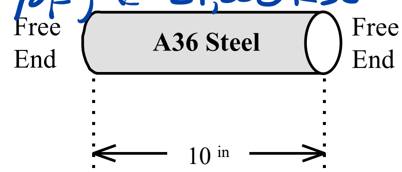
Worksheet #6C
Axial & Thermal Deformation

A solid, circular rod is made of A36 steel with a total length of 10 inches. The bar experiences a temperature increase of 50 degrees Fahrenheit. For each of the three cases below, determine the total axial deformation and the normal stress (σ). $\alpha_T = 6.5\ \mu/{}^\circ F$, $E = 29{,}000\ ksi$

a. Case 1: **Unconstrained**: Ends are free; no applied load.



The ends are free, no load applied

$\delta_{TOT} = \delta_{Temp} + \cancel{\delta_{LOAD}} \rightarrow 0$

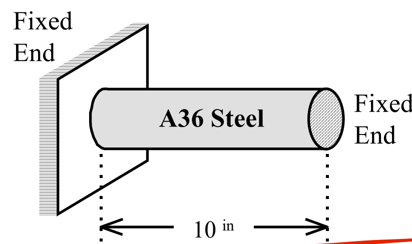
$\delta_{TOT} = \delta_{Temp} = \alpha_T \Delta T L_o$

$= (6.5\ \mu/{}^\circ F)(50^\circ F)(10 in)$

$\boxed{\delta_{Tot} = \delta_{Temp} = 0.00325 in}$

$\boxed{\sigma = 0 ksi}$

b. Case 2: **Fully Constrained**: Both ends are fixed.



Ends are fixed, no load applied

$\delta_{Tot} = \delta_{Temp} + \delta_{Load}$

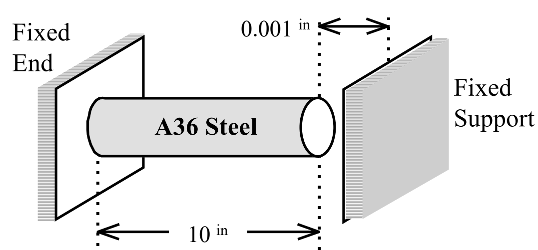
$0 = \alpha_T \Delta T L_o + \frac{PL}{AE} = \alpha_T \Delta T L_o + \frac{\sigma L_o}{E}$; $\frac{P}{A} = \sigma$

Solve for σ...

$\sigma = -E\alpha_T \Delta T$ (L_o cancels) $= (-29000 ksi)(6.5\times10^{-6} \frac{in/in}{{}^\circ F})(50^\circ F) = -9.47 ksi$

$\boxed{\sigma = 9.47 ksi (C)}$

c. Case 3: **Partially Constrained**: One end is fixed but with a fixed support located 0.001 inches from the free end.



$\delta_{Tot} = \delta_{Temp} + \delta_{Load} = .001 in$

$.001 in = \alpha_T \Delta T L_o + \frac{\sigma L_o}{E}$

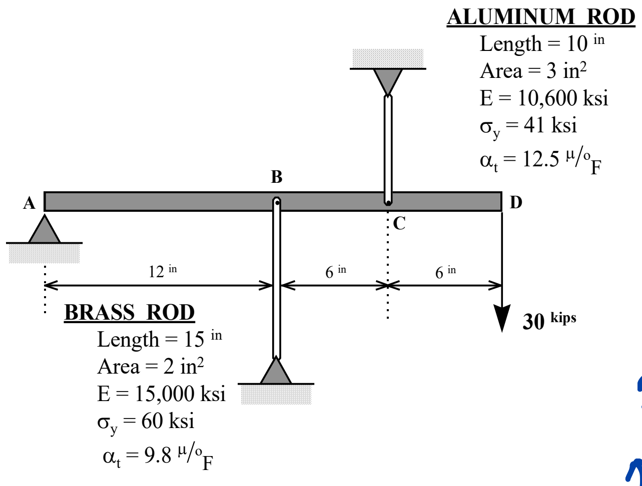
$.001 in = (6.5\times10^{-6} \frac{in/in}{{}^\circ F})(50^\circ F)(10 in) + \frac{\sigma (10 in)}{29000 ksi}$

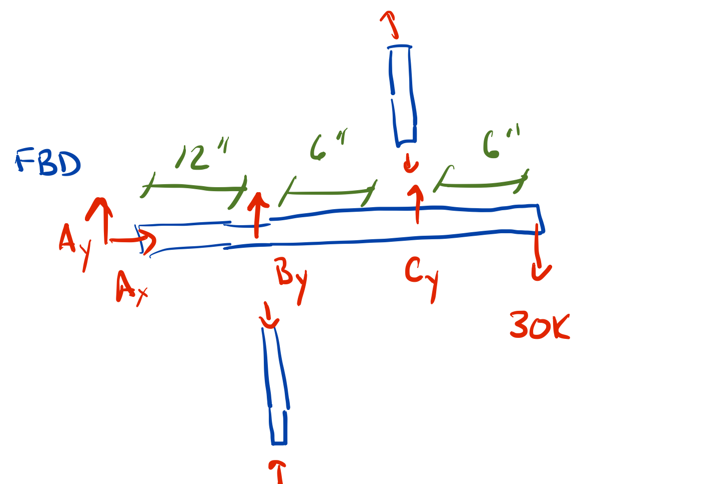
$\sigma = -6.53 ksi = 6.53 ksi (C)$

$\boxed{\delta_{tot} = .001'' \quad \sigma = 6.53 ksi (C)}$

Worksheet #6D
Statically Indeterminate Structures

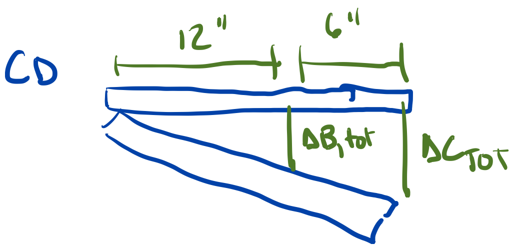
The rigid bar ABCD is supported by brass and 2014-T6 aluminum rods as shown and must support a 30 kip load at D. Determine the axial stress in the two rods when the temperature <u>decreases</u> 50 degrees Fahrenheit.





$$\uparrow \sum F_y = 0 \quad A_y + B_y + C_y - 30K = 0 \qquad (1)$$

$$\circlearrowleft \sum M_A = 0 \quad B_y(12) + C_y(18) - 30(24) = 0 \qquad (2)$$



$$\Delta B_{TOT} = \delta_{B\,tot} = \delta_{B,load} + \delta_{B,temp}$$

$$\Delta C_{TOT} = \delta_{C\,tot} = \delta_{C,load} + \delta_{C,temp}$$

$$\delta_{B,tot} = \alpha_T \Delta T L_0 + \frac{PL}{AE} = \left(9.8\times10^{-6}\,\frac{in/in}{°F}\right)(50)(15\,in) + \frac{B_y(15\,in)}{(2\,in^2)(15000\,ksi)} = .00735 + 5\times10^{-4} B_y$$

$$\delta_{C,tot} = \frac{PL}{AE} - \alpha_T \Delta T L_0 = \frac{C_y(10\,in)}{(3\,in^2)(10{,}600\,ksi)} - \left(12.5\times10^{-6}\,\frac{in/in}{°F}\right)(50°F)(10\,in)$$

$$= 3.14465\times10^{-4} C_y - .00625$$

$$\frac{.00735 + .0005 B_y}{12} = \frac{.000314465 C_y - .00625}{18}$$

$$.1323 + .009 B_y = .00377352\, C_y - .075$$

$$.009\, B_y - .00377352\, C_y = -.2063 \qquad (3)$$

2 & 3 contain B & C ... solve via matrix or substitute & solve.

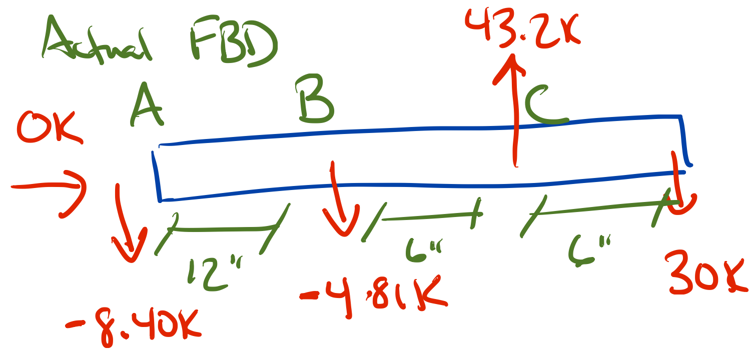
$$\begin{bmatrix} .009 & -.00377352 \\ 12 & 18 \end{bmatrix} \begin{Bmatrix} B_y \\ C_y \end{Bmatrix} = \begin{Bmatrix} -.2063 \\ 720 \end{Bmatrix}$$

$B_y = -4.80706\ K$

$C_y = 43.2047\ K$

$A_y = -8.39765\ K$

Actual FBD



Check Elastic Assumptions

$\sigma_{Brass} = \frac{B_y}{2in^2} = 2.45\,ksi \leq \sigma_y = 60\,ksi$ ✓

$\sigma_{AL} = \frac{C_y}{3in^2} = 14.42\,ksi \leq \sigma_y = 41\,ksi$ ✓

$\therefore\ \sigma_B = 2.45\,ksi\ (T)$ $\sigma_C = 14.42\,ksi\ (T)$

Comment:

Overall $\delta_{B,tot} = .0024035\,in - .00735\,in = .00495\,in. \downarrow$

$\delta_{C,Tot} = 0.13586\,in - .00625\,in = .00734\,in \downarrow$