Problem Set 5.2
Cardiac Work

1. The work done by the heart in each beat contains at least two components. The first is the amount of work done to generate and maintain the pressure in the ventricle above that in the arterial outlet port. This is defined as **pressure–volume work**. As blood is ejected, it acquires a velocity. Hence there is **kinetic work** done to propel the blood forward.

   The pressure–volume work can be found by integrating the cardiac pressure–volume loop. This requires detailed knowledge of the form of the loop. However, we can *estimate* this area easily if we use some simple assumptions. Instead of working against a variable pressure, for example, we can assume that all of the blood is ejected against a constant *average* pressure of 93 mmHg. What counts is not the absolute pressure, of course, but the difference in pressure. Assume that the end diastolic pressure is 5 mmHg and the end systolic pressure is the same. This estimation is supported by the mean value theorem of integrals:

   [5.P2.1] $$\frac{\int_A^B f(x)\mathrm{d}x}{B-A} = \overline{f(\varsigma)}$$
   $$A \le \varsigma \le B$$

   Here the overline denotes the average value of the function. Thus, the average value of a function is *defined* as the value that, when multiplied by the interval, gives the definite integral. If you substitute $\mathrm{d}x = \mathrm{d}V$, the volume increment, and $f(x) = P$, you can see what is meant by "average" pressure.

   The kinetic energy of the ejected blood is calculated as

   $$\text{Kinetic work} = \frac{1}{2}mv^2$$

   where m = stroke volume × density, v = flow/area; stroke volume = end diastolic volume (EDV) − end systolic volume (ESV); flow = stroke volume/duration of systole.

   Numbers you may need to know: duration of ventricular systole = QT interval on ECG = 0.4 s. You should recognize this as the time over which the blood is ejected. Aortic radius = 1 cm; assume a heart rate of 70 bpm; assume an EDV = 150 mL and an ESV = 75 mL. Density of blood is approximately 1.055 g cm^{-3}.

   Calculate the work done by the heart and compare the two forms of work, i.e., pressure–volume versus kinetic. Which is greater?
2. Using the results from the problem above, calculate the mechanical efficiency of the heart using its O_2 consumption as a measure of energy input. O_2 consumption at rest (which is the situation in Problem #1) is 0.09 mL O_2 (at STPD) per g of tissue per min. Assume a left ventricular weight of 250 g. The average metabolic energy derived from oxidation of substrates by oxygen is 20.3 J mL^{-1} of O_2 (at STPD).
3. A. Calculate the approximate length of time for a molecule of oxygen to diffuse from your lungs to your calf muscles. Assume that $D_{O_2} = 1.78 \times 10^{-5}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$, the value of the O_2 diffusion coefficient in water.

   B. Compare the time of oxygen diffusion to the time required to move blood from the heart to the muscles. Estimate this from the velocity of blood as it leaves the heart (as calculated in Problem #1) and the distance from the heart to the calf muscle.
4. The radius of the aorta in a test subject determined by MRI is 1.2 cm. At rest, his end diastolic volume is 140 mL and his end systolic volume is 55 mL. His diastolic pressure is 70 mmHg and his systolic pressure is 115 mmHg. If we start the cardiac cycle at the opening of the mitral valve, the aortic valve in this person opens at 0.55 s and closes at 0.9 s. The entire cycle lasts 1.1 s in this person. The density of blood is 1.055 g mL^{-1} and its viscosity is 3.0×10^{-3} Pa s. A pascal is one N m^{-2}. Using this information, answer the following questions:

   A. What is the stroke volume?
   B. What is the heart rate?
   C. What is the cardiac output?
   D. What is the period of ejection?
   E. What is the *average* velocity of blood in the aorta?
   F. What is the Reynolds number for blood during ejection?
   G. Is blood flow in the aorta laminar or turbulent?
   H. What is the kinetic energy of the ejected blood?

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I. What pressure–volume work is done during ejection?

5. For the test person in Problem #4, the total oxygen content of blood at the root of the aorta where the coronary arteries come off is 20 mL%, or 20 mL of O_2 per dL of whole blood. Most of this is carried by hemoglobin inside erythrocytes and a small amount is dissolved. Cardiac venous blood contains 10 mL% of O_2. The heart weighed 250 g and blood flow at rest is 275 mL min^{-1}. Assume that two-thirds of the heart mass forms the left ventricle.
   A. What is the resting oxygen consumption of the heart, in mL O_2 per g heart per min?
   B. The average metabolic energy derived from oxygen consumption is 20.3 J mL^{-1} of O_2. Assuming that all parts of the heart consume oxygen equally, and using the answer to parts H and I of Problem #4, what is the mechanical efficiency of the left ventricle?
6. A person has a mass of 50 kg and 7.5% of her body weight is blood. The density of her blood is 1.053 g mL^{-1}, and her hematocrit is 40%. Her hemoglobin concentration is 14 g%.
   A. What is her blood volume?
   B. What is her plasma volume?
   C. Assume that the average life span of the red blood cells is 120 days. What volume of blood is being destroyed each day? On average, what volume of blood is being made each day?
   D. How much hemoglobin is degraded each day?
   E. The molecular weight of hemoglobin is 64,500 g mol^{-1} and each hemoglobin binds 4 Fe atoms. How much iron is liberated from hemoglobin each day? Give the answer in mg/day and mol/day.
   F. The molecular weight of bilirubin is 584.7 g mol^{-1}. How much bilirubin is liberated from hemoglobin each day in this person, in mg/day?
   G. How much total hemoglobin is in her blood? How much iron?
7. Injection of KCl raises its concentration in the plasma to 20 mM. What happens to E_K? What do you expect would happen to the heart?
8. At the top of the *R* wave, lead I reads 0.35 mV and lead III reads 0.75 mV. What is the electrical axis of the heart?
9. The end diastolic volume in a patient was estimated by echocardiography to be 170 mL and the end systolic volume was 120 mL. Calculate the ejection fraction. Is this normal?
10. During exercise, an athlete consumed 1.0 L of O_2 per min. Arterial content of O_2 was 20.5 mL % and mixed venous blood had 12.5 mL%. What is the athlete's cardiac output during exercise?
11. Estimate the distance from the AV node to the apex of the heart. Compare the time it would take for conduction of an action potential to travel from the AV node to the apex if it were conducted only by nodal cells, atrial cells, ventricular cells, or Purkinje fibers respectively.
12. Assume that the diameter of a biceps muscle is 4 cm.
   A. How long would it take O_2 to diffuse from the surface of the muscle to its interior if its diffusion coefficient was equal to that in water, $D = 1.78 \times 10^{-5}\ cm^2\ s^{-1}$?
   B. Each muscle fiber generally contacts several capillaries. If the muscle fiber is 50 μm in diameter, how long would it take O_2 to diffuse to its interior, given the same diffusion coefficient as in part A?
13. At steady state A, blood flow through a muscle is 0.025 mL $min^{-1}\ g^{-1}$. Arteriolar O_2 content is 20 mL% and venous O_2 content is 14 mL%. This question deals with the relationship between perfusion and extracellular concentration of O_2.
   A. What is the rate of O_2 consumption of the muscle?
   B. Suppose that blood flow increases from 0.025 to 0.1 mL $min^{-1}\ g^{-1}$. If arteriolar and venous O_2 content is unchanged, what happens to O_2 consumption?
   C. Under the new condition of higher flow, what is the venous O_2 content if O_2 consumption does ***not*** change?
14. The end diastolic volume of a heart is 140 mL. Assume that it is a sphere. At end diastole, the intraventricular pressure is 7 mmHg. The wall thickness at this time is 1.1 cm. At the end of isovolumetric contraction, the intraventricular pressure is 80 mmHg.
   A. What is the wall tension at end diastole?
   B. What is the wall tension at the end of the isovolumetric contraction?
   C. At the end of systole, the intraventricular volume is 65 mL, the pressure is 100 mmHg, and its wall thickness is 1.65 cm. What is the wall tension at this time?
   D. The wall stress is related to tension by $\sigma = T/w$, where σ is the wall stress, T is the tension, and w is the wall thickness. Calculate the wall stress from A, B, and C.
15. At the top of the *R* wave, lead I reads 0.55 mV and lead III reads 0.70 mV. What is the electrical axis of the heart?
16. The concentration of TnC in heart muscle cells is estimated at about 70 nmol g^{-1} wet weight of tissue. O_2 consumption is estimated to be about 0.09 mL O_2 per min per g wet weight of tissue. Assume the TnC is 60% saturated during normal, resting contractions. Assume that 80% of the activating Ca^{2+} originates from SR stores. Assume a heart rate of 70 min^{-1}. Assume free energy of ATP hydrolysis is 57 kJ mol^{-1} and that metabolism obtains 20.3 kJ L^{-1} of O_2 consumed.
   A. How much Ca^{2+} is released and taken back up per heart beat, per g of tissue?

B. How much energy is consumed to take the Ca^{2+} back up into the SR, per heart beat?
C. What fraction of the cell's energy consumption is used for this purpose?
D. How much Ca^{2+} enters and leaves the cell across the SL membrane?
E. Assuming that all of the Ca^{2+} exits the SL over the NCX, how much Na^+ enters the cell over the NCX?
F. How many ATP are consumed to remove the Na^+ that enters over the NCX?
G. What fraction of the cell's energy consumption is used for this purpose?