

# Problem Set

## Blood 5.1

1. The value of  $L_p$  for the red blood cell is about  $1.8 \times 10^{-11} \text{ cm}^3 \text{ dyne}^{-1} \text{ s}^{-1}$ . Its surface area is about  $1.35 \times 10^{-6} \text{ cm}^2$  (Solomon, *Methods Enzymol.* 192–222, 1989).
  - A. What is the initial osmotic flow if the osmolarity inside is initially 300 mOsm and the osmolarity outside is 275 mOsm? (Assume  $\sigma = 1.0$  for all solutes).
  - B. If the volume of the cell is  $100 \times 10^{-12} \text{ cm}^3$ , how long would it take to double its volume provided that the osmotic pressure and area of the membrane and  $L_p$  did not change?
  - C. In the case described, how much water would be required to enter the cell to equilibrate the osmotic pressure between inside and outside? Assume that the outside bath is essential infinite so that its osmotic pressure is kept constant.
2. A. Assume that water movement is fast compared to solute movement when red blood cells are placed in contact with solutions of different tonicity. Derive an equation that relates the volume of the cell,  $V_c$ , after equilibrating with the outside solution, in terms of the original volume,  $V_0$ ; the osmotic pressure of the solution under isotonic conditions,  $\pi_{\text{isotonic}}$ ; the osmotic pressure of the new solution,  $\pi$ .

Hints: In this situation where water movement is fast compared to solute movement, the total amount of osmotically active solutes inside the cell will be constant. Write this in terms of the original volume and the total osmotic pressure. After equilibration with a new solution, the volume changes but the total number of osmotically active solutes inside the cell does not. At this new volume,  $V_c$ , the cell's osmotic pressure equilibrates with the osmotic pressure of the outside solution. From this you should be able to derive an expression for a perfect osmometer.

  - B. Because some of the cell's volume is occupied by structures, it appears to have an "osmotically inactive" volume, which we will denote as  $V_b$ . How would this volume alter your equation in Part A?
3. A. The hematocrit in most healthy adult males is about 45%. If the number of red blood cells in whole blood is about  $5.2 \times 10^6$  per  $\text{mm}^3$ , calculate the apparent volume per cell. How does this compare to the volume given in problem 1?
- B. The red blood cell is a biconcave disk about  $7.5 \mu\text{m}$  in diameter and about  $1.9 \mu\text{m}$  at its thickest point. Estimate the surface area from these dimensions. How does this calculation compare to the surface area given in problem 1?
4. A. Assume that the mass of a person is 70 kg and that 8% of the body mass is blood. If the density of blood is 1.055 g/mL, what is the blood volume?
  - B. Assume that the hematocrit ratio is 0.45. What is the plasma volume in this person?
  - C. Assume that the average life span of the red blood cell is 120 days. What volume of packed red blood cells is being destroyed each day? This corresponds to what volume of whole blood? On average, what volume of whole blood is being made each day?
  - D. Assume that the hemoglobin concentration in the whole blood is 16 g%. How much hemoglobin is degraded each day?
  - E. The molecular weight of hemoglobin is  $66,600 \text{ g mol}^{-1}$  and each hemoglobin binds four Fe atoms. How much iron is liberated from hemoglobin each day? Give the answer in mg/day and mol/day.
  - F. The molecular weight of bilirubin is  $584.7 \text{ g mol}^{-1}$ . How much bilirubin is liberated from hemoglobin each day in this person, in mg/day?
  - G. From Part A and Part D, calculate the total amount of hemoglobin in the blood. From this calculate the total amount of iron in the blood.
5. A patient's blood was centrifuged in a capillary tube. The total height of the column of blood was 3.5 cm and the column of packed cells was 1.8 cm. Analysis of whole blood gave a hemoglobin concentration of 12 g%, and counting of the RBCs in a hemacytometer gave a count of  $4 \times 10^6$  cells per  $\text{mm}^3$ .
  - A. What is the hematocrit?
  - B. What is the MCV?
  - C. What is the MCHC?
6. Assume that the average density of hemoglobin is  $1.34 \text{ g cm}^{-3}$ . If the hemoglobin concentration of whole blood is 15 g% and the hematocrit is 525

- 45%, what fraction of the red blood cell volume is occupied by hemoglobin?
7. Assume that the shear rate in the aorta is about  $120 \text{ s}^{-1}$ . Assume further that the aorta has a diameter of 1 inch and that the stroke volume is 70 mL and that the viscosity of whole blood is  $3 \times 10^{-3} \text{ Pa s}$ .
    - A. What is the drag force of the vessel wall on each stroke volume?
    - B. If the density of whole blood is  $1.05 \text{ g cm}^{-3}$  and its average velocity in the aorta is  $0.2 \text{ m s}^{-1}$ , what is the initial deceleration of the blood flow in the aorta?
  8. Normal dimensions of the human heart for live persons can be determined noninvasively using magnetic resonance imaging (MRI) or two-dimensional echocardiography (2DE). (S. Kaul, et al., Measurement of normal left heart dimensions using optimally oriented MR images, *Am. J. Radiol.* 146:75–79, 1986.) During systole the left ventricular cavity has a diameter of about 34 mm and the wall thickness is about 16 mm. If systolic pressure is 130 mmHg, estimate the wall tension assuming that the wall stress is constant and that the ventricle is a sphere of the dimensions given. Estimate the wall stress.
  9. A. If the left ventricle contains 120 mL at end diastole, and we assume it is a sphere, what should its cavity radius be?
    - B. If the left ventricle contains 50 mL at end systole, and we assume it is a sphere, what should its cavity radius be?
    - C. How do these answers compare to measurements of cavity diameter of normal persons of about 46 mm at end diastole and about 34 mm at end systole? What causes the discrepancy?
  10. You sliced your leg accidentally with an axe and lost 1.5 L of blood before you stopped the bleeding. Assume that you had 15 mg% hemoglobin, 45% hematocrit, and 5 L of blood before your accident.
    - A. The blood volume is generally made up quickly by fluid entering plasma from the ISF and from fluid intake (persons who lose blood get thirsty). What would the new hemoglobin concentration be after you have recovered a blood volume of 5 L?
    - B. What would the new hematocrit be?
    - C. How much iron did you lose in the accident?
    - D. The rate of formation of red blood cells depends in part on the recycling of iron from destroyed red blood cells, but can be increased by erythropoietin and mobilization of Fe from ferritin stores in the liver. Suppose that the life span of the red blood cells is a constant 120 days. Suppose further that the rate of incorporation of Fe into Hb is a constant 21 mg per day. Write an equation for the recovery of the hemoglobin concentration after the loss of blood. How long does it take to recover one-half of the lost Hb?
  11. The red blood cell count is typically  $5 \times 10^6 \text{ mm}^{-3}$  and the blood volume is 5 L, and Hct is 45%.
    - A. How many red blood cells are there in the total circulation?
    - B. If the average life span is 126 days, how many red blood cells are destroyed each day?
    - C. How many red blood cells are made each day?
    - D. If the white blood cell count is  $5 \times 10^3 \text{ mm}^{-3}$  and the volume of the white blood cell is the same as that of the red blood cell, what is the total number of white blood cells in the circulation?
    - E. What is the total volume of white blood cells in the circulation?
  12. The end systolic volume in an athlete's heart is 50 mL and his end diastolic volume is 140 mL.
    - A. What is his stroke volume?
    - B. Is this larger or smaller than normal?
    - C. What explains the difference?
    - D. What is his ejection fraction?
    - E. Is the ejection fraction larger or smaller than normal?
    - F. What explains the difference?
  13. The last time you gave blood your Hct was 40% and your hemoglobin was 14 g%. That was about 3 months ago. In the blood bank they determined you are type A–. Today you have been in an accident and your doctors suspect internal bleeding. They took a sample of your blood and found a Hct of 25%.
    - A. What would you expect your hemoglobin to be?
    - B. Can you tell how much blood you have lost? If you cannot, why not?
    - C. Can you determine the fraction of blood that you have lost?
    - D. What types of blood can you receive without a transfusion reaction?
  14. During a hot August, a football player weighing 100 kg lost 5 L of hypotonic sweat. His Hct before the workout was 45% and RBC count was  $5 \times 10^6 \text{ mm}^{-3}$ . During the workout he drank 1.5 L of water which had been absorbed into the blood from the GI tract. Assume that the blood mass is 8% of his body mass and that the density of blood is  $1.05 \text{ g cm}^{-3}$ . Further assume that the fluids lost from his body were distributed proportionately from the extracellular volume and intracellular volume, which make up 20% and 40% of the body mass, respectively (assume the density of the fluids is  $1.0 \text{ g cm}^{-3}$ ).
    - A. How much volume did he lose?
    - B. How much of the lost volume came from the extracellular fluid compartment? How much from the intracellular volume?
    - C. What was the MCV before the workout?

- D. What is the MCV after the workout? (The red blood cell has an intracellular volume)
- E. What is the hematocrit after the workout?
15. A backpacker weighs 220 pounds. Assume his blood weight is about 7% of his body weight. His hematocrit at rest is 40% and total plasma proteins is 6 g%. After hiking on a hot day, the backpacker loses a net 4 L of body fluid by sweating and drinking, where the drinking is insufficient to match his sweating. Assume that the plasma makes up 6% of his total body water and that the fluid lost from each compartment is in proportion to the size of each fluid compartment.
- A. Assuming no loss of blood, what is his hematocrit after his hot hike?
- B. What is his total plasma protein concentration after hiking?
- C. What effect would these changes have on his blood viscosity?
16. A person has a total Hb concentration of 14 g%, a hematocrit of 40%, and a RBC count of  $5 \times 10^6$  cells  $\text{mm}^{-3}$ .
- A. What is the MCV?
- B. What is the MCHC?
- C. Calculate the Hb content of an average RBC.
- D. Calculate the number of molecules of Hb in an average RBC.
- E. If the density of Hb is  $1.34 \text{ g cm}^{-3}$ , what volume of the average RBC is occupied by Hb?