \frac{g_{Cl}}{(g_{Na} + g_K + g_{Cl})} E_{Cl}$$

[3.1.22]

where E_m is the resting membrane potential and g_i is the chord conductance of ion i .

53. Transmembrane Resistance:

The transmembrane resistance is given as:

$$[3.3.4] \quad R = \frac{R_m}{A}$$

where R_m is the transmembrane resistance of a unit area of membrane and A is the area of the membrane. R is in Ω ; R_m is in units of $\Omega \text{ cm}^2$.

54. Axoplasmic Resistance:

The axoplasmic resistance is given by:

$$[3.3.5] \quad R_i = \frac{\rho_i d}{A}$$

where R_i is the internal resistance, d is the distance down the axon, A is the cross-sectional area, and ρ_i is the specific resistance in $\Omega \text{ cm}$.

55. Electrotonus:

Injection of current in an axon causes the membrane potential to vary with distance and time from the point of current injection. At infinite time, a steady state is reached, electrotonus. The voltage depends on the distance from the point of current injection as:

$$[3.3.17] \quad V = (V_0 - V_r)e^{-\frac{x}{\lambda}} + V_r$$

where V is the voltage at point x , V_0 is the voltage at the point of current injection, V_r is the voltage infinitely far away (the resting potential), x is the distance from the point of current injection, and λ is the space constant, in units of distance.

56. Space Constant (or length constant) for an Axon:

$$[3.3.15] \quad \lambda = \sqrt{\frac{R_m \pi a^2}{\rho_i 2 \pi a}} = \sqrt{\frac{R_m}{\rho_i}} a$$

where λ is the space constant, R_m is the specific transmembrane resistance, a is the radius of the axon, and ρ_i is the specific resistance of the axoplasm.

57. Decay Under Space Clamp:

If the internal resistance is made small (=space clamp), temporal decay of small departures from the resting membrane potential is given by:

$$[3.3.22] \quad V = (V_0 - V_r)e^{-(t/\tau)} + V_r$$

where V is the voltage at time t , V_0 is the voltage at the beginning of the decay, and V_r is the voltage at infinite time, the resting membrane potential; t is the time and τ is the time constant, in units of time.

58. Time Constant for an Axon:

$$[3.3.15] \quad \tau = R_m C_m$$

where τ is the time constant in s^{-1} , R_m is the specific resistance of the axon membrane (in $\Omega \text{ cm}^2$), and C_m is the specific capacitance (in F cm^{-2}).

59. Weber–Fechner Law:

The Weber–Fechner Law attempts to relate sensory perception to stimulus intensity. It is written as:

$$[4.3.1] \quad \Delta S = k \frac{\Delta W}{W}$$

where k is a constant, W is the stimulus strength, and S is the sensory perception.

60. Stevens' Power Law:

Stevens' Power Law attempts to relate sensory perception to stimulus intensity. It is written as:

$$[4.3.3] \quad \Psi = k \Phi^n$$

where Ψ is the psychological sensation, k is a constant, n is a constant, and Φ is the stimulus intensity.

61. Decibel:

The Decibel is defined as:

$$[4.7.1] \quad \text{Loudness (decibels)} = 10 \log \frac{I_{\text{sound}}}{I_{\text{ref}}}$$

$$[4.7.2] \quad \text{Loudness (dB)} = 20 \log \frac{\Delta P_{\text{sound}}}{\Delta P_{\text{ref}}}$$

where I is the intensity in W cm^{-2} and ΔP is the increment in pressure over ambient pressure. The reference intensity that physicists use is $10^{-16} \text{ W cm}^{-2} = 10^{-12} \text{ W m}^{-2}$.

62. Sound Intensity:

The intensity of sound, in W m^{-2} , is related to the pressure increment according to:

$$[4.7.A1.32] \quad I = \frac{\Delta P_0^2}{2 \rho c}$$

where I is the intensity, ΔP_0 is the pressure amplitude, ρ is the density of the air, and c is the velocity of sound.

63. Snell's Law of Refraction:

Snell's law of refraction describes the relationship between the angle of incidence of incoming light, θ_1 , and the angle of refraction, θ_2 , as being given by:

$$[4.8.A1.4] \quad n_i \sin \theta_i = n_r \sin \theta_r$$

where n_1 is the index of refraction in medium 1 and n_2 is the index of refraction in medium 2.

64. The Thin Lens Formula:

The thin lens formula gives the relationship between the distance to the object from the nodal point, the distance to its image, and the focal length. It is

$$[4.8.A1.12] \quad \frac{1}{O} + \frac{1}{I} = \frac{1}{f}$$

where O is the distance (in m) to the object, I is the distance (in m) to the image, and f is the focal length.

65. Pressure-Driven Flow:

Pressure drives flow with the following relation:

$$[5.1.3] \quad Q = \frac{\Delta P}{R}$$

where Q is the flow, ΔP is the pressure difference that drives the flow, and R is the resistance to the flow. This is the hydraulic analogy to Ohm's law.

66. Mechanical Compliance:

Mechanical compliance is defined as:

$$[5.1.4] \quad C = \frac{\Delta V}{\Delta P}$$

where C is the compliance, ΔV is a volume increment, and ΔP is the pressure increment accompanying ΔV .

67. Hematocrit:

The hematocrit is defined as

$$\text{Hct} = \frac{\text{packed cell volume}}{\text{total blood volume}} \times 100$$

which is a number between 0 and 100, or it is defined as the hematocrit ratio, which is the same ratio but without multiplying it by 100 to convert it to percent.

68. Addition of Cardiac Leads I, II, and III:

The three Einthoven leads are defined as:

$$\begin{aligned} \text{Left arm} - \text{right arm} &= \text{LEAD I} \\ \text{Left leg} - \text{right arm} &= \text{LEAD II} \\ \text{Left leg} - \text{left arm} &= \text{LEAD III} \end{aligned}$$

Because of their definition, Kirchhoff's voltage law is applied as:

$$[5.6.3] \quad \text{I} + \text{III} = \text{II}$$

69. Definition of Stroke Volume:

The stroke volume is the difference between end diastolic volume and end systolic volume:

$$[5.8.1] \quad \text{SV} = \text{EDV} - \text{ESV}$$

where SV is the stroke volume, EDV is the end diastolic volume, and ESV is the end systolic volume.

70. Calculation of Cardiac Output:

Cardiac output can be most easily calculated as:

$$[5.8.2] \quad \text{CO} = \text{SV} \times \text{HR}$$

where CO is the cardiac output, SV is the stroke volume, and HR is the heart rate.

71. Fick Principle of Estimation of Cardiac Output:

The cardiac output can be measured from the total rate of oxygen consumption and the difference in arterial and venous blood total oxygen concentration:

$$[5.8.7] \quad Q_a = \frac{Q_{O_2}}{[O_2]_a - [O_2]_v}$$

where Q_a is the cardiac output, Q_{O_2} is the rate of oxygen consumption, $[O_2]_a$ is the total arterial concentration of oxygen, and $[O_2]_v$ is the total venous concentration of oxygen.

72. Estimation of Cardiac Output by the Indicator Dilution Method:

Cardiac output can be estimated by injecting m moles of indicator into the venous side of the circulation and measuring the concentration of indicator on the opposite side of the heart. The equation gives the cardiac output

$$[5.8.12] \quad Q_a = \frac{m}{\int_0^t C_m dt}$$

where Q_a is the cardiac output, m is the moles of injected indicator, and C_m is the concentration of the indicator in the arterial side of the circulation; t is the time.

73. Total Energy of a Fluid and Equivalent Pressure:

The total energy of a moving fluid has components of pressure, kinetic energy, and gravitational potential energy:

$$[5.9.2] \quad E = E_p + E_k + E_g = PV + 1/2\rho Vv^2 + \rho ghV$$

The equivalent pressure is obtained by dividing by the volume, V :

$$[5.9.3] \quad P' = P + 1/2\rho v^2 + \rho gh$$

where P' is the total equivalent pressure, P is the pressure due to PV work, ρ is the density, v is the velocity, g is the acceleration due to gravity, and h is the height above an arbitrary zero reference point.

74. Definition of Pulse Pressure:

The pulse pressure is defined as:

$$[5.9.7] \quad \Delta P_{\text{pulse}} = P_{\text{systolic}} - P_{\text{diastolic}}$$

This is the difference between the systolic pressure and the diastolic pressure.

75. Mean Arterial Pressure:

The mean arterial pressure is given by:

$$[5.9.8] \quad P_A = \frac{\int_{t_1}^{t_2} P dt}{t_2 - t_1}$$

where P_A is the mean arterial pressure, P is the pressure that varies in the arteries with time, and t is the time. This equation is often approximated as:

$$[5.9.9] \quad P_A = P_{\text{diastolic}} + \frac{\Delta P_{\text{pulse}}}{3}$$

where $P_{\text{diastolic}}$ is the diastolic pressure and ΔP_{pulse} is the pulse pressure.

76. The Reynolds Number:

The Reynolds number is:

$$[5.9.12] \quad Re = \frac{2a \langle V \rangle \rho}{\eta}$$

where Re is the Reynolds number, a is the radius of the tube, $\langle V \rangle$ is the average velocity in the tube, ρ is the density of the fluid, and η is the viscosity. This is taken as an indicator of laminar versus chaotic or turbulent flow.

77. Venous Concentration of Solute:

The venous concentration of solute is given as:

$$[5.10.10] \quad C_v = C_i + (C_a - C_i)e^{(-Sp/Q_v)}$$

where C_v is the venous concentration, C_i is the interstitial fluid concentration, presumably constant, C_a is the arterial concentration, S is the surface area of the capillaries through which exchange occurs, p is the permeability, and Q_v is the flow through the capillaries.

78. Interstitial Fluid Concentration of Solute:

The interstitial fluid concentration of solute is given as:

$$[5.10.15] \quad C_i = \frac{C_a + Q_m}{Q_v E}$$

where C_i is the interstitial fluid concentration, C_a is the arterial concentration, Q_m is the rate of metabolism, which could be positive if the substance is produced by the tissue, or negative if the substance is consumed; Q_v is the flow through the capillaries and E is the extraction. The extraction is given as:

$$[5.10.14] \quad [1 - e^{-(Sp/Q_v)}] = E$$

79. Starling Forces for Filtration of Fluid Out of the Microcirculation:

The net filtration of fluid out of the microcirculation is given as:

$$[5.10.20] \quad Q_v = Ak_f[(P_C - P_i) - \sigma(\pi_C - \pi_i)]$$

where Q_v is the flow out of the capillary (the filtration flow), A is the area through which the flow occurs (the

surface area of the capillary), k_f is the filtration coefficient, P_C is the hydrostatic pressure within the capillary, and P_i is the hydrostatic pressure in the interstitial fluid; π_C is the oncotic pressure within the capillary and π_i is the oncotic pressure in the interstitial fluid; σ is the reflection coefficient of those substances contributing to the oncotic pressure.

80. Capillary Hydrostatic Pressure:

The capillary hydrostatic pressure is given as:

$$[5.11.5] \quad P_C = P_A \left[\frac{R_V}{R_A + R_V} \right] + P_V \left[\frac{R_A}{R_A + R_V} \right]$$

where P_C is the capillary hydrostatic pressure, P_A is the arterial hydrostatic pressure, P_V is the venous hydrostatic pressure, R_A is the resistance of the arteries to flow, and R_V is the resistance of the veins to flow.

81. Definition of Total Peripheral Resistance:

The total peripheral resistance can be determined from

$$[5.12.2] \quad Q_{\text{veins}} = \frac{P_A - P_{RA}}{\text{TPR}}$$

where Q_{veins} is the flow through the veins, which at steady state is the cardiac output, P_A is the arterial pressure, P_{RA} is the right atrial pressure, and TPR is the total peripheral resistance.

82. Vascular Function Curve:

The linear portion of the vascular function curve is given as:

$$[5.12.13] \quad Q_{\text{veins}} = -\frac{[1 + (C_V/C_A)]}{\text{TPR}} [P_{RA} - P_{MS}]$$

where Q_{veins} is the flow through the vasculature, C_V is the compliance of the veins, C_A is the compliance of the arteries, P_{RA} is the right atrial pressure, P_{MS} is the mean systemic pressure, and TPR is the total peripheral resistance. This relationship describes only the linear portion of the vascular function curve and fails at low pressures when the veins collapse.

83. The Ideal Gas Law:

Ideal gases obey

$$[6.1.3] \quad PV = nRT$$

where P is the pressure, V is the volume, R is the gas constant ($= 0.082 \text{ L atm mol}^{-1} \text{ K}^{-1}$), and T is the absolute temperature.

84. Surface Tension:

The surface tension relates the free energy of the surface to its area according to:

$$[6.1.5] \quad dG = \gamma dA$$

where dG is the change in surface free energy, γ is the surface tension (energy per unit area or force per unit length), and dA is the change in surface area.

85. Definition of Respiratory Quotient:

The respiratory quotient is defined as:

$$[6.3.1] \quad R = \frac{Q_{CO_2}}{Q_{O_2}}$$

where R is the respiratory quotient, Q_{CO_2} is the rate of CO_2 production by the body, and Q_{O_2} is the rate of oxygen consumption by the body.

86. Definition of Partial Pressure:

The partial pressure of a gas in a mixture of gases is given by:

$$[6.3.6] \quad P_A = \frac{n_A}{n} P_B = f_A P_B$$

where P_A is the partial pressure of gas A, n_A is the number of moles of gas A, n is the total number of moles of gas in the mixture, f_A is the mole fraction, and P_B is the barometric or total pressure of the mixture of gases.

87. Partial Pressure in Moist Air:

The partial pressure of a gas in moist tracheal air is given by:

$$[6.3.8] \quad P_A = f_A (P_B - P_{H_2O})$$

where P_A is the partial pressure in the moist air, f_A is the mole fraction in dry air, P_B is the total pressure of the gas mixture, and P_{H_2O} is the partial pressure of water in the moist air. At body temperature, $P_{H_2O} = 47$ mmHg.

88. Henry's Law:

Henry's Law relates the concentration of gas dissolved in a fluid to the partial pressure of the gas in the air in equilibrium with the fluid:

$$[6.3.9] \quad x_A = \beta_A P_A$$

where x_A is the mole fraction of gas A in the fluid, β_A is the solubility of the gas in the fluid, and P_A is the partial pressure of the gas in the air. This can be converted, approximately, to:

$$[6.3.11] \quad [A] = \alpha_A P_A$$

where $[A]$ is the concentration of gas in fluid phase, P_A is the partial pressure of the gas in the air in equilibrium with the fluid, and α_A is the solubility in different units from β .

89. Diffusing Capacity:

The diffusing capacity of a gas across the lungs is given as:

$$[6.3.15] \quad Q_s = D_L \Delta P$$

where Q_s is the flow of gas, D_L is the diffusing capacity, and ΔP is the partial pressure difference that drives diffusion.

90. Alveolar Ventilation:

Alveolar ventilation can be calculated as:

$$[6.3.16] \quad Q_A = \nu_R (V_T - V_D)$$

where Q_A is the alveolar ventilation (in $L \text{ min}^{-1}$), ν_R is the respiratory rate (in breaths min^{-1}), V_T is the tidal volume (in L), and V_D is the dead space volume, part of the tidal volume that does not exchange gas because it does not reach the alveoli (in L).

91. Physiological Dead Space:

The physiological dead space can be calculated from:

$$[6.3.19] \quad V_D = V_T \left[1 - \frac{f_{ECO_2}}{f_{ACO_2}} \right]$$

where V_D is the dead space volume, V_T is the tidal volume, f_{ECO_2} is the mole fraction of CO_2 in expired air, and f_{ACO_2} is the mole fraction of CO_2 in the alveolar air.

92. Alveolar Ventilation Equation:

The alveolar ventilation can be determined by

$$[6.3.21] \quad Q_A = \frac{Q_{CO_2}}{P_{ACO_2}} (P_B - 47)$$

where Q_A is the alveolar ventilation, Q_{CO_2} is the rate of CO_2 production, P_{ACO_2} is the partial pressure of CO_2 in alveolar air, P_B is the barometric or atmospheric pressure, in mmHg, and 47 is P_{H_2O} at body temperature, in mmHg.

93. Alveolar Gas Equation:

The partial pressure of O_2 in the alveolar can be calculated from:

$$[6.3.A1.15] \quad P_{AO_2} = P_{I_{O_2}} - \frac{1}{R} P_{ACO_2} + f_{I_{O_2}} \left(\frac{(1-R)}{R} \right) P_{ACO_2}$$

where P_{AO_2} is the partial pressure of O_2 in the alveoli, $P_{I_{O_2}}$ is the partial pressure of O_2 in the inspired air, R is the respiratory quotient, $f_{I_{O_2}}$ is the mole fraction of O_2 in dry inspired air, and P_{ACO_2} is the partial pressure of CO_2 in alveolar air.

94. Conversion of V_{BTPS} to V_{STPD} :

$$[6.3.A2.3] \quad \begin{aligned} V_{BTPS} &= \frac{310}{273} \frac{760}{(760 - 47)} V_{STPD} \\ &= 1.2104 V_{STPD} \end{aligned}$$

where V_{BTPS} is the volume at body temperature and pressure, saturated with water vapor, and V_{STPD} is the volume of gas at standard temperature and pressure, dry.

95. The Hill Equation for O_2 Binding to Hemoglobin:

The Hill equation for O_2 binding to hemoglobin is given as:

$$[6.4.2] \quad [Hb \cdot O_2] = \frac{[O_2]^h}{K + [O_2]^h} [Hb \cdot O_2]_{max}$$

where $[Hb \cdot O_2]$ is the concentration of hemoglobin with oxygen bound, $[O_2]$ is the oxygen concentration (or partial pressure), K is the affinity constant, h is the exponent of the O_2 concentration, and $[Hb \cdot O_2]_{max}$ is the maximum O_2 binding capacity of hemoglobin.

96. Determination of Oxygen Consumption:

Oxygen consumption can be calculated from the cardiac output and A–V difference in total [O₂]:

$$[6.4.4] \quad Q_a([O_2]_a - [O_2]_v) = Q_{O_2}$$

where Q_a is the flow through the arteries (assumed equal to the flow through the veins, equal to the cardiac output); [O₂]_a is the total arterial content of O₂, in mL of O₂ at STPD per 100 mL of blood; [O₂]_v is the total venous content of O₂ at STPD per 100 mL of blood; Q_{O_2} is the rate of oxygen consumption, given as a positive value as mL O₂ at STPD per min.

Oxygen consumption can also be calculated from the respiratory gases:

$$[6.4.5] \quad Q_T^* f_{O_2} = Q_T f_{E_{O_2}} + Q_{O_2}$$

where Q_T^* is the flow of inspired air, f_{O_2} is the mole fraction of O₂ in the inspired air, Q_T is the flow of expired air, $f_{E_{O_2}}$ is the mole fraction of O₂ in the expired air, and Q_{O_2} is the consumption of O₂, in mL O₂ at STPD per min. Q_T^* and Q_T should be in volumes at STPD per min.

97. The Henderson–Hasselbalch Equation:

For the dissociation of any acid HA to H⁺ and the conjugate base, A[−], the Henderson–Hasselbalch Equation is written as:

$$[6.5.12] \quad pH = pK + \log \frac{[A^-]}{[HA]}$$

where pH = −log[H⁺], and pK = −log K, where K is the acid dissociation constant of the acid.

98. The Henderson–Hasselbalch Equation for the HCO₃–Buffer System:

The Henderson–Hasselbalch Equation for the bicarbonate buffer system is written as:

$$[6.5.23] \quad pH = 6.10 + \log \frac{[HCO_3^-]}{0.0308 P_{CO_2}}$$

here [HCO₃[−]] must be in units of mM and P_{CO_2} must be in units of mmHg.

99. Calculation of ICF Volume:

The intracellular fluid compartment can be calculated by:

$$[7.1.2] \quad ICF = TBW - ECF$$

where ICF is the intracellular fluid volume, TBW is the total body water, and ECF is the extracellular fluid volume.

100. Calculation of Interstitial Fluid Volume:

The interstitial fluid volume can be calculated as:

$$[7.1.3] \quad ISF = ECF - plasma$$

where ISF is the interstitial fluid volume, ECF is the extracellular fluid volume, and plasma is the plasma fluid volume.

101. Lean Body Mass:

The lean body mass, LBM, can be calculated from the total body water, TBW, by:

$$[7.1.4] \quad \text{Lean body mass} = \frac{\text{TBW}}{0.73}$$

102. Macroscopic Electroneutrality:

Because of the strength of electric forces, solutions are neutral, and so

$$[7.1.6] \quad \sum_i C_i^+ = \sum_i C_i^-$$

where C_i^+ refers to the concentration of the i -th positively charged ion (cation) and C_i^- refers to the concentration of the i -th negatively charged ion (anion).

103. Gibbs–Donnan Equilibrium:

At equilibrium, a membrane that separates a solution of NaCl from NaP will produce a membrane potential given by the Nernst Equation and a distribution of Na and Cl ions such that

$$[7.1.12] \quad \frac{[Na^+]_o}{[Na^+]_i} = \frac{[Cl^-]_i}{[Cl^-]_o} = r$$

Here r is called the Gibbs–Donnan ratio; [Na⁺]_o is the concentration of Na⁺ in the compartment without impermeant anion; [Na⁺]_i is the concentration of Na⁺ in the compartment with the impermeant anion.

104. Material Balance Equation for the Kidney:

$$[7.3.3] \quad \begin{aligned} &\text{Rate of filtration} + \text{rate of secretion} \\ &= \text{rate of excretion} + \text{rate of reabsorption} \end{aligned}$$

105. GFR as Inulin Clearance:

The glomerular filtration rate can be calculated as the inulin clearance as:

$$[7.3.6] \quad GFR = \frac{Q_u U_{\text{inulin}}}{P_{\text{inulin}}}$$

where GFR is the glomerular filtration rate, usually in mL min^{−1}, Q_u is the rate of urine flow, in mL min^{−1}, U_{inulin} is the urinary concentration of inulin in a timed sample, and P_{inulin} is the plasma concentration of inulin at the time of the urinary sample.

106. Renal Plasma Flow as PAH Clearance:

The renal plasma flow can be calculated as:

$$[7.3.10] \quad RPF = \frac{Q_u U_{PAH}}{RA_{PAH} - RV_{PAH}}$$

where RPF is renal plasma flow, typically in mL min^{−1}, Q_u is the rate of urine flow, in mL min^{−1}, U_{PAH} is the urinary concentration of PAH (para aminohippuric acid); RA_{PAH} is the concentration of PAH in the renal artery; RV_{PAH} is the concentration of PAH in the renal vein.

107. Renal Clearance:

The renal clearance of any substance, x , is defined as:

$$[7.3.12] \quad C_x = \frac{Q_u U_x}{P_x}$$

where C_x is the clearance of substance x ; Q_u is the urine flow rate, in mL min^{-1} ; U_x is the urinary concentration of x ; P_x is the plasma concentration of x .

108. Definition of Filtration Fraction:

The filtration fraction is defined as:

$$[7.3.13] \quad \text{Filtration fraction} = \frac{\text{GFR}}{\text{RPF}}$$

where GFR is the glomerular filtration rate and RPF is the renal plasma flow.

109. Definition of the Sieving Coefficient:

The sieving coefficient for any substance is defined as:

$$[7.3.14] \quad \text{Sieving coefficient} = \Theta = \frac{C_B}{C_P}$$

where Θ is the sieving coefficient of a substance, C_B is the concentration of the substance in Bowman's space, and C_P is the concentration of the substance in the plasma.

110. Forces Producing Ultrafiltration:

The forces producing glomerular filtration are written as:

$$[7.3.18] \quad \text{GFR} = K_f [P_{GC} - P_{BS} - \pi_{GC}]$$

where GFR is the glomerular filtration rate, K_f is the filtration coefficient of the renal corpuscle, P_{GC} is the hydrostatic pressure within the glomerular capillaries, P_{BS} is the hydrostatic pressure within Bowman's space, and π_{GC} is the oncotic pressure (=colloid osmotic pressure) of the blood in the glomerular capillaries.

111. Filtered Load:

The filtered load of any substance is calculated by:

$$[7.4.1] \quad \text{Filtered load}_x = \text{GFR} \Theta_x P_x$$

where GFR is the glomerular filtration rate, Θ_x is the sieving coefficient for substance x , and P_x is the plasma concentration of substance x .

112. Fraction of Water Reabsorbed:

The fraction of water reabsorbed from the glomerulus to any segment of the nephron can be estimated by:

$$[7.4.15] \quad \frac{V_R}{V_T} = 1 - \frac{1}{(\text{TF}/P)_{\text{inulin}}}$$

where V_R is the volume of fluid reabsorbed, V_T is the volume of fluid filtered, $(\text{TF}/P)_{\text{inulin}}$ is the ratio of the tubular fluid inulin concentration to its concentration in plasma.

113. Fraction of Filtered Load Remaining: The Double Ratio:

The fraction of filtered load of substance x remaining at any locus in the nephron can be calculated by:

$$[7.4.18] \quad \text{Fraction remaining} = \frac{(\text{TF}/P)_x}{(\text{TF}/P)_{\text{inulin}}}$$

where $(\text{TF}/P)_x$ is the ratio of the tubular fluid concentration of x to its plasma concentration, and $(\text{TF}/P)_{\text{inulin}}$ is the ratio of the tubular fluid concentration of inulin to its plasma concentration.

114. Net Gain of HCO_3^- from the Kidney:

The net gain (or loss) of HCO_3^- from renal processes is calculated as:

$$\text{Net } \text{HCO}_3^- \text{ gain} = \text{titratable acid} + \text{NH}_4^+ - \text{HCO}_3^-$$

$$[7.7.8]$$

where titratable acid is the amount of base necessary to titrate the urine to pH 7.4; NH_4^+ is the amount of ammonium excreted in a known amount of time; and HCO_3^- is the amount of bicarbonate in the urine.

115. Energetic Equivalence of Oxygen:

The rate of energy expenditure can be estimated from the rate of oxygen consumption as:

$$[8.6.3] \quad M = 4.85 \text{ kcal L}^{-1} \times Q_{O_2}$$

$$= 20.3 \text{ kJ L}^{-1} \times Q_{O_2}$$

where M is the total metabolic rate (in energy per unit time) and Q_{O_2} is the rate of oxygen consumption in L (STPD) per min.

116. Gas Consumption and Production as a Function of Fuel Use:

The amount of O_2 consumed or CO_2 produced can be calculated from the amount of fats, carbohydrates, and proteins that are used for fuel, using the following:

$$[8.6.7] \quad V_{O_2} = 0.746c + 2.02f + 1.01p$$

$$V_{CO_2} = 0.746c + 1.43f + 0.844p$$

here V_{O_2} is the volume of O_2 , in L at STPD, consumed; V_{CO_2} is the volume of CO_2 in L at STPD; c is the amount of carbohydrate burned, in g; f is the amount of fats burned, in g; and p is the amount of protein consumed, in g.

117. Metabolic Clearance Rate:

The metabolic clearance rate is the amount of hormone removed per unit time divided by its plasma concentration:

$$[9.1.6] \quad \text{MCR} = \frac{kH}{H/V_d} = kV_d$$

where MCR is the metabolic clearance rate, in mL min^{-1} or L min^{-1} , k is the first-order rate constant for hormone degradation, in min^{-1} , H is the amount of hormone, in moles, and V_d is the volume of distribution of hormone, in mL or L .

118. The Scatchard Equation:

The basis for the Scatchard plot is written as:

$$[9.1.\text{A1.13}] \quad \frac{B}{[L]} = K(B_{\max} - B)$$

where B is the amount of ligand bound, $[L]$ is the free ligand concentration, K is the intrinsic association constant, B_{\max} is the maximum amount of ligand that can be bound.

119. The Hill Plot:

The Hill Equation is given as:

$$[9.1.\text{A1.21}] \quad \nu = \frac{[P \cdot L_h]}{[P] + [P \cdot L_h]} = \frac{K[L]^h}{1 + K[L]^h}$$

where ν is the fractional saturation of binding (restricted to $0 \leq \nu \leq 1$), K is the association constant, $[L]$ is the free ligand concentration, and h is the Hill coefficient. This can be rearranged to:

$$[9.1.\text{A1.22}] \quad \log \left[\frac{\nu}{1 - \nu} \right] = \log K + h \log [L]$$

which suggests the Hill plot of $\log[\nu/(1 - \nu)]$ against $\log[L]$.