"Hybrid Stars in the Octet QMC Model.".

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Outline

Motivation

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- We wish to obtain models of stellar objects possessing these properties and make predictions about the stellar structure

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with relevant symmetries according to QFT.

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- Hightly compact sphere of matter
- Leftovers from a supernova event
- Further collapse prevented by Pauli blocking - neutron degeneracy (*c.f. white dwarf*)
- Originally modelled as neutrons only
- Extension can be thought of as a ‘giant nucleus’

Hybrid Stars

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- A hybrid star is somewhere in between - part baryon, part quark.
- This is not particularly easy...

Field Theory:

Many Body Field Theory

Quark–Meson Coupling (QMC)

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QHD/QMC;

- Relativistic many body field theory
- Mean-Field Approximation (Hartree)
- Exhibits natural saturation
- Balance between scalar attraction and vector repulsion
- Approximation to QCD

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- The building blocks of matter are described using quarks and gluons via Quantum Field Theory.
- We can make an approximation to this using mesons as the exchange particles.

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"A relation between state variables; a thermodynamic equation describing the state of matter under a given set of physical conditions"

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Conditions

- $\rho_{\rm total} = \sum_{i\in\{B\}} \rho_i\ , \quad \rho_i = \frac{2}{(2\pi)^3}\int \theta(k_{F_i} - |\vec{k}|)d^3k = \frac{k_{F_i}^3}{3\pi^2}$
- $Q_{\rm total} = \sum_{i\in\{B,\ell\}} Q_i\ \rho_i = 0$
- $\mu_i = B_i\ \mu_n - Q_i\ \mu_e \ \sim\ \sqrt{(k_{F_i})^2 + (M_i^*)^2}$

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- Calculate energy density, pressure
- Calculate species fractions $Y_i = \dfrac{\rho_i}{\rho_{\rm total}}$

Quantum Hadrodynamics (QHD)

- In QHD, the effective baryon masses are linear in the applied scalar field

$$\boxed{M_B^* = M_B - g_{\sigma B}\langle\sigma\rangle}$$

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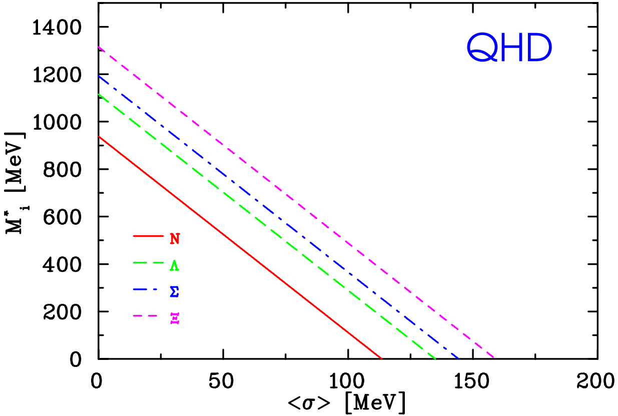

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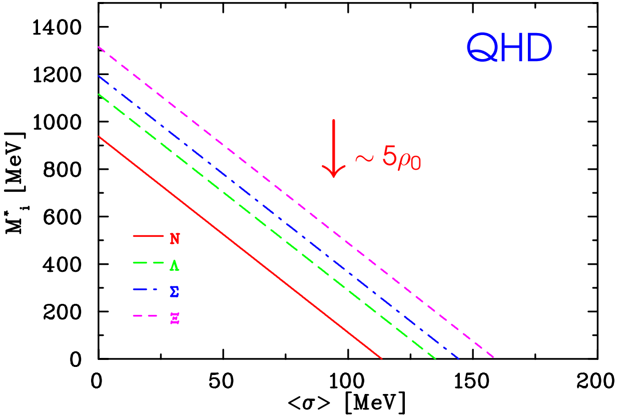

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> *"d represents the response of the baryon to the applied scalar field, analagous to the electric polarizability of QED."*

Quark–Meson Coupling (QMC)

- In QMC, we include the quadratic contribution via the `scalar polarizability,' *d*

$$\boxed{M_B^* = M_B - w_B^\sigma g_{\sigma N}\langle\sigma\rangle + \frac{d}{2}\tilde{w}_B^\sigma (g_{\sigma N}\langle\sigma\rangle)^2}$$

- The additional factors w_B^σ and $\tilde{w}_B^\sigma$ have been determined by *Rikovska-Stone et. al.*

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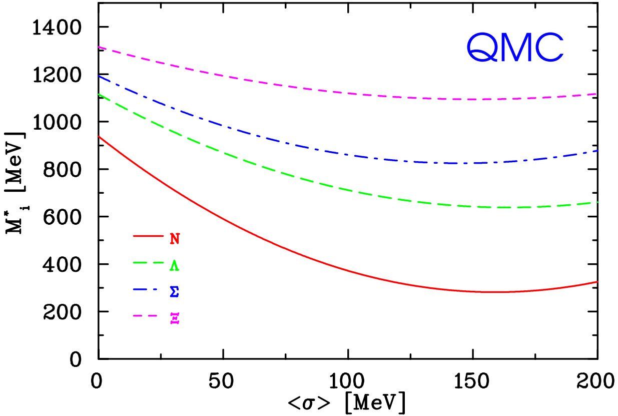

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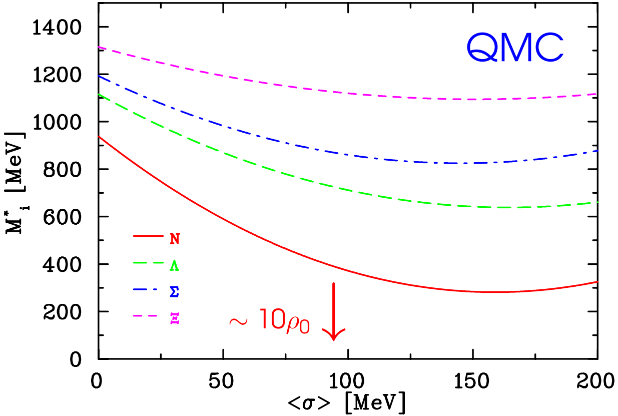

Comparison to Known Results

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Some empirical data are known for finite nuclei. We aim for our model to be able to reproduce these. The measurements of interest are;

	K	$M^*_{\rm sat}$
Empirical:	$200 - 300\ \mathrm{MeV}$	$563\ \mathrm{MeV}$
QHD Theory:	$525\ \mathrm{MeV}$	$523\ \mathrm{MeV}$
QMC Theory:	$280\ \mathrm{MeV}$	$735\ \mathrm{MeV}$

The coupling constants used are found via a fit to (experimental) saturation properties of finite nuclei.

Quark–Meson Coupling (QMC)

We now have a quark-level model for the octet of baryons in β-equilibrium

Building a Star

TOV

Tolman-Oppenheimer-Volkoff Equation

Connection:

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Note!

This applies to static, non-rotating stars only.

RESULTS

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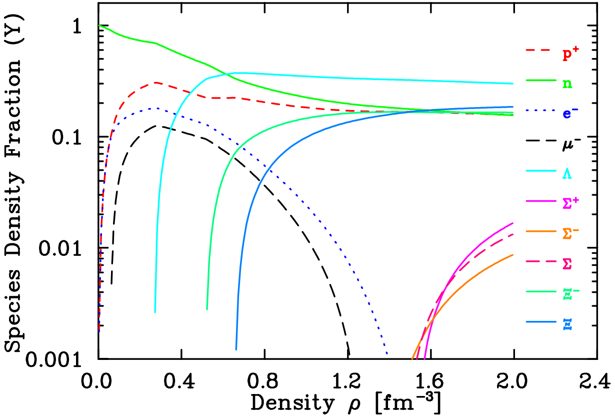

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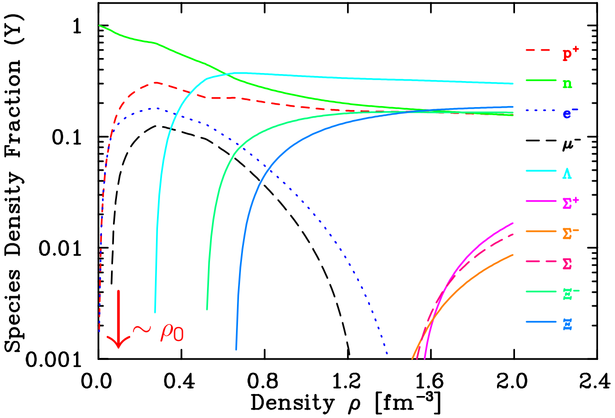

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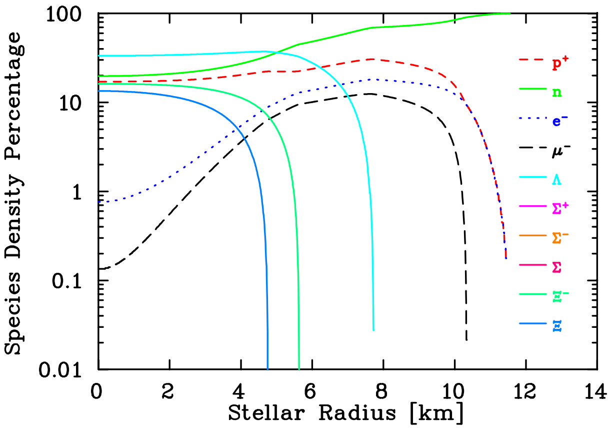

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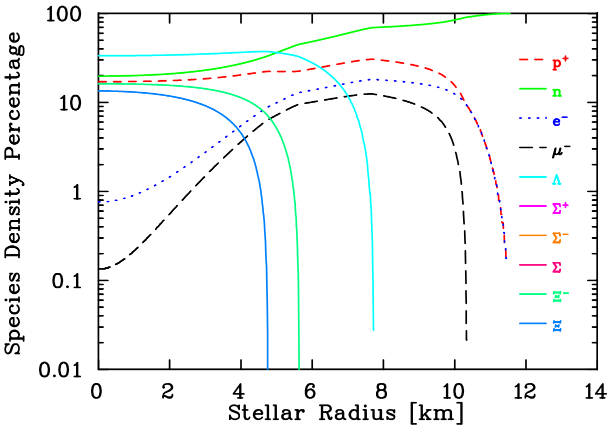
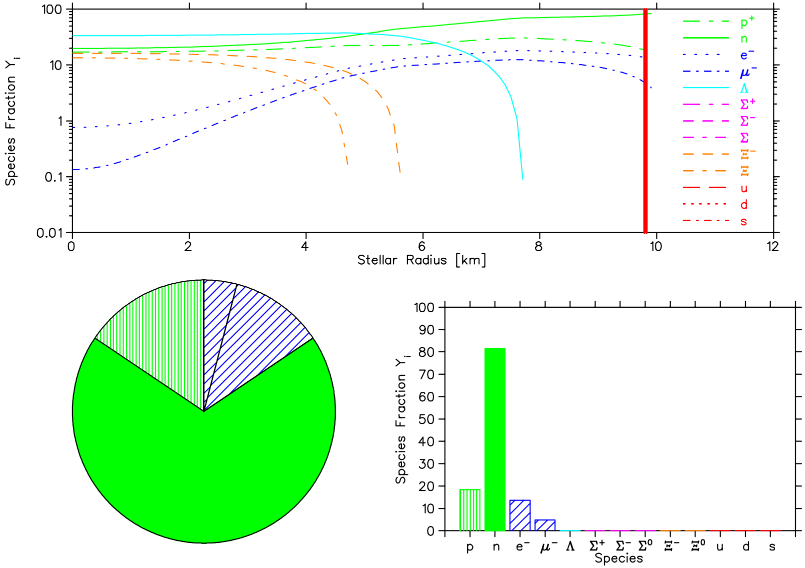


An alternative view. . .

These diagrams can be difficult to interpret, so here are some alternatives. . .



Species Fraction Y_i
Stellar Radius [km]
100
10
1
0.1
0.01
0
2
4
6
8
10
12
p^+
n
e^-
μ^-
Λ
Σ^+
Σ^-
Σ
Ξ^-
Ξ
u
d
s
Species Fraction Y_i
Species
p
n
e^-
μ^-
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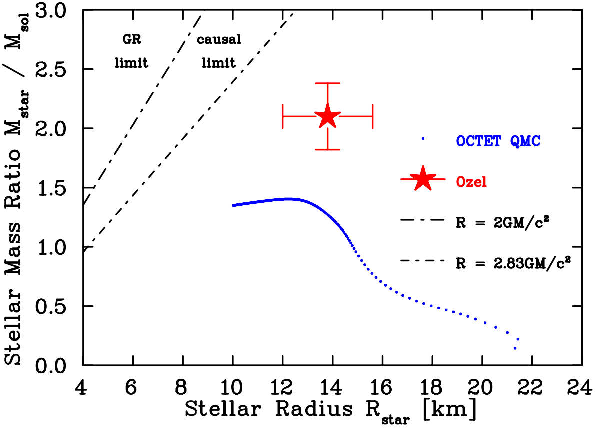

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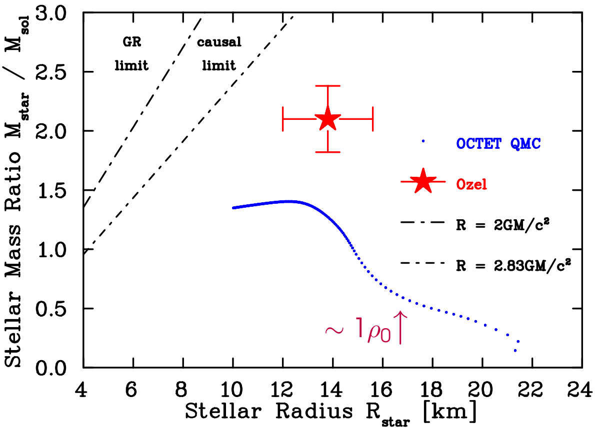

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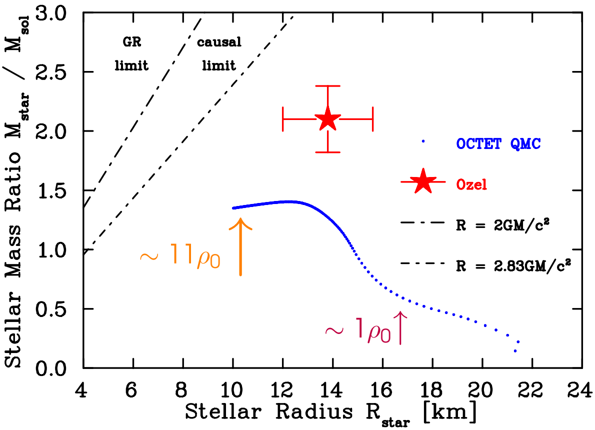

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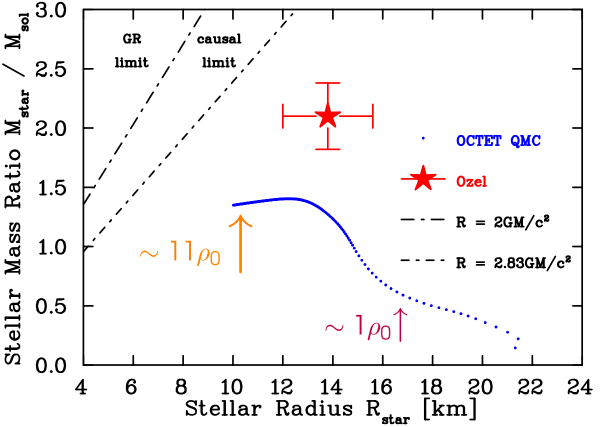
SOL: (700,000;1)



Phase Transitions

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Equilibrium Conditions

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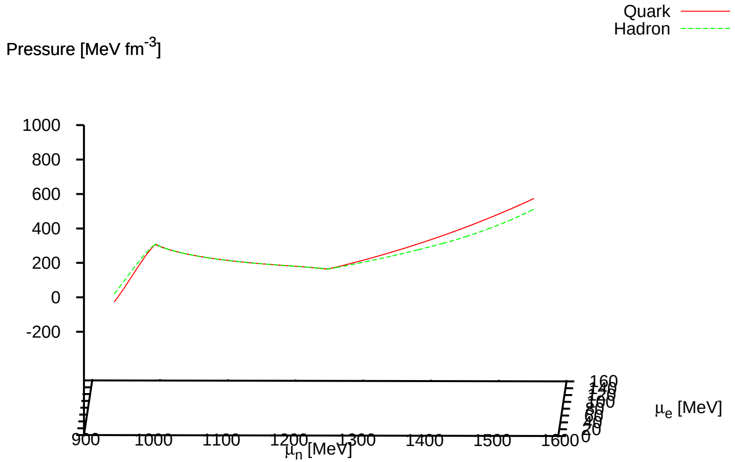
At $T = 0$, independent chemical potentials ($\mu_n,\ \mu_e$) and pressure (p) determine the phase transition point



Quark
Hadron
Pressure [MeV fm^{-3}]
1000
800
600
400
200
0
-200
900
1000
1100
1200
1300
1400
1500
1600
μ_n [MeV]
μ_e [MeV]

Glendenning Style Phase Transition

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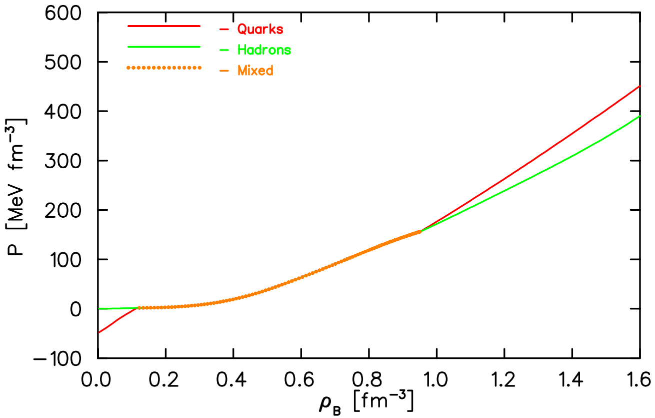

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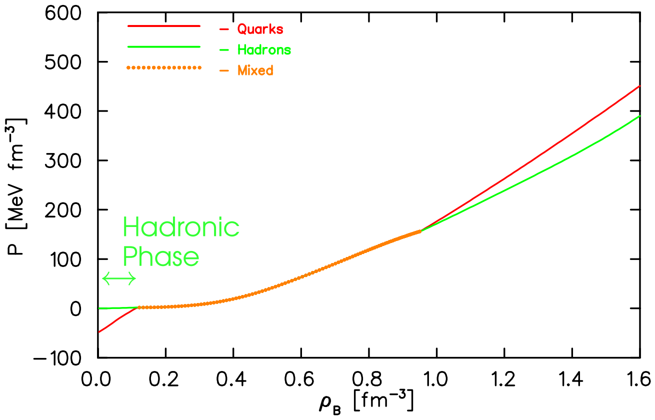

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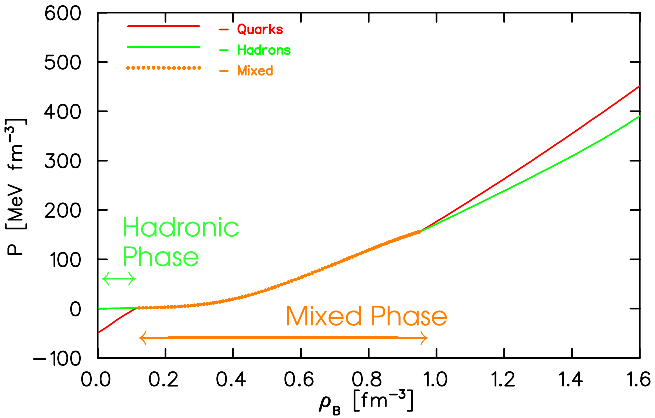

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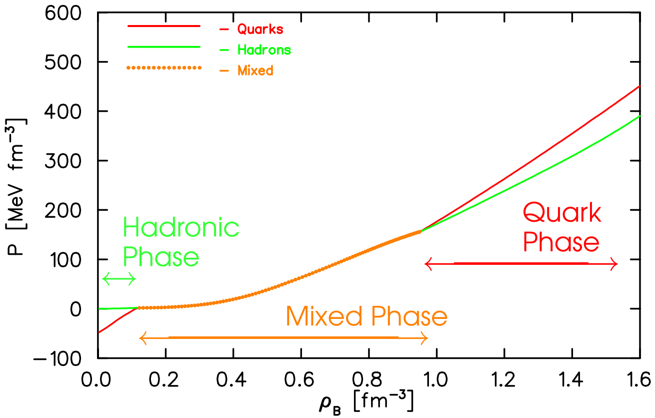

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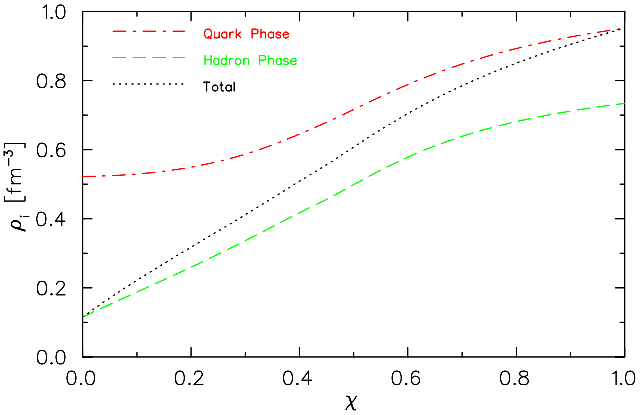

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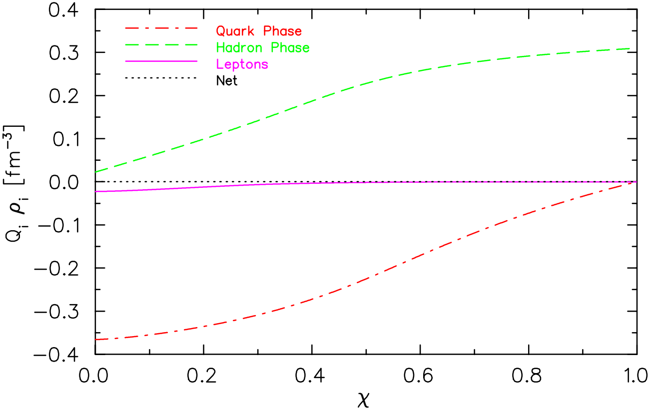

Quark Phase
Hadron Phase
Leptons
Net
$Q_i\ \rho_i\ [\mathrm{fm}^{-3}]$
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.0
0.2
0.4
0.6
0.8
1.0
χ

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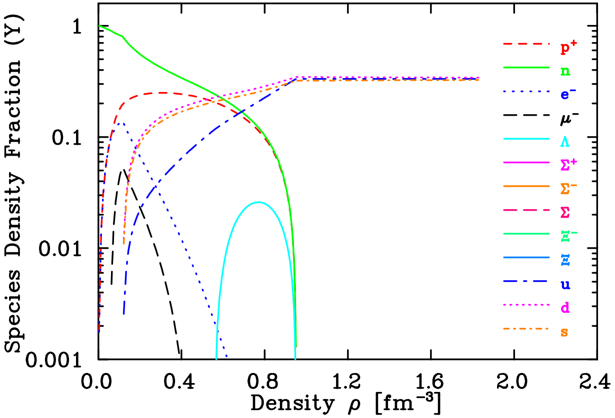

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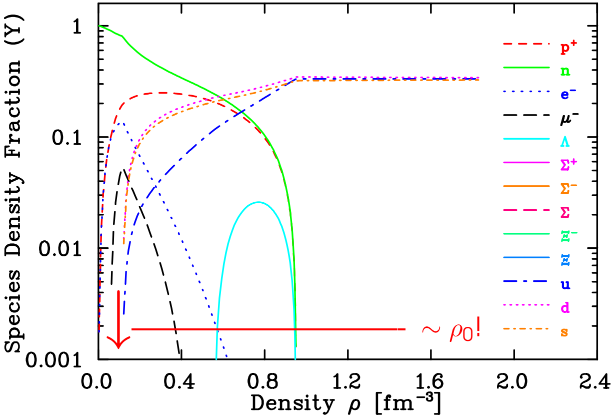

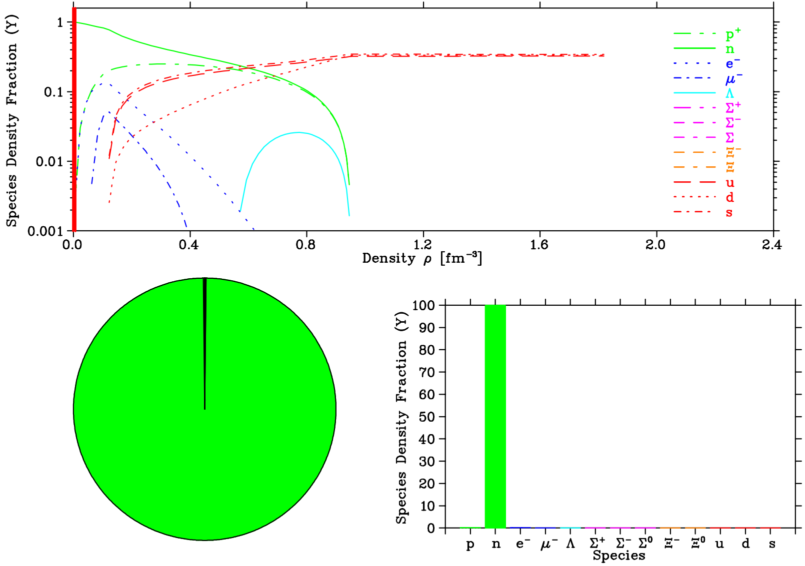

Species Density Fraction (Y)
1
0.1
0.01
0.001
0.0
0.4
0.8
1.2
1.6
2.0
2.4
Density ρ [fm⁻³]
p⁺
n
e⁻
μ⁻
Λ
Σ⁺
Σ⁻
Σ
Ξ⁻
Ξ
u
d
s
Species Density Fraction (Y)
100
90
80
70
60
50
40
30
20
10
0
p
n
e⁻
μ⁻
Λ
Σ⁺
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d
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Species

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- First order model - can be expanded
- Trying not to introduce 'fudge terms'

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$\Rightarrow$ Explore the parameters?

"Explore the parameters. . . "

NJL (Simplified)

We have used constant (current) quark masses. but DχSB suggests that we should have constituent quarks at low density. . .

$\Rightarrow$ Simplified NJL Model...

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$\longrightarrow$ We also need to think about confinement.

Hybrid Stars

Using the MIT bag model, if we once again solve the TOV equation, we obtain stellar solutions

Hybrid Star

Comparison between OCTET QMC star and Hybrid Star

Hybrid Star

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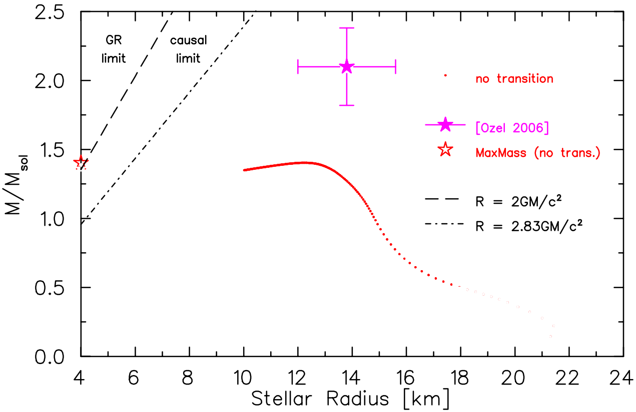

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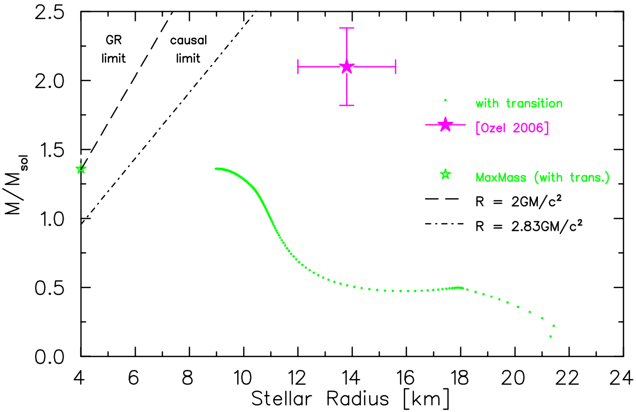

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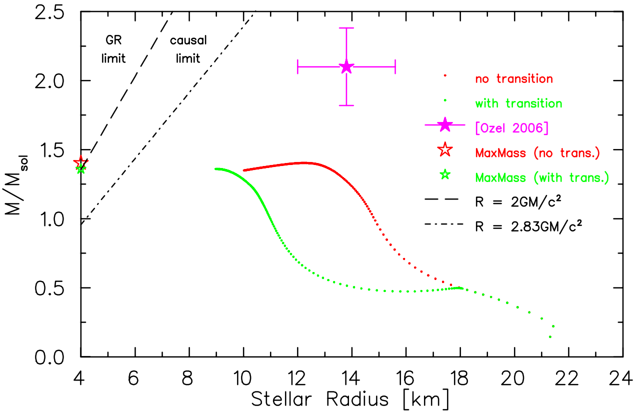

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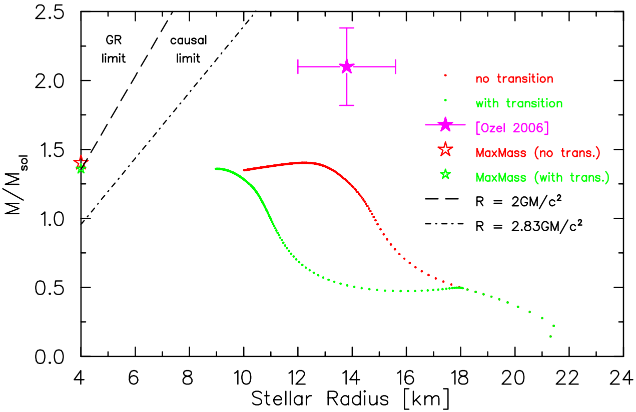

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- Stellar masses slightly low, but may be improved soon (Hartree-Fock / Full NJL)
- Good results for compression modulus K
- DχSB will have consequences for high-density matter
- For a given stellar mass, Hybrid stars are predicted to be less massive than Nucleon/Hyperon stars

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Thankyou.

Any Questions?

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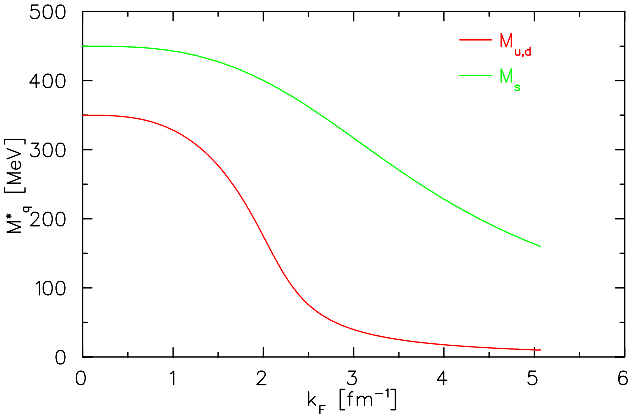

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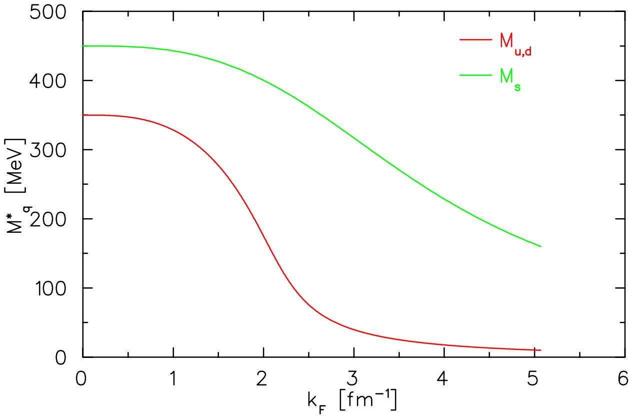

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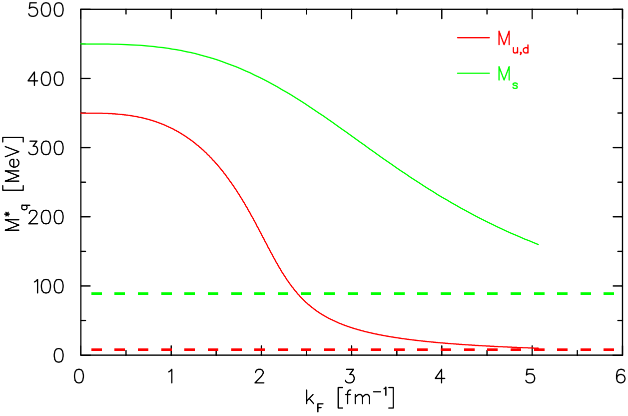

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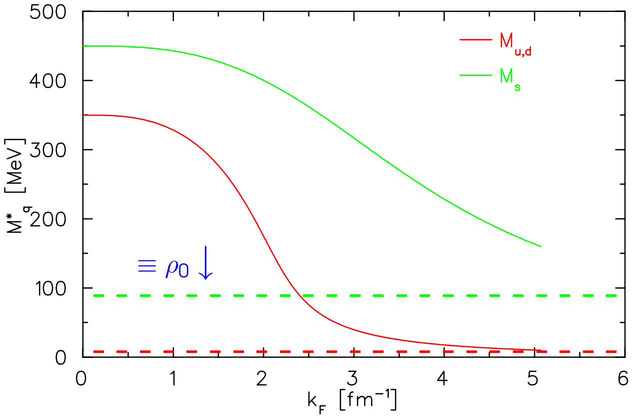

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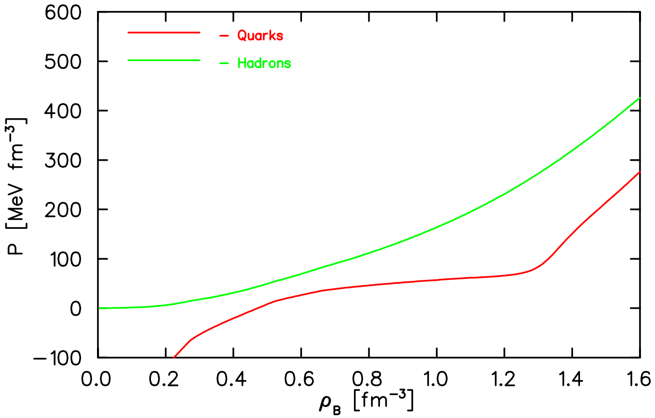

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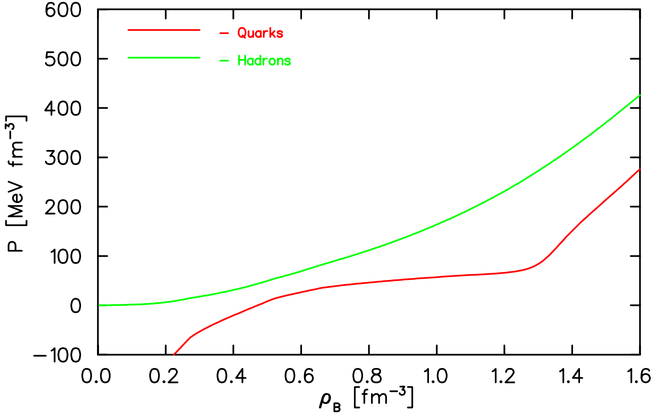

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Note

$\rho_{\mathrm{Quark}} \overset{!}{>} \rho_{\mathrm{Hadron}}$ to ensure ρ_{Total} increases monotonically