Types of Rigid Body Motion

1. Translation
2. Rotation About a Fixed Axis (RAFA)
3. General Plane Motion (GPM)

Hint: Your first step with any kinematic problem is to identify the motion

Translation

$$\Delta\vec{r}_A = \Delta\vec{r}_B$$

$$\vec{v}_A = \vec{v}_B$$

$$\vec{a}_A = \vec{a}_B$$

Rotation About a Fixed Axis (RAFA)

$v = \omega r$ $\qquad \vec{v}_B = \vec{\omega}_{BODY} \times \vec{r}_{B/A}$

$a_T = \alpha r_{B/A}$ $\qquad \vec{a}_T = \vec{\alpha}_{BODY} \times \vec{r}_{B/A}$

$a_N = \omega^2 r_{B/A}$ $\qquad \vec{a}_N = \vec{\omega}_{BODY} \times \vec{\omega}_{BODY} \times \vec{r}_{B/A}$

$$\vec{a} = \vec{\alpha}_{BODY} \times \vec{r}_{B/A} + \vec{\omega}_{BODY} \times \vec{\omega}_{BODY} \times \vec{r}_{B/A}$$

Key points to remember:

- Velocity, $\boldsymbol{v}$, is always perpendicular to the radius, $\boldsymbol{r}$
- Velocity, $\boldsymbol{v}$, is consistent with the direction of **ω,** i.e. it moves in the same direction
- $\boldsymbol{a_T}$ is always perpendicular to $\boldsymbol{r}$ and consistent with direction of angular acceleration, $\boldsymbol{\alpha}$
- $\boldsymbol{a_N}$ is always parallel to $\boldsymbol{r}$ and always points toward fixed axis

Point Properties: $\qquad r_B, v_B, a_B$
Body Properties: $\qquad \theta_B, \omega_{AB}, \alpha_{AB}$

Note:

- Two points on the same body have different r, v, a, but have the same θ, ω, α
- $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ can have different directions

General Plane Motion (GPM)

For all of the following equations, points A & B **must** be on the ***same body.***

Position: $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

Velocity: $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

Velocity derived from:

$$\frac{\Delta\vec{r}_B}{\Delta t} = \frac{\Delta\vec{r}_A}{\Delta t} + \frac{\Delta\vec{r}_{B/A}}{\Delta t}$$

where, $\vec{v}_B$ is due to both translation & rotation (GPM)

$\vec{v}_A$ is due to translation of A

$\vec{v}_A$ & $\vec{v}_B$ are absolute velocities measured with respect to x & y

$\vec{v}_{B/A}$ is velocity of B due to rotation about A, i.e. it is the velocity of B as if A were fixed and B was rotating about A (RAFA!). Note: it is very important that you recognize that $\vec{v}_{B/A}$ is modeled as a RAFA stick; this will help you with the analysis!

No slip wheel: $v_{PC} = 0$, where PC is the Point of Contact of the wheel with the ground

$$v_O = \omega_{WH} r_{WH}$$

$$a_O = \alpha_{WH} r_{WH}$$

General Plane Motion (GPM) cont.

Instantaneous Center of Velocity – ICZV: $v_{ICZV} = 0$

ICZV is a point of zero velocity (within or outside a body) about which the GPM body rotates at that instant

Locating ICZV of a Body:

IF	THEN
$V_{PT} = 0$ (no slip wheel)	ICZV is at Point of Contact
Lines drawn perpendicular to $\vec{v}_A$ & $\vec{v}_B$ intersect @ C	ICZV is at C
Lines drawn perpendicular to $\vec{v}_A$ & $\vec{v}_B$ are collinear	use $\omega = \frac{v_A}{r_{A/IC}} = \frac{v_B}{r_{B/IC}}$
$\vec{v}_A$ & $\vec{v}_B$ are parallel velocities (do not intersect)	$r_{IC(pt)} \rightarrow \infty$

General Plane Motion (GPM) cont.

Acceleration: $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_A \equiv translation$

$\vec{a}_{B/A} \equiv RAFA, tangential \; \& \; normal$

$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_T + (\vec{a}_{B/A})_N$

A & B must be on the same body!

Solution Procedures for Rigid Body Kinematics

1. Classify motion
2. Write relative velocity equation
    a. Look for two points on the same body
3. Draw kinematic diagrams
4. Write scalar equations
5. Solve for unknowns
    a. May need to repeat step 2 and write additional relative velocity equations